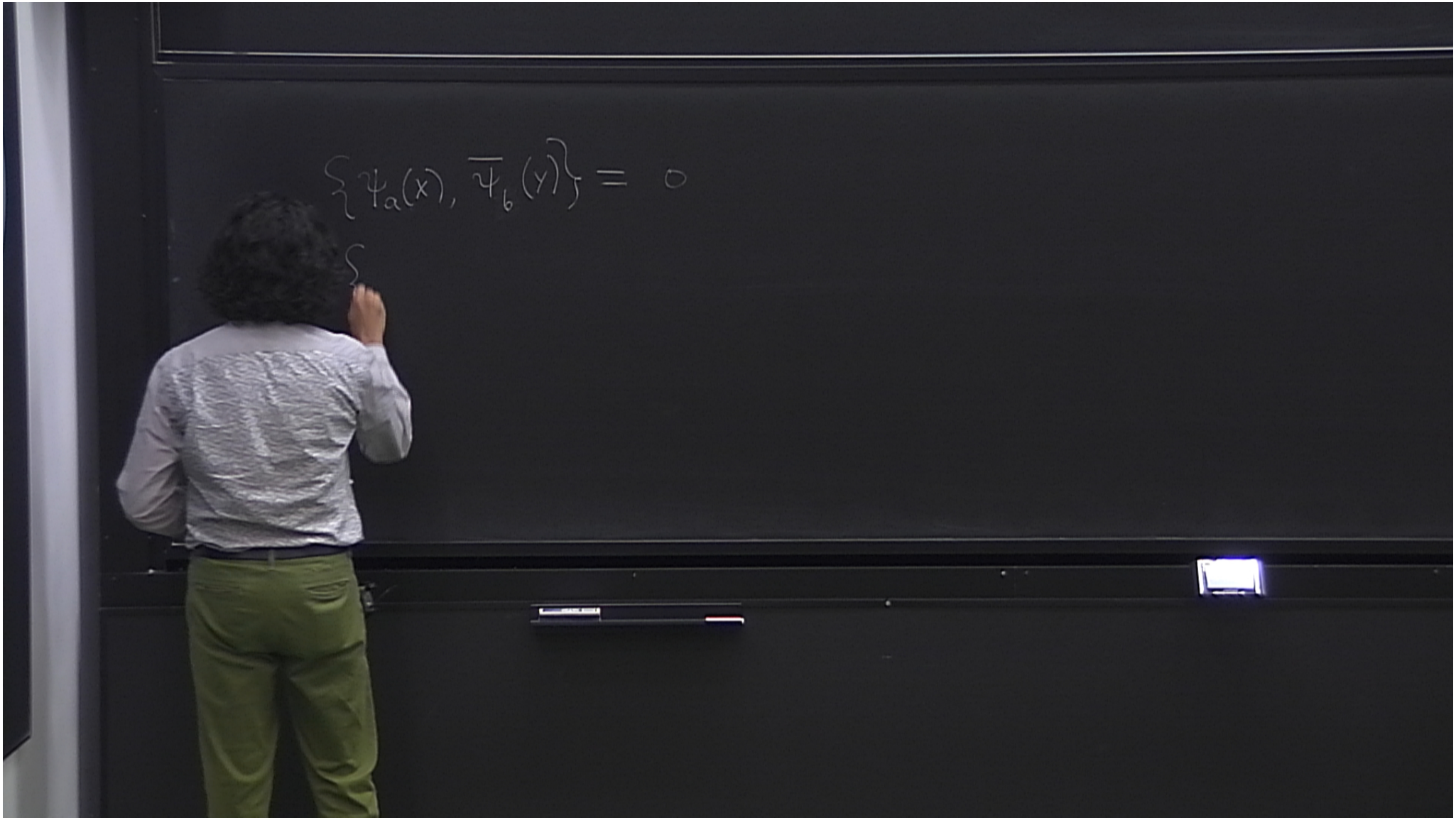
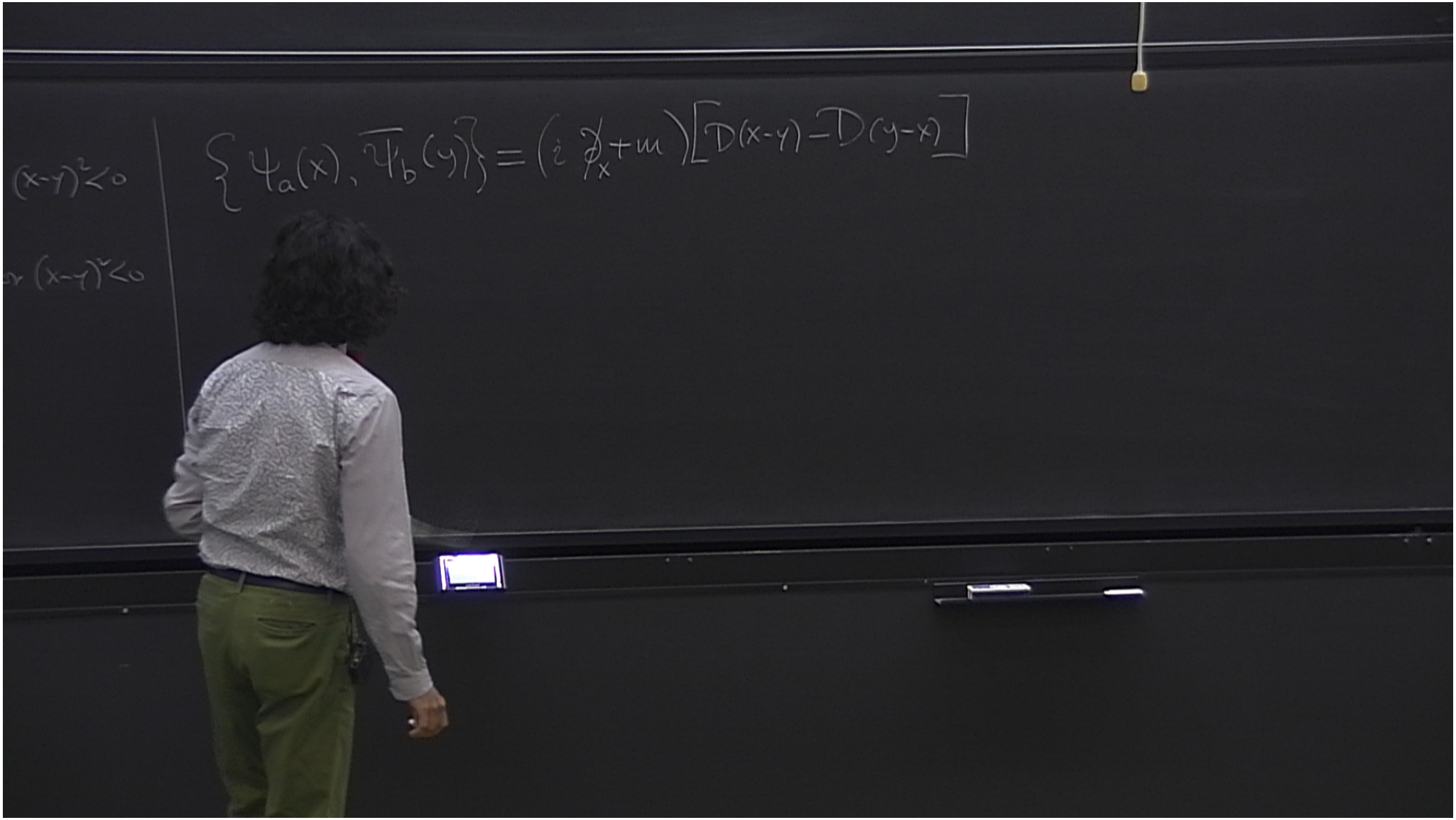
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 12

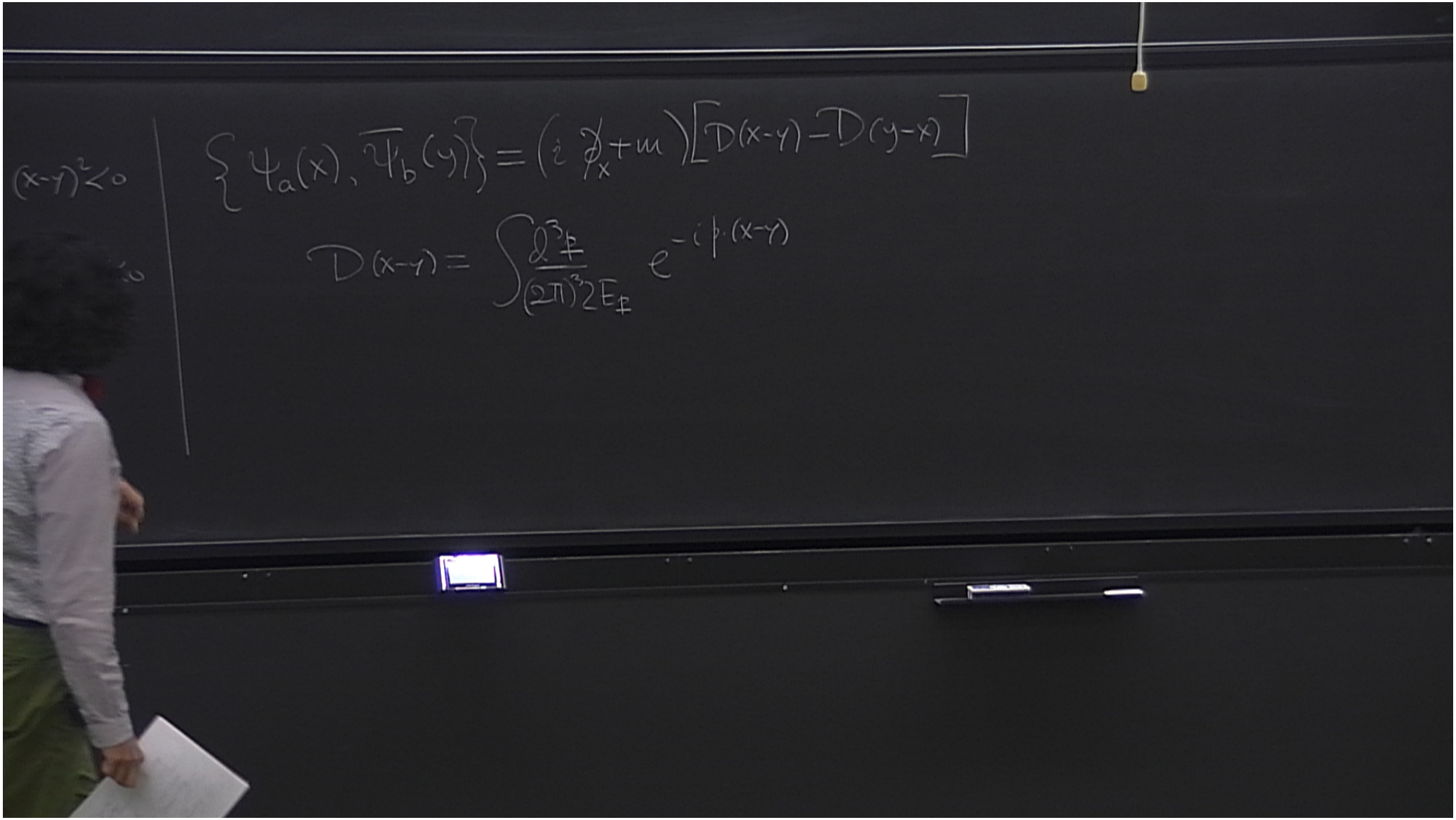
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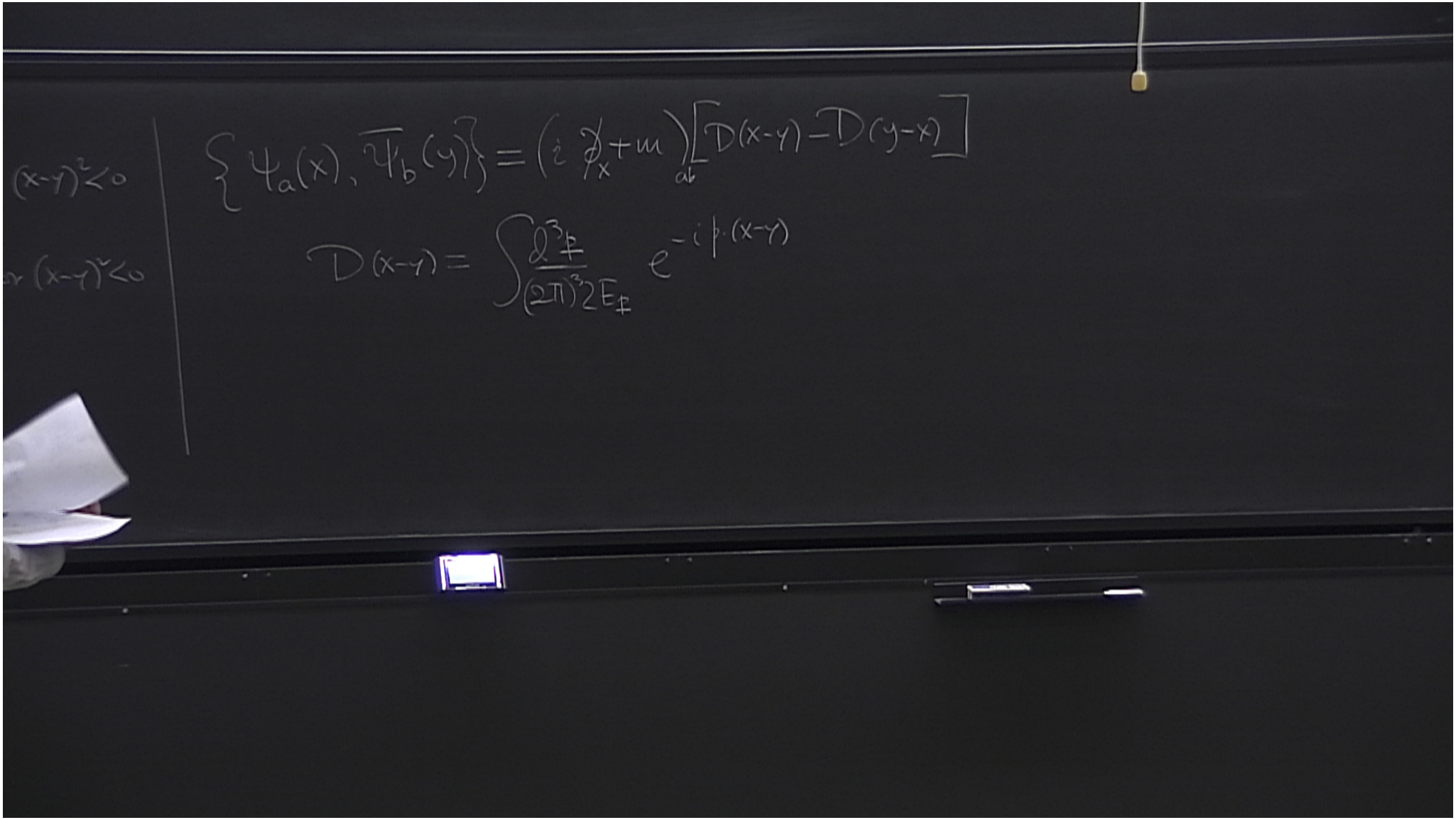
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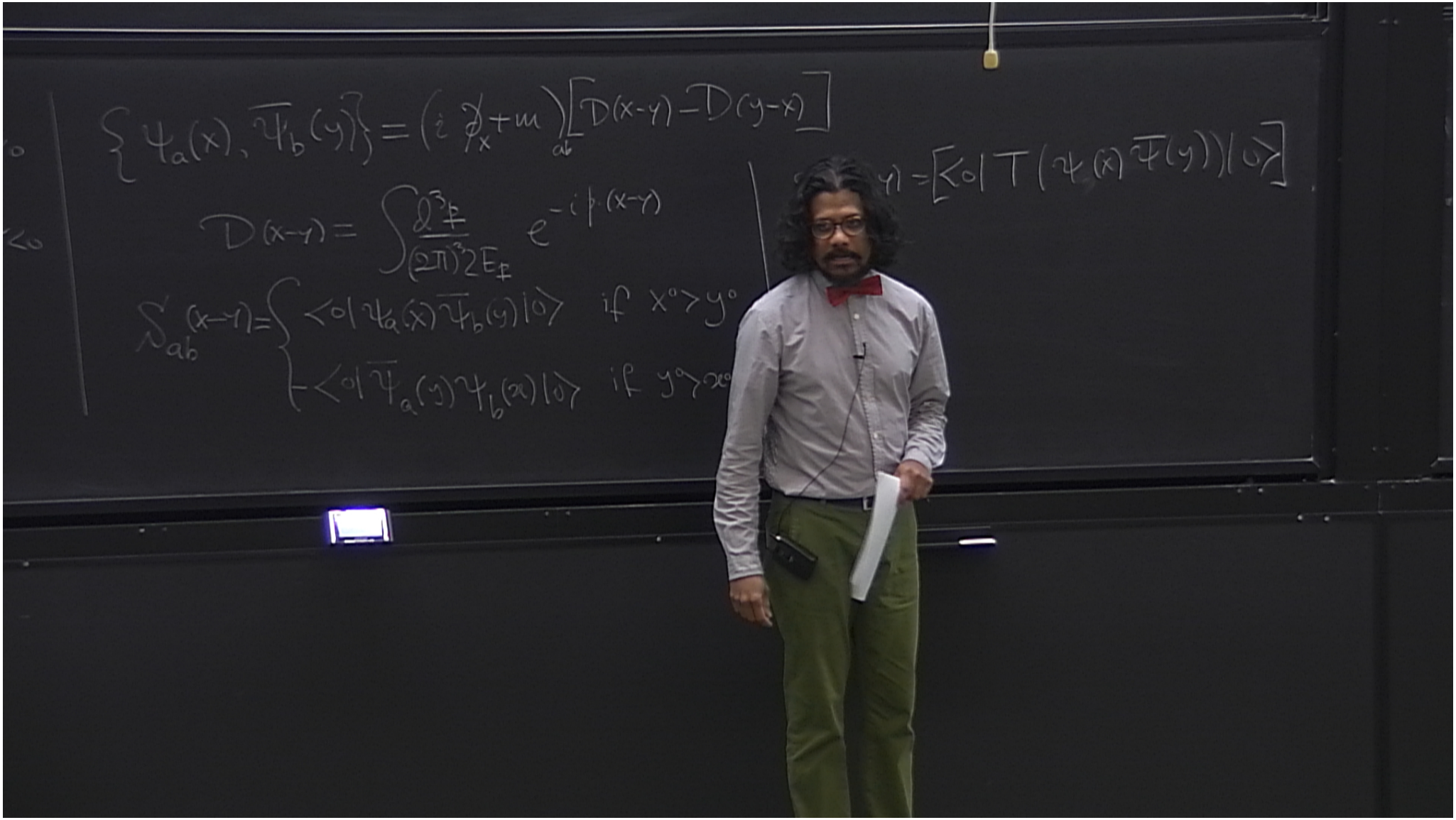
Abstract:


$\{\psi_a(x), \overline{\psi}_b(y)\} = 0$


$(x-y)^2<0$
or $(x-y)^2<0$
$\{\psi_a(x), \bar{\psi}_b(y)\} = (i\partial\!\!\!/_x + m)[D(x-y) - D(y-x)]$


$(x-y)^2<0$
$\{\psi_a(x), \bar{\psi}_b(y)\} = (i\partial\!\!\!/_x + m)[D(x-y) - D(y-x)]$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)}$


$(x-y)^2<0$
$(x-y)^2<0$
$\{\psi_a(x), \bar{\psi}_b(y)\} = (i\partial\!\!\!/_x + m)_{ab}[D(x-y) - D(y-x)]$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)}$


$\{\psi_a(x), \bar{\psi}_b(y)\} = (i\partial_x + m)_{ab}[D(x-y) - D(y-x)]$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)}$
$S_{ab}(x-y) = \begin{cases} \langle 0|\psi_a(x)\bar{\psi}_b(y)|0\rangle & \text{if } x^0 > y^0 \\ -\langle 0|\bar{\psi}_a(y)\psi_b(x)|0\rangle & \text{if } y^0 > x^0 \end{cases}$
$= [\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle]$

$(x-y)^2 < 0$

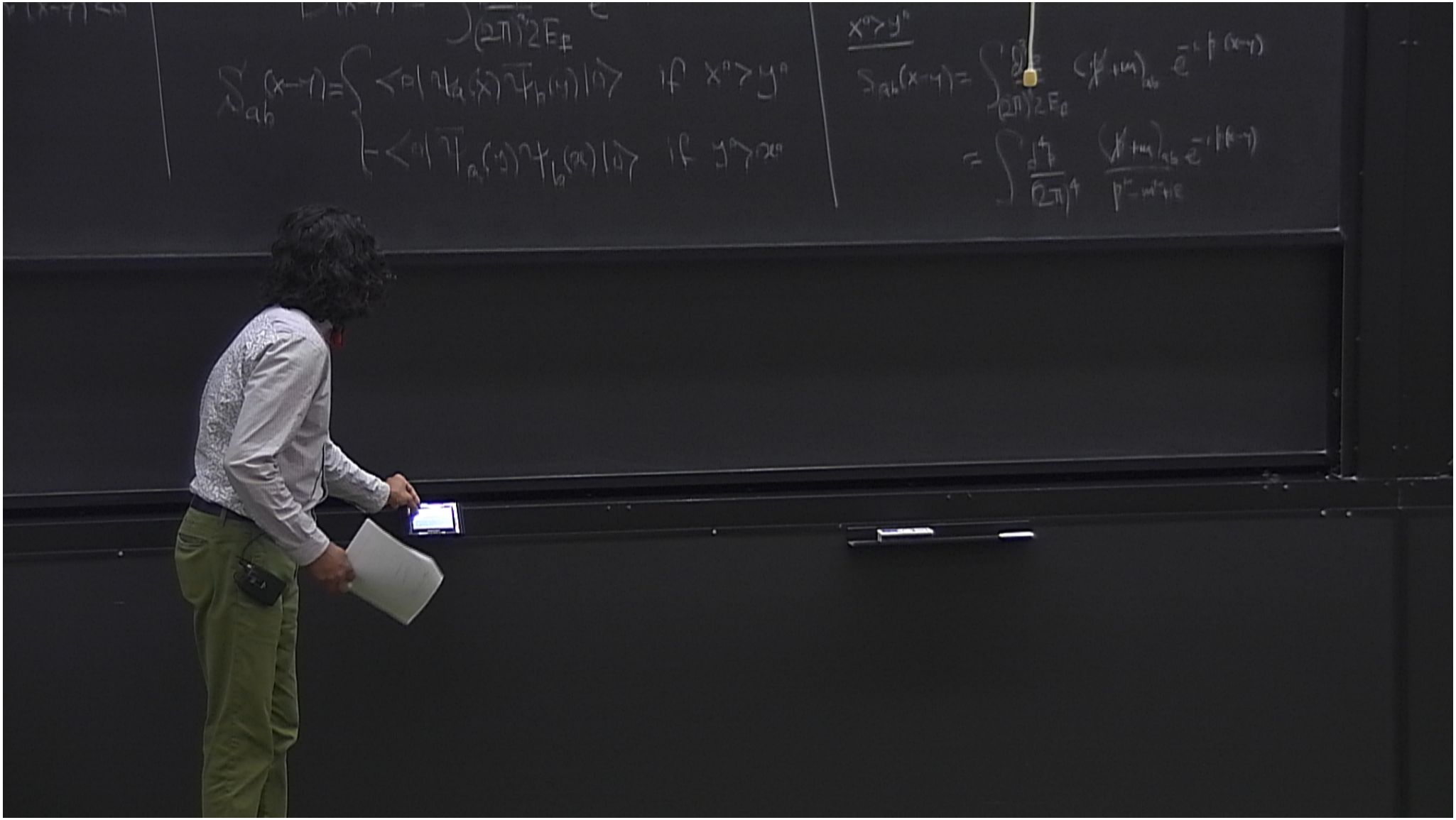
or $(x-y)^2 < 0$

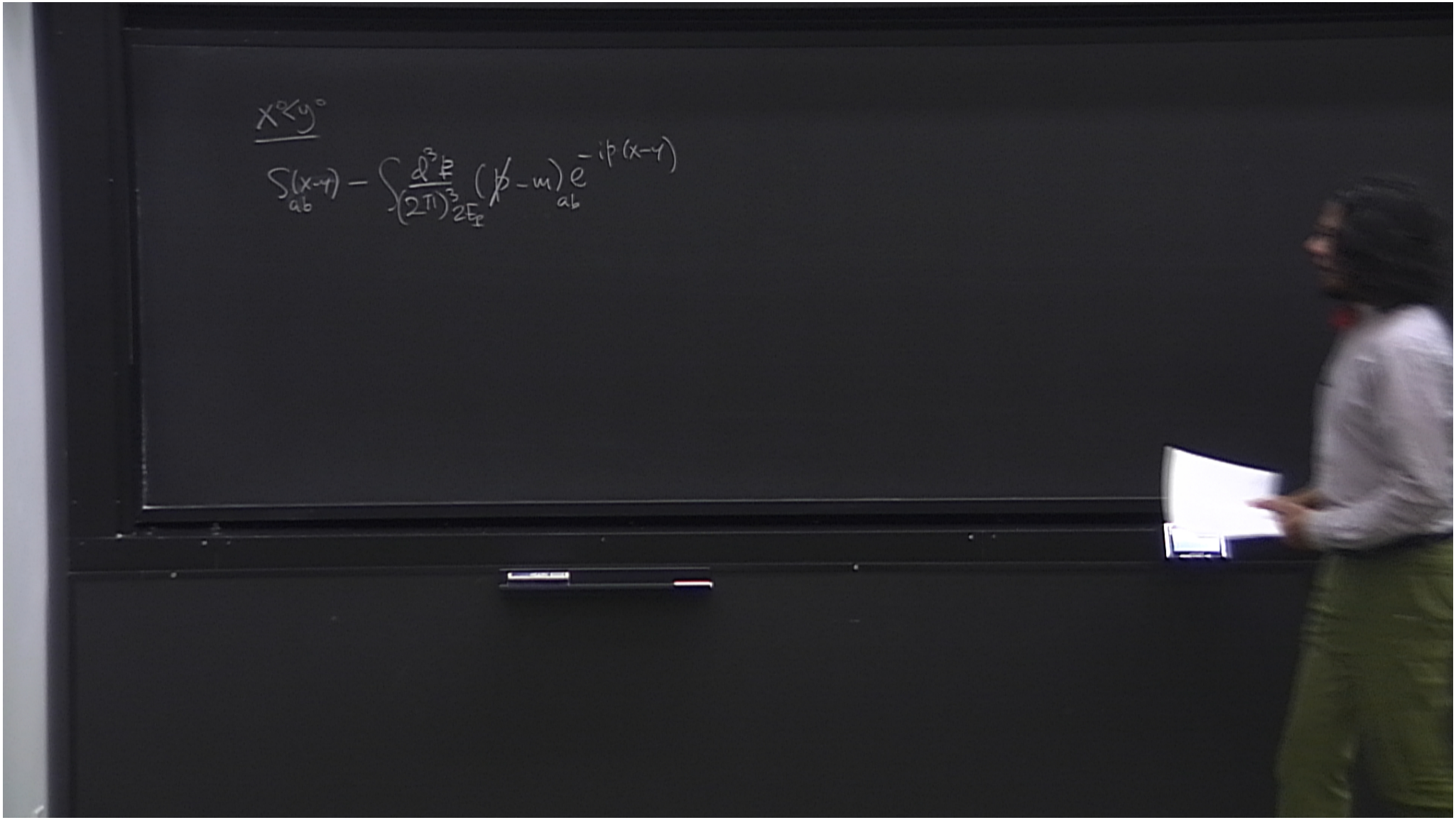
$$\{\psi_a(x), \bar{\psi}_b(y)\} = (i\partial\!\!\!/_x + m)_{ab}\left[D(x-y) - D(y-x)\right]$$

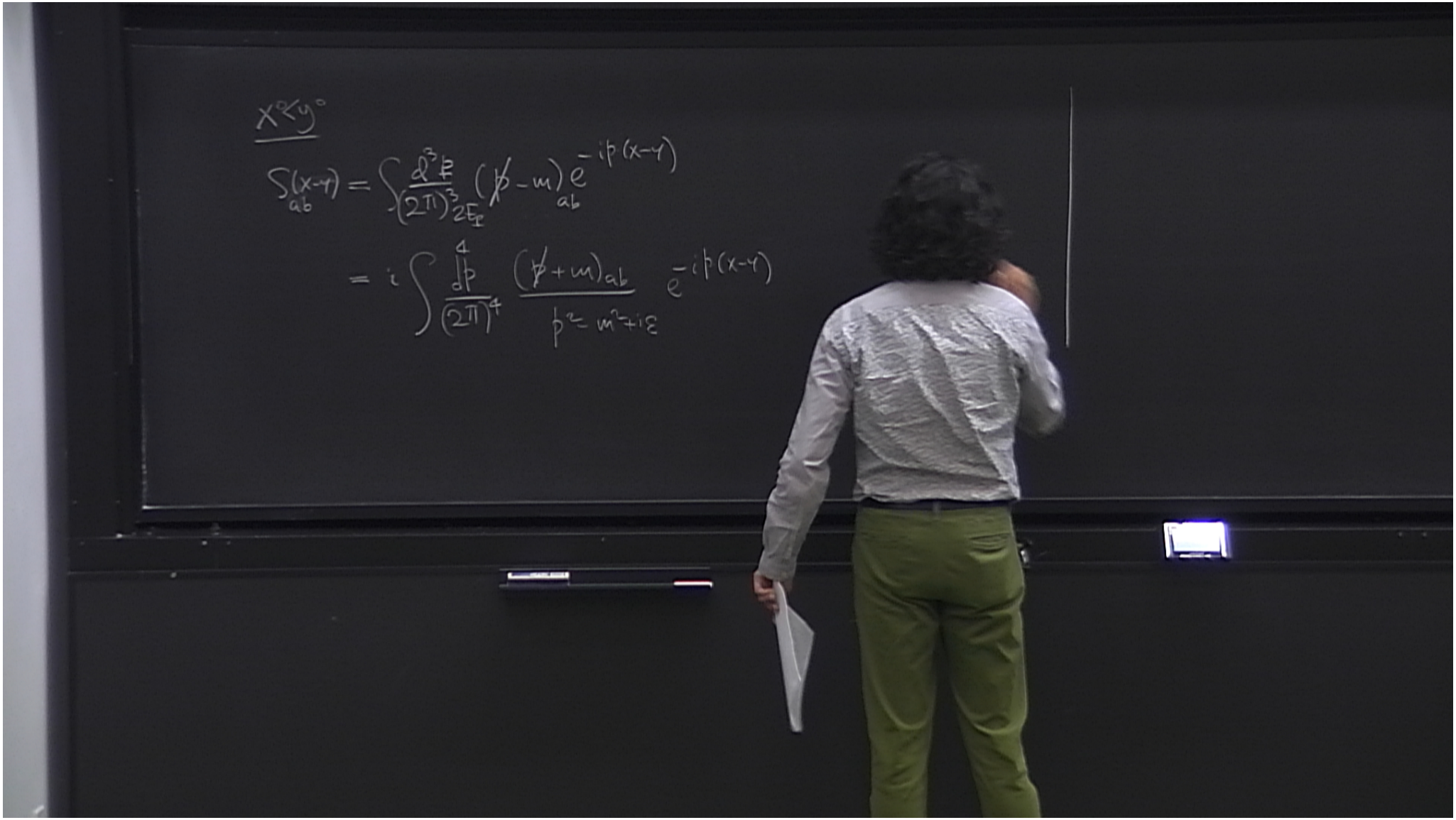
$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip(x-y)}$$

$$S_{ab}(x-y) = \begin{cases} \langle 0|\psi_a(x)\bar{\psi}_b(y)|0\rangle & \text{if } x^0 > y^0 \\ -\langle 0|\bar{\psi}_a(y)\psi_b(x)|0\rangle & \text{if } y^0 > x^0 \end{cases}$$

$$S_{ab}(x-y) = \left[\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle\right]_{ab}$$


$x^0 > y^0$

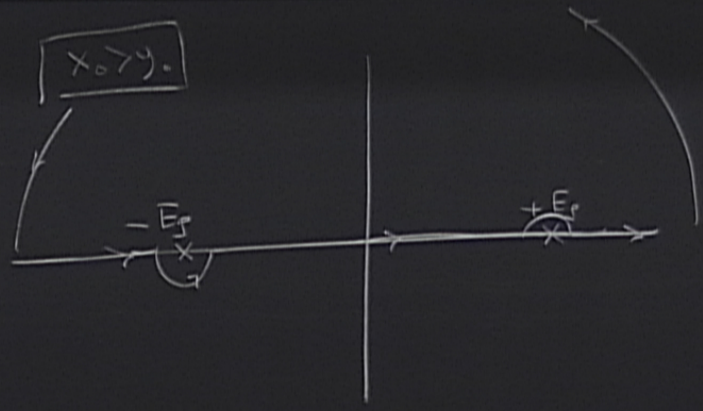

$x^0 < y^0$
$S_{ab}(x-y) - \int \frac{d^3p}{(2\pi)^3 2E_p} (\not{p} - m)_{ab} e^{-ip(x-y)}$

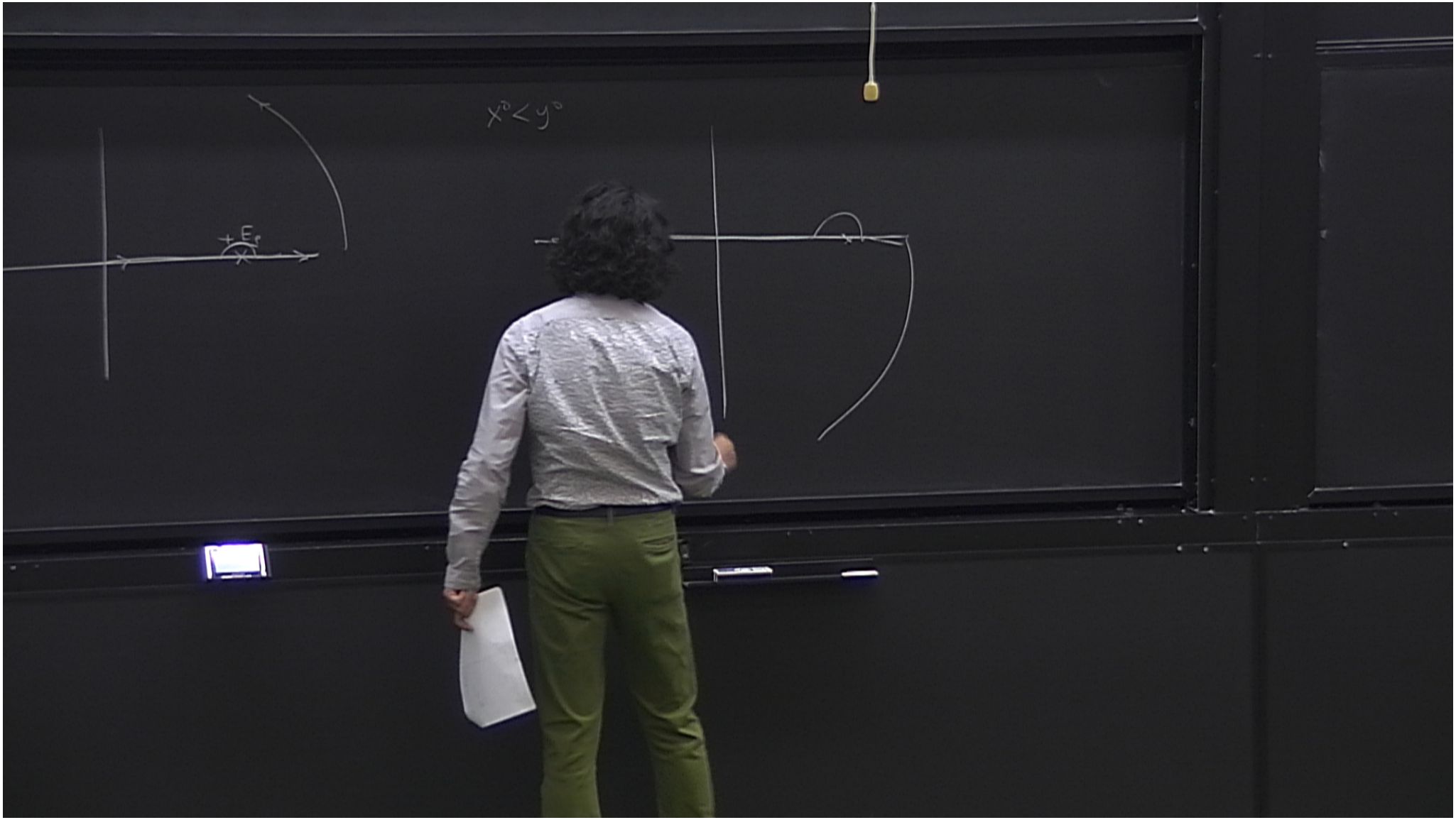

$x^\circ y^\circ$
$S_{ab}(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} (\not{p} - m)_{ab} e^{-ip(x-y)}$
$= i \int \frac{d^4p}{(2\pi)^4} \frac{(\not{p} + m)_{ab}}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$

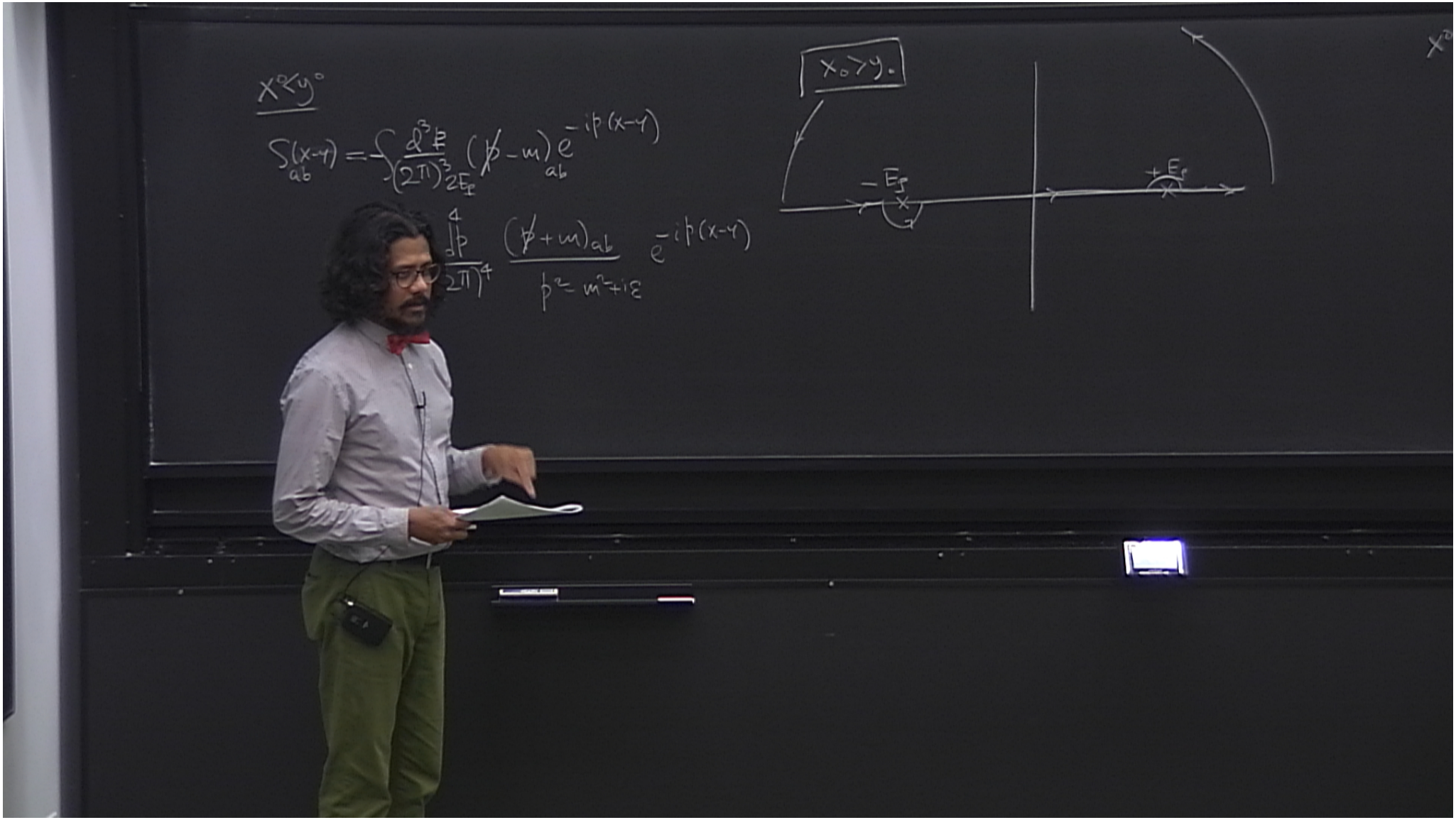
$x^0 < y^0$

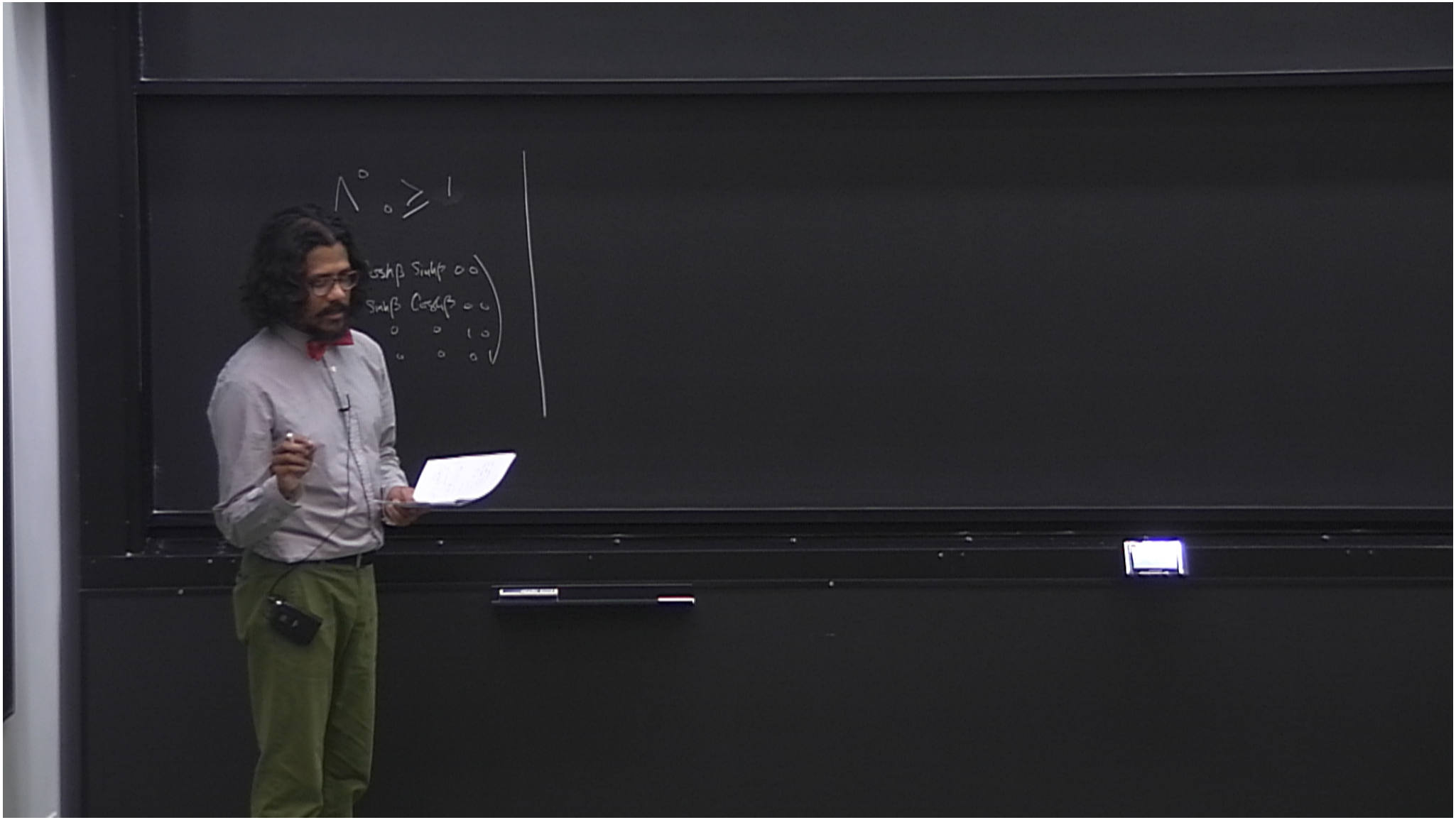
$$S_{ab}(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} (\not{p} - m)_{ab} e^{-ip(x-y)}$$

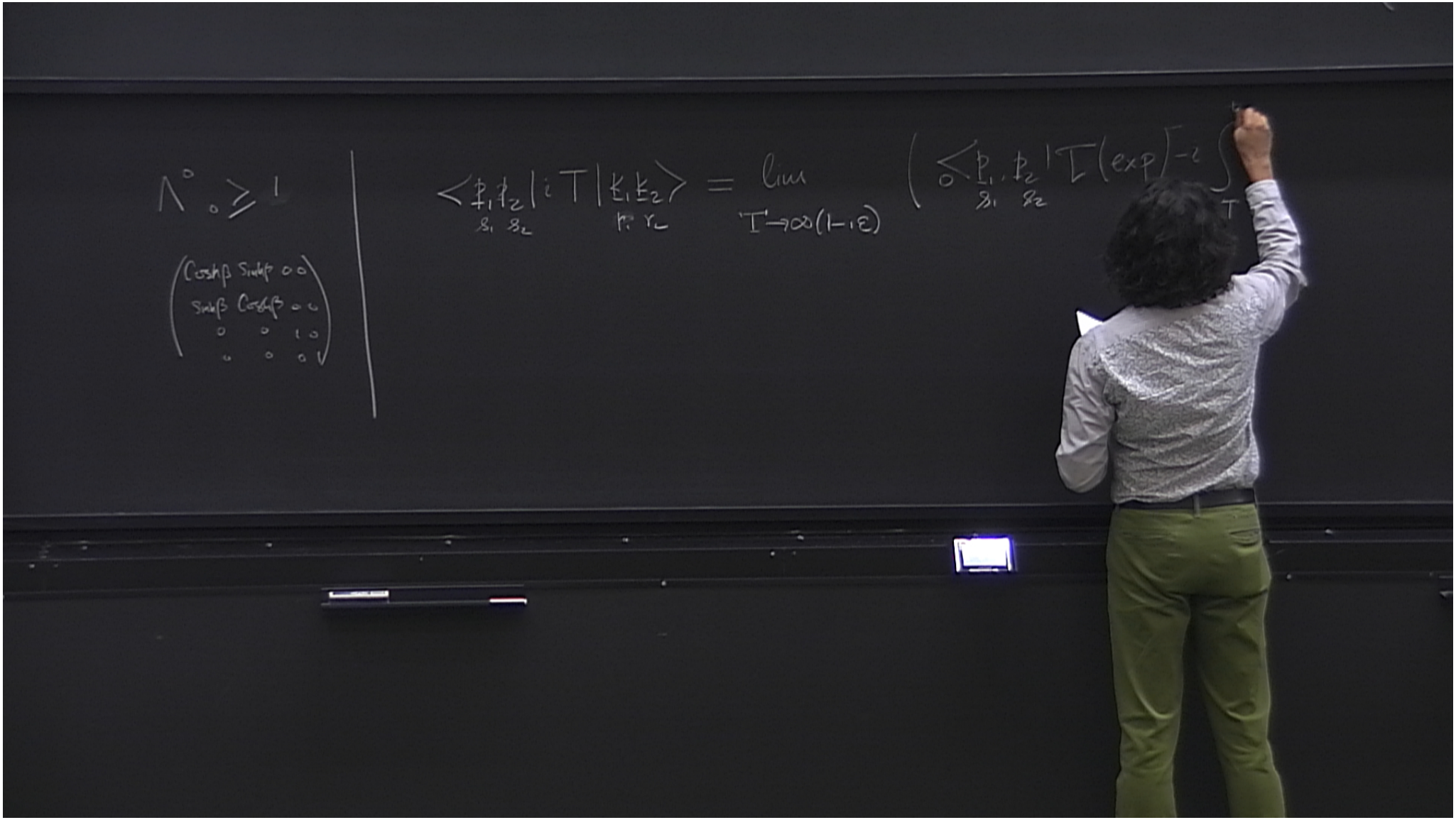
$$= i \int \frac{d^4p}{(2\pi)^4} \frac{(\not{p} + m)_{ab}}{p^2 = m^2 + i\varepsilon} e^{-ip(x-y)}$$

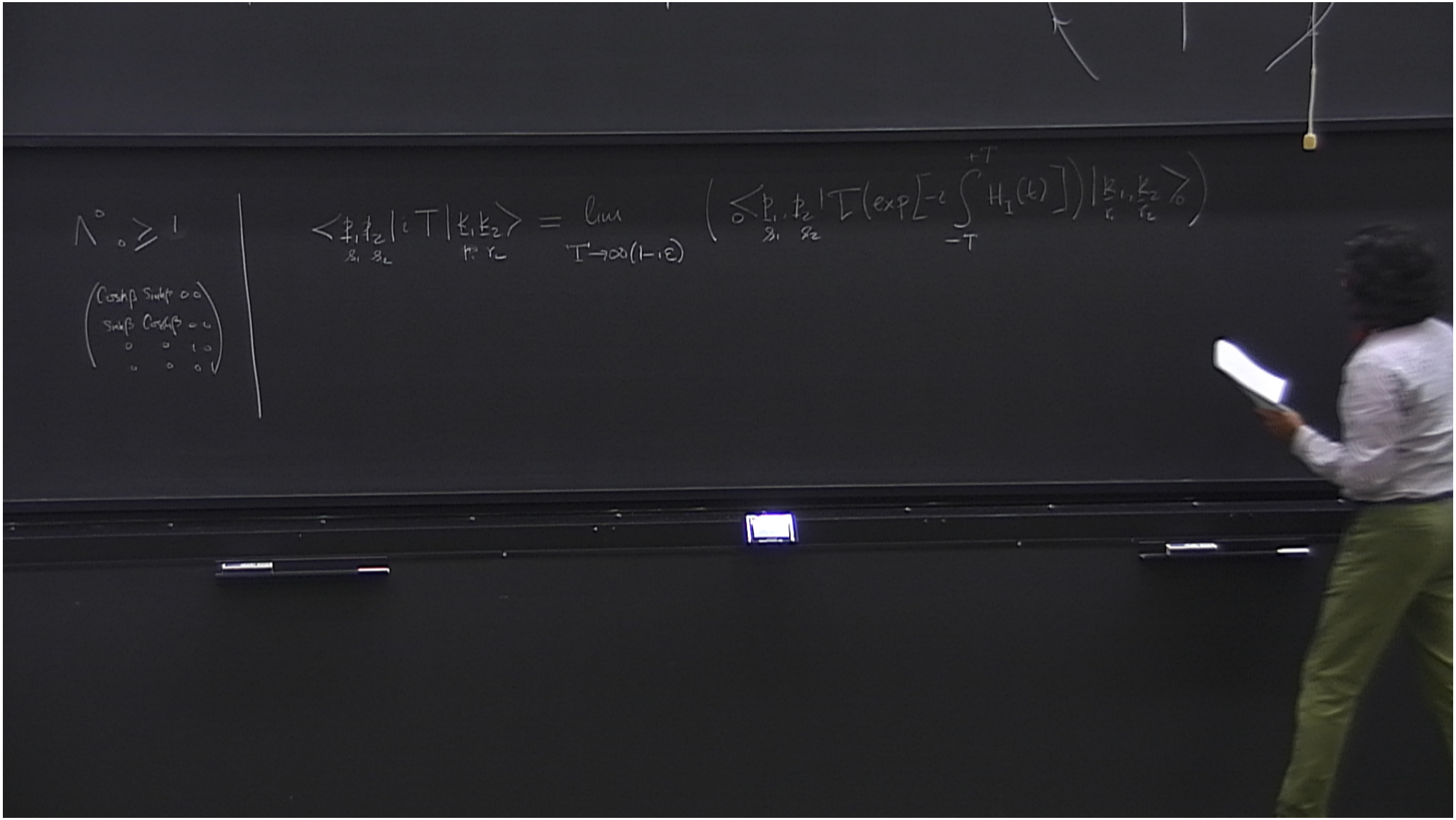


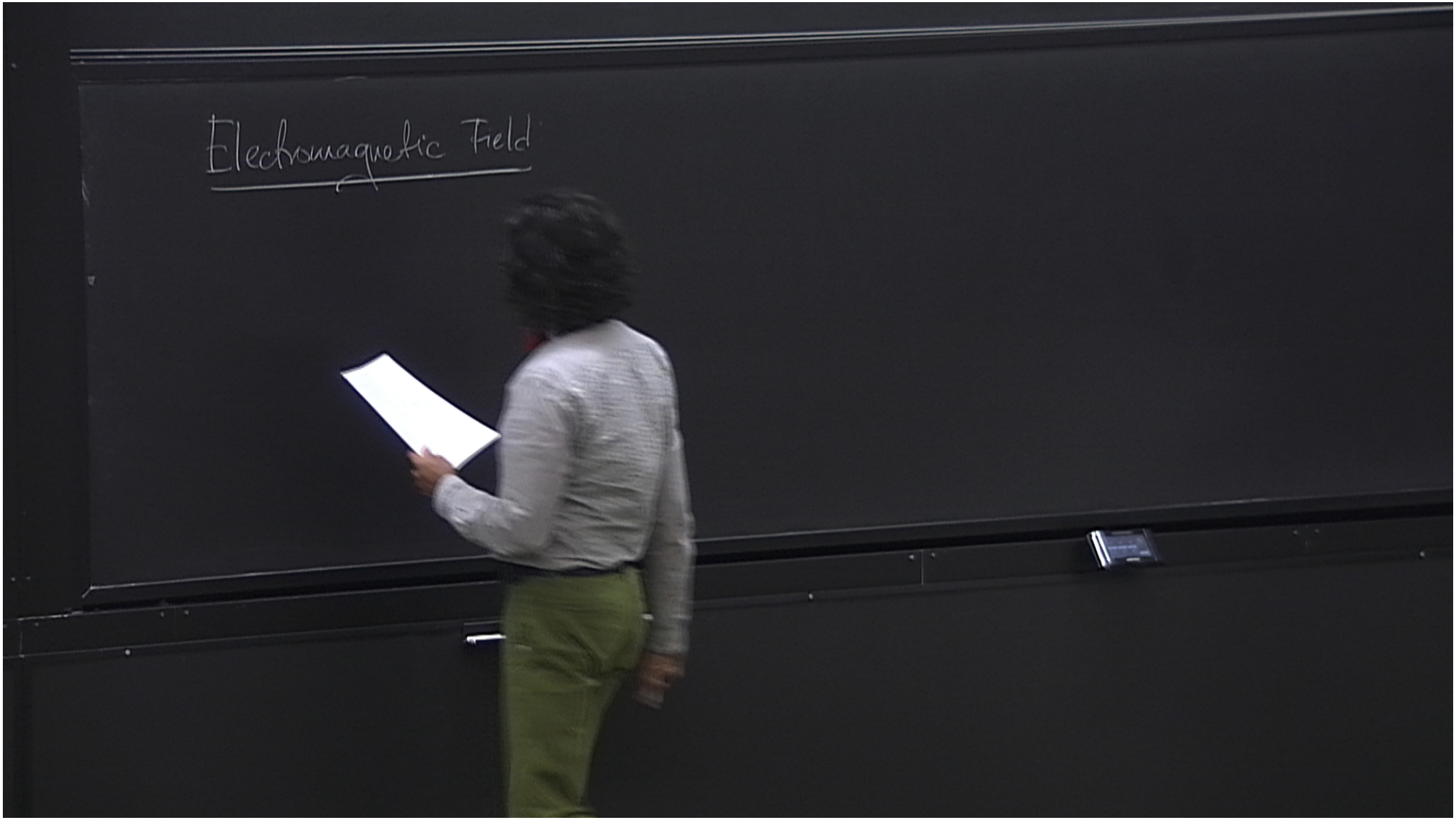

$x^0 < y^0$
$+E_r$

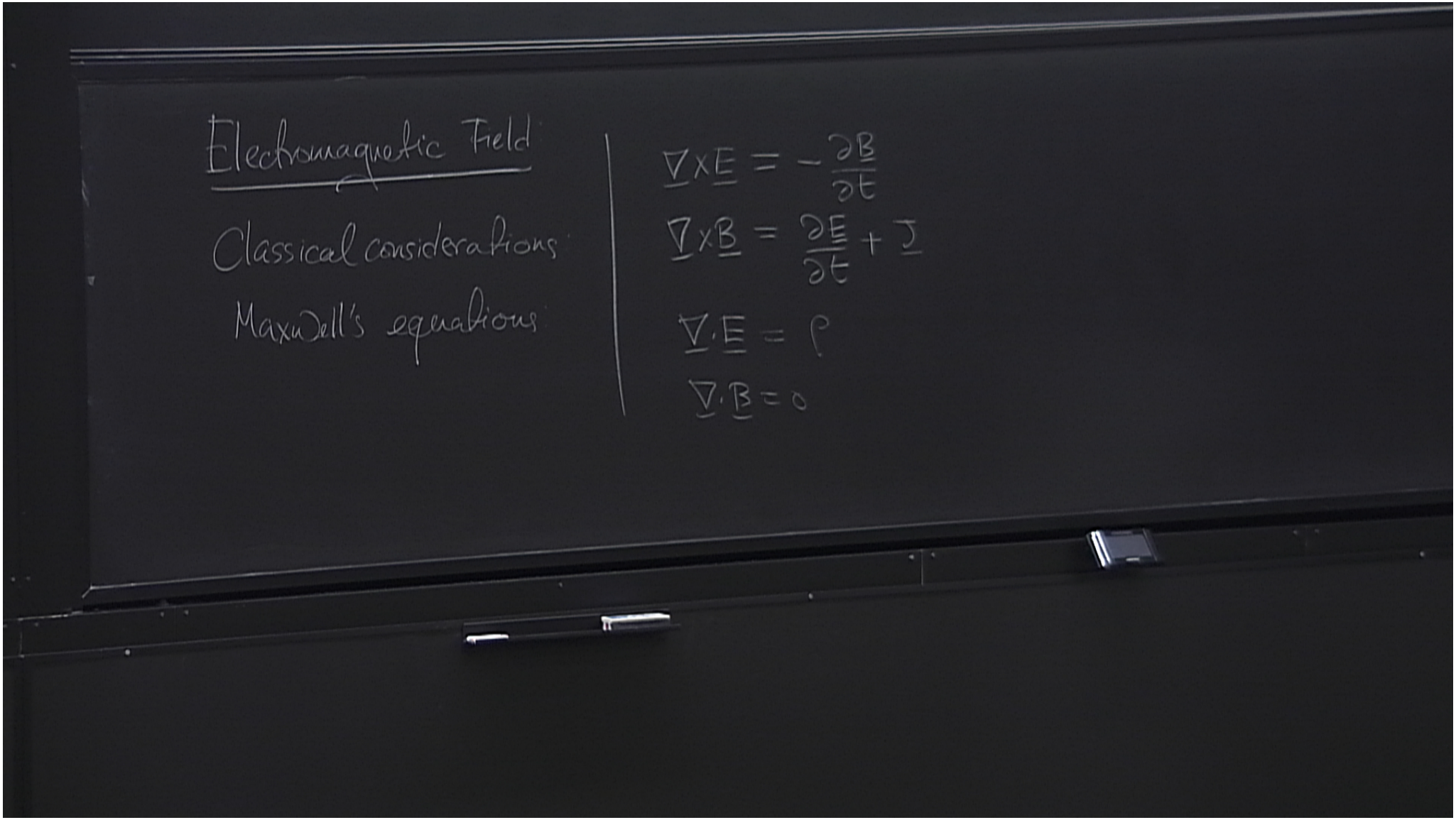

$x^0 < y^0$
$S_{ab}(x-y) = -\int \frac{d^3p}{(2\pi)^3 2E_p} (\not{p} - m)_{ab} e^{-ip(x-y)}$
$= \int \frac{d^4p}{(2\pi)^4} \frac{(\not{p}+m)_{ab}}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$
$x_0 > y_0$
$-E_p$
$+E_p$
x^0

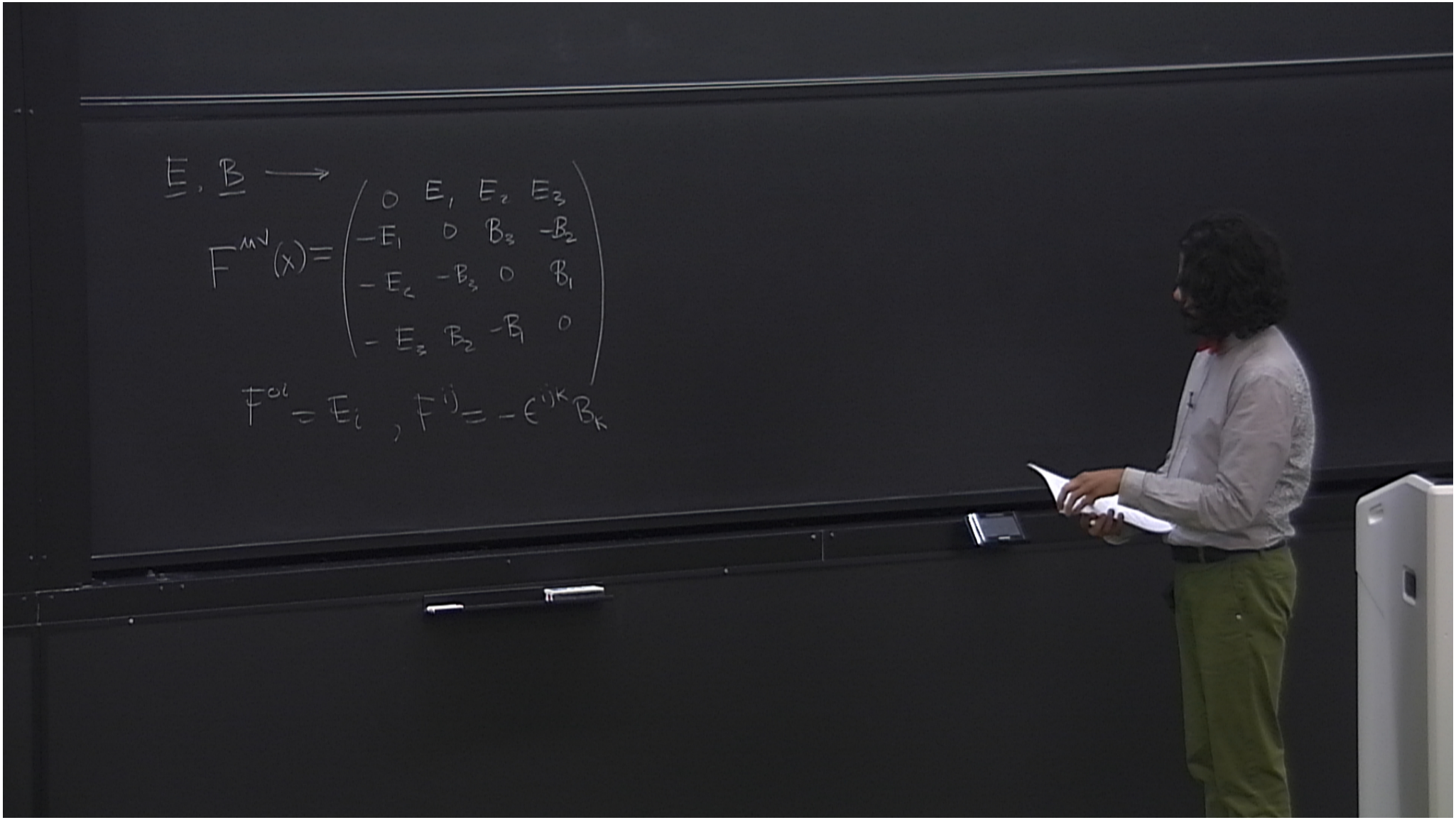


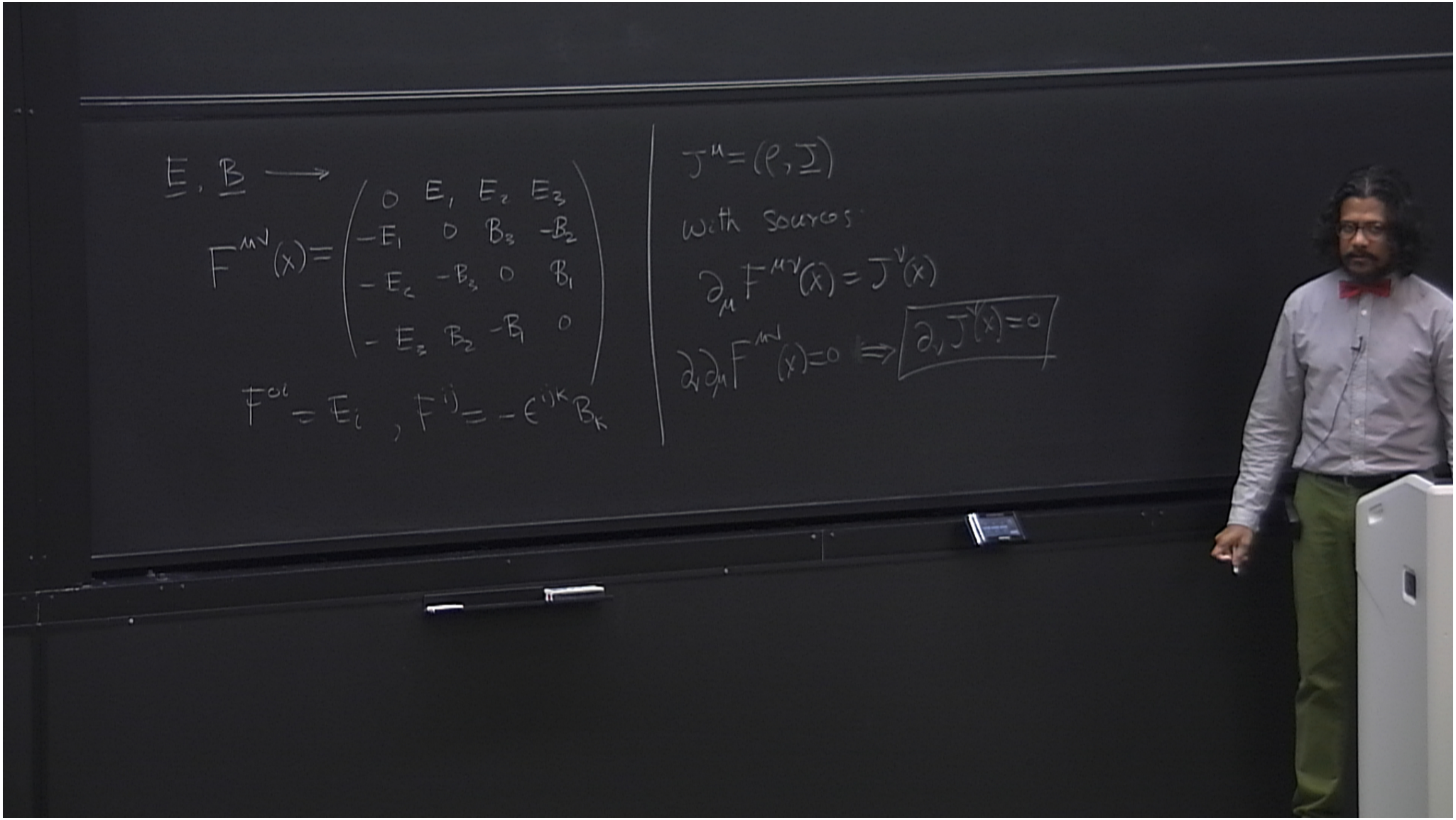

$\Lambda^0{}_0 \geq 1$
$\begin{pmatrix} \cosh\beta & \sinh\beta & 0 & 0 \\ \sinh\beta & \cosh\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$\langle p_1 p_2 | iT | k_1 k_2 \rangle = \lim_{T \to \infty(1-i\varepsilon)}$

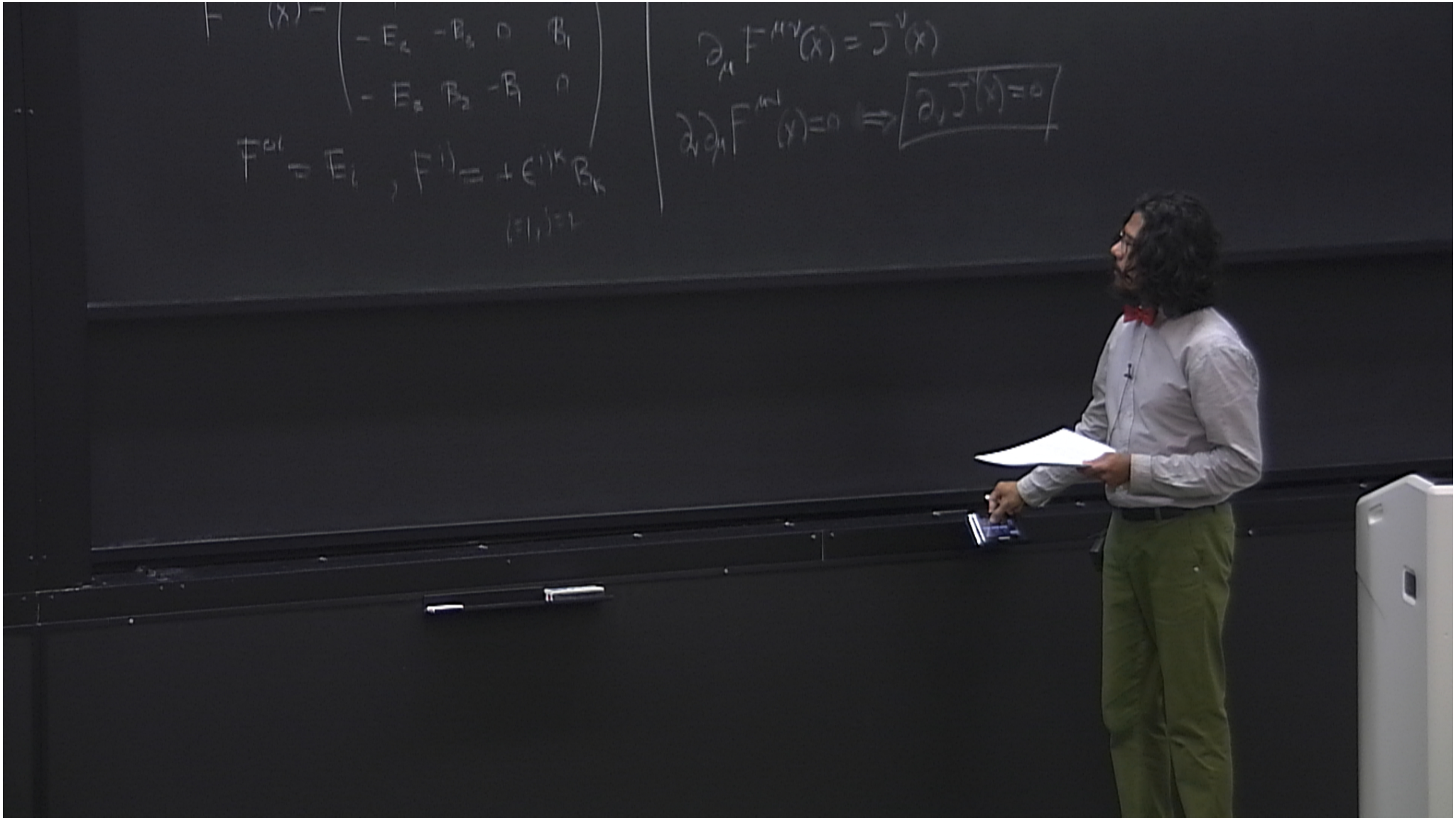

$\Lambda^0{}_0 \geq 1$
$\begin{pmatrix} \cosh\beta & \sinh\beta & 0 & 0 \\ \sinh\beta & \cosh\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$\langle \underset{s_1}{p_1}\, \underset{s_2}{p_2} | iT | \underset{r_1}{k_1}\, \underset{r_2}{k_2} \rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(\langle 0 | \underset{s_1}{p_1}\, \underset{s_2}{p_2} | T\left(\exp\left[-i\int_{-T}^{+T} H_I(t)\right]\right) | \underset{r_1}{k_1}\, \underset{r_2}{k_2} \rangle_0 \right)$

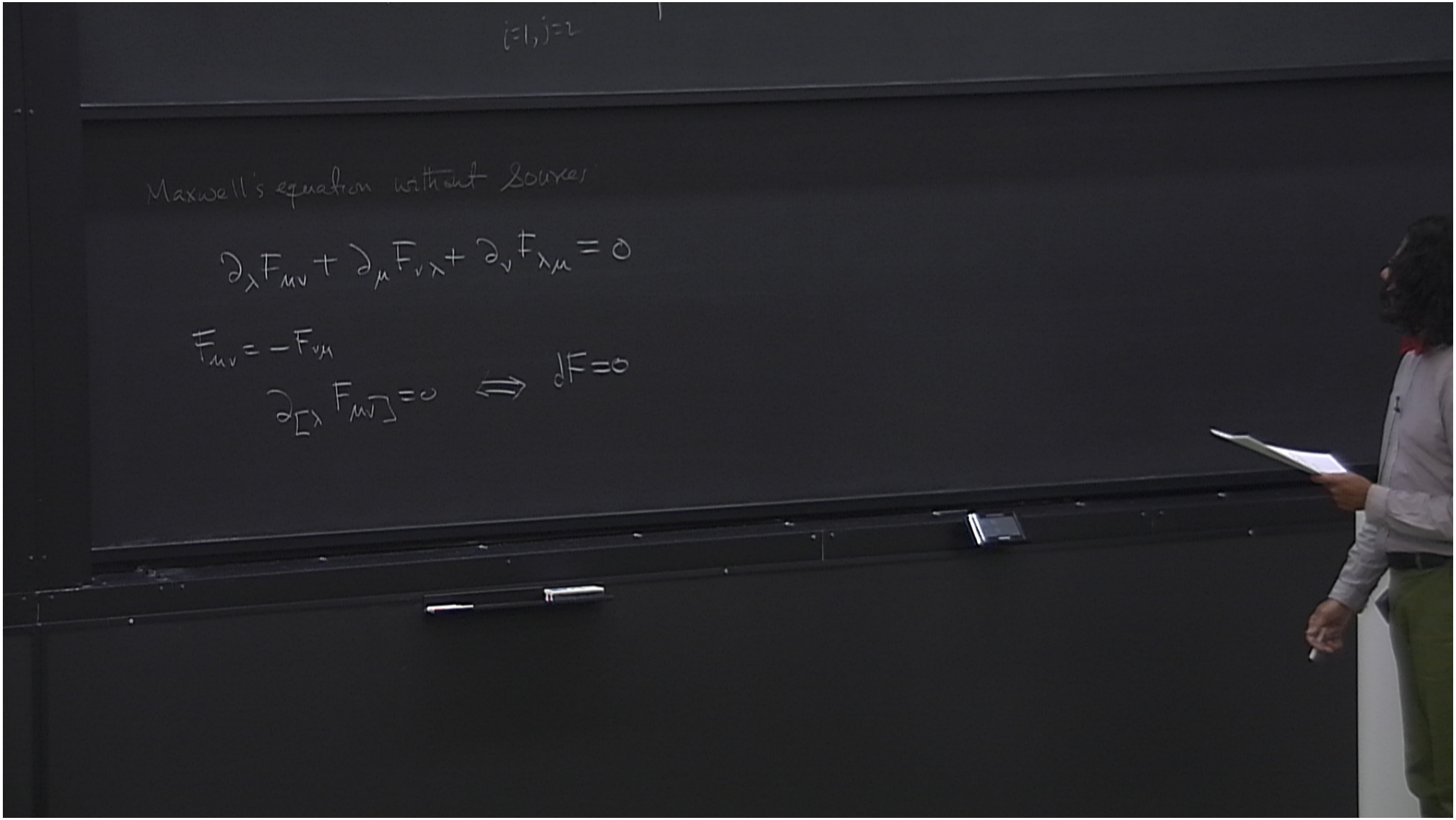

Electromagnetic Field

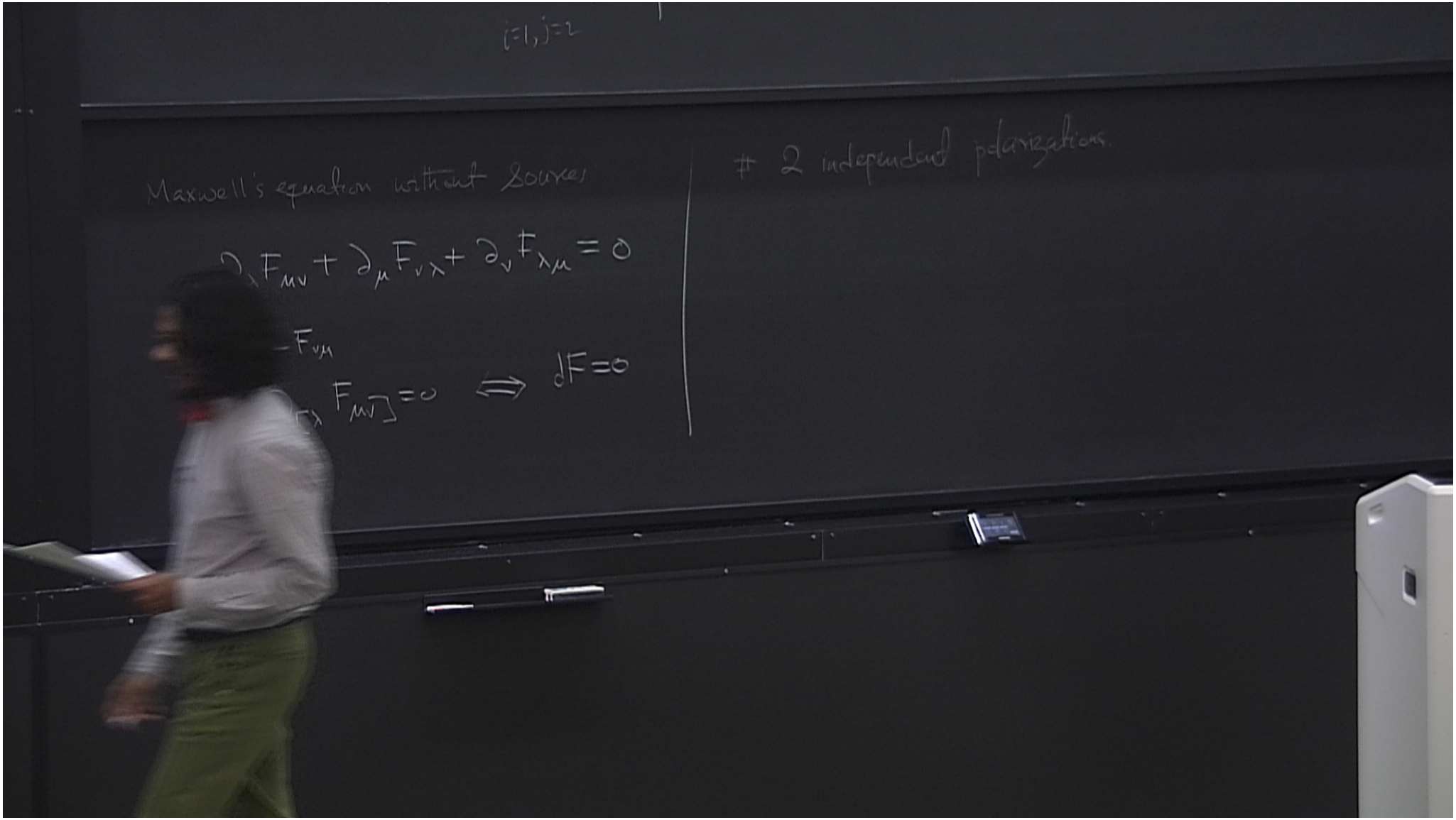

Electromagnetic Field
Classical considerations
Maxwell's equations
$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$
$\nabla \times \underline{B} = \frac{\partial \underline{E}}{\partial t} + \underline{J}$
$\nabla \cdot \underline{E} = \rho$
$\nabla \cdot \underline{B} = 0$


$\underline{E}, \underline{B} \longrightarrow$
$F^{\mu\nu}(x) = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$
$F^{0i} = E_i \;,\; F^{ij} = -\epsilon^{ijk} B_k$


$\underline{E}, \underline{B} \longrightarrow$
$F^{\mu\nu}(x) = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$
$F^{0i} = E_i \,, \quad F^{ij} = -\epsilon^{ijk} B_k$
$J^\mu = (\rho, \underline{J})$
with sources
$\partial_\mu F^{\mu\nu}(x) = J^\nu(x)$
$\partial_\nu \partial_\mu F^{\mu\nu}(x) = 0 \Rightarrow \boxed{\partial_\nu J^\nu(x) = 0}$


$\partial_\mu F^{\mu\nu}(x) = J^\nu(x)$
$\partial_\nu \partial_\mu F^{\mu\nu}(x) = 0 \Rightarrow \partial_\nu J^\nu(x) = 0$


i=1, j=2
Maxwell's equation without Sources
$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$
$F_{\mu\nu} = -F_{\nu\mu}$
$\partial_{[\lambda} F_{\mu\nu]} = 0 \iff dF = 0$


i=1, j=2
Maxwell's equation without sources
$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$
$-F_{\nu\mu}$
$F_{\mu\nu}] = 0 \iff dF = 0$
2 independent polarizations.

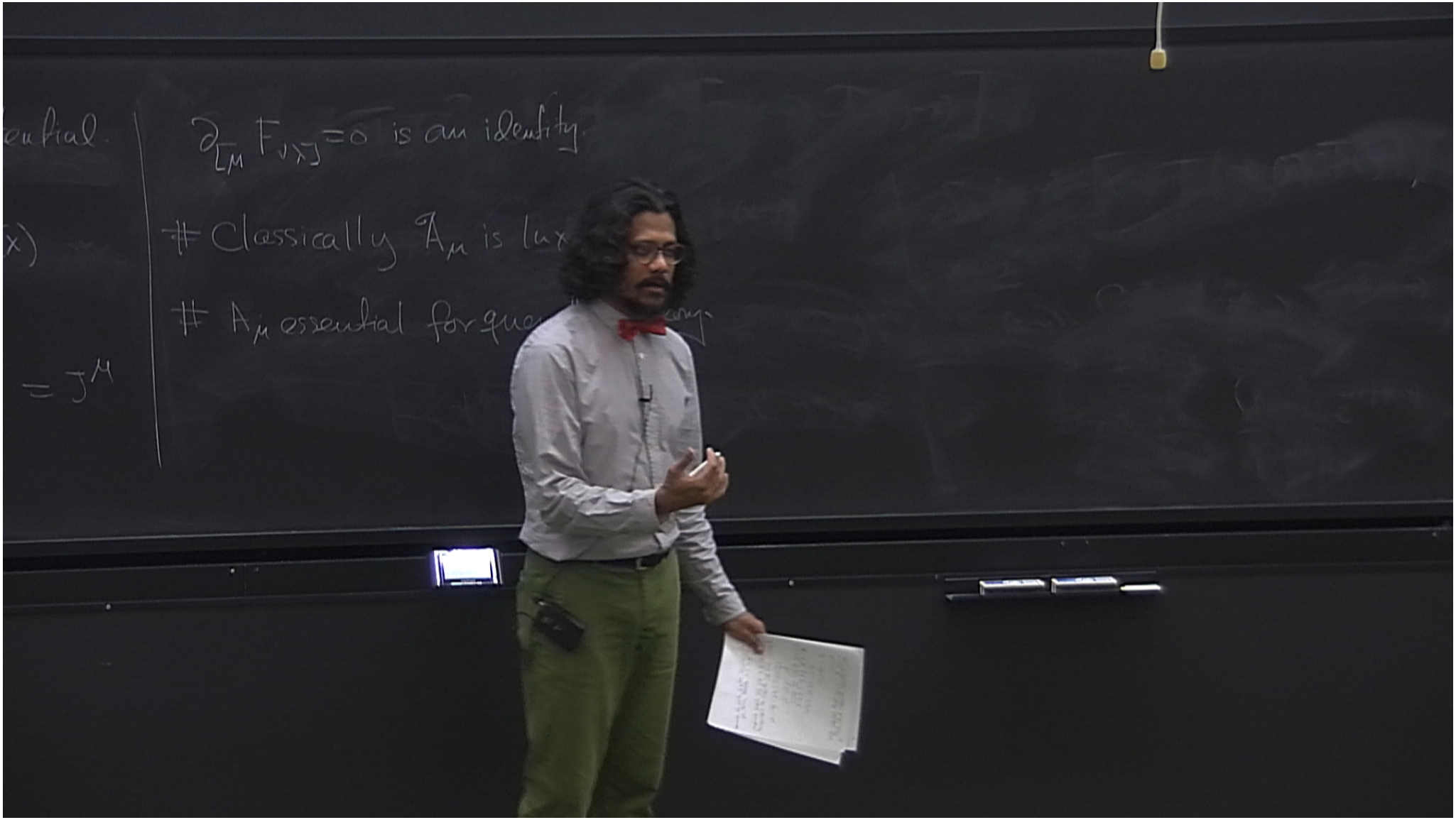
Introduce $A_\mu(x) \rightarrow$ 4 vector potential

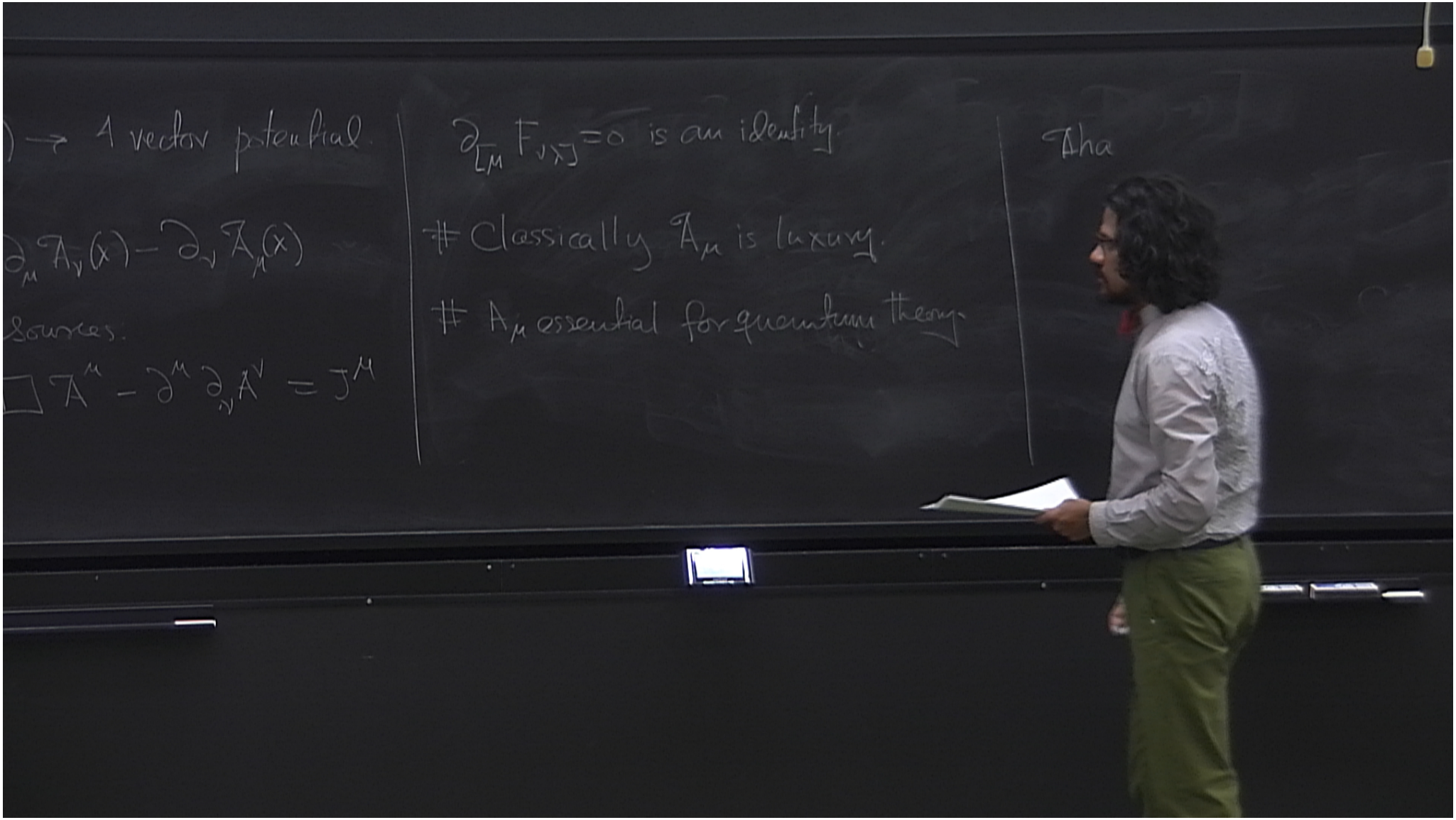
$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

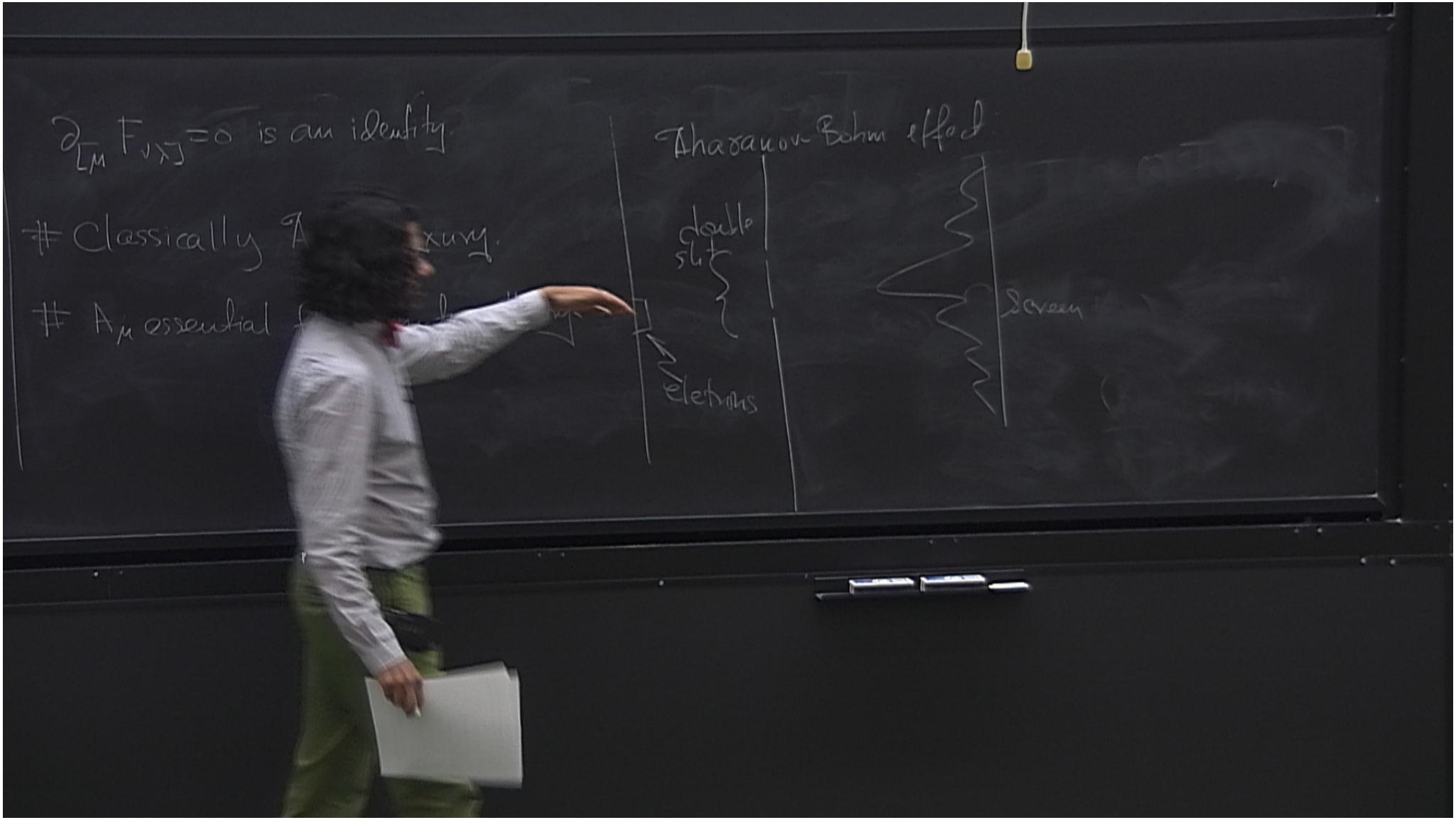
Maxwell with Sources.

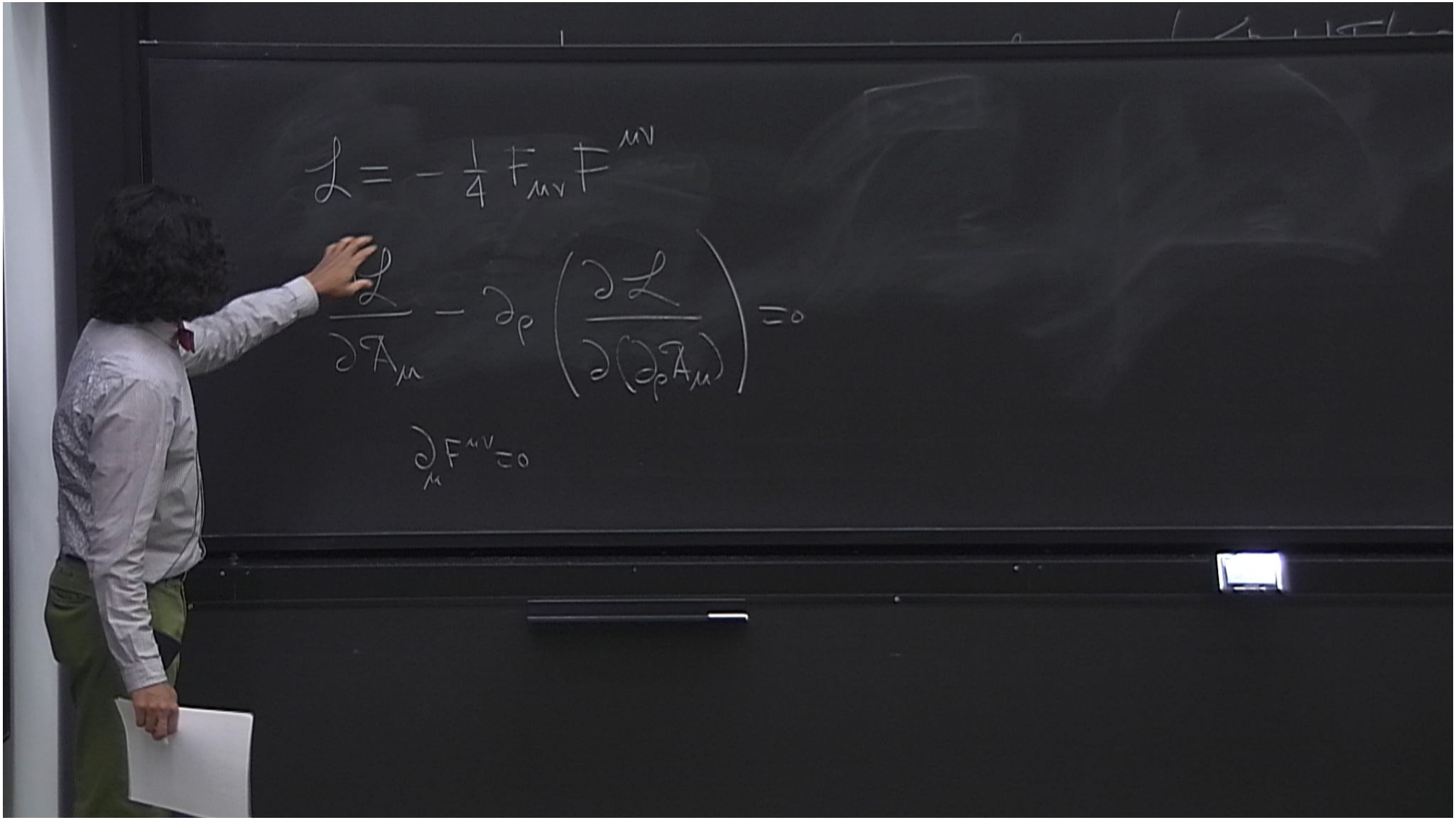
$$\Box A^\mu - \partial^\mu \partial_\nu A^\nu = J^\mu$$

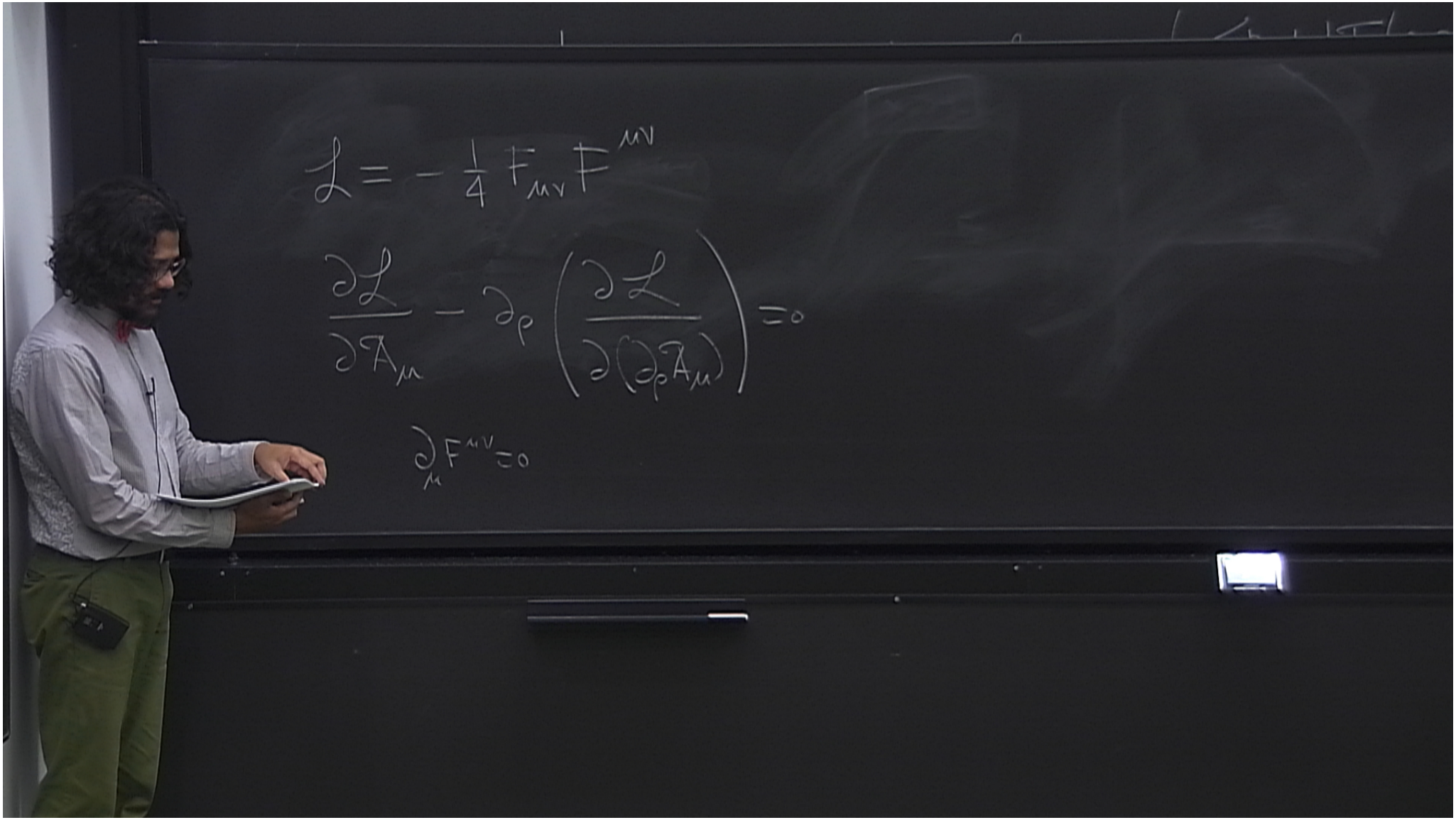
$$\Box = \partial_\nu \partial^\nu$$

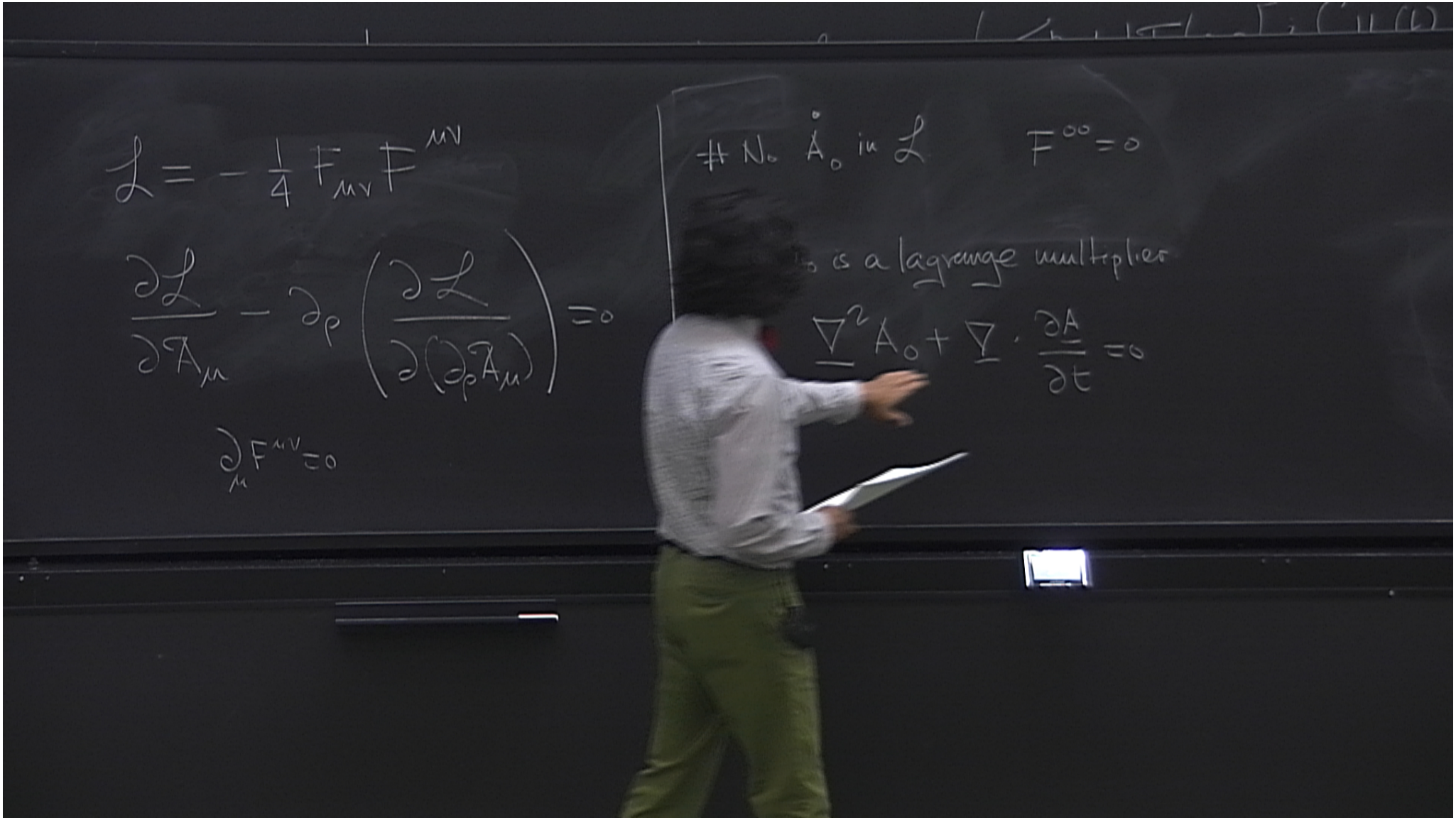

$\partial_{[\mu} F_{\nu\lambda]} = 0$ is an identity.
Classically A_μ is lux
A_μ essential for

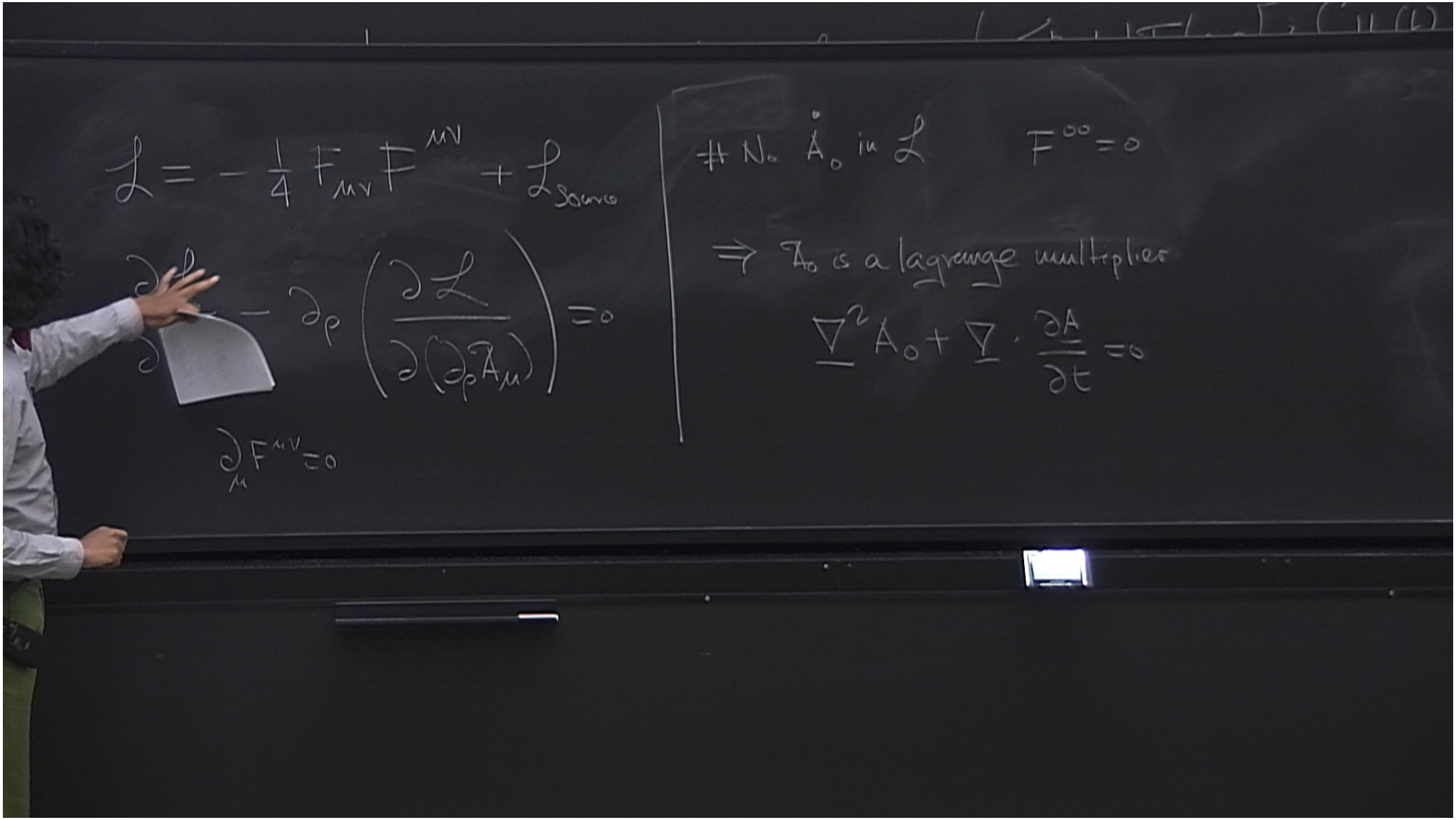

→ A vector potential.
$\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$
Sources:
$\Box A^\mu - \partial^\mu \partial_\nu A^\nu = J^\mu$
$\partial_{[\mu} F_{\nu\lambda]} = 0$ is an identity.
Classically A_μ is luxury.
A_μ essential for quantum theory.
Tha

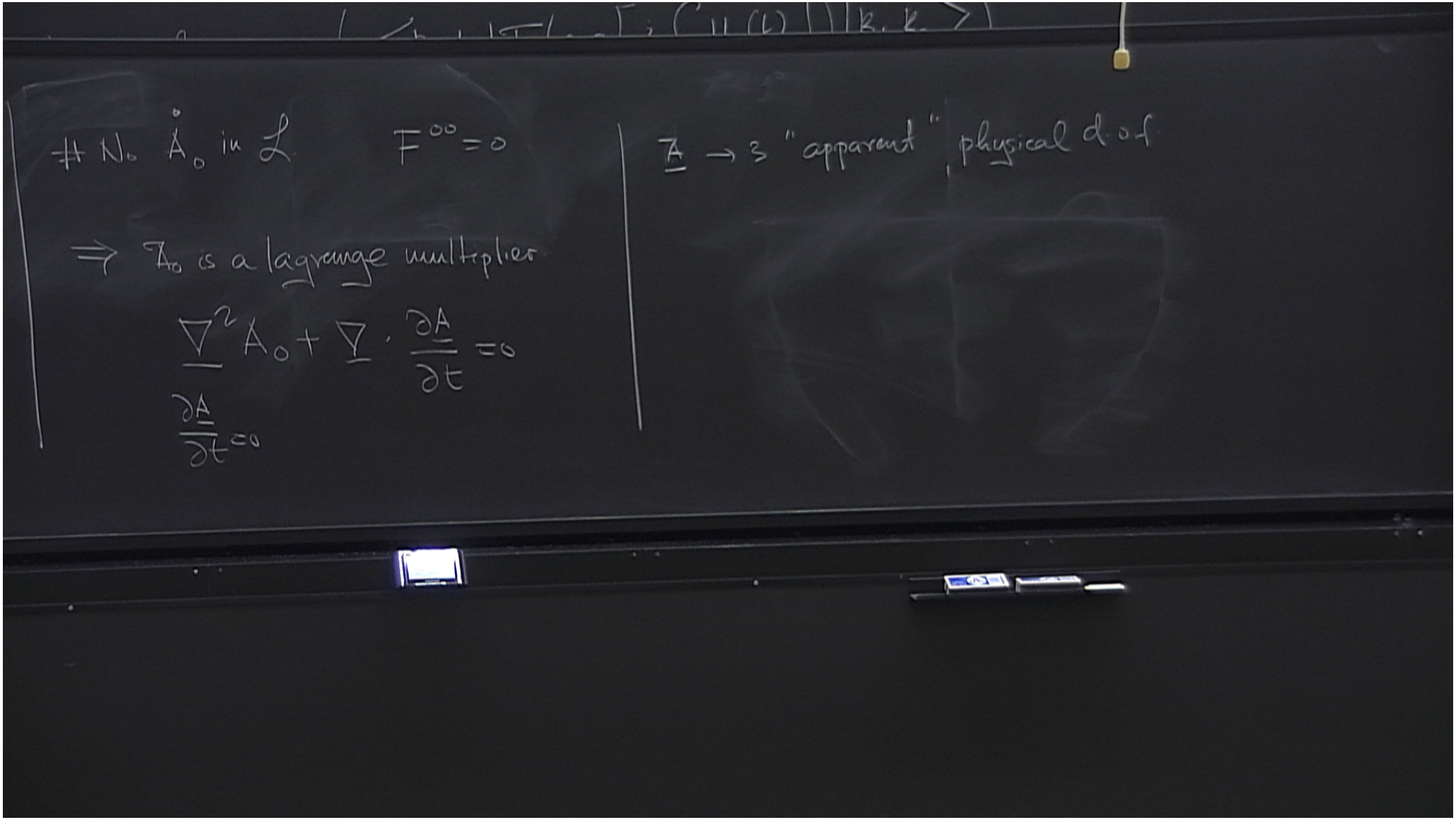

$\partial_{[\mu} F_{\nu\lambda]} = 0$ is an identity
Classically
A_μ essential
Aharanov-Bohm effect
double slit
electrons
screen

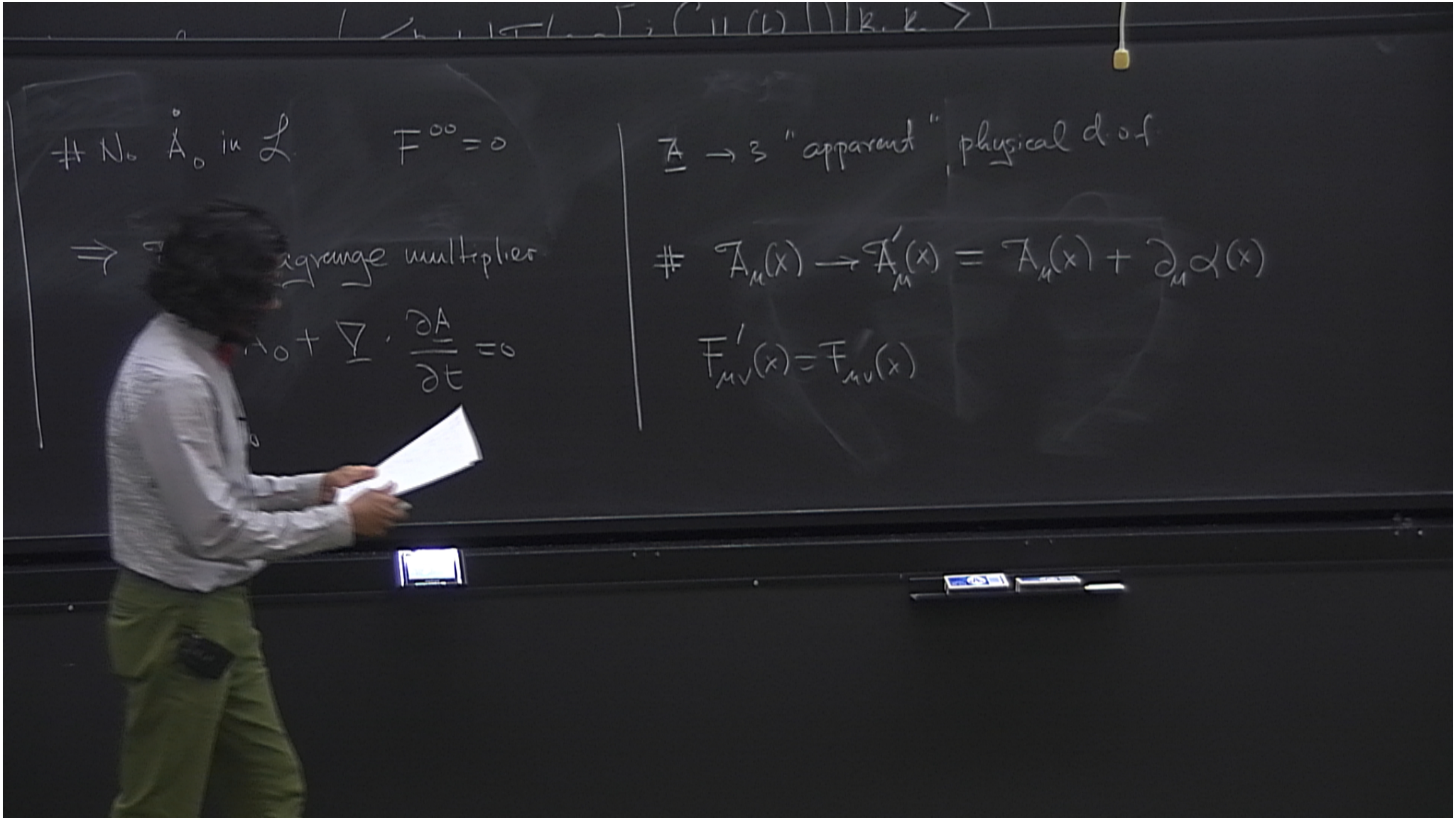

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\frac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\rho \left(\frac{\partial \mathcal{L}}{\partial (\partial_\rho A_\mu)} \right) = 0$
$\partial_\mu F^{\mu\nu} = 0$

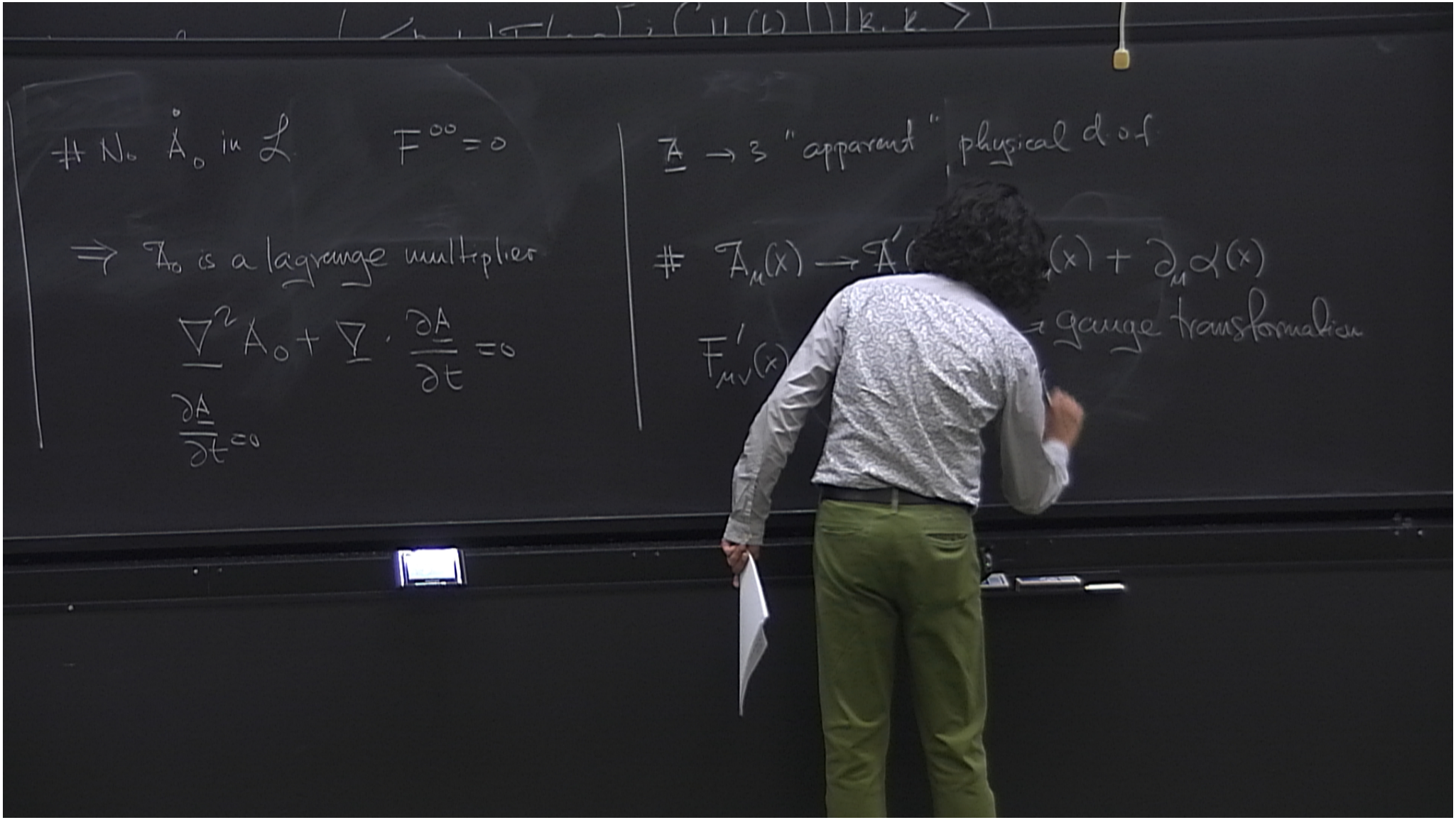

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\frac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\rho \left(\frac{\partial \mathcal{L}}{\partial (\partial_\rho A_\mu)} \right) = 0$
$\partial_\mu F^{\mu\nu} = 0$

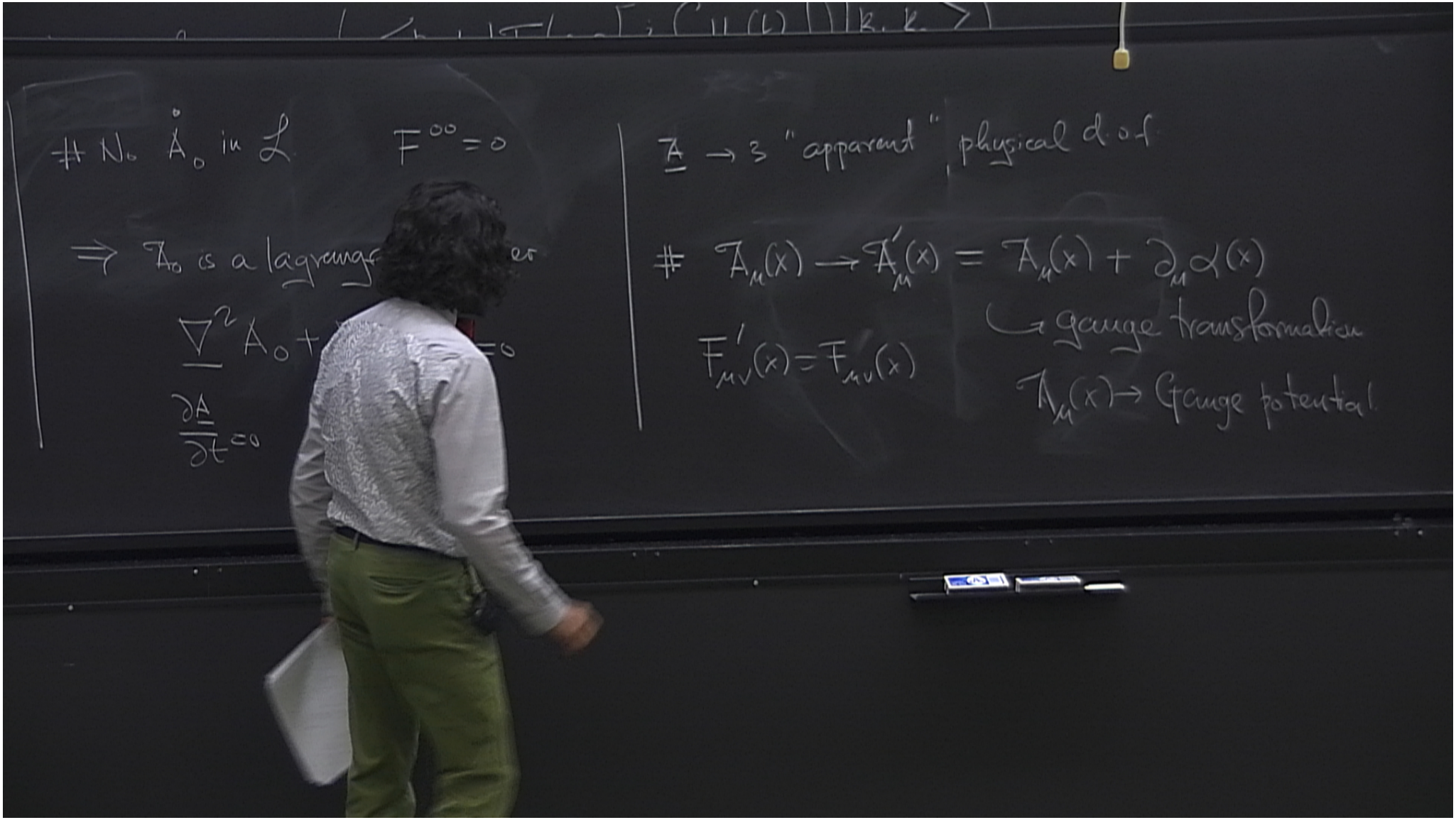

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu} - \partial_\rho \left(\frac{\partial \mathcal{L}}{\partial (\partial_\rho A_\nu)} \right) = 0$
$\partial_\mu F^{\mu\nu} = 0$
No. $\dot{A}_0$ in $\mathcal{L}$
$F^{00} = 0$
A_0 is a lagrange multiplier
$\nabla^2 A_0 + \nabla \cdot \frac{\partial \vec{A}}{\partial t} = 0$

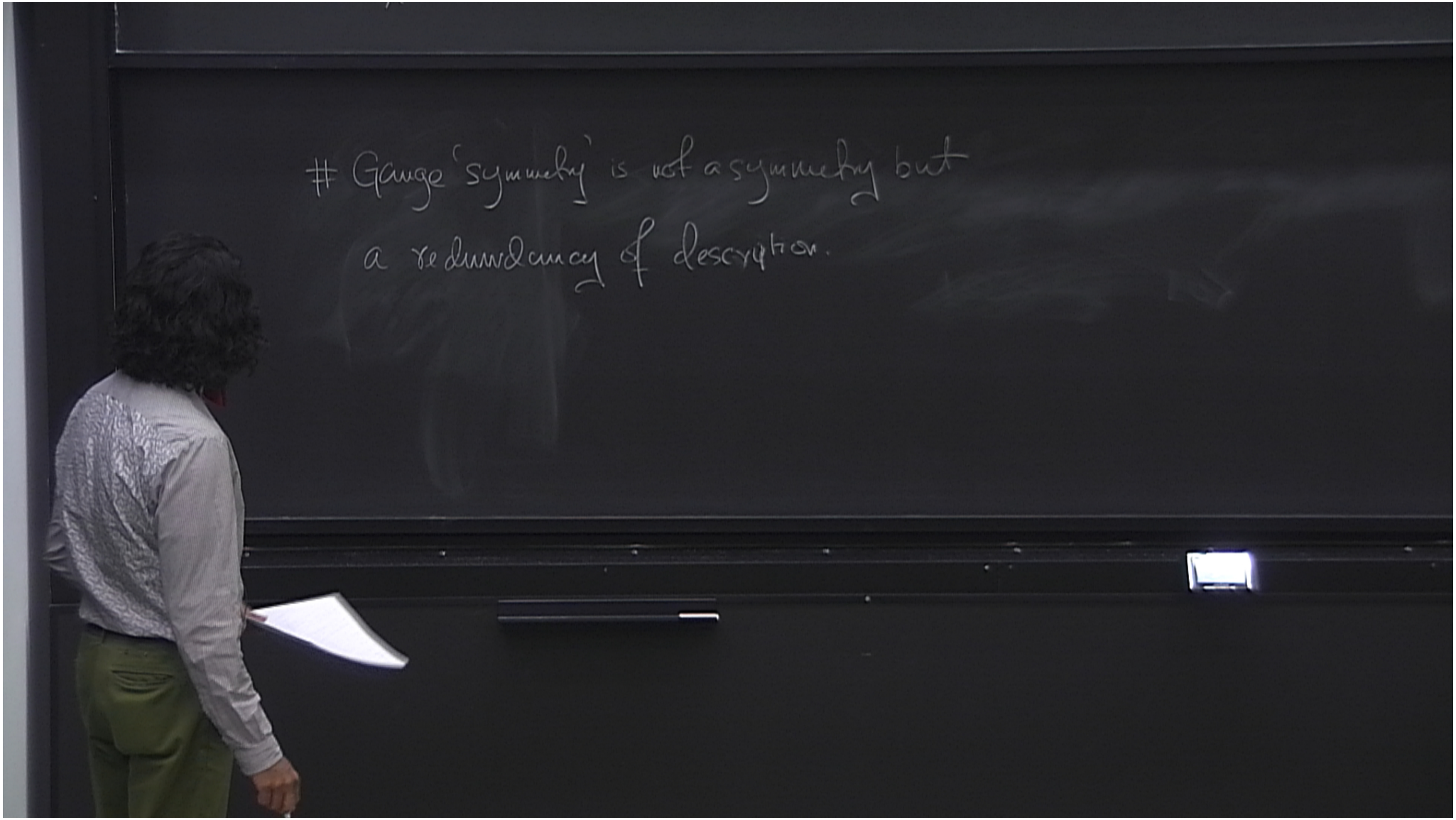

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{source}$
$\partial_\rho \left(\frac{\partial \mathcal{L}}{\partial(\partial_\rho A_\mu)} \right) = 0$
$\partial_\mu F^{\mu\nu} = 0$
No. $\dot{A}_0$ in $\mathcal{L}$
$F^{00} = 0$
$\Rightarrow A_0$ is a lagrange multiplier
$\nabla^2 A_0 + \nabla \cdot \frac{\partial \vec{A}}{\partial t} = 0$

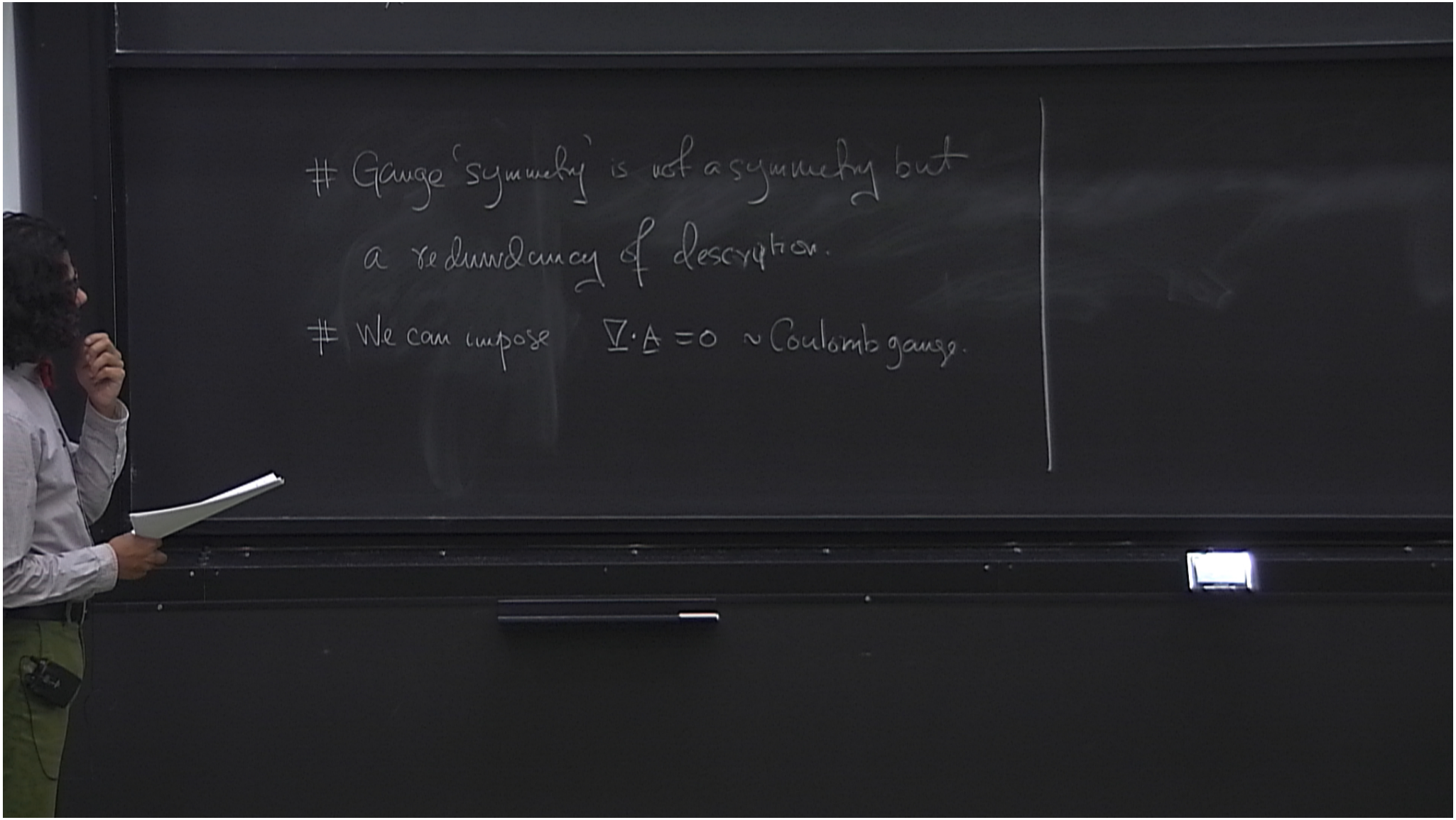

$\#$ No. $\dot{A}_0$ in $\mathcal{L}$ $F^{00}=0$
$\Rightarrow$ A_0 is a lagrange multiplier
$\nabla^2 A_0 + \nabla \cdot \frac{\partial \vec{A}}{\partial t} = 0$
$\frac{\partial \vec{A}}{\partial t} = 0$
$\vec{A} \rightarrow$ 3 "apparent" physical d.o.f

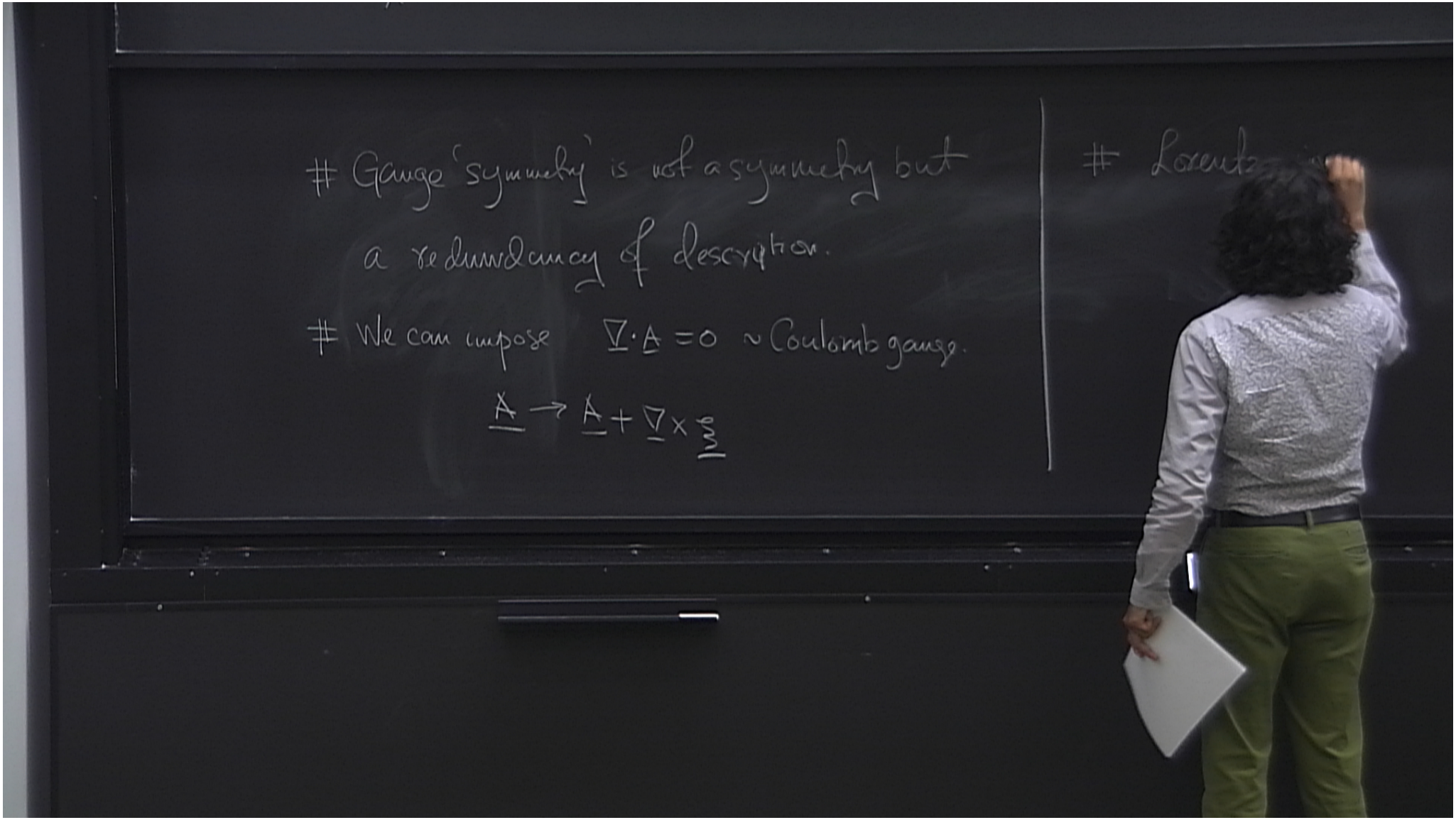

$\#$ No. $\dot{A}_0$ in $\mathcal{L}$ $F^{00}=0$
$\vec{A} \to$ 3 "apparent" physical d.o.f
$\#$ $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x)$
$F'_{\mu\nu}(x) = F_{\mu\nu}(x)$

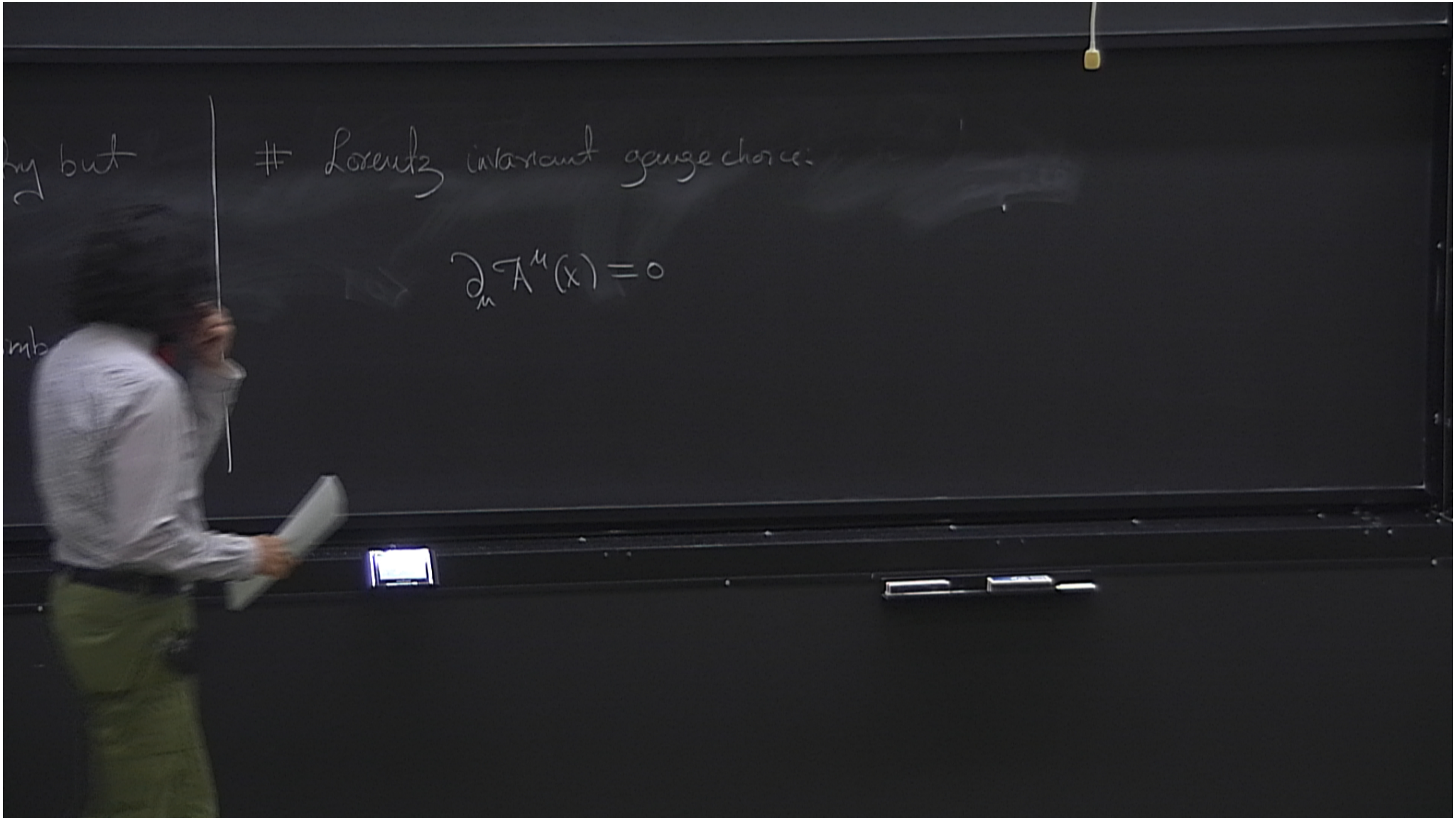

$\# N_o \dot{A}_0$ in $\mathcal{L}$ $F^{00} = 0$
$\Rightarrow A_0$ is a lagrange multiplier
$\nabla^2 A_0 + \nabla \cdot \frac{\partial \underline{A}}{\partial t} = 0$
$\frac{\partial \underline{A}}{\partial t} = 0$
$\underline{A} \rightarrow 3$ "apparent" physical d.o.f
$\# A_\mu(x) \rightarrow A'$ $(x) + \partial_\mu \alpha(x)$
$F'_{\mu\nu}(x)$
gauge transformation

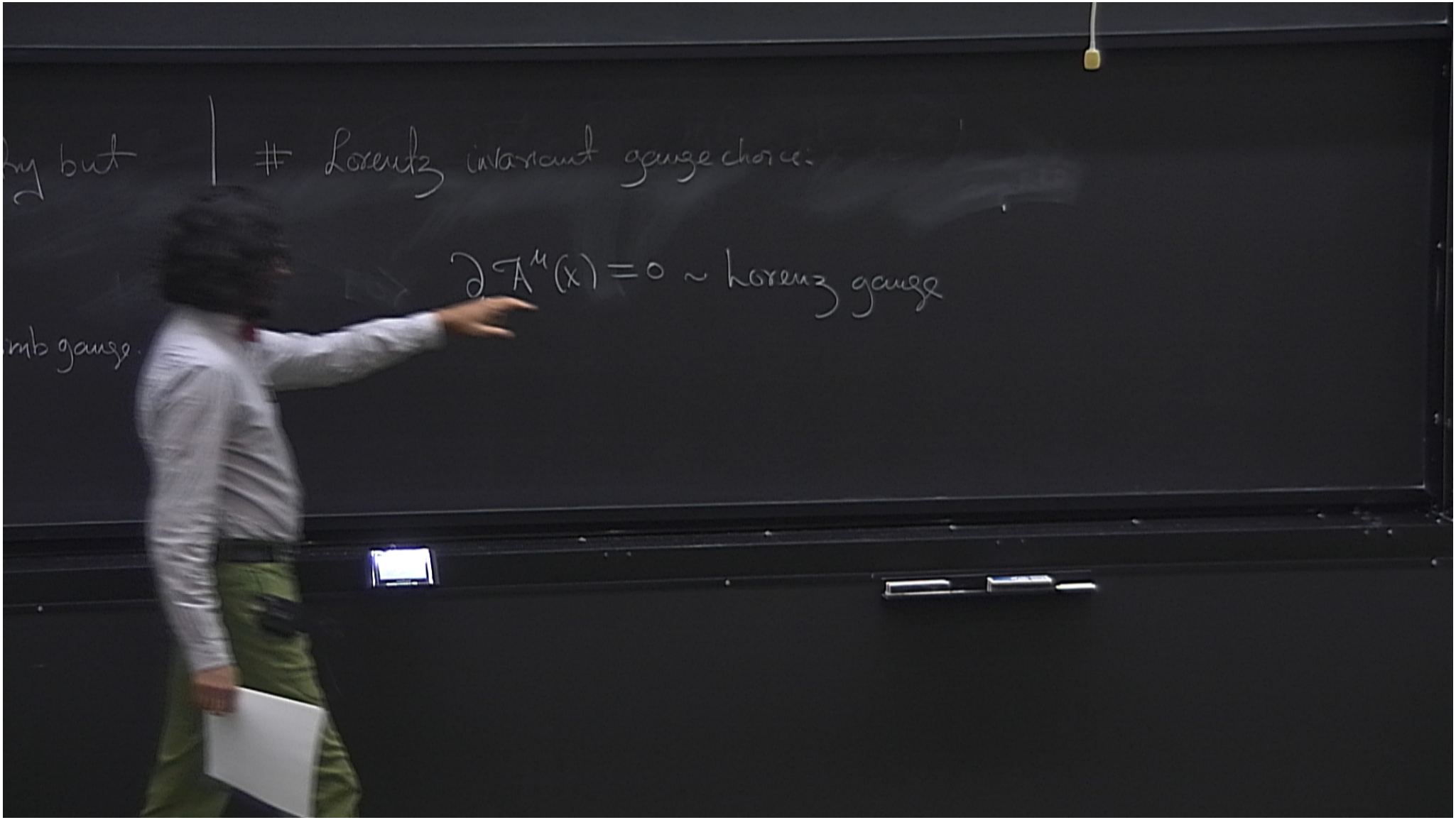

$\#$ No. $\dot{A}_0$ in $\mathcal{L}$ $F^{00}=0$
$\Rightarrow$ A_0 is a lagrange
$\nabla^2 A_0 +$ $=0$
$\frac{\partial \underline{A}}{\partial t}=0$
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$\# A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x)$
$F'_{\mu\nu}(x) = F_{\mu\nu}(x)$
↳ gauge transformation
$A_\mu(x) \rightarrow$ Gauge potential

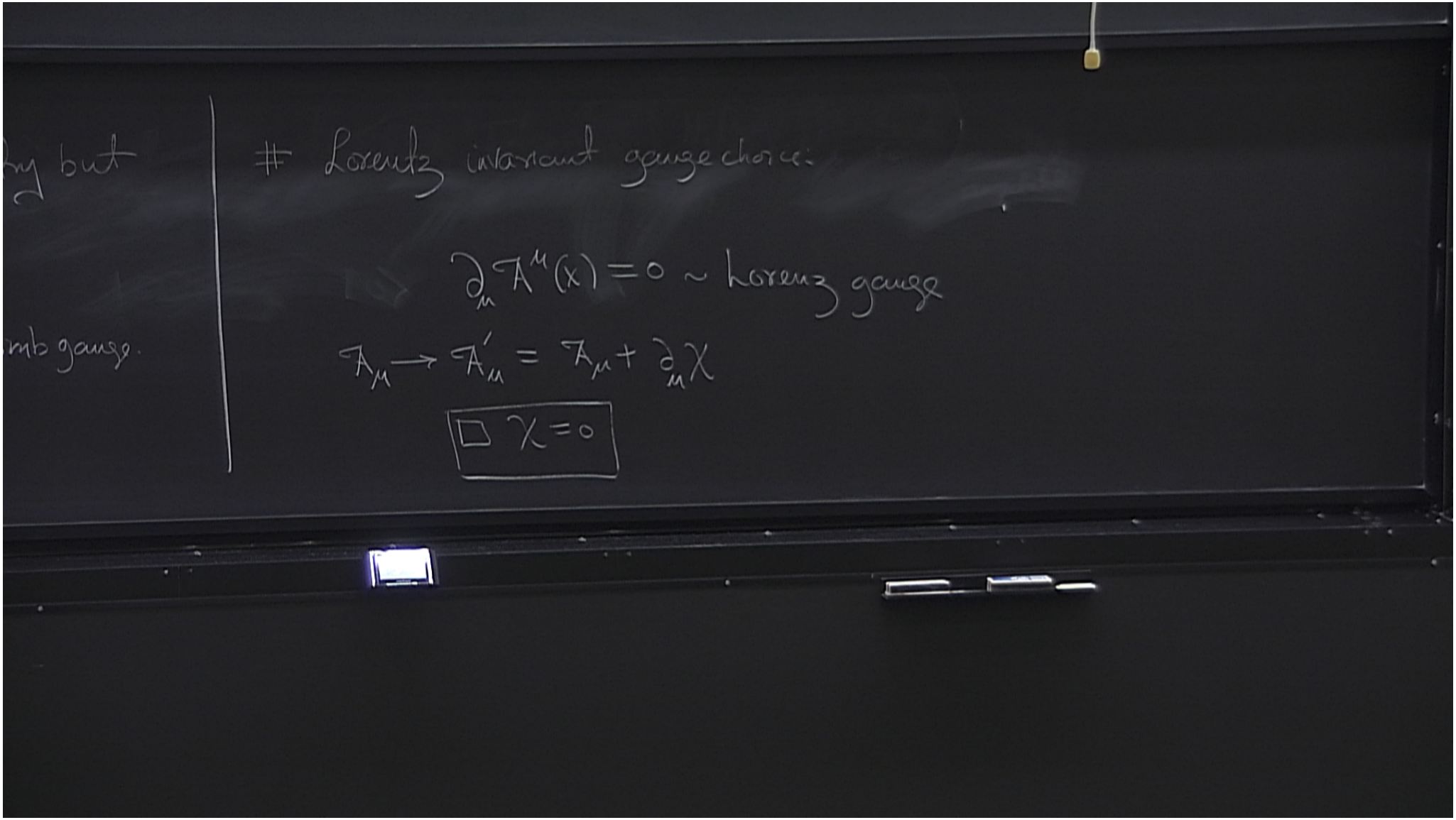

Gauge 'symmetry' is not a symmetry but
a redundancy of description.

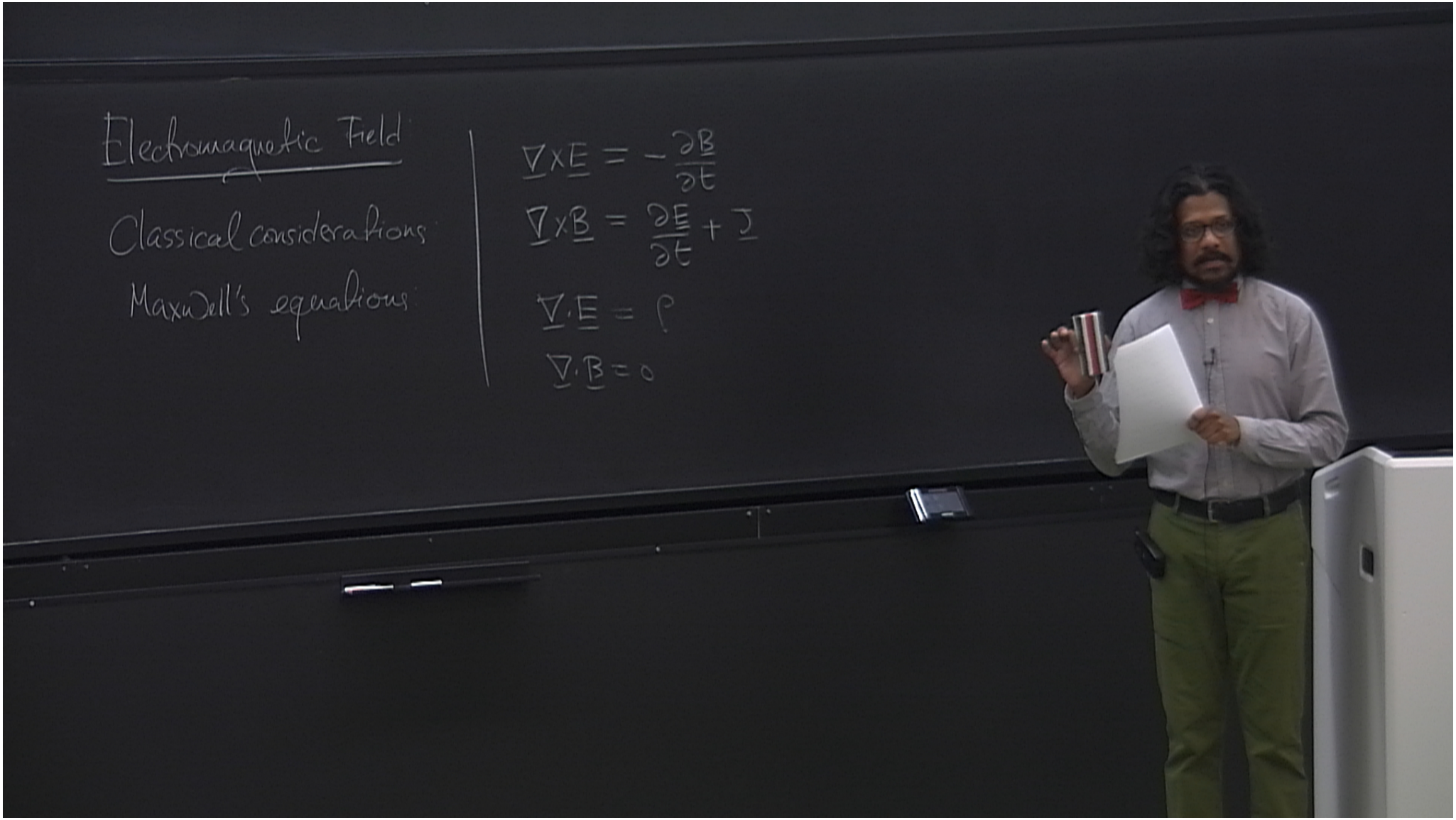

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We can impose $\nabla \cdot \underline{A} = 0$ ~ Coulomb gauge.

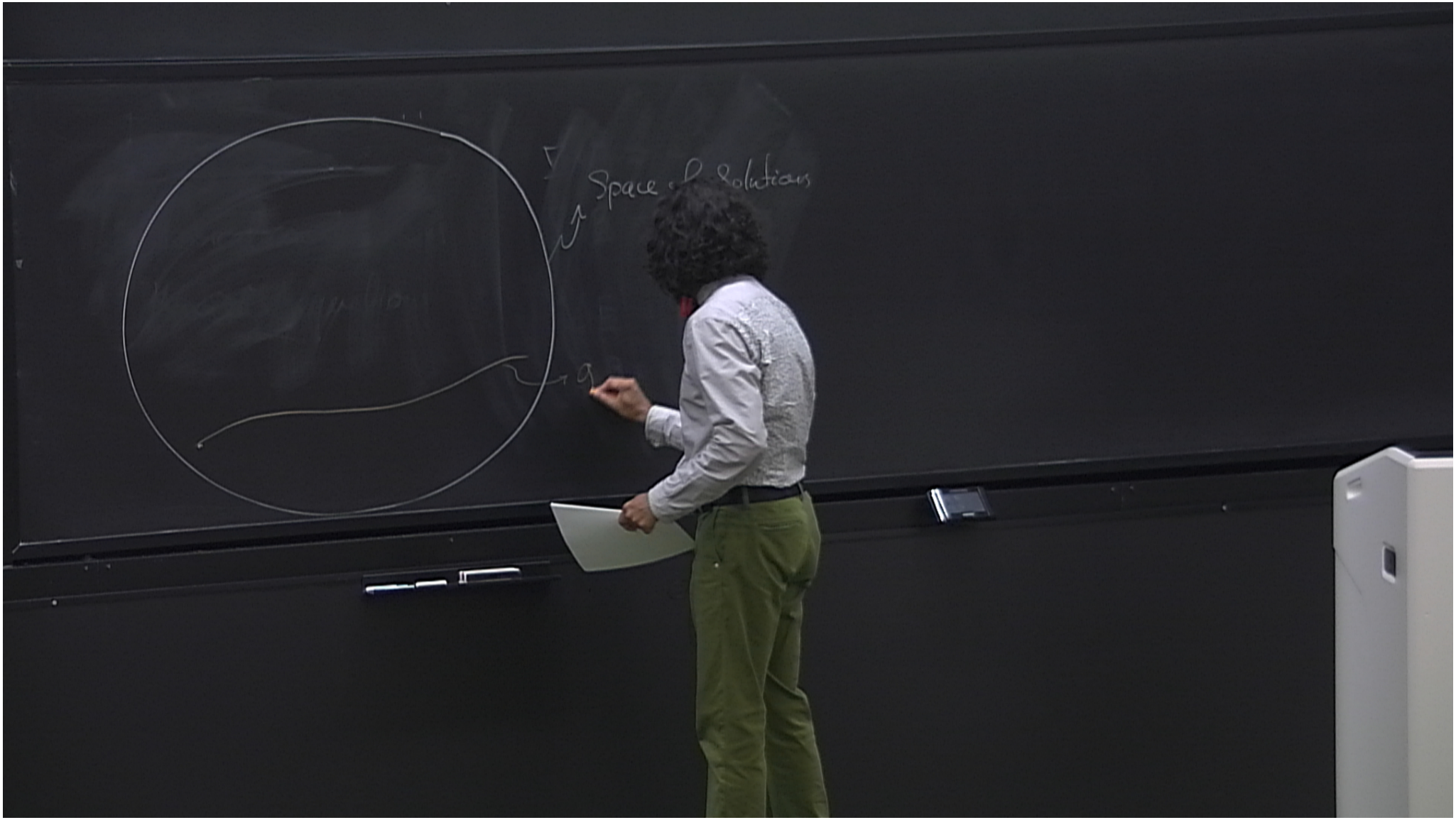

Gauge 'symmetry' is not a symmetry but
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We can impose $\nabla \cdot \underline{A} = 0$ ~ Coulomb gauge.
$\underline{A} \rightarrow \underline{A} + \nabla \times \underline{\psi}$
Lorentz

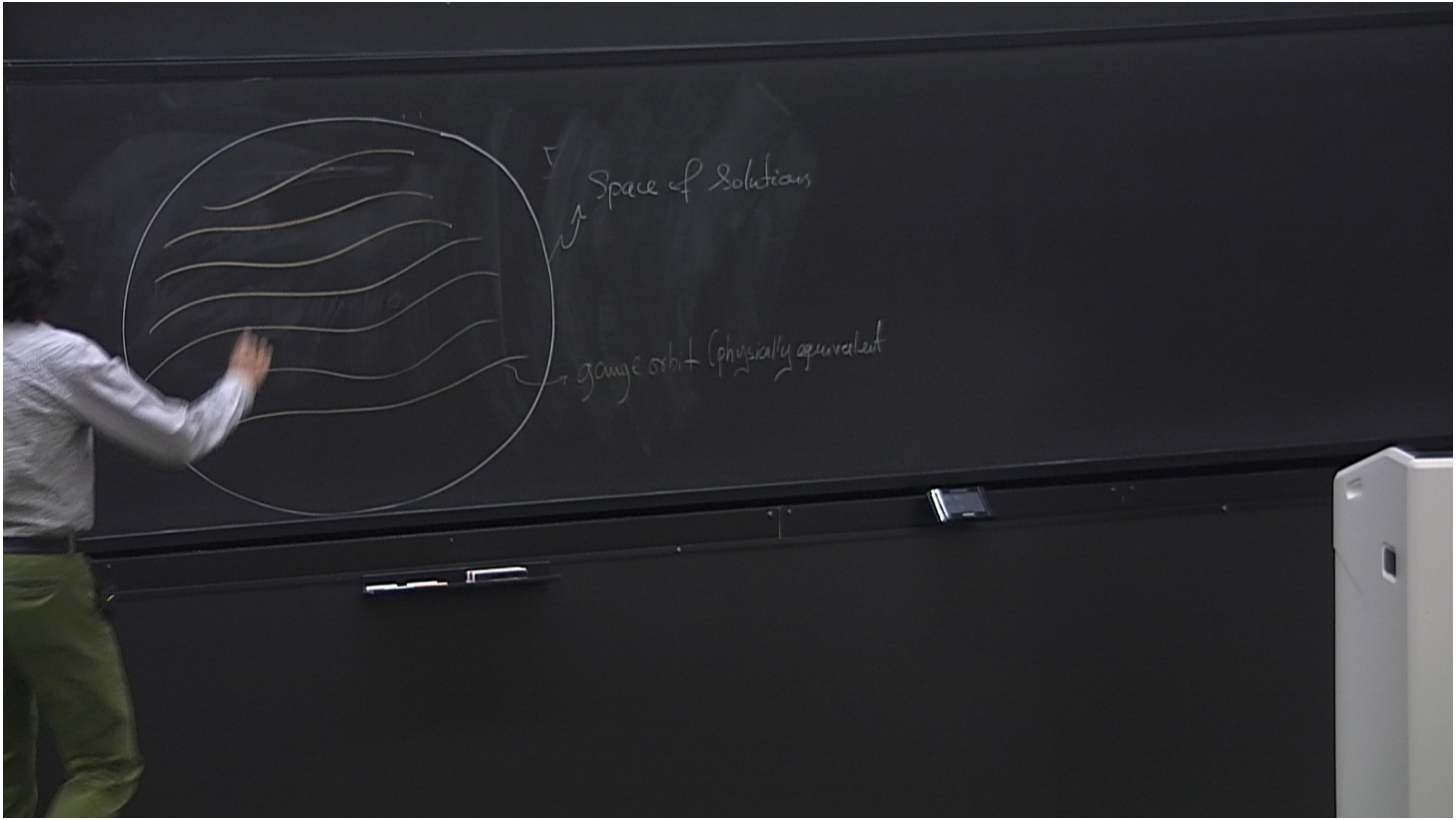

by but
Lorentz invariant gauge choice:
$\partial_\mu A^\mu(x) = 0$

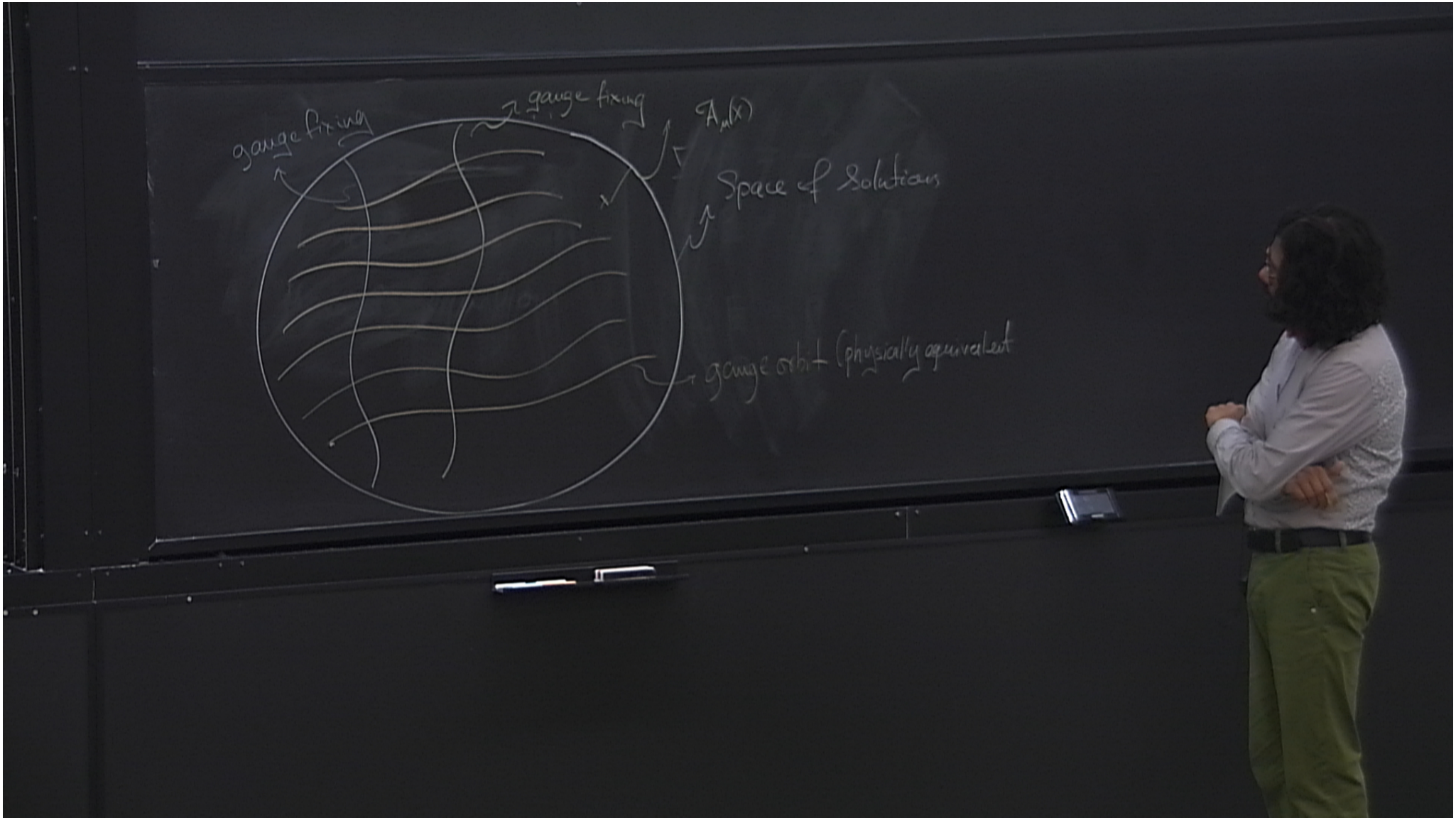

by but | # Lorentz invariant gauge choice:
$\partial^\mu A_\mu(x) = 0 \sim$ Lorenz gauge
mb gauge.

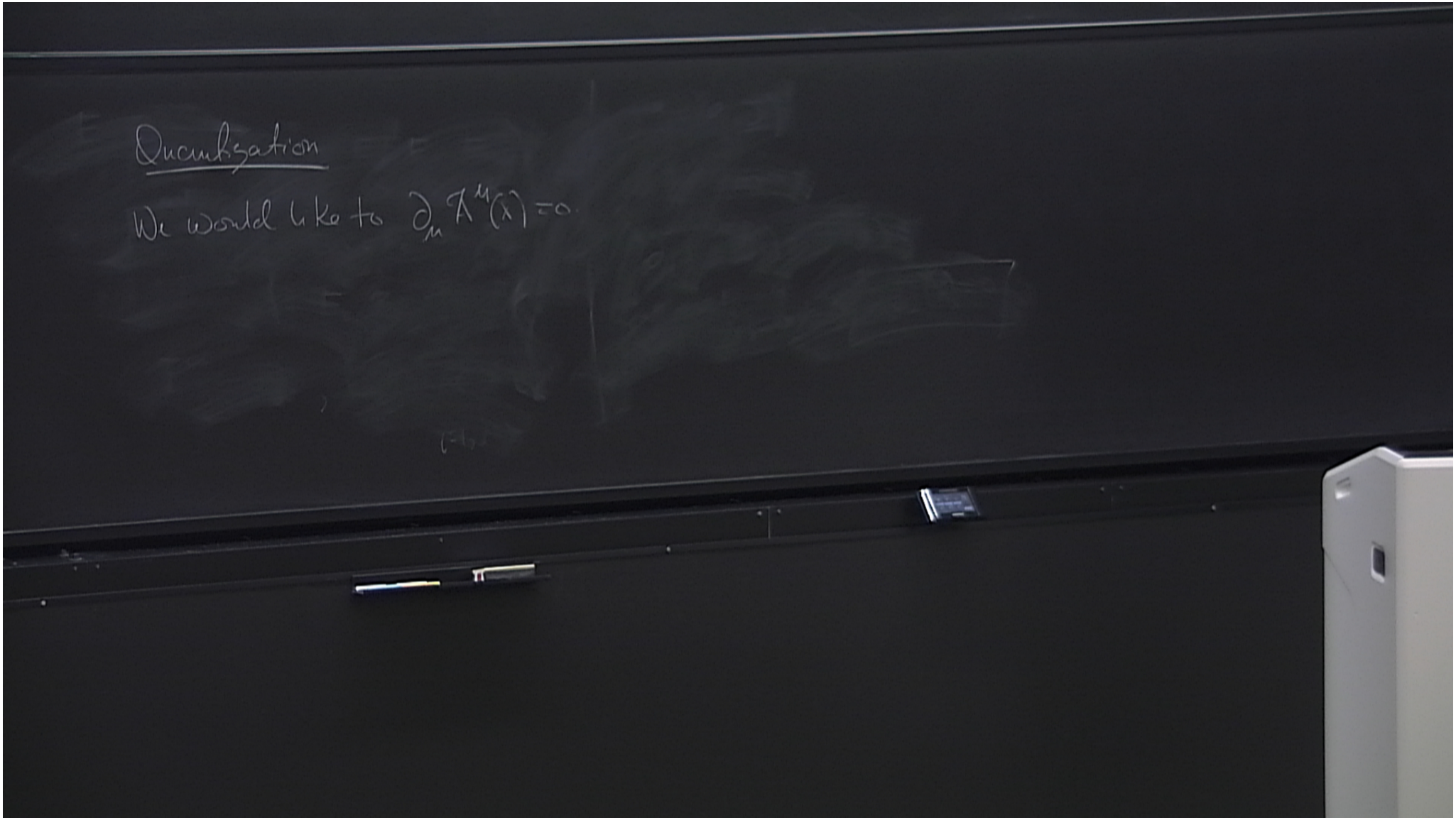

by but
mb gauge.
Lorentz invariant gauge choice:
$\partial_\mu A^\mu(x) = 0 \sim$ Lorenz gauge
$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$
$\Box \chi = 0$

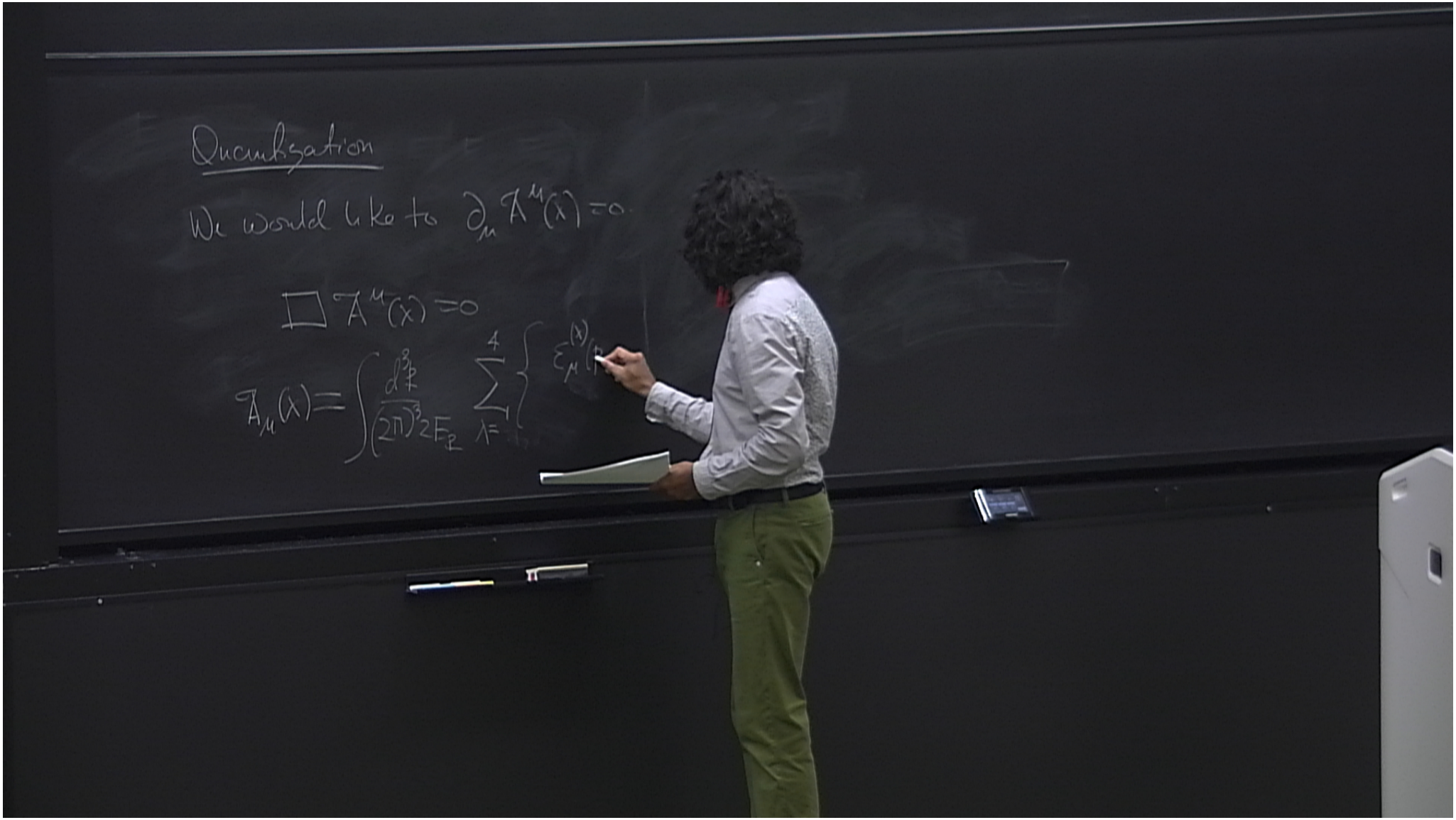

Electromagnetic Field
Classical considerations
Maxwell's equations
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times B = \frac{\partial E}{\partial t} + J$
$\nabla \cdot E = \rho$
$\nabla \cdot B = 0$

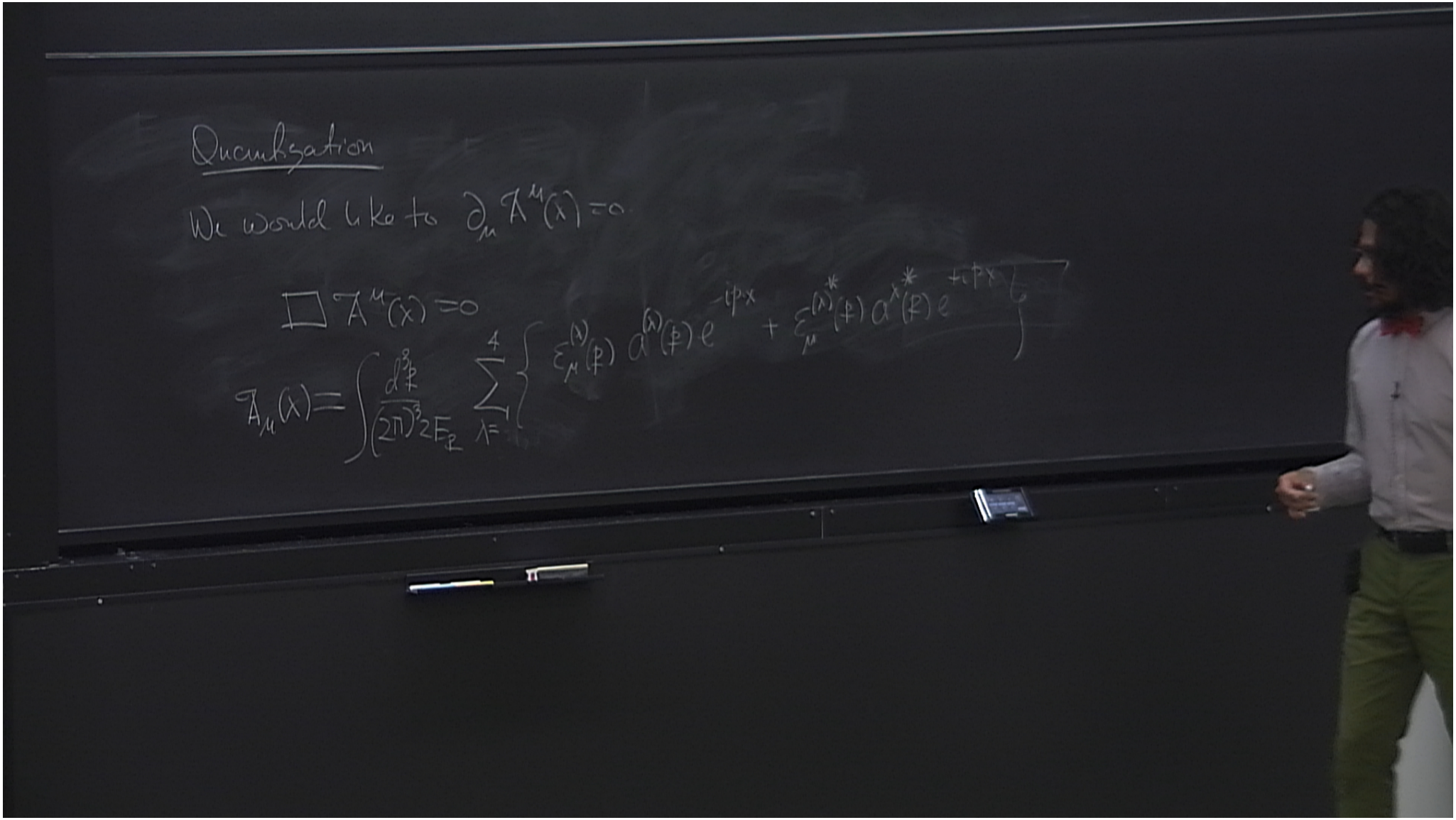

Space of Solutions

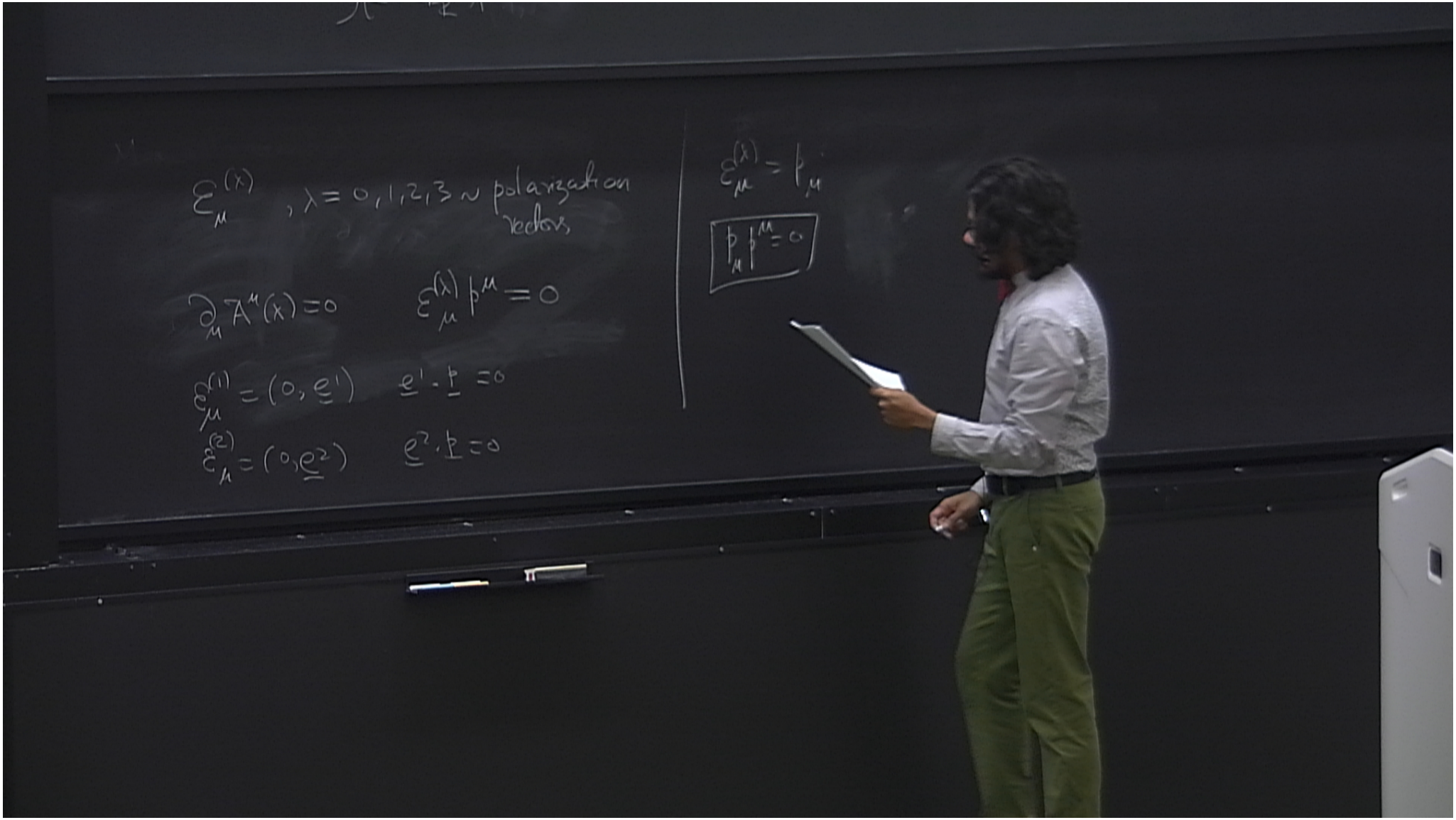

Space of Solutions
gauge orbit (physically equivalent

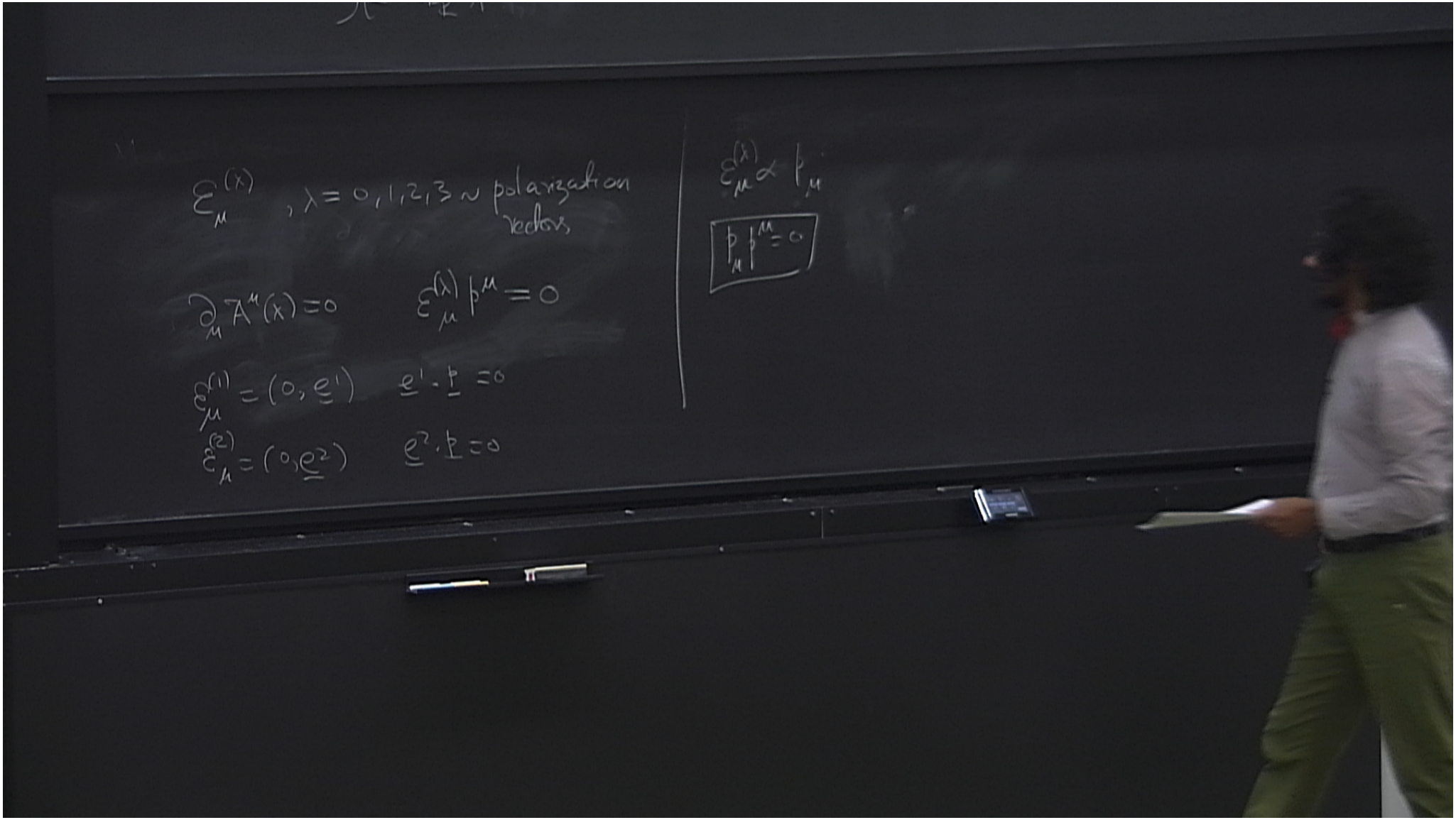

gauge fixing
gauge fixing
$A_\mu(x)$
Space of Solutions
gauge orbit (physically equivalent

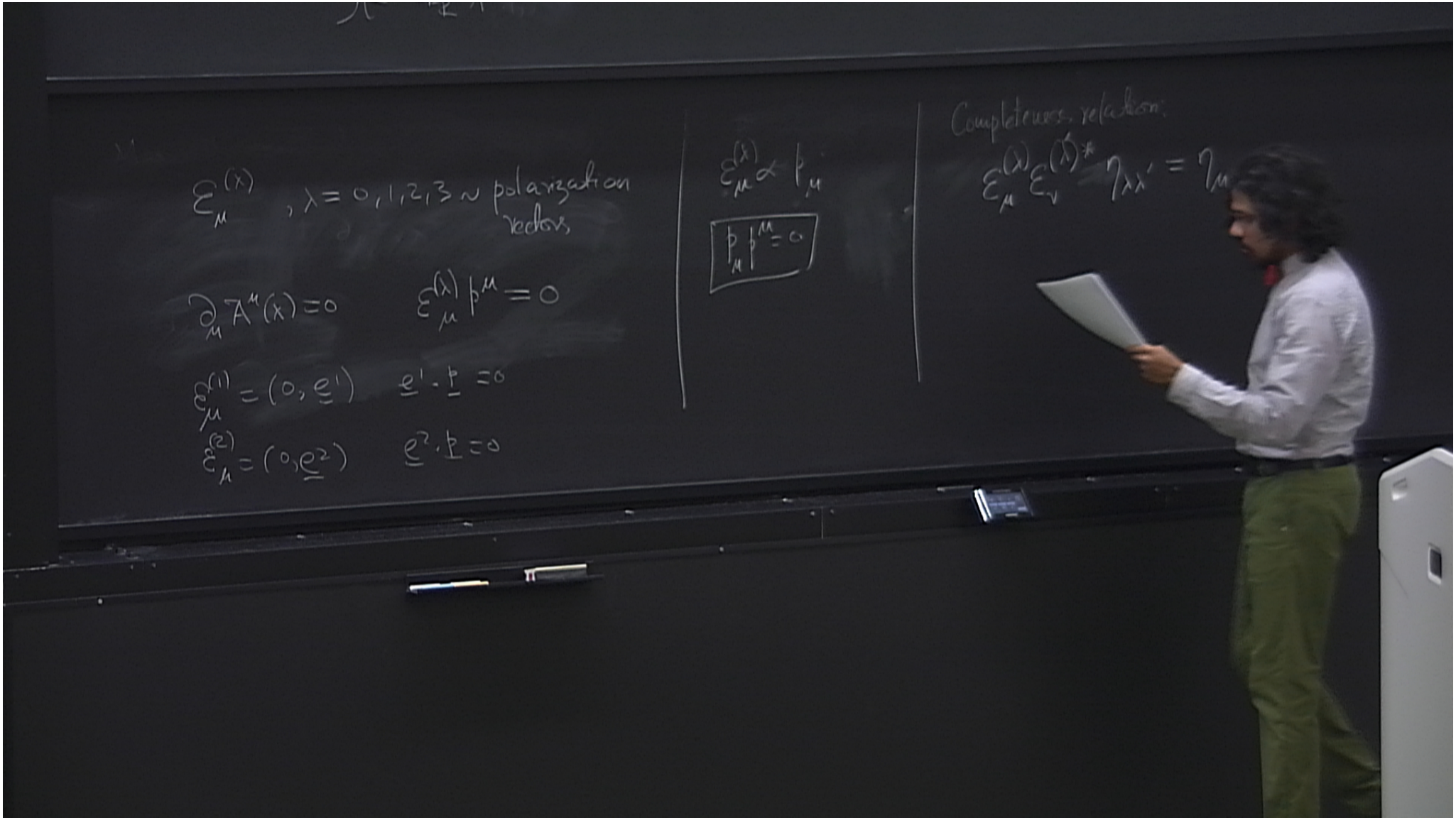

Quantisation
We would like to $\partial_\mu A^\mu(x) = 0$.

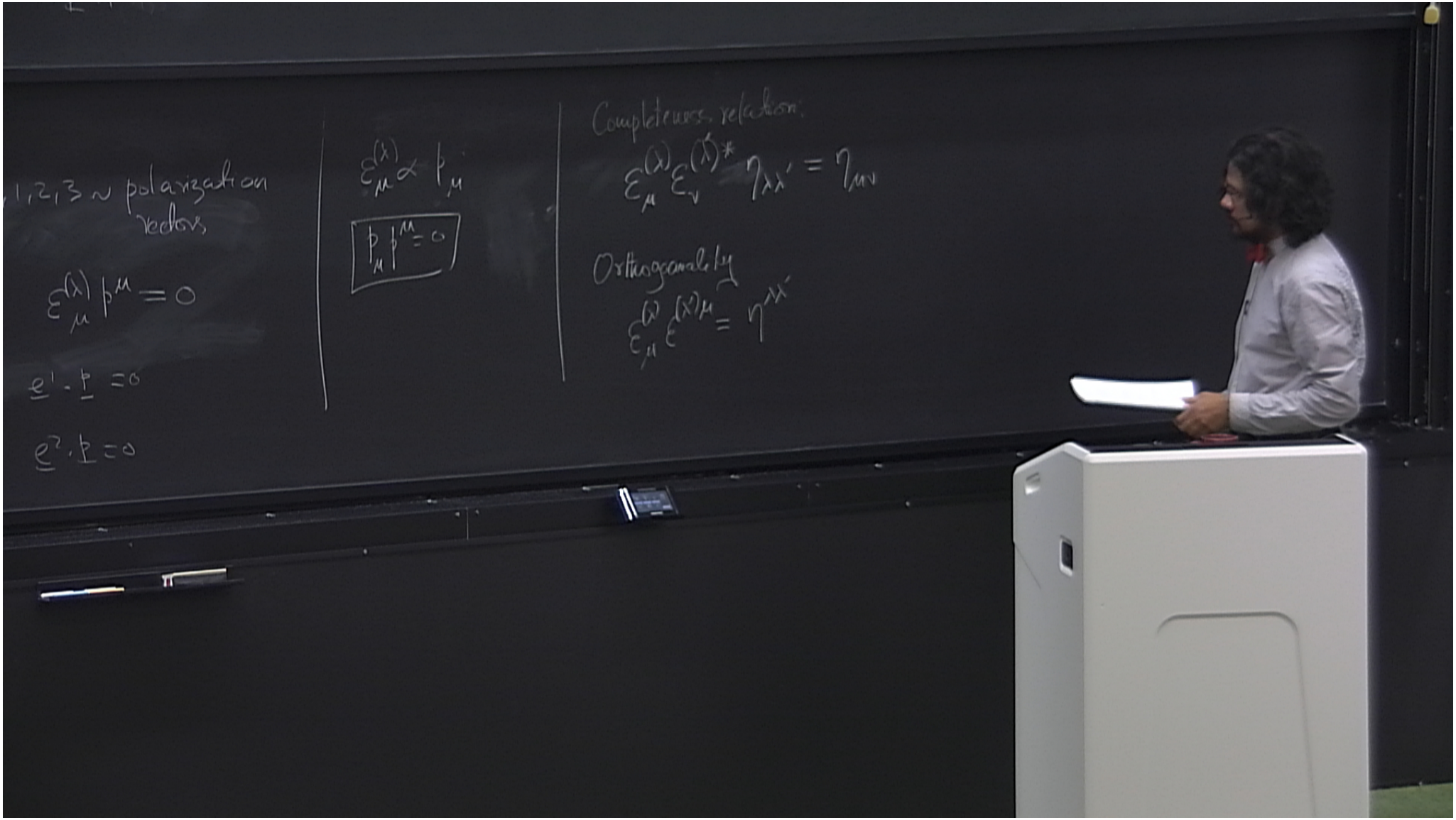

Quantization
We would like to $\partial_\mu A^\mu(x) = 0$
$\Box A^\mu(x) = 0$
$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_{\lambda=1}^{4} \{ \varepsilon_\mu^{(\lambda)}(k)$

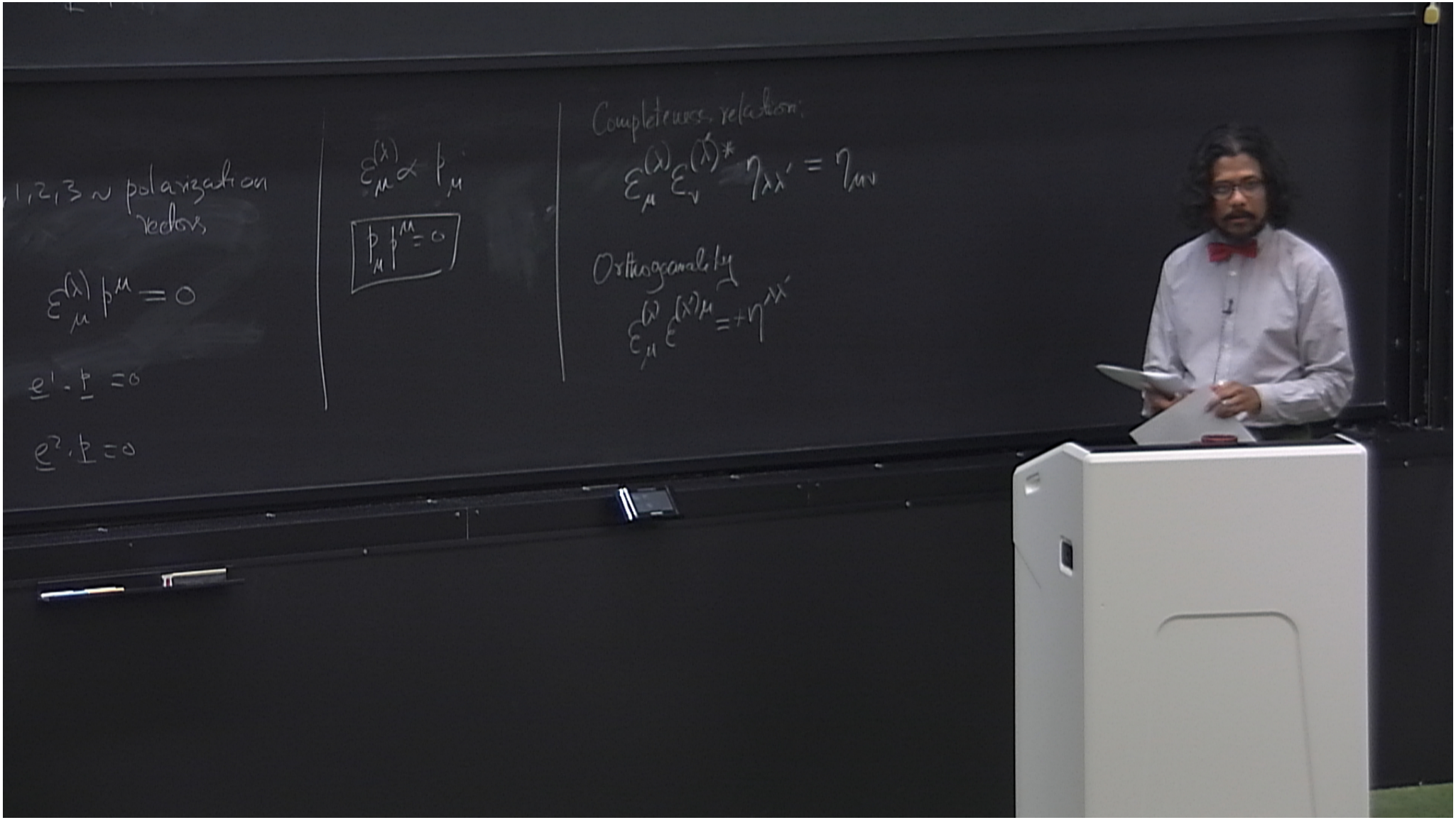

Quantization
We would like to $\partial_\mu A^\mu(x) = 0$
$\Box A^\mu(x) = 0$
$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{\lambda=1}^{4} \{ \varepsilon_\mu^{(\lambda)}(p)\, a^{(\lambda)}(p)\, e^{-ip\cdot x} + \varepsilon_\mu^{(\lambda)*}(p)\, a^{\dagger(\lambda)*}(p)\, e^{+ip\cdot x} \}$

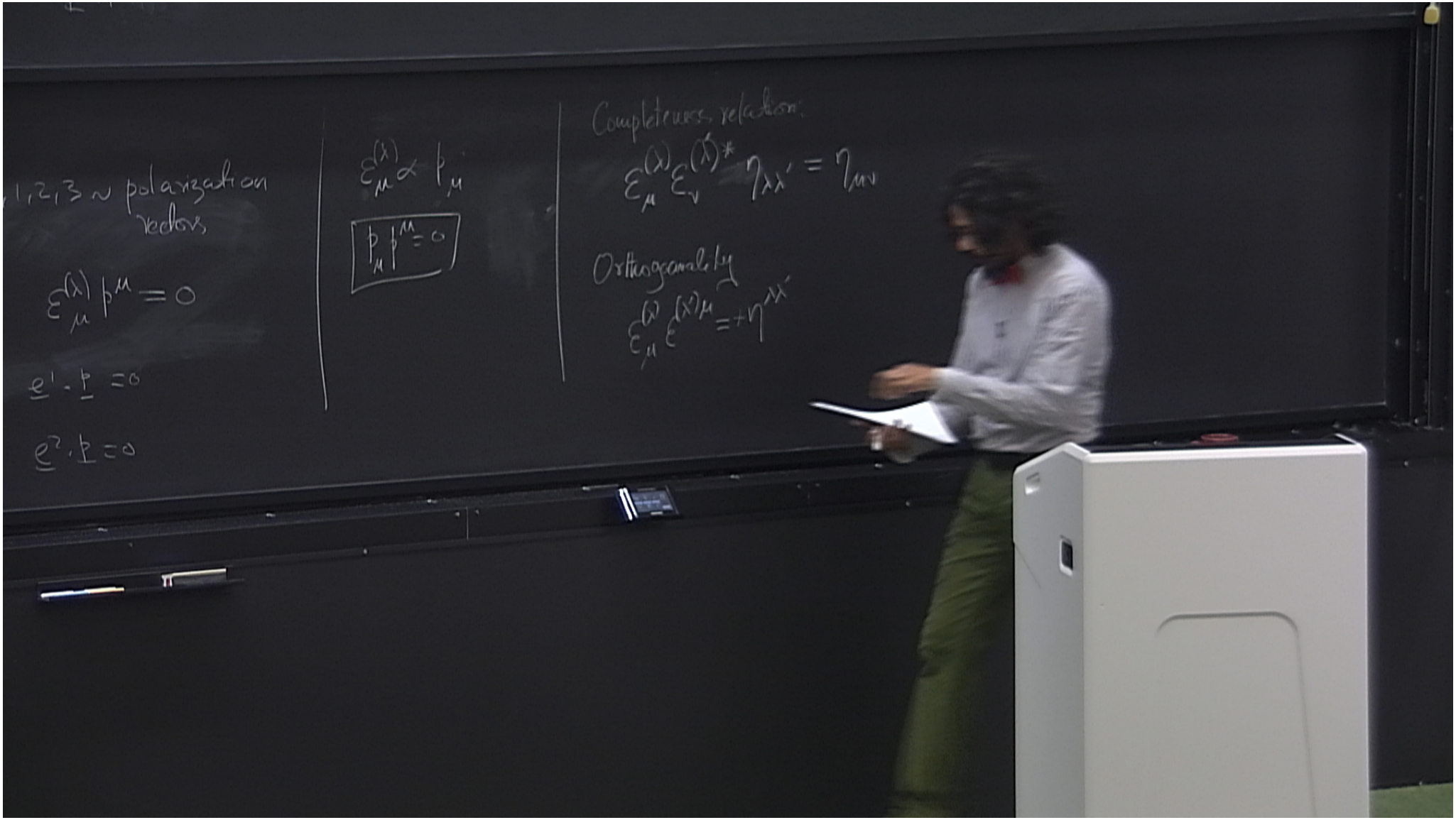

$\varepsilon_\mu^{(\lambda)}$, $\lambda = 0,1,2,3$ ~ polarization vectors
$\partial_\mu A^\mu(x) = 0$
$\varepsilon_\mu^{(\lambda)} p^\mu = 0$
$\varepsilon_\mu^{(1)} = (0, \underline{e}^1)$
$\underline{e}^1 \cdot \underline{p} = 0$
$\varepsilon_\mu^{(2)} = (0, \underline{e}^2)$
$\underline{e}^2 \cdot \underline{p} = 0$
$\varepsilon_\mu^{(\lambda)} = p_\mu$
$p_\mu p^\mu = 0$

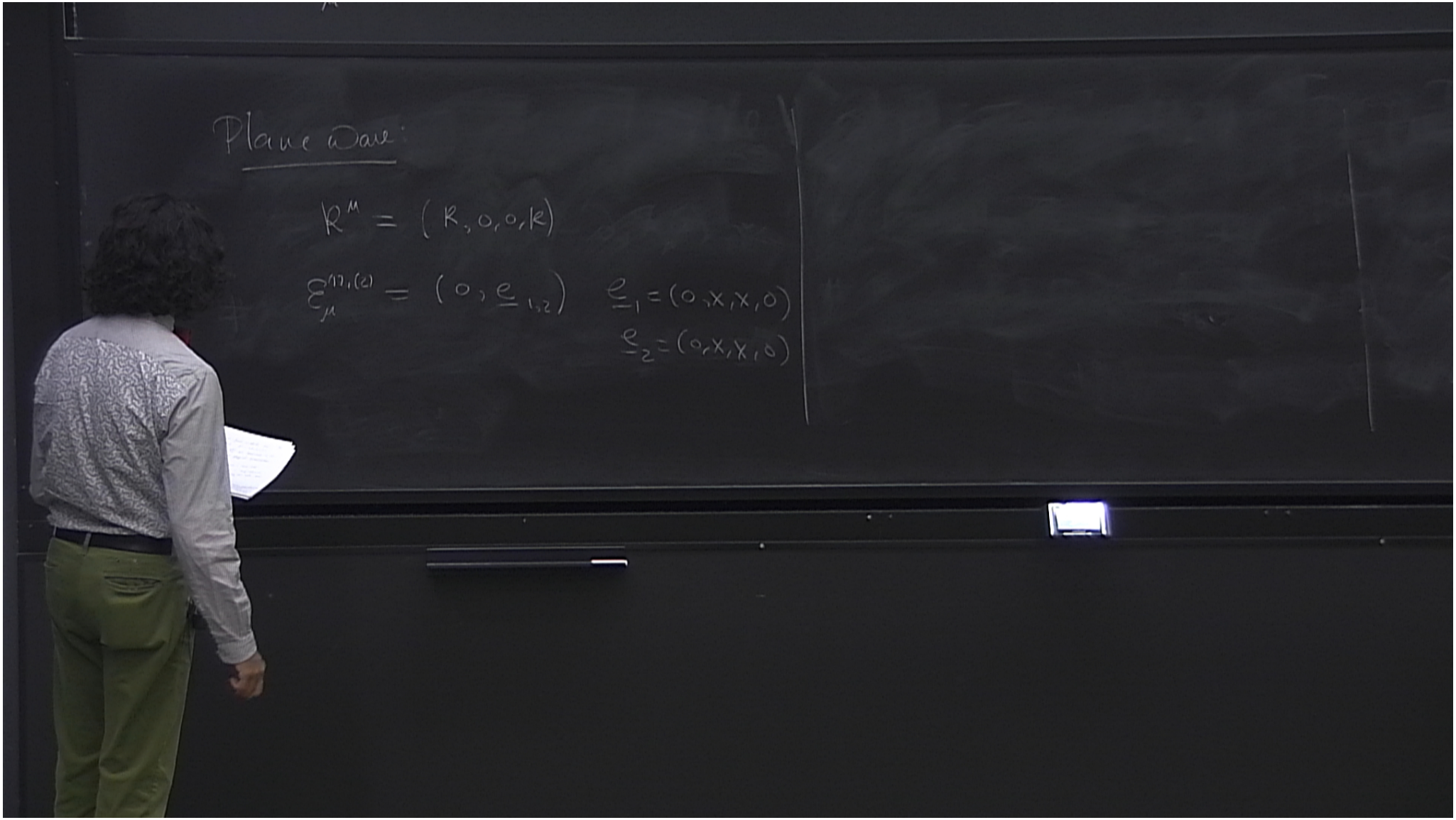

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$\varepsilon_\mu^{(\lambda)} \propto p_\mu$
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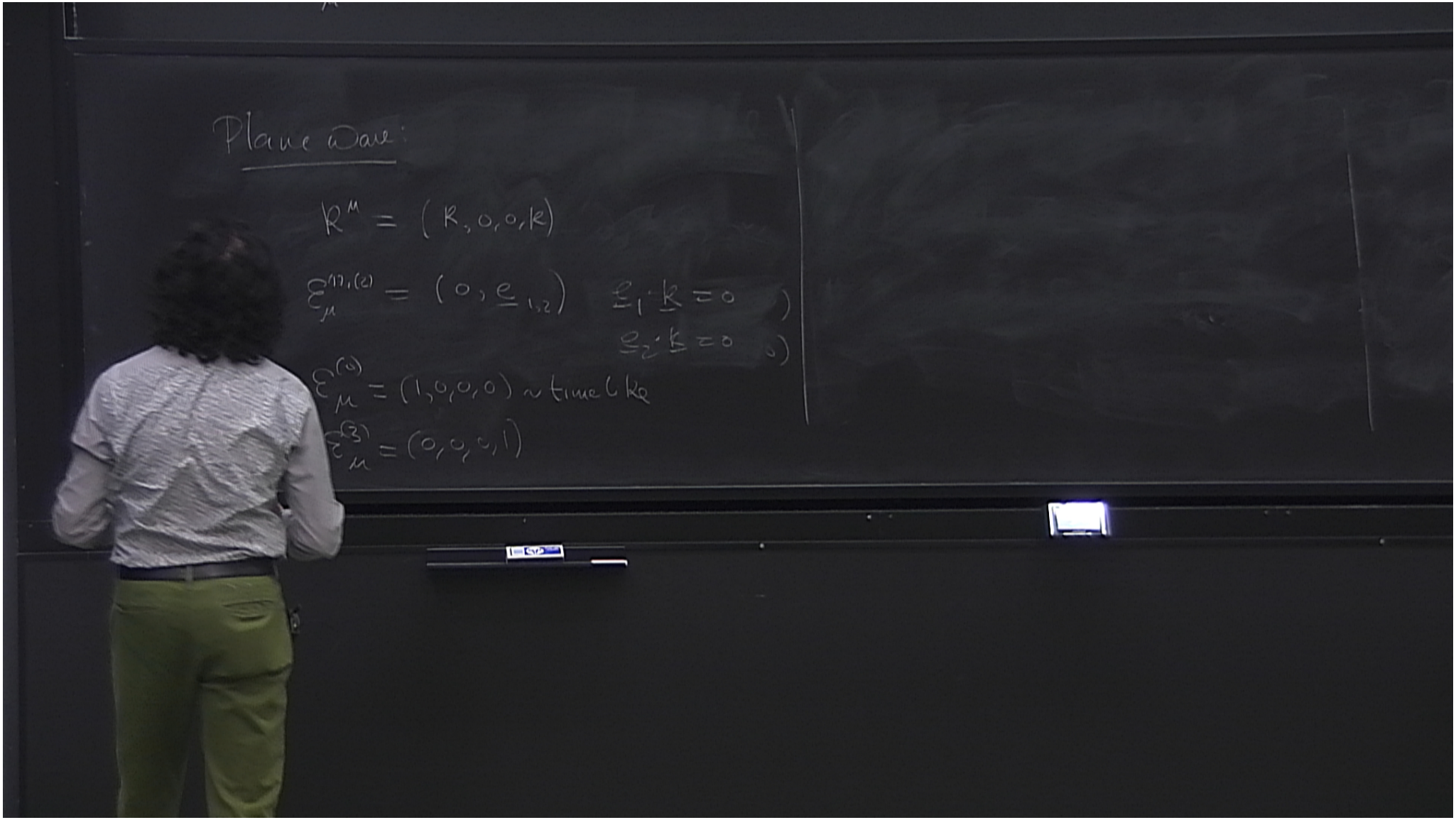

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$\varepsilon_\mu^{(\lambda)} \propto p_\mu$
$p_\mu p^\mu = 0$
Completeness relation:
$\varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda')*} \eta_{\lambda\lambda'} = \eta_\mu$

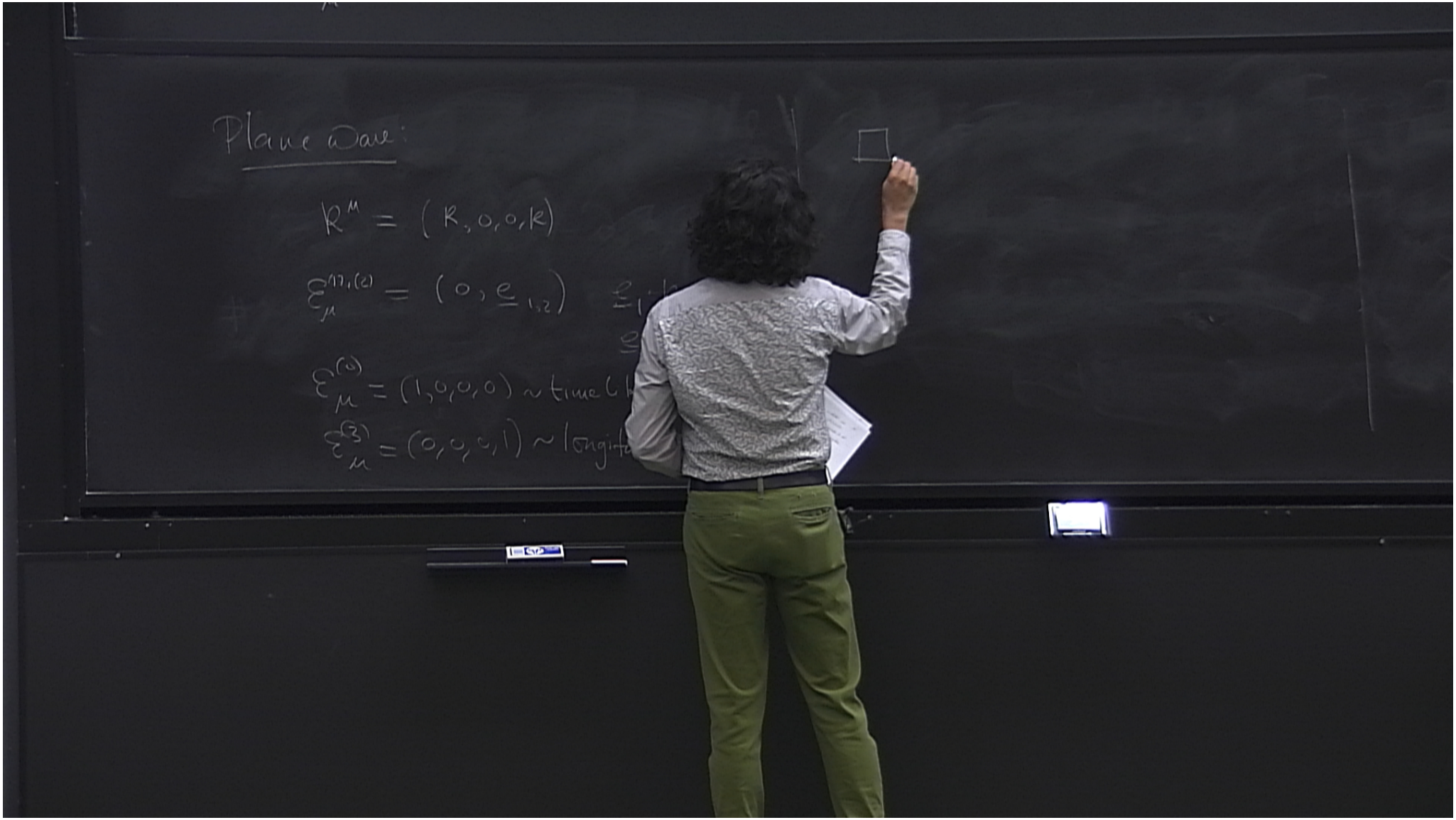

1,2,3 ~ polarization vectors
$\varepsilon^{(\lambda)}_\mu p^\mu = 0$
$\underline{e}^1 \cdot \underline{p} = 0$
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$p_\mu p^\mu = 0$
Completeness relation:
$\varepsilon^{(\lambda)}_\mu \varepsilon^{(\lambda')*}_\nu \eta_{\lambda\lambda'} = \eta_{\mu\nu}$
Orthogonality
$\varepsilon^{(\lambda)}_\mu \varepsilon^{(\lambda')\mu} = \eta^{\lambda\lambda'}$

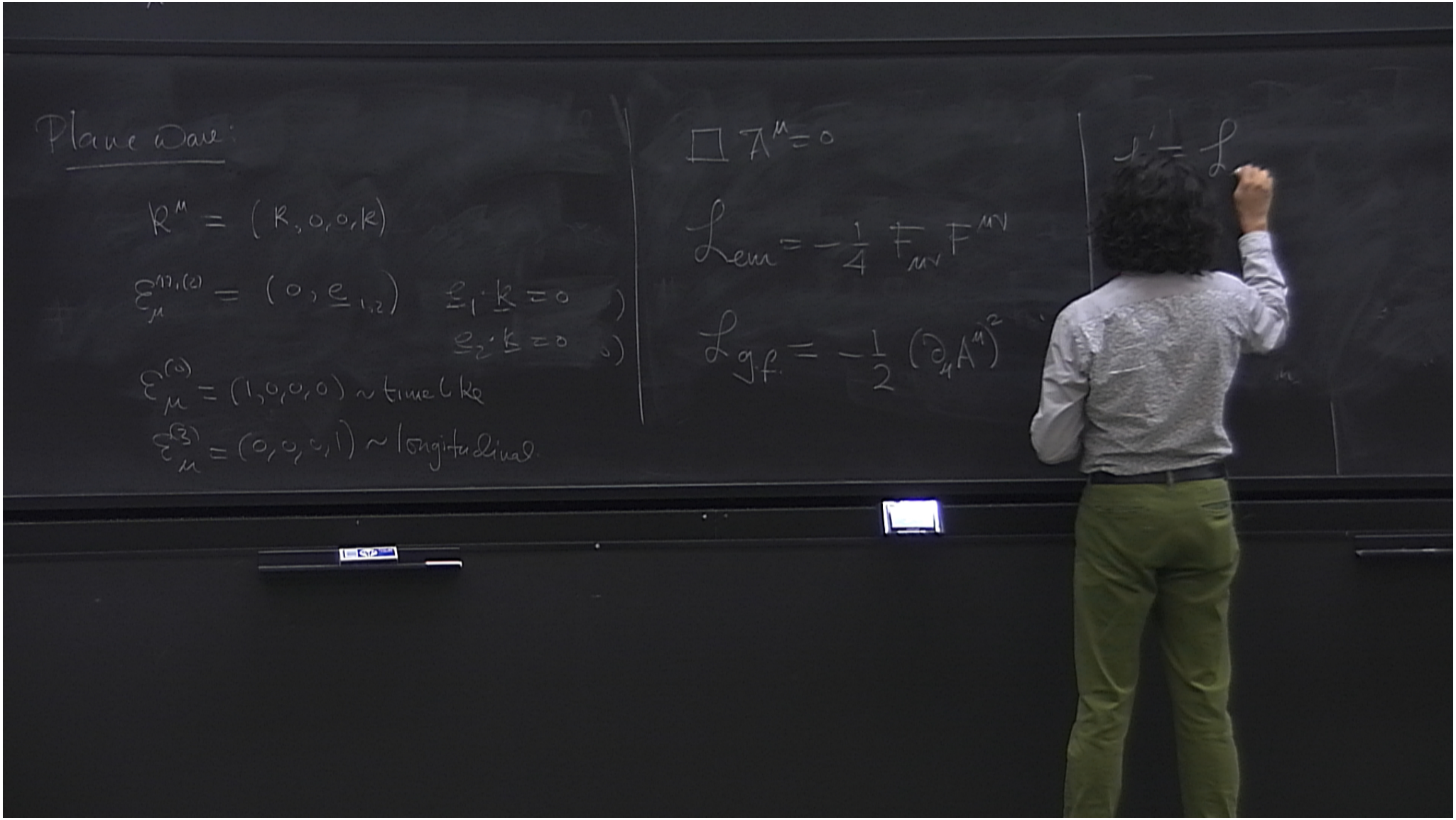

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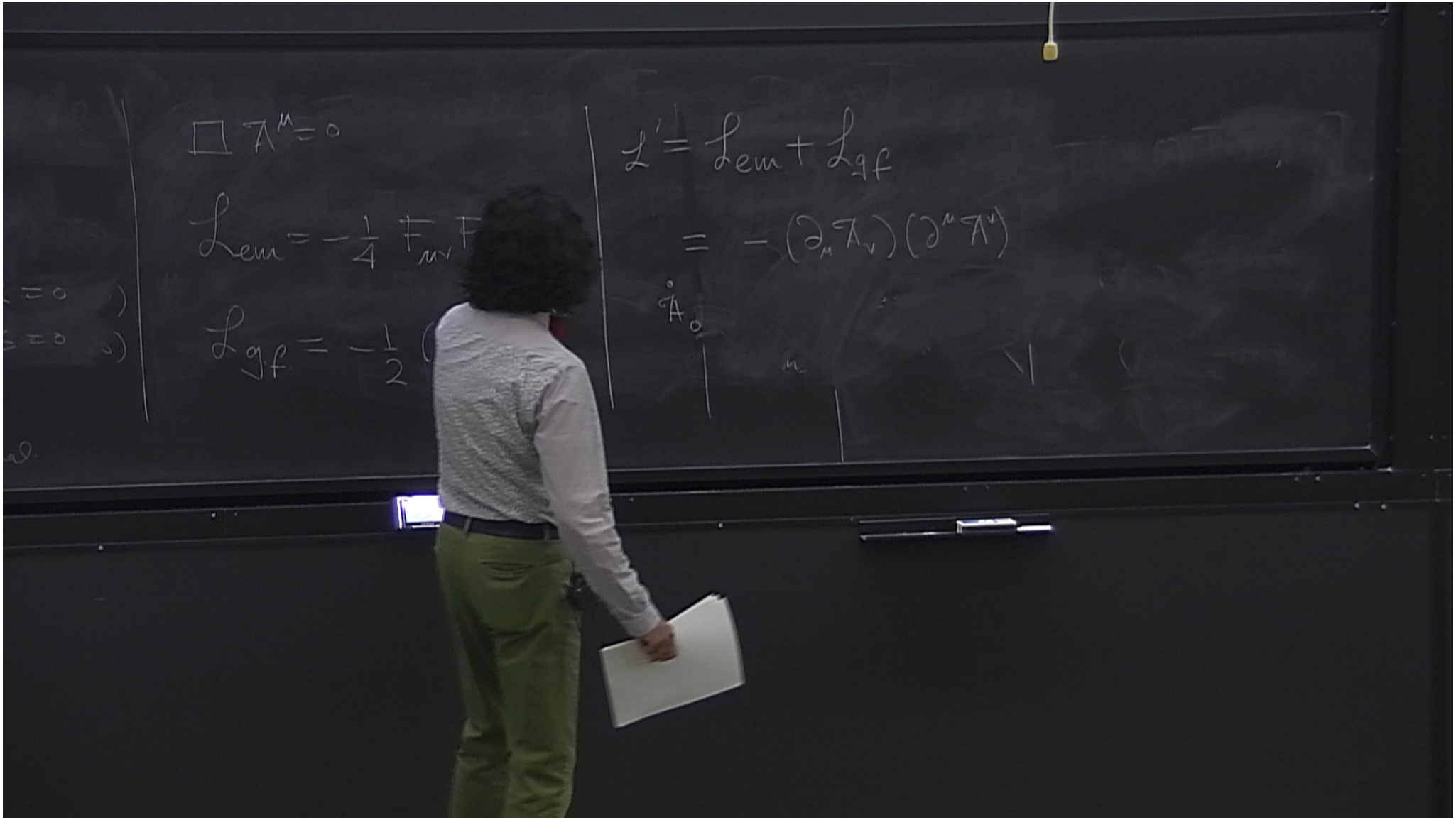

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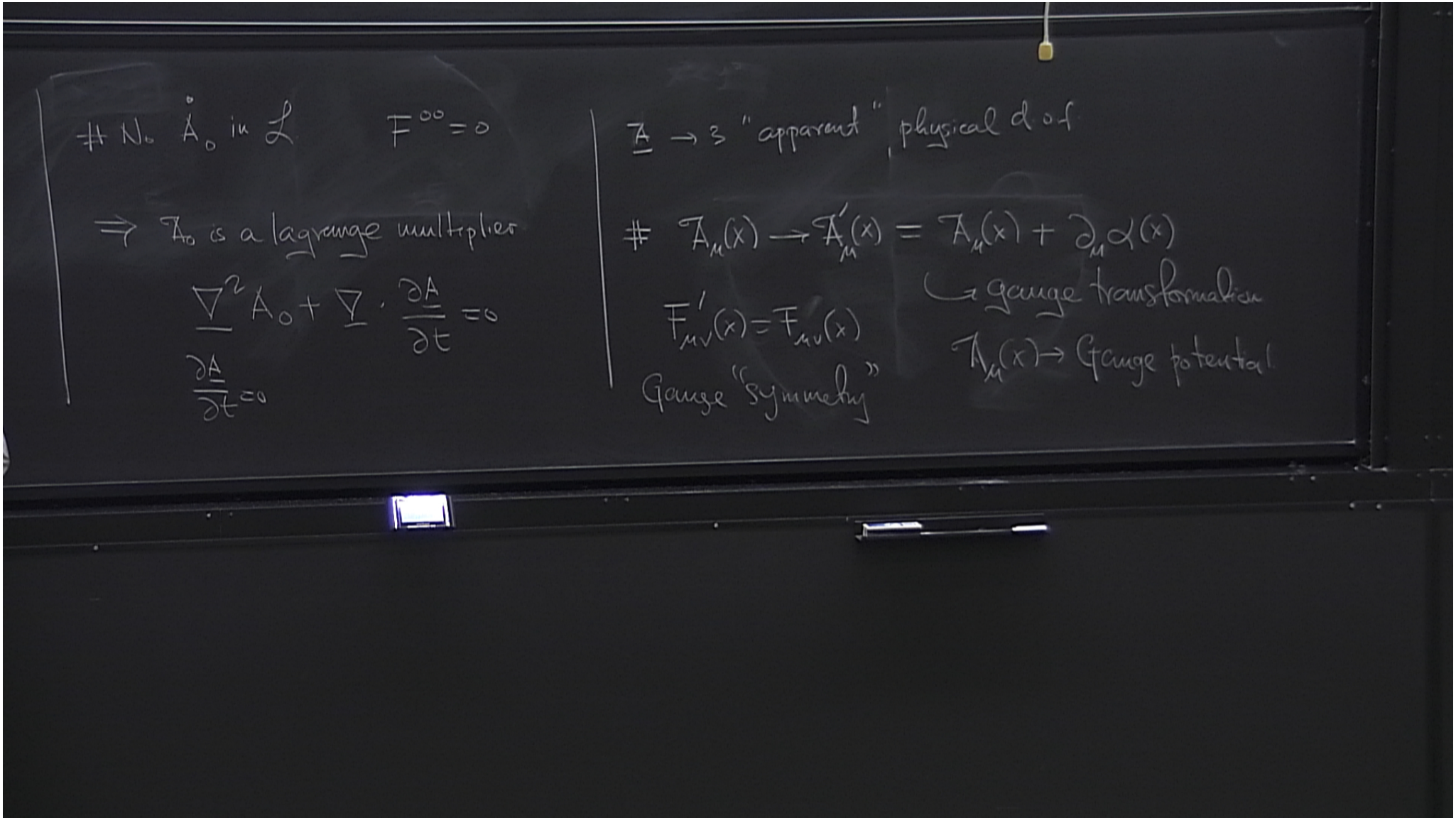

Plane Wave:
$k^\mu = (k, 0, 0, k)$
$\varepsilon_\mu^{(1),(2)} = (0, \underline{e}_{1,2})$
$\underline{e}_1 = (0, x, x, 0)$
$\underline{e}_2 = (0, x, x, 0)$

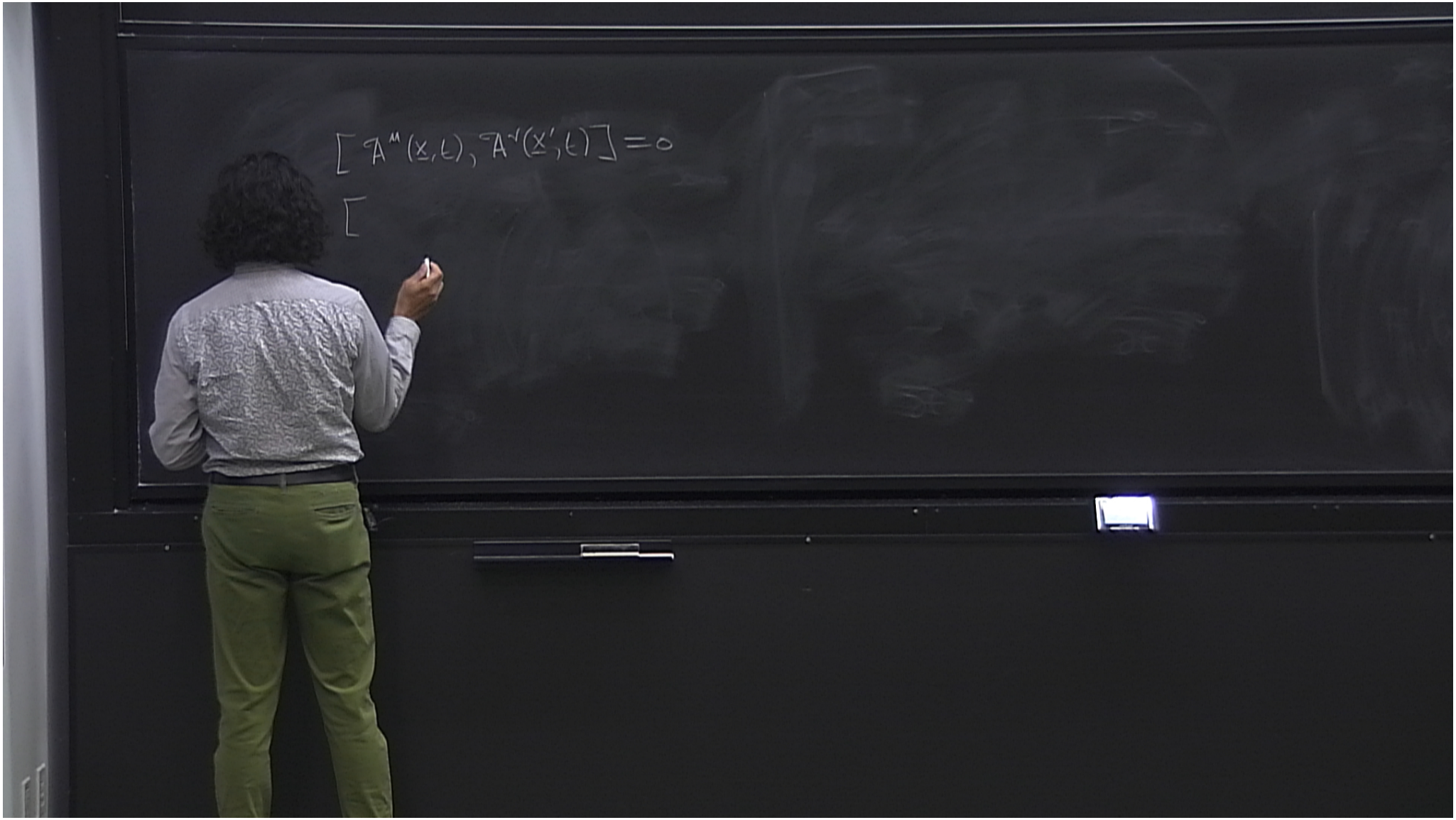

Plane wave:
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$\varepsilon_\mu^{(1),(2)} = (0, \underline{e}_{1,2})$ $\underline{e}_1 \cdot \underline{k} = 0$)
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$\varepsilon_\mu^{(0)} = (1,0,0,0)$ ~ time like
$\varepsilon_\mu^{(3)} = (0,0,0,1)$

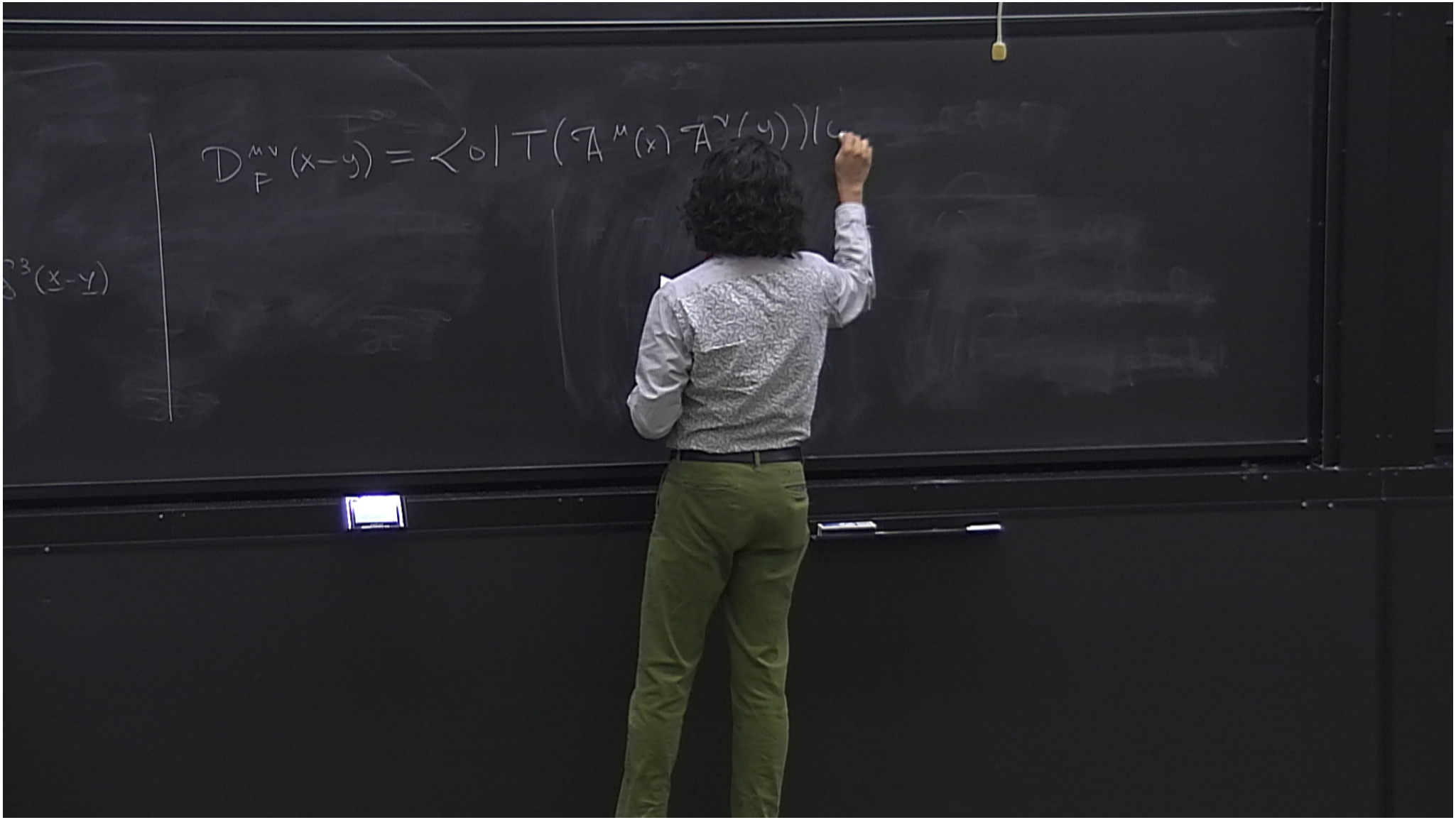

Plane wave:
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$\varepsilon_\mu^{(0)} = (1,0,0,0) \sim$ time
$\varepsilon_\mu^{(3)} = (0,0,0,1) \sim$ longit

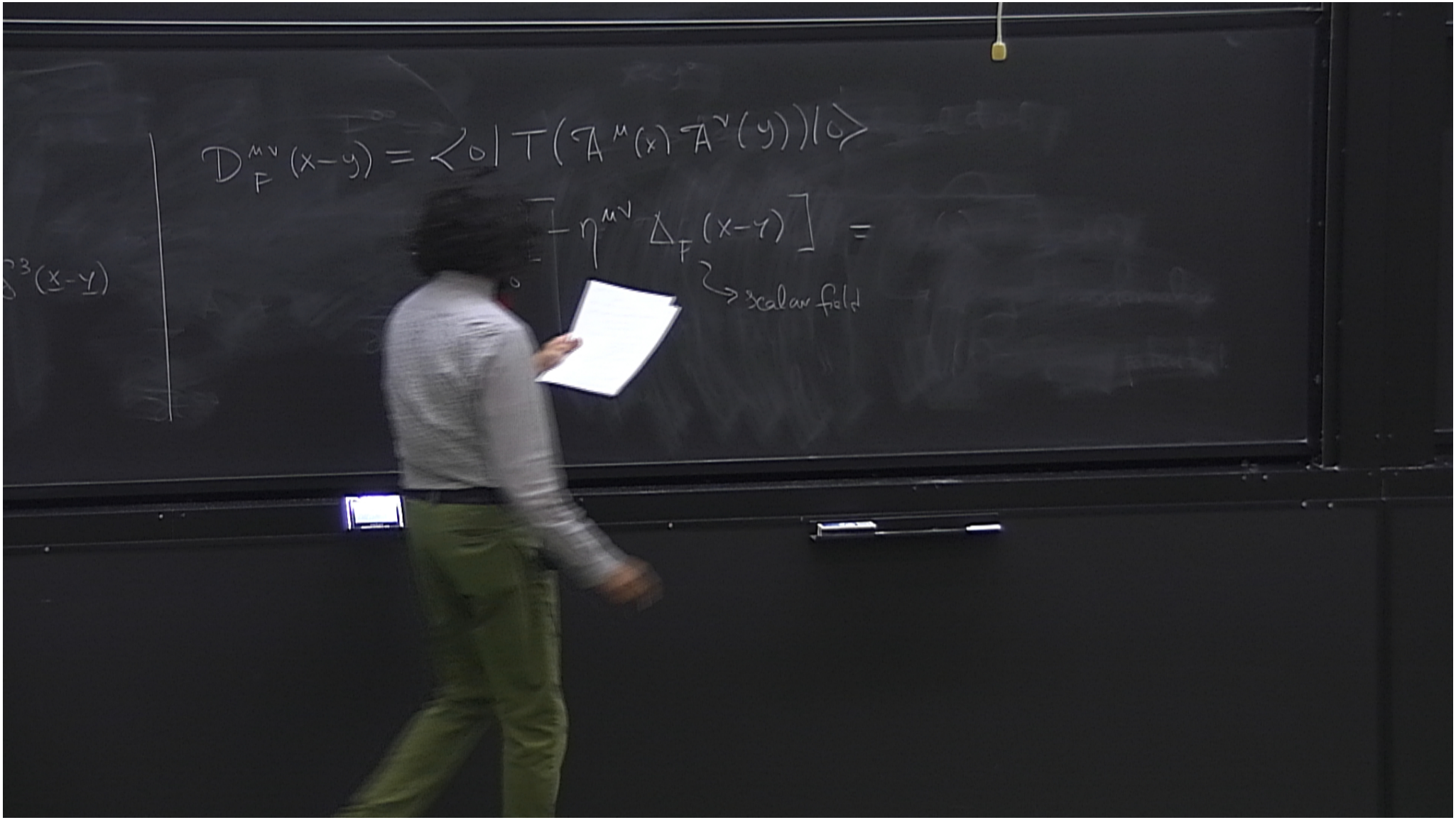

Plane wave:
$k^\mu = (k, 0, 0, k)$
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$\underline{e}_2 \cdot \underline{k} = 0$)
$\varepsilon_\mu^{(0)} = (1,0,0,0) \sim$ time like
$\varepsilon_\mu^{(3)} = (0,0,0,1) \sim$ longitudinal
$\Box A^\mu = 0$
$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\mathcal{L}_{g.f.} = -\frac{1}{2} (\partial_\mu A^\mu)^2$
$\mathcal{L}' = \mathcal{L}$

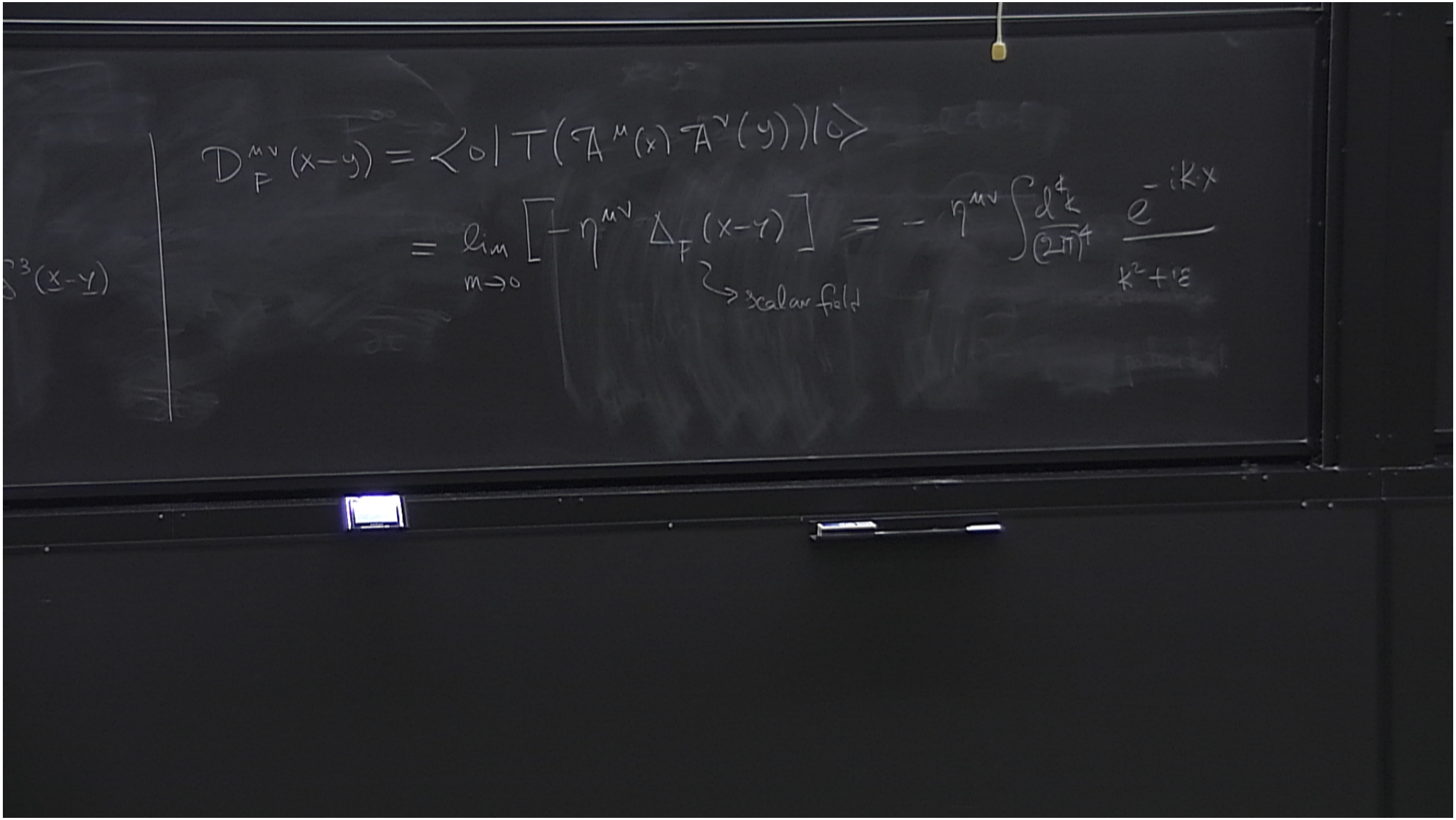

$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F$
$\mathcal{L}_{gf} = -\frac{1}{2}$
$\mathcal{L}' = \mathcal{L}_{em} + \mathcal{L}_{gf}$
$= -(\partial_\mu A_\nu)(\partial^\mu A^\nu)$


No. $\dot{A}_0$ in $\mathcal{L}$ $F^{00}=0$
$\Rightarrow A_0$ is a lagrange multiplier
$\nabla^2 A_0 + \nabla \cdot \frac{\partial \underline{A}}{\partial t} = 0$
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$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x)$
↳ gauge transformation
$A_\mu(x) \rightarrow$ Gauge potential
$F'_{\mu\nu}(x) = F_{\mu\nu}(x)$
Gauge "symmetry"


$[\bar{A}^{\mu}(\underline{x},t), \bar{A}^{\nu}(\underline{x}',t)] = 0$
[


$D_F^{\mu\nu}(x-y) = \langle 0|T(A^\mu(x)A^\nu(y))|$
$\delta^3(\underline{x}-\underline{y})$


$D_F^{\mu\nu}(x-y) = \langle 0|T(A^\mu(x)A^\nu(y))|0\rangle$
$= [-\eta^{\mu\nu}\Delta_F(x-y)] =$
Δ_F
scalar field


$D_F^{\mu\nu}(x-y) = \langle 0|T(A^\mu(x)A^\nu(y))|0\rangle$
$= \lim_{m\to 0}\left[-\eta^{\mu\nu}\Delta_F(x-y)\right] = -\eta^{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2+i\varepsilon}$
scalar field
$\delta^3(\underline{x}-\underline{y})$

$$D_F^{\mu\nu}(x-y) = \langle 0|T(A^\mu(x)A^\nu(y))|0\rangle$$

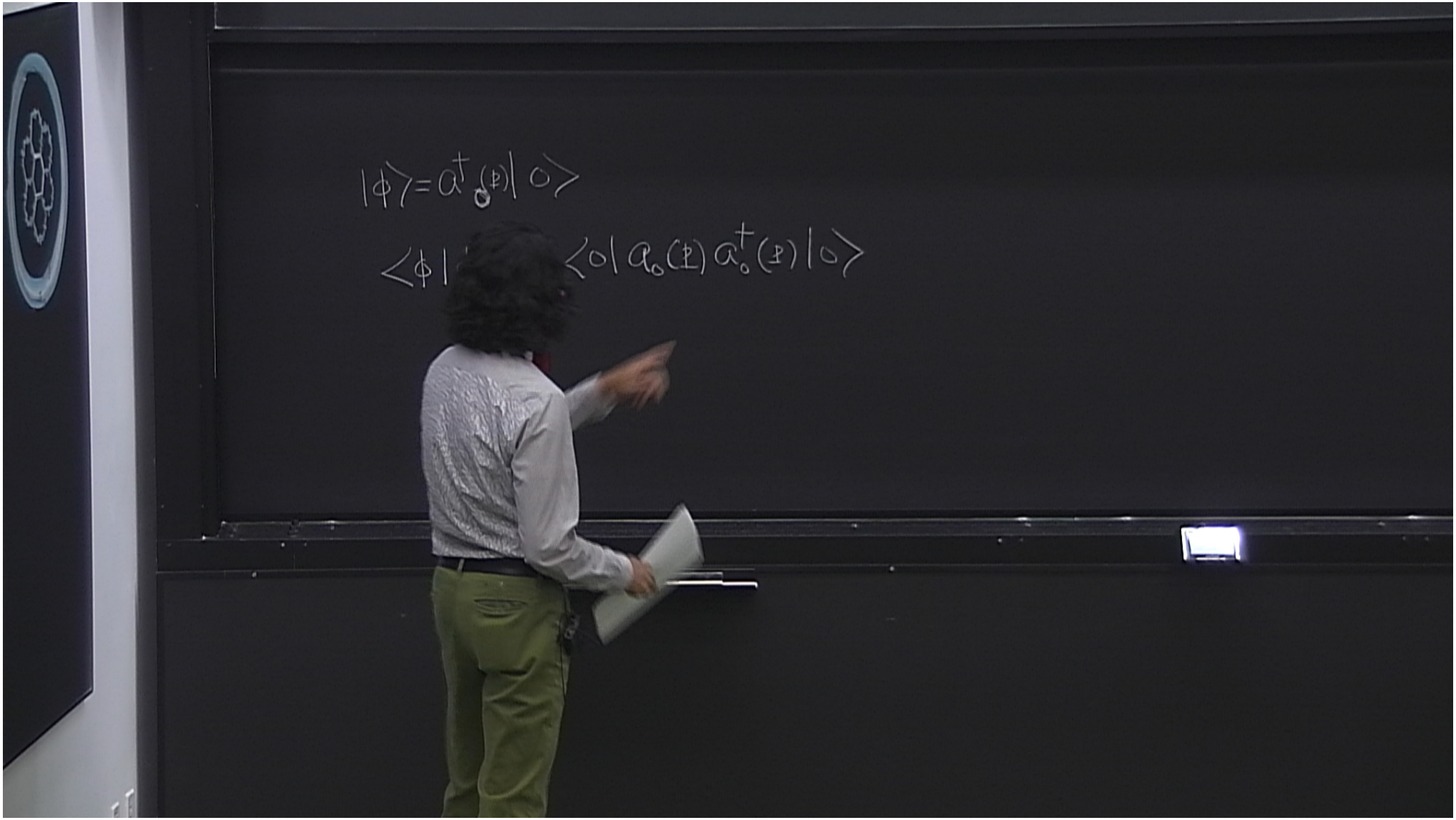
$$= \lim_{m\to 0}\left[-\eta^{\mu\nu}\Delta_F(x-y)\right] = -\eta^{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2+i\varepsilon}$$

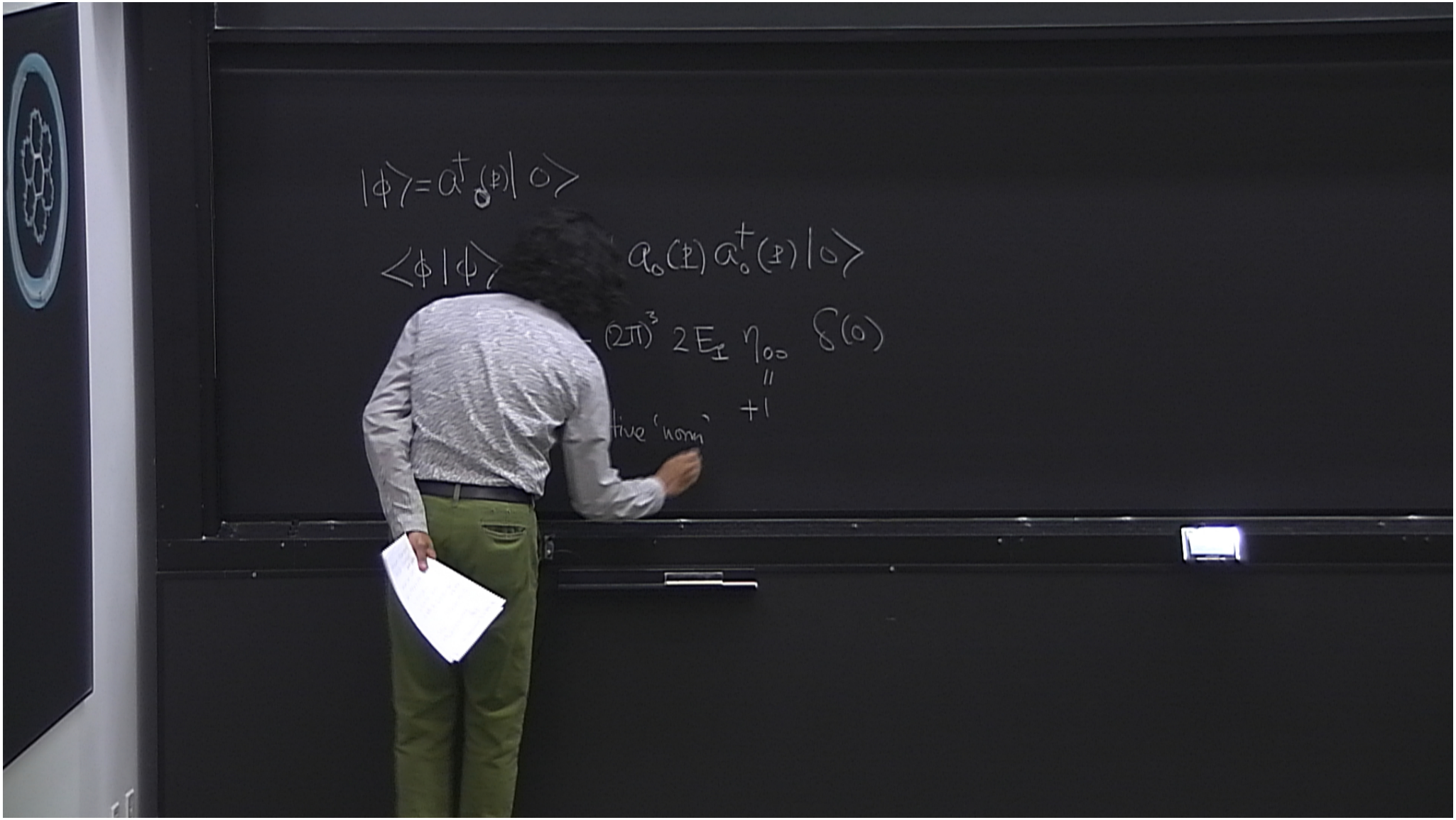
Δ_F → scalar field

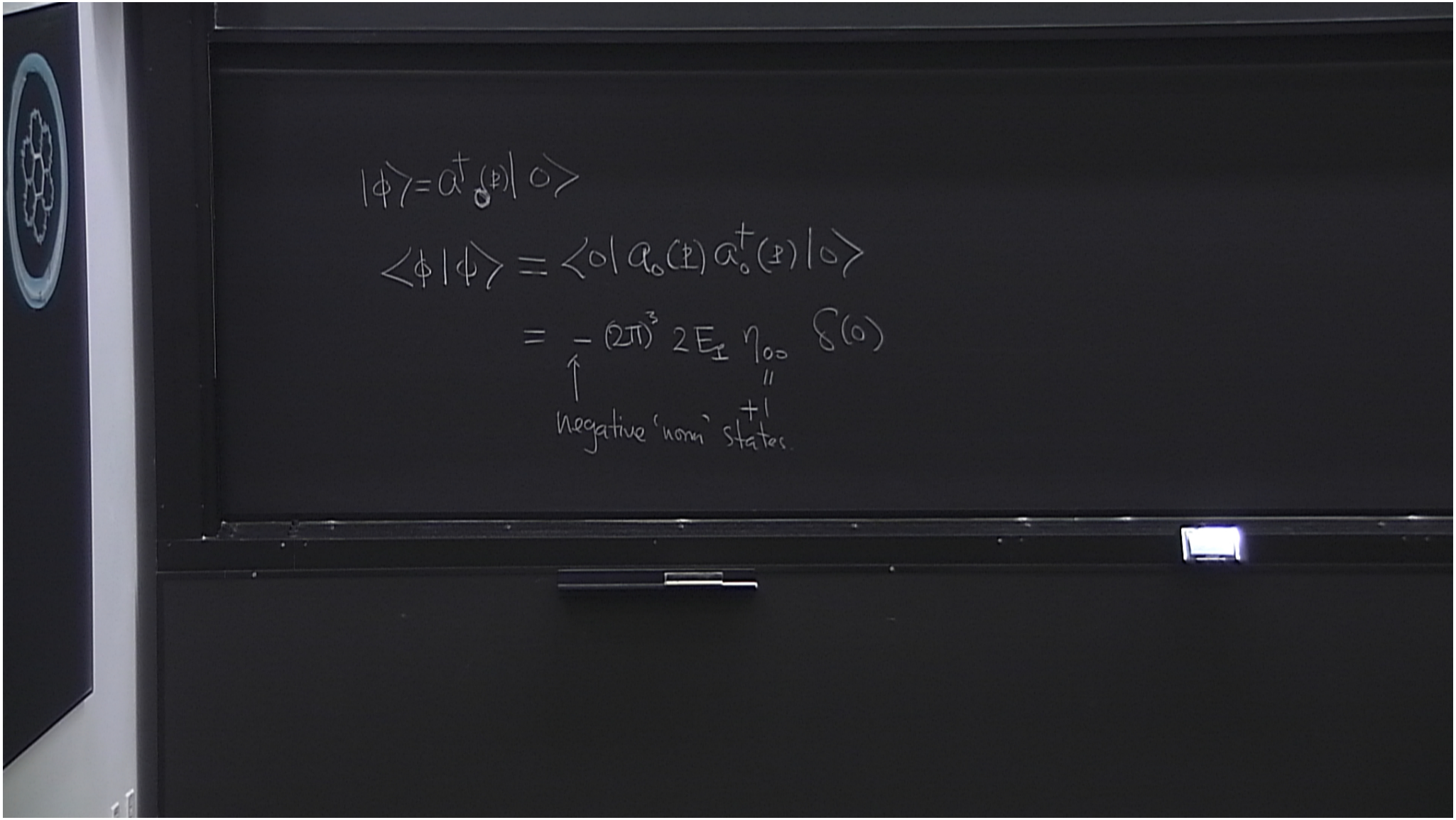
$$[a_\lambda(\underline{k}), a^\dagger_{\lambda'}(\underline{k}')] = -(2\pi)^3 2E_{\underline{k}}\,\eta_{\lambda\lambda'}\,\delta^3(\underline{k}-\underline{k}')$$

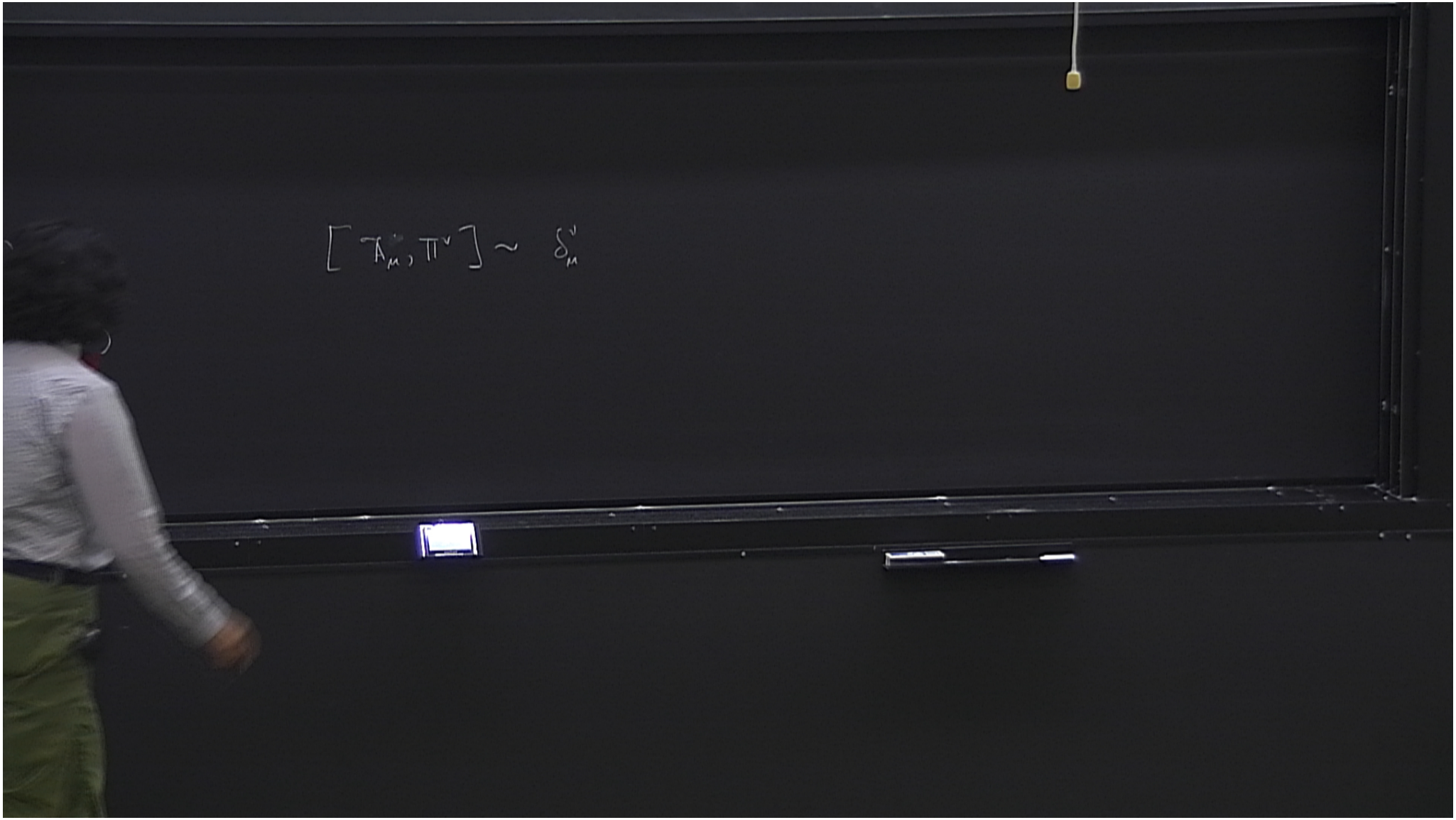
$$[a,a] = [a^\dagger, a^\dagger] = 0$$

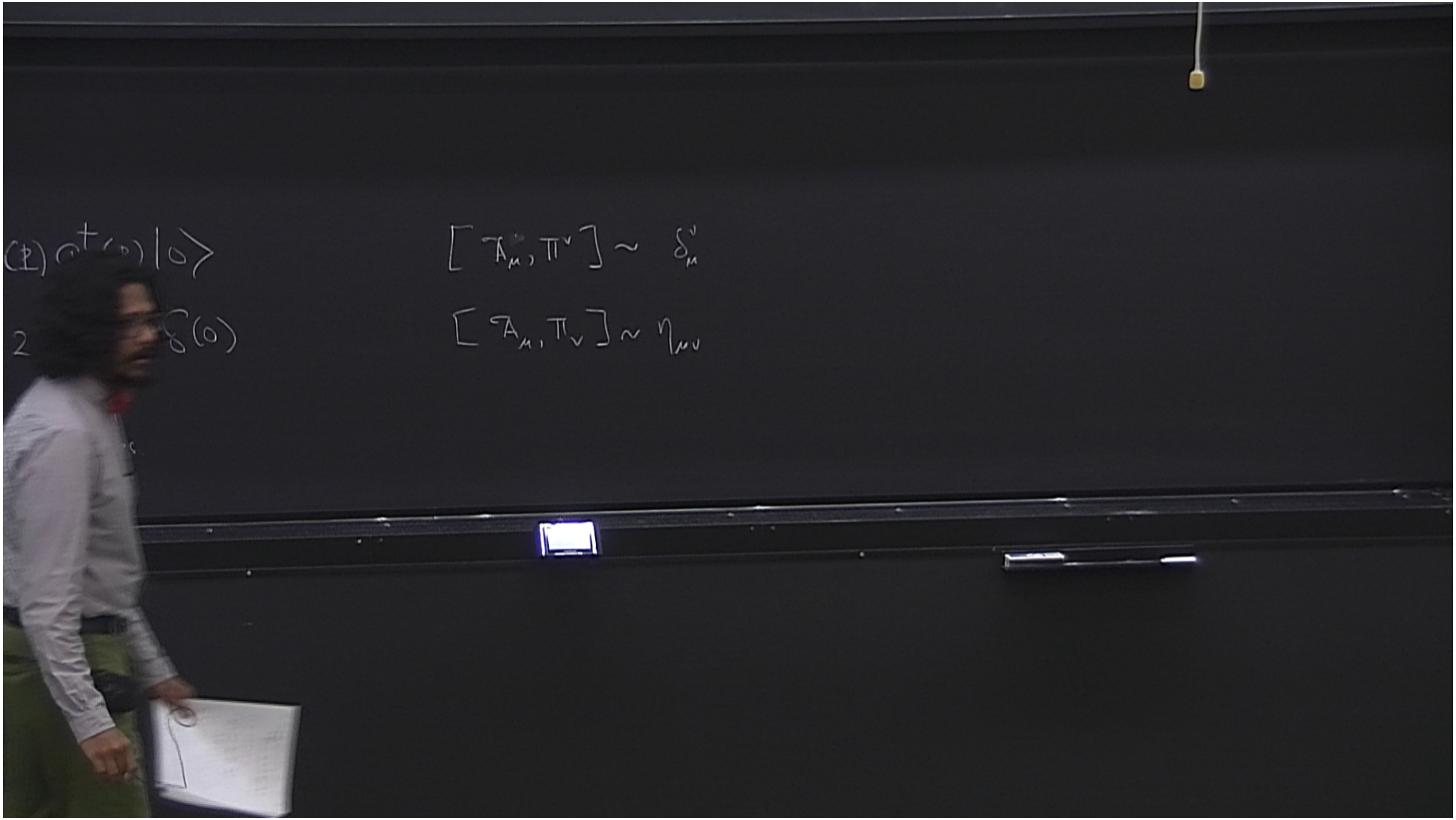
$\delta^3(\underline{x}-\underline{y})$


$|\phi\rangle = a^\dagger(\vec{p})|0\rangle$
$\langle 0|a_0(\vec{k})\, a_0^\dagger(\vec{p})|0\rangle$


$|\phi\rangle = a^\dagger(\underline{k})|0\rangle$
$\langle\phi|\phi\rangle$
$a_0(\underline{k})\, a_0^\dagger(\underline{k})|0\rangle$
$(2\pi)^3\, 2E_{\underline{k}}\, \eta_{00}\, \delta(0)$
$\eta_{00} = +1$
tive 'norm'


$|\phi\rangle = a^\dagger(\underline{k})|0\rangle$
$\langle\phi|\phi\rangle = \langle 0|a_0(\underline{k})\, a_0^\dagger(\underline{k})|0\rangle$
$= -(2\pi)^3\, 2E_{\underline{k}}\, \eta_{00}\, \delta(0)$
$\eta_{00} = +1$
negative 'norm' states


$[\hat{A}_\mu, \Pi^\nu] \sim \delta_\mu^\nu$


$[\mathcal{A}_\mu, \Pi^\nu] \sim \delta_\mu^\nu$
$[\mathcal{A}_\mu, \Pi_\nu] \sim \eta_{\mu\nu}$