Title: Phase-Locked Cascade in the AdS Stability Problem

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Abstract: <p>I will first briefly report the current status on the stability problem of global AdS space under gravitational self-interaction. I will then present evidence that the possibility of a blackhole-forming instability is strongly connected to phase-locked cascade, which is different from the usual energy cascade in turbulent flow.</p>

① AdS_{d+1} (In) Stability

Status:

Tools:

② Phase-Locked Cascade

Empirical:

Analytical:

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_{d-1}^2$$

⊕ small perturbations

$\Downarrow$ evolve

Something different

for example:

$$ds^2 = -\left(1+r^2-\frac{2M}{r^{d-2}}\right)dt^2 + \frac{dr^2}{1+r^2-\frac{2M}{r^{d-2}}} + r^2 d\Omega_{d-1}^2$$

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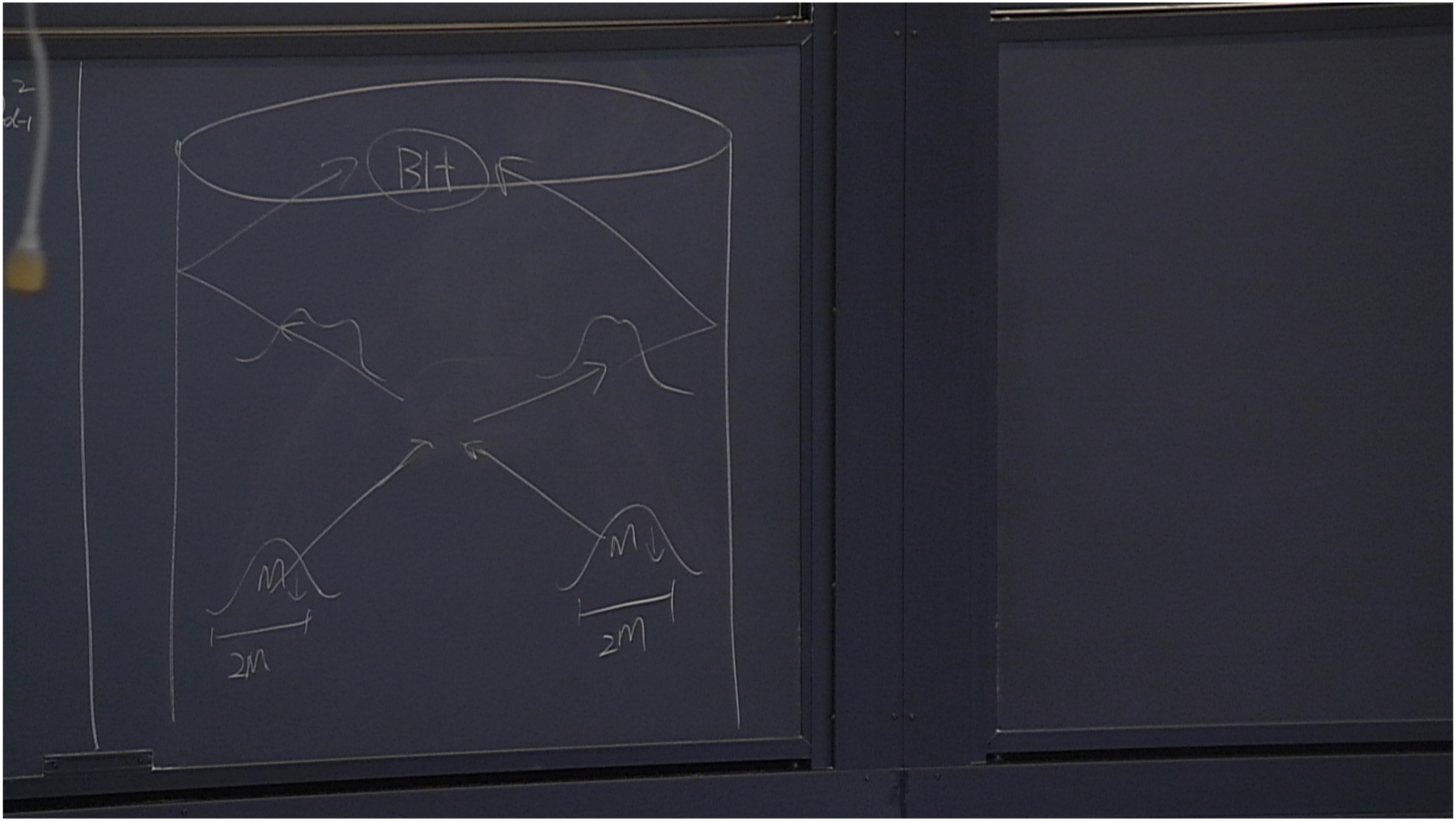
BH
M
M
2M
2M

① AdS_{d+1} (In) Stability

Status: $T \sim \varepsilon^{-2}$

BH

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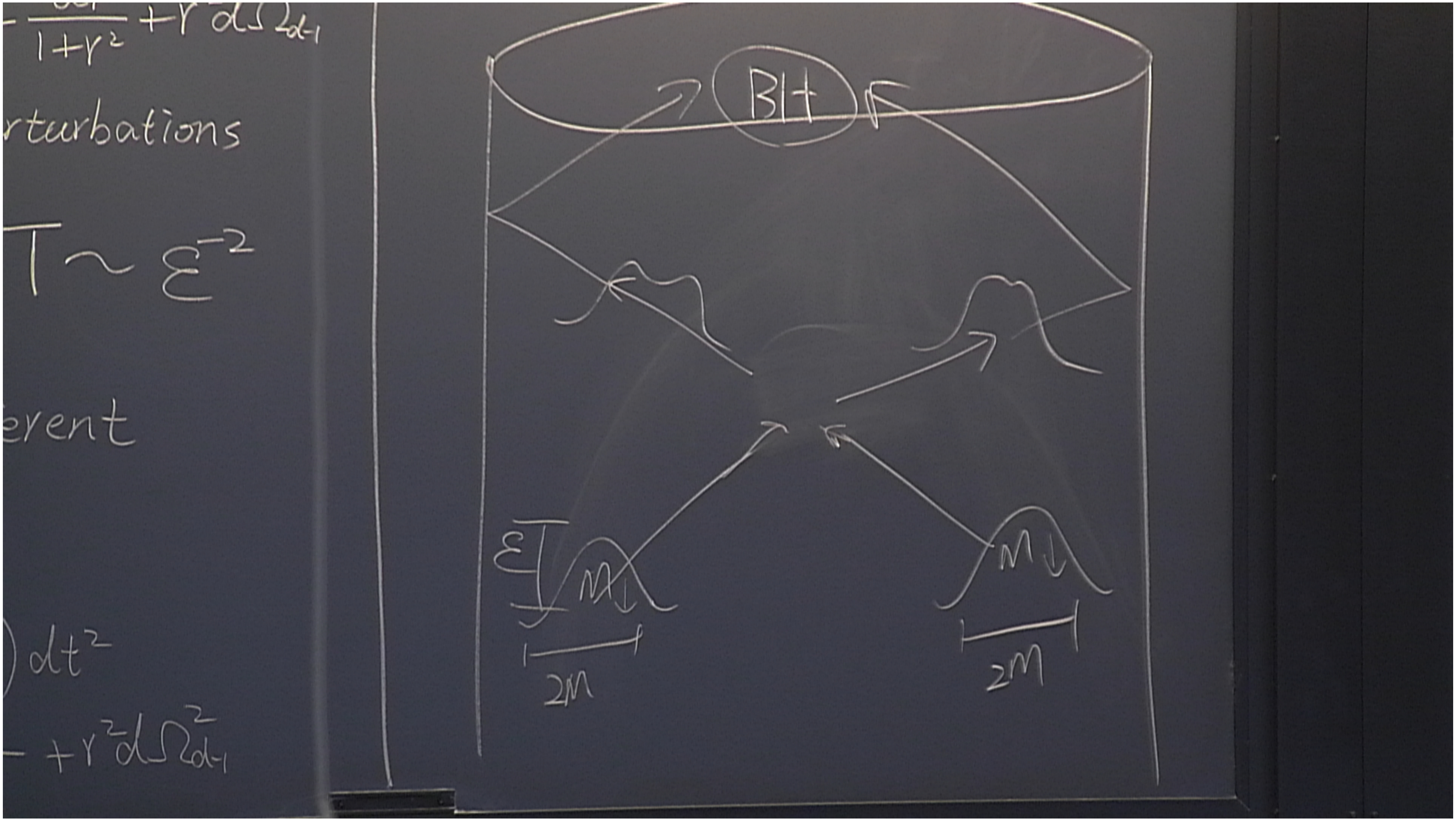


1+r²
+r²dΩ_{d-1}
rturbations
T ~ ε⁻²
erent
dt²
+r²dΩ²_{d-1}
BH
ε
M
M
2M
2M

① define $\|\phi\|$: $\|c\phi\| = c\|\phi\|$ and $\|\phi_1 + \phi_2\| \leq \|\phi_1\| + \|\phi_2\|$

② evolve $\phi(0) \Rightarrow \phi(t)$

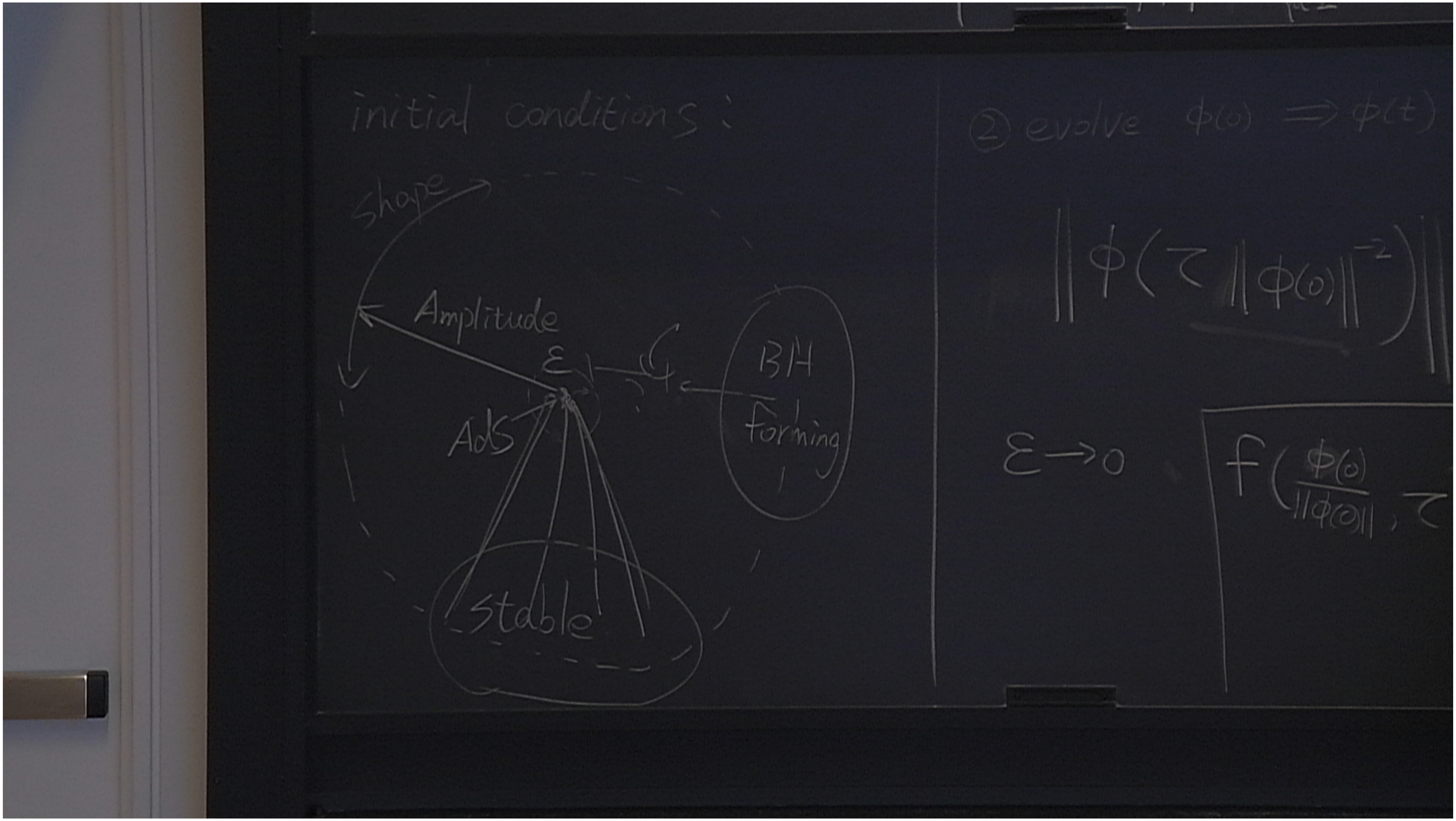
$$\left\| \phi\left(\tau \|\phi(0)\|^{-2}\right) \right\| = \|\phi(0)\| \left[f\left(\frac{\phi(0)}{\|\phi(0)\|}, \tau\right) + \Theta\left(\phi(0 \leq \tau' \leq \tau)\right) \right]$$

② evolve $\phi(0) \Longrightarrow \phi(t)$

$$\left\| \phi\left(\tau \|\phi(0)\|^{-2}\right) \right\| = \|\phi(0)\| \left(f\left(\frac{\phi(0)}{\|\phi(0)\|}, \tau\right) + \Theta\left(\phi(0 \leq \tau' \leq \tau)\right) \right.$$

$\varepsilon \to 0$

$$f\left(\frac{\phi(0)}{\|\phi(0)\|}, \tau\right) \begin{cases} \text{finite} \Longrightarrow \text{stable} \\ \infty \Longrightarrow \text{unstable} \end{cases}$$


initial conditions:
shape
Amplitude
ε
AdS
BH
forming
stable
② evolve φ(0) ⟹ φ(t)
ε → 0

$$\|\phi\| \equiv \sqrt{|\Delta g_{tt}|^2 + |\Delta g_{rr}|^2 + 2|\Delta g_{tr}|^2}$$

near $r=0$:

$$g_{tt} = -\left(1 + \frac{2M}{r^{d-2}} + 4V + \underline{r^2}\right)$$

$$g_{rr} = \left(1 + \frac{2M}{r^{d-2}} - \underline{r^2}\right)$$

ignore

$$M(r) = \int_0^r \frac{1}{2}\left(\dot{\phi}^2 + \phi'^2\right)\bar{r}^{d-1}\,d\bar{r}$$

$$V(r) = -\int_r^\infty (d-2)\frac{M(\bar{r})}{\bar{r}^{d-1}}\,d\bar{r}$$

$$\phi(r,t) = \sum \phi_n(t)\, e$$

$$\ddot{\phi}_n + \omega_n^2\phi_n = \sum_{k,l,m} C_{klmn}$$

$$\phi_n = A_n\cos(\omega_n t +$$

$$2\omega_n\frac{dA_n}{dt} = \sum_{k,l,m}^{k+l=m+n} C_{klmn} A_k A$$

$$2\omega_n\frac{dB_n}{dt} = A_n^{-1}\sum_{k,l,m}^{k+l=m+n} C_{klmn}$$

$$\bar{A}_n \equiv A_n\|\phi(0)\|^{-1}, \quad \tau \equiv t\|$$

$$\ddot{\phi}_n + \omega_n^2 \phi_n = \sum_{k,l,m} C_{klmn} \phi_k \phi_l \phi_m + O($$

$$\phi_n = A_n \cos(\omega_n t + B_n), \qquad (A_n, B$$

$$2\omega_n \frac{dA_n}{dt} = \sum_{k,l,m}^{\boxed{k+l=m+n}} C_{klmn} A_k A_l A_m \sin(B_n + B$$

$$2\omega_n \frac{dB_n}{dt} = A_n^{-1} \sum_{k,l,m}^{k+l=m+n} C_{klmn} A_k A_l A_m \cos(B$$

$$\ddot{\phi}_n + \omega_n^2 \phi_n = \sum_{k,l,m} C_{klmn}\, \phi_k \phi_l \phi_m + O(\phi^5)$$

$$\phi_n = \|\phi(0)\|\, \bar{A}_n \cos(\omega_n t + B_n), \qquad (\bar{A}_n, \bar{B}_n) \text{ slowly changing}$$

$$2\omega_n \frac{d\bar{A}_n}{d\tau} = \sum_{k,l,m}^{\boxed{k+l=m+n}} C_{klmn}\, \bar{A}_k \bar{A}_l \bar{A}_m \sin(B_n + B_m - B_l - B_k)$$

$$2\omega_n \frac{dB_n}{d\tau} = \bar{A}_n^{-1} \sum_{k,l,m}^{k+l=m+n} C_{klmn}\, \bar{A}_k \bar{A}_l \bar{A}_m \cos(B_n + B_m - B_k - B_l)$$

$$\bar{A}_n \equiv A_n \|\phi(0)\|^{-1}, \quad \tau \equiv t\, \|\phi(0)\|^2$$

① AdS_{d+1} (In) Stability

Status: $T \sim \varepsilon^{-2}$

BH

?

Tools: $\|\phi\|$
$\phi \to \phi(0)$ $\Big\}\, f(\tau)$

② Phase-Locked Cascade

Empirical:

Analytical:

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_{d-1}^2$$

⊕ small perturbations

$\Downarrow$ evolve $T \sim \varepsilon^{-2}$

Something different

for example:

$$ds^2 = -\left(1+r^2-\frac{2M}{r^{d-2}}\right)dt^2 + \frac{dr^2}{1+r^2-\frac{2M}{r^{d-2}}} + r^2 d\Omega_{d-1}^2$$

Kolmogorov, in turbulence.

similarities:

1. same e.o.m.
2. powerlaw

difference:

1. $C_{\lambda m \lambda n \lambda k \lambda l} = \lambda^{\beta} C_{mnkl}$
2. $T \sim \varepsilon^{-4}$!

① AdS_{d+1} (In)Stability

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$\left.\begin{matrix}\|\phi\| \\ \phi \to \phi(0)\end{matrix}\right\} f(\tau)$

② Phase-Locked Cascade

Empirical: not Kolmogorov

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$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_{d-1}^2$$

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$$M(\bar{r}) = \int_0^{\bar{r}} d\bar{r} \left[\sum_{n=0}^{\infty} \omega_n A_n e_n(r) \cos(\omega_n t + B_n) \right]^2$$

$$= \begin{cases} \displaystyle\int_0^{\bar{r}} dr \sum_{n=0}^{\infty} \omega_n^2 A_n^2 e_n(0)^2 \Rightarrow \text{random } B_n \\ \displaystyle\int_0^{\bar{r}} dr \left(\sum \omega_n A_n e_n(0) \right)^2 \Rightarrow \text{coherent } B_n \end{cases}$$

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$\left.\begin{array}{l} \|\phi\| \\ \phi \rightarrow \phi(0) \end{array}\right\} f(\tau)$

② Phase-Locked Cascade

Empirical: not Kolmogorov

$\|\phi\| \Rightarrow$ BH requires coherent.

Analytical: $\alpha(\beta) \Rightarrow$ phase-locked

$ds^2 = -(1+r^2)dt^2 + \frac{\quad}{1+r^2}$

⊕ small perturba

$\Big\Downarrow$ evolve $T \sim$

Something different

for example:

$$ds^2 = -\left(1 + r^2 - \frac{2M}{r^{d-2}}\right) dt^2 + \frac{dr^2}{1 + r^2 - \frac{2M}{r^{d-2}}} + r^2 d$$

$+r^2 d\Omega_{d-1}^2$

bations

$\sim \mathcal{E}^{-2}$

t

$^2 d\Omega_{d-1}^2$

rical observation:

ormation:

$A_n \sim n^{-\alpha} = A_0 n^{-\alpha}$

$\alpha \sim \frac{6}{5}$ in AdS_{3+1}

$\alpha \sim 2$ in AdS_{4+1}

$(\omega_n$

$2n$

$$(\omega_n t + B_n)$$

$$nt - nC(z) \equiv 0 \pmod{2\pi}$$

$$B_n \sim nC(z) \quad + \quad \frac{dA_n}{dz} = 0, \quad A_n = A_0 n^{-\alpha}$$

$nx = k$
$ny = l$
$nz = m$

$$2n\frac{dB_n}{dz} = A_n^{-1} \sum C_{klmn} A_k A_l A_m$$

$$= A_0^2 n^{\alpha} \int_0^{\infty} dk\,dl\,dm\; \delta(k+m-l-n)\, C_{klmn} (klm)^{-\alpha}$$

$$= A_0^2 n^{\alpha} \int n^3\, dx\,dy\,dz\; \delta[n(x+y-z-1)]\, n^{\beta} C_{xyz1}\, n^{-3\alpha} (xyz)$$

$$(\omega_n t + B_n) \qquad nt - nC(\tau) \equiv 0 \pmod{2\pi}$$

$$\boxed{B_n \sim nC(\tau) \quad + \quad \frac{dA_n}{d\tau} = 0, \quad A_n = A_0 n^{-\alpha}}$$

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dz

$$= A_0^2 n^\alpha \int_0^\infty dk\, dl\, d$$

$$= A_0^2 n^\alpha \int n^3 dx\, d$$

$$\boxed{n^{3+\beta-2\alpha}} = n^2$$

Status: $T \sim \varepsilon^{-2}$

BH

Tools: $\|\phi\|$

$\left.\begin{array}{l} \\ \phi \to \phi(0) \end{array}\right\} f(\tau)$

② Phase-Locked Cascade

Empirical: not Kolmogorov

$\|\phi\| \Rightarrow$ BH requires coherent.

Analytical: $\underline{\alpha(\beta) \Rightarrow \text{phase-locked}}$

$\oplus$ small perturbations

$\Downarrow$ evolve $T \sim \varepsilon^{-2}$

Something different

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$$ds^2 = -\left(1 + r^2 - \frac{2M}{r^{d-2}}\right) dt^2 + \frac{dr^2}{1 + r^2 - \frac{2M}{r^{d-2}}} + r^2 d\Omega_{d-1}^2$$

$$A_0 = A_1 = \varepsilon \,, \quad A_{n>1} = 0$$

$$\ddot{\phi}_2 + \omega_2^2 \phi_2 = \boxed{C_{1102}}\, \phi_1^2 \phi_0$$

$$= \varepsilon^3 \cos\left[(2\omega_1 - \omega_0)t + (2B_1 - B_0)\right]$$

$$\phi_2 \sim \varepsilon^3\, t \cos\Big[\omega_2 t + \underbrace{(2B_1 - B_0) - \frac{\pi}{2}}_{B_2}\Big]$$

$$\ddot{\phi}_3 + \omega_3 \phi = C_{1203}\underbrace{\phi_1\phi_2\phi_0}_{\Downarrow\ \varepsilon^5} + C_{2213}\underbrace{\phi_2^2\phi_1}_{\Downarrow\ \varepsilon^7 \times}$$

$$\vdots$$

$$\phi_3 \sim \varepsilon^5 t^2 \cos\left[\omega_3 t + \underbrace{(B_1 + B_2 - B_0) - \frac{\pi}{2}}_{3B_1 - 2B_0 - 2\frac{\pi}{2} \,=\, B_3}\right]$$

$$\phi_{n\geq 1} \sim \varepsilon(\varepsilon^2 t)^{n-1} \cos\left[\omega_n t + \left(B_1 - B_0 - \frac{\pi}{2}\right)n + B_0\right]$$

② Phase-Locked Cascade

Empirical : not Kolmogorov

$||\phi|| \Rightarrow BH$ requires coherent.

Analytical : $\boxed{\alpha(\beta) \Rightarrow \text{phase-locked}}$

$\Rightarrow$ 2-mode $\Rightarrow$ initial $\boxed{B_n \text{ coherent}}$

Som

for

$ds^2 =$