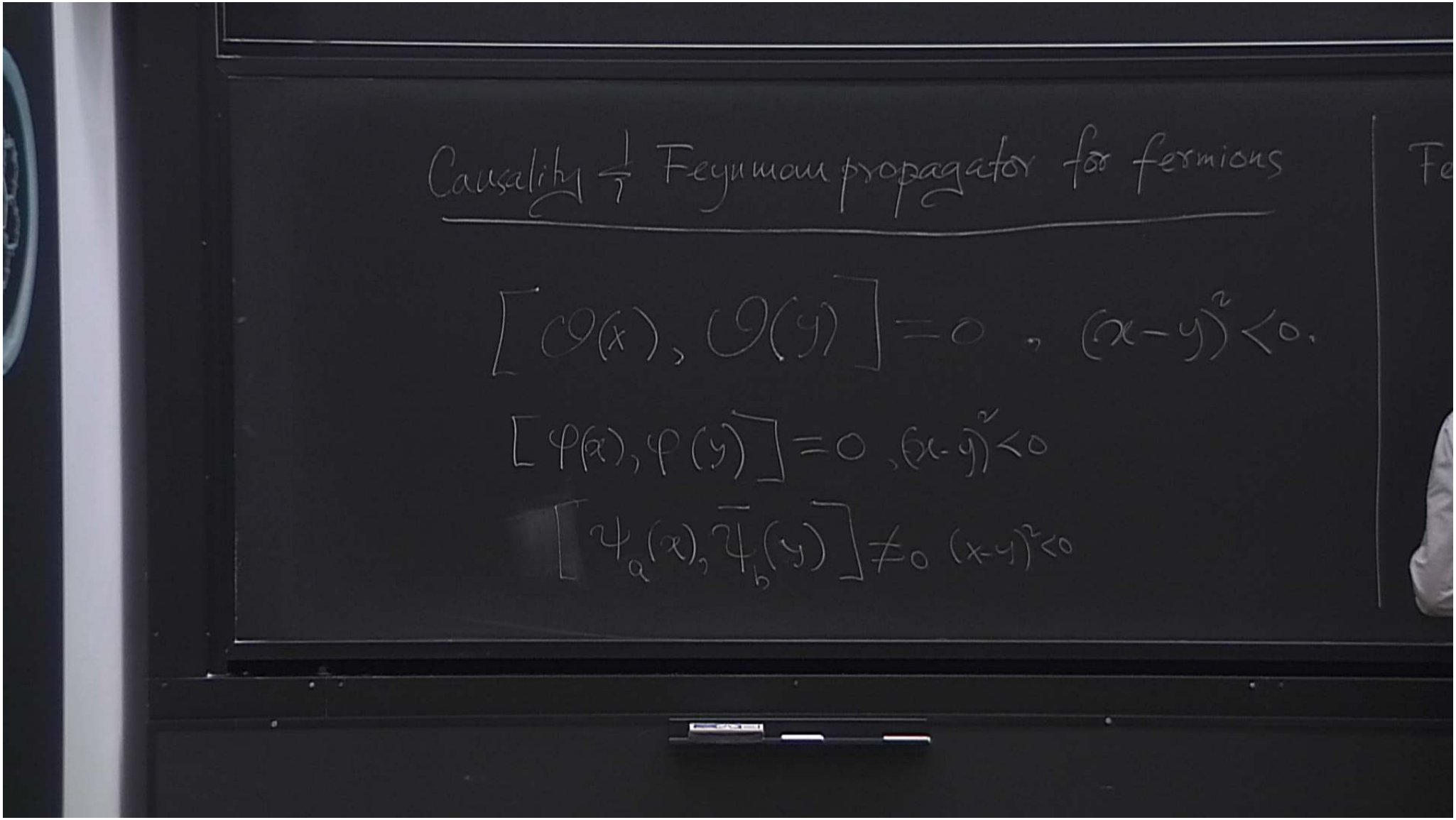
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 11

Date: Oct 27, 2015 09:00 AM

URL: http://pirsa.org/15100046

Abstract:


Causality ⇐ Feynman propagator for fermions
$[\mathcal{O}(x), \mathcal{O}(y)] = 0 \;,\; (x-y)^2 < 0.$
$[\varphi(x), \varphi(y)] = 0 \;,\; (x-y)^2 < 0$
$[\psi_a(x), \bar{\psi}_b(y)] \neq 0 \;\; (x-y)^2 < 0$

Fermionic observables: $\mathcal{O}(x) = \bar{\psi}(x) \Gamma \psi(x)$

↳ Combination of $\gamma$-matrices.

$\{\psi_a(x), \bar{\psi}_b(y)\} = 0$ for $(x-y)^2 < 0$.

Fermionic observables: $\mathcal{O}(x) = \overline{\psi}(x) \Gamma \psi(x)$

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$$\{\psi_a(x), \overline{\psi}_b(y)\} = 0 \quad \text{for } (x-y)^2 < 0.$$

$$\{\psi_a(x), \overline{\psi}_b(y)\} = \left(i \not{\partial}_x + m\right)_{ab} \left[D(x-y) - D(y-x)\right]$$

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)}$$

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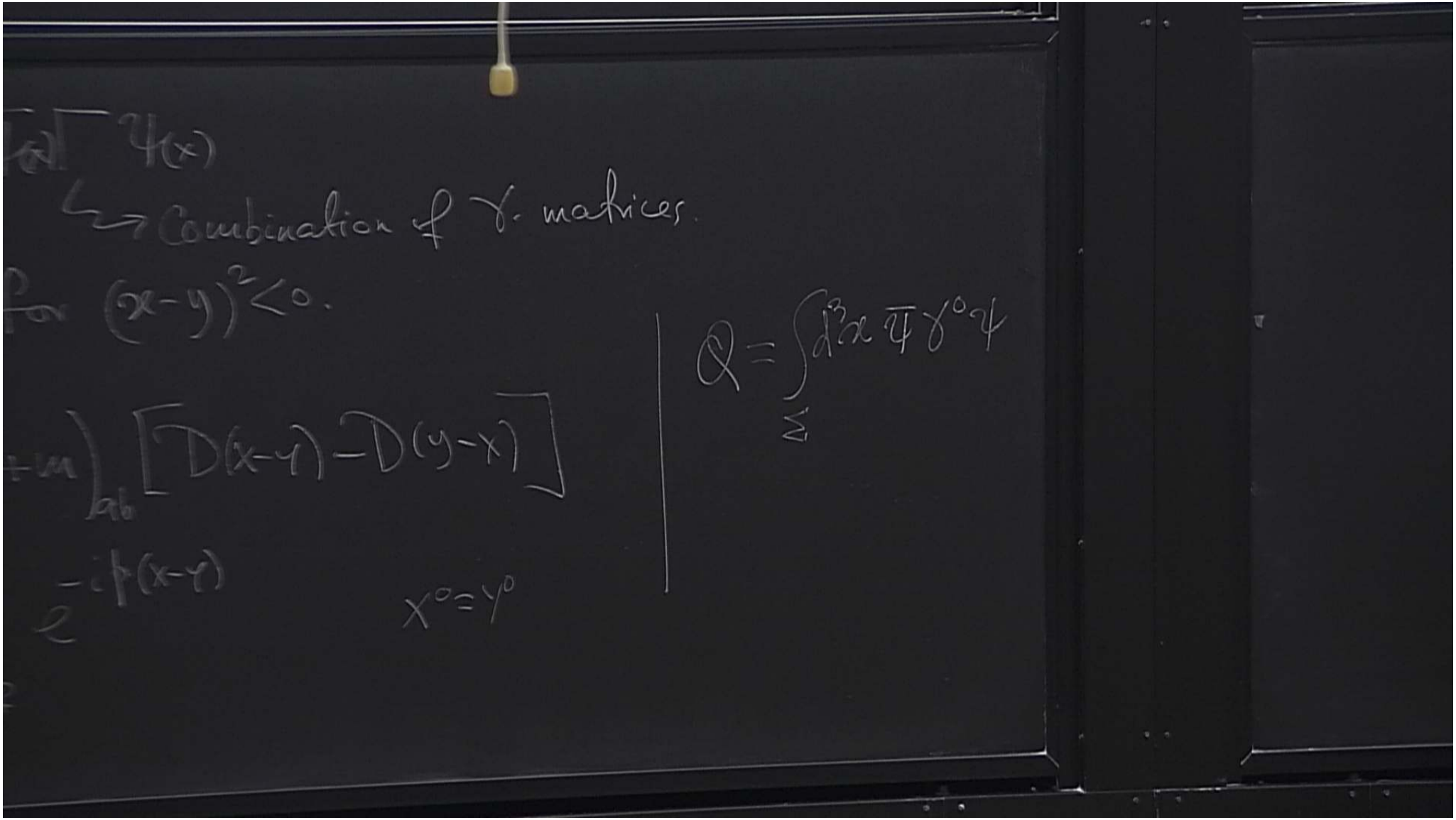
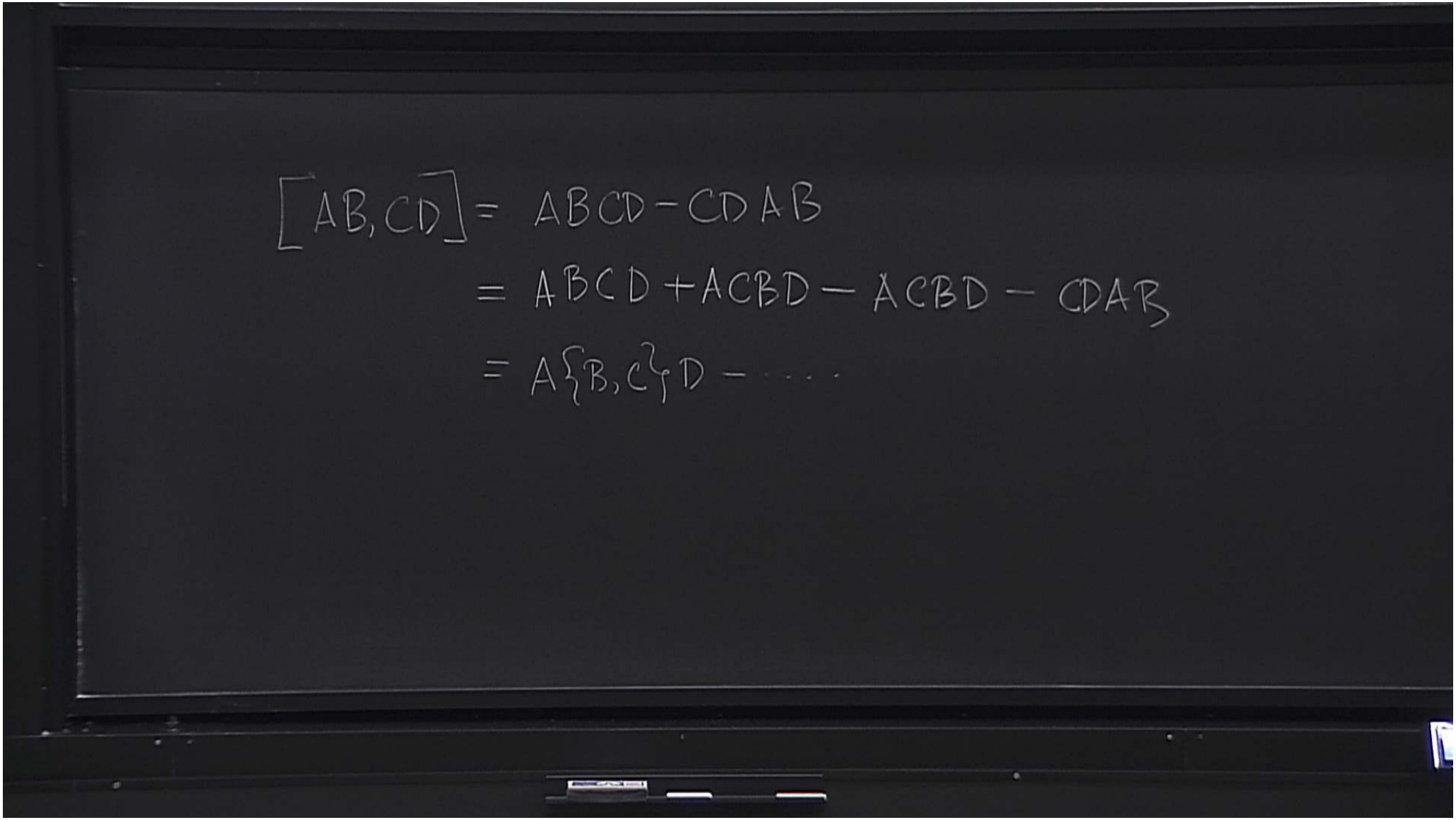
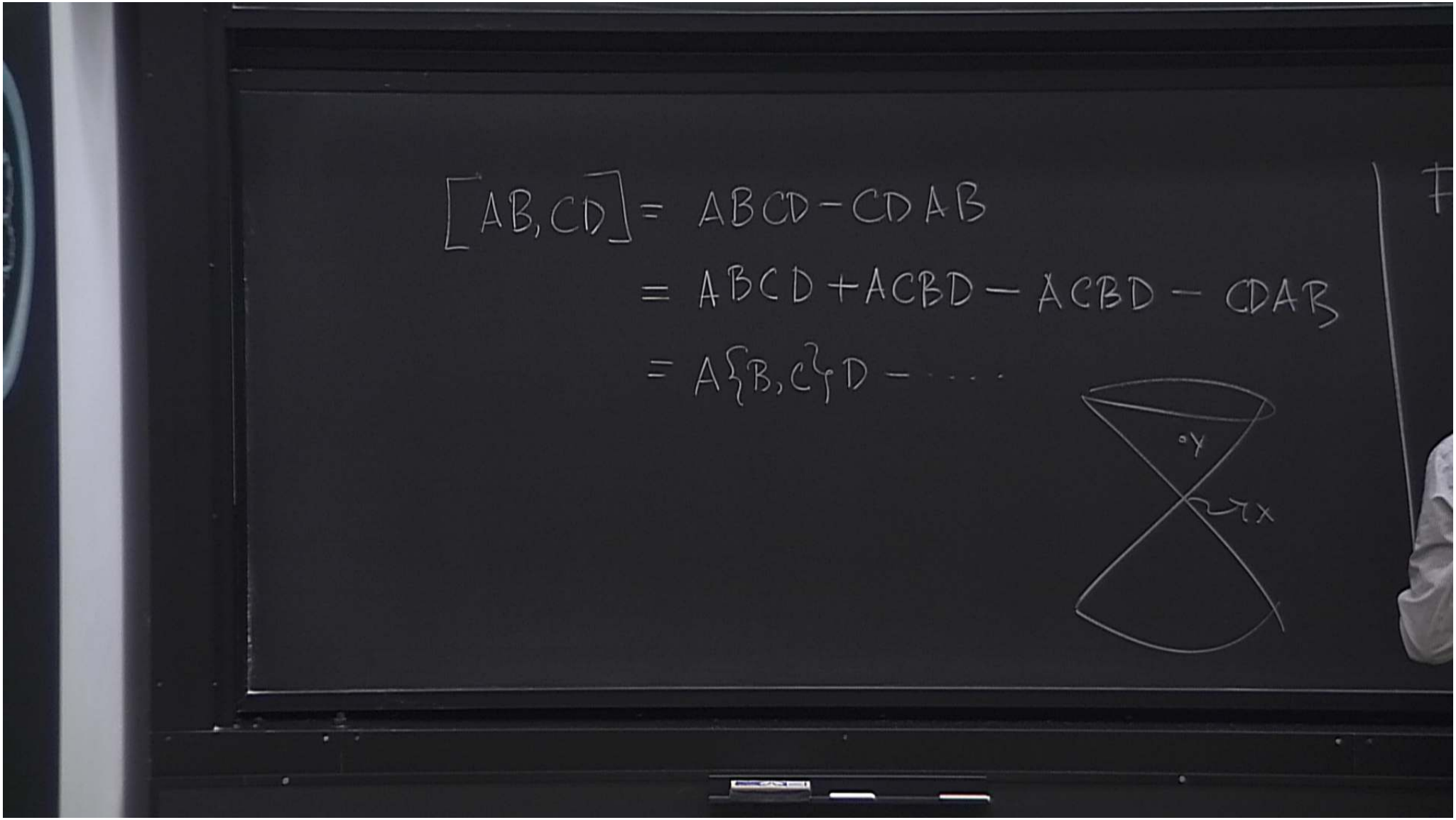
$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)} \qquad x^0 = y^0$$


$\psi(x)$
↳ Combination of $\gamma$-matrices.
for $(x-y)^2 < 0$.
$[D(x-y) - D(y-x)]$
$x^0 = y^0$
$Q = \int_\Sigma d^3x\, \bar{\psi} \gamma^0 \psi$


[AB, CD] = ABCD − CDAB
= ABCD + ACBD − ACBD − CDAB
= A{B, C}D − ...


[AB, CD] = ABCD − CDAB
= ABCD + ACBD − ACBD − CDAB
= A{B, C}D − . . . .
Y
X

Feynman propagator:

$$S_F(x-y)_{ab} = [\langle 0|T(\psi_a(x)\bar{\psi}_b(y))|0\rangle]$$

$$= \begin{cases} \langle 0|\psi_a(x)\bar{\psi}_b(y)|0\rangle & \text{if } x^0 > y^0 \\ -\langle 0|\bar{\psi}_a(y)\psi_b(x)|0\rangle & \text{if } y^0 > x^0 \end{cases}$$

$$S_F = \langle 0|T(\psi\bar{\psi})|0\rangle$$

Feynman propagator:

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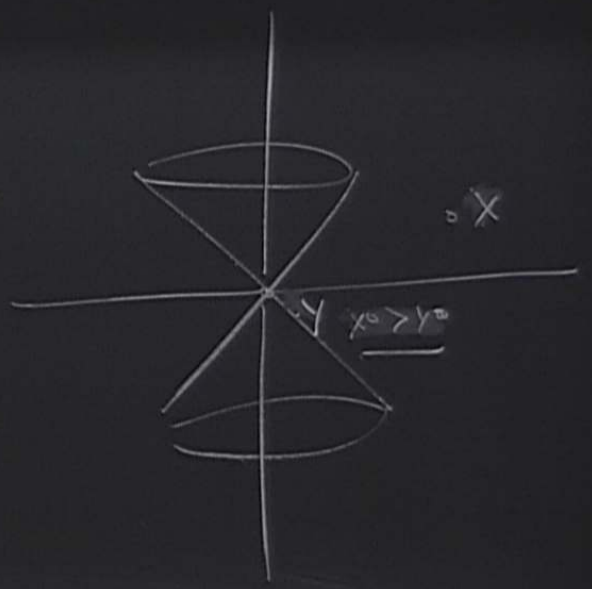
$$S_F = \langle 0|T(\psi\bar{\psi})|0\rangle$$



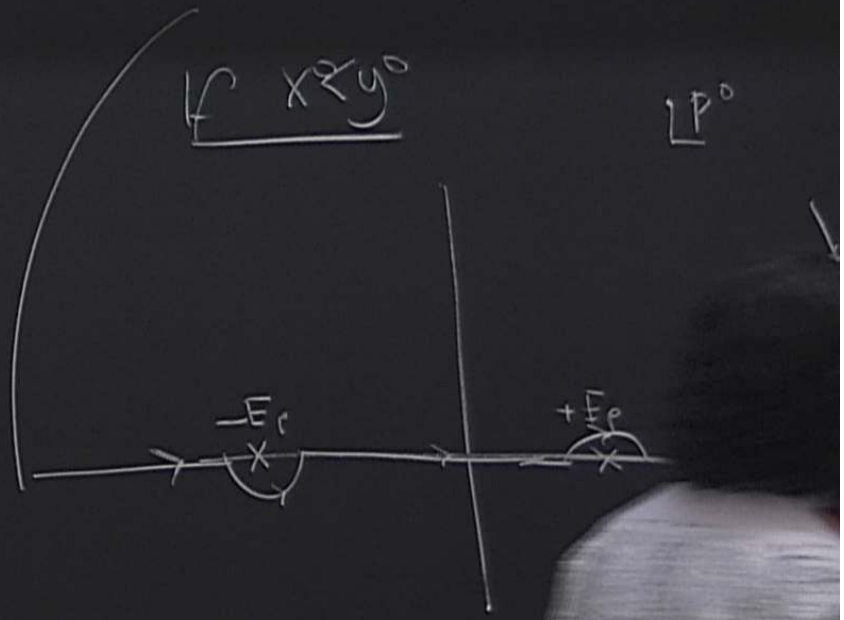
Explicitly:

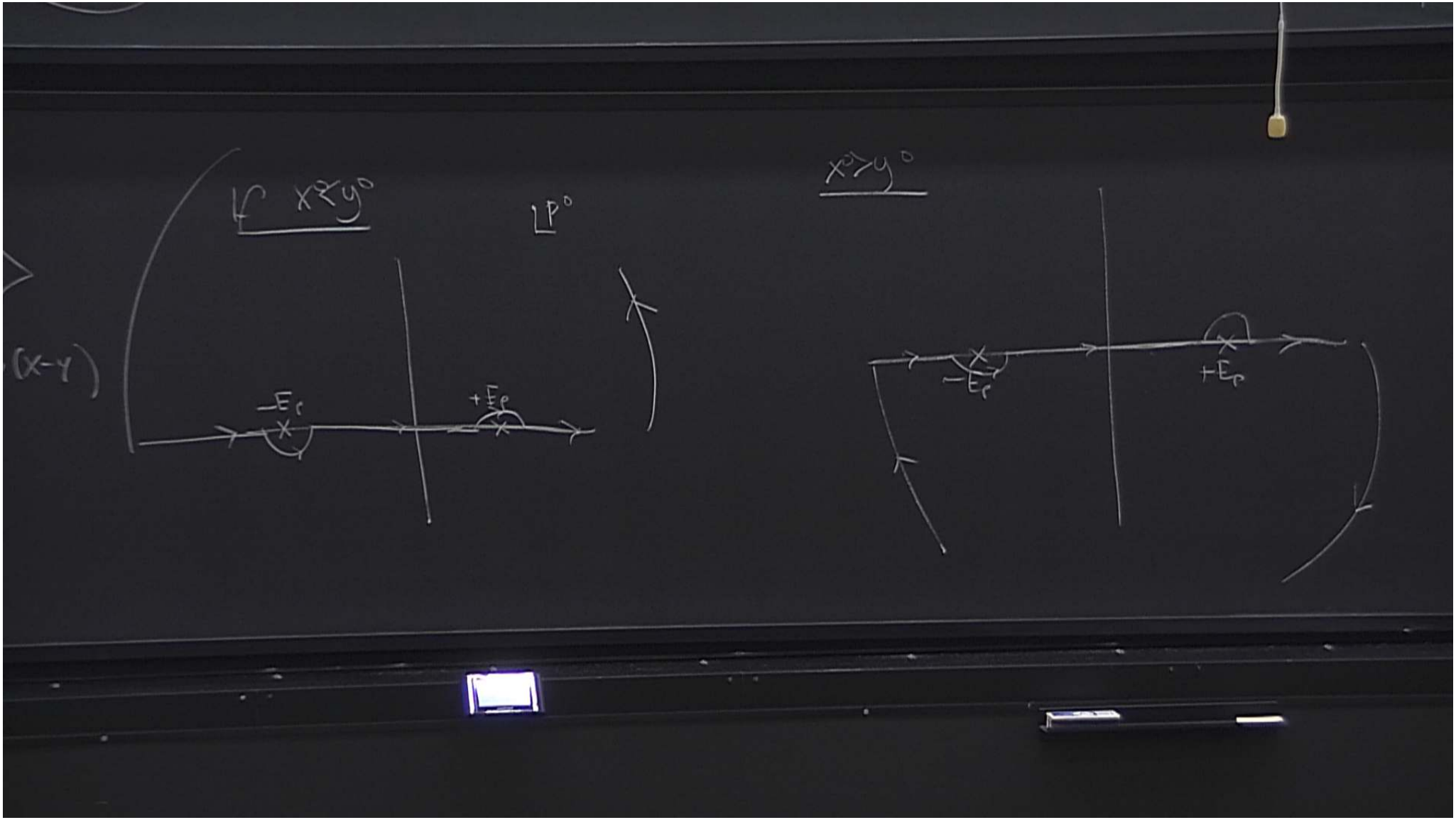
$$S_F = \langle 0 | T(\psi(x)\overline{\psi}(y)) | 0 \rangle$$

$$= i \int \frac{d^4p}{(2\pi)^4} \frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$$

If $x^0 > y^0$

$p^0$




If x<y°
LP°
x>y°
(x-y)
-Ep
+Ep
-Ep
+Ep

Explicitly:

$$S_F = \langle 0 | T(\psi(x)\overline{\psi}(y)) | 0 \rangle$$

$$= i \int \frac{d^4p}{(2\pi)^4} \frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$$

If $x^0 < y^0$

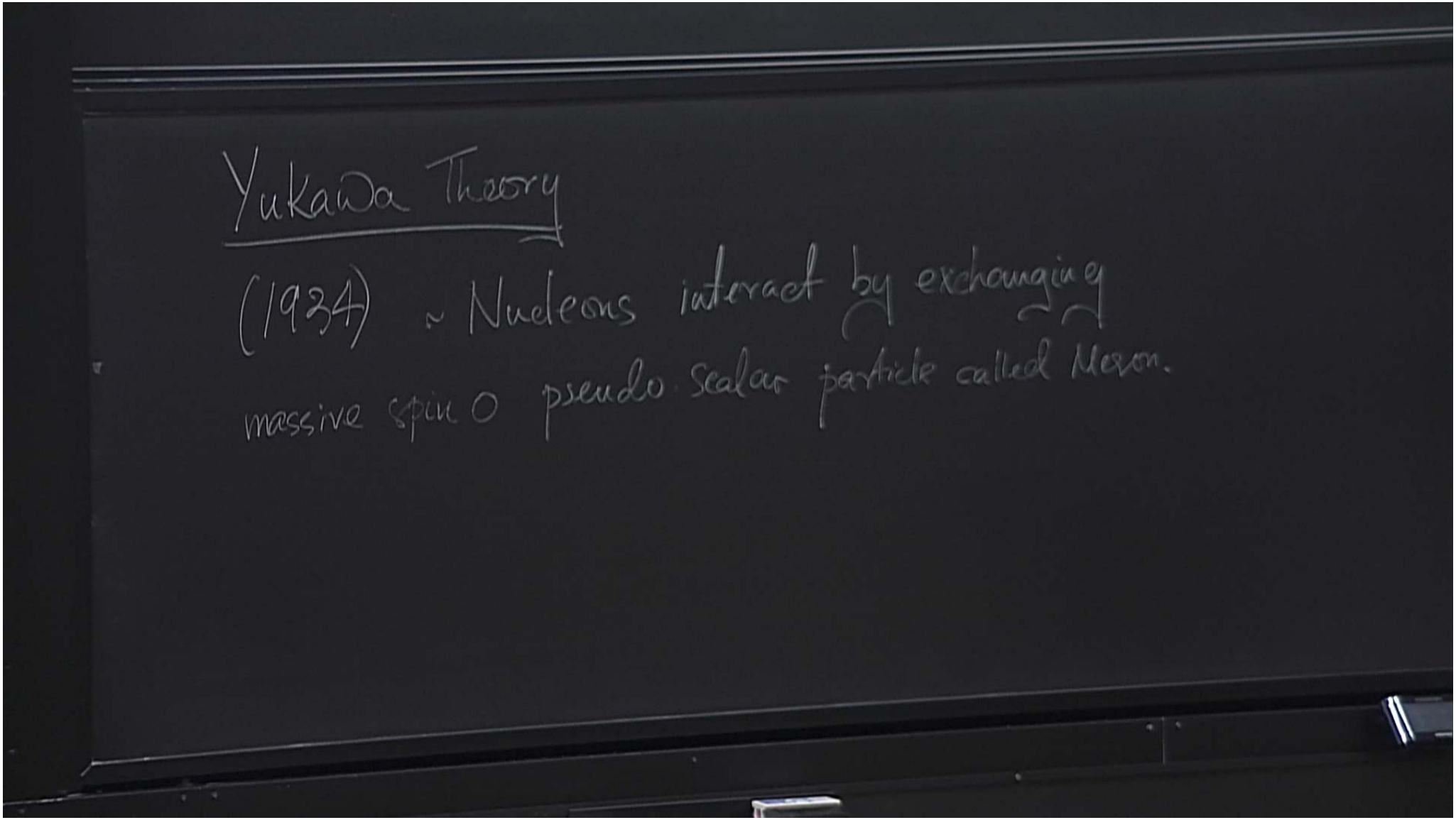
$x^0$

$-E_p$

$+E_p$

x

x


Yukawa Theory
(1934) ~ Nucleons interact by exchanging
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# Yukawa Theory

# (1934) ~ Nucleons interact by exchanging massive spin 0 pseudo scalar particle called Meson.

# A generalization of Yukawa interaction plays the role renormalizable coupling between Chiral fermions

and

and Higgs field in the SM which

respects the gauge symmetries of

the theory.

$$\mathcal{L}_{incl.} = \mathcal{L}_f + \mathcal{L}_\varphi + \mathcal{L}_{yuk}$$

$$\mathcal{L}_f = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

$$\mathcal{L}_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M^2\varphi^2 \quad , m > M$$

$$\mathcal{L}_{yuk} = -\lambda\varphi\bar{\psi}\psi$$

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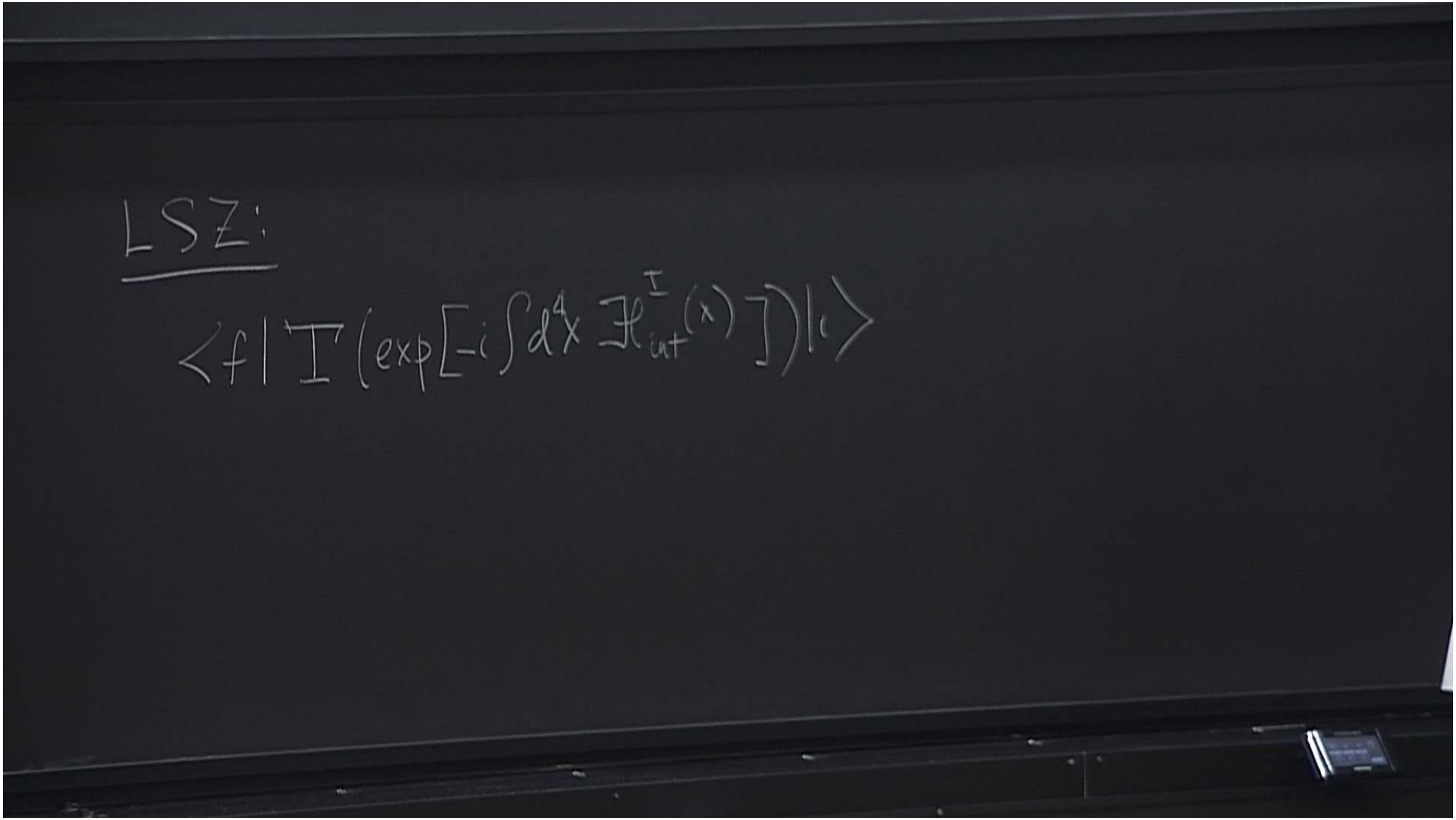
$$\mathcal{L}_{yuk} = -\lambda\varphi\bar{\psi}\psi \qquad [\lambda] = 0$$

U(1) symmetry

$$\psi \to e^{i\alpha}\psi$$

$$\bar{\psi} \to e^{-i\alpha}\psi$$


LSZ:
$\langle f | T\left(\exp\left[-i\int d^4x\, \mathcal{H}_{int}^{I}(x)\right]\right)|i\rangle$

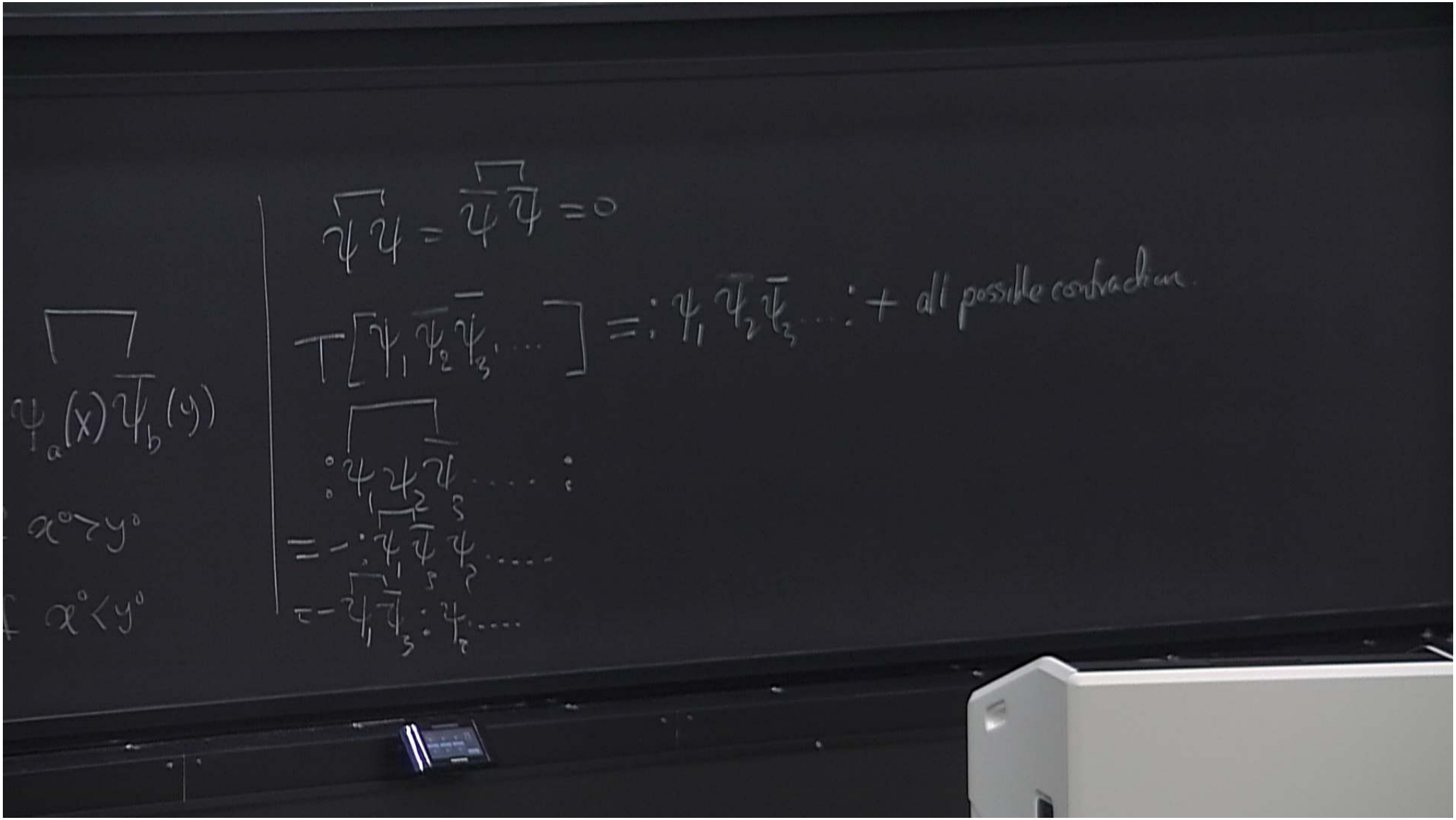
LSZ:

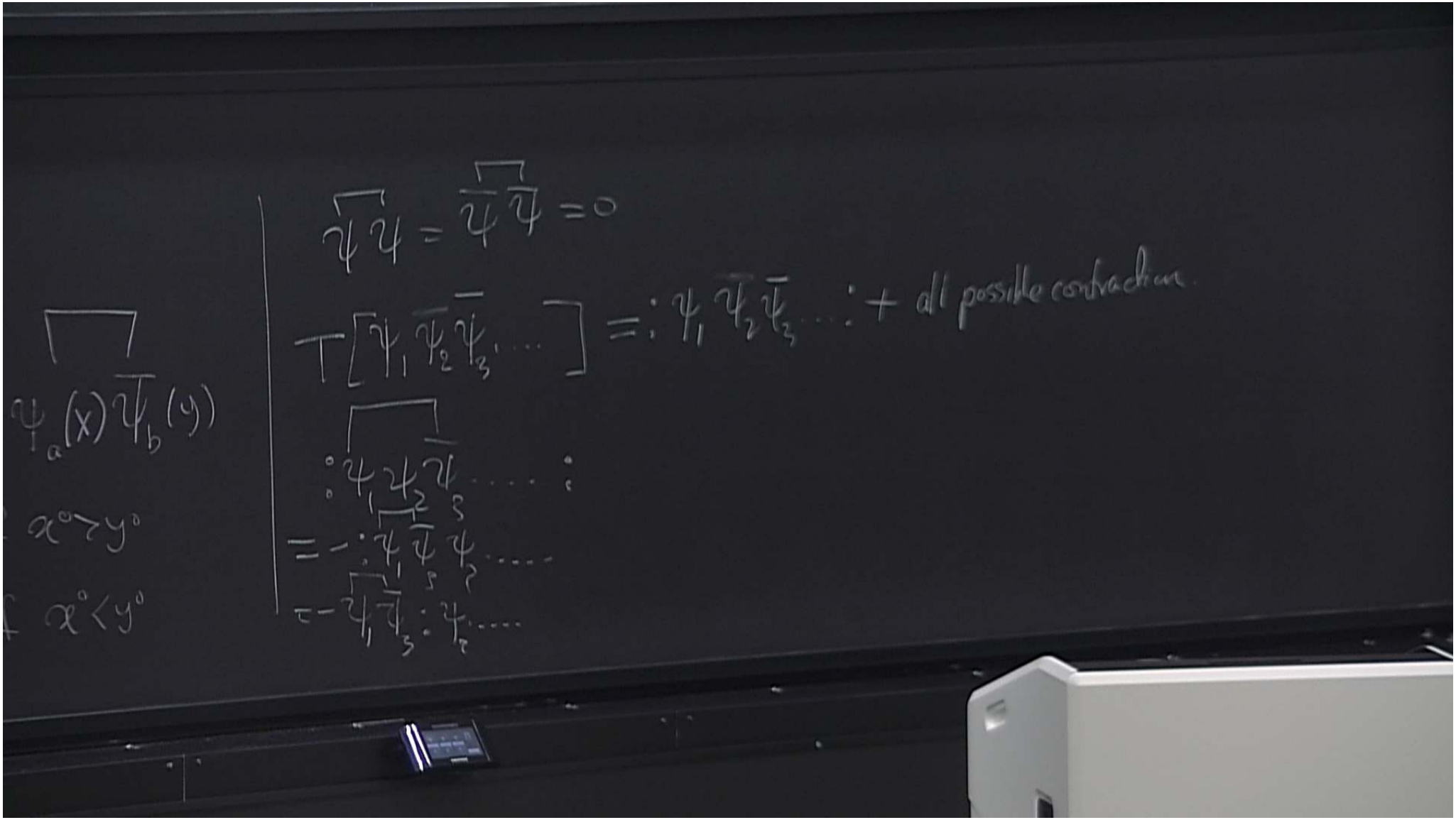
$$\langle f | T\left(\exp\left[-i\int d^4x\, \mathcal{H}_{int}^{I}(x)\right]\right)|i\rangle$$

$$T[\psi_a(x)\bar{\psi}_b(y)] = :\psi_a(x)\bar{\psi}_b(y): + \psi_a(x)\bar{\psi}_b(y)$$

$$\overbrace{\psi_a(x)\bar{\psi}_b(y)} = \begin{cases} \{\psi_a^{(+)}(x), \bar{\psi}_b^{(-)}(y)\}, & \text{if } x^0 > y^0 \\ -\{\bar{\psi}_a^{(+)}(y), \psi_b^{(-)}(x)\} & \text{if } x^0 < y^0 \end{cases}$$

$\overbrace{\psi\bar{\psi}}$


$\psi_a(x)\bar{\psi}_b(y)$
$x^0 > y^0$
$x^0 < y^0$
$\psi\psi = \bar{\psi}\bar{\psi} = 0$
$T[\psi_1 \bar{\psi}_2 \psi_3 \cdots] =: \psi_1 \bar{\psi}_2 \psi_3 \cdots :$ + all possible contractions
$: \psi_1 \psi_2 \bar{\psi}_s \cdots :$
$= - : \psi_1 \bar{\psi}_s \psi_2 \cdots$
$\mp \cdots$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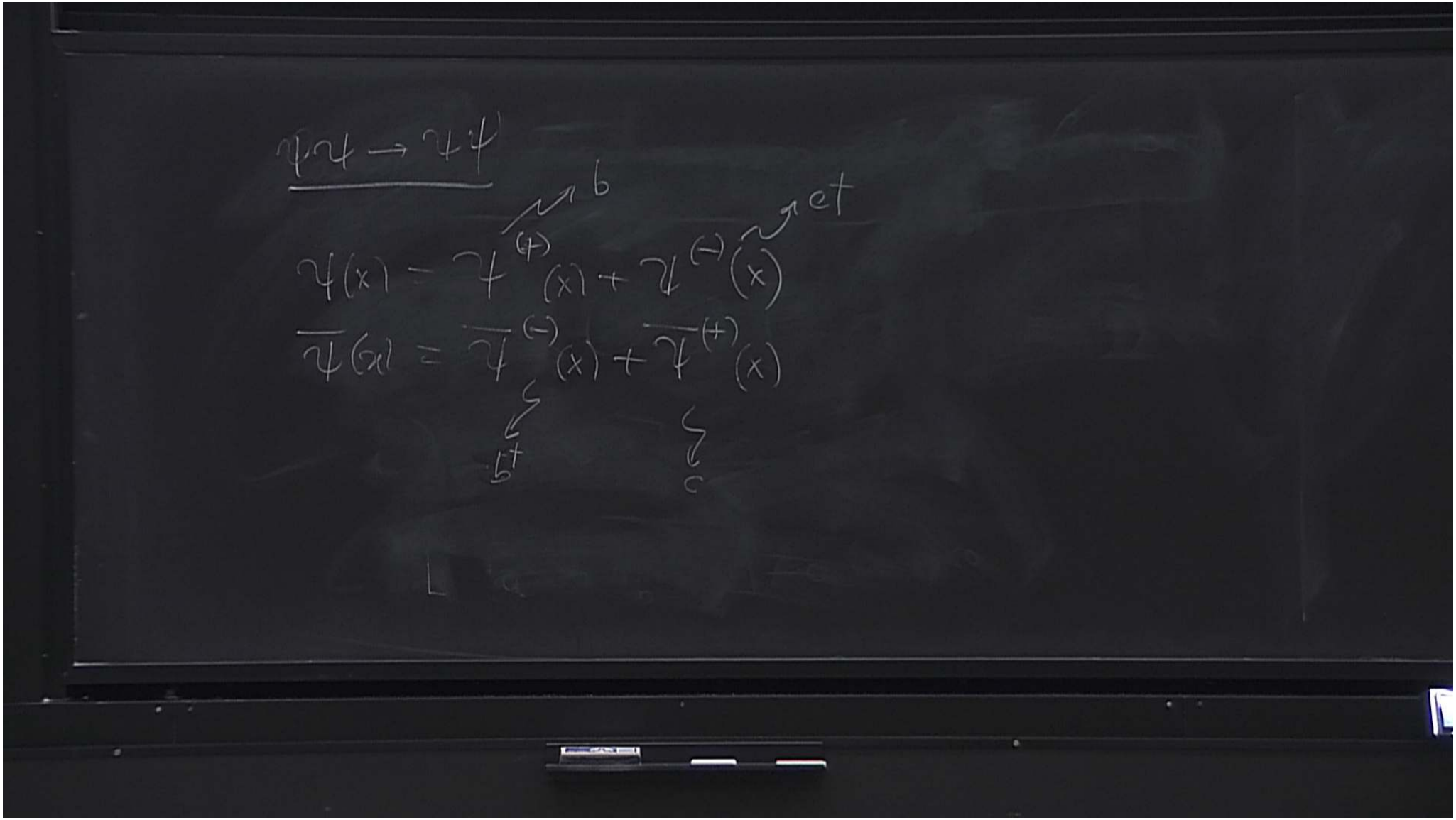

ψ·ψ → ψψ
ψ(x) = ψ(+)(x) + ψ(−)(x)
b
c
ψ̄(x) = ψ̄(−)(x) + ψ̄(+)(x)
b†
c

$\varphi, \psi \to \psi, \psi$

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x)$$

$$\overline{\psi}(x) = \overline{\psi}^{(-)}(x) + \overline{\psi}^{(+)}(x)$$

$$\varphi(x) = \varphi^{(+)} + \varphi^{(-)}$$

$|in\rangle = b_s^\dagger(k)\, b_r^\dagger(p)\,|0\rangle$

$|final\rangle = b_{s'}^\dagger(k')\, b_{r'}^\dagger(p')\,|0\rangle$

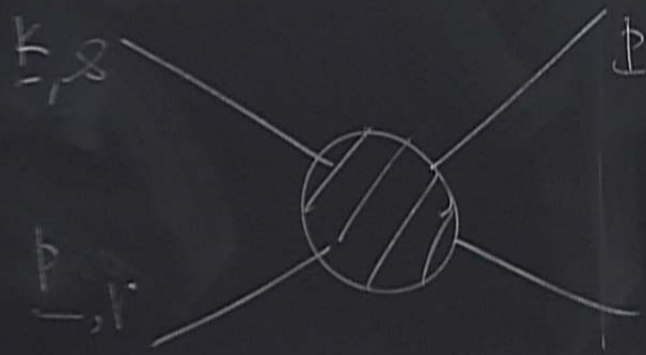
$\langle final| = \langle 0|\, b_{r'}(p')\, b_{s'}(k')$

$H_{int} = \lambda \int d^3x\; \varphi(x)\,\bar{\psi}(x)\,\psi(x)$

$|in\rangle = b^\dagger_s(\underline{k})\, b^\dagger_r(\underline{p})\,|0\rangle$

$|final\rangle = b^\dagger_{s'}(\underline{k}')\, b^\dagger_{r'}(\underline{p}')\,|0\rangle$

$\langle final| = \langle 0|\, b_{r'}(\underline{p})\, b_{s'}(\underline{k}')$




$H_{int} = \lambda \int d^3x\; \varphi(x)\,\bar{\psi}(x)\,\psi(x)$

$\varphi(x)$

$\varphi^{(+)}|in\rangle = 0$

$\langle final|\,\varphi^{(-)} = 0$

$\mathcal{O}(\lambda^2)$:

$$iT = \frac{(-i\lambda)^2}{2!}\langle f|T\int d^4x_1 d^4x_2\,(\varphi\bar{\psi}\psi)_1(\varphi\bar{\psi}\psi)_2|i\rangle$$

$$= \frac{(-i\lambda)^2}{2!}\langle f|\int d^4x_1 d^4x_2\,:(\bar{\psi}\psi)_1(\bar{\psi}\psi)_2:|in\rangle\,\overbrace{\varphi_1\varphi_2}$$

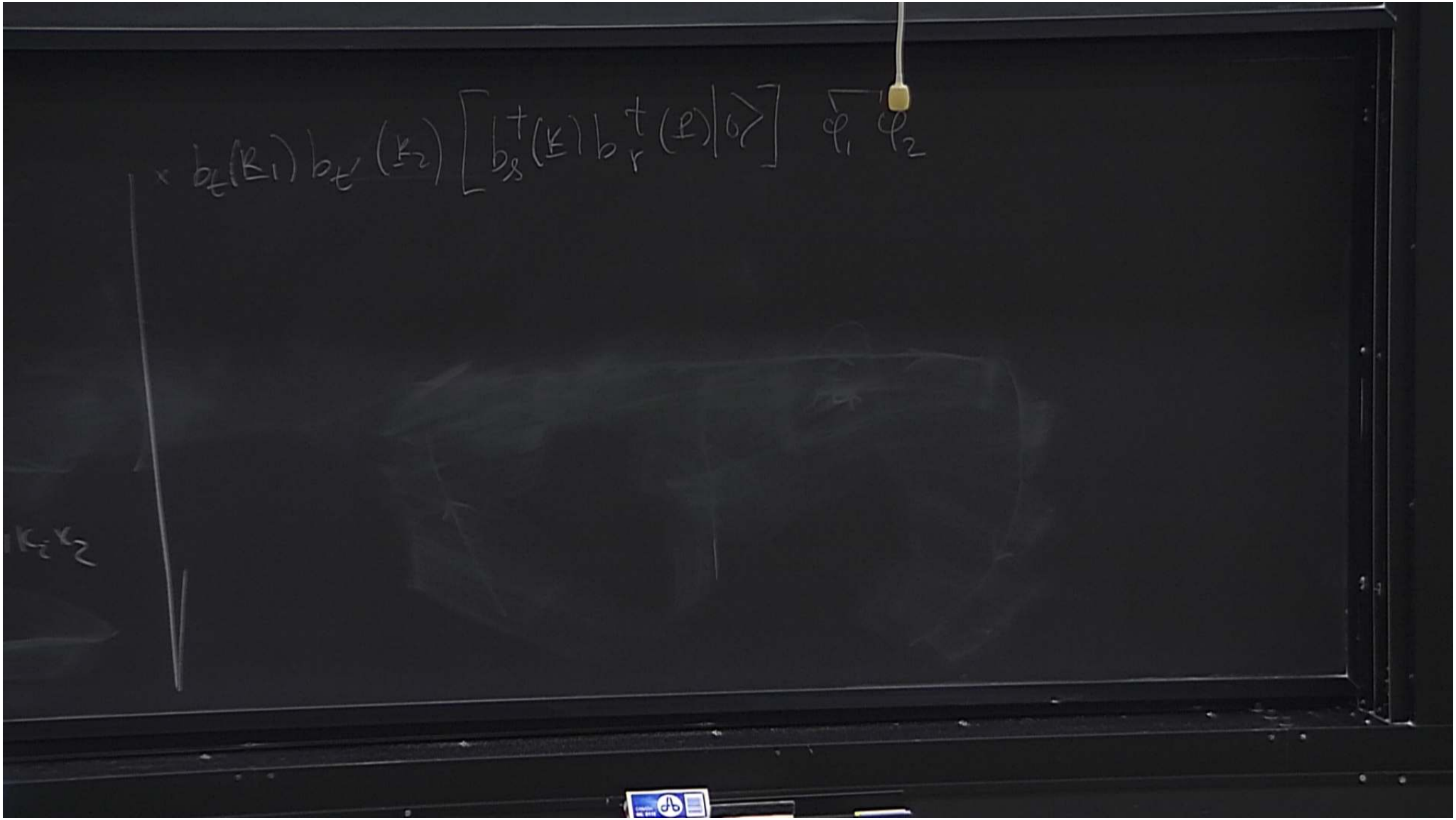
$$= -\frac{(i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 2E_{k_1}E_{k_2}}\langle f|\sum_{t,t'}\overline{\psi_1^{(-)}\cdot u_t(k_1)}\,\overline{\psi_2^{(-)}\cdot u_{t'}(k_2)}\,e^{-ik}$$

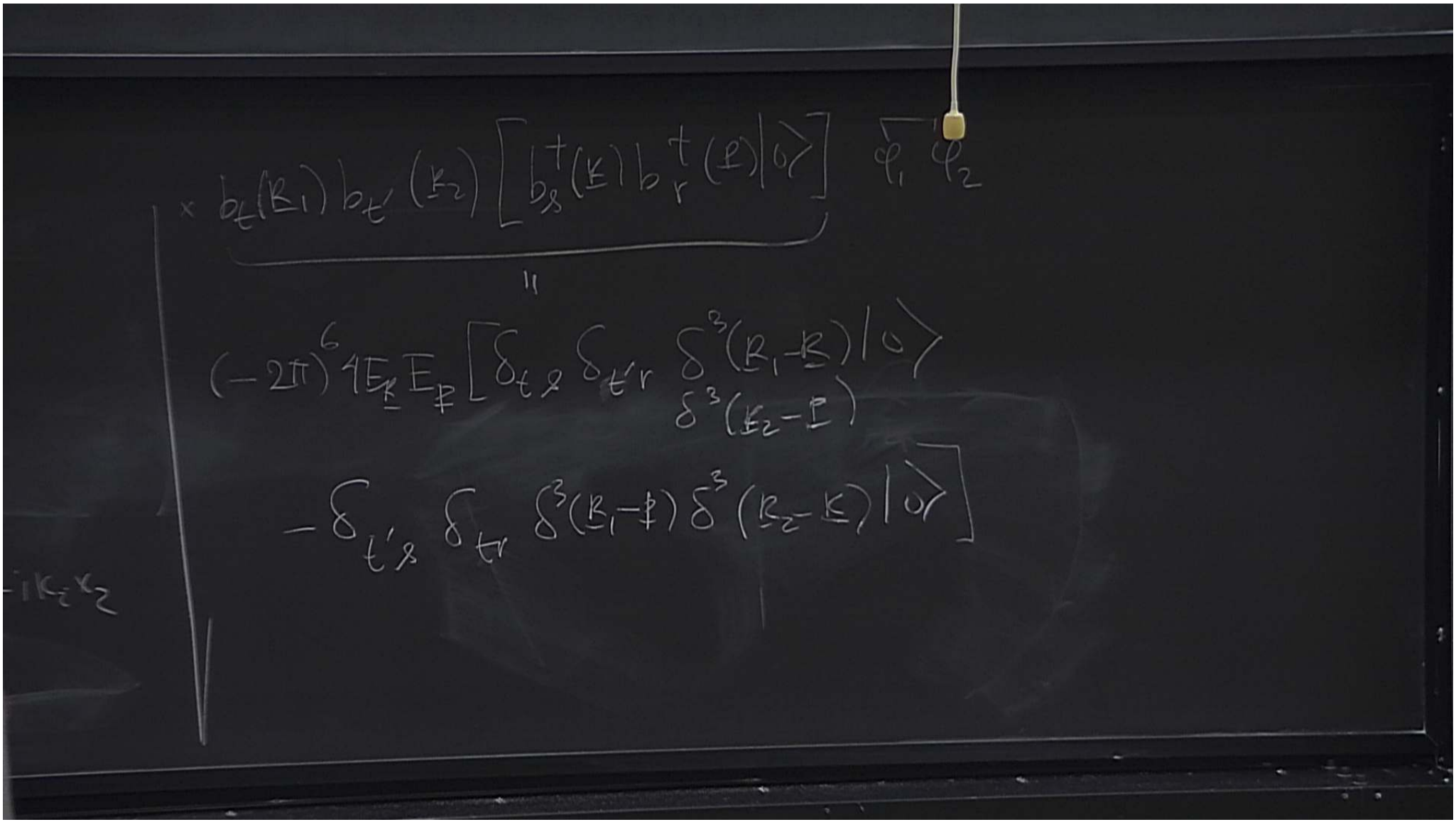
$\mathcal{O}(\lambda^2)$:

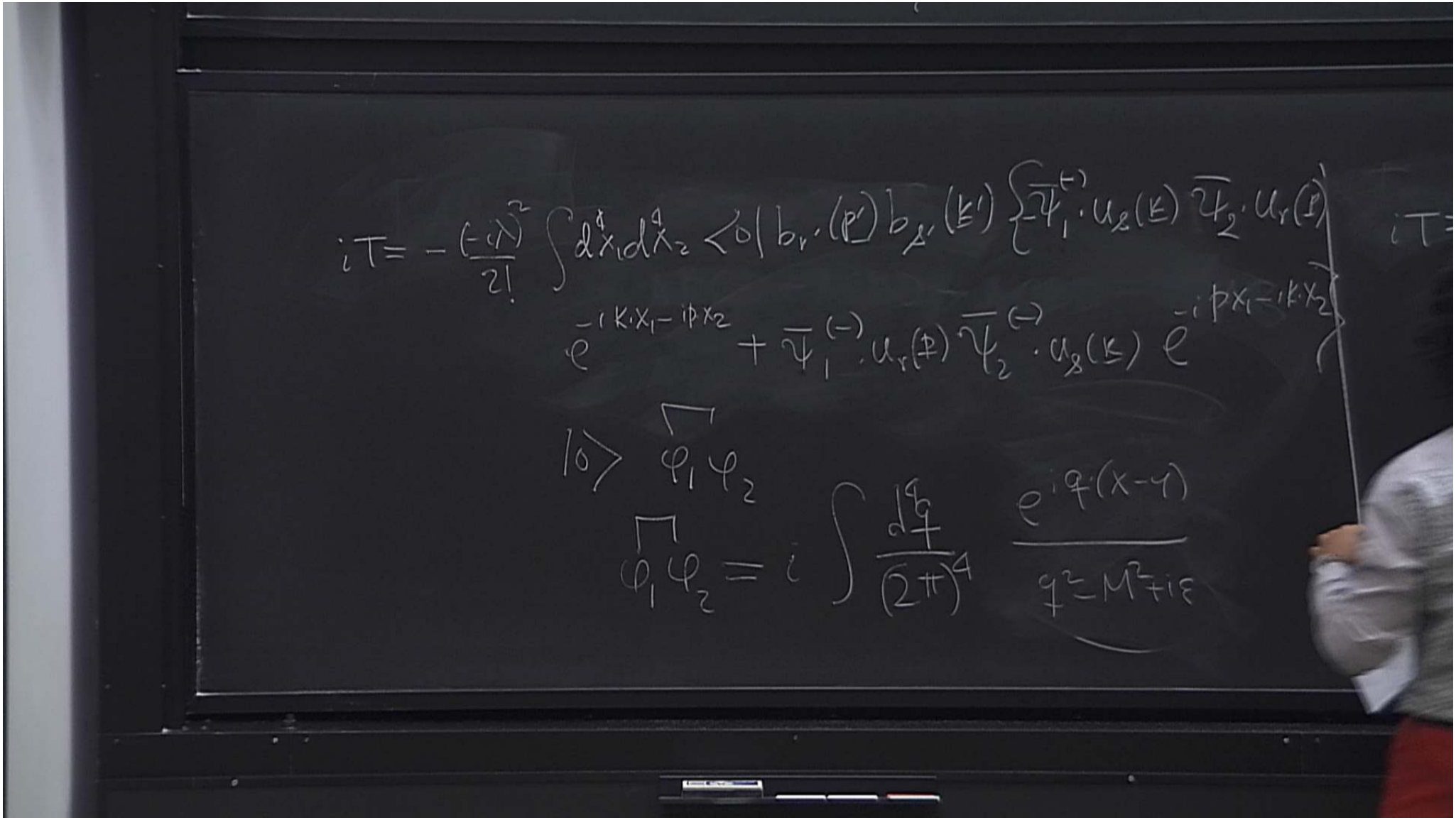
$$iT = \frac{(-i\lambda)^2}{2!} \langle f| T \int d^4x_1 d^4x_2 \, (\varphi\bar{\psi}\psi)_1 (\varphi\bar{\psi}\psi)_2 |i\rangle$$

$$= \frac{(-i\lambda)^2}{2!} \langle f| \int d^4x_1 d^4x_2 \, :(\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2: |in\rangle \, \varphi_1 \varphi_2$$

$$= -\frac{(i\lambda)^2}{2!} \int d^4x_1 d^4x_2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 2E_{k_1} 2E_{k_2}} \langle f| \sum_{t,t'} \bar{\psi}_1^{(-)} \cdot u_t(k_1) \, \bar{\psi}_2^{(-)} \cdot u_{t'}(k_2) \, e^{-ik_1x_1 - ik_2x_2}$$


$\times b_t(k_1)\, b_{t'}(k_2) \left[ b_s^\dagger(k)\, b_r^\dagger(p)|0\rangle \right] \varphi_1\, \varphi_2$


$\times b_t(k_1)\, b_{t'}(k_2) \underbrace{\left[ b_s^\dagger(k)\, b_r^\dagger(p)|0\rangle \right]}_{=} \varphi_1\, \varphi_2$
$(-2\pi)^6 4E_k E_p \left[ \delta_{ts}\delta_{t'r}\, \delta^3(k_1-k)\, \delta^3(k_2-p) \right.$
$\left. - \delta_{t's}\delta_{tr}\, \delta^3(k_1-p)\, \delta^3(k_2-k) \right]|0\rangle$
$-ik_2x_2$


$iT = -\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \langle 0|\, b_r(p')\, b_s(k')\, \{\bar{\psi}_1^{(-)} \cdot u_s(k)\, \bar{\psi}_2 \cdot u_r(p)\, e^{-ipx_1 - ikx_2} + \bar{\psi}_1^{(-)} \cdot u_r(p)\, \bar{\psi}_2^{(-)} \cdot u_s(k)\, e^{-ipx_1 - ikx_2}\}$
$|0\rangle\ \overbrace{\varphi_1 \varphi_2}$
$\overbrace{\varphi_1 \varphi_2} = i \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq(x-y)}}{q^2 - M^2 + i\varepsilon}$

$$iT = (-i\lambda)^2 \int \frac{d^4q}{q^2 - M^2 + i\varepsilon} \frac{(2\pi)^4}{} \Big[ \big(\bar{u}_{r'}(p') u_s(p)\big)\big(\bar{u}_{s'}(k') u_r(k)\big)\, \delta^4(p' - k + q)\, \delta^4(k' - p - q)$$

$$- \big(\bar{u}_{s'}(k') u_s(k)\big)\big(\bar{u}_{r'}(p') u_r(p)\big)\, \delta^4(k' - k + q)\, \delta^4(p' - p + q)$$

$$= (2\pi)^4 \delta^4(p + k - p' + k') \Big\{ (-i\lambda)^2 \Big[ \frac{\big(\bar{u}_{r'}(p') u_s(k)\big)\big(\bar{u}_{s'}(k') u_r(p)\big)}{(k' - p)^2 - M^2 + i\varepsilon}$$

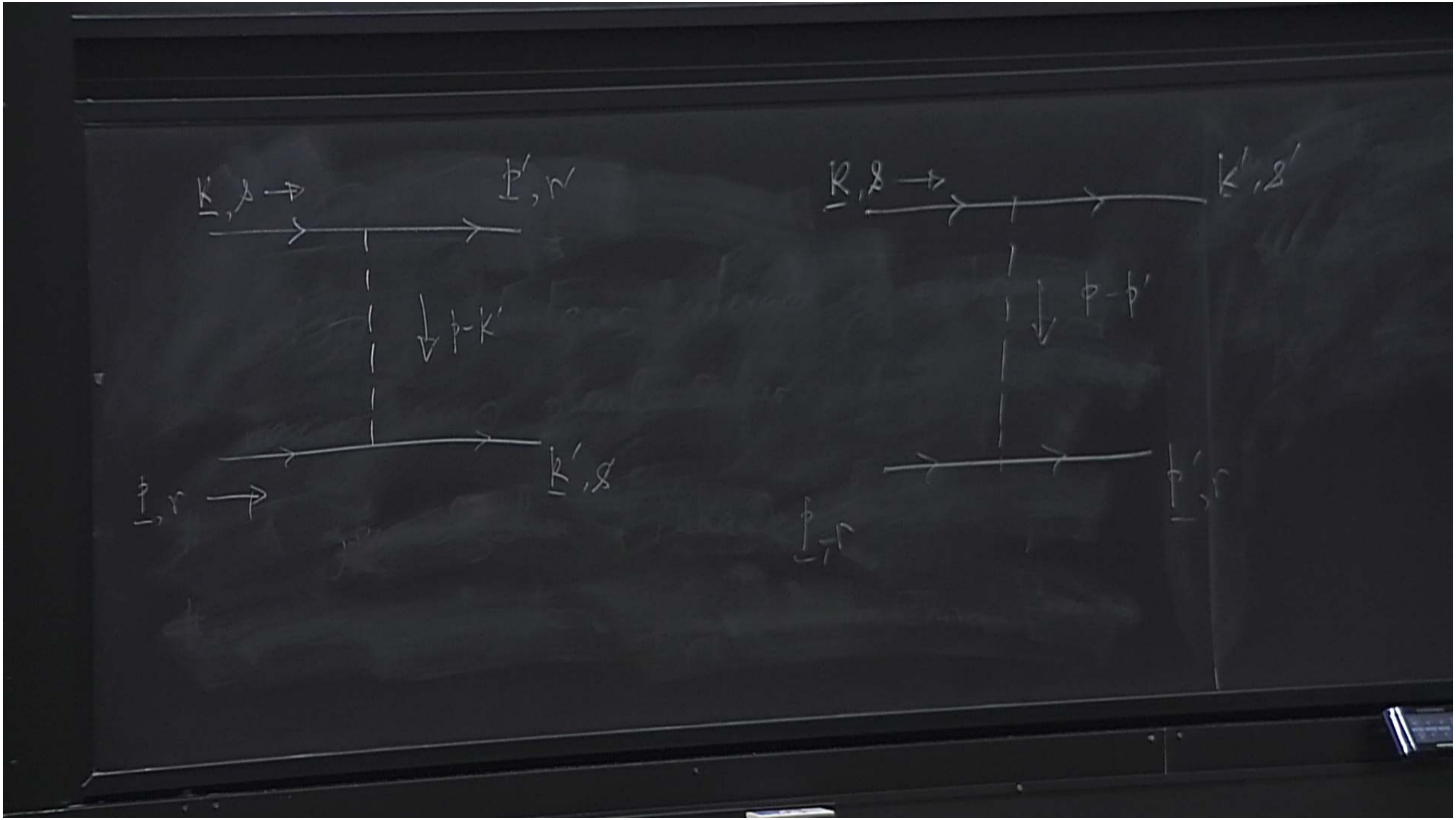
$$- \frac{\big(\bar{u}_{s'}(k') u_s(k)\big)\big(\bar{u}_{r'}(p') u_r(p)\big)}{(p' - p)^2 - M^2 + i\varepsilon} \Big\} \rightsquigarrow i\mathcal{M}$$

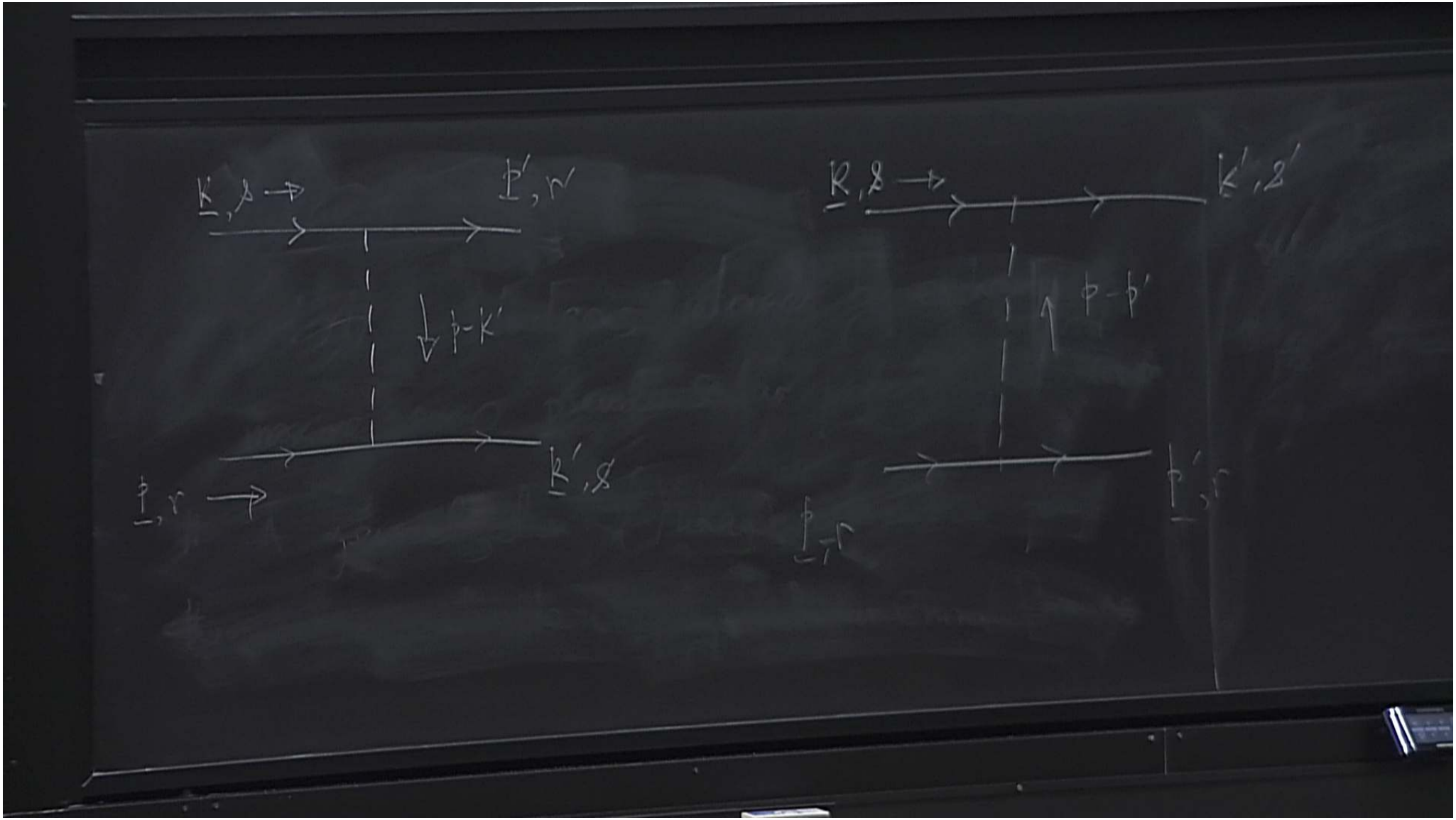
$$= (-i\lambda)^2 \int \frac{d^4q}{q^2 - M^2 + i\varepsilon} \frac{(2\pi)^4}{} \Big[ (\bar{u}_{r'}(k') u_s(k)) (\bar{u}_{s'}(k') u_r(p)) \, \delta^4(p' - k + q) \, \delta^4(k' - p - q)$$

$$- (\bar{u}_{s'}(k') u_s(k)) (\bar{u}_{r'}(p') u_r(p)) \, \delta^4(k' - k + q) \, \delta^4(p' - p + q) \Big]$$

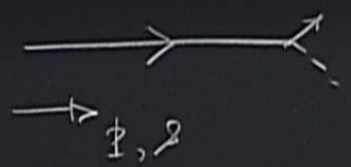
$$= (2\pi)^4 \delta^4(p + k - p' - k') \Big\{ (-i\lambda)^2 \Big[ \frac{(\bar{u}_{r'}(p') u_s(k)) (\bar{u}_{s'}(k') u_r(p))}{(k' - p)^2 - M^2 + i\varepsilon}$$

$$- \frac{(\bar{u}_{s'}(k') u_s(k)) (\bar{u}_{r'}(p') u_r(p))}{(p' - p)^2 - M^2 + i\varepsilon} \Big\} \rightsquigarrow i\mathcal{M}$$

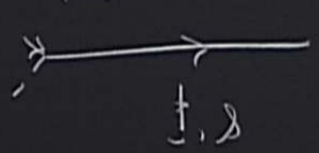

k, λ
p', ν
p-k'
p, τ
k', δ
R, δ
k', δ'
P-P'
p, τ
p', τ


k, λ
p', ν
p-k'
p, τ
k', δ
R, β
k', α'
p-p'
p, τ
p', τ

1. For each incoming fermion $(\not{p}, s)$ we write $u_s(p)$

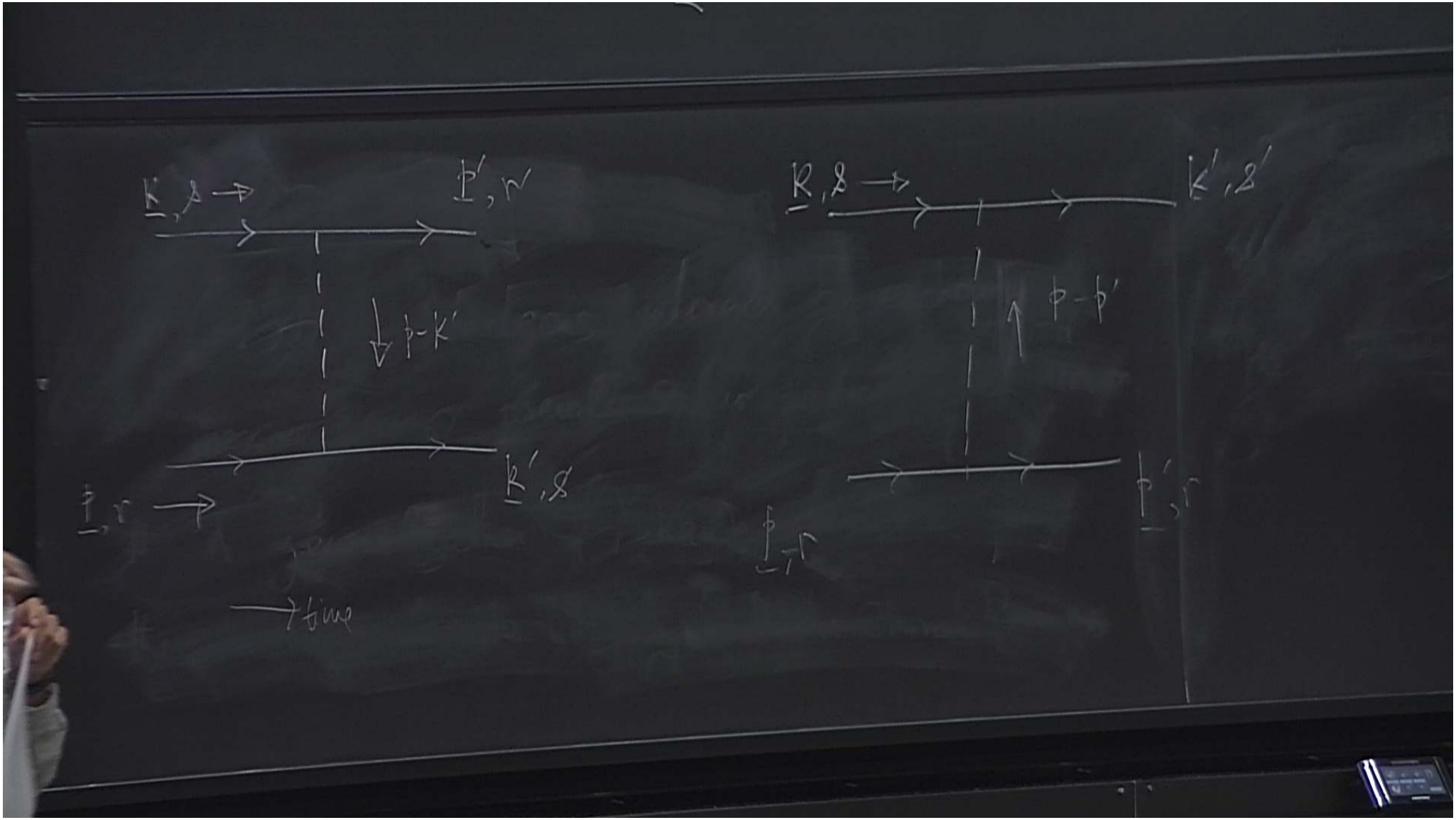



2. Outgoing



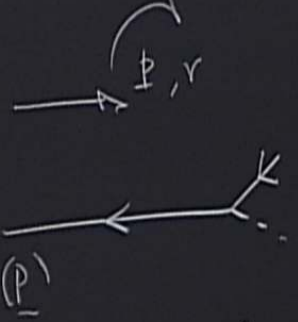

3. We label fermion lines by an arrow. Arrow follows direction of time. We write down the above factors from left to right

while

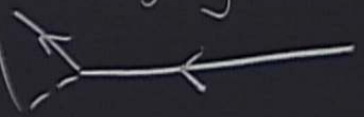

k, λ
p', ν
p-k'
k', δ
p, ν
time
K, δ
k', δ'
p-p'
p', ν
p, ν

while following the Fermion line
opposite to the direction of the arrow.

4. Incoming anti-fermion $\bar{v}_r(p)$




5. Outgoing anti-fermion $v_s(p)$

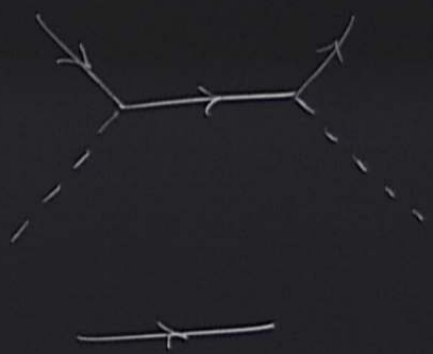


6. Associate $(-iX)$ with

6. Associate $(-i\lambda)$ with

7.

8.





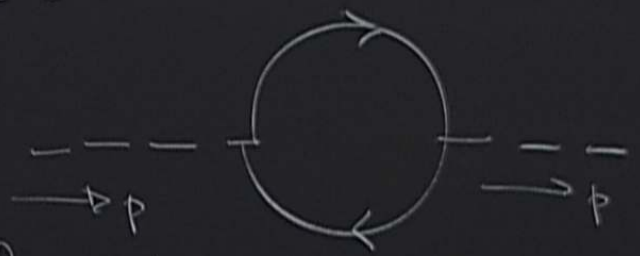
$$\frac{i(\not{p}+m)}{p^2-m^2+i\varepsilon}$$

9. Impose

9. Impose momentum conservation at each vertex.

10. minus sign due to Fermi-statistics.

11. Integrate over undetermined loop momentum




12. $(-1)$ with fermion loops.