Title: Near-Horizon Magnetospheres of Rapidly Spinning Black Holes

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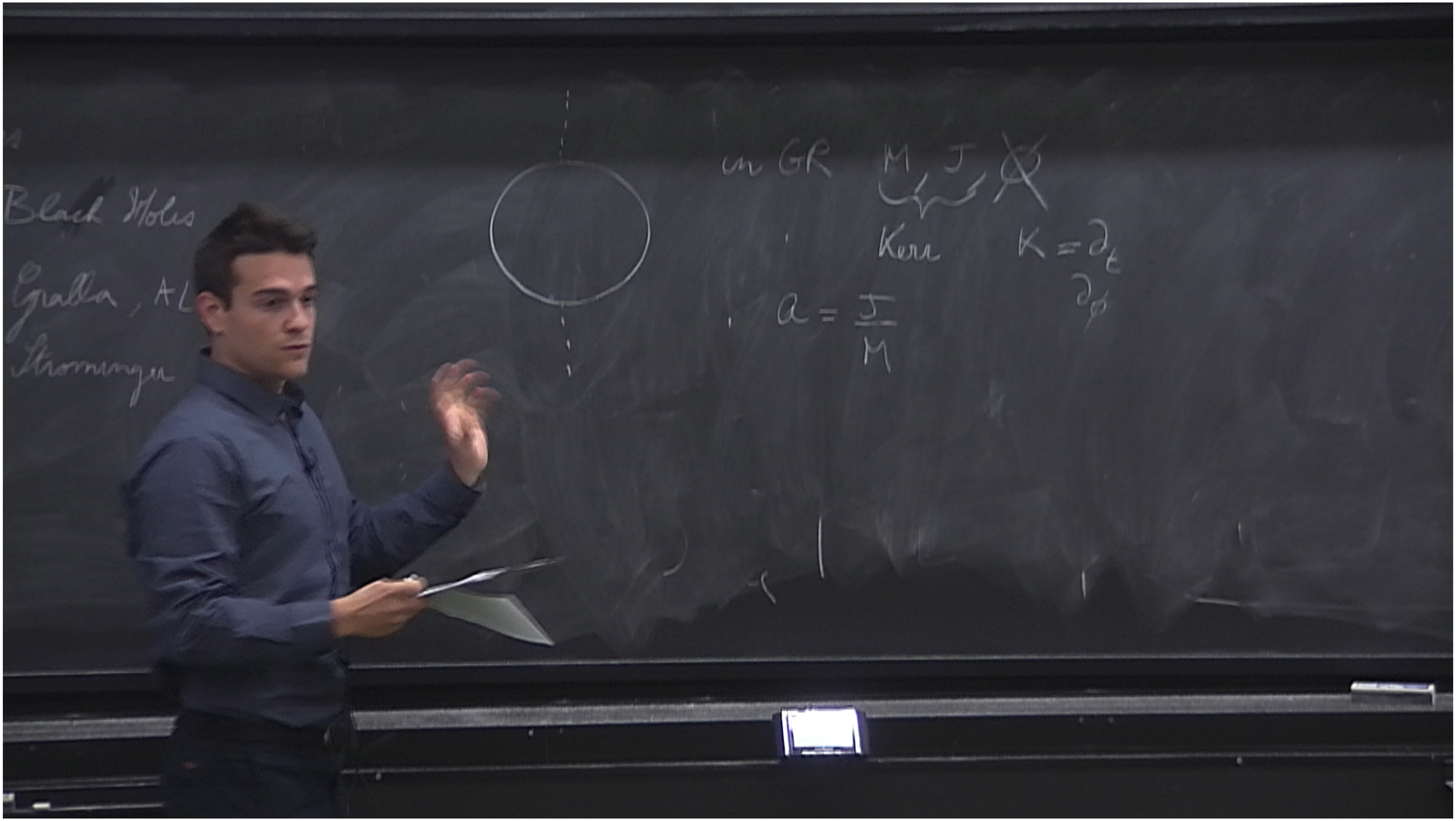
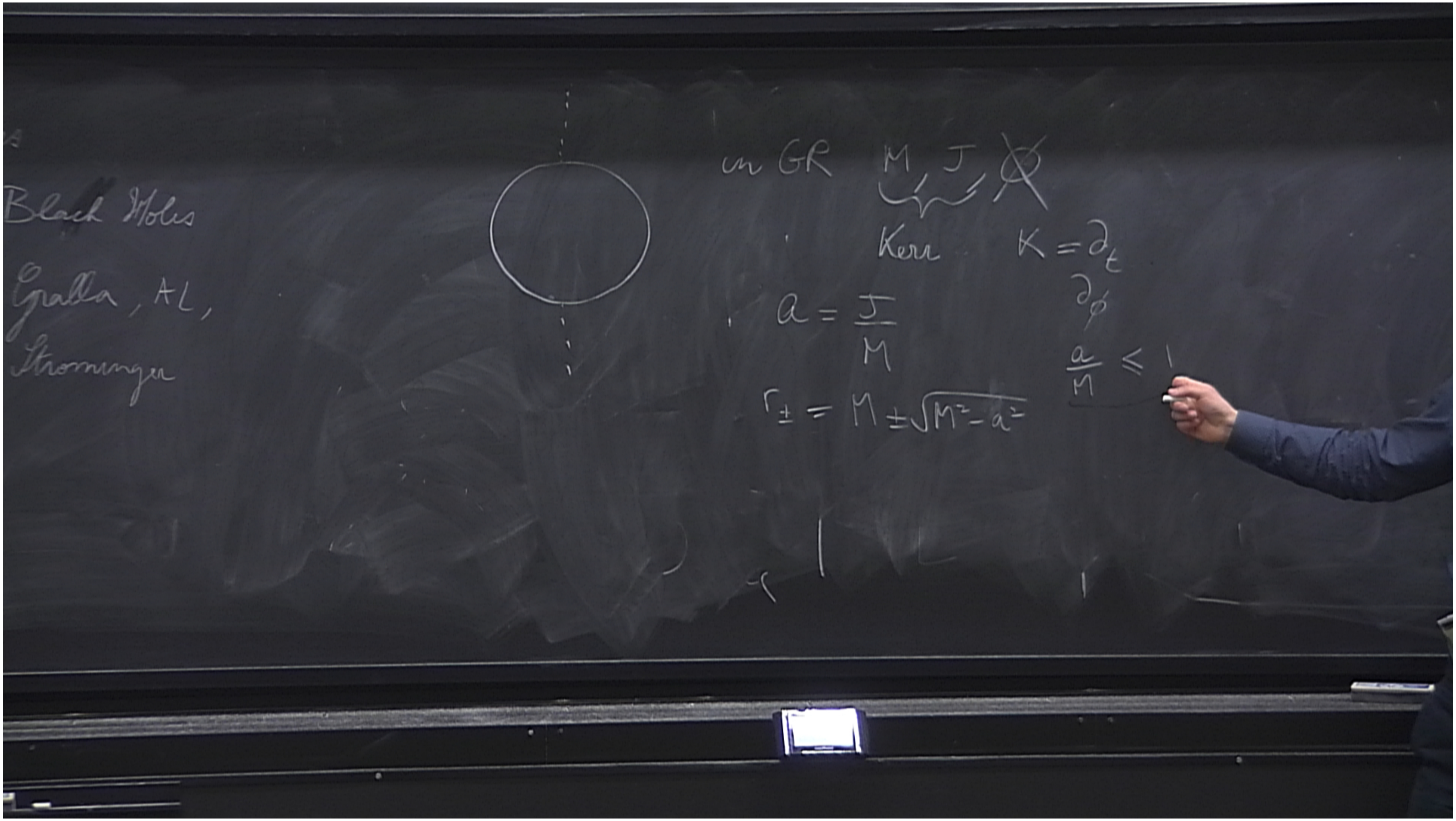
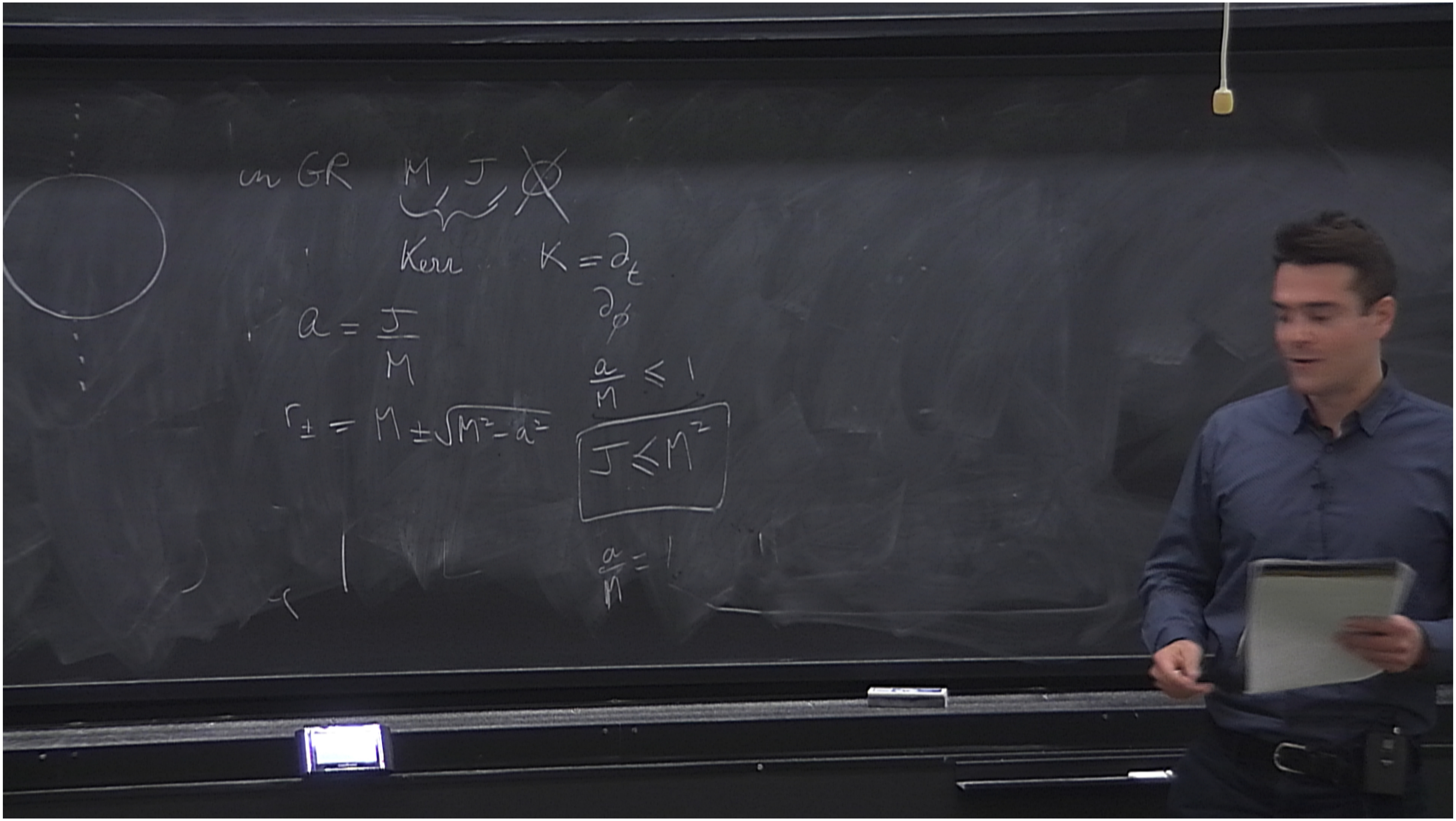
Abstract: <p>Plasma-filled magnetospheres can extract energy from a spinning black hole and provide the power source for a variety of observed astrophysical phenomena. These magnetospheres are described by the highly nonlinear equations of force-free electrodynamics. Typically these equations can only be solved numerically, but they become amenable to analytic solution in the extremal limit when the black hole achieves maximal angular momentum and an infinite-dimensional conformal symmetry emerges in the high-redshift region near its horizon. This constitutes an example of critical behavior in astronomy. We use the near-horizon scaling symmetry of the extreme Kerr metric to determine universal properties of physics near rapidly spinning black holes.</p>

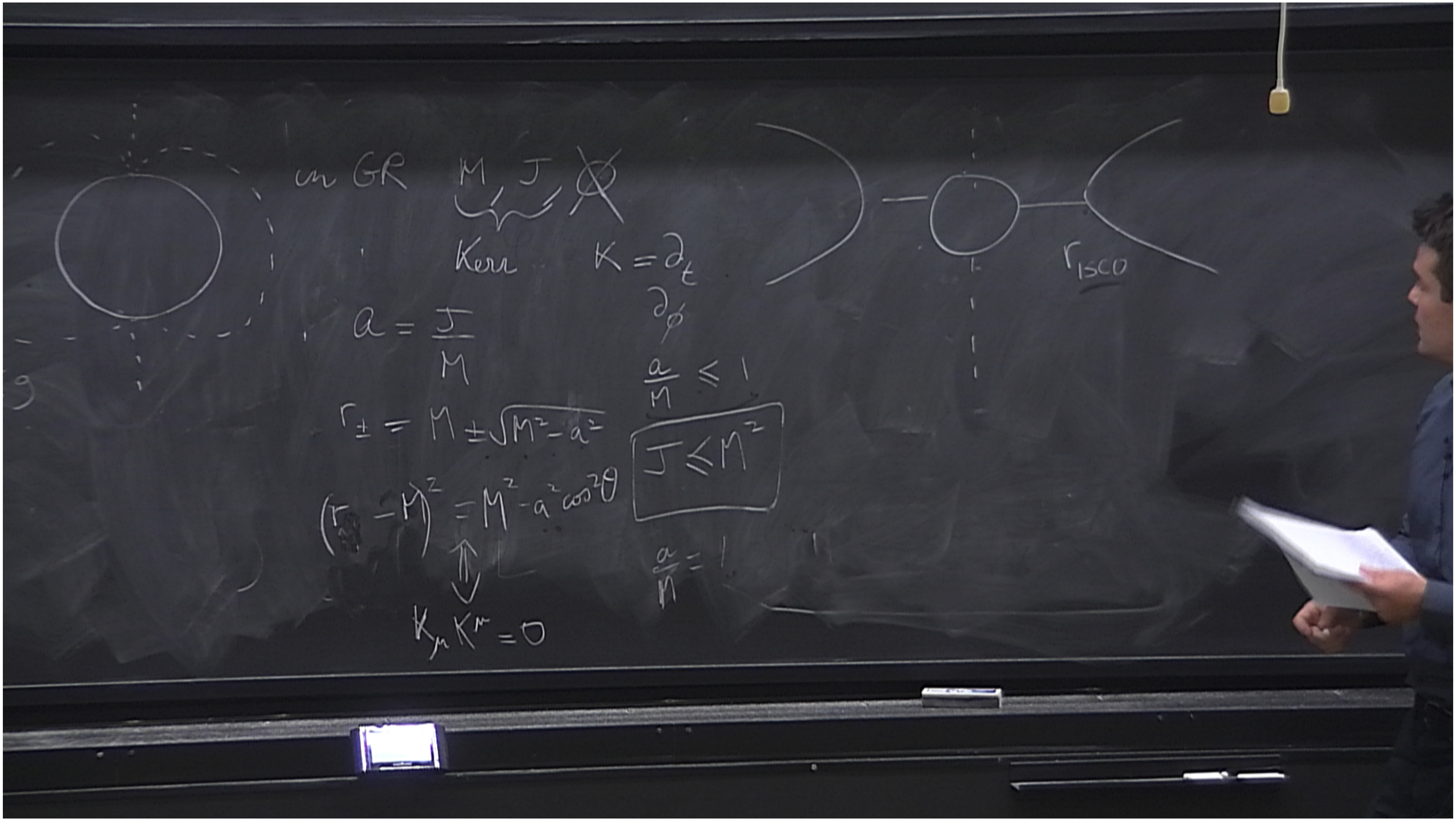
Near Horizon Magnetospheres
of Rapidly Spinning Black Holes

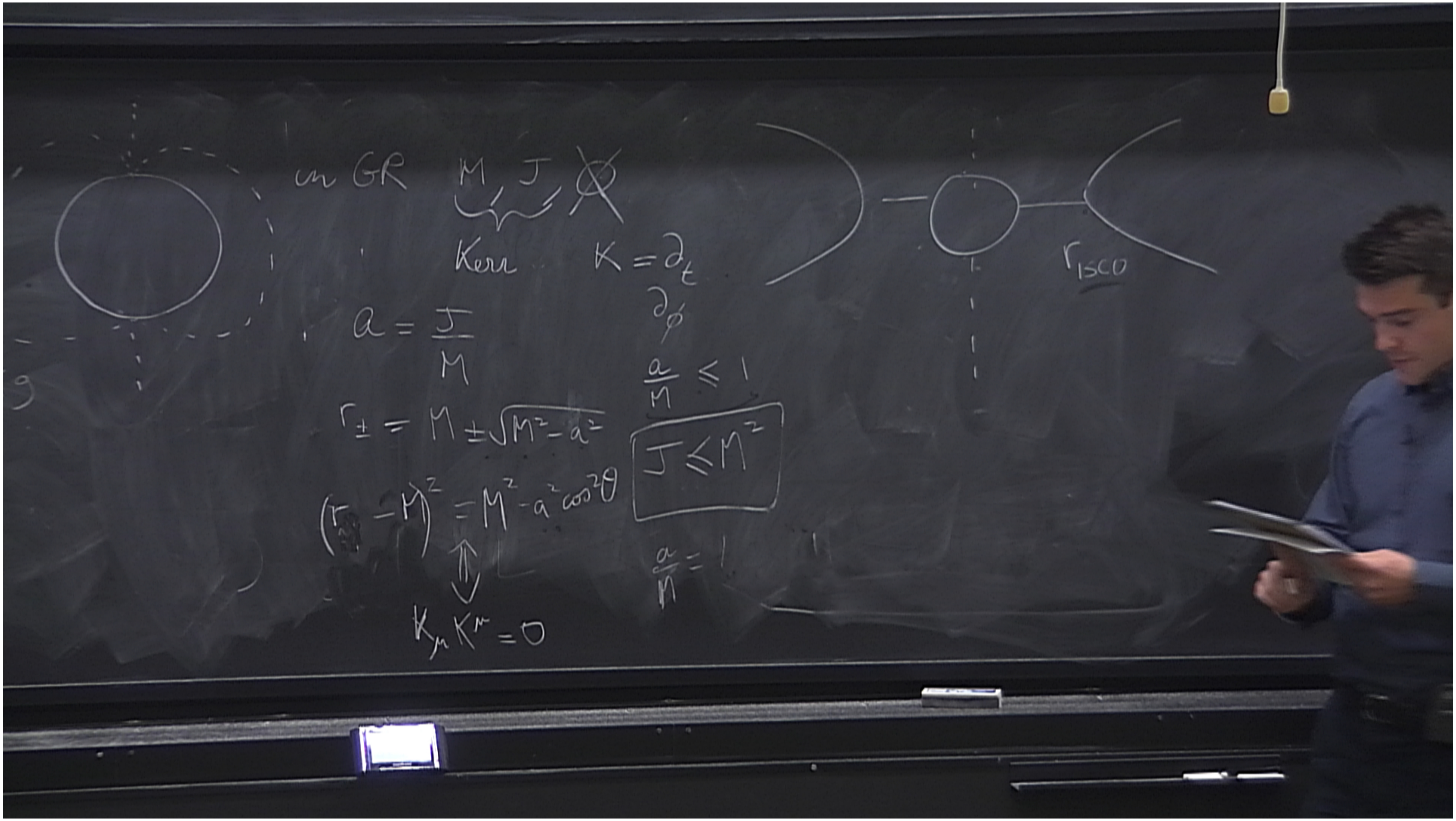
with Sam Gralla, AL,
Andy Strominger

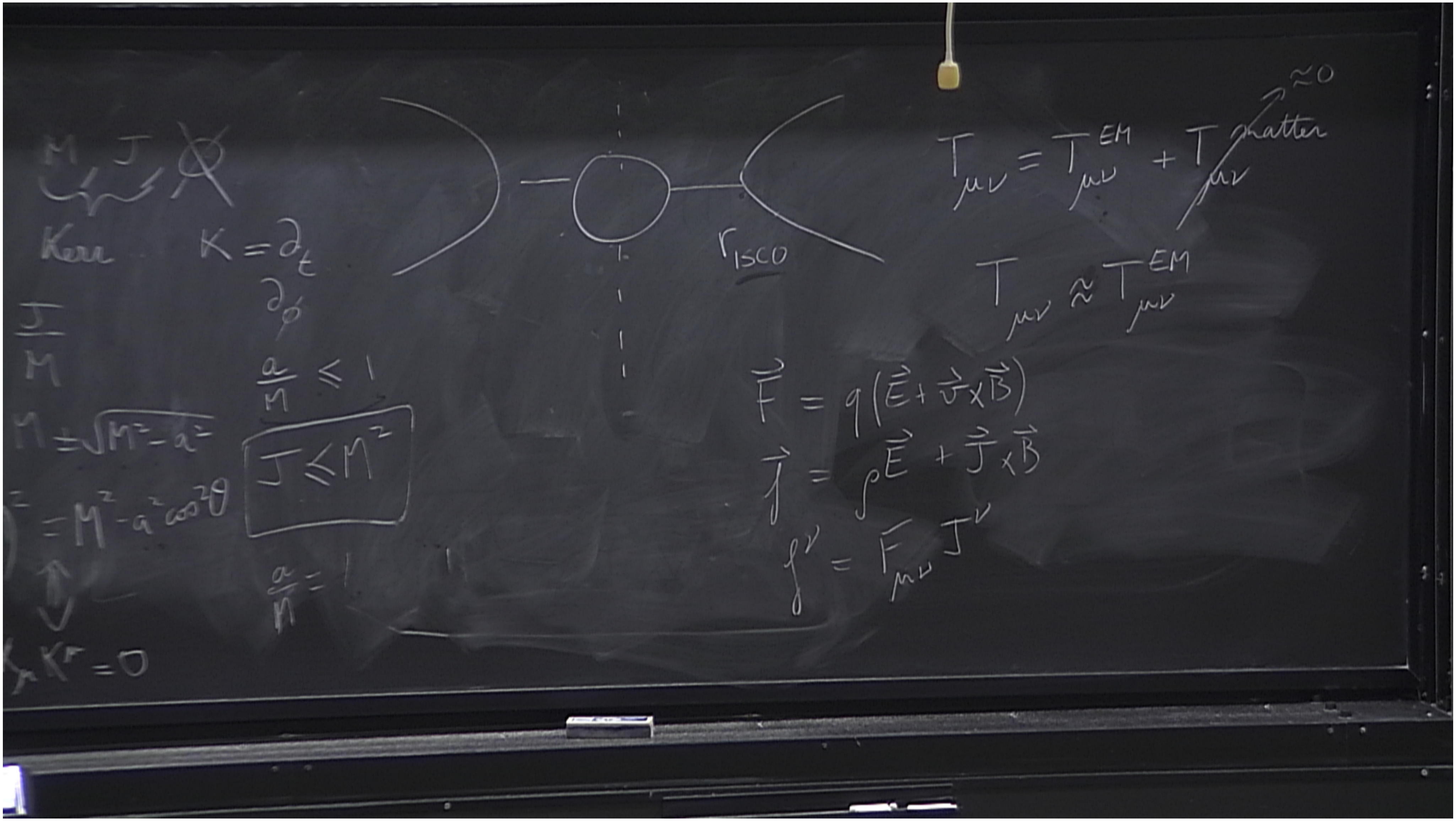

Black Holes
Gralla, AL
Strominger
in GR M J Q
Kerr
K = ∂t
∂φ
a = J/M

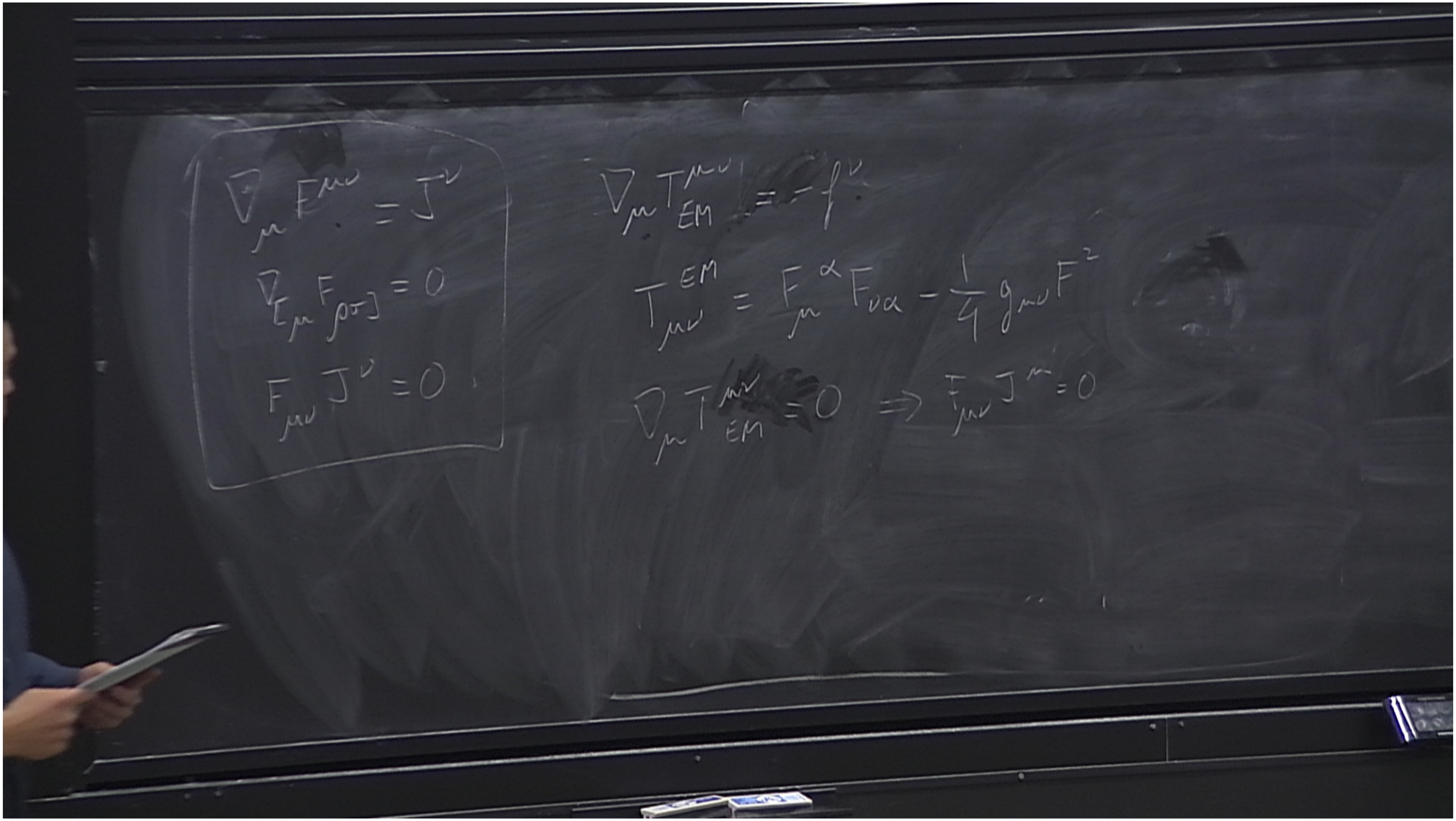

Black Holes
Gralla, AL,
Strominger
in GR M, J, $\cancel{Q}$
Kerr
$K = \partial_t$
∂_ϕ
$a = \frac{J}{M}$
$\frac{a}{M} \leq 1$
$r_\pm = M \pm \sqrt{M^2 - a^2}$


in GR $M, J, \cancel{Q}$
Kerr $K = \partial_t$
∂_ϕ
$a = \frac{J}{M}$
$\frac{a}{M} \leq 1$
$r_\pm = M \pm \sqrt{M^2 - a^2}$
$\boxed{J \leq M^2}$
$\frac{a}{M} = 1$


in GR M, J, $\cancel{Q}$
Kerr
$K = \partial_t$
∂_ϕ
$a = \frac{J}{M}$
$\frac{a}{M} \leq 1$
$r_\pm = M \pm \sqrt{M^2 - a^2}$
$\boxed{J \leq M^2}$
$(r - M)^2 = M^2 - a^2 \cos^2\theta$
$\frac{a}{M} = 1$
$K_\mu K^\mu = 0$
r_{ISCO}


in GR M, J, ~~Q~~
Kerr
K = ∂_t
∂_φ
a = J/M
a/M ≤ 1
r_± = M ± √(M² − a²)
J ≤ M²
(r − M)² = M² − a² cos²θ
a/M = 1
K_μ K^μ = 0
r_ISCO


M J
Kerr
K = ∂_t
∂_φ
J/M
a/M ≤ 1
J ≤ M^2
M ± √(M^2 − a^2)
= M^2 − a^2 cos^2θ
a/M = 1
r_ISCO
≃ 0
T_μν = T_μν^EM + T_μν^matter
T_μν ≈ T_μν^EM
F = q(E + v × B)
j = ρE + J × B
f^ν = F_μν J^ν


$\nabla_\mu F^{\mu\nu} = J^\nu$
$\nabla_{[\mu} F_{\rho\sigma]} = 0$
$F_{\mu\nu} J^\nu = 0$
$\nabla_\mu T^{\mu\nu}_{EM} = -f^\nu$
$T^{EM}_{\mu\nu} = F_\mu{}^\alpha F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F^2$
$\nabla_\mu T^{\mu\nu}_{EM} = 0 \Rightarrow F_{\mu\nu} J^\mu = 0$

$$\nabla_\mu F^{\mu\nu} = J^\nu$$

$$\nabla_{[\mu} F_{\rho\sigma]} = 0$$

$$F_{\mu\nu} J^\nu = 0$$

$$\nabla_\mu F^{\mu\nu} = J^\nu$$

$$\nabla_{[\mu} F_{\rho\sigma]} = 0$$

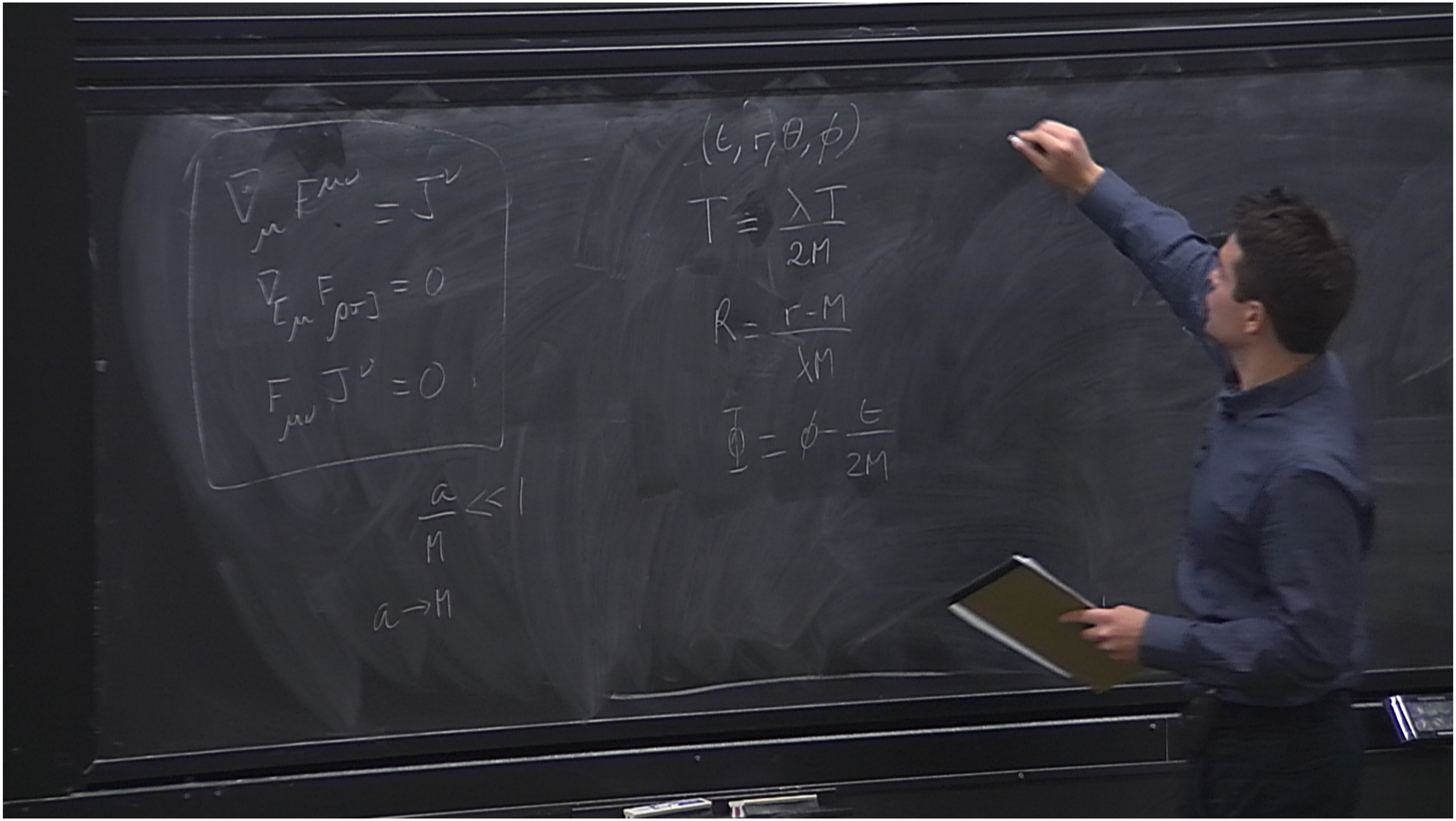
$$F_{\mu\nu} J^\nu = 0$$

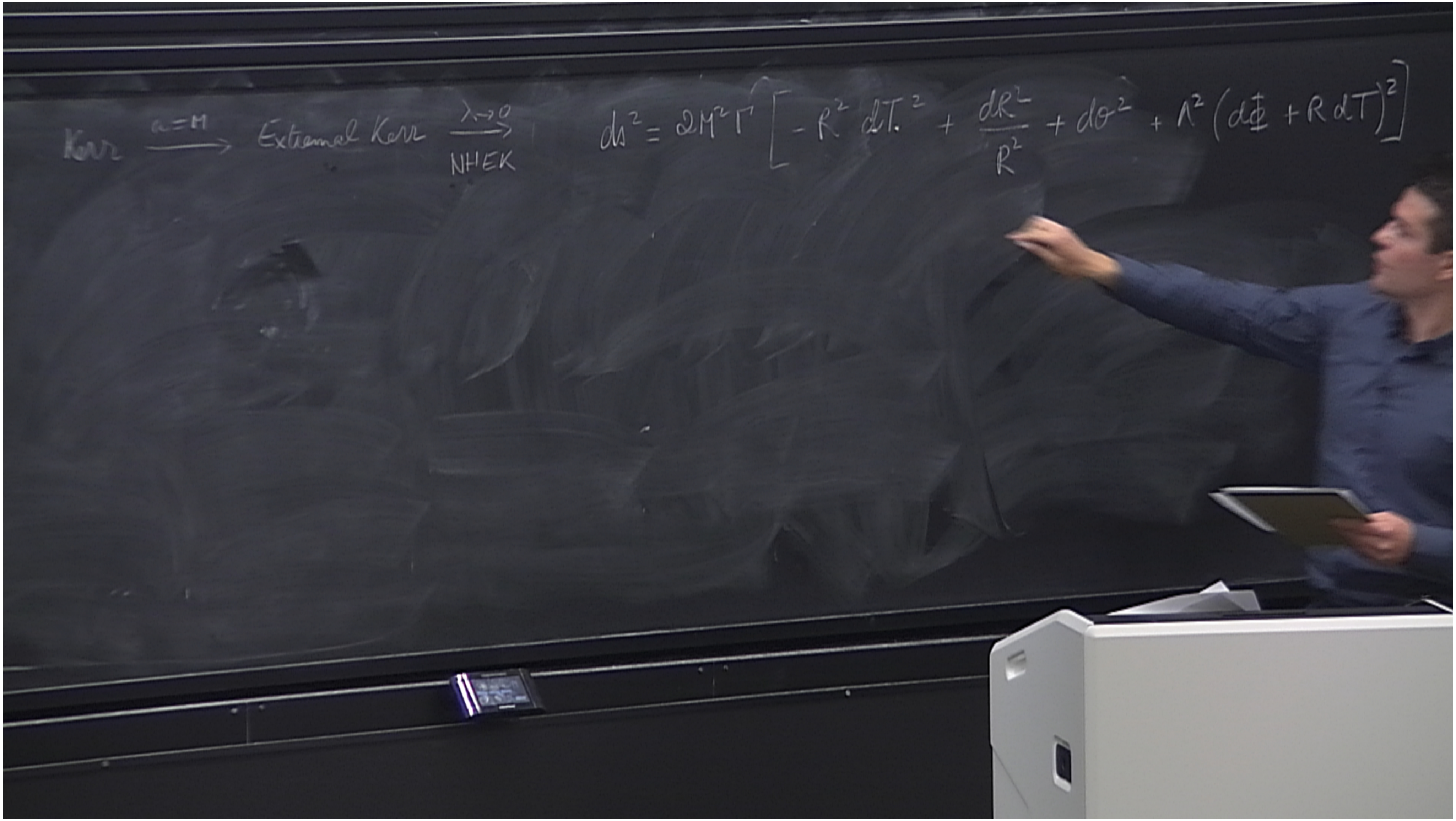
$$\frac{a}{M} \ll 1$$

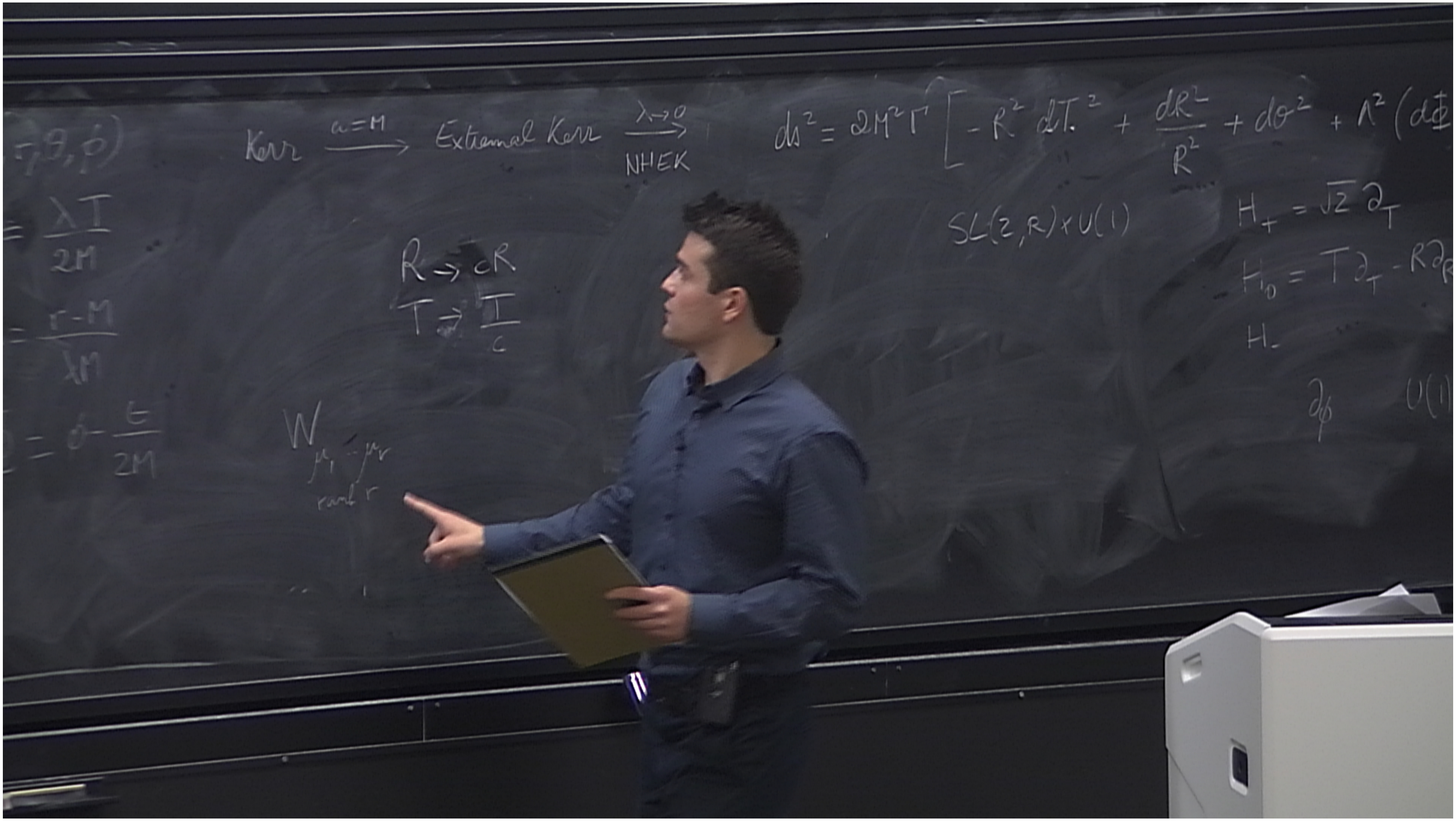
$$a \to M$$

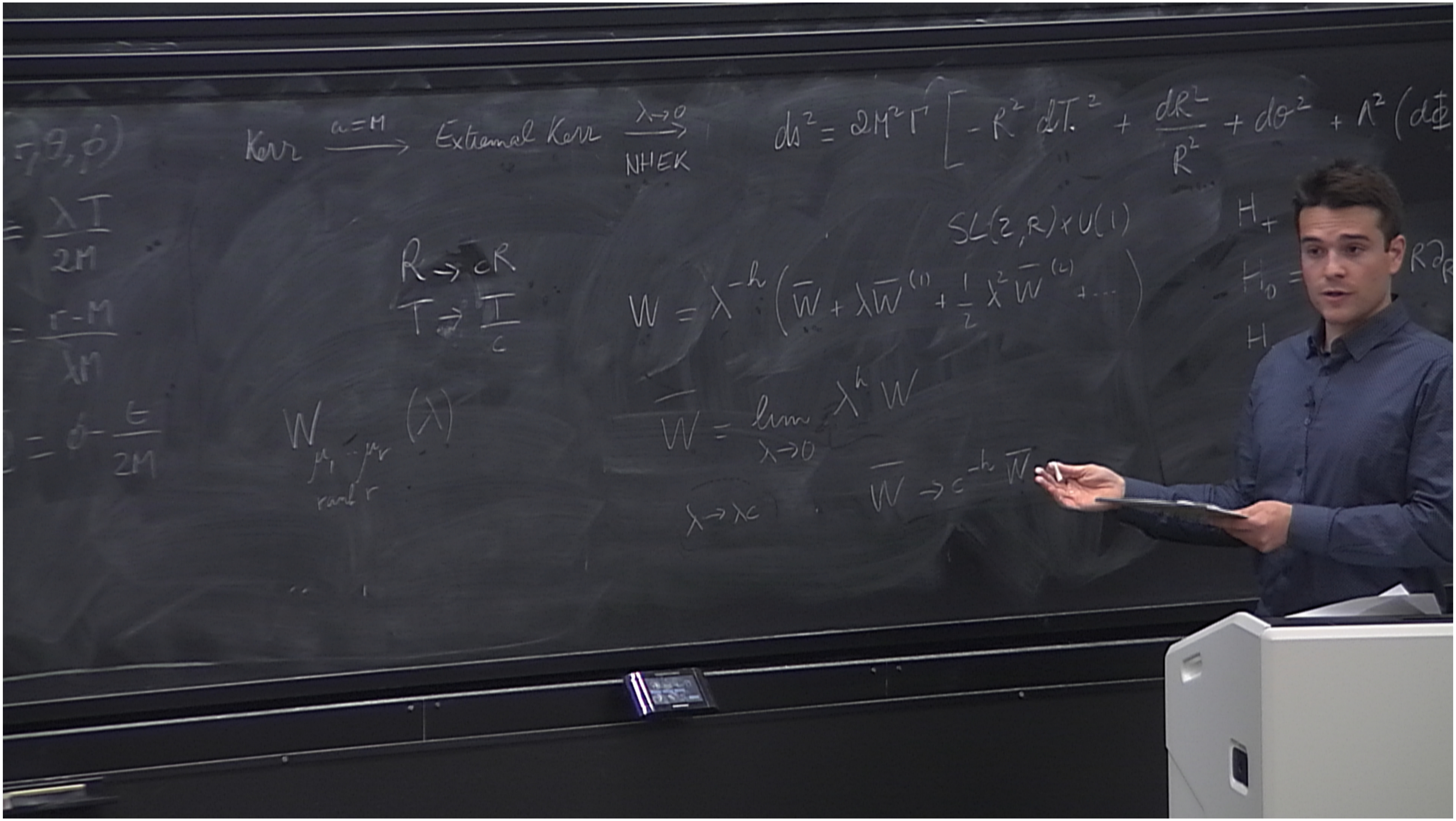
$$T_H = \frac{r_+ - M}{4\pi M r_+}$$

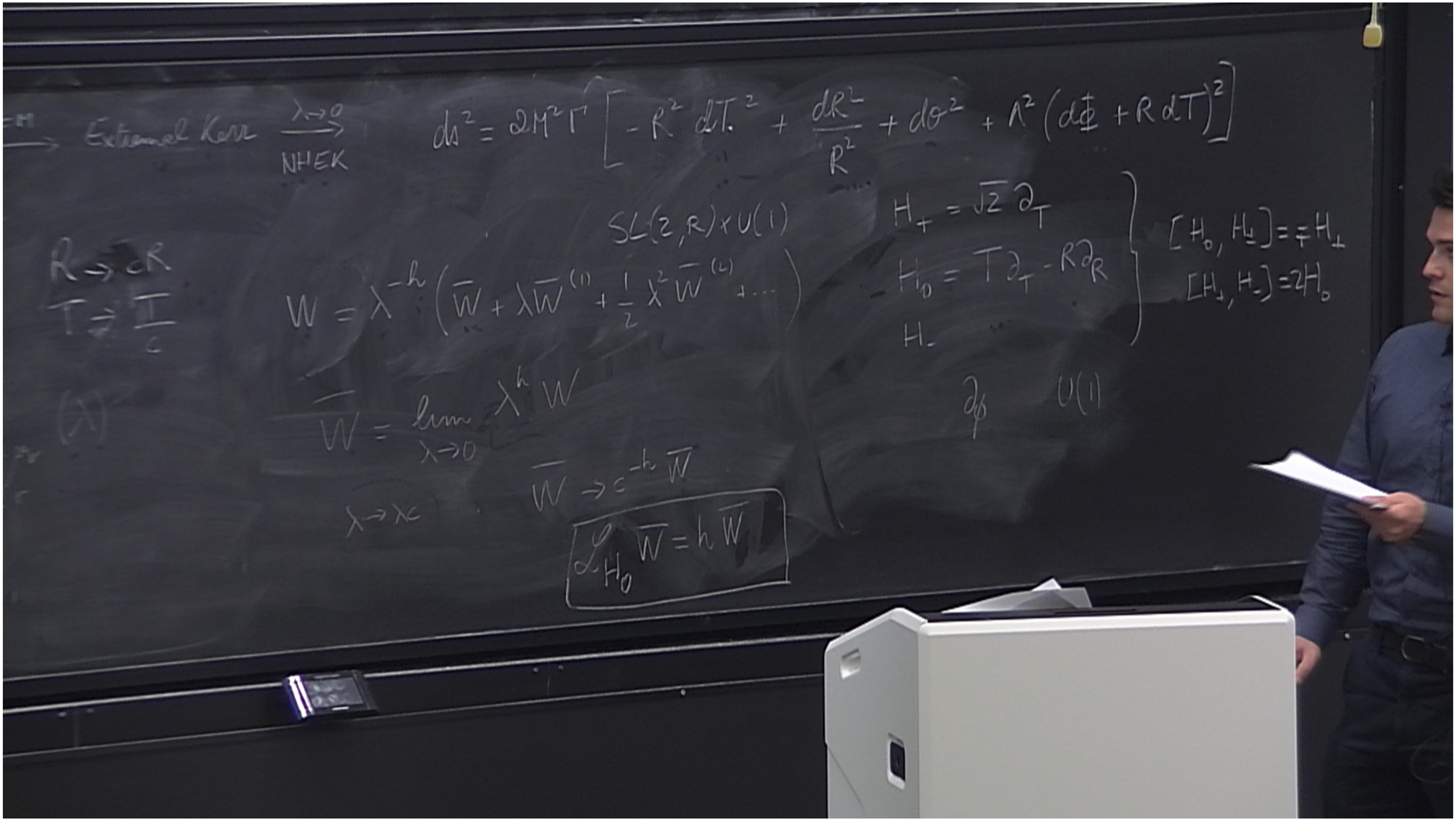
$$T_H \to 0$$

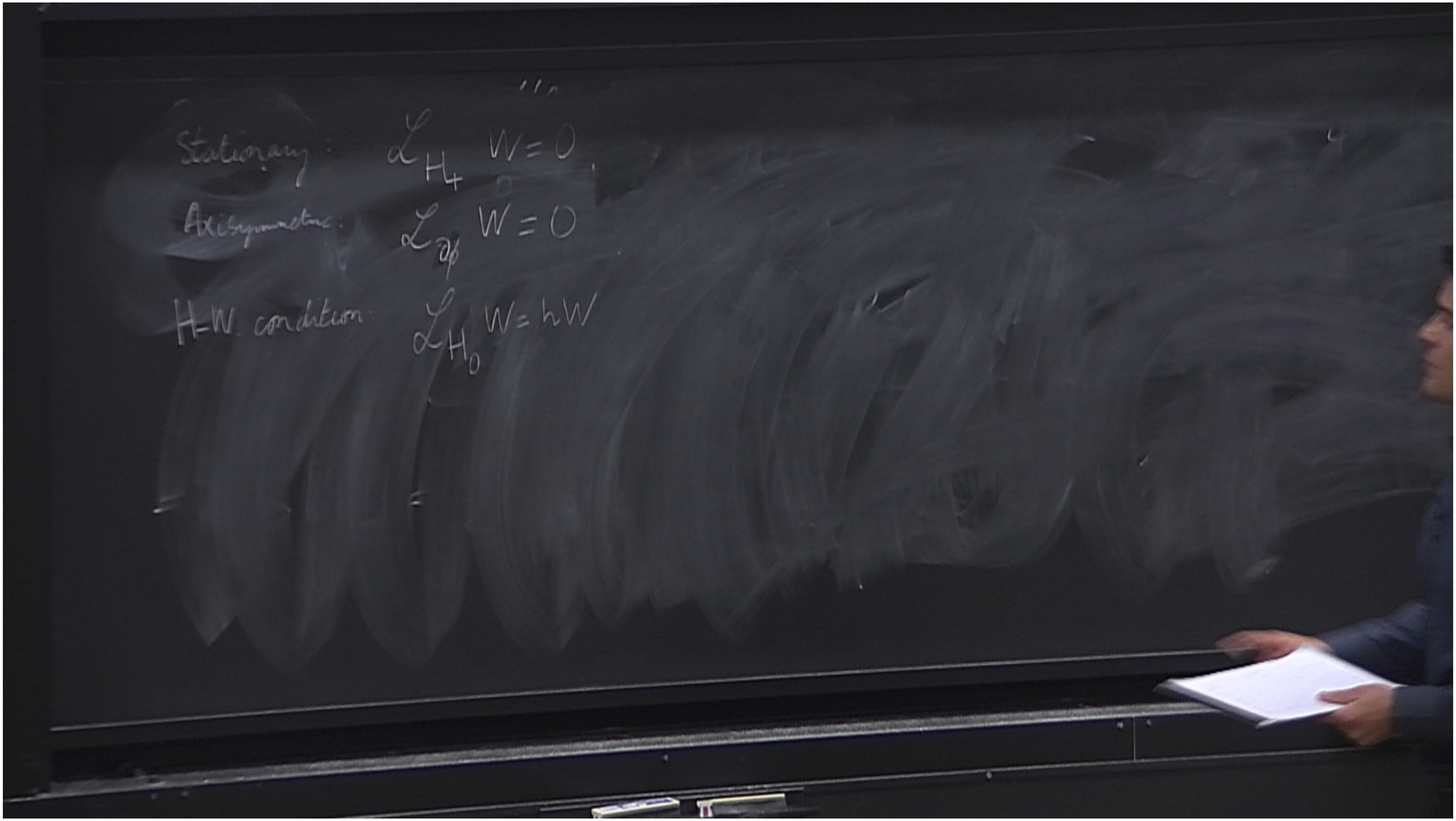

$\nabla_\mu F^{\mu\nu} = J^\nu$
$\nabla_{[\mu} F_{\rho\sigma]} = 0$
$F_{\mu\nu} J^\nu = 0$
$\frac{a}{M} \ll 1$
$a \to M$
(t, r, θ, ϕ)
$T = \frac{\lambda T}{2M}$
$R = \frac{r-M}{\lambda M}$
$\Phi = \phi - \frac{t}{2M}$

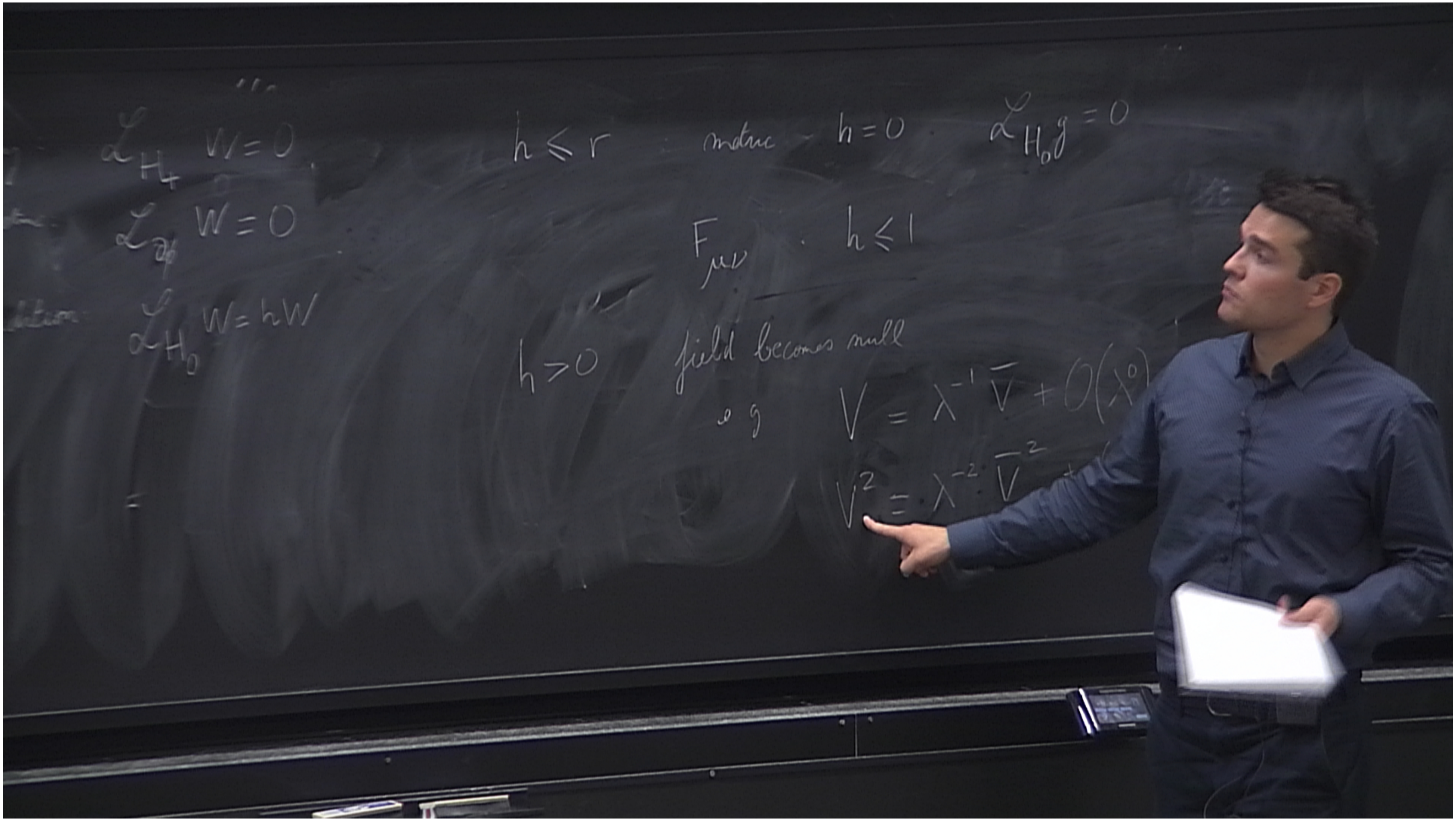

Kerr a=M Extremal Kerr λ→0 NHEK
ds² = 2M²Γ [-R² dT² + dR²/R² + dθ² + Λ² (dΦ + R dT)²]


Kerr $\xrightarrow{a=M}$ Extremal Kerr $\xrightarrow[\text{NHEK}]{\lambda\to 0}$
$ds^2 = 2M^2\Gamma\left[-R^2 dT^2 + \frac{dR^2}{R^2} + d\theta^2 + \Lambda^2(d\Phi\right.$
$R \to cR$
$T \to \frac{T}{c}$
$SL(2,R)\times U(1)$
$H_+ = \sqrt{2}\,\partial_T$
$H_0 = T\partial_T - R\partial_R$
H_-
∂_ϕ
$U(1)$
$W_{\mu_1 \cdots \mu_r}$
rank r

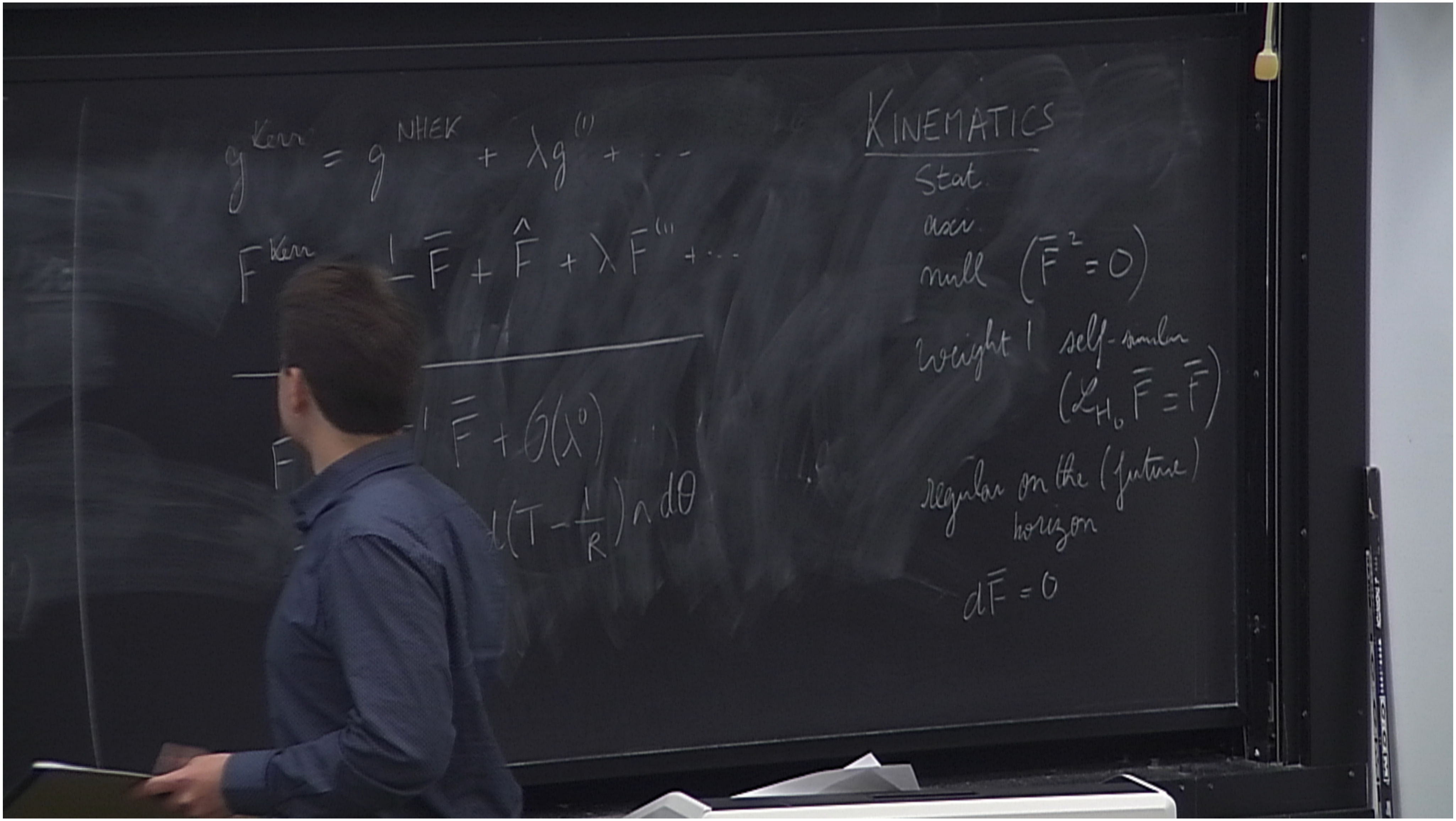

Kerr a=M Extremal Kerr λ→0 NHEK
ds² = 2M²Γ [-R² dT² + dR²/R² + dθ² + Λ² (dΦ
SL(2,R) × U(1)
R → cR
T → T/c
W = λ^{-h} (W̄ + λW̄^{(1)} + ½ λ² W̄^{(2)} + ...)
W̄ = lim λ→0 λ^h W
W̄ → c^{-h} W̄
λ → λc
rank r


Extremal Kerr λ→0 NHEK
ds² = 2M²Γ [-R² dT² + dR²/R² + dθ² + Λ²(dΦ + R dT)²]
SL(2,R)×U(1)
H+ = √2 ∂T
H0 = T∂T - R∂R
H-
[H0, H±] = ∓H±
[H+, H-] = 2H0
R → cR
T → T/c
(λ)
W = λ^-h (W̄ + λW̄^(1) + ½ λ² W̄^(2) + ...)
W̄ = lim λ→0 λ^h W
∂φ U(1)
W̄ → c^-h W̄
λ → λc
L_H0 W̄ = h W̄


Stationary: $\mathcal{L}_{H_4} W = 0$
Axisymmetric: $\mathcal{L}_{\partial\phi} W = 0$
H-W condition: $\mathcal{L}_{H_0} W = hW$



W = 0
W = 0
W = hW
h ≤ r
metric
h = 0
= 0
h ≤ 1
h > 0
field becomes null
e.g.



$g^{Kerr} = g^{NHEK} + \lambda g^{(1)} + \dots$
$F^{Kerr} = \frac{1}{\lambda}\bar{F} + \hat{F} + \lambda F^{(1)} + \dots$
$\bar{F} + \mathcal{O}(\lambda^0)$
$d(T - \frac{1}{R}) \wedge d\theta$
KINEMATICS
Stat.
axi.
null $(\bar{F}^2 = 0)$
weight 1 self-similar
$(\mathcal{L}_{H_0}\bar{F} = \bar{F})$
regular on the (future) horizon
$d\bar{F} = 0$

$\Rightarrow \bar{V}^2 = 0$

weight $h=0$ $\rightarrow$ $F = \hat{F} + O(X)$

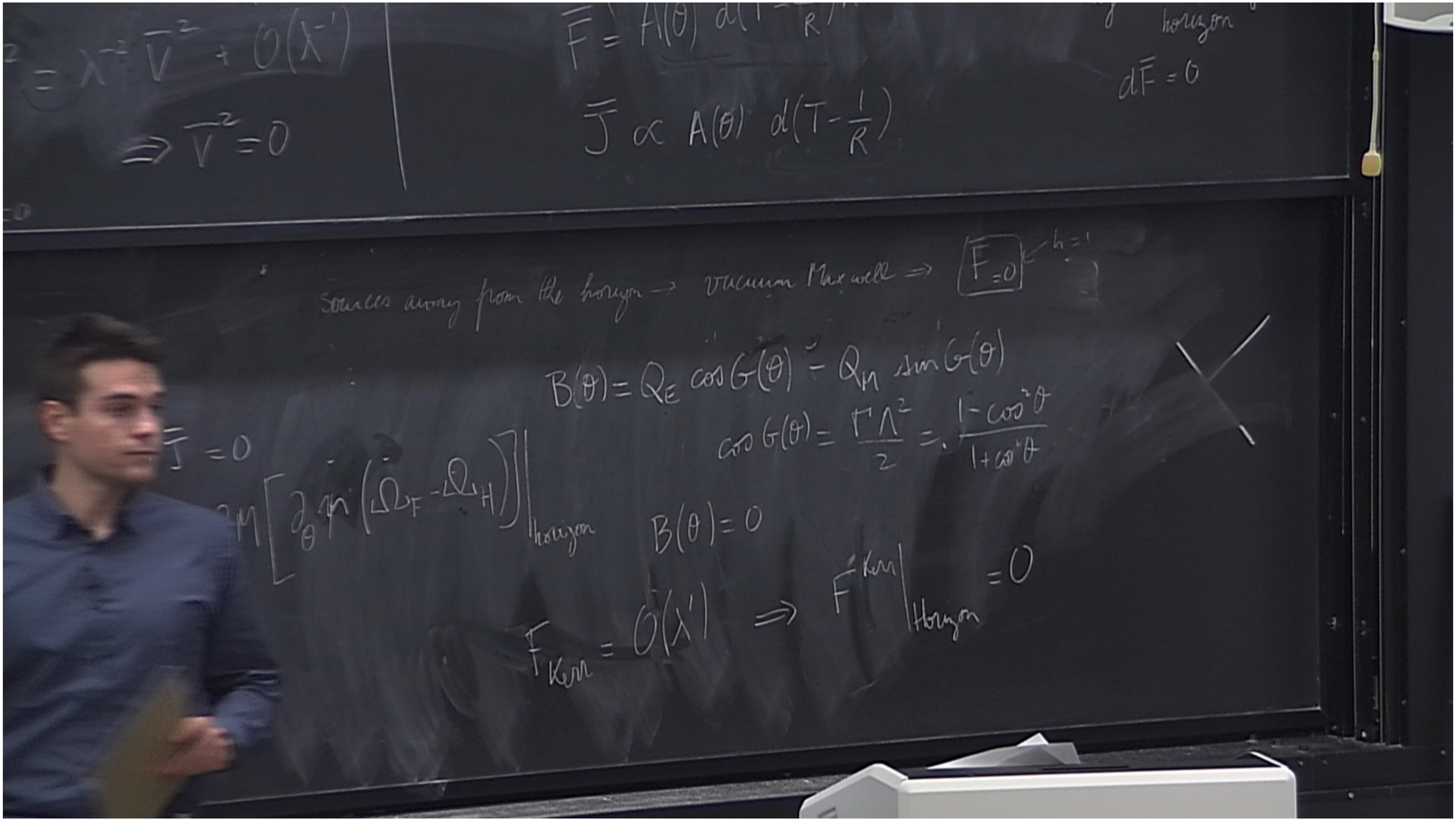
$\mathcal{L}_{H_0} \hat{F} = 0$

∂_ϕ^μ

$$\hat{F} = B(\theta)\, dT \wedge dR + B'(\theta)\left(R\, dT + d\Phi\right) \wedge d\theta$$

$SL(2,R) \times U(1)$ invariant

$$\hat{J}^\mu \propto \partial_\phi^\mu$$


⇒ V̄² = 0
J̄ ∝ A(θ) d(T − 1/R)
horizon
dF = 0
Sources away from the horizon → vacuum Maxwell ⇒ F = 0
B(θ) = Q_E cos G(θ) − Q_M sin G(θ)
cos G(θ) = Γ Λ²/2 = (1 − cos²θ)/(1 + cos²θ)
J̄ = 0
horizon
B(θ) = 0
F_Kerr = O(λ¹) ⇒ F^Kerr|_Horizon = 0