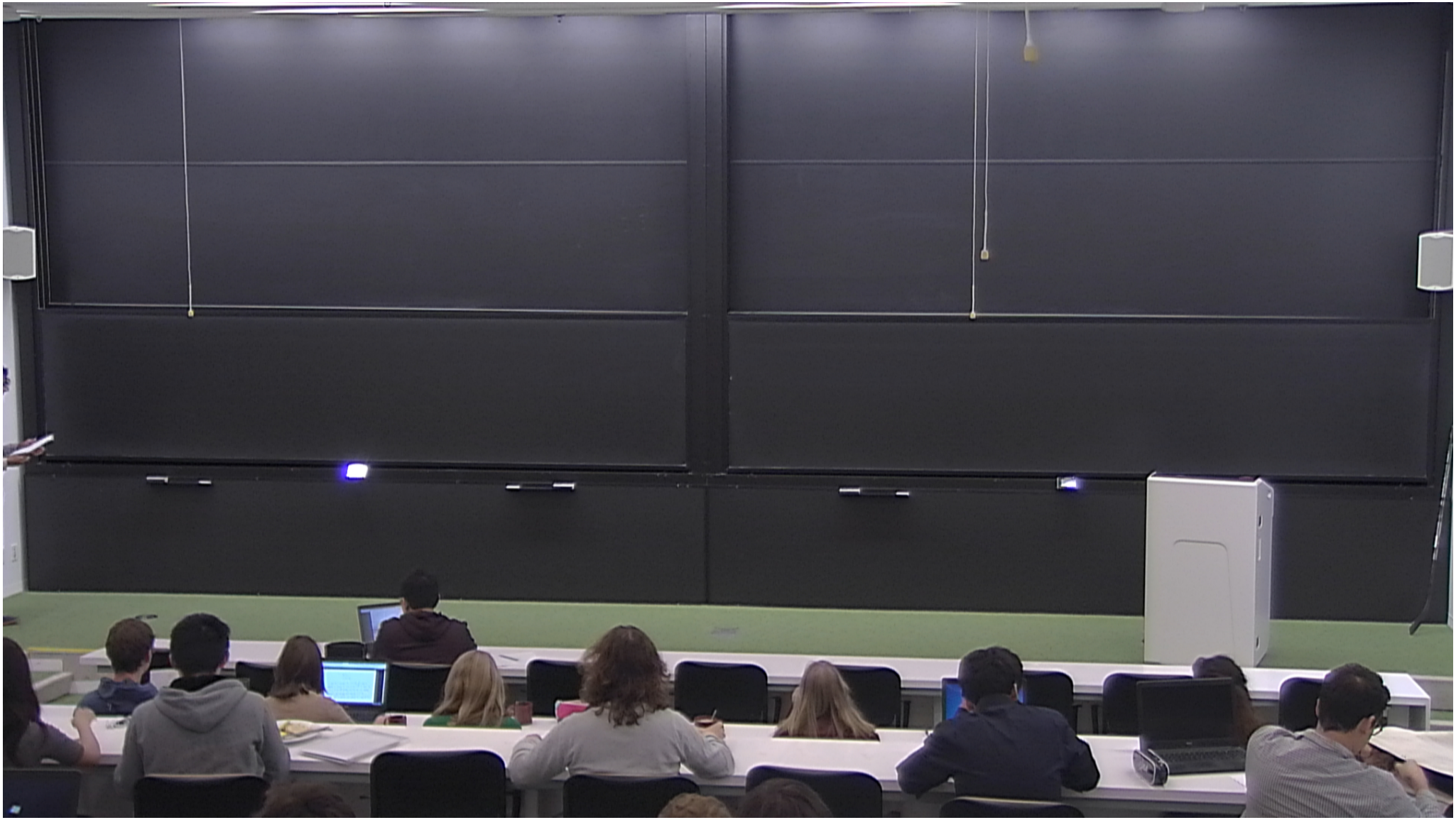
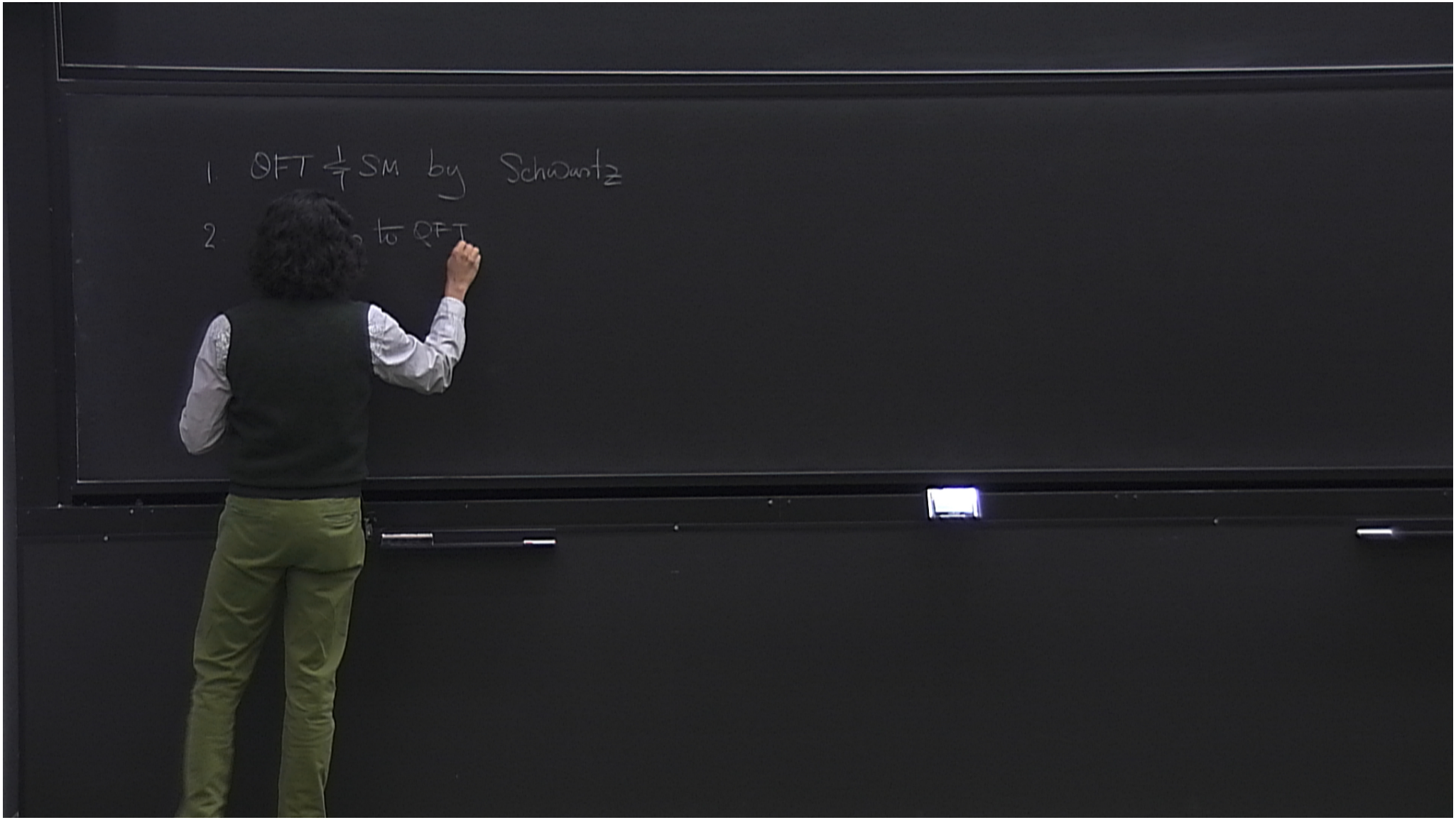
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 8

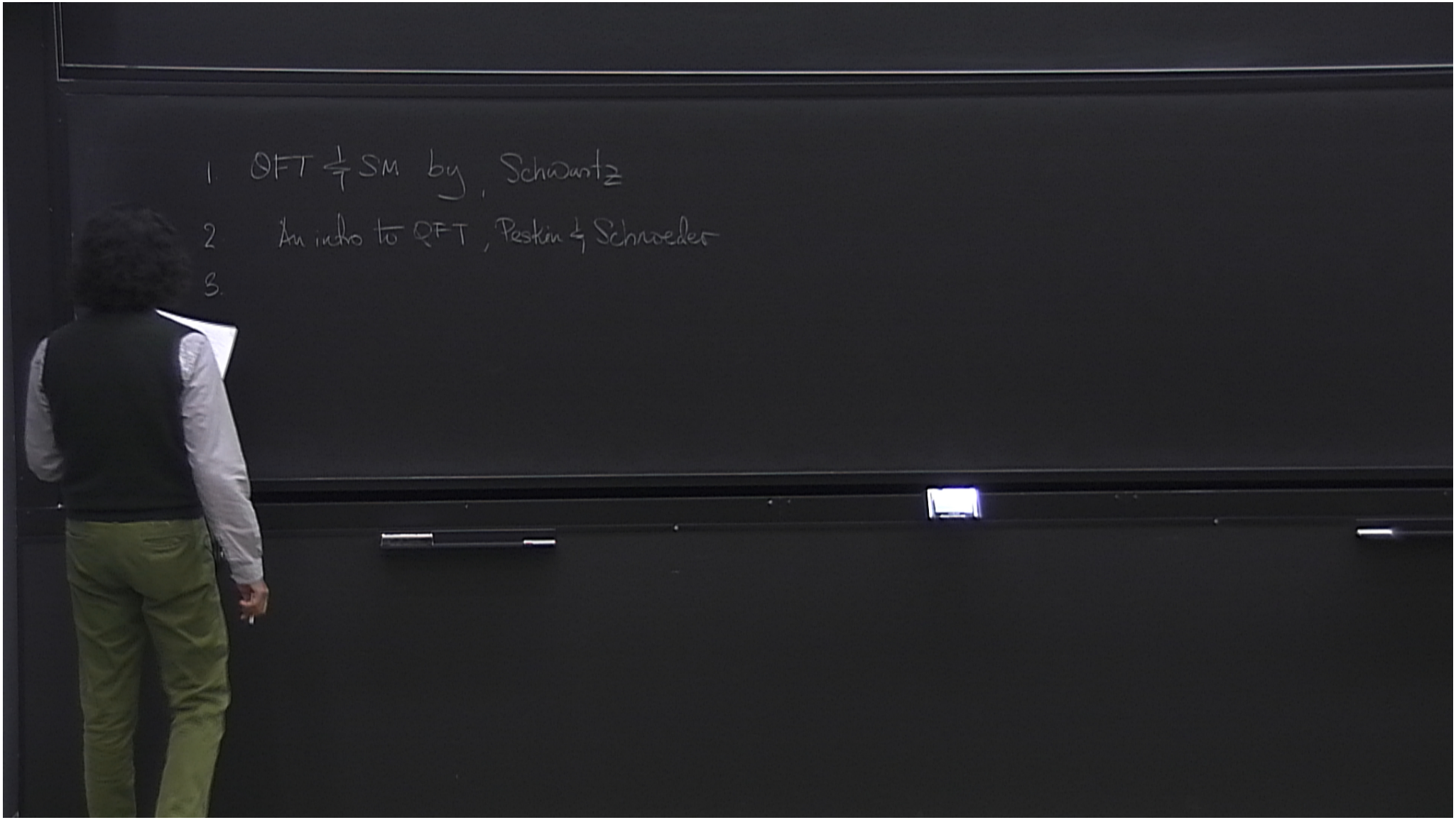
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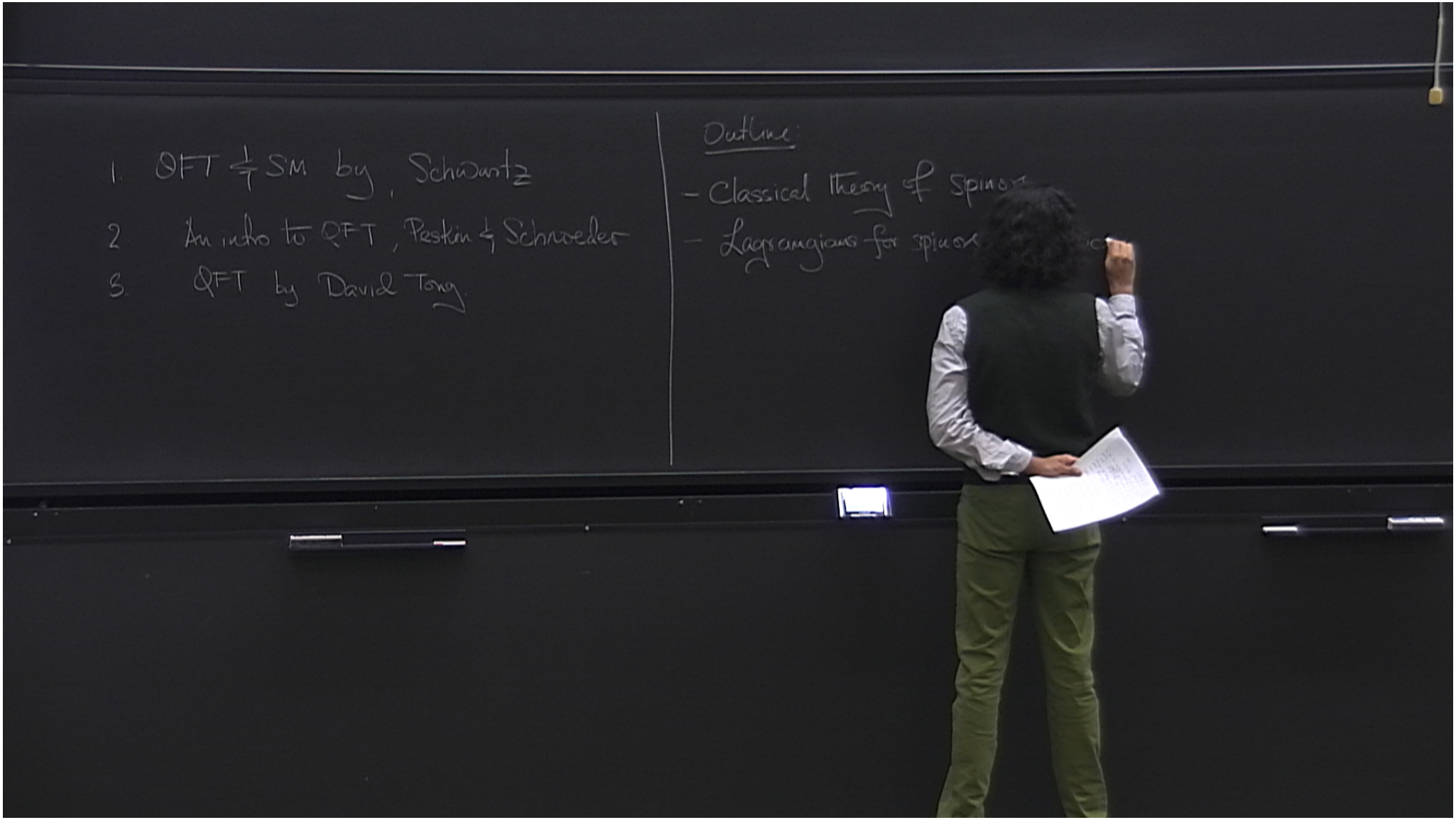
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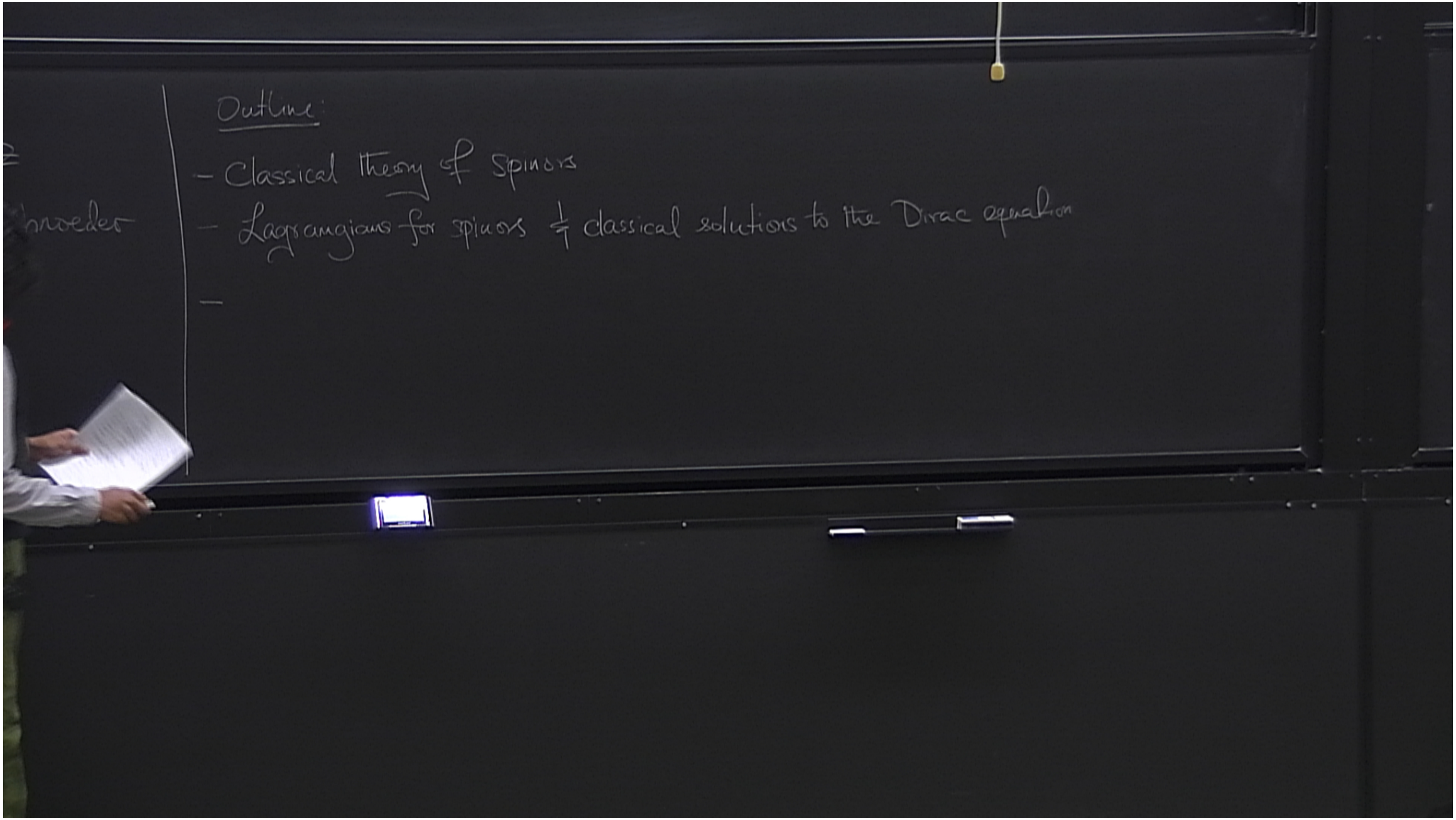
Abstract:

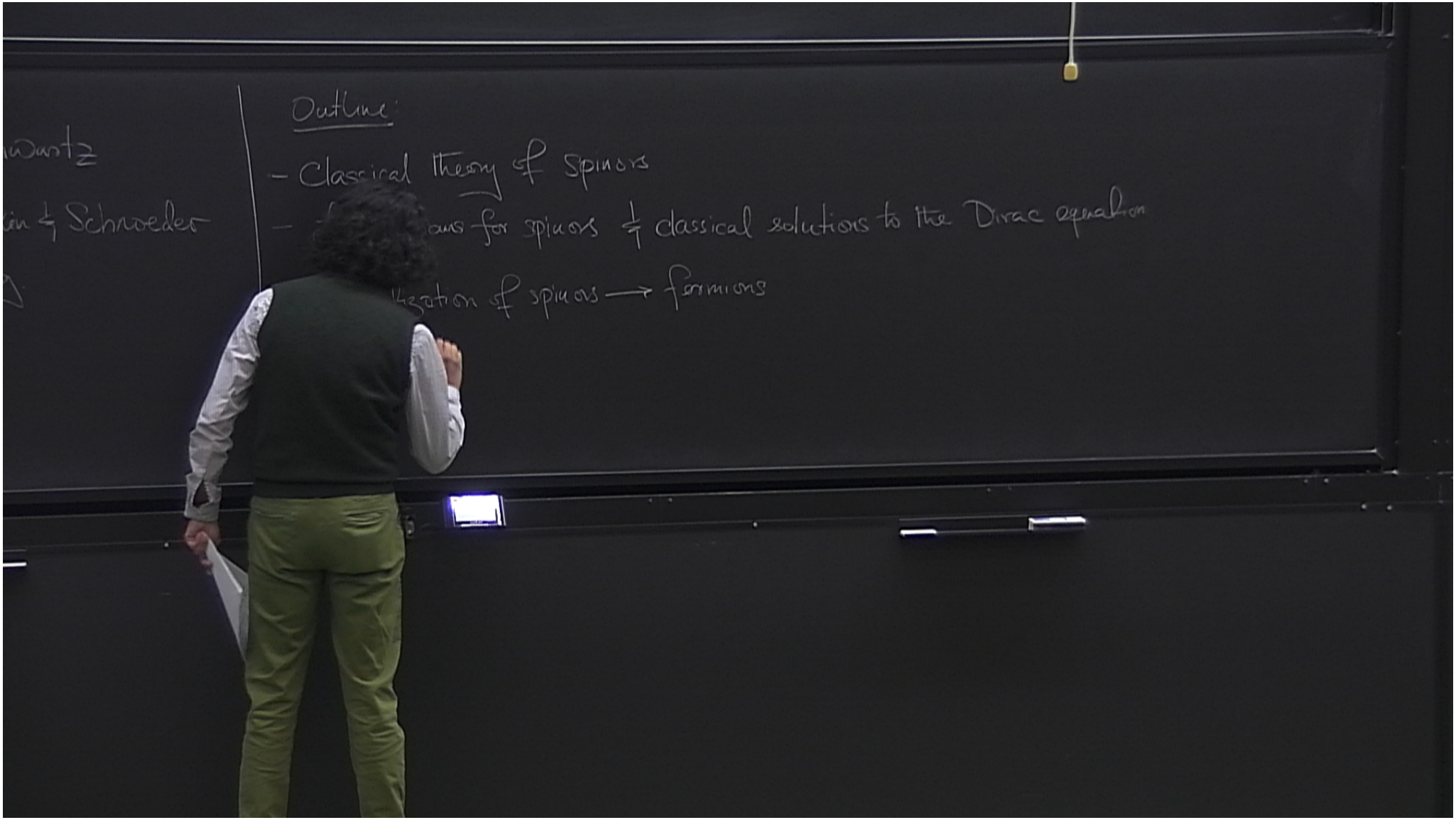


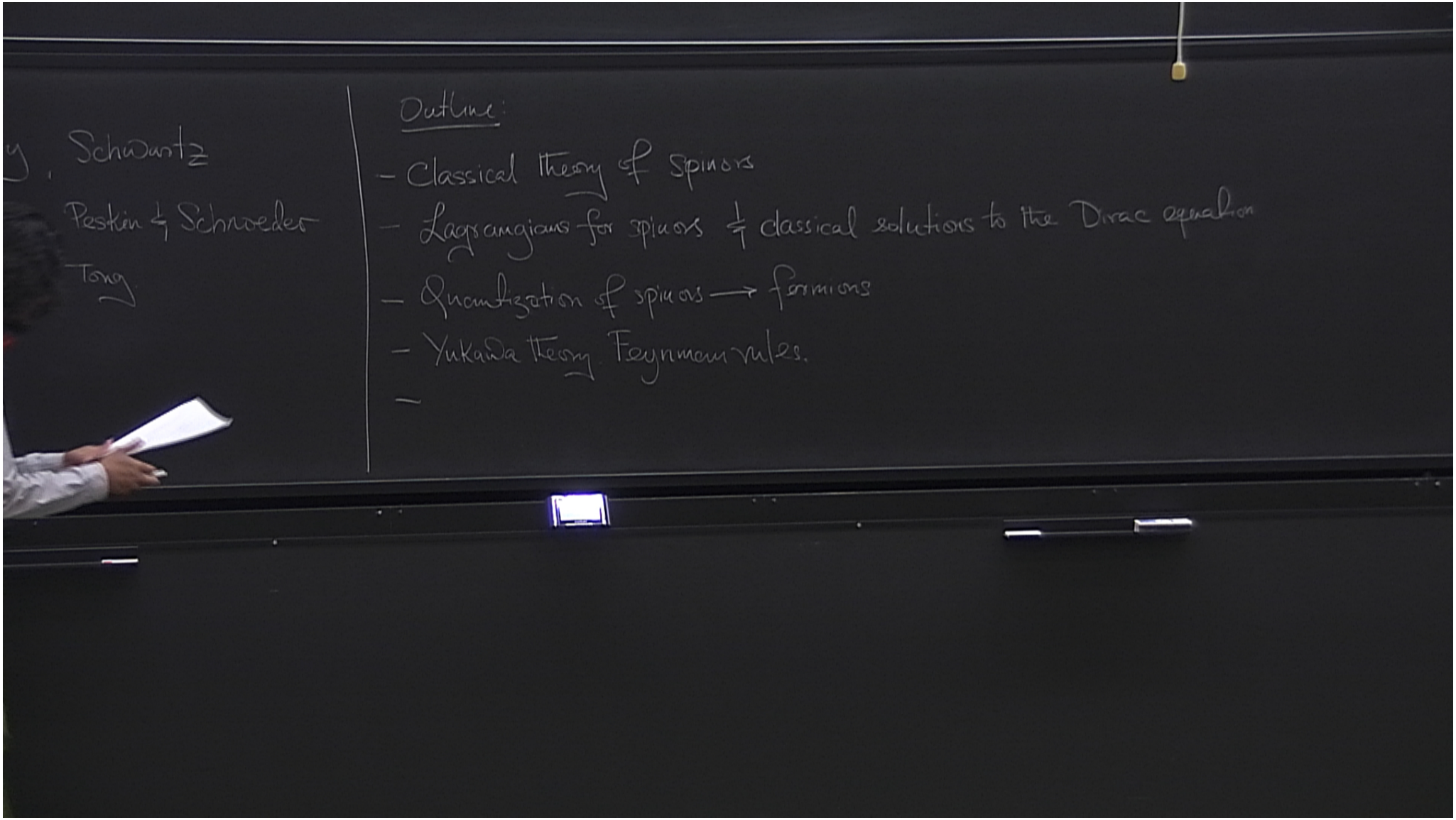

1. QFT ≠ SM by Schwartz
2 to QFT

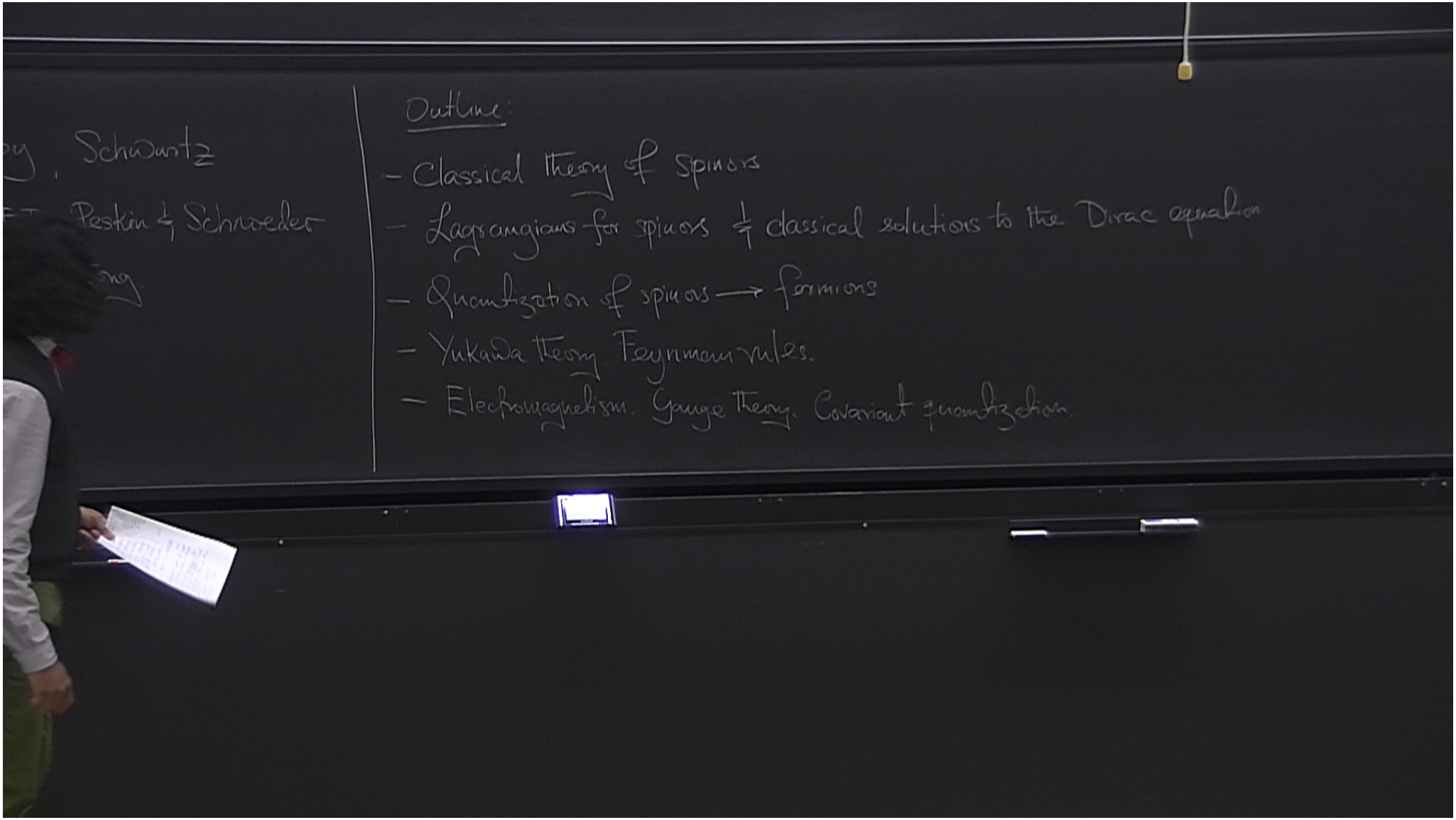

1. QFT & SM by Schwartz
2 An intro to QFT, Peskin & Schroeder
3.

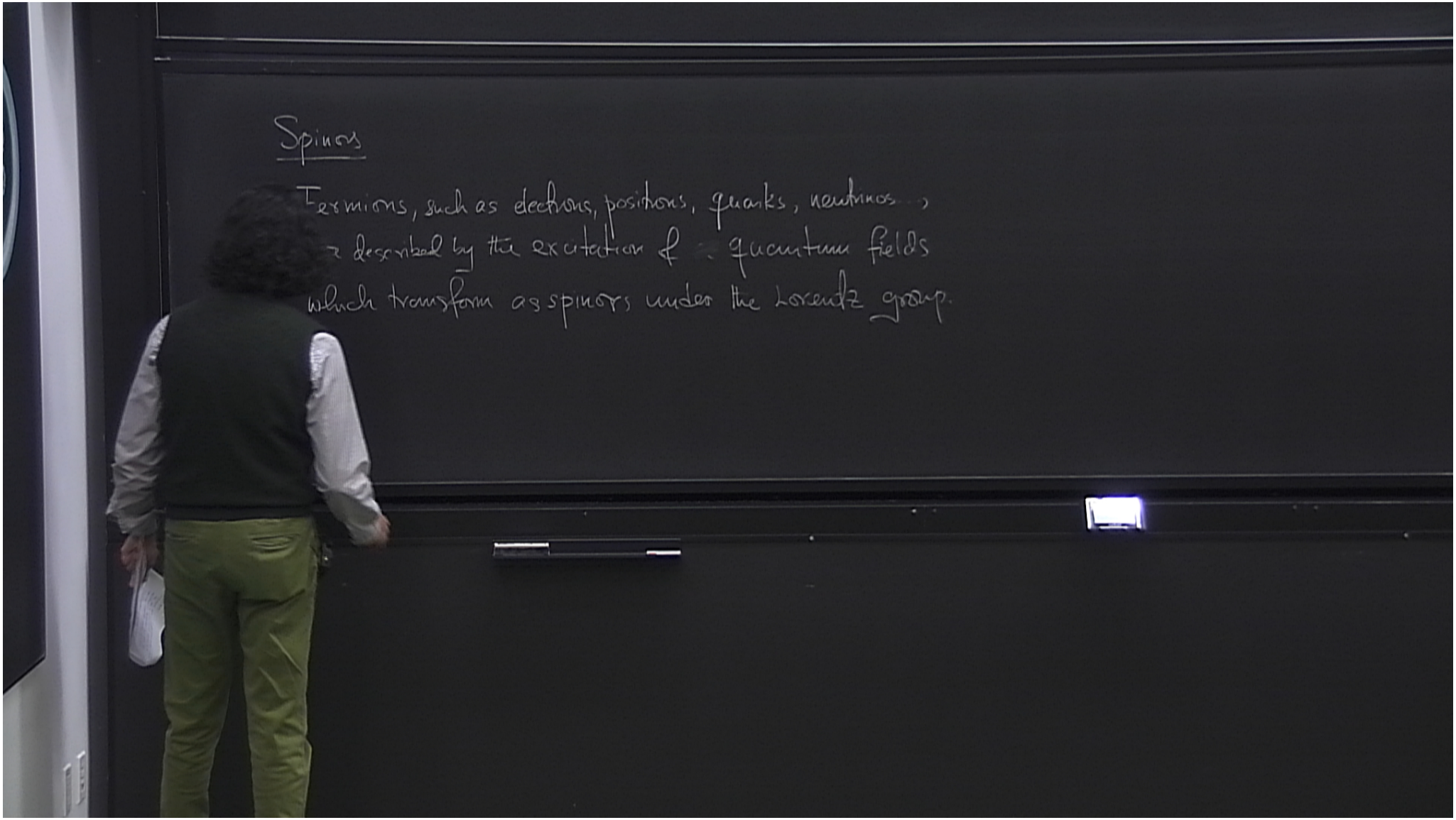

1. QFT & SM by Schwartz
2. An intro to QFT, Peskin & Schroeder
3. QFT by David Tong
Outline:
- Classical theory of spinors
- Lagrangians for spinors

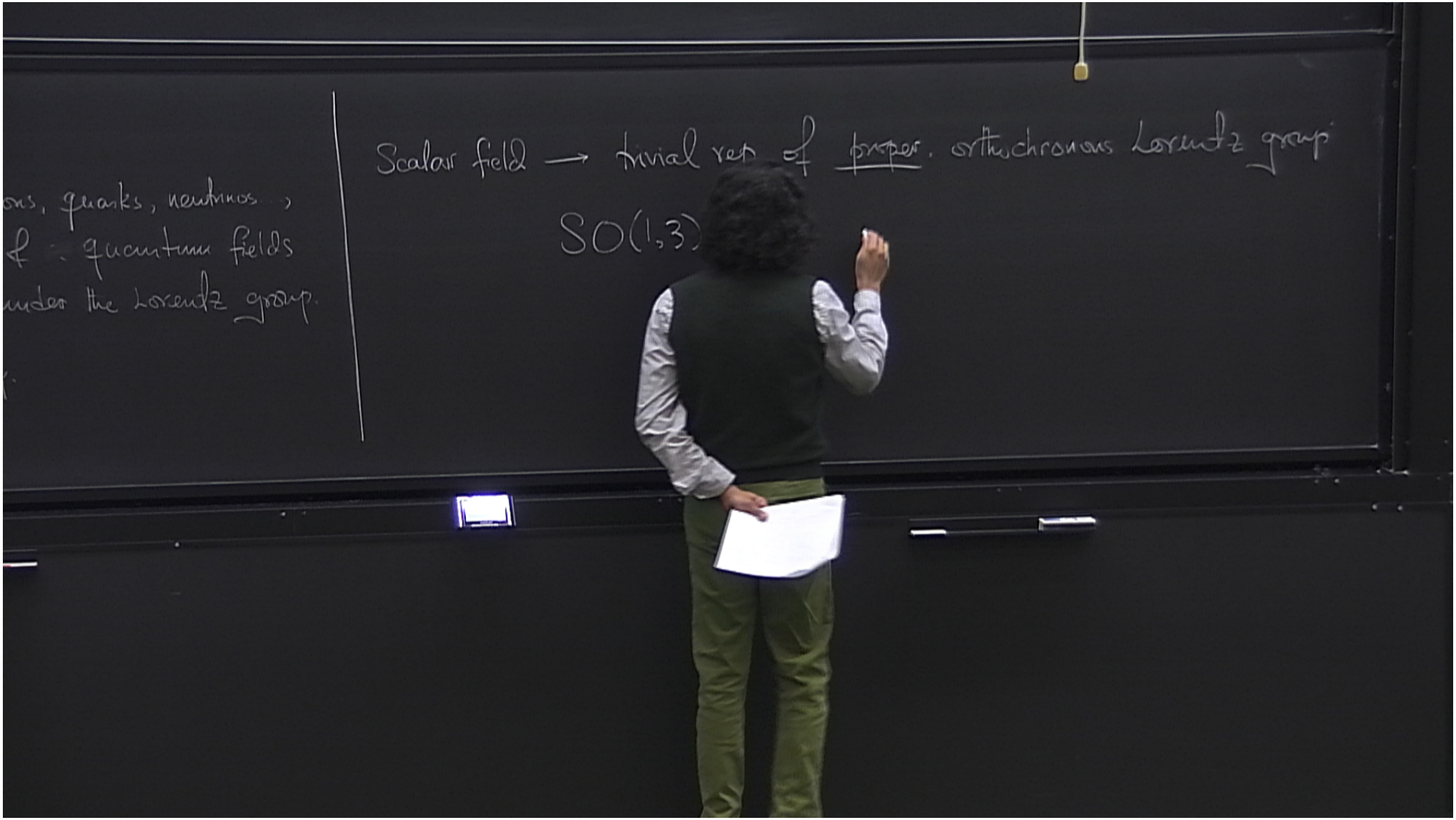

Outline:
- Classical theory of spinors
- Lagrangians for spinors & classical solutions to the Dirac equation

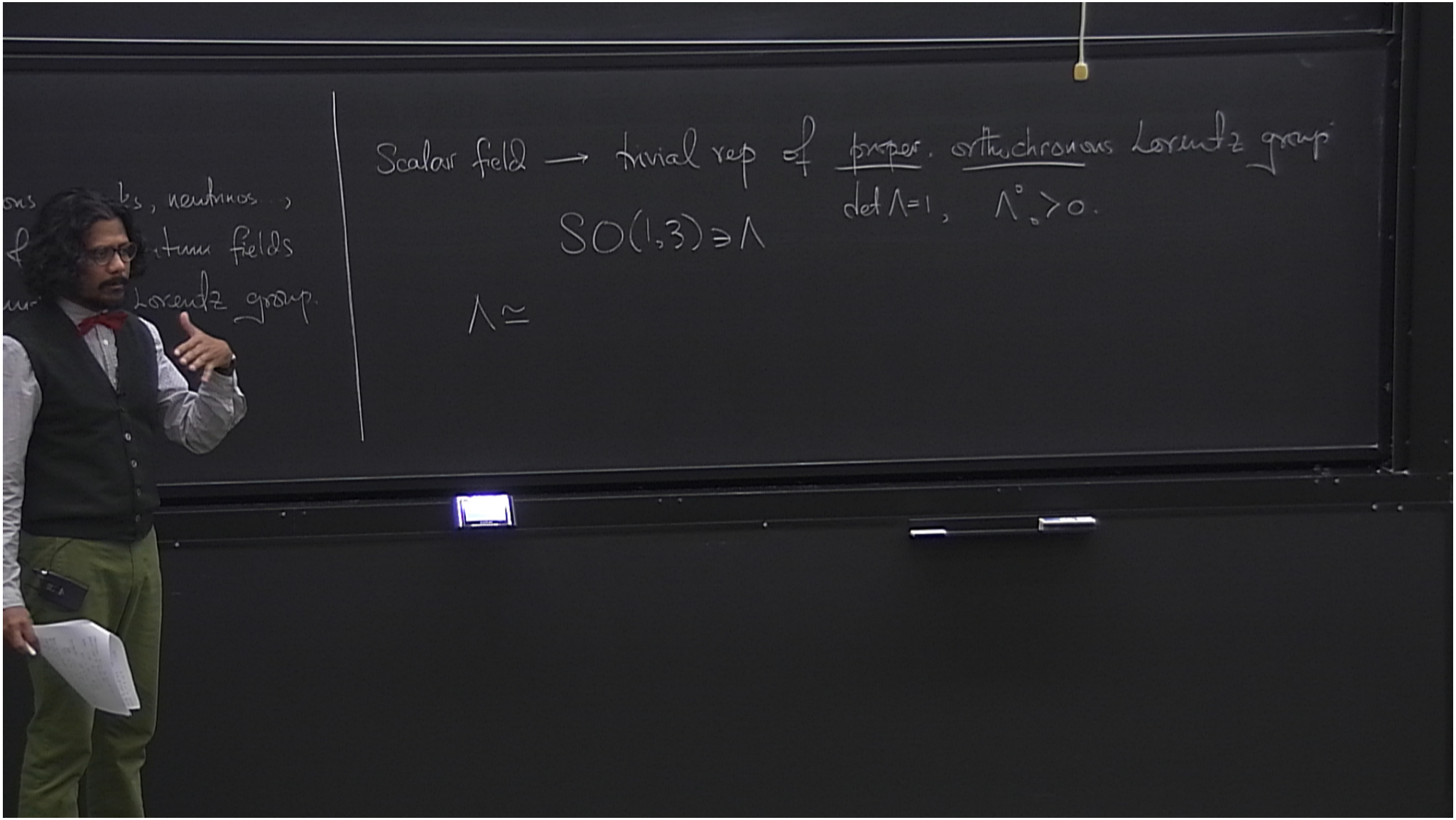

Outline:
- Classical theory of spinors
- ... ans for spinors ≠ classical solutions to the Dirac equation
...ization of spinors → fermions

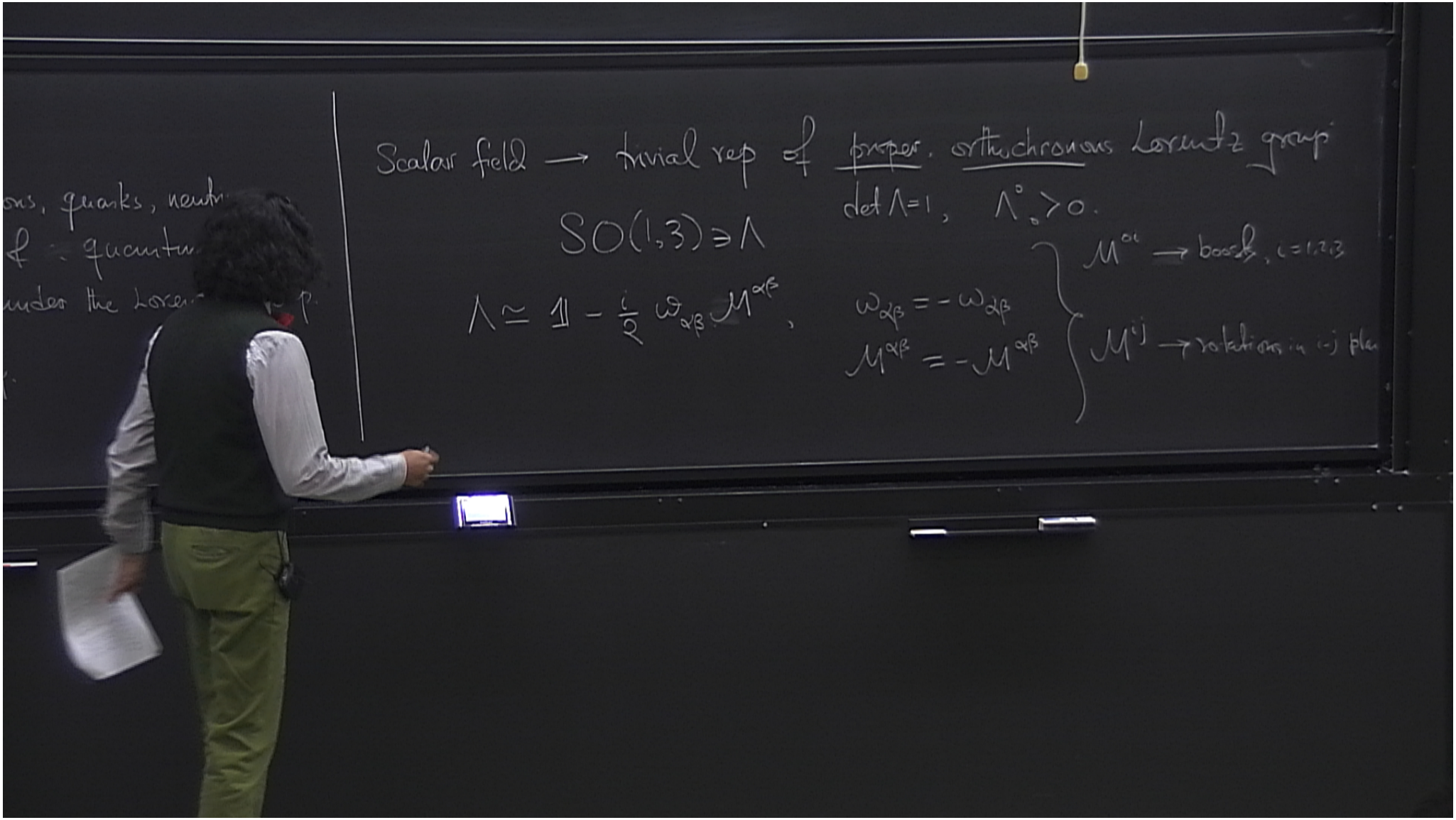

y. Schwartz
Peskin & Schroeder
Tong
Outline:
- Classical theory of spinors
- Lagrangians for spinors & classical solutions to the Dirac equation
- Quantization of spinors → fermions
- Yukawa theory. Feynman rules.
-

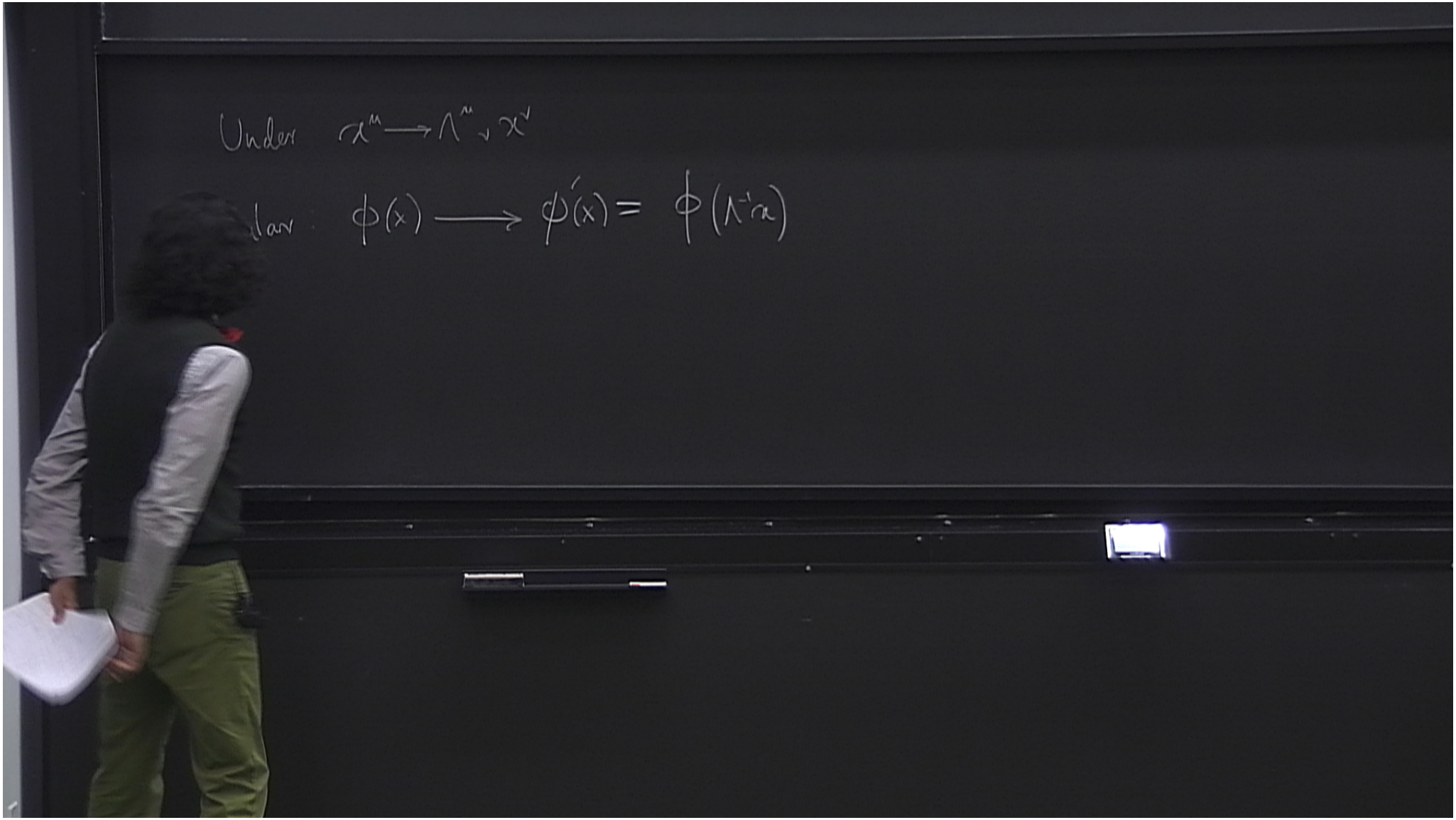

Schwartz
Peskin & Schroeder
Outline:
- Classical theory of spinors
- Lagrangians for spinors & classical solutions to the Dirac equation
- Quantization of spinors → fermions
- Yukawa theory. Feynman rules.
- Electromagnetism. Gauge theory. Covariant quantization.

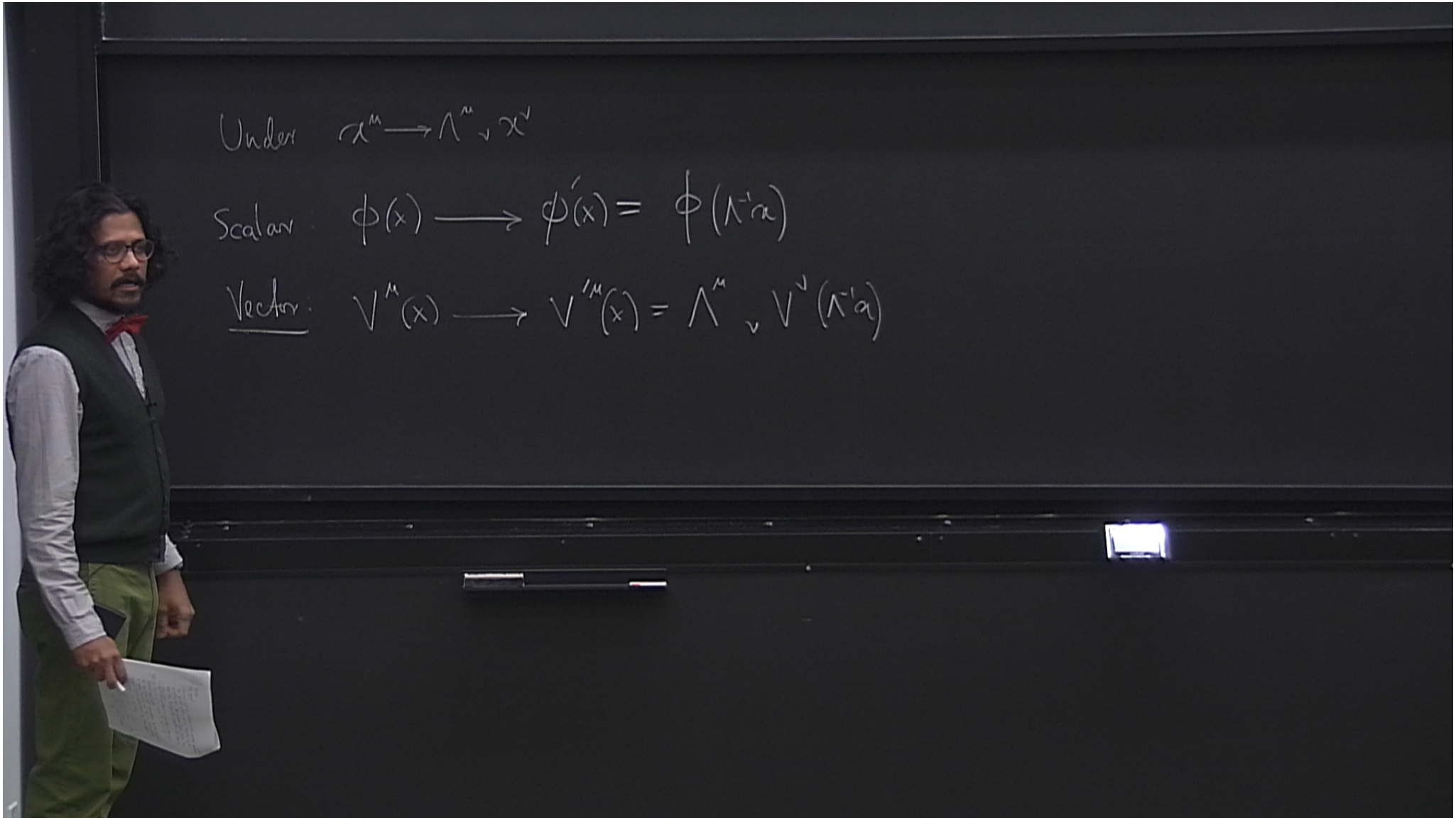

Spinors
Fermions, such as electrons, positrons, quarks, neutrinos ...
= described by the excitation of quantum fields
which transform as spinors under the Lorentz group.

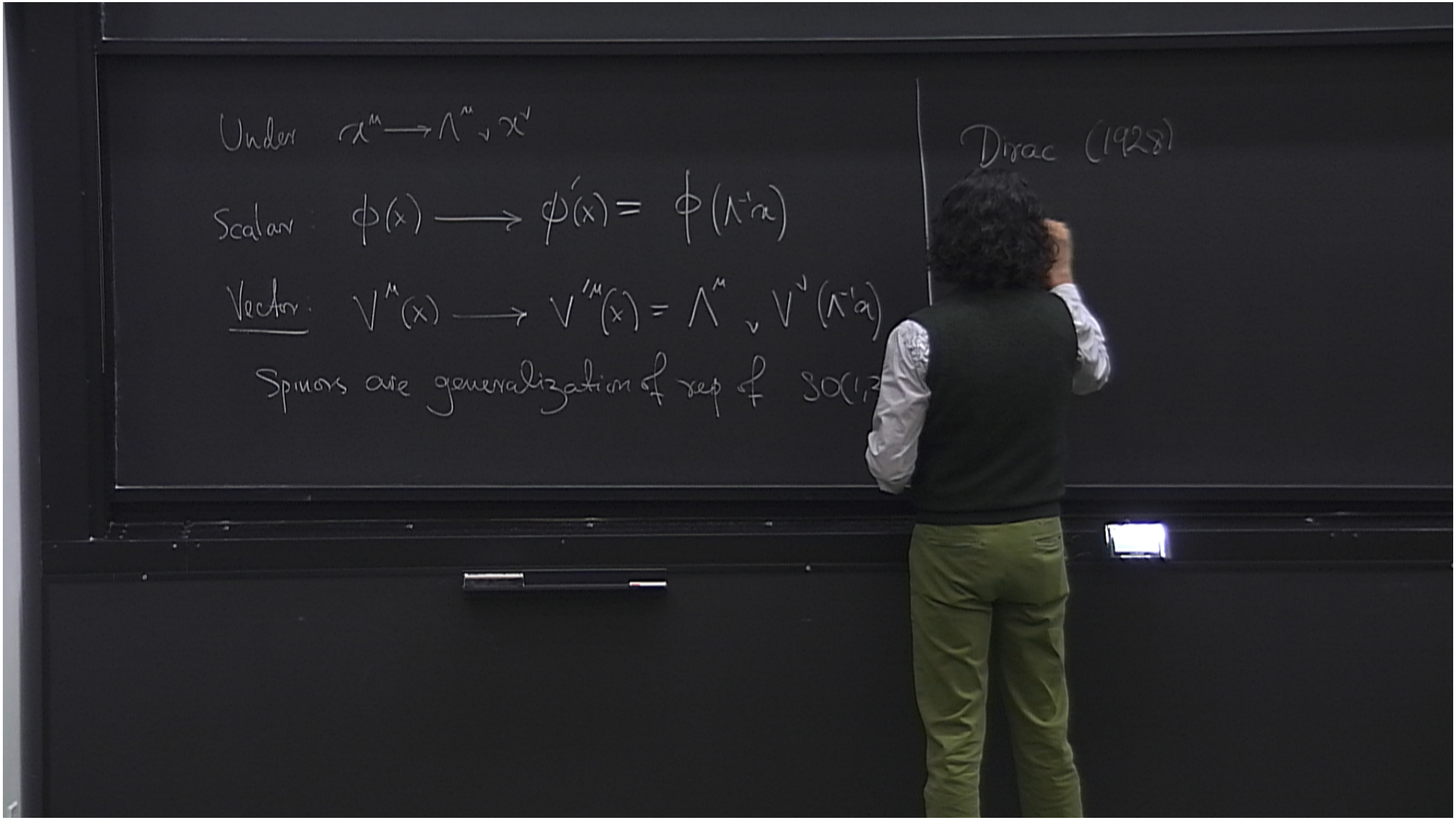

ons, quarks, neutrinos ,
f : quantum fields
under the Lorentz group.
Scalar field → trivial rep of proper, orthochronous Lorentz group
SO(1,3)

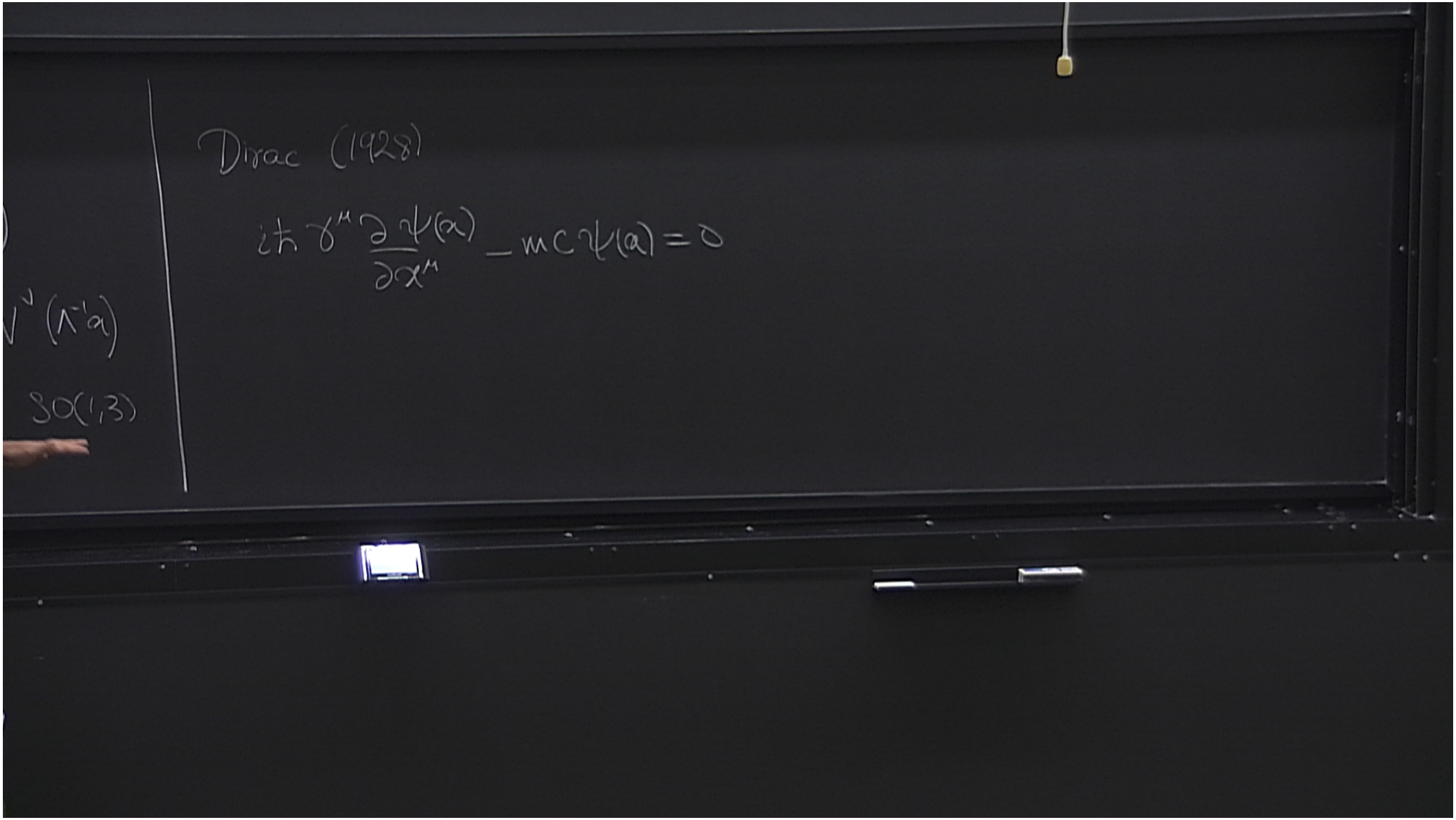

Scalar field → trivial rep of proper, orthochronous Lorentz group
det Λ=1, Λ⁰₀>0.
SO(1,3) ∋ Λ
Λ≃
Lorentz group.

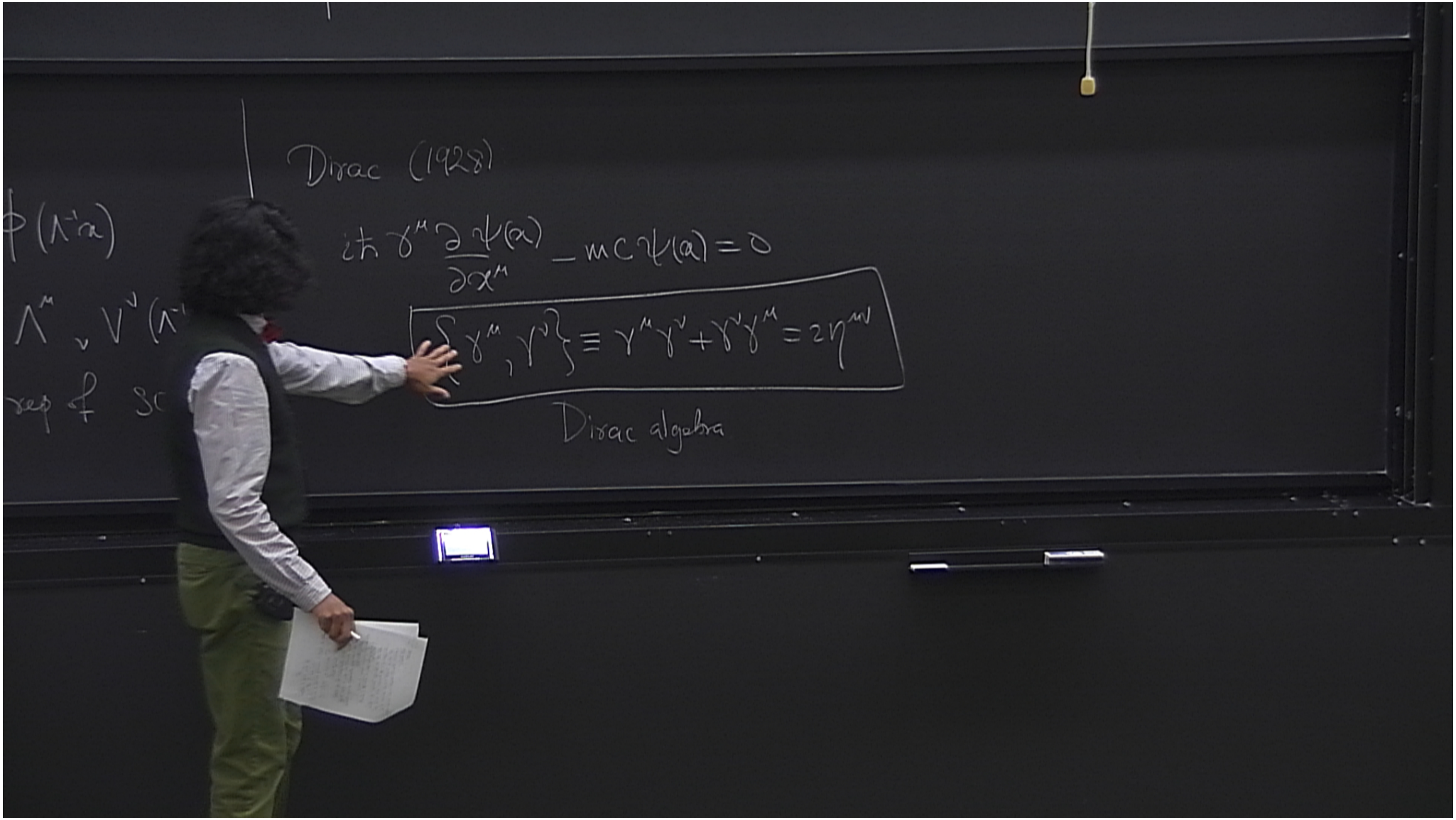

Scalar field → trivial rep of proper, orthochronous Lorentz group
det Λ = 1, Λ⁰₀ > 0.
SO(1,3) ∋ Λ
Λ ≃ 𝟙 − (i/2) ω_αβ M^αβ,
ω_αβ = − ω_αβ
M^αβ = − M^αβ
M^0i → boosts, i = 1,2,3
M^ij → rotations in i-j plan

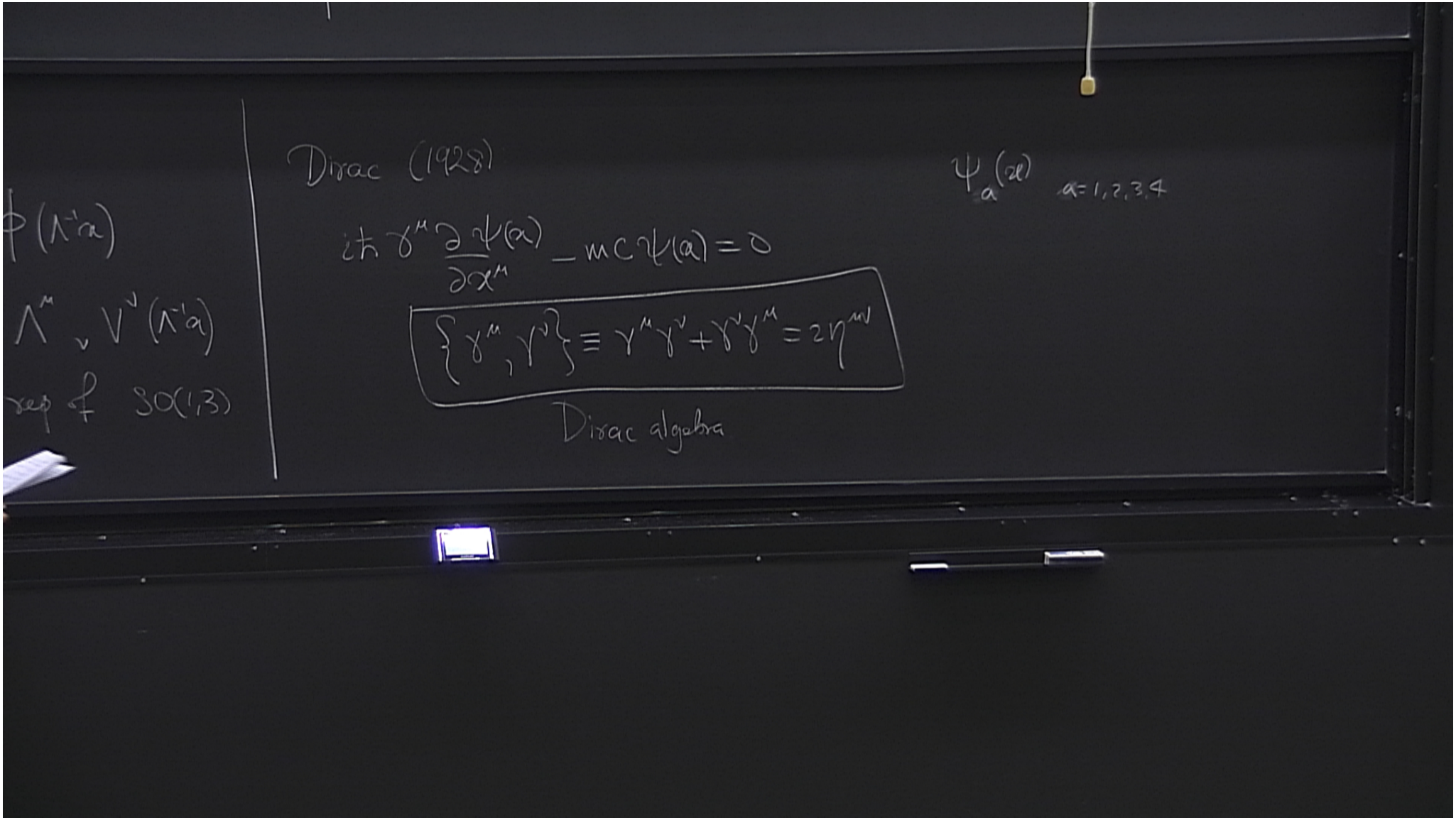

Under $x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu$
$\phi(x) \longrightarrow \phi'(x) = \phi(\Lambda^{-1}x)$


Under $x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu$
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Vector: $V^\mu(x) \longrightarrow V'^\mu(x) = \Lambda^\mu{}_\nu V^\nu(\Lambda^{-1}x)$


Under $x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu$
Scalar $\phi(x) \longrightarrow \phi'(x) = \phi(\Lambda^{-1}x)$
Vector $V^\mu(x) \longrightarrow V'^\mu(x) = \Lambda^\mu{}_\nu V^\nu(\Lambda^{-1}a)$
Spinors are generalization of rep of SO(1,
Dirac (1928)


Dirac (1928)
$i\hbar\gamma^\mu \frac{\partial \psi(x)}{\partial x^\mu} - mc\psi(x) = 0$
$V'(\Lambda^{-1}a)$
$SO(1,3)$


Dirac (1928)
$i\hbar\gamma^\mu \frac{\partial \psi(x)}{\partial x^\mu} - mc\psi(x) = 0$
$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$
Dirac algebra


$\phi(\Lambda^{-1}x)$
$\Lambda^{\mu}{}_{\nu} V^{\nu}(\Lambda^{-1}x)$
rep of $SO(1,3)$
Dirac (1928)
$i\hbar\gamma^{\mu}\frac{\partial\psi(x)}{\partial x^{\mu}} - mc\psi(x) = 0$
$\{\gamma^{\mu},\gamma^{\nu}\} \equiv \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}$
Dirac algebra
$\psi_a(x)$ $a=1,2,3,4$

$\phi(\Lambda^{-1}a)$

$= \Lambda^{\mu}{}_{\nu} V^{\nu}(\Lambda^{-1}a)$

Λ rep of $SO(1,3)$

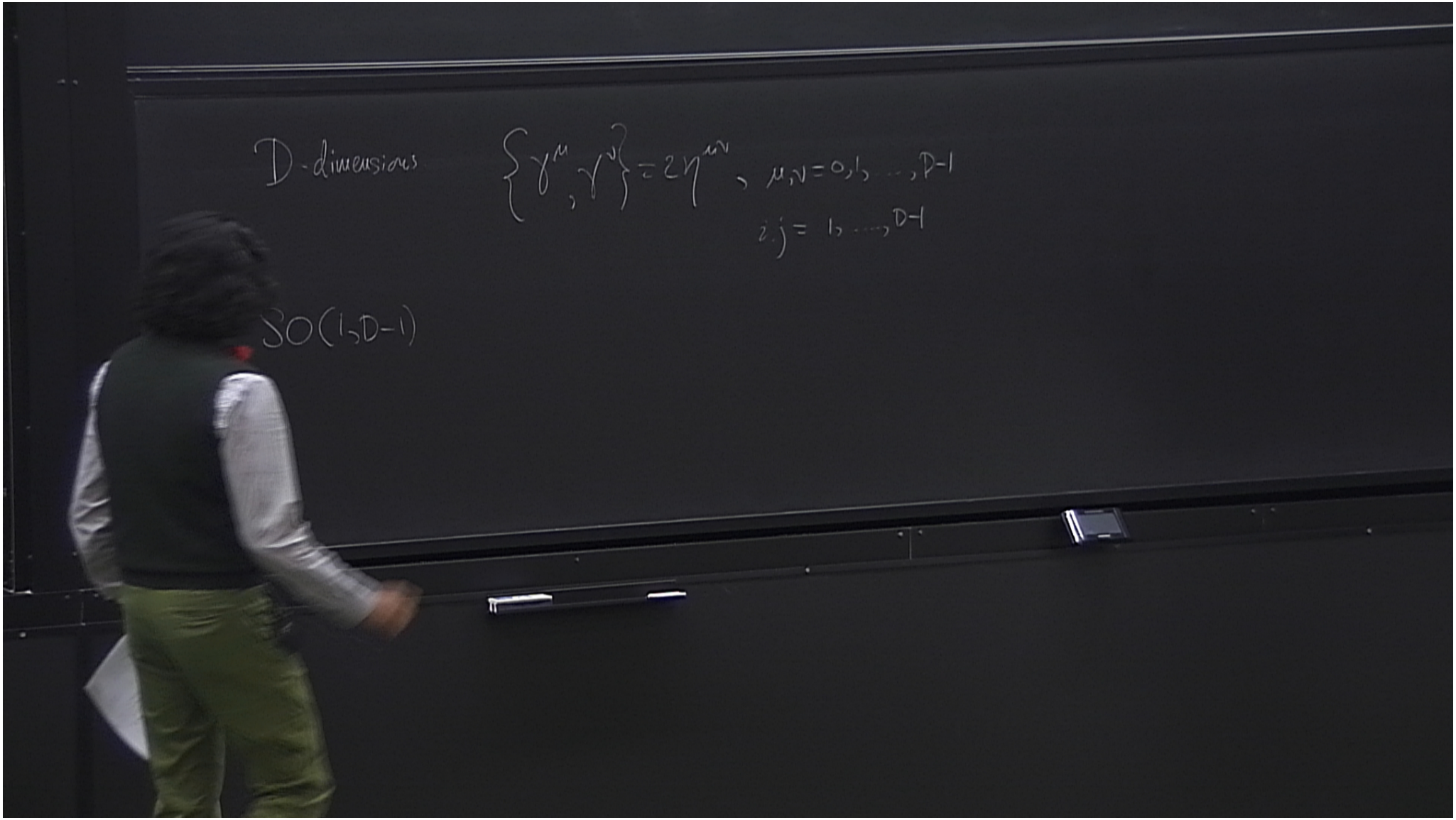
Dirac (1928)

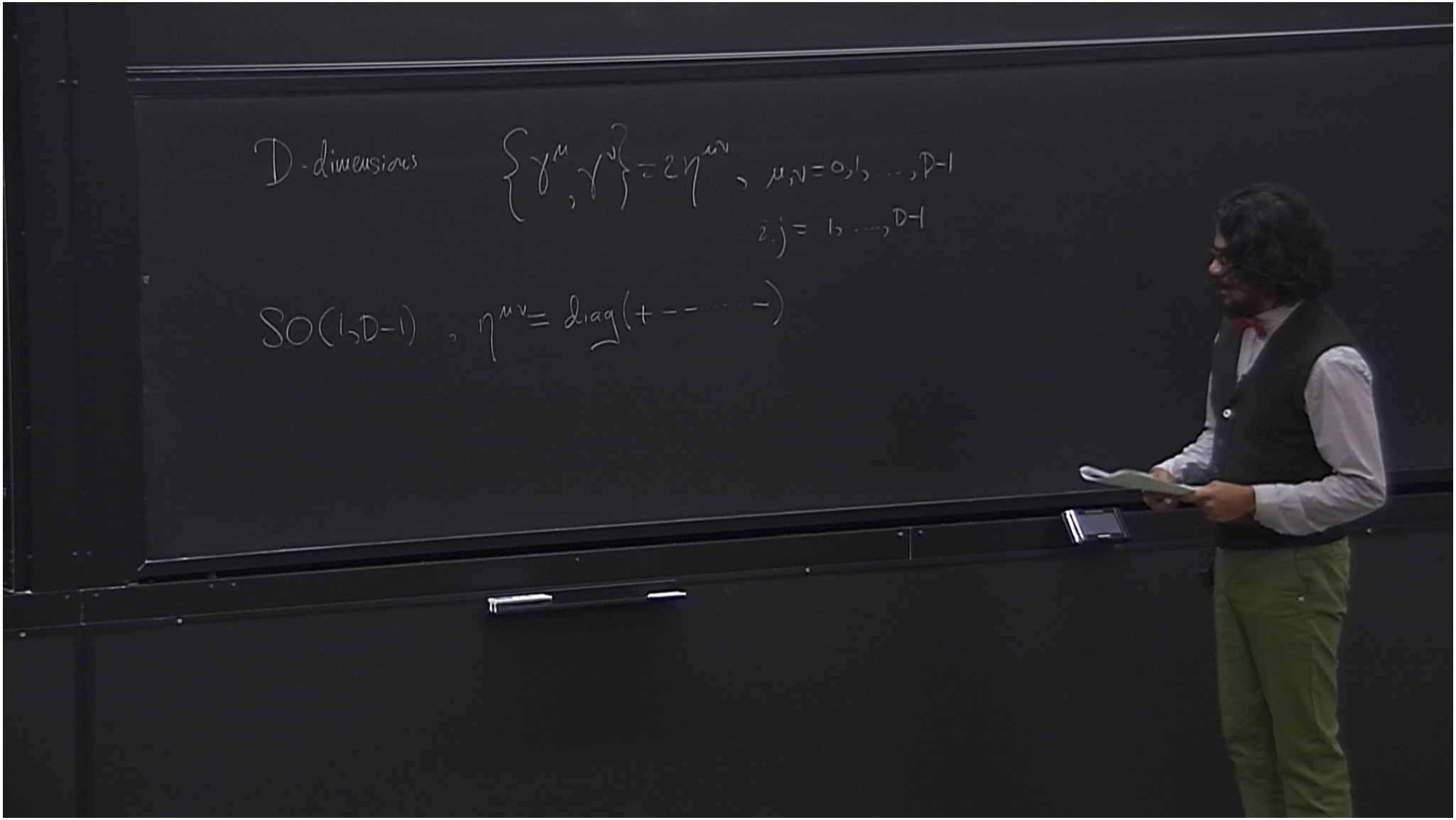
$i\hbar \gamma^{\mu} \frac{\partial \psi(a)}{\partial x^{\mu}} - mc\psi(a) = 0$

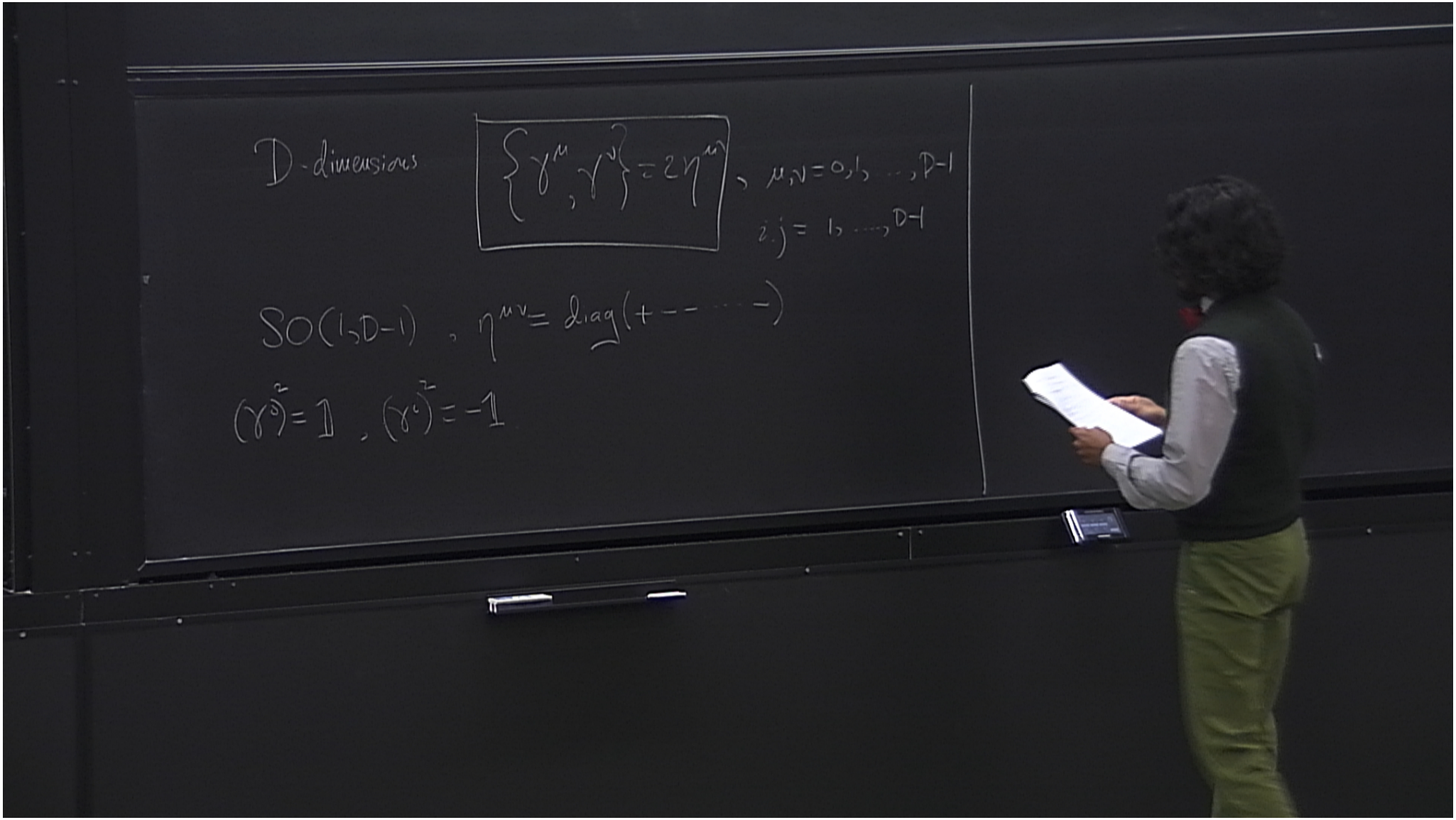
$\{\gamma^{\mu}, \gamma^{\nu}\} \equiv \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}$

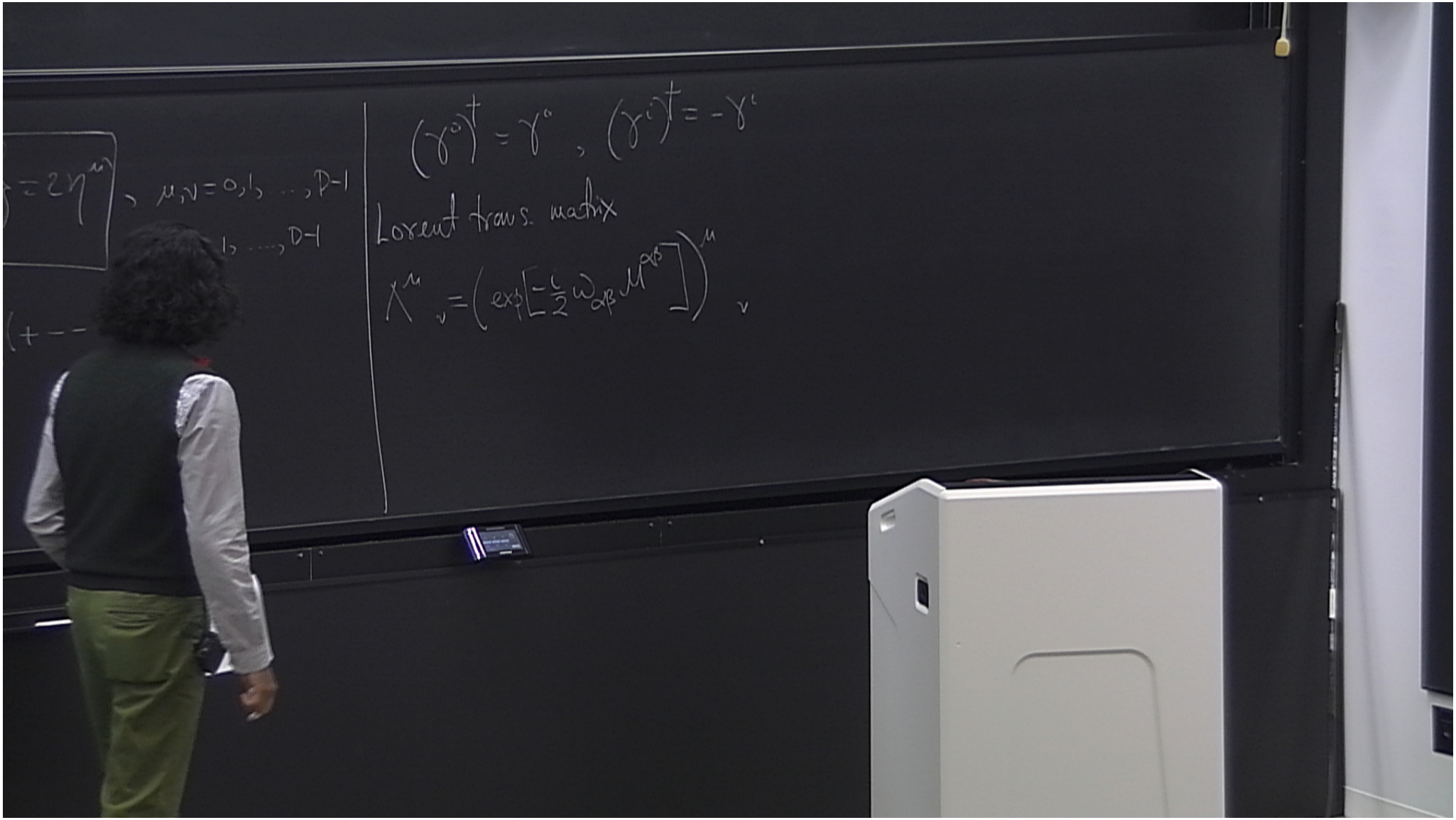
Dirac algebra

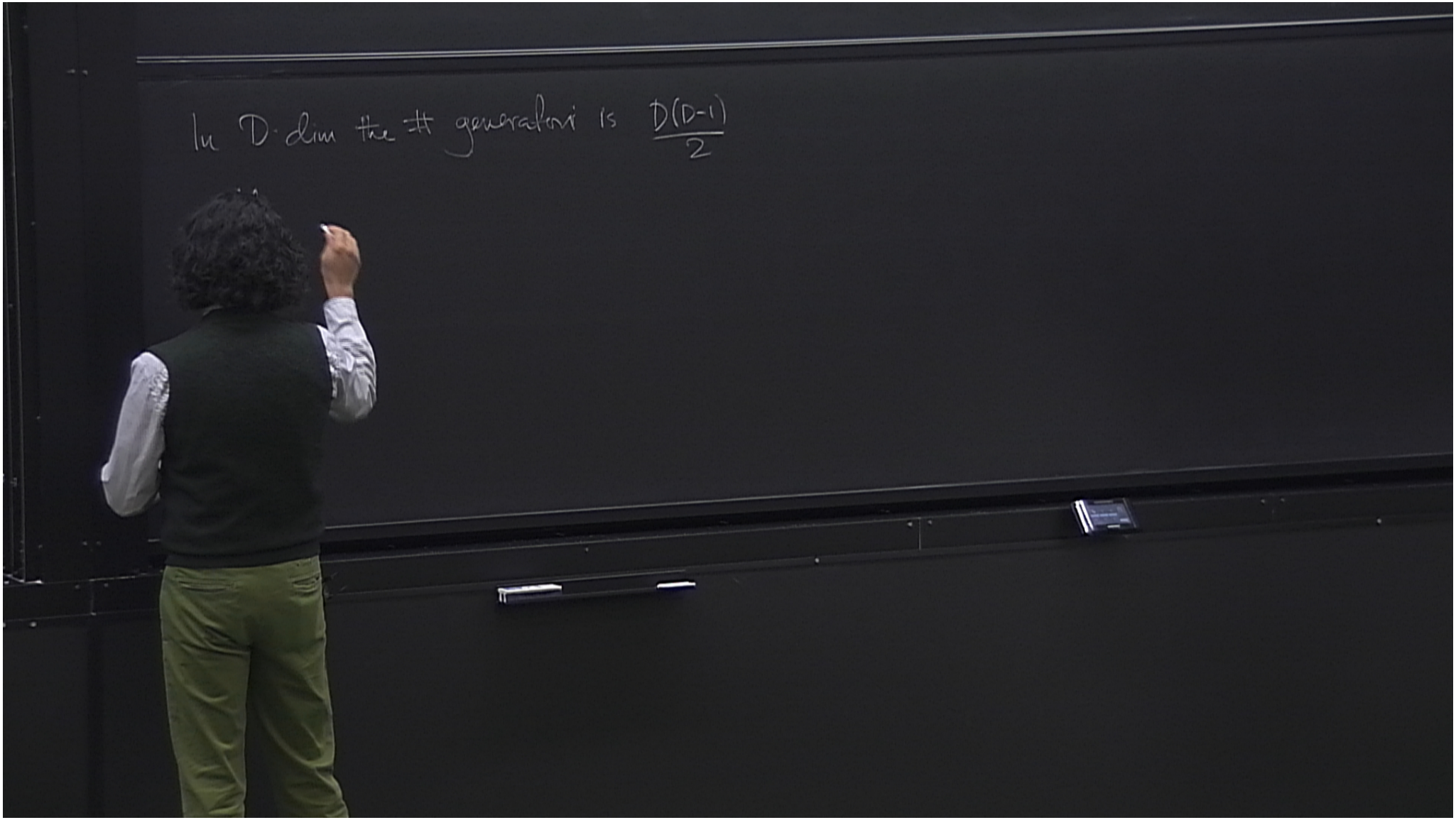
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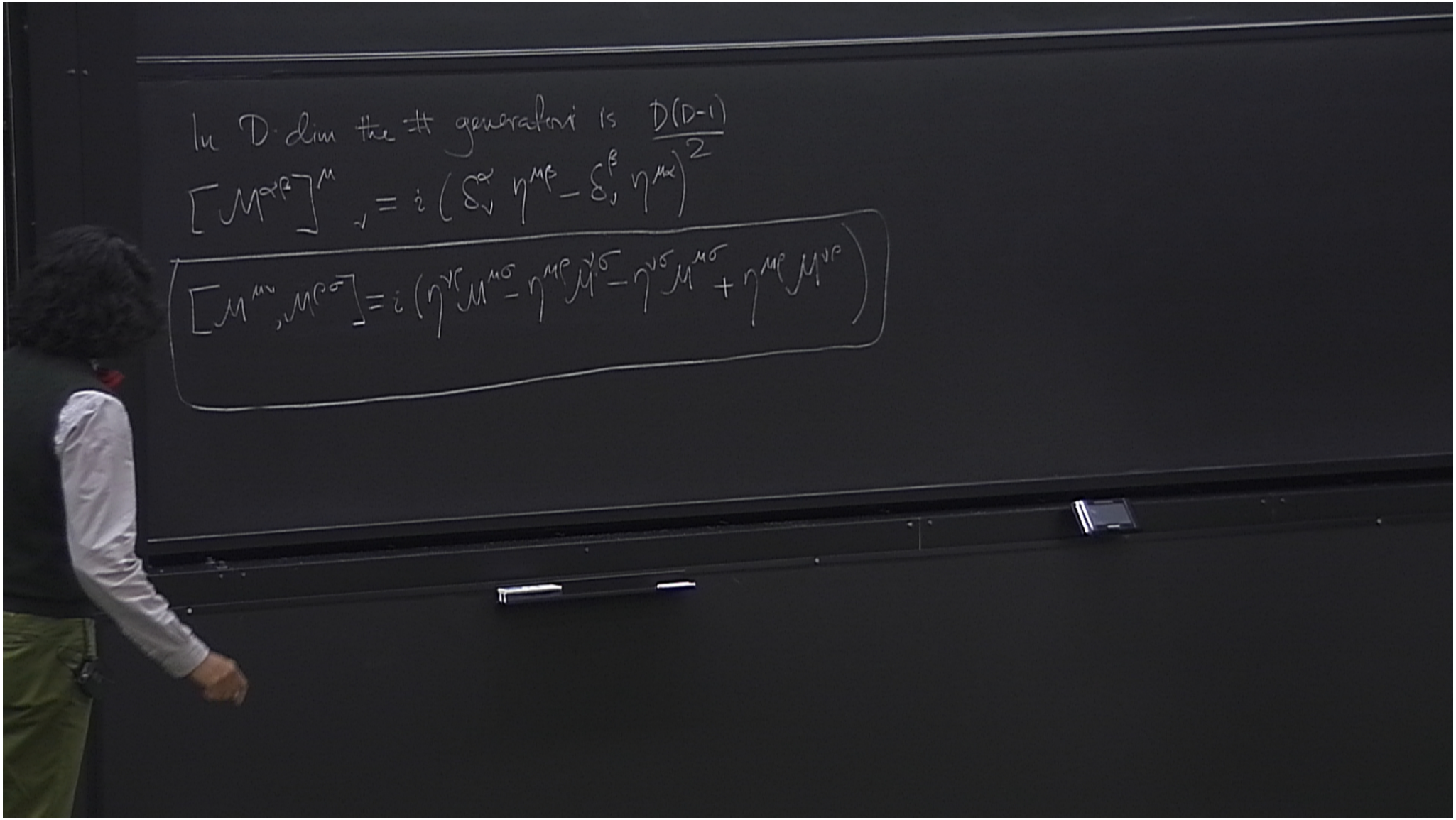

D - dimensions
$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, $\mu,\nu = 0,1,\ldots,D-1$
$i,j = 1,\ldots,D-1$
$SO(1,D-1)$

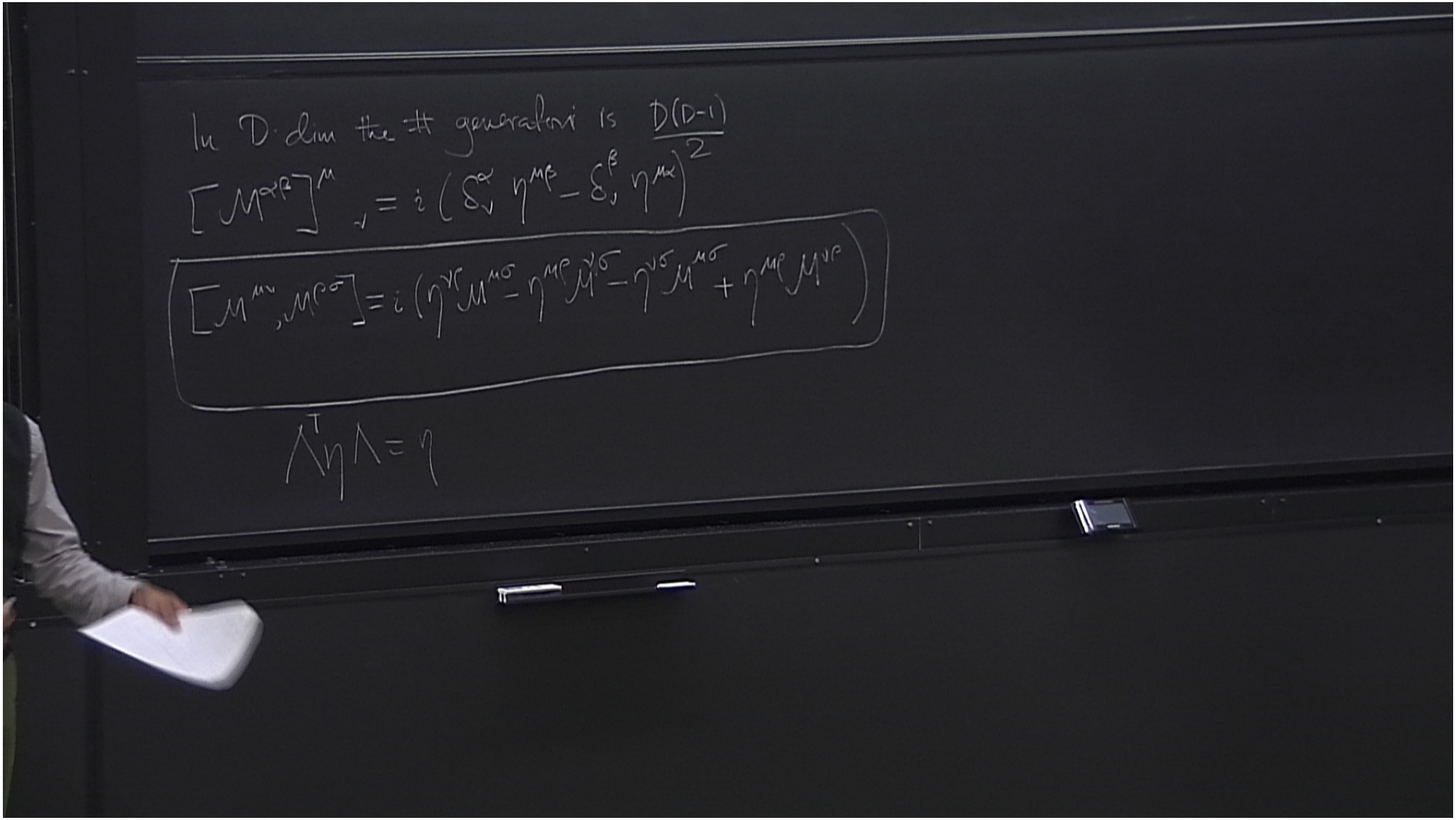

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$SO(1,D-1)$. $\eta^{\mu\nu} = diag(+ - - \cdots -)$
$(\gamma^0)^2 = 1$. $(\gamma^i)^2 = -1$


$= 2\eta^{\mu\nu}$, $\mu,\nu = 0,1,\ldots,D-1$
$1,\ldots,D-1$
$(+---$
$(\gamma^0)^\dagger = \gamma^0$, $(\gamma^i)^\dagger = -\gamma^i$
Lorent trans. matrix
$\Lambda^\mu{}_\nu = \left(\exp\left[-\frac{i}{2}\omega_{\alpha\beta}M^{\alpha\beta}\right]\right)^\mu{}_\nu$


In D-dim the # generators is $\frac{D(D-1)}{2}$


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$[M^{\alpha\beta}]^{\mu}{}_{\nu} = i\left(\delta^{\alpha}_{\nu}\eta^{\mu\beta} - \delta^{\beta}_{\nu}\eta^{\mu\alpha}\right)$
$[M^{\mu\nu}, M^{\rho\sigma}] = i\left(\eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho} + \eta^{\mu\rho}M^{\nu\sigma}\right)$

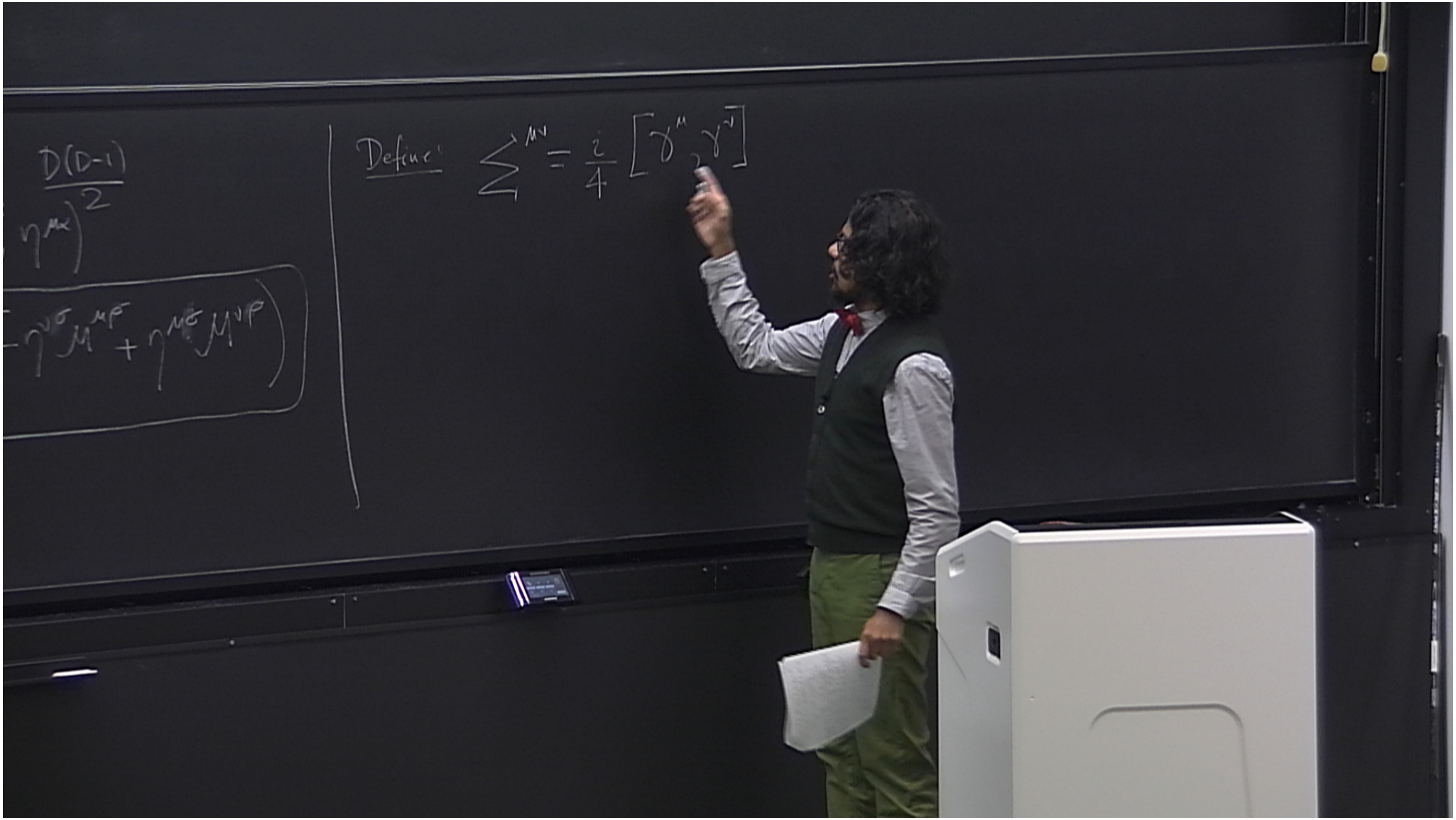

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$\Lambda^T\eta\Lambda = \eta$

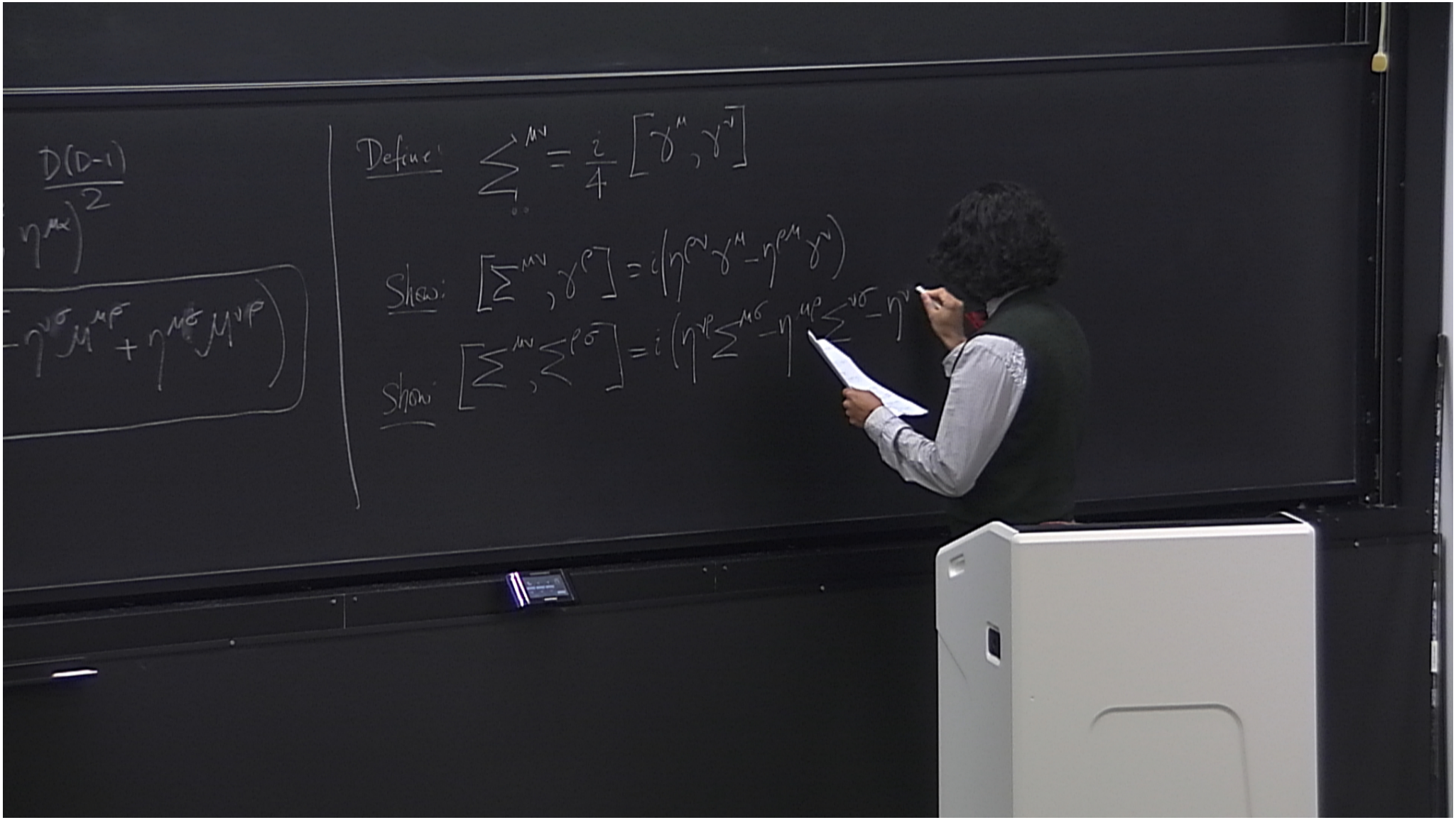
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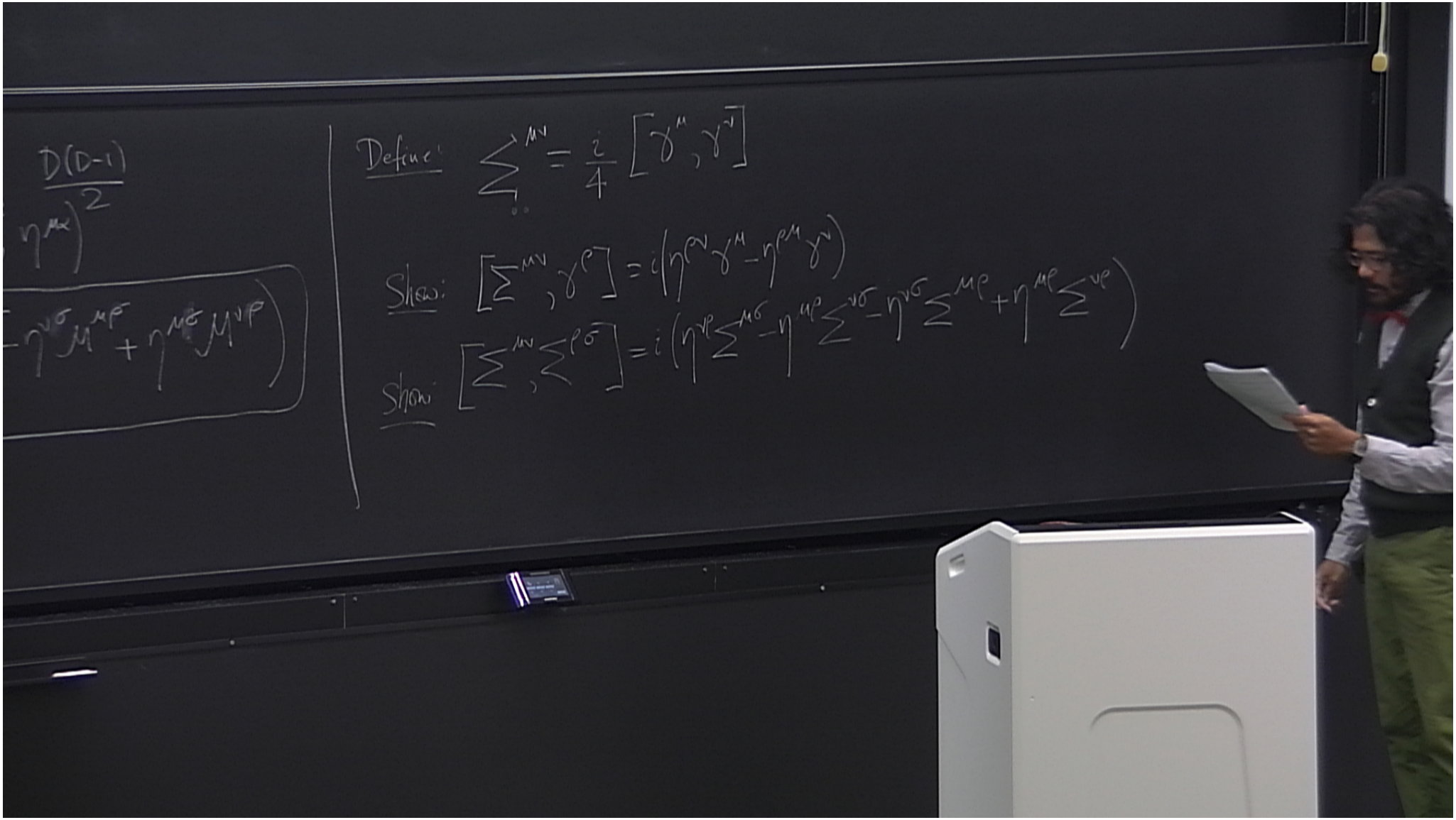
$$[M^{\alpha\beta}]^{\mu}{}_{\nu} = i\left(\delta^{\alpha}_{\nu}\eta^{\mu\beta} - \delta^{\beta}_{\nu}\eta^{\mu\alpha}\right)$$

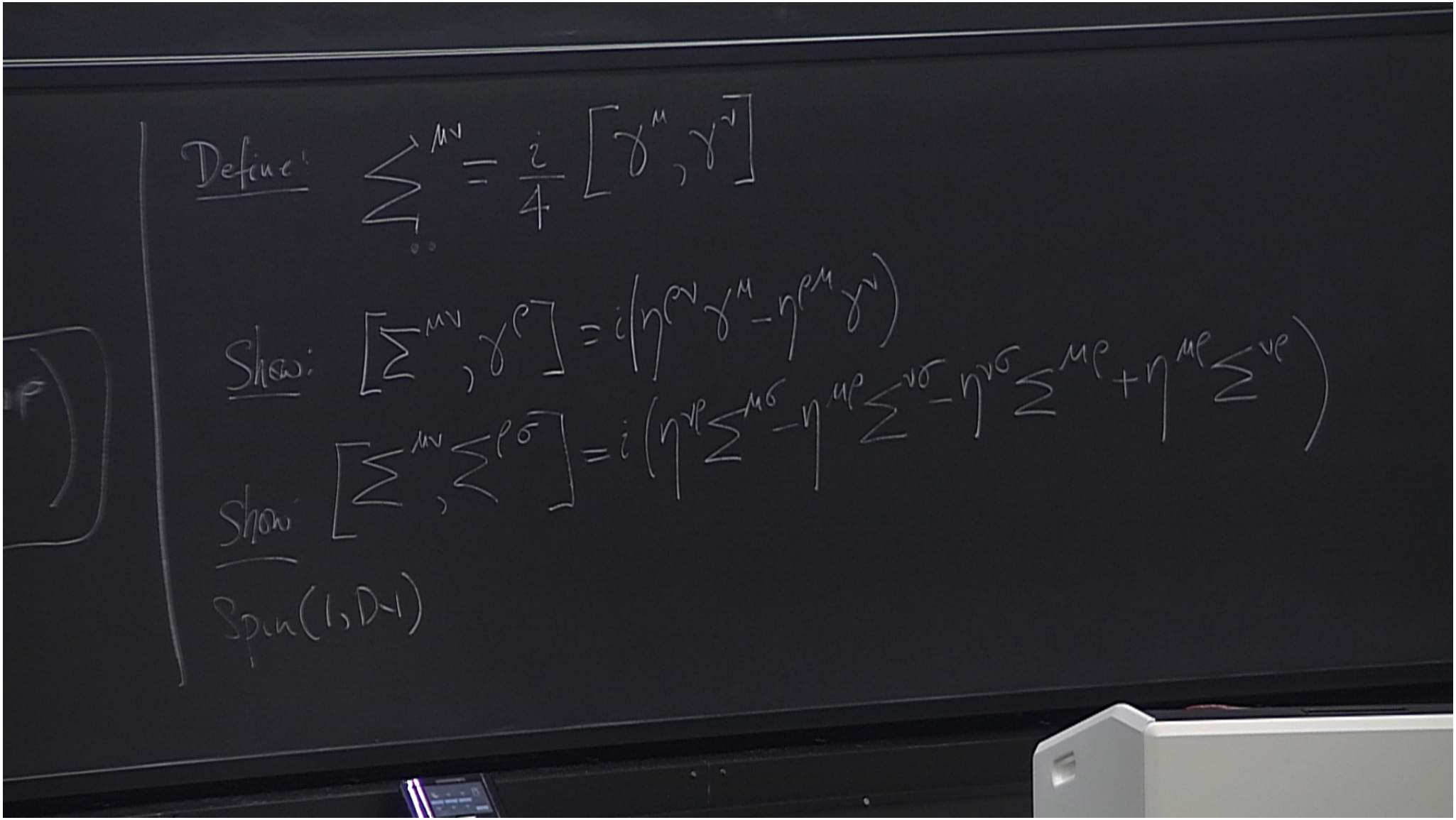
$$\boxed{[M^{\mu\nu}, M^{\rho\sigma}] = i\left(\eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\sigma} + \eta^{\mu\rho}M^{\nu\rho}\right)}$$

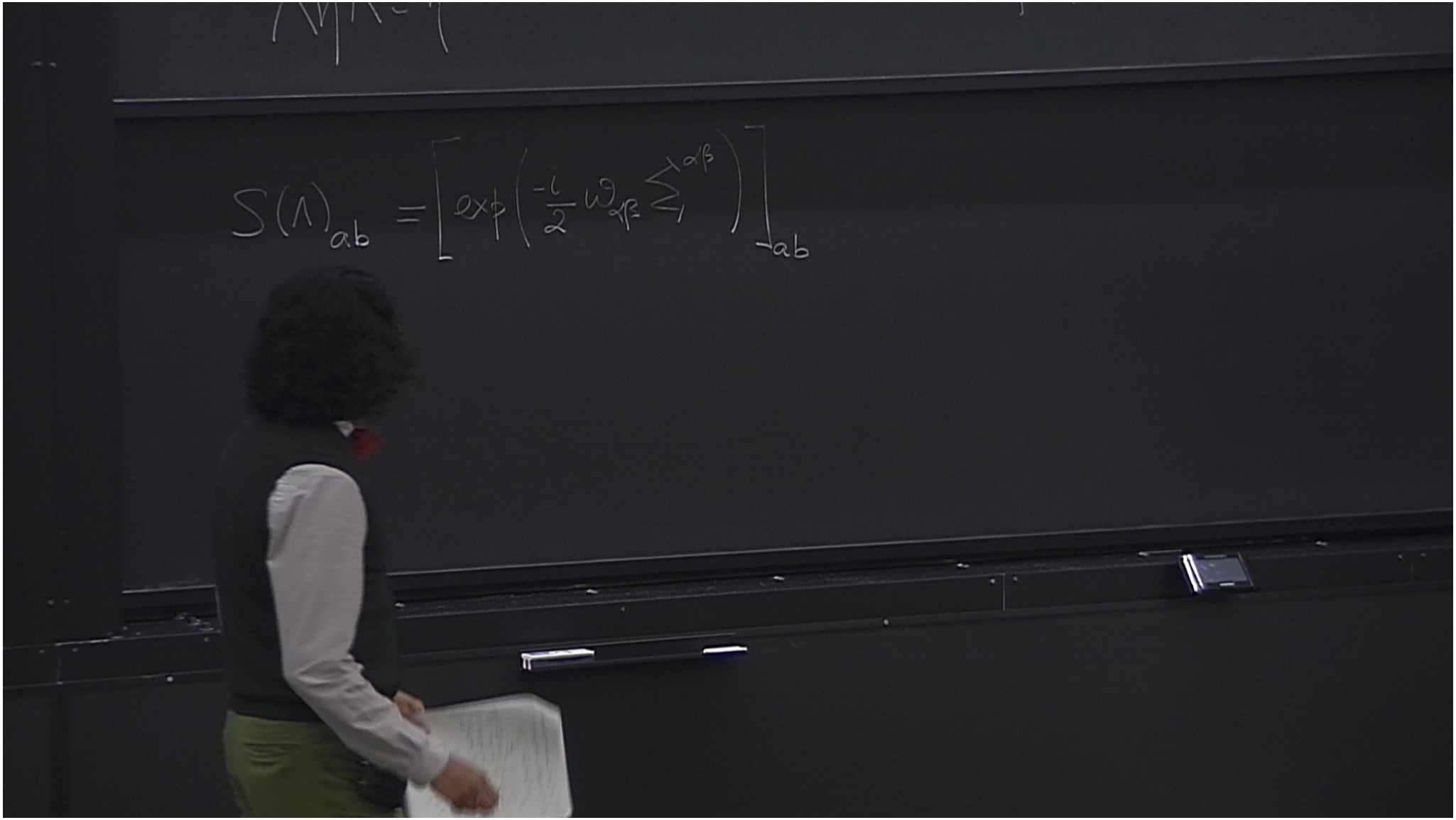
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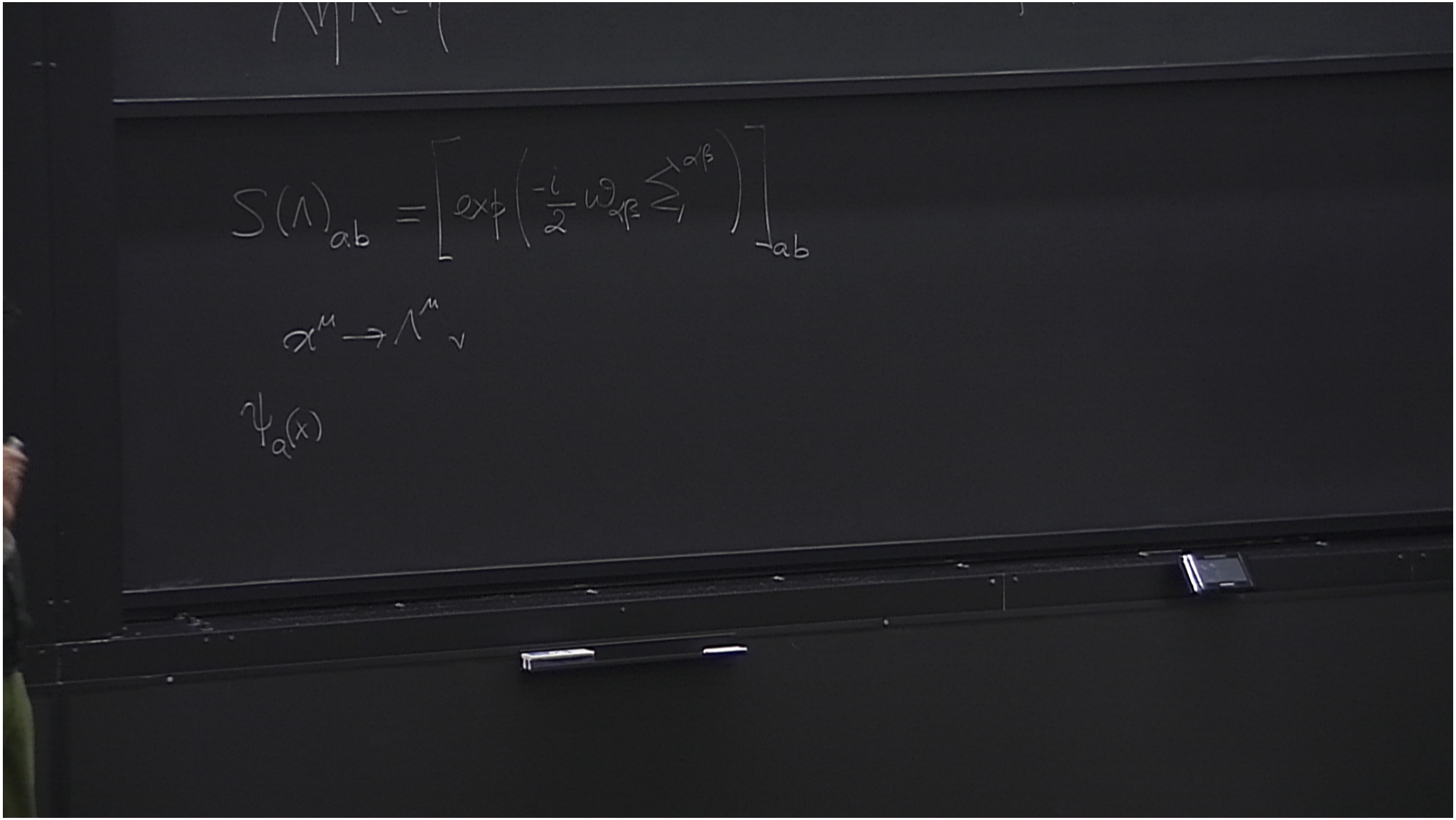

Define: $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$

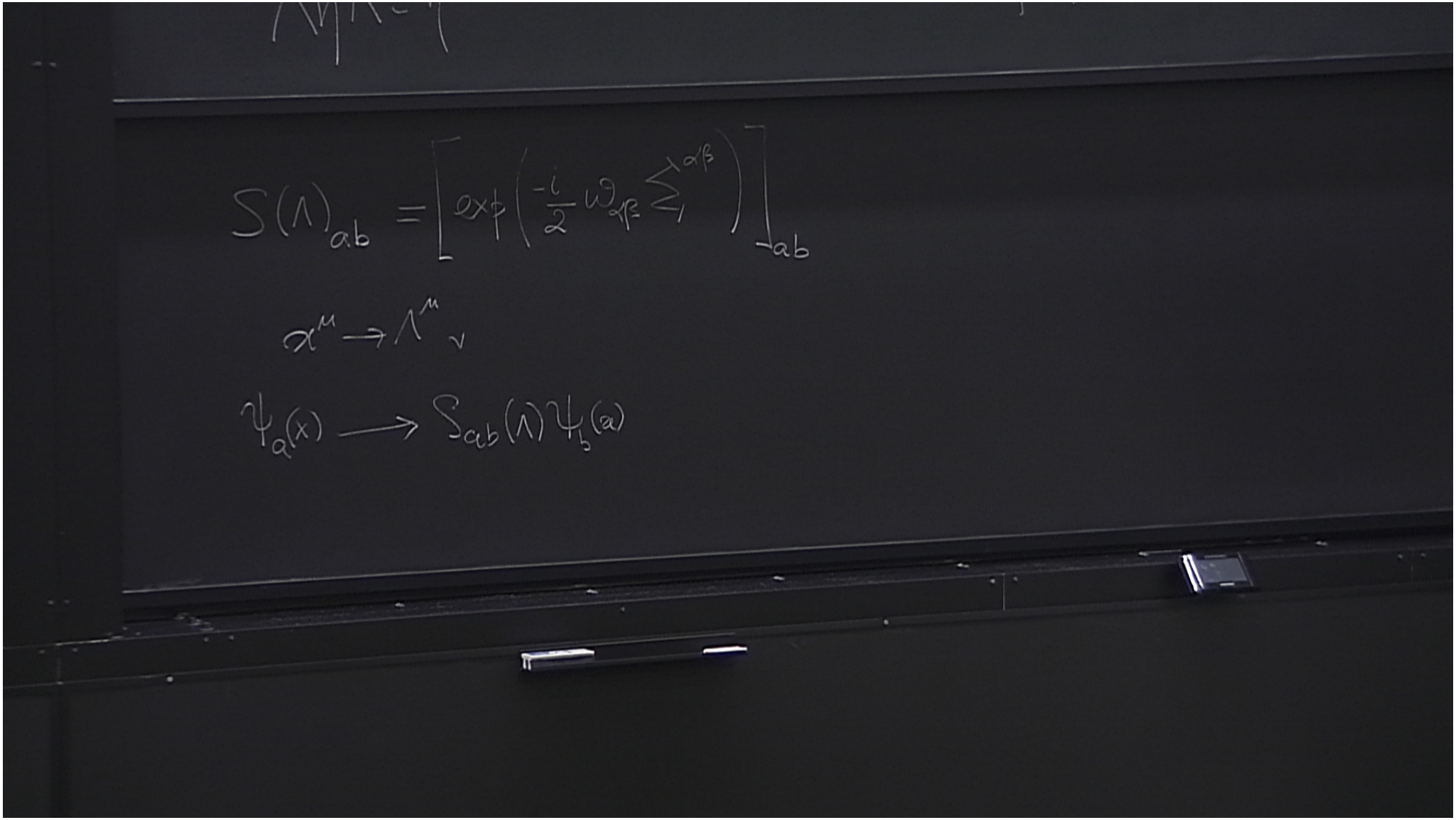

D(D-1)/2
Define: $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$
Show: $[\Sigma^{\mu\nu}, \gamma^\rho] = i(\eta^{\rho\nu}\gamma^\mu - \eta^{\rho\mu}\gamma^\nu)$
Show: $[\Sigma^{\mu\nu}, \Sigma^{\rho\sigma}] = i(\eta^{\nu\rho}\Sigma^{\mu\sigma} - \eta^{\mu\rho}\Sigma^{\nu\sigma} - \eta^{\nu}$

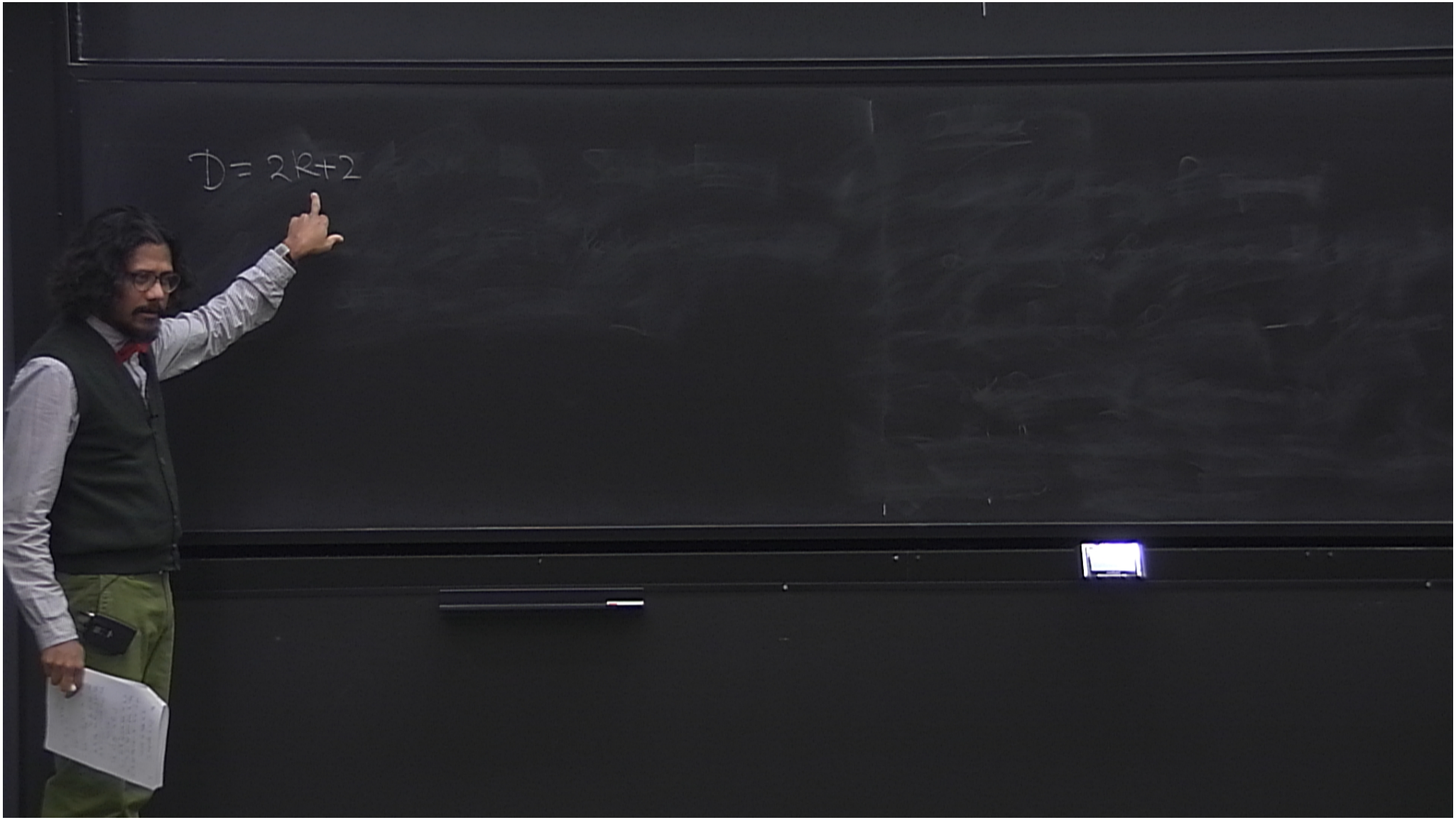

D(D-1)
2
Define: $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$
Show: $[\Sigma^{\mu\nu}, \gamma^\rho] = i(\eta^{\rho\nu}\gamma^\mu - \eta^{\rho\mu}\gamma^\nu)$
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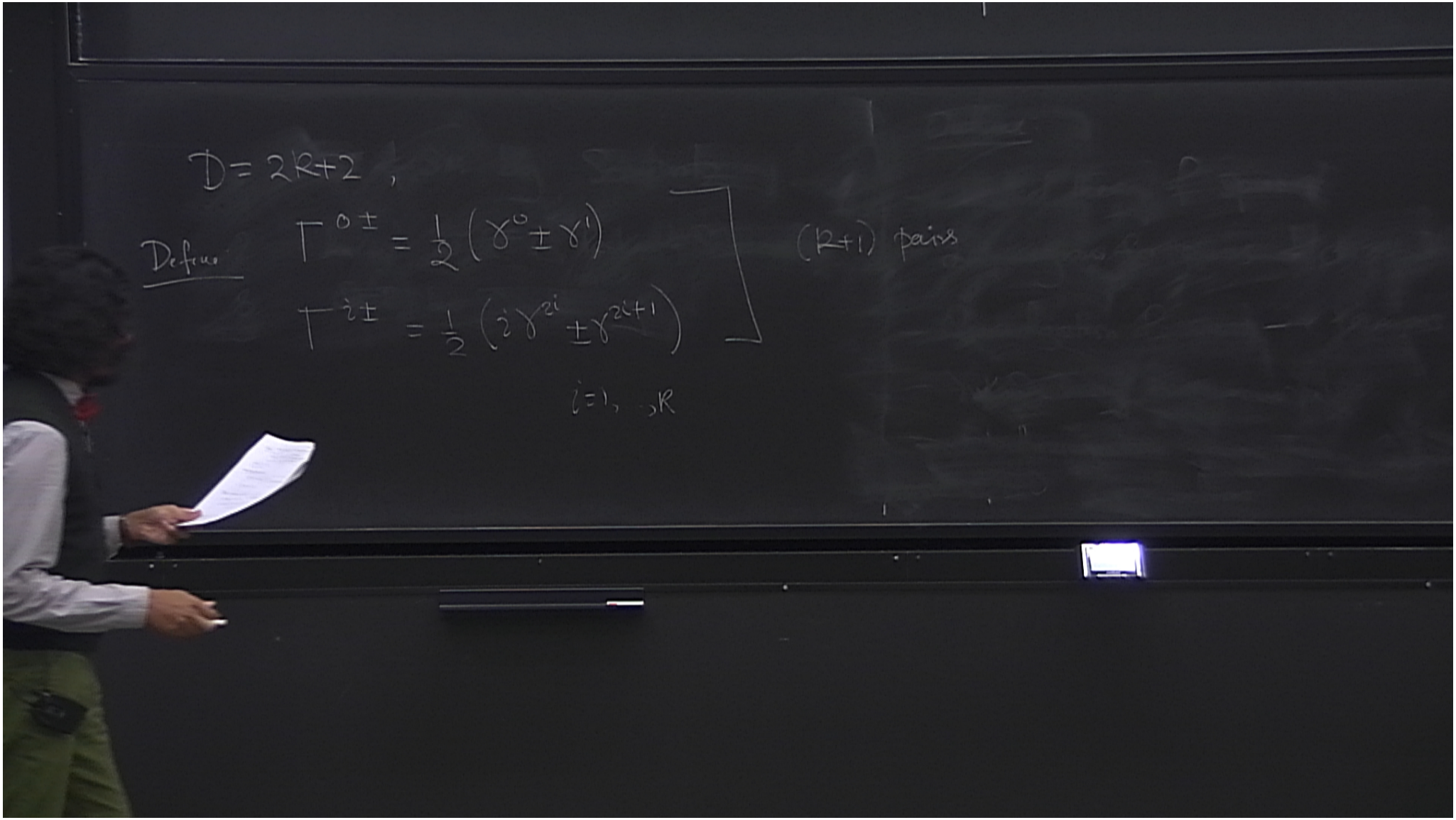

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Spin(1, D-1)

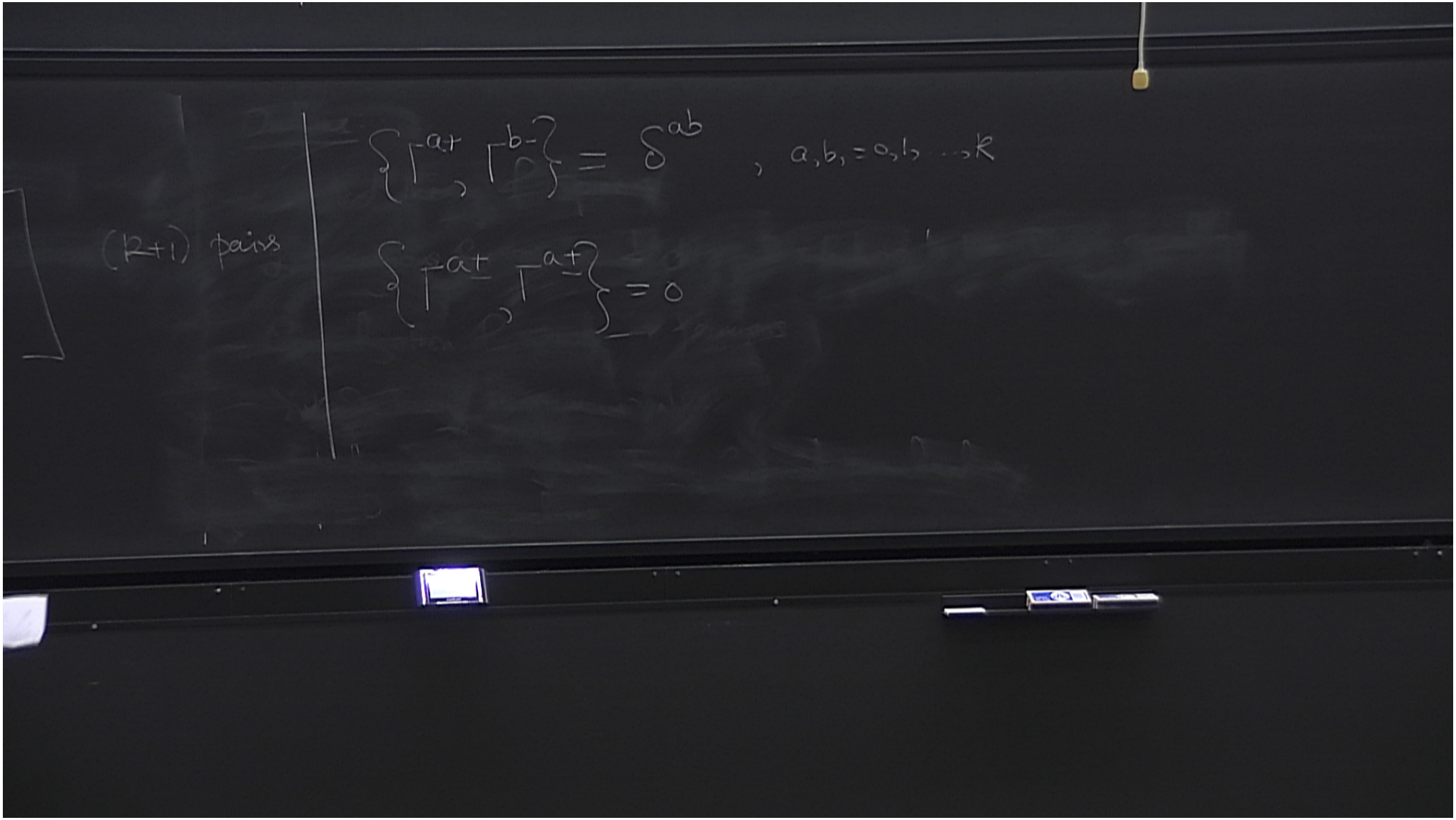

$S(\Lambda)_{ab} = \left[\exp\left(\frac{-i}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right) \right]_{ab}$

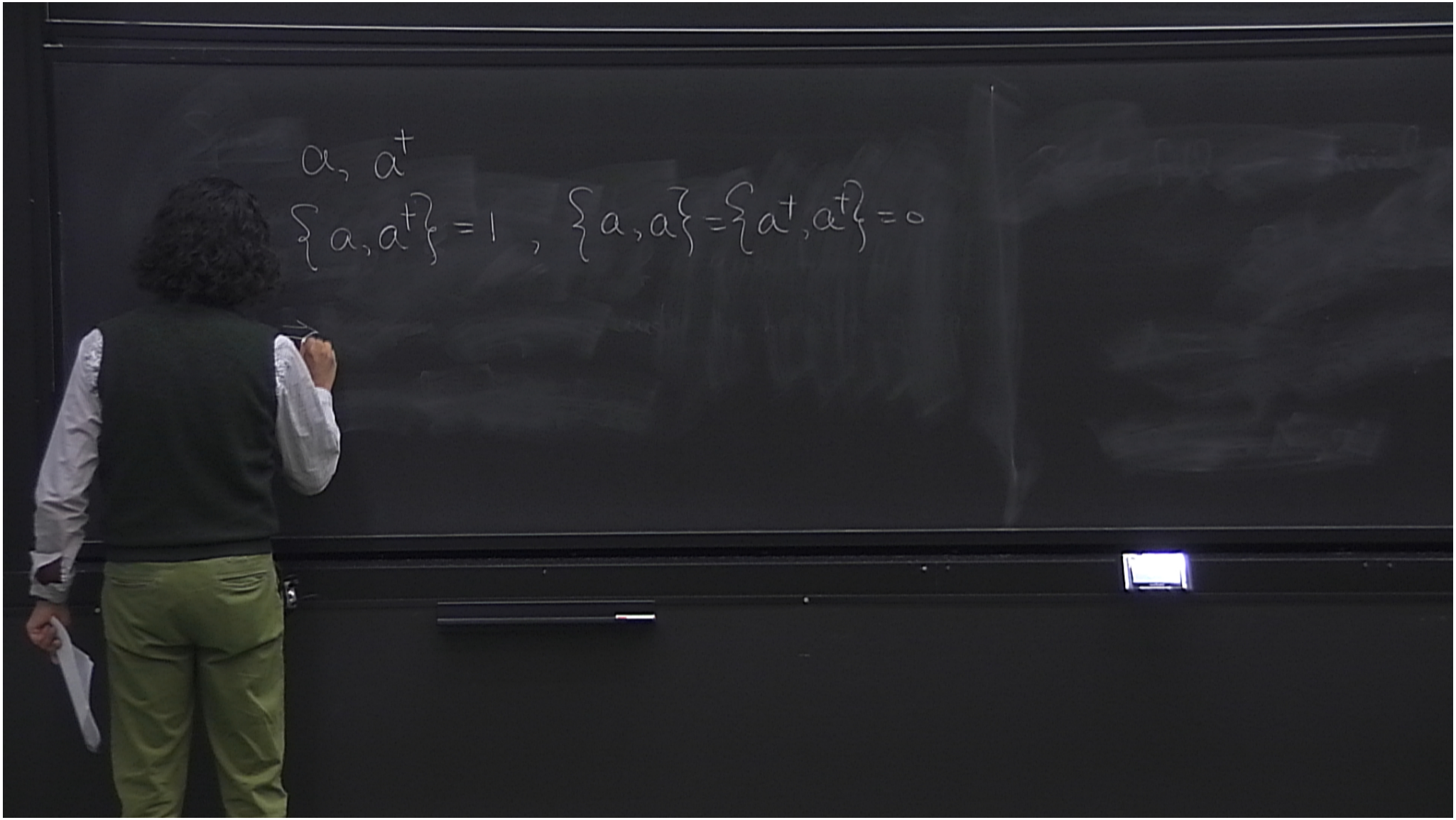

$S(\Lambda)_{ab} = \left[\exp\left(\frac{-i}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right) \right]_{ab}$
$x^\mu \to \Lambda^\mu{}_\nu$
$\psi_a(x)$

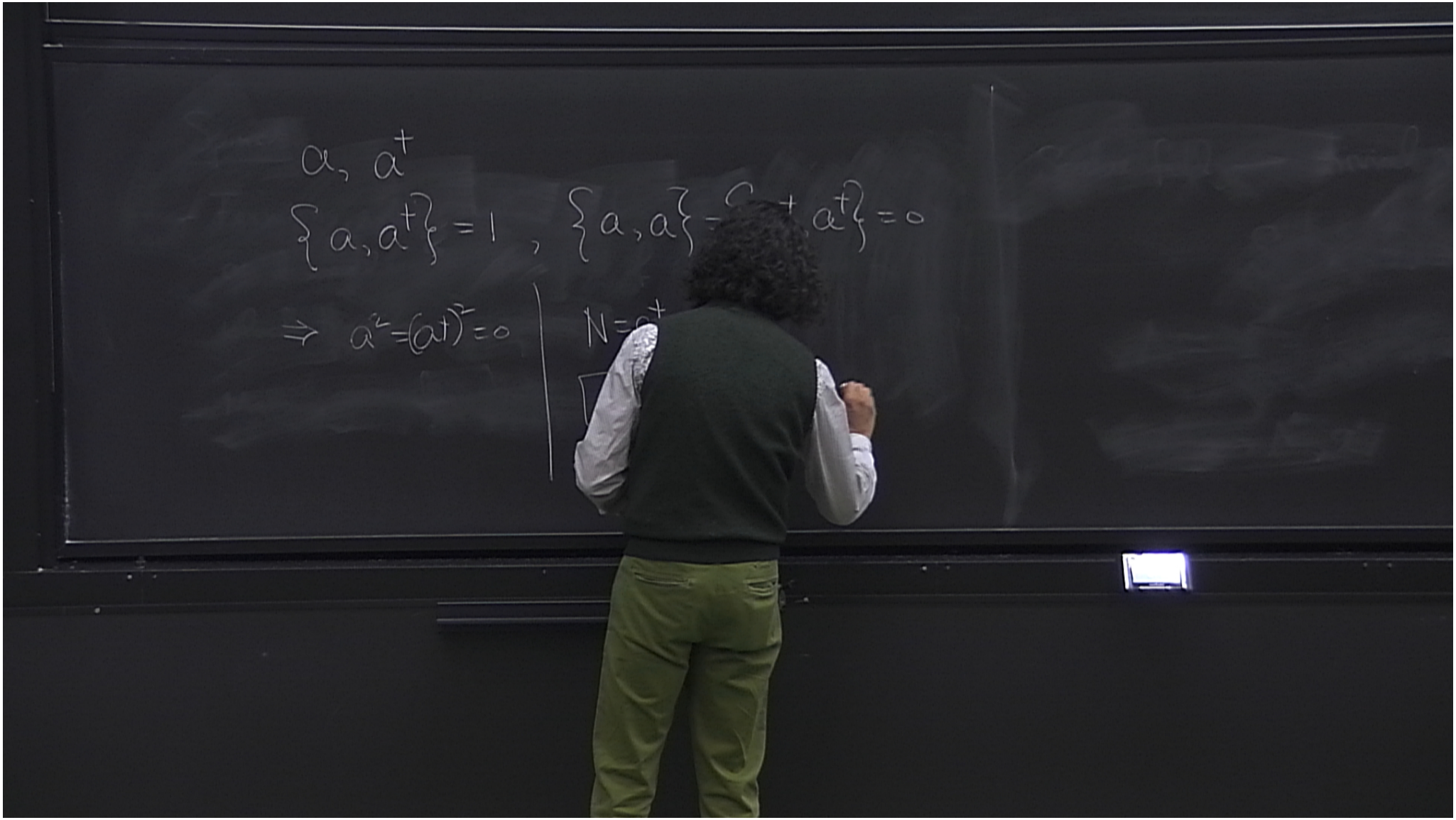

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$x^\mu \to \Lambda^\mu{}_\nu$
$\psi_a(x) \longrightarrow S_{ab}(\Lambda) \psi_b(x)$

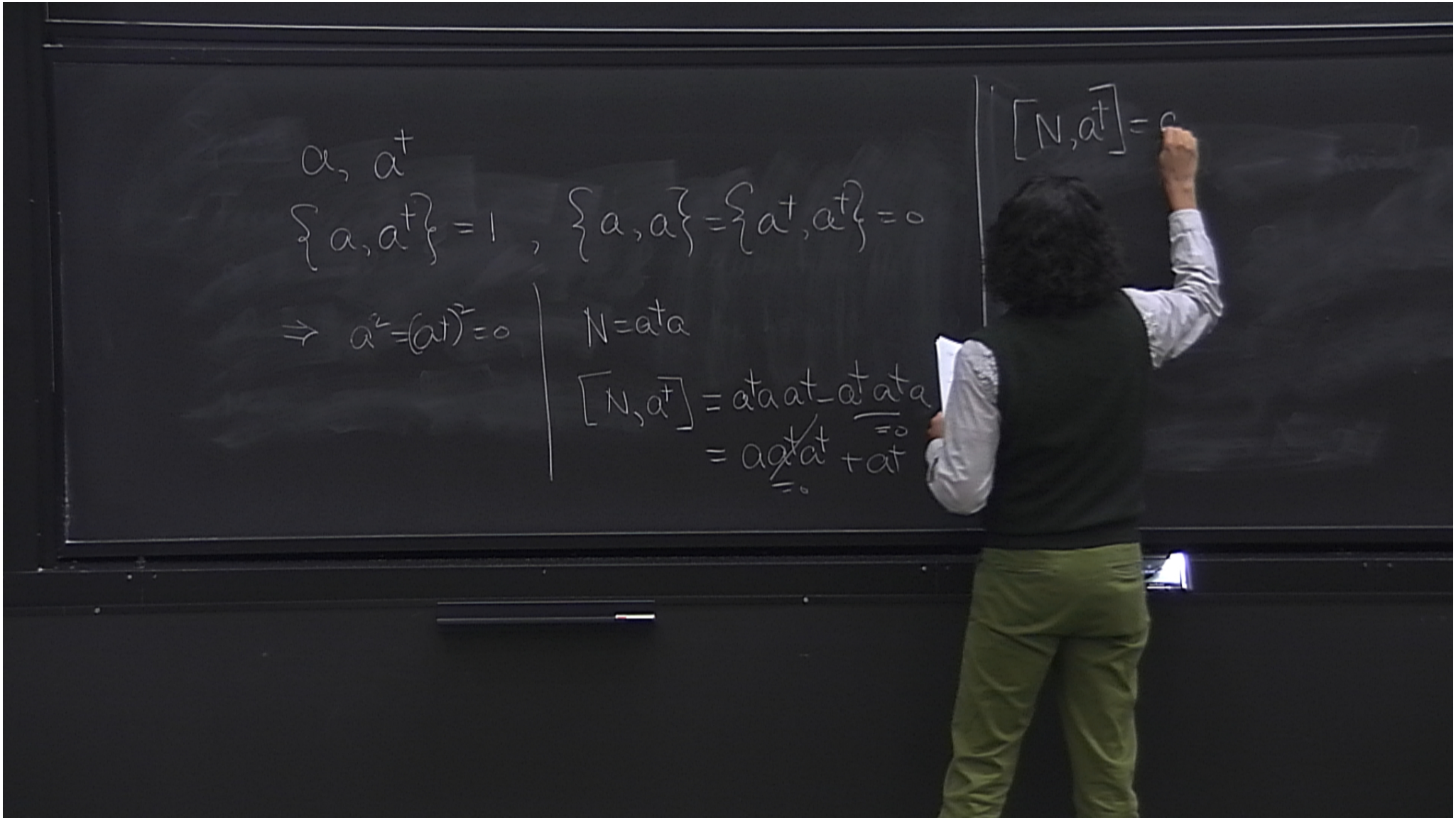

D = 2R+2

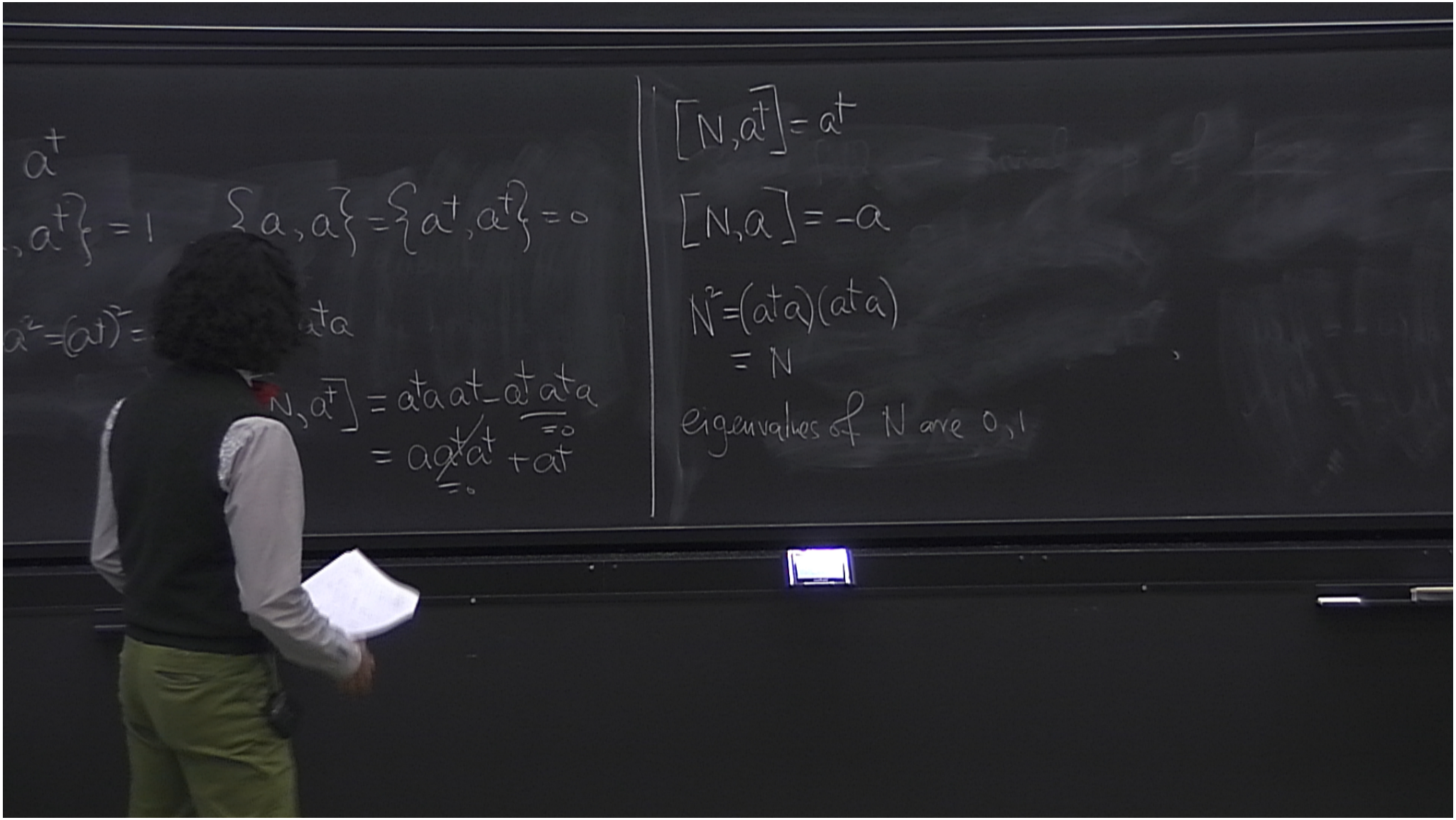

D = 2R+2 ,
Define
$\Gamma^{0\pm} = \frac{1}{2}(\gamma^0 \pm \gamma^1)$
$\Gamma^{i\pm} = \frac{1}{2}(i\gamma^{2i} \pm \gamma^{2i+1})$
(R+1) pairs
i = 1, ..., R

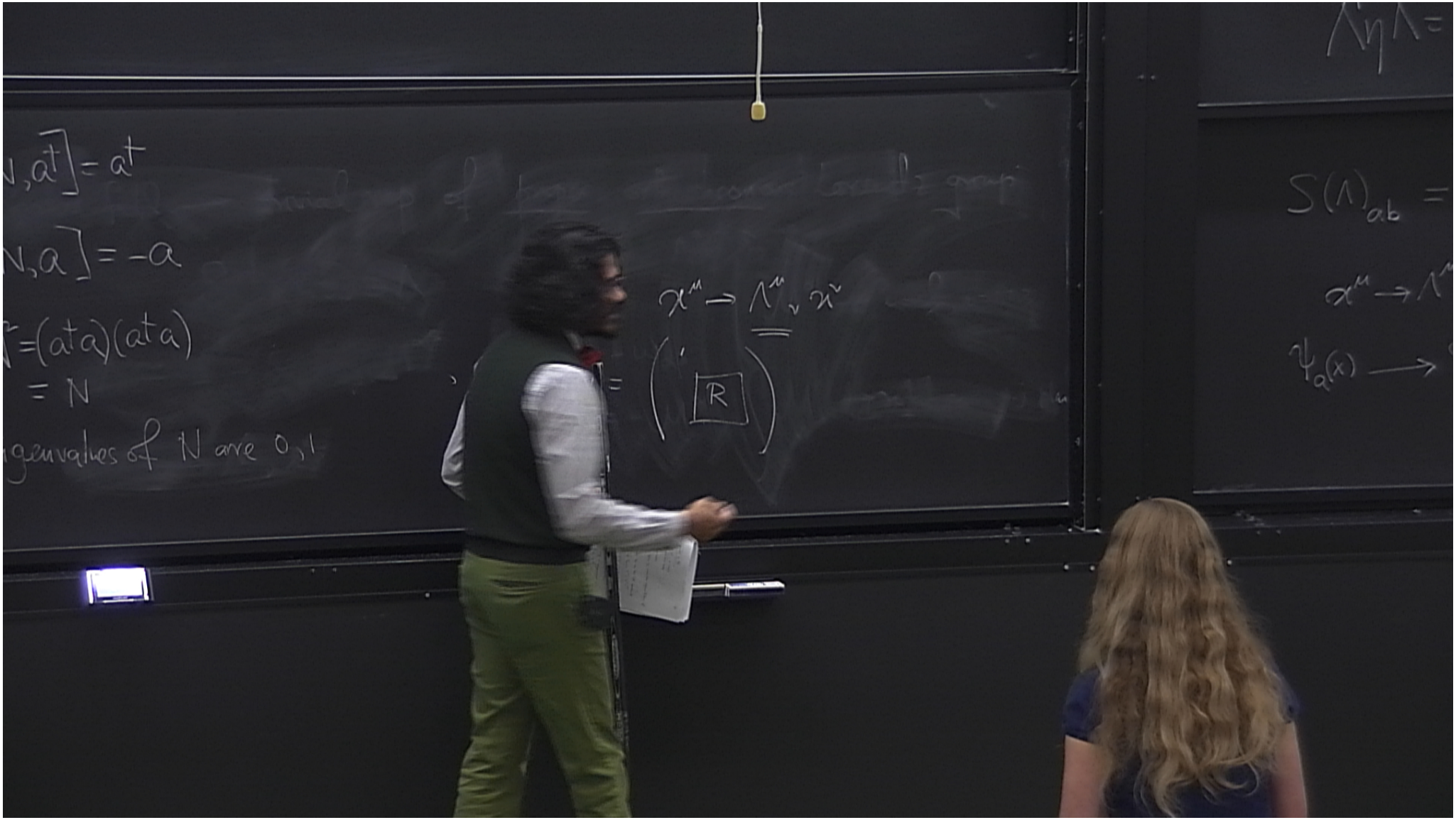

(k+1) pairs
$\{\Gamma^{a+}, \Gamma^{b-}\} = \delta^{ab}$, $a, b, = 0, 1, \ldots, k$
$\{\Gamma^{a+}, \Gamma^{a+}\} = 0$

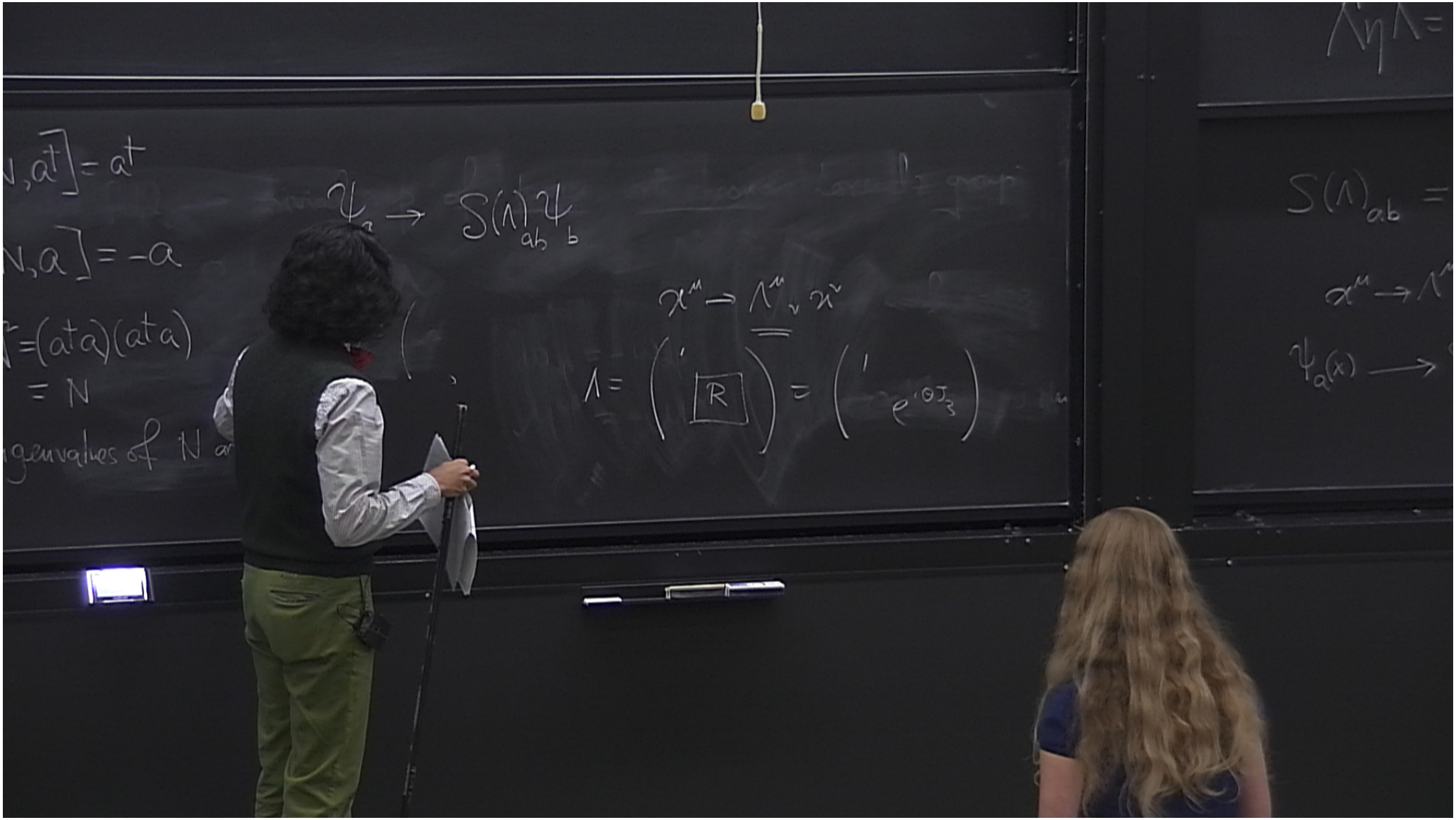

a, a†
{a, a†} = 1 , {a, a} = {a†, a†} = 0

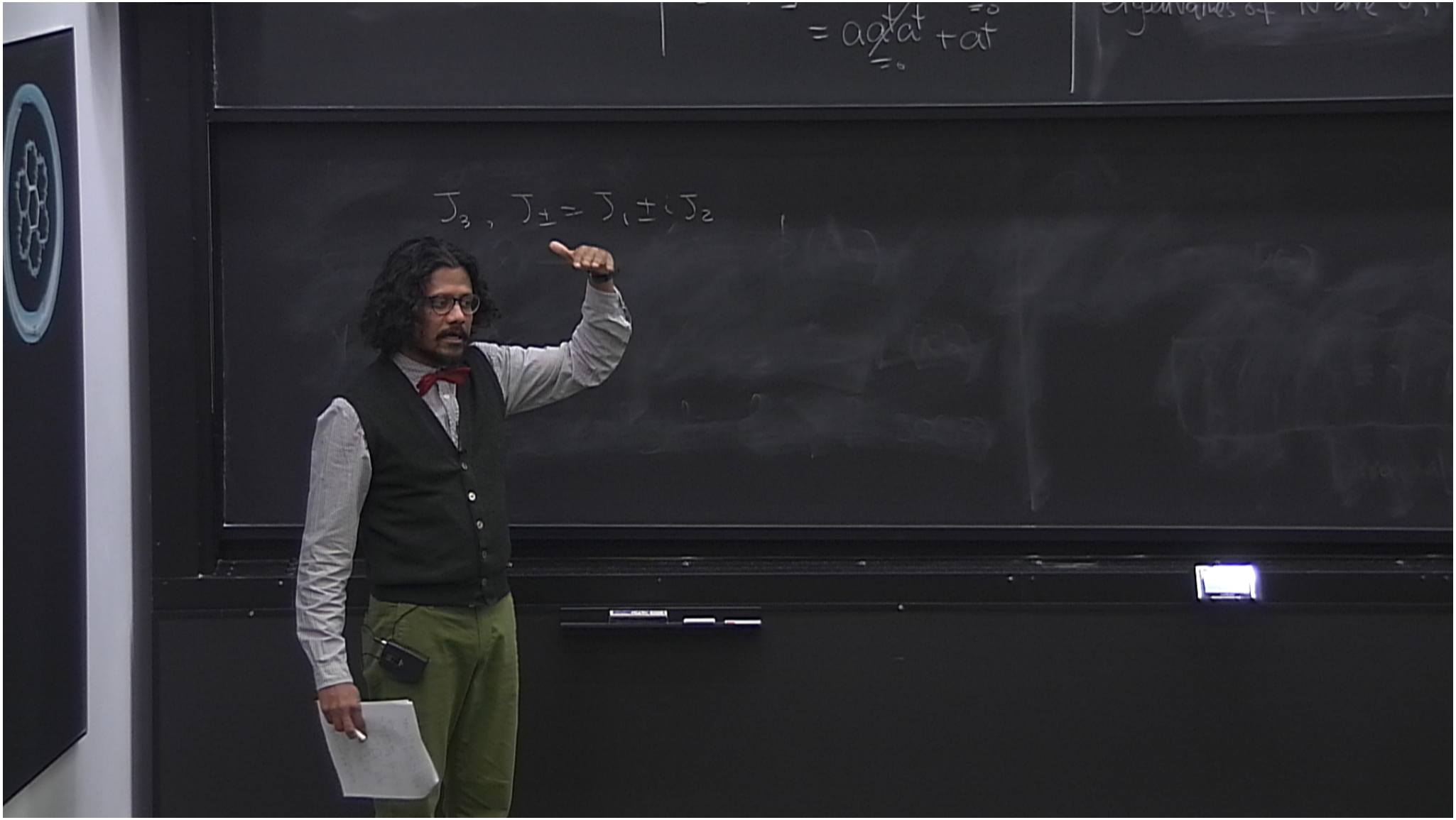

a, a†
{a, a†} = 1 , {a, a} = {a†, a†} = 0
⇒ a² = (a†)² = 0
N = a†

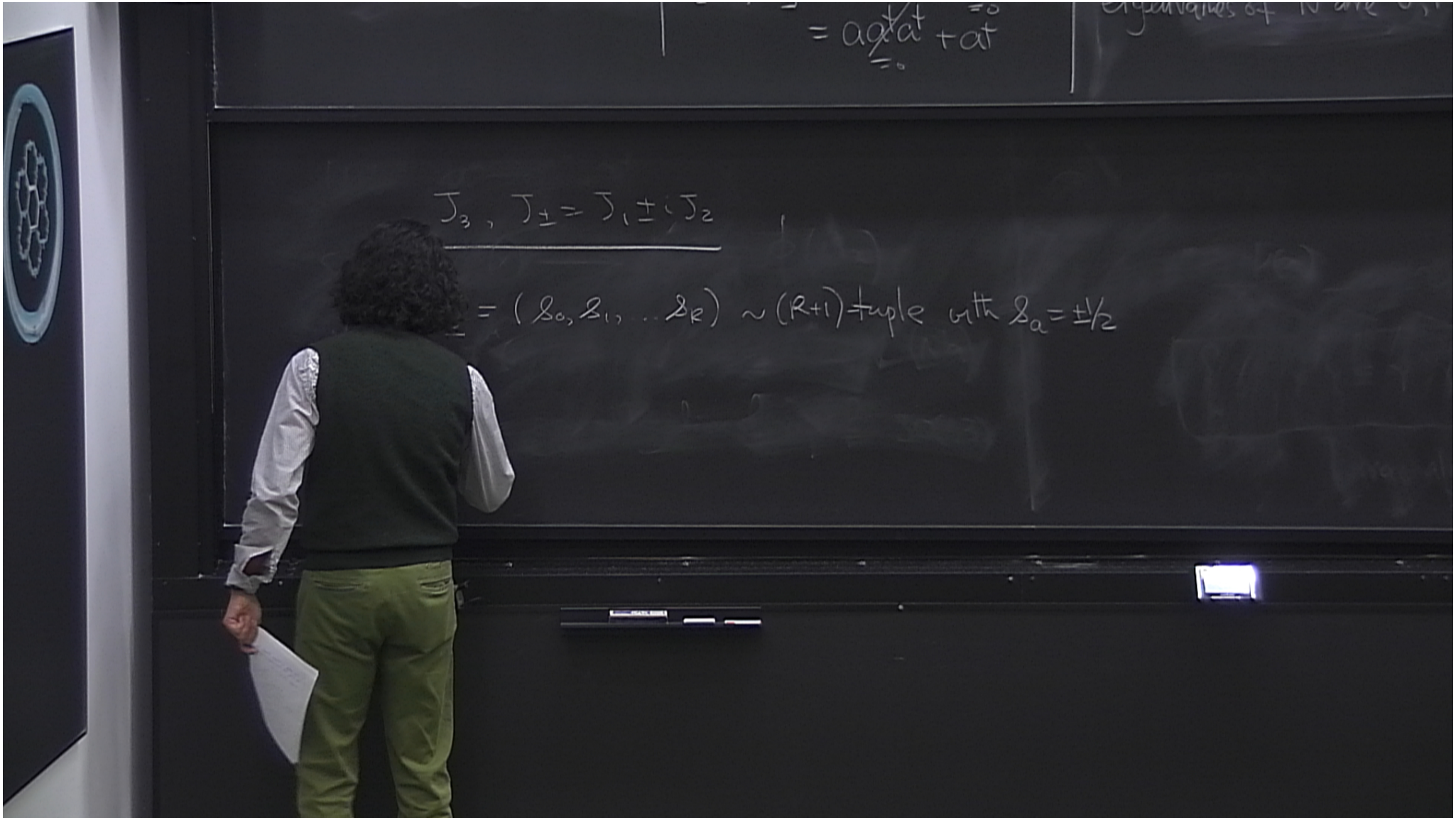

a, a†
{a, a†} = 1 , {a, a} = {a†, a†} = 0
⇒ a² = (a†)² = 0
N = a†a
[N, a†] = a†aa† − a†a†a
= a(a†a†) + a†
[N, a†] =

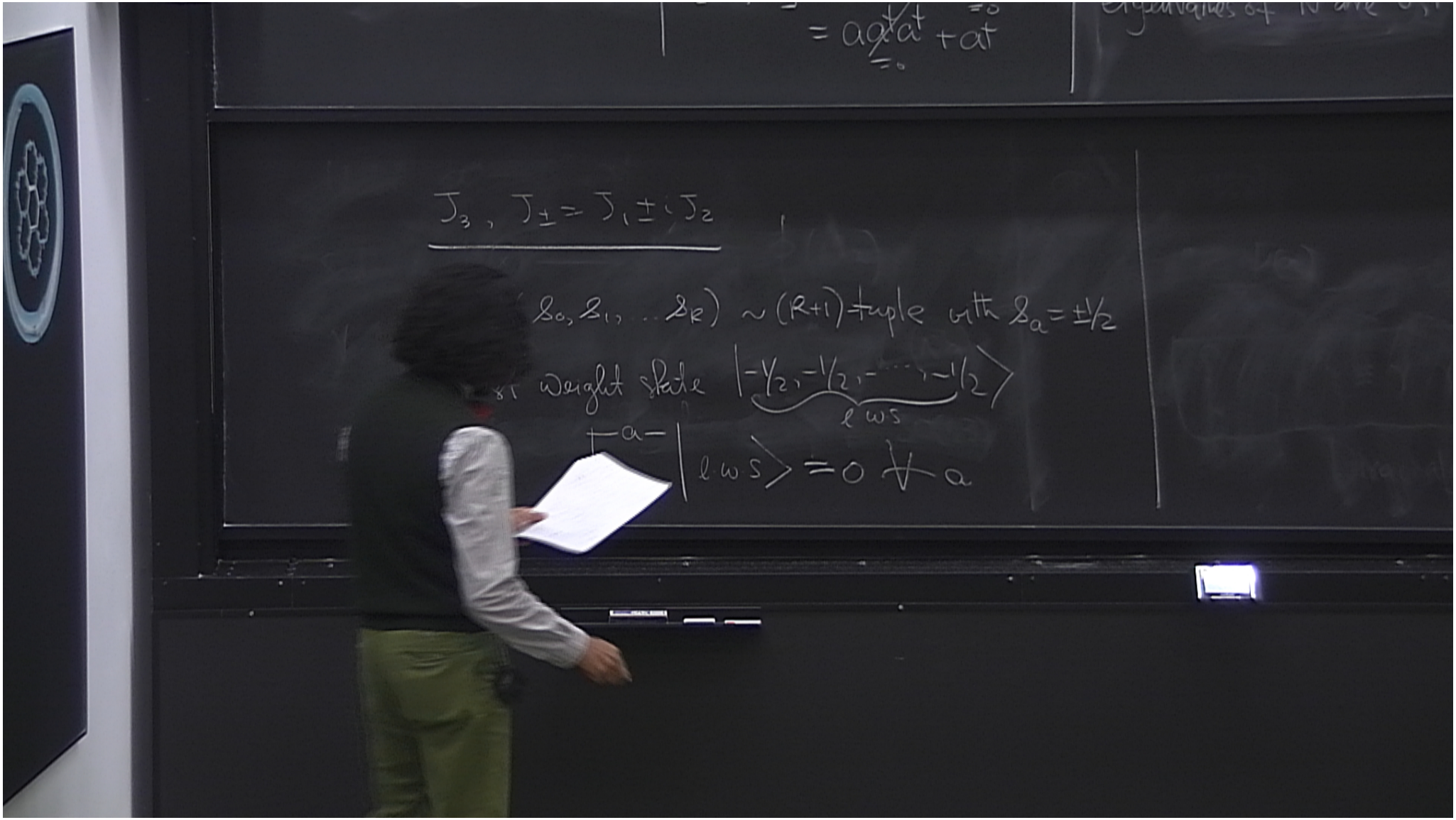

$a^\dagger$
$\{a, a^\dagger\} = 1$
$\{a, a\} = \{a^\dagger, a^\dagger\} = 0$
$a^2 = (a^\dagger)^2 =$
$a^\dagger a$
$N, a^\dagger] = a^\dagger a a^\dagger - a^\dagger a^\dagger a a$
$= a a^\dagger a^\dagger + a^\dagger$
$[N, a^\dagger] = a^\dagger$
$[N, a] = -a$
$N^2 = (a^\dagger a)(a^\dagger a)$
$= N$
eigenvalues of N are $0, 1$

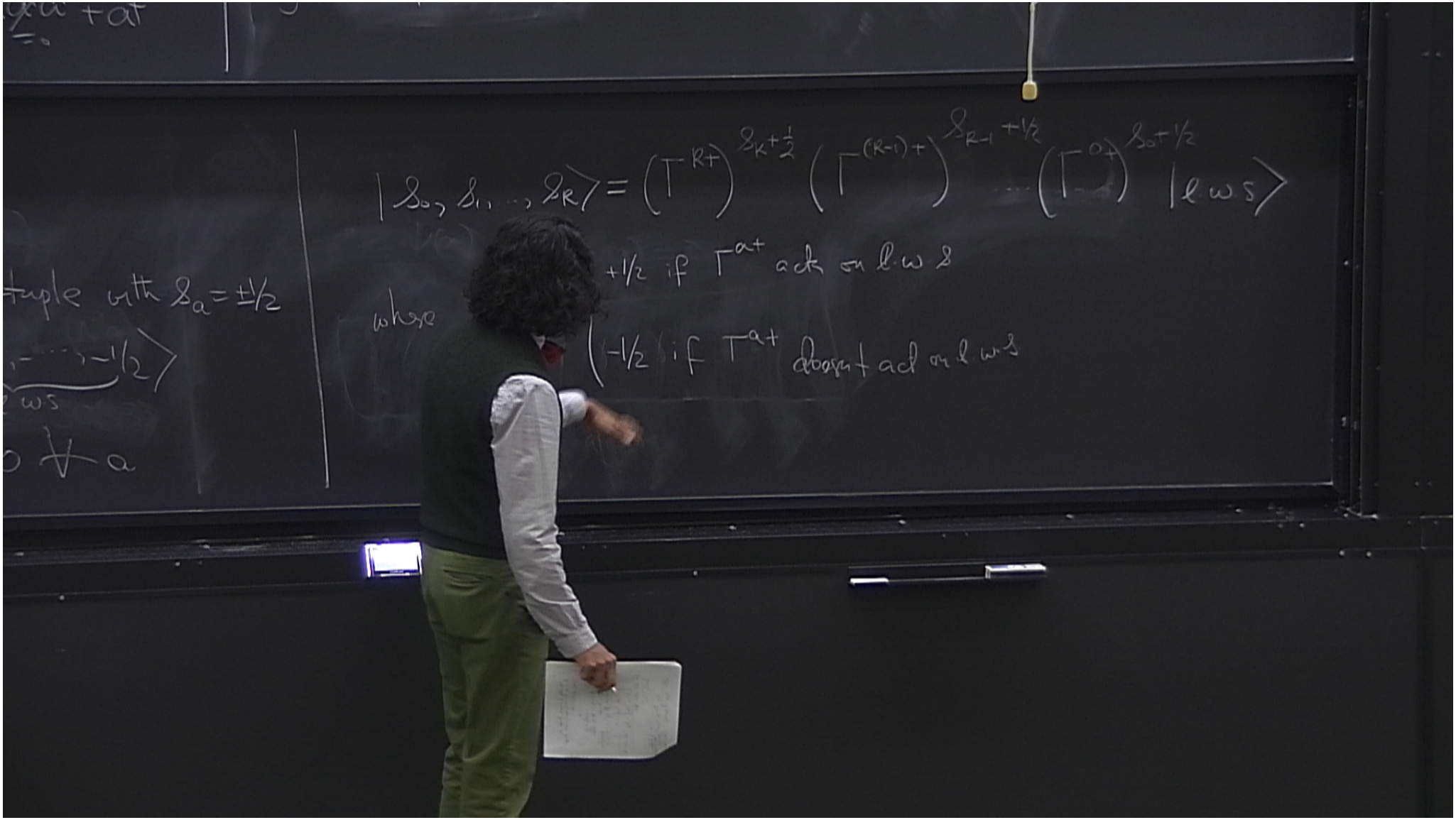

$N, a^\dagger] = a^\dagger$
$N, a] = -a$
$= (a^\dagger a)(a^\dagger a)$
$= N$
eigenvalues of N are $0, 1$
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
R
$S(\Lambda)_{ab}$
$\psi_a(x) \longrightarrow$

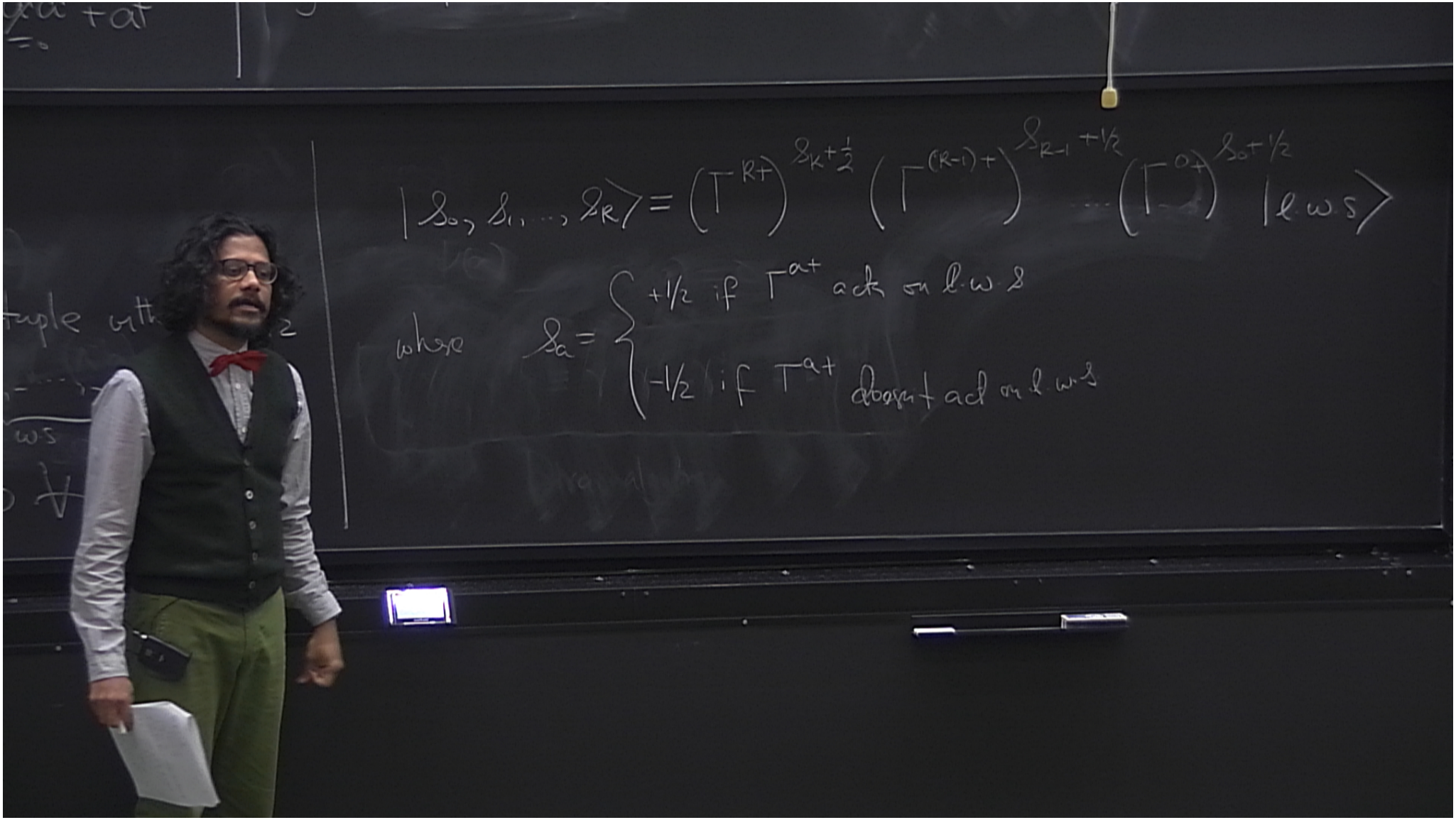

$\psi_a \to S(\Lambda)_{ab} \psi_b$
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\Lambda = \begin{pmatrix} 1 & \\ & R \end{pmatrix} = \begin{pmatrix} 1 & \\ & e^{i\theta J_3} \end{pmatrix}$
$S(\Lambda)_{ab}$
$\psi_a(x) \to$

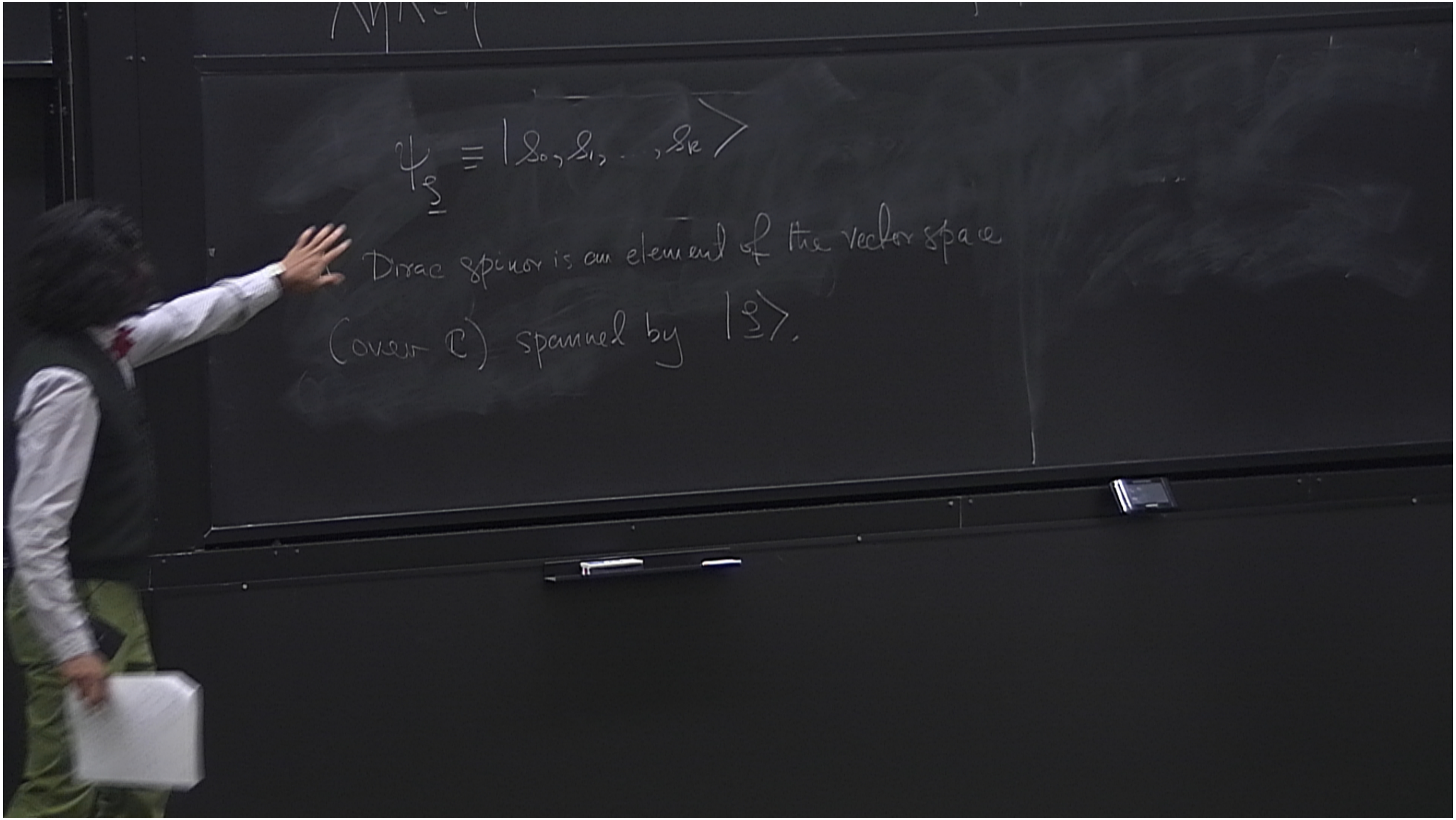

$J_3, J_\pm = J_1 \pm i J_2$

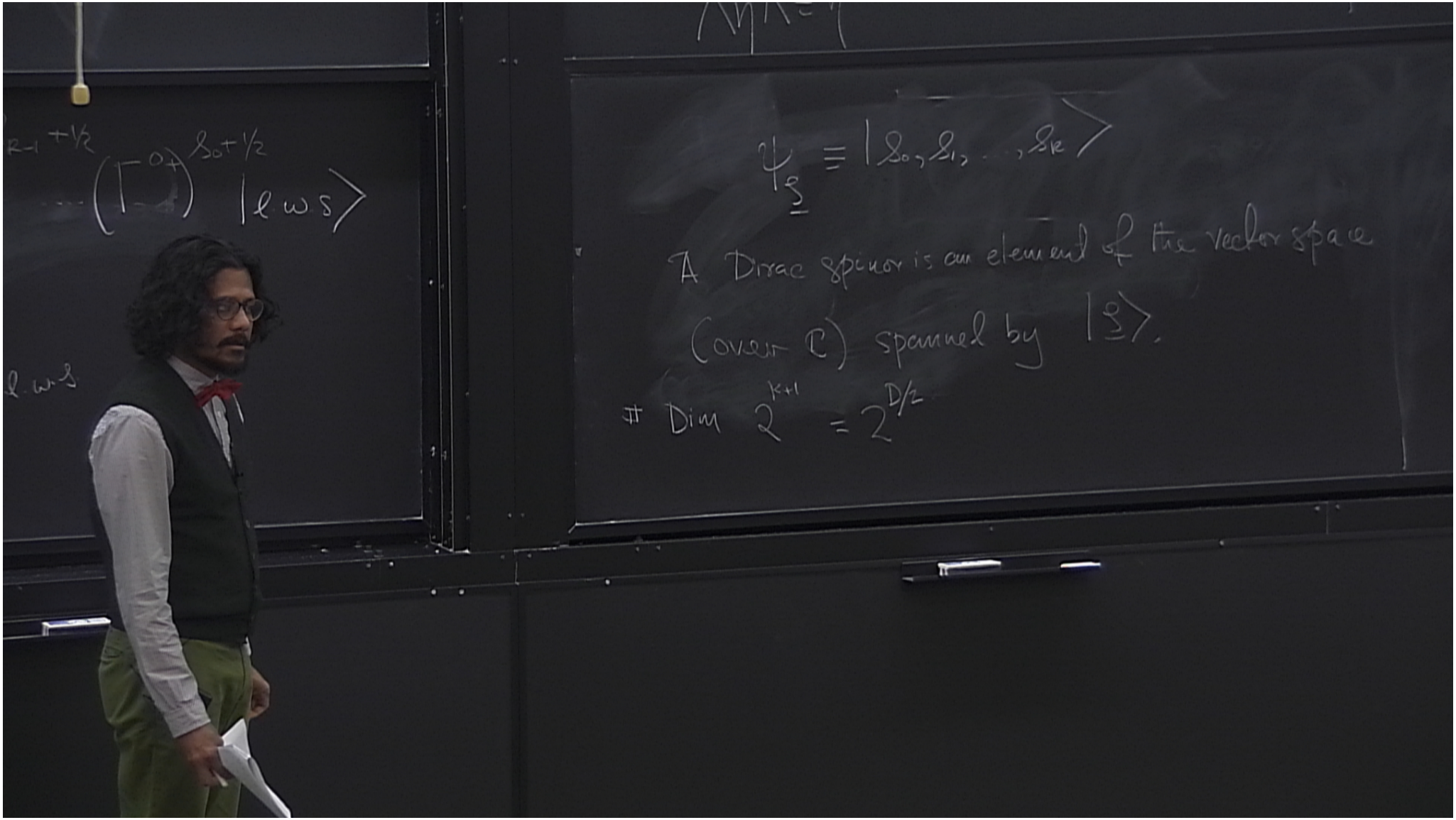

$= a\dot{a}^{\dagger}a + \dot{a}^{\dagger}$
$J_3, J_\pm = J_1 \pm i J_2$
$= (s_0, s_1, \ldots s_R) \sim (R+1)$-tuple with $s_a = \pm 1/2$

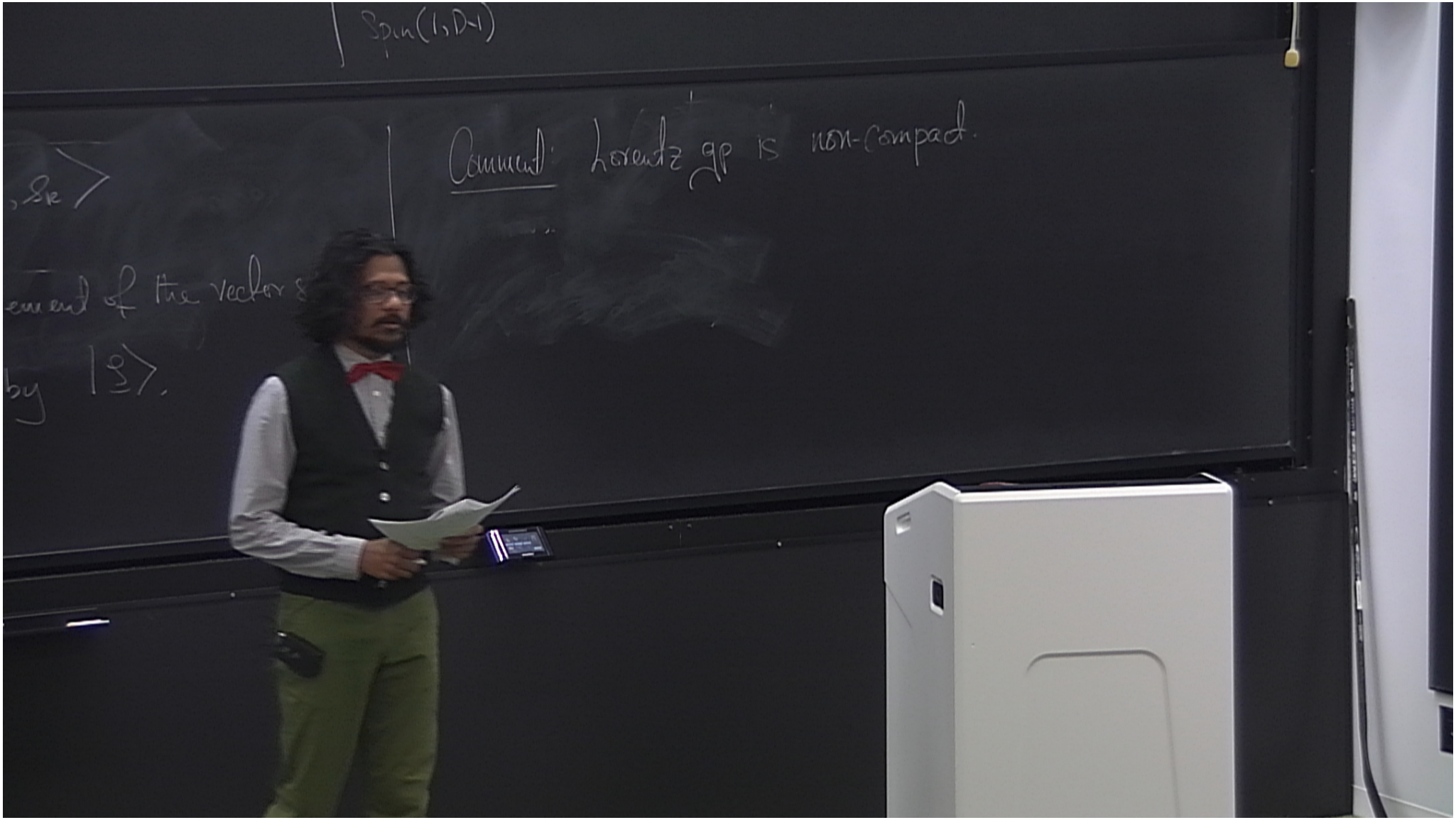

$= a a^\dagger a + a^\dagger$
$J_3, J_\pm = J_1 \pm i J_2$
$s_0, s_1, \ldots s_R) \sim (R+1)$-tuple with $s_a = \pm 1/2$
weight state $|-1/2, -1/2, \ldots, -1/2\rangle$
$\ell w s$
$-a-|\ell w s\rangle = 0 \; \forall \, a$

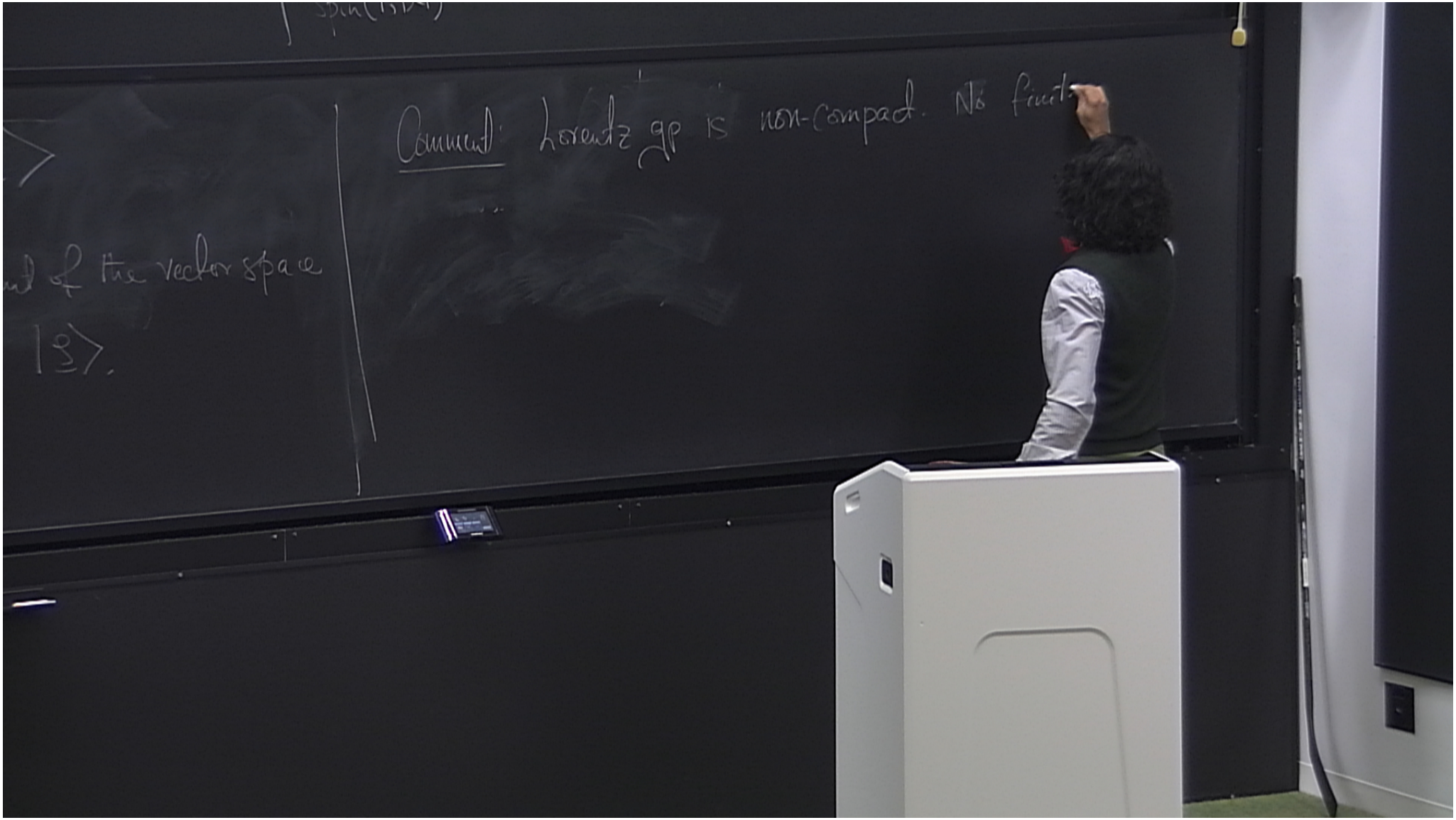

$|s_0, s_1, \ldots, s_R\rangle = (\Gamma^{R+})^{s_R+\frac{1}{2}} (\Gamma^{(R-1)+})^{s_{R-1}+\frac{1}{2}} \cdots (\Gamma^{0+})^{s_0+\frac{1}{2}} |\ell\, \omega\, s\rangle$
$+1/2$ if Γ^{a+} acts on $\ell \cdot \omega \cdot s$
whose
$-1/2$ if Γ^{a+} doesn't act on $\ell \cdot \omega \cdot s$
tuple with $s_a = \pm 1/2$
$\forall\, a$

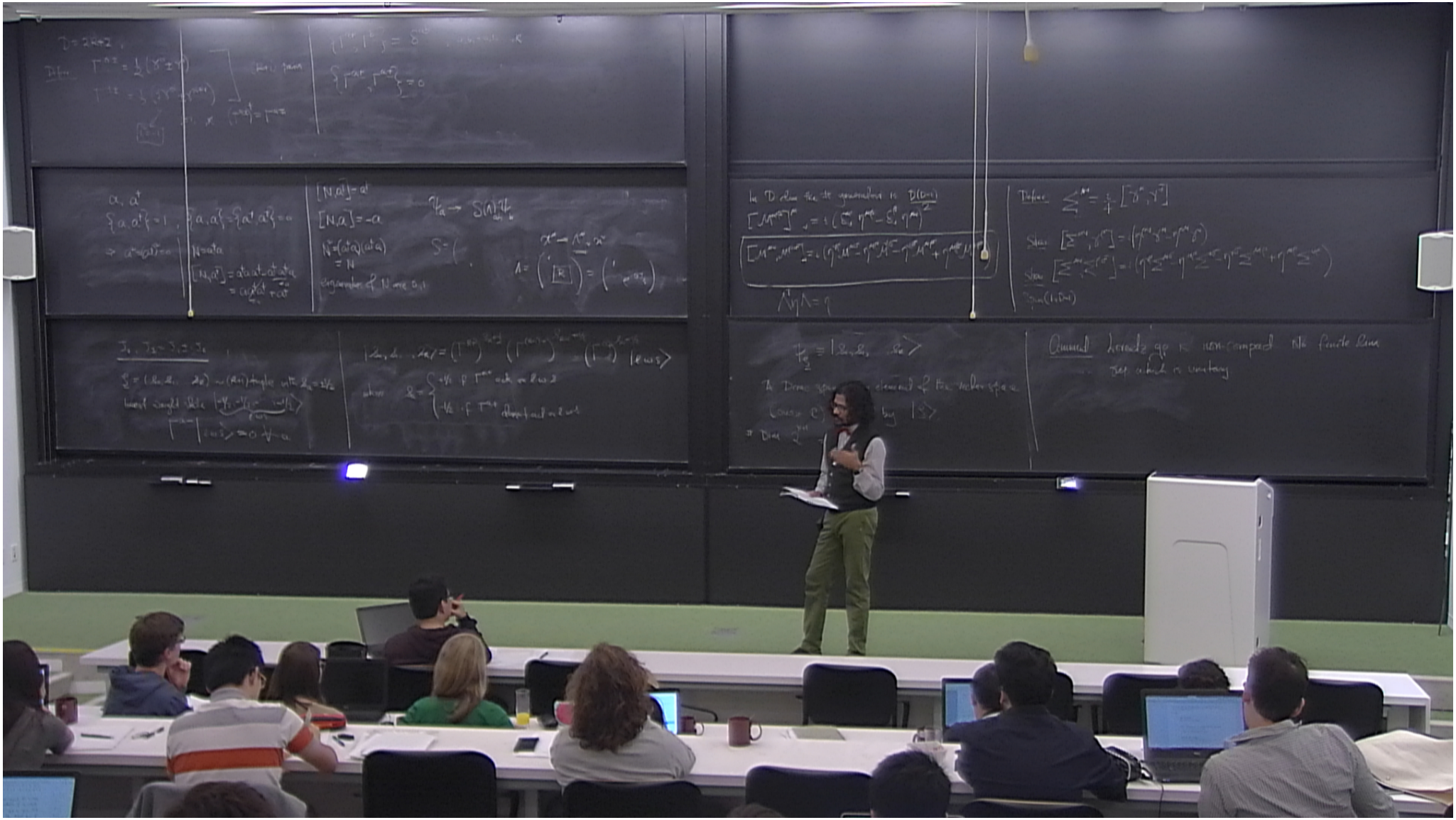

$|s_0, s_1, \ldots, s_R\rangle = (\Gamma^{R+})^{s_R+\frac{1}{2}} (\Gamma^{(R-1)+})^{s_{R-1}+1/2} \cdots (\Gamma^{0+})^{s_0+1/2} |\ell\, \omega\, s\rangle$
where $s_a = \begin{cases} +1/2 \text{ if } \Gamma^{a+} \text{ acts on } \ell\cdot\omega\, s \\ -1/2 \text{ if } \Gamma^{a+} \text{ doesn't act on } \ell\cdot\omega\, s \end{cases}$

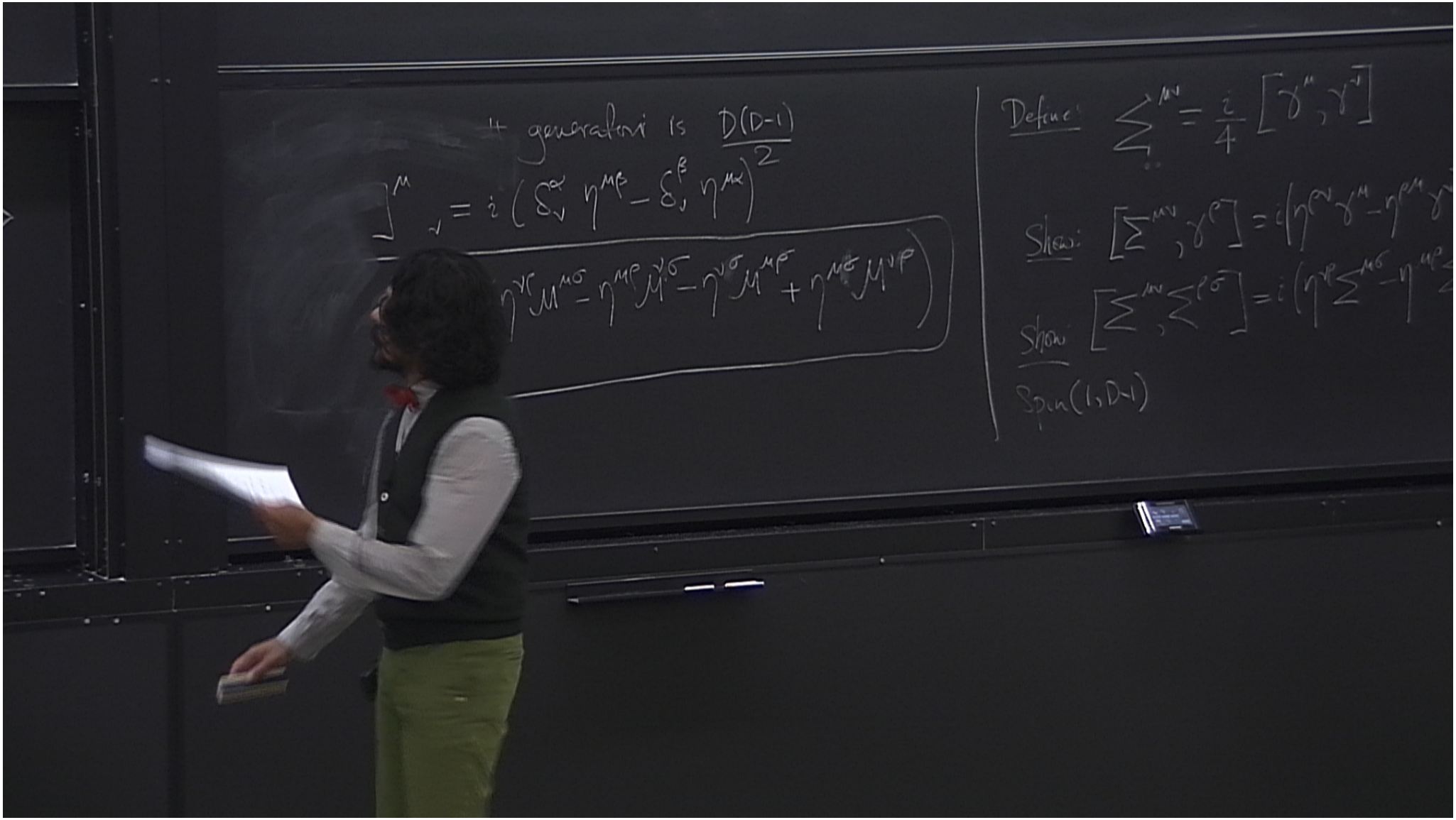

$\psi_{\underline{s}} \equiv |s_0, s_1, \ldots, s_k\rangle$
A Dirac spinor is an element of the vector space
(over $\mathbb{C}$) spanned by $|\underline{s}\rangle$.

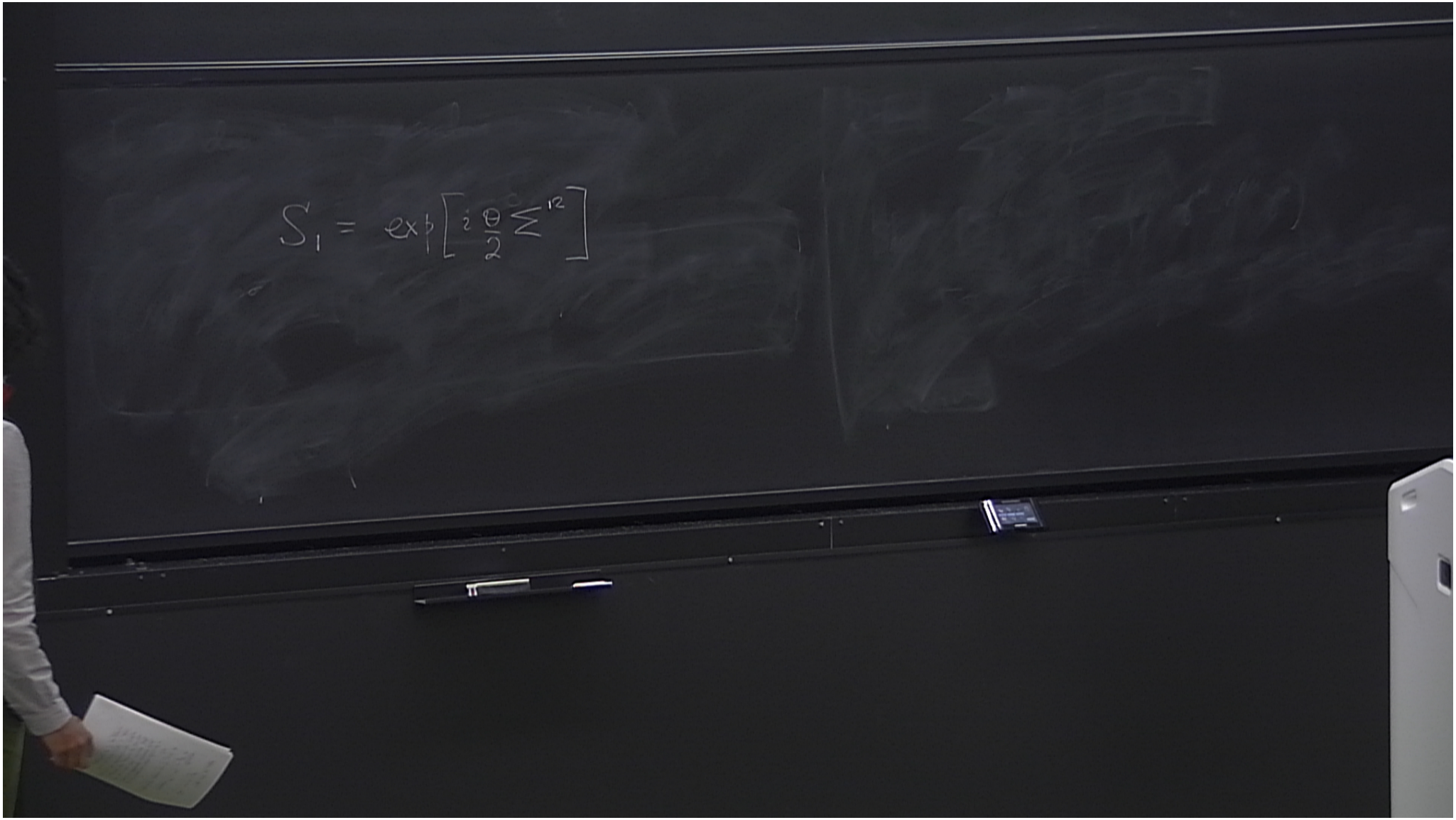

$\psi_{\underline{s}} \equiv |s_0, s_1, \ldots, s_k\rangle$
A Dirac spinor is an element of the vector space
(over $\mathbb{C}$) spanned by $|\underline{s}\rangle$.
Dim $2^{k+1} = 2^{D/2}$

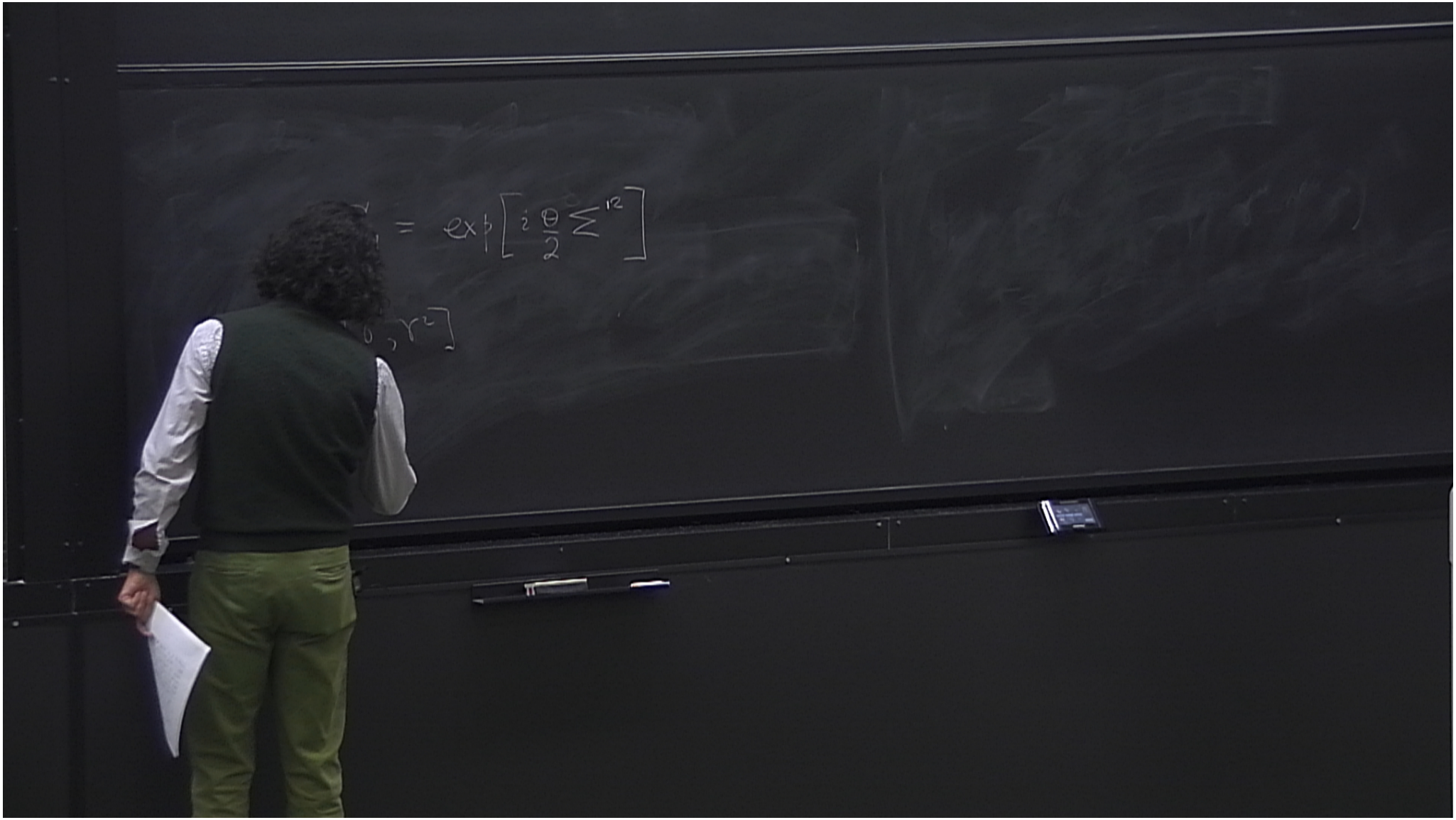

Spin(1,D-1)
Comment: Lorentz gp is non-compact.

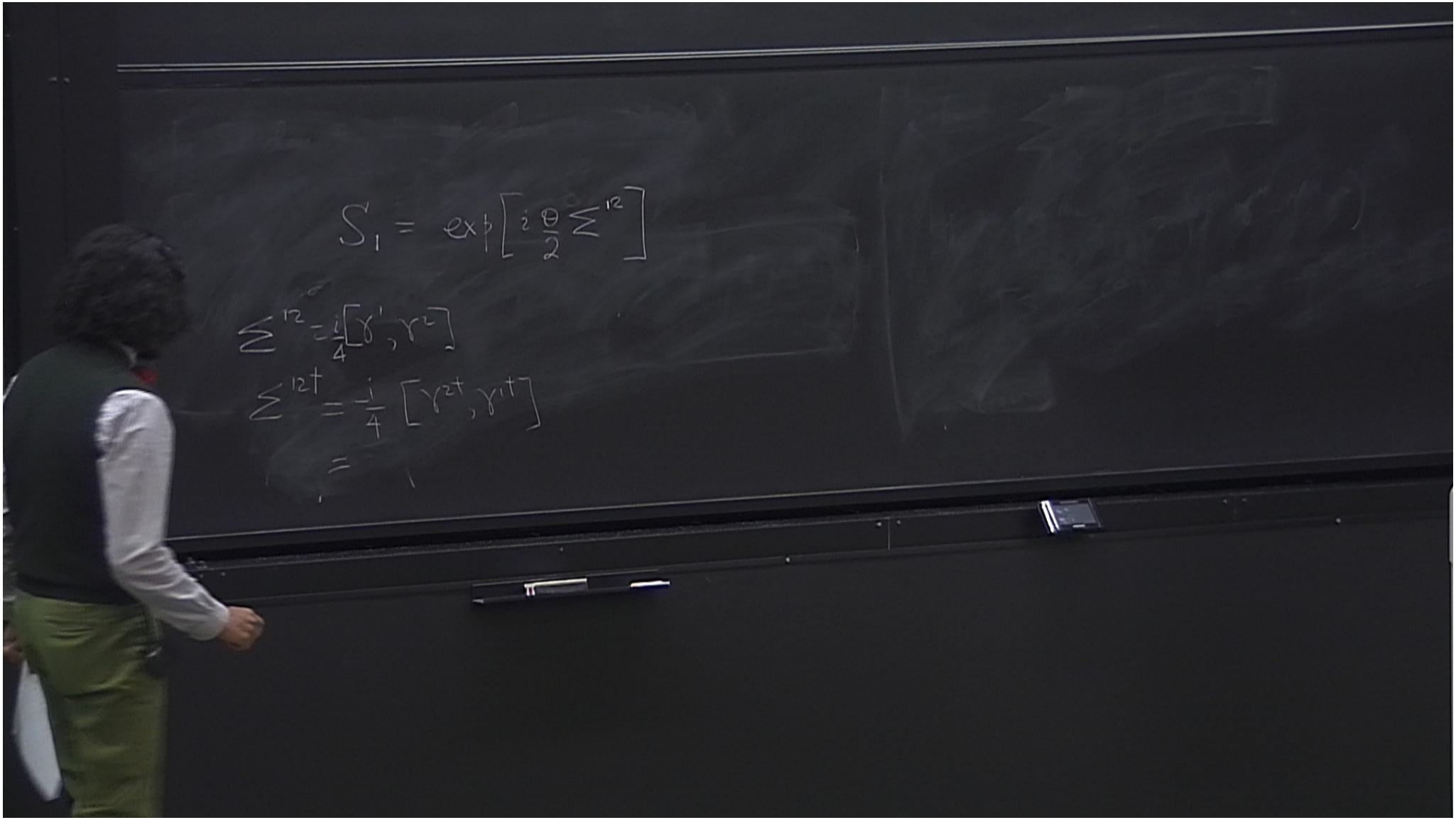

Comment: Lorentz gp is non-compact. No finite
of the vector space
|3⟩.

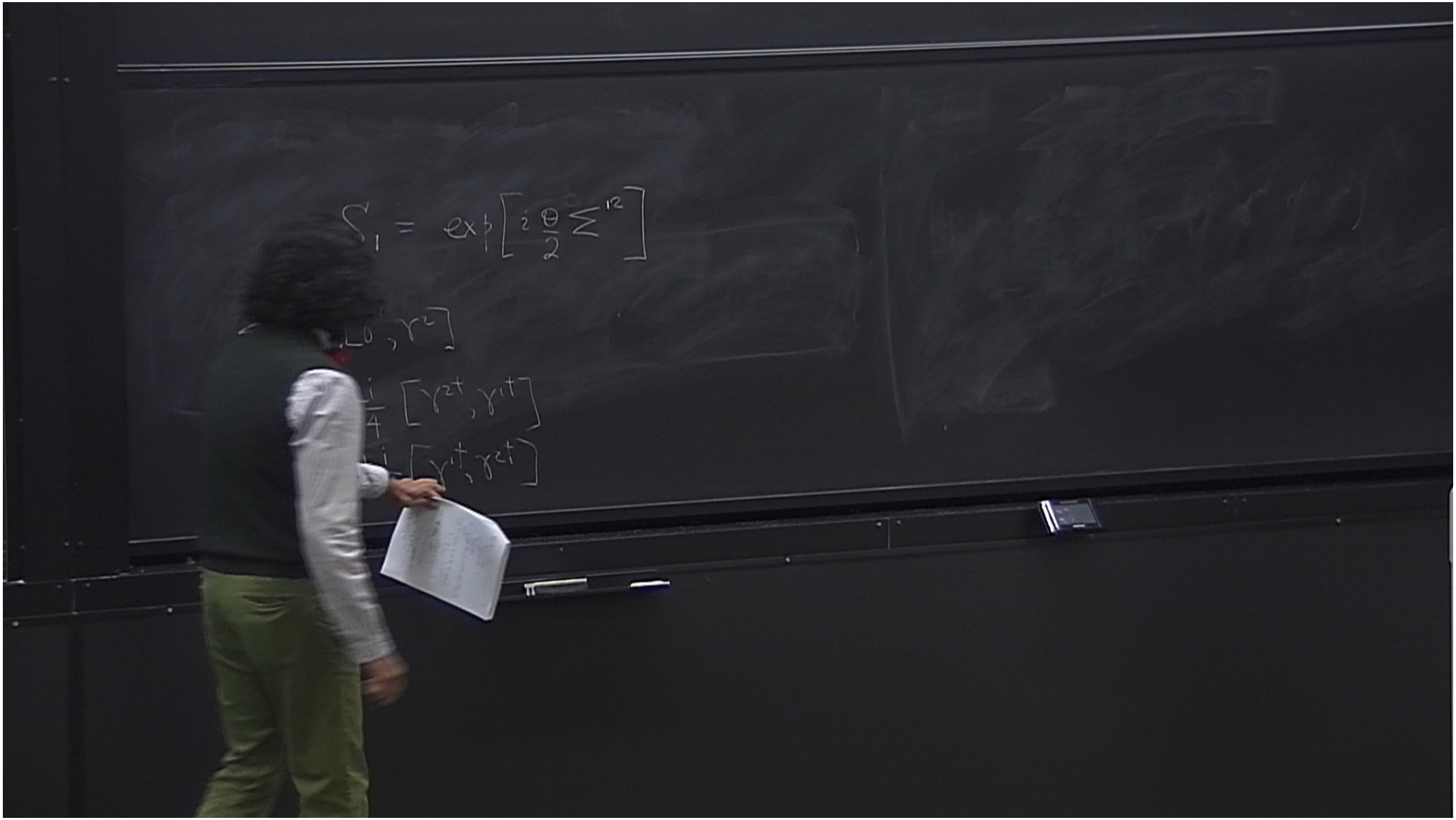


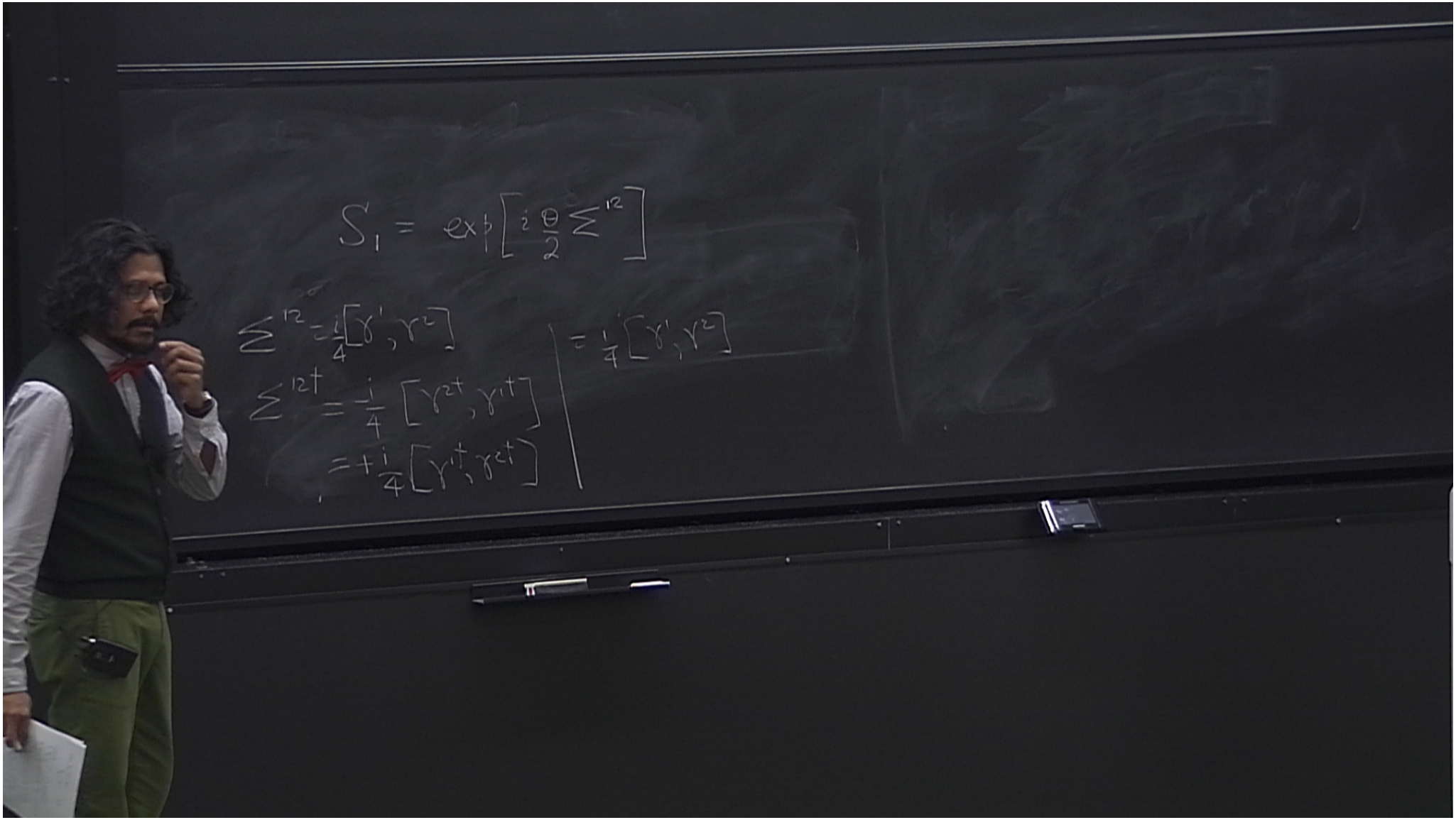

+ generators is D(D-1)/2
Define:
Show:
Show:
Spin(1,D-1)

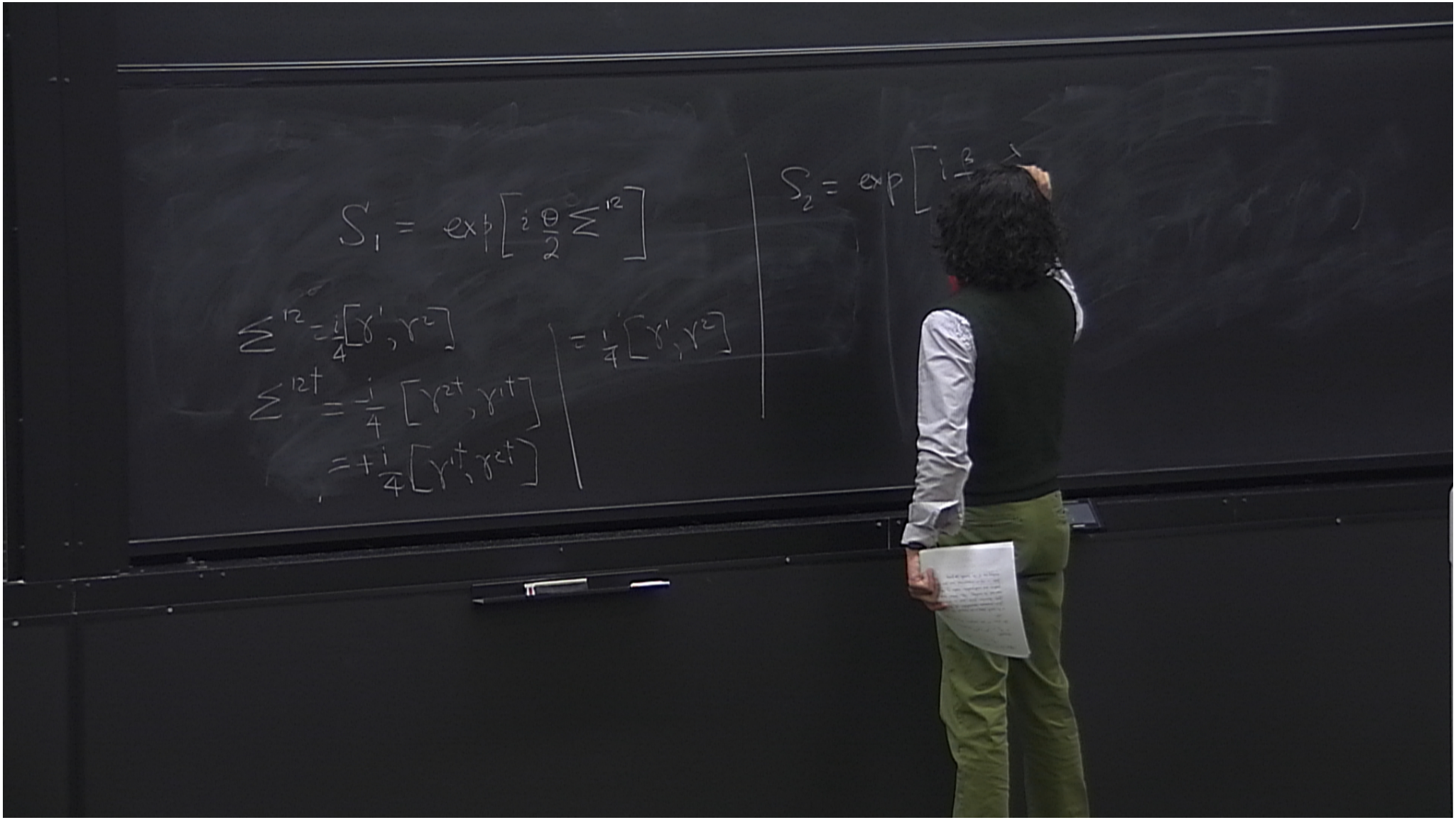

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$

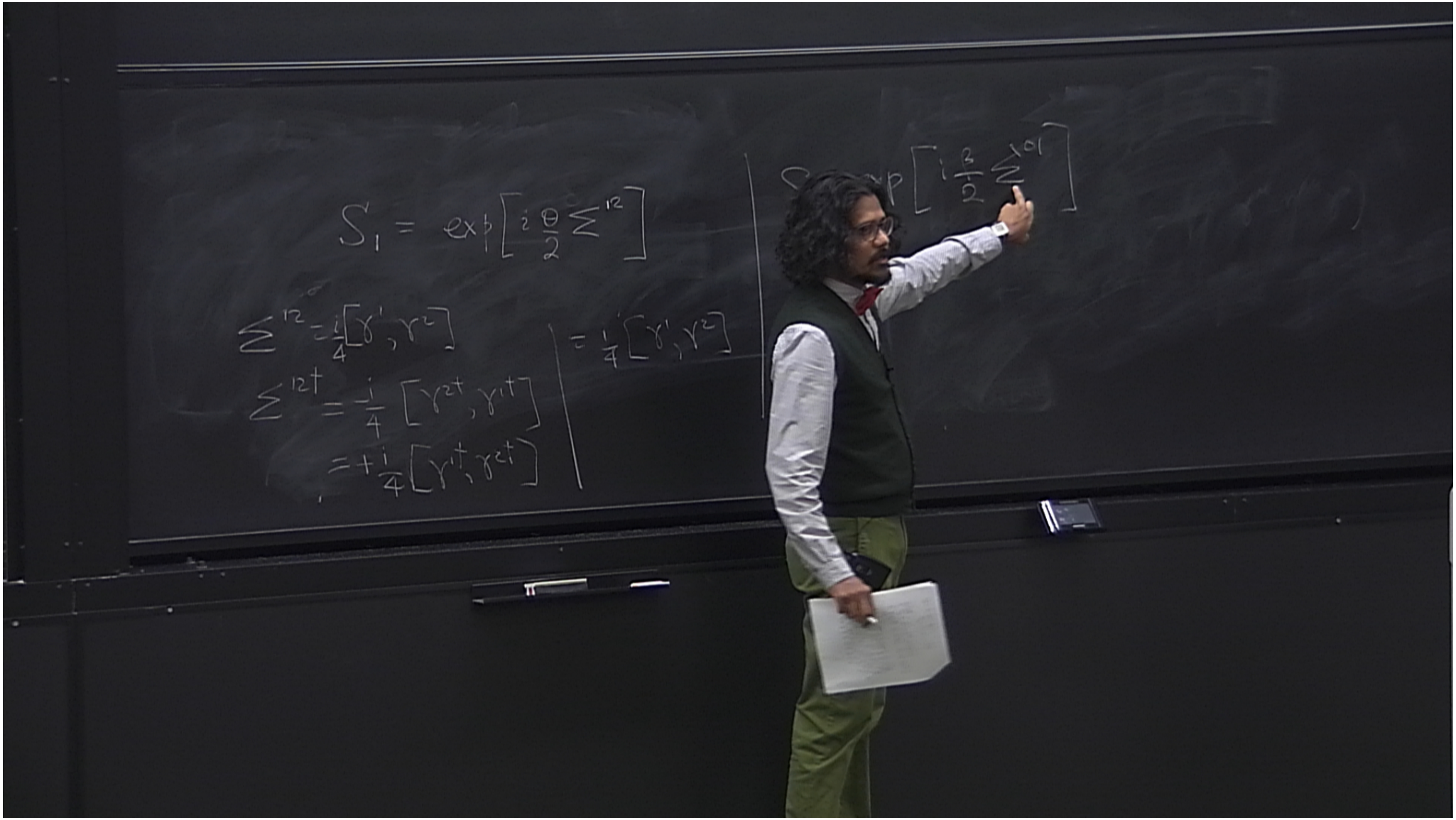

$= \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$

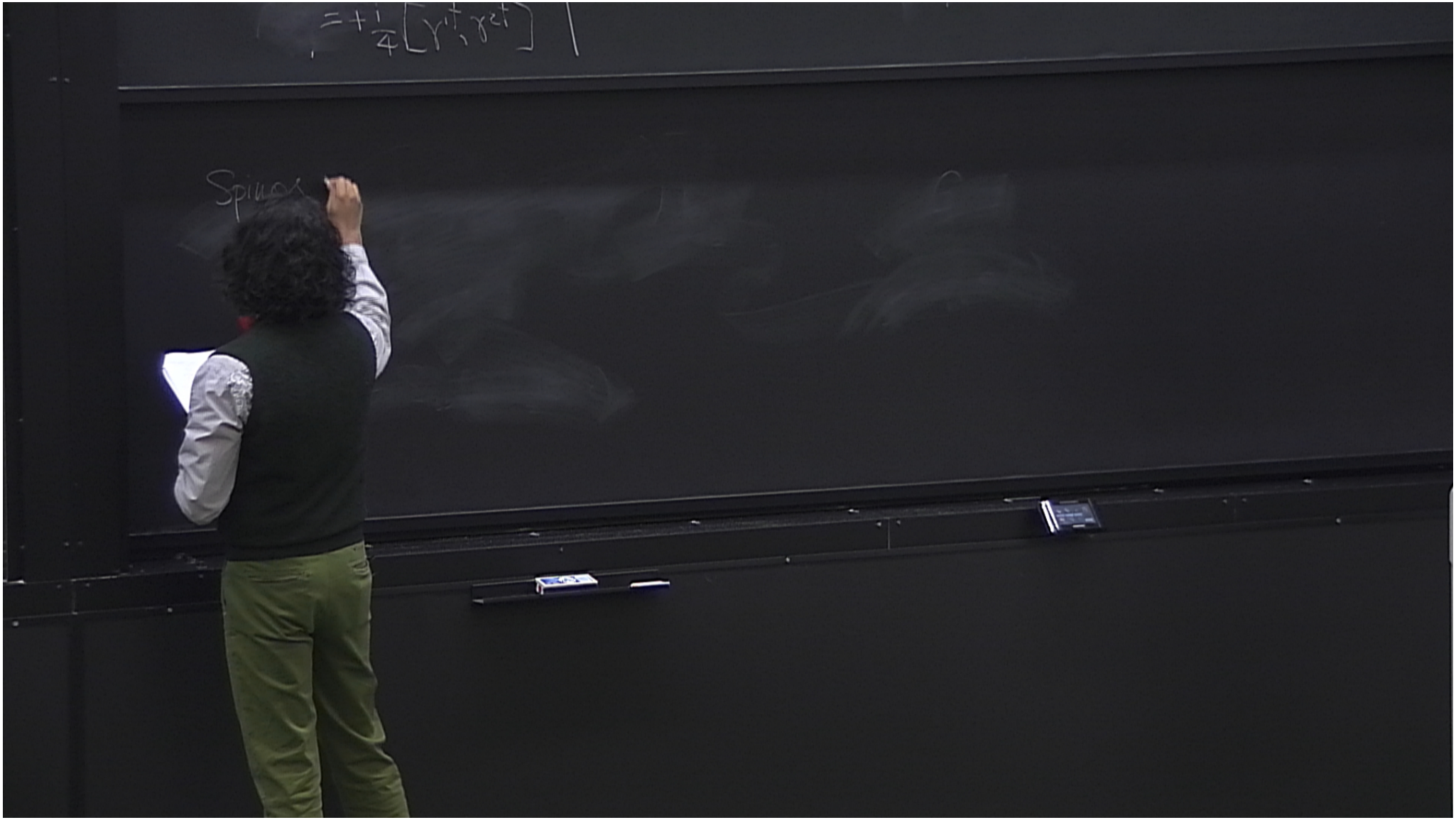

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$
$\Sigma^{12} = \frac{i}{4}[\gamma^1, \gamma^2]$
$\Sigma^{12\dagger} = -\frac{i}{4}[\gamma^{2\dagger}, \gamma^{1\dagger}]$
$=$

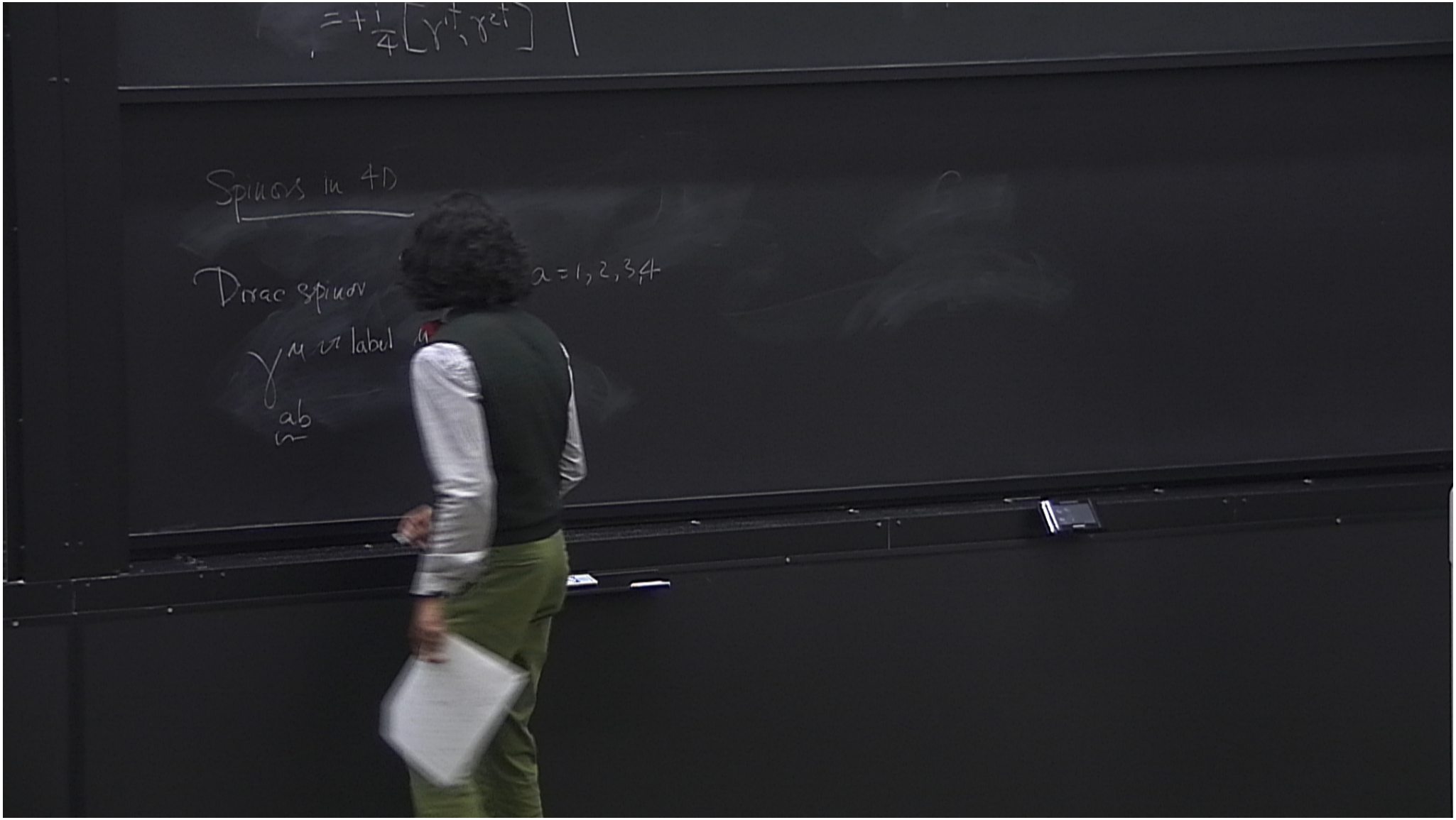

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$
$\frac{i}{4}[\gamma^2, \gamma^1]$

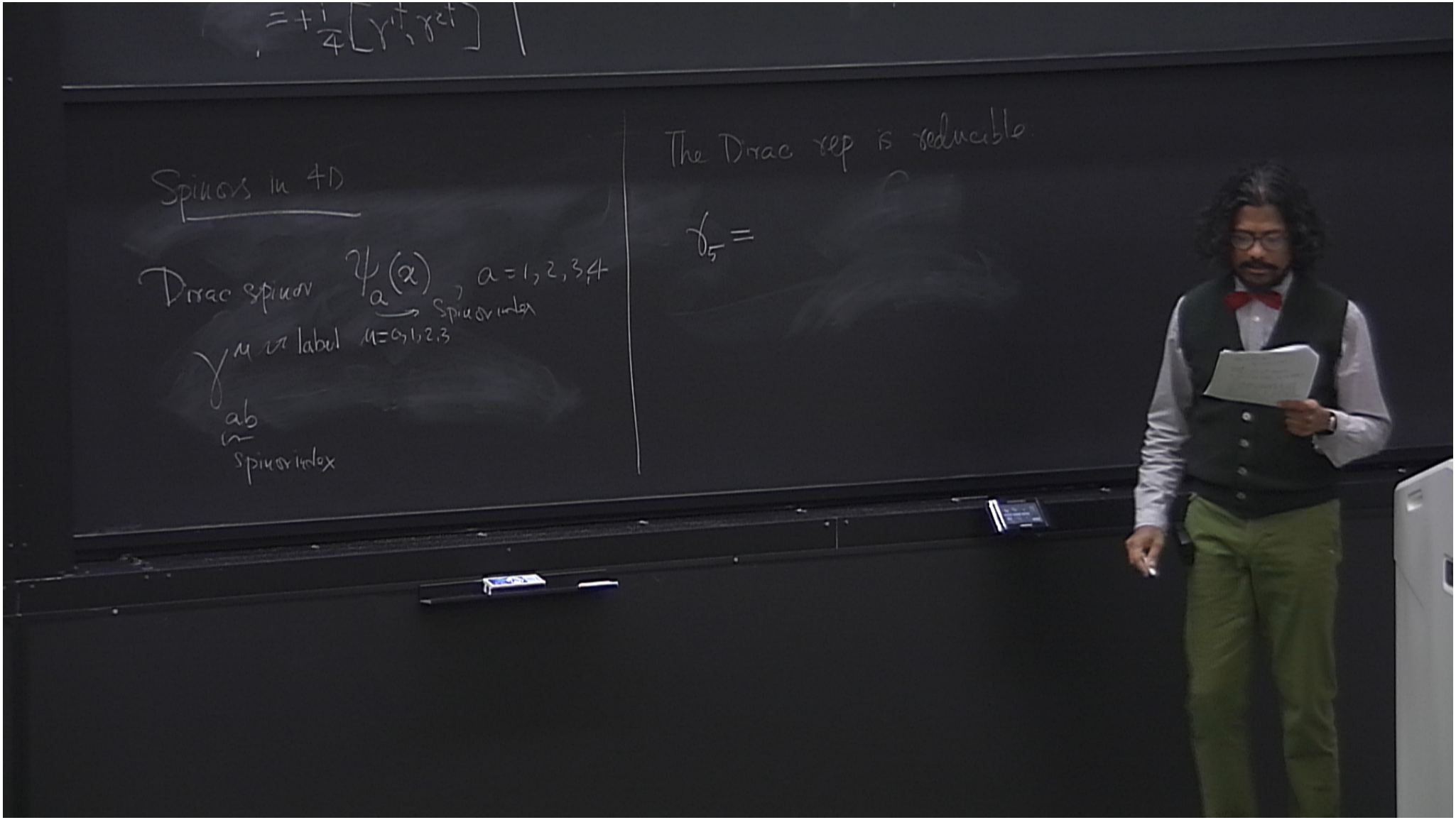

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$
$\Sigma^{12} = \frac{i}{4}[\gamma^1, \gamma^2]$
$\Sigma^{12\dagger} = -\frac{i}{4}[\gamma^{2\dagger}, \gamma^{1\dagger}]$
$= +\frac{i}{4}[\gamma^{1\dagger}, \gamma^{2\dagger}]$
$= \frac{i}{4}[\gamma^1, \gamma^2]$

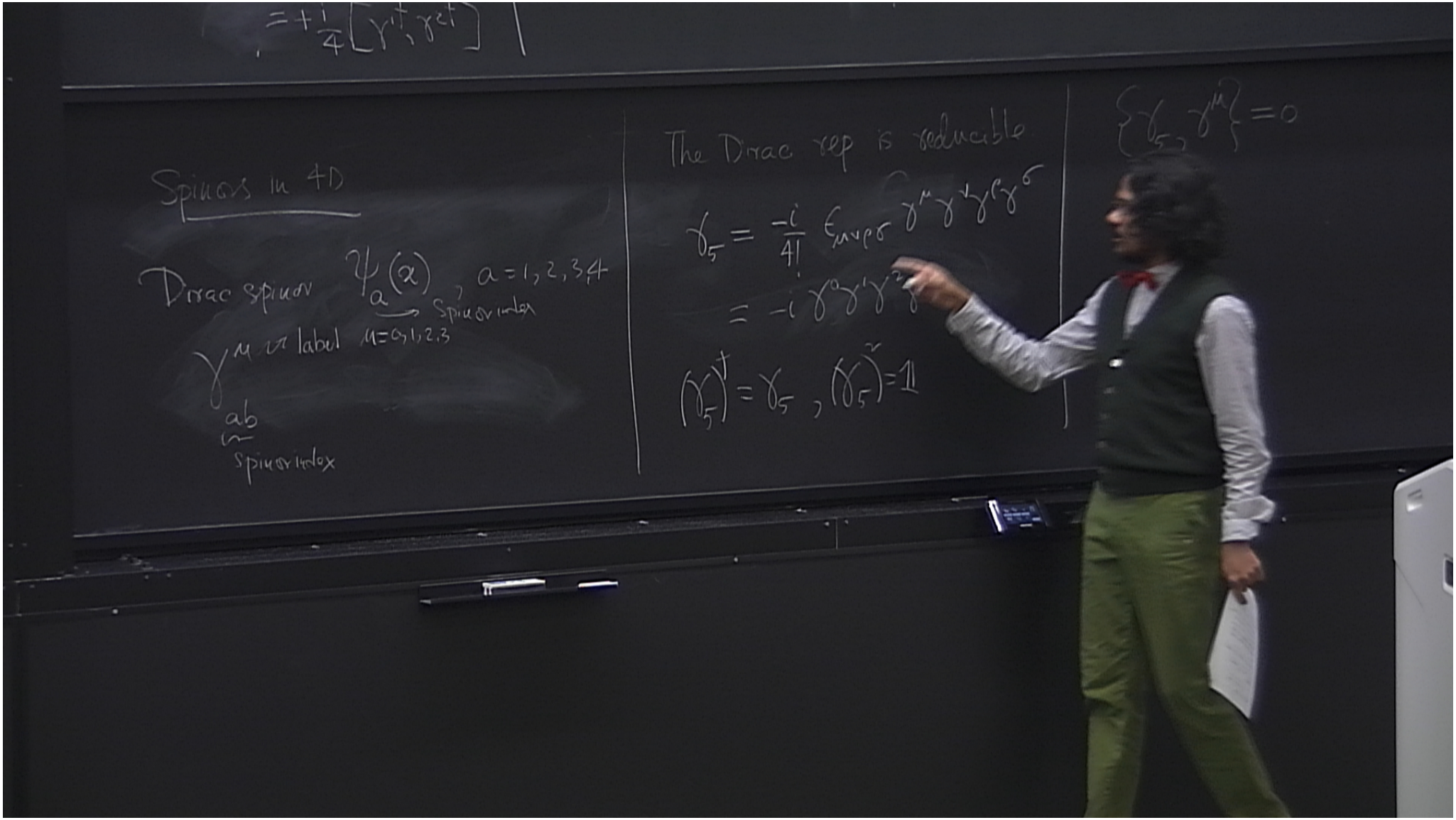

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$
$S_2 = \exp\left[i\frac{3}{}\right.$
$\Sigma^{12} = \frac{i}{4}[\gamma^1, \gamma^2]$
$= \frac{i}{4}[\gamma^1, \gamma^2]$
$\Sigma^{12\dagger} = \frac{-i}{4}[\gamma^{2\dagger}, \gamma^{1\dagger}]$
$= +\frac{i}{4}[\gamma^{1\dagger}, \gamma^{2\dagger}]$

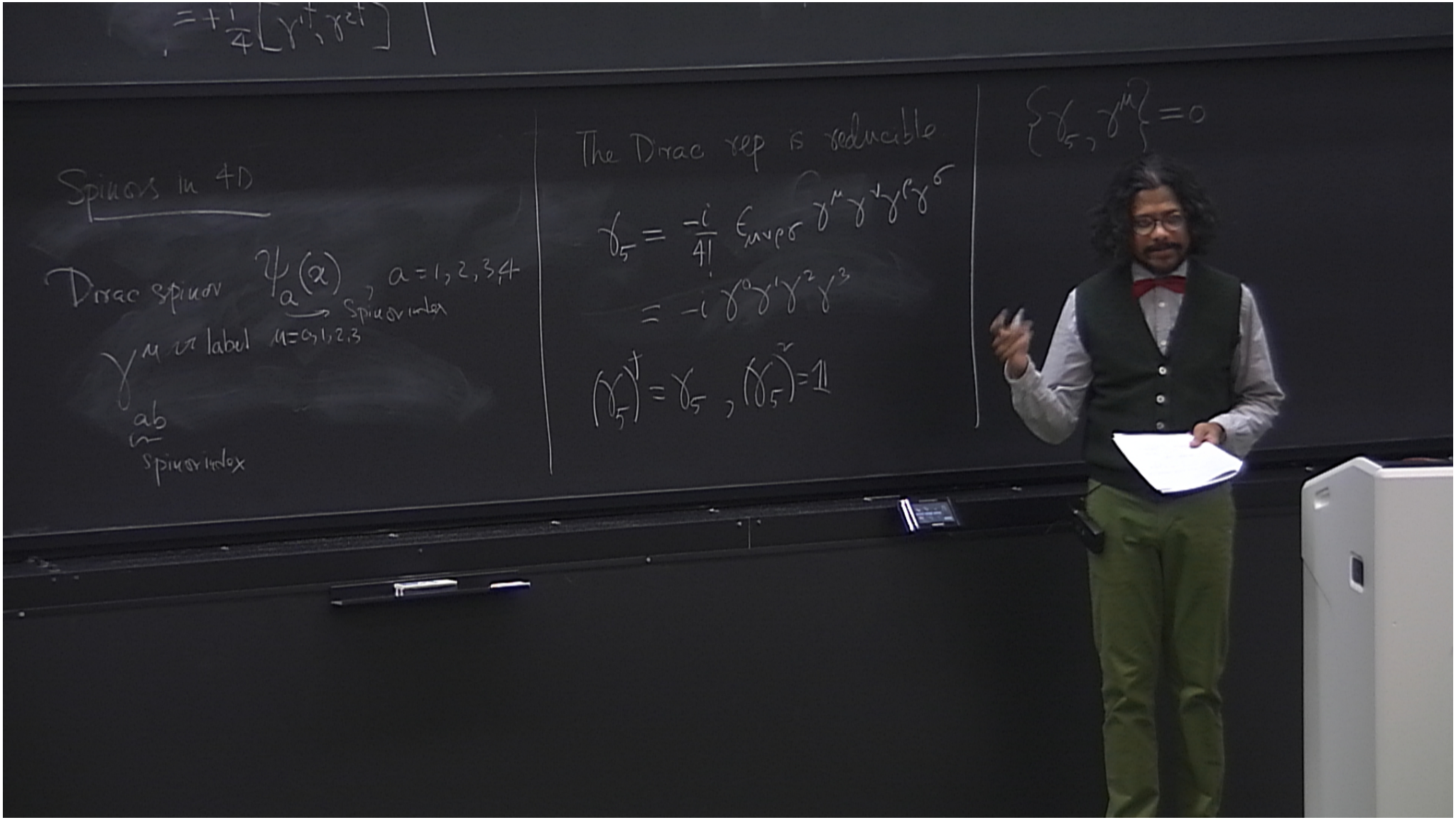

$S_1 = \exp\left[i\frac{\theta}{2}\Sigma^{12}\right]$
$\Sigma^{12} = \frac{i}{4}[\gamma^1, \gamma^2]$
$\Sigma^{12\dagger} = -\frac{i}{4}[\gamma^{2\dagger}, \gamma^{1\dagger}]$
$= +\frac{i}{4}[\gamma^{1\dagger}, \gamma^{2\dagger}]$
$= \frac{1}{4}[\gamma^1, \gamma^2]$
$\exp\left[i\frac{3}{2}\Sigma^{01}\right]$

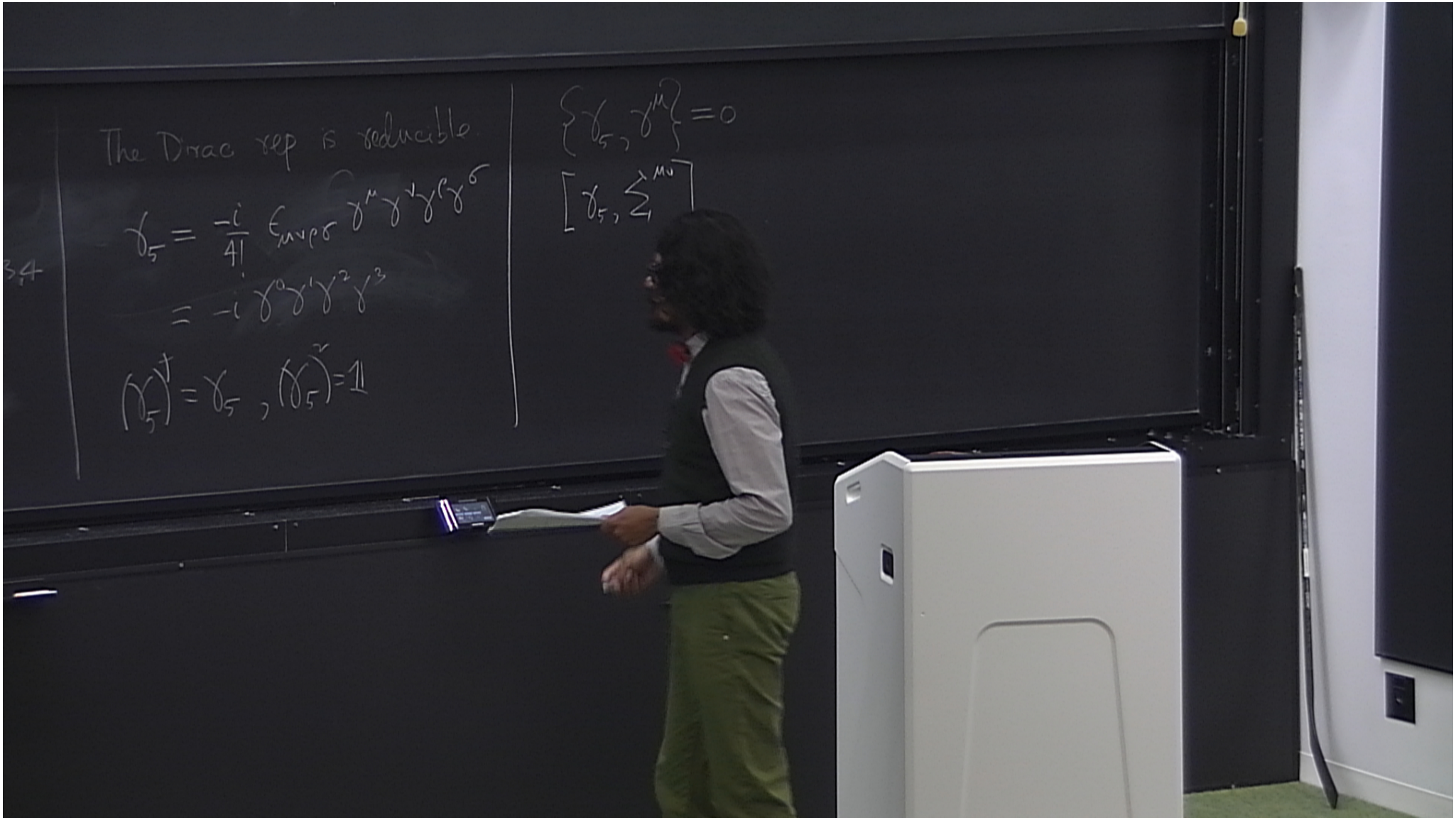

Spinors

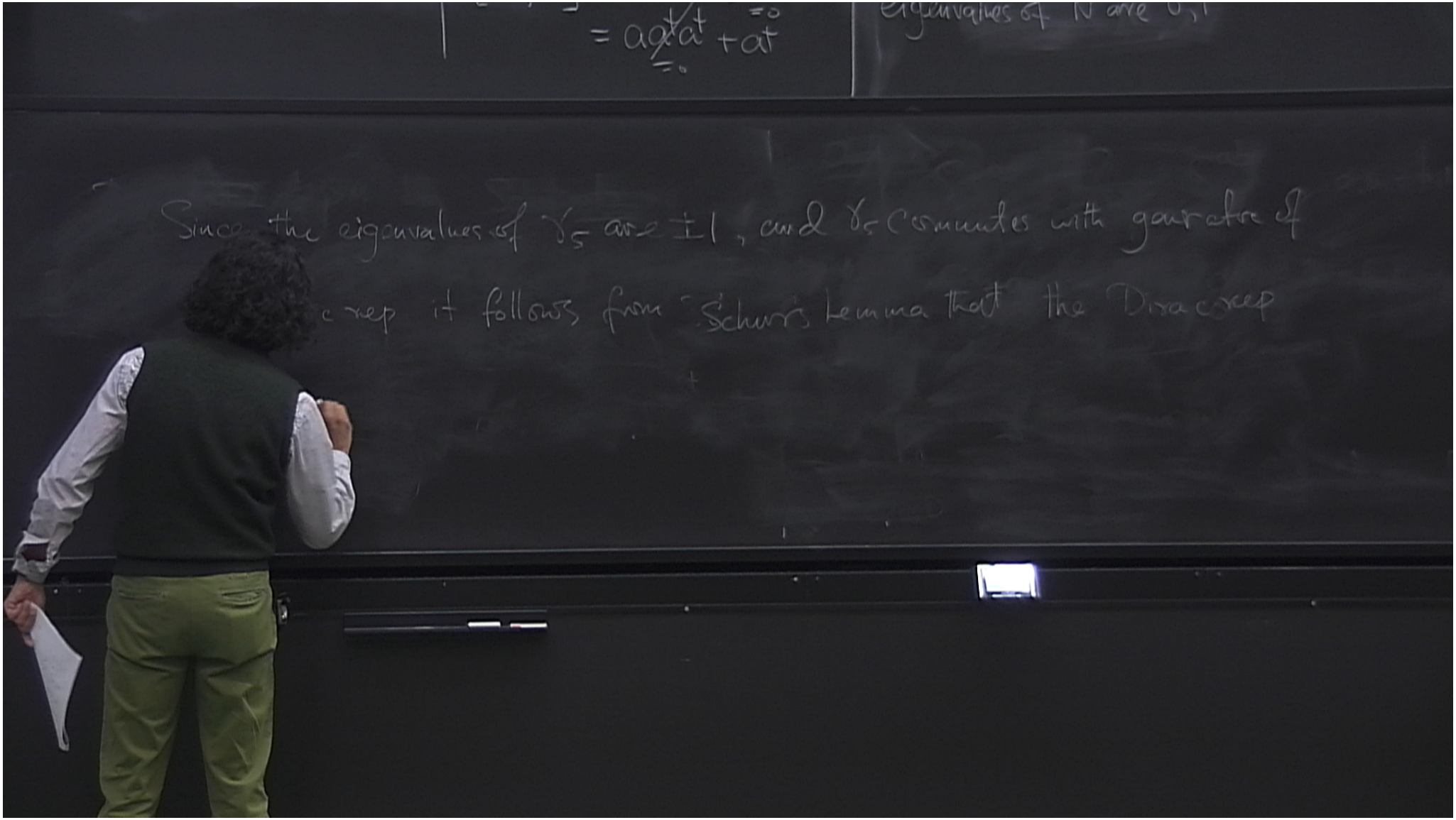

Spinors in 4D
Dirac spinor
α = 1, 2, 3, 4
γ^μ ν label
ab

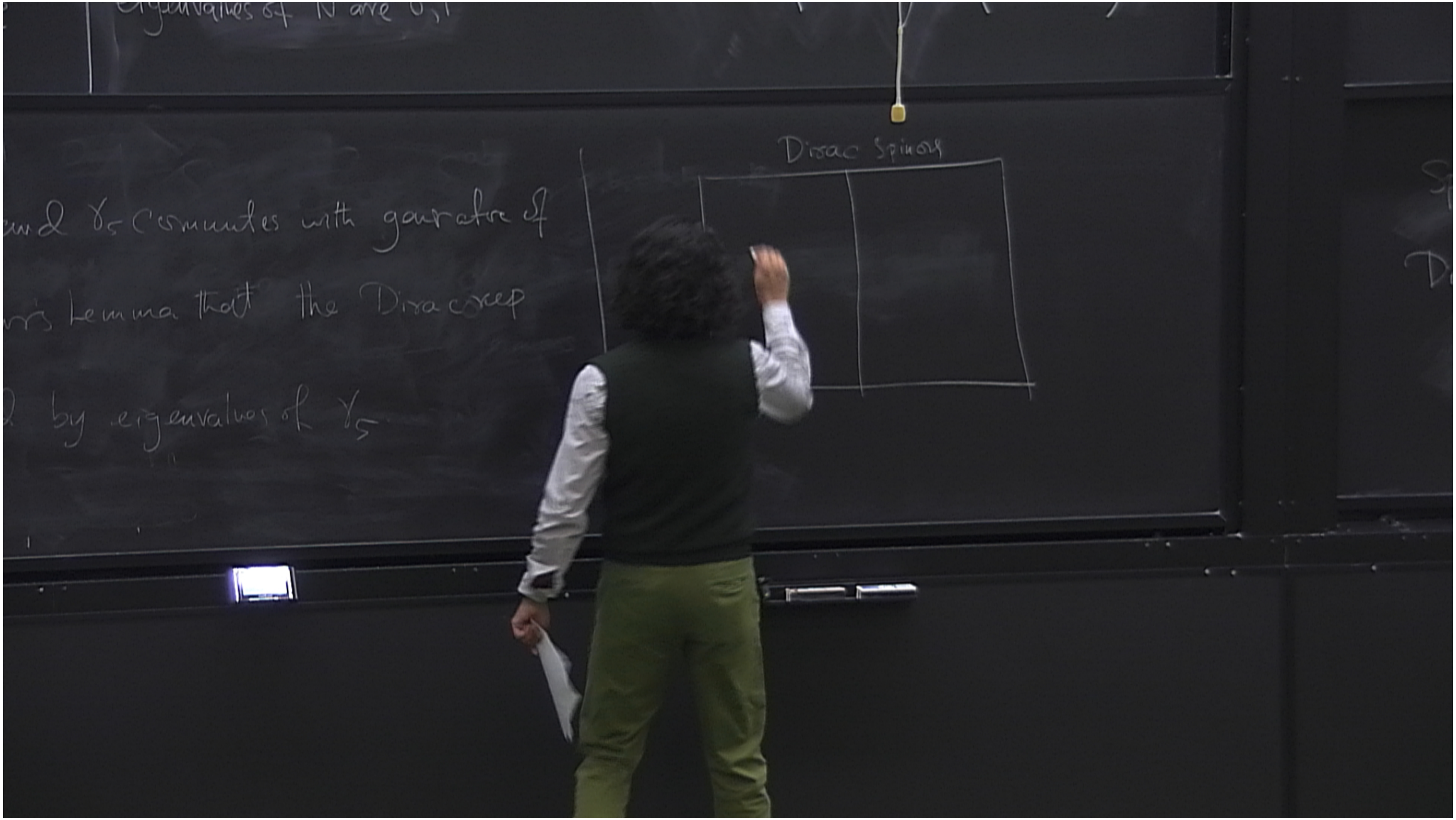

$= +\frac{1}{4}[\gamma^\mu, \gamma^\nu]$
Spinors in 4D
Dirac spinor $\psi_a(x)$, $a = 1,2,3,4$
$a \rightarrow$ Spinor index
$\gamma^\mu \rightarrow$ label $\mu = 0,1,2,3$
ab
$\sim$
Spinor index
The Dirac rep is reducible
$\gamma_5 =$

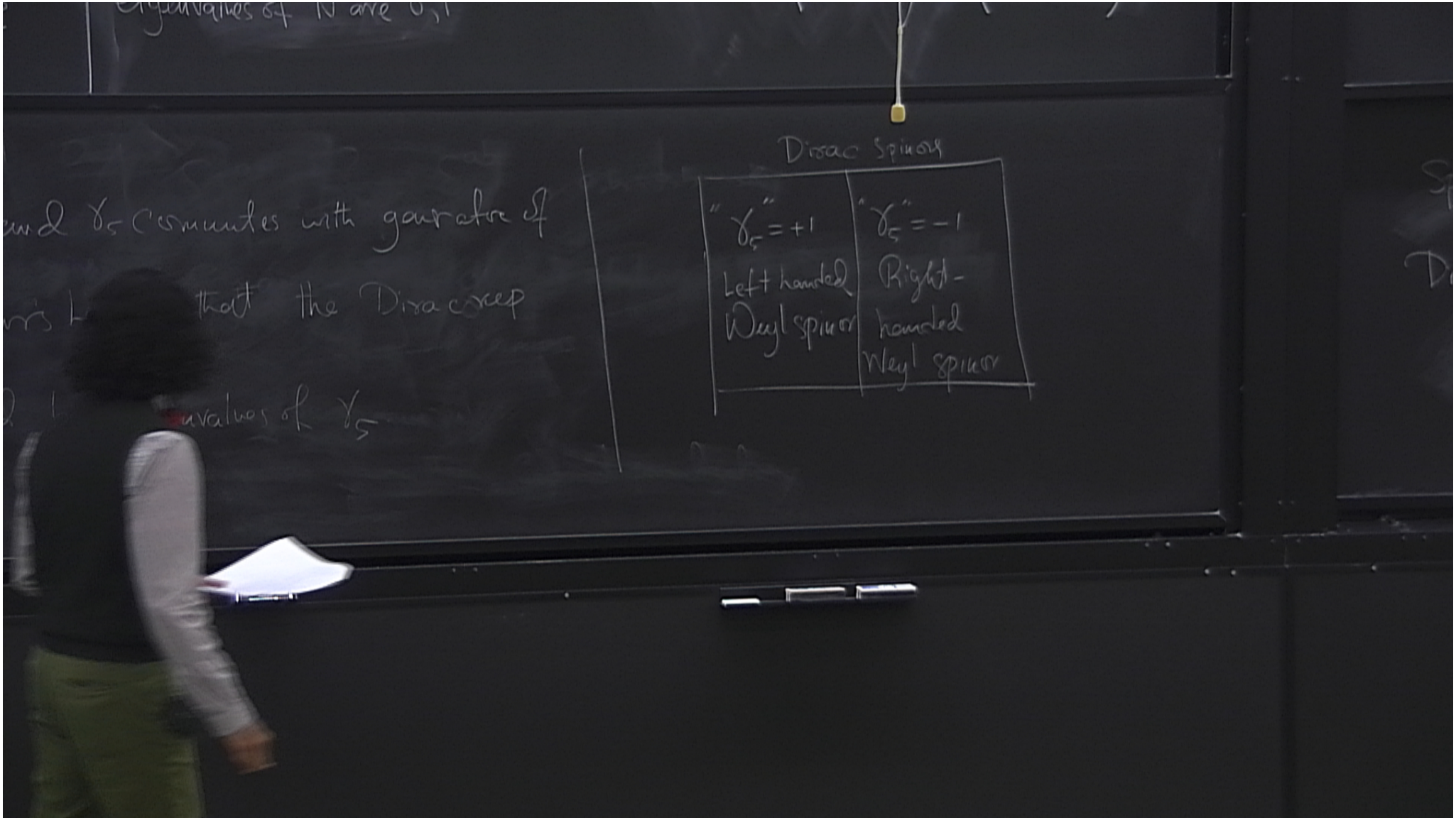

$= +\frac{1}{4}[\gamma^\mu, \gamma^\nu]$
Spinors in 4D
Dirac spinor $\psi_a(x)$, $a = 1,2,3,4$
Spinor index
γ^μ ~ label $\mu = 0,1,2,3$
γ^{ab}
Spinor index
The Dirac rep is reducible
$\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
$= -i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma_5)^\dagger = \gamma_5$, $(\gamma_5)^2 = \mathbb{1}$
$\{\gamma_5, \gamma^\mu\} = 0$

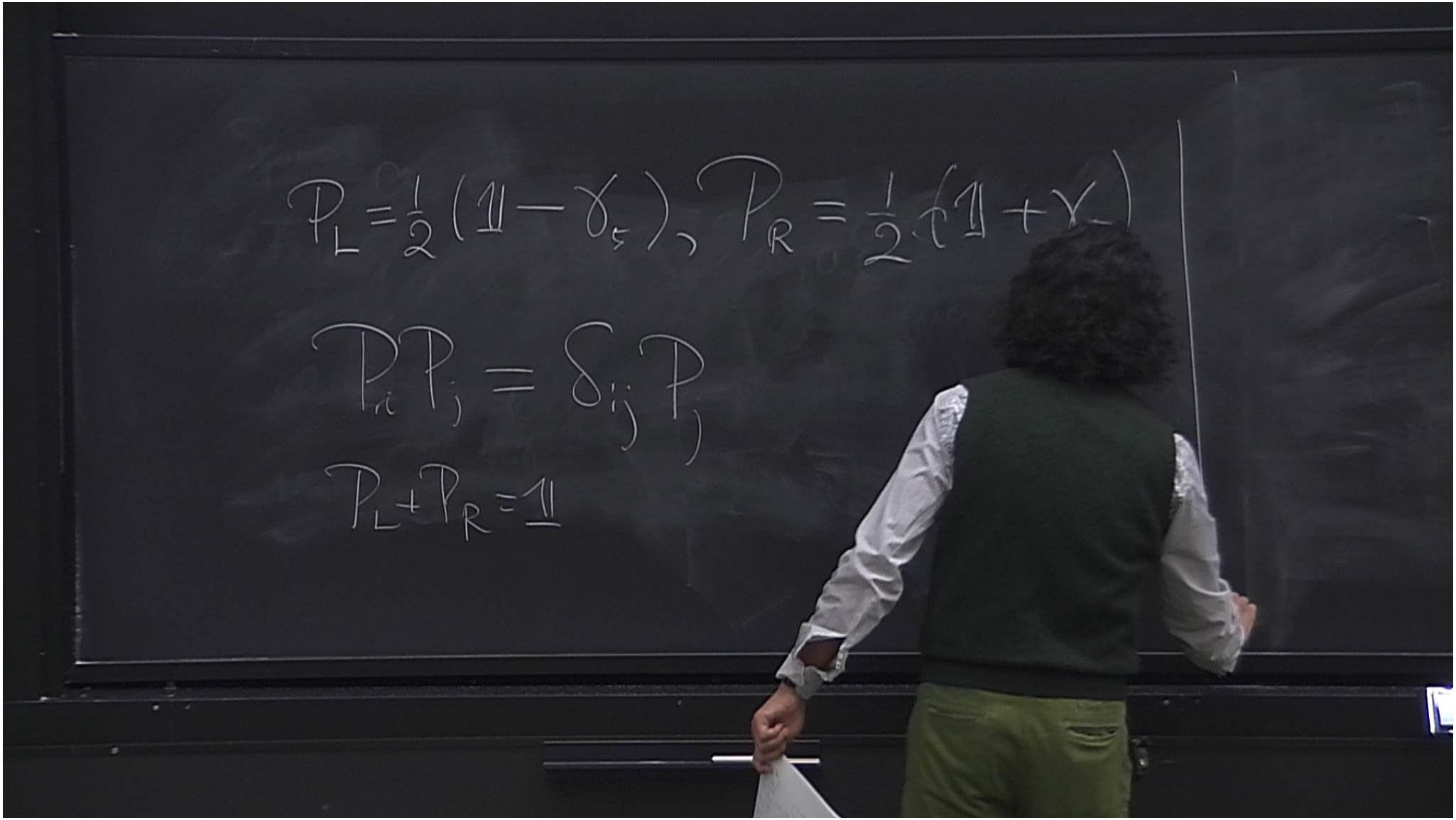

$= +\frac{i}{4}[\gamma^\mu, \gamma^\nu]$
Spinors in 4D
Dirac spinor $\psi_a(x)$, $a = 1,2,3,4$
$a \rightarrow$ Spinor index
γ^μ : label $\mu = 0,1,2,3$
γ^{ab}
ab : Spinor index
The Dirac rep is reducible
$\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
$= -i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma_5)^\dagger = \gamma_5$, $(\gamma_5)^2 = \mathbb{1}$
$\{\gamma_5, \gamma^\mu\} = 0$

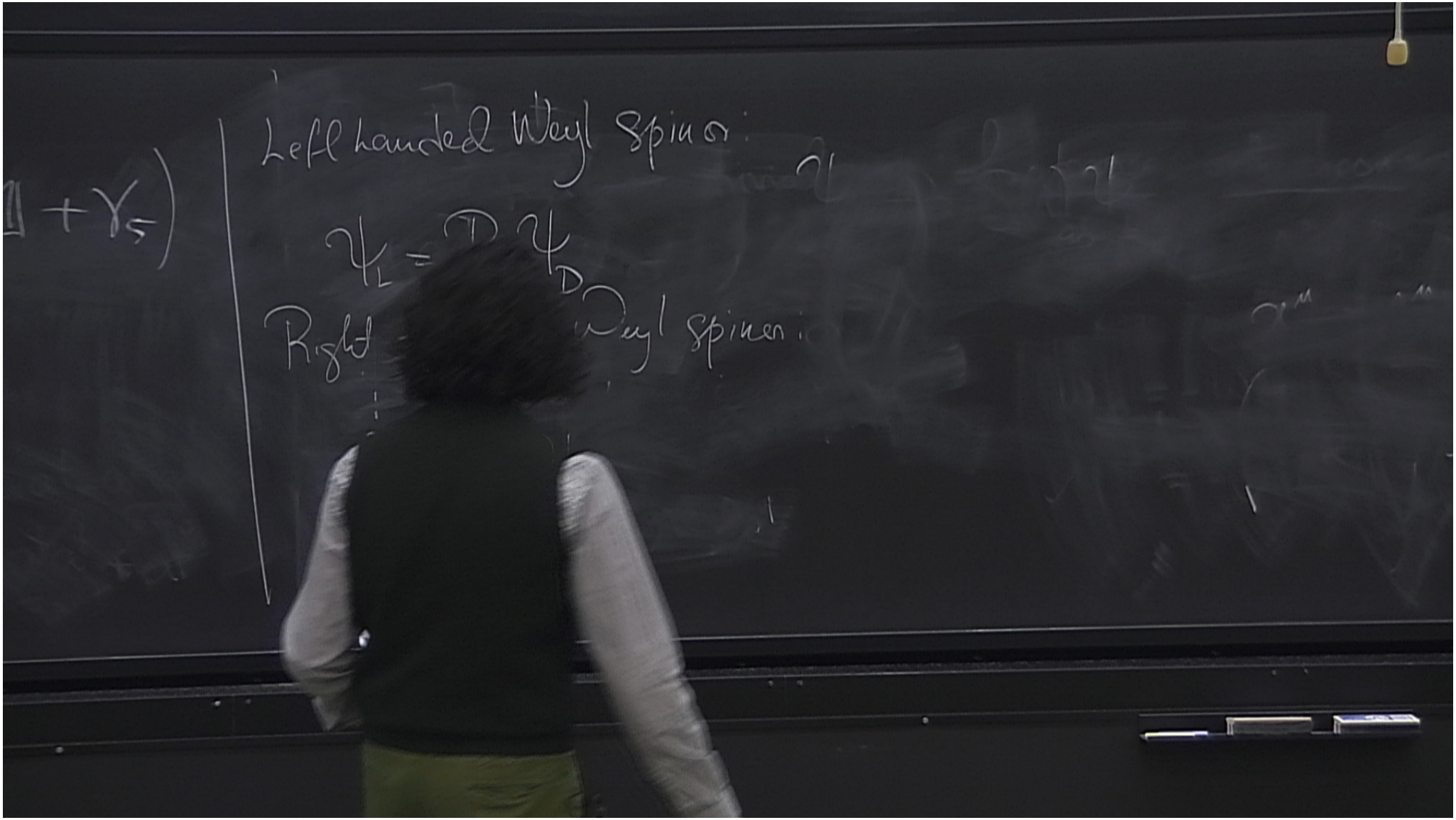

The Dirac rep is reducible
$\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
$= -i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma_5)^\dagger = \gamma_5$, $(\gamma_5)^2 = \mathbb{1}$
$\{\gamma_5, \gamma^\mu\} = 0$
$[\gamma_5, \Sigma^{\mu\nu}]$

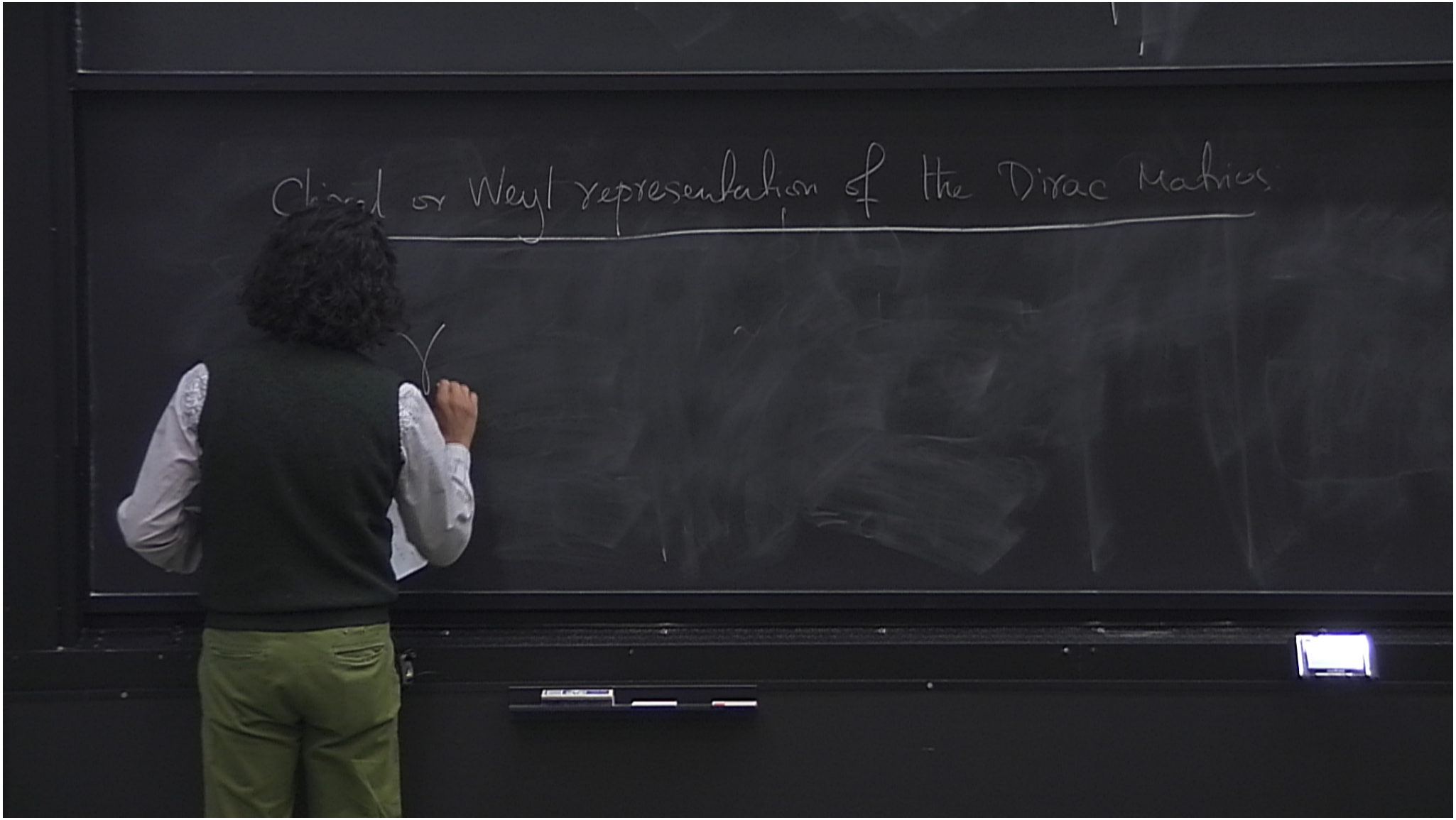

$= a\underline{\dot{a}^t} + a\dot{a}^t$
$=0$
$=0$
eigenvalues of N are 0,1
Since the eigenvalues of γ_5 are ± 1, and γ_5 commutes with gauratee of
c rep it follows from Schur's Lemma that the Dirac rep

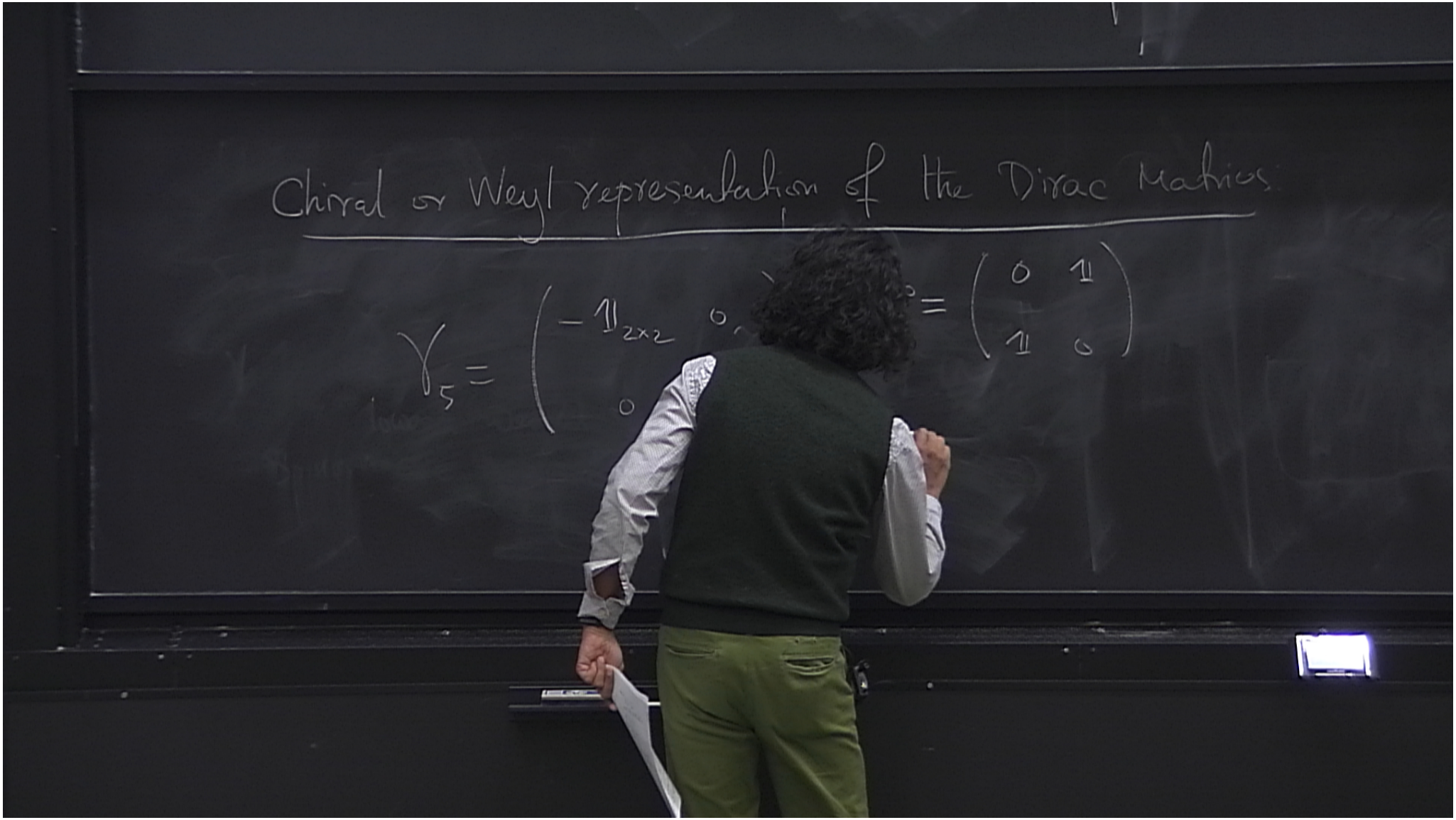

Dirac Spinors

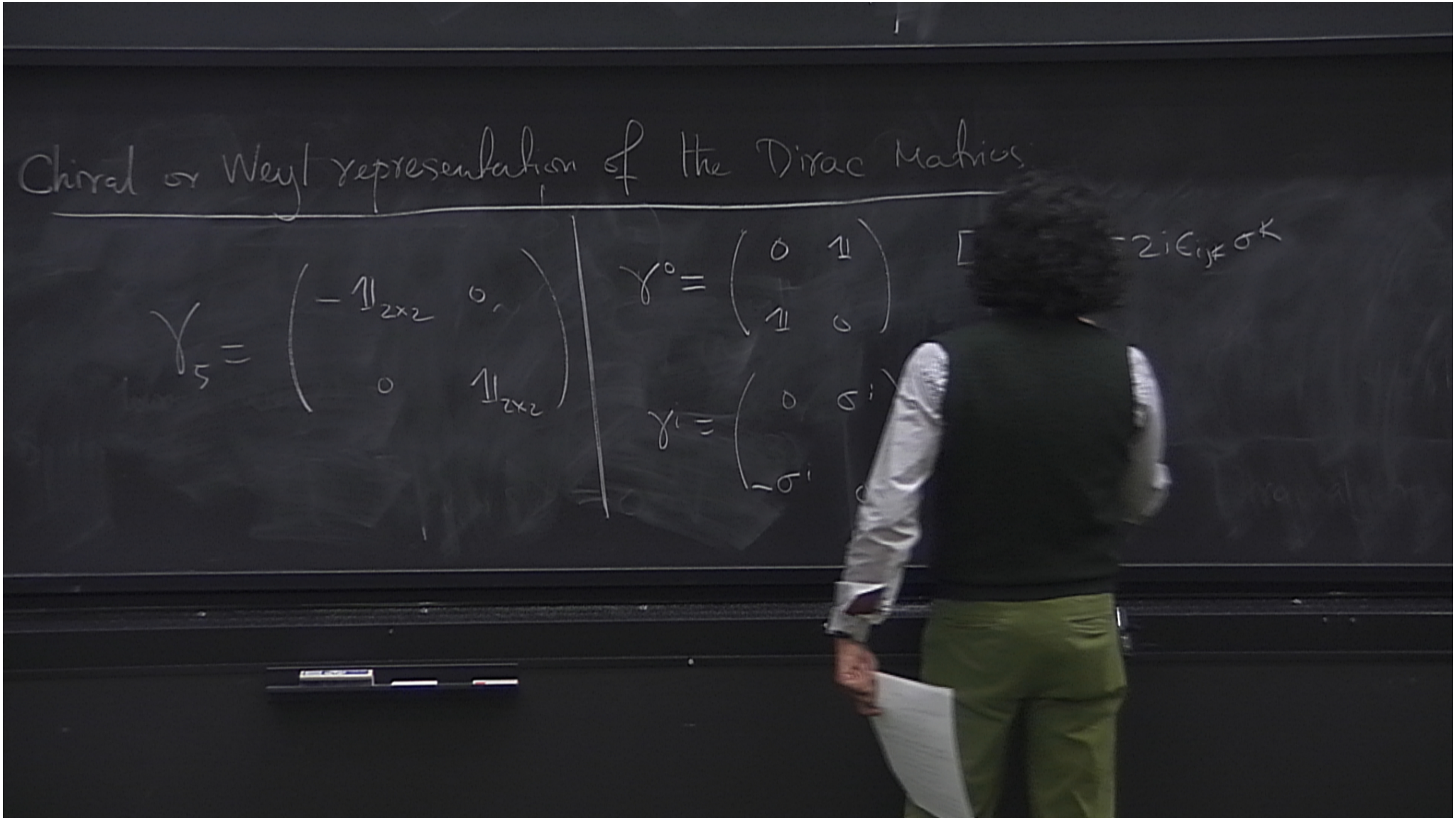

Dirac Spinors
"γ_5" = +1
Left handed Weyl spinor
"γ_5" = −1
Right-handed Weyl spinor

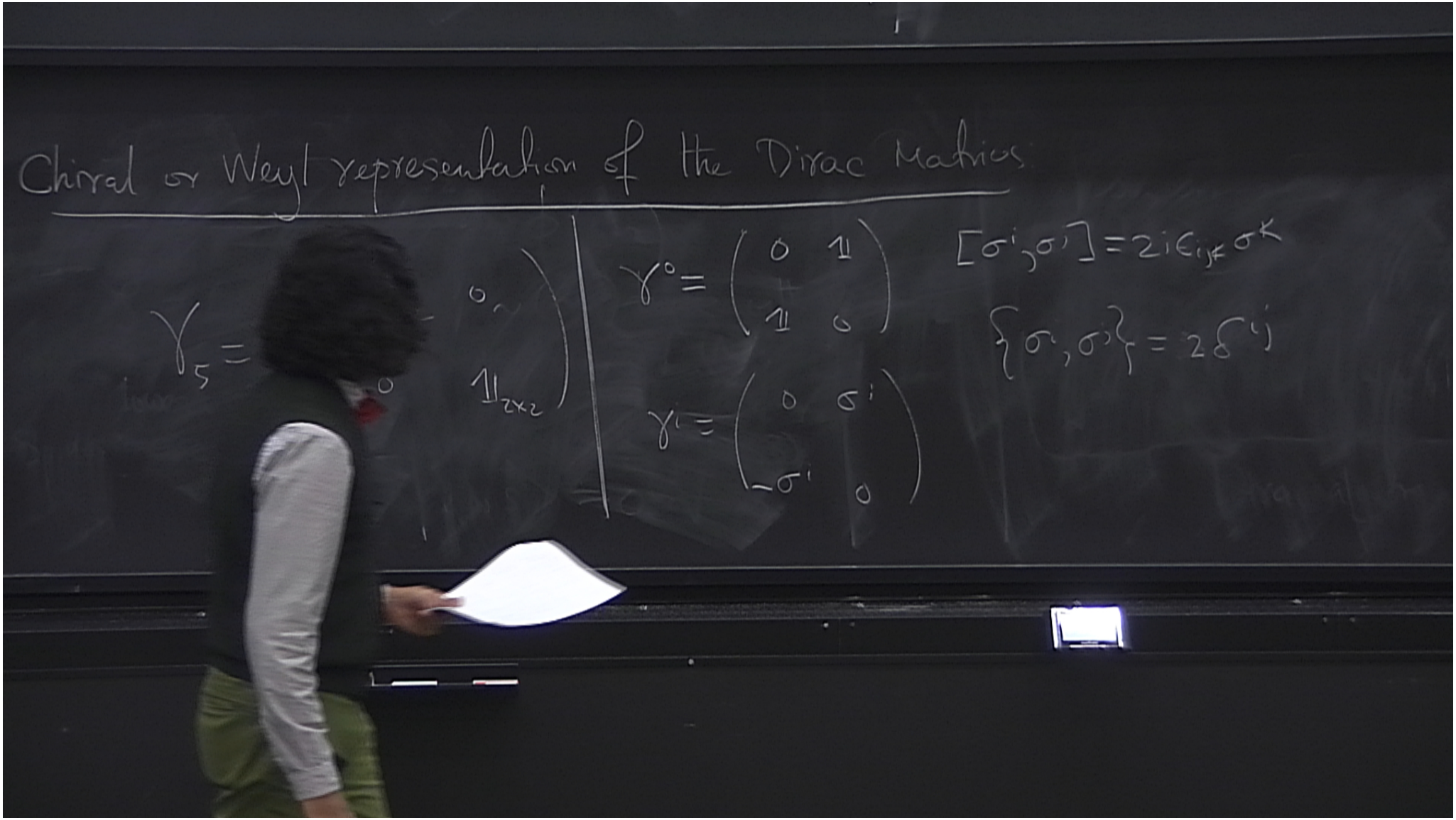

$P_L = \frac{1}{2}(\mathbb{1} - \gamma_5)$, $P_R = \frac{1}{2}(\mathbb{1} + \gamma_5)$
$P_i P_j = \delta_{ij} P_j$
$P_L + P_R = \mathbb{1}$

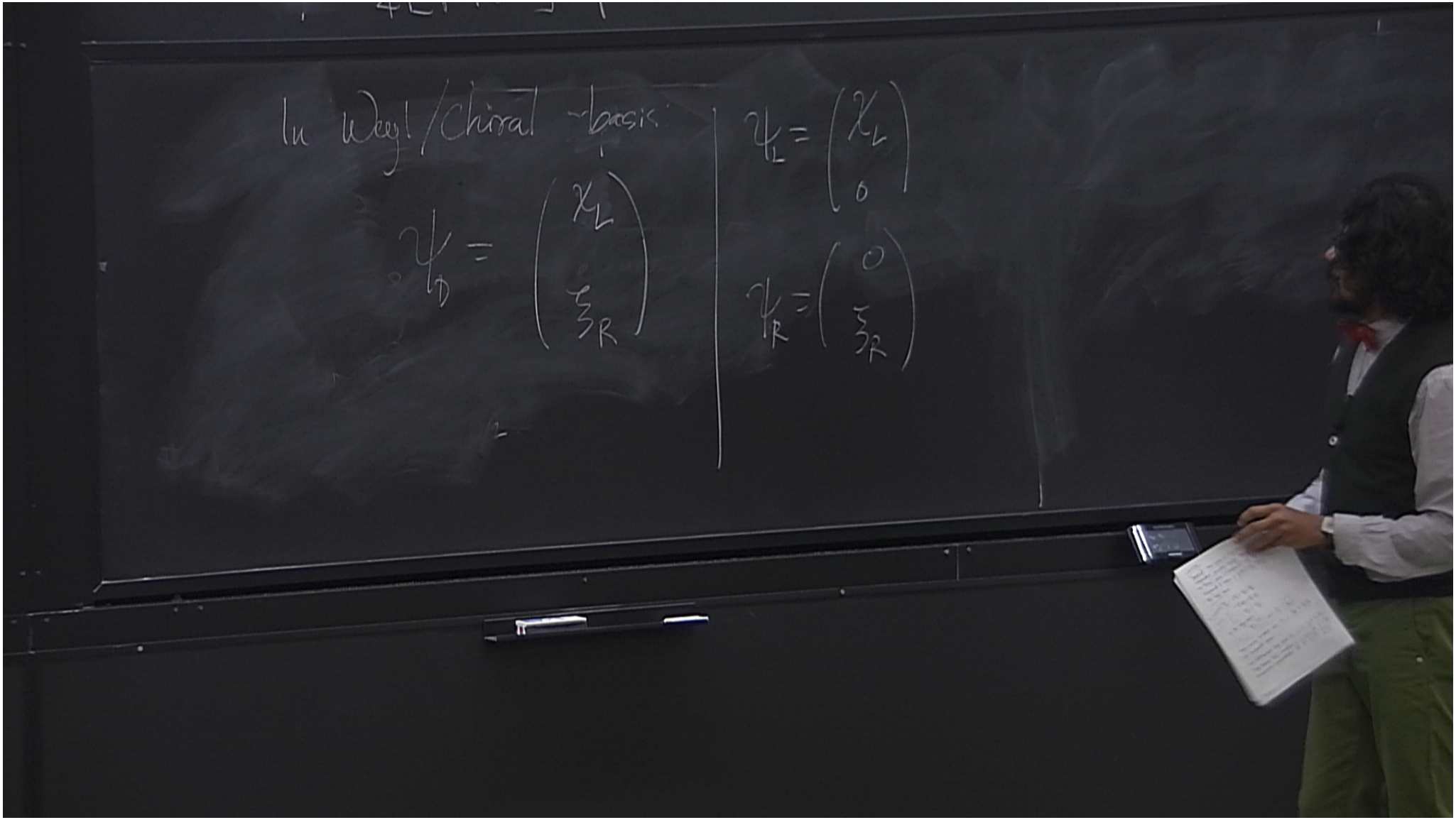

$(1+\gamma_5)$
Left handed Weyl spinor:
$\psi_L = \mathcal{P}\,\psi_D$
Right
Weyl spinor

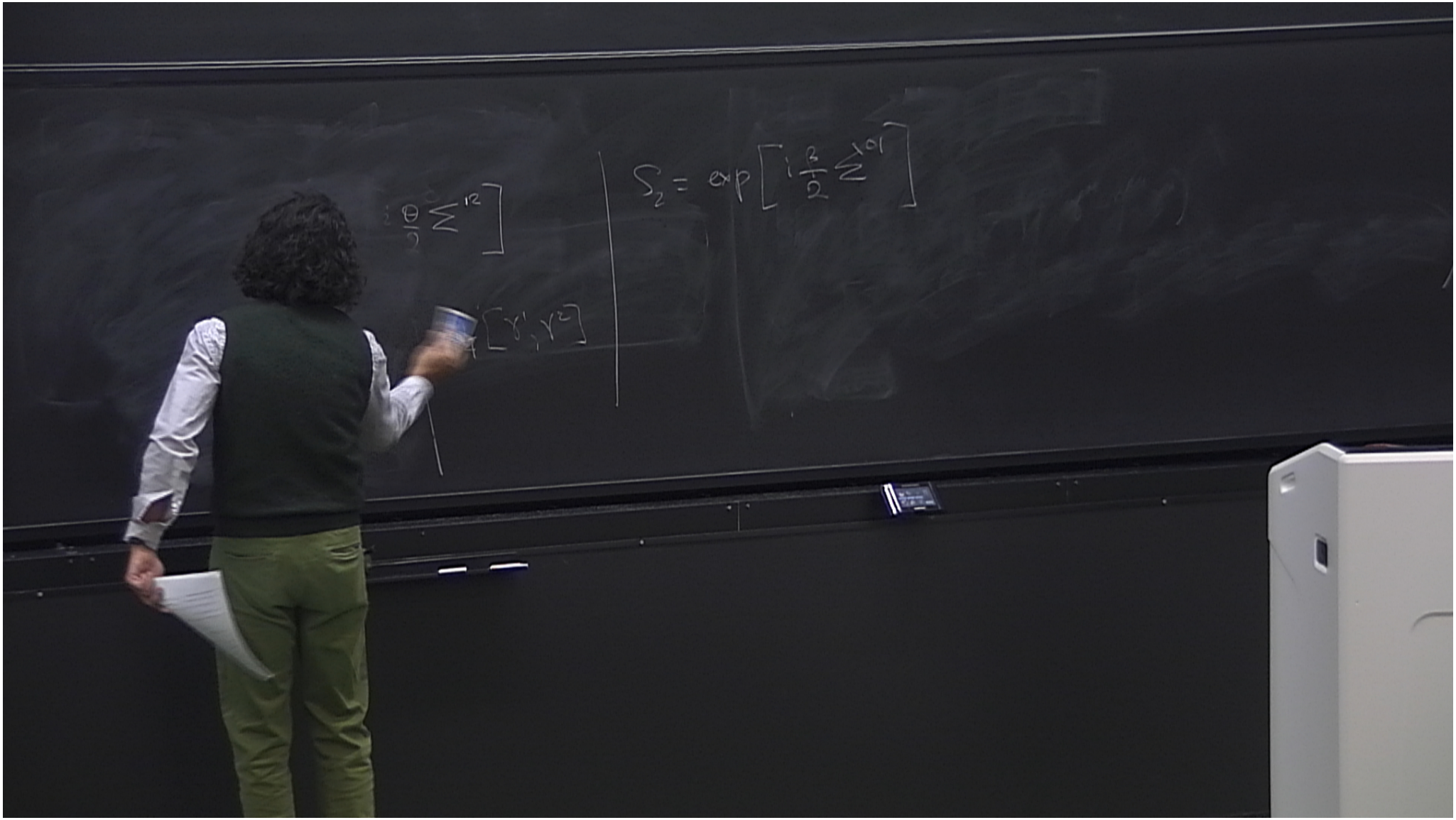

Chiral or Weyl representation of the Dirac Matrices

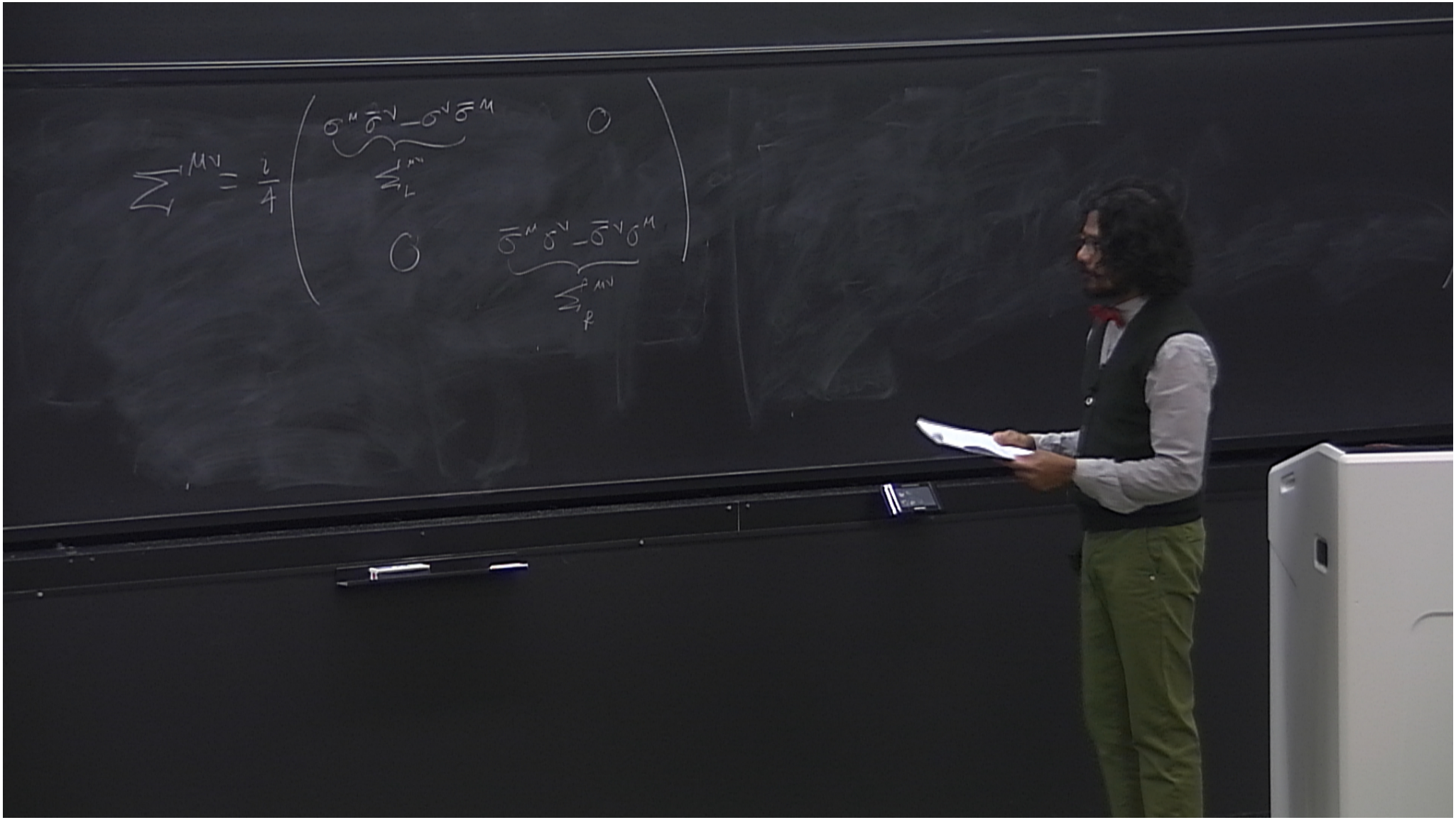

Chiral or Weyl representation of the Dirac Matrices
$\gamma_5 = \begin{pmatrix} -\mathbb{1}_{2\times 2} & 0 \\ 0 & \end{pmatrix}$
$= \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$

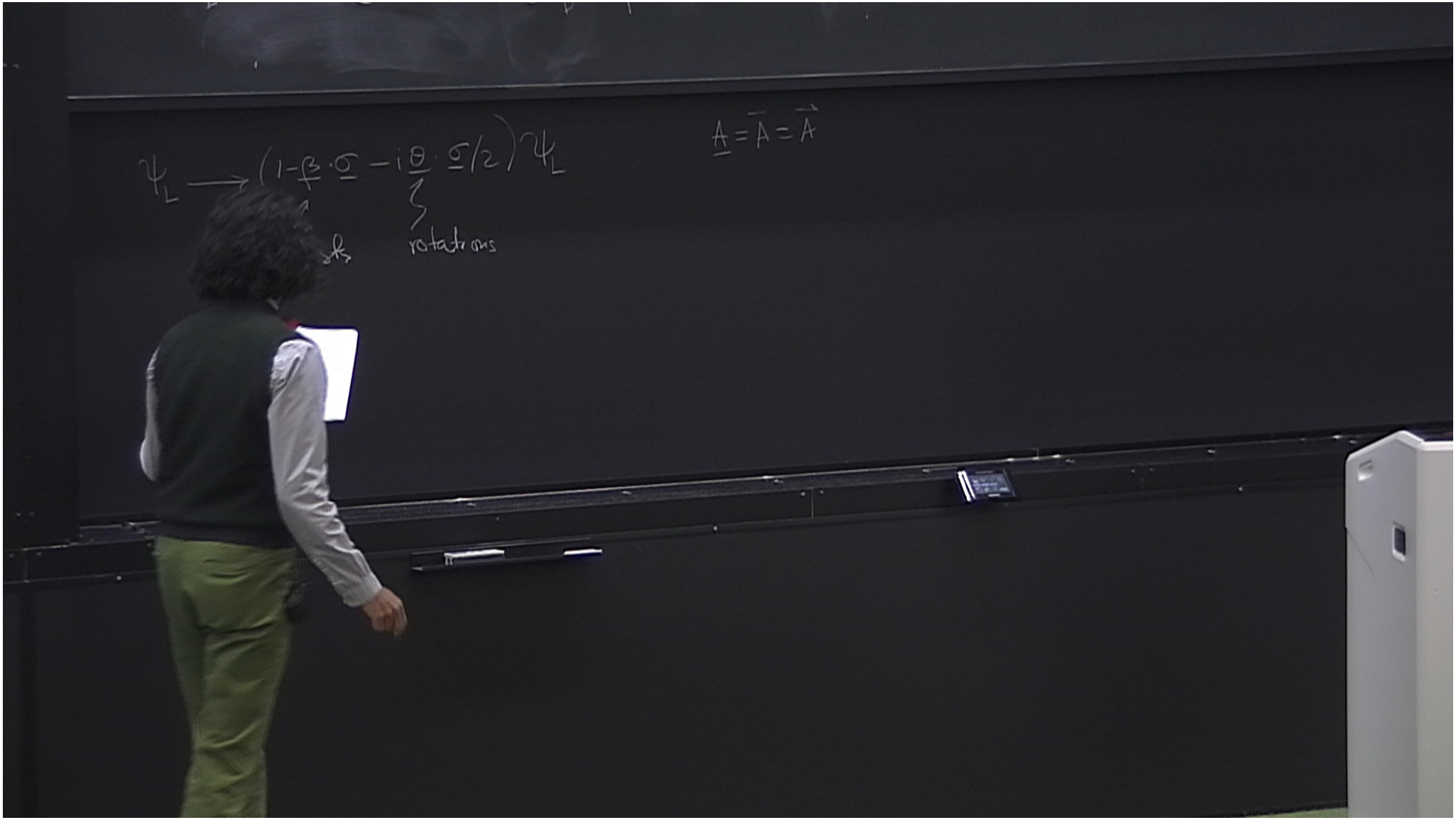

Chiral or Weyl representation of the Dirac Matrices
$\gamma_5 = \begin{pmatrix} -\mathbb{1}_{2\times2} & 0 \\ 0 & \mathbb{1}_{2\times2} \end{pmatrix}$
$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$
$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$
$-2i\epsilon_{ijk}\sigma^k$

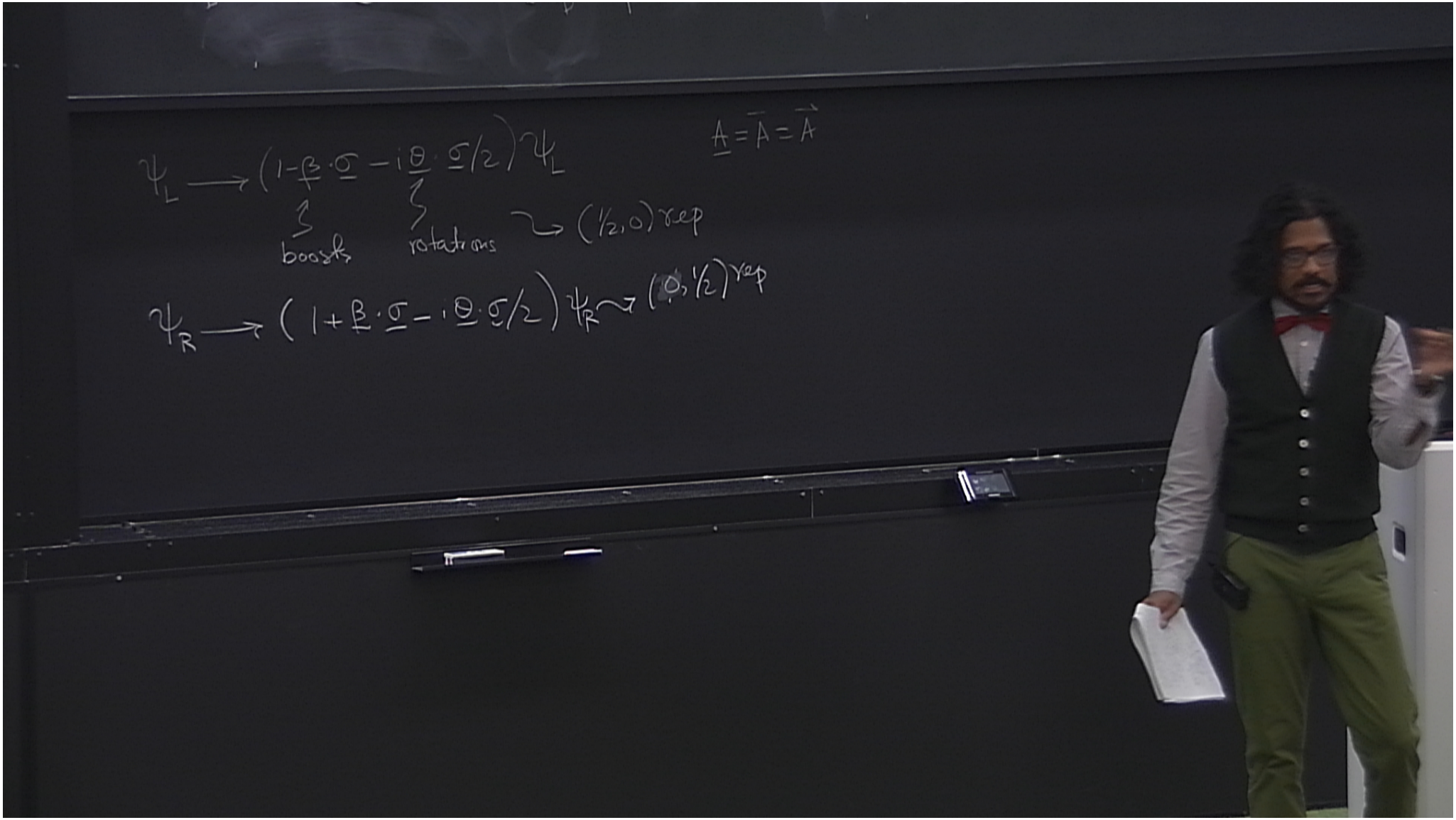

Chiral or Weyl representation of the Dirac Matrices
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$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$
$[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k$
$\{\sigma^i, \sigma^j\} = 2\delta^{ij}$


In Weyl/Chiral basis:
$\psi_D = \begin{pmatrix} \chi_L \\ \xi_R \end{pmatrix}$
$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$
$\psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix}$


$S_2 = \exp\left[i\frac{\theta}{2}\Sigma^{01}\right]$


$\Sigma^{\mu\nu} = \frac{i}{4}$
$\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu$
$\Sigma_L^{\mu\nu}$
0
0
$\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu$
$\Sigma_R^{\mu\nu}$


$\psi_L \longrightarrow (1 - \beta \cdot \sigma - i\theta \cdot \sigma/2)\psi_L$
$\underline{A} = \bar{A} = \vec{A}$
rotations


$\psi_L \longrightarrow (1 - \underline{\beta}\cdot\underline{\sigma} - i\underline{\theta}\cdot\underline{\sigma}/2)\psi_L$
$\underline{A} = \bar{A} = \vec{A}$
boosts
rotations
$\rightsquigarrow (1/2, 0)$ rep
$\psi_R \longrightarrow (1 + \underline{\beta}\cdot\underline{\sigma} - i\underline{\theta}\cdot\underline{\sigma}/2)\psi_R \rightsquigarrow (0, 1/2)$ rep