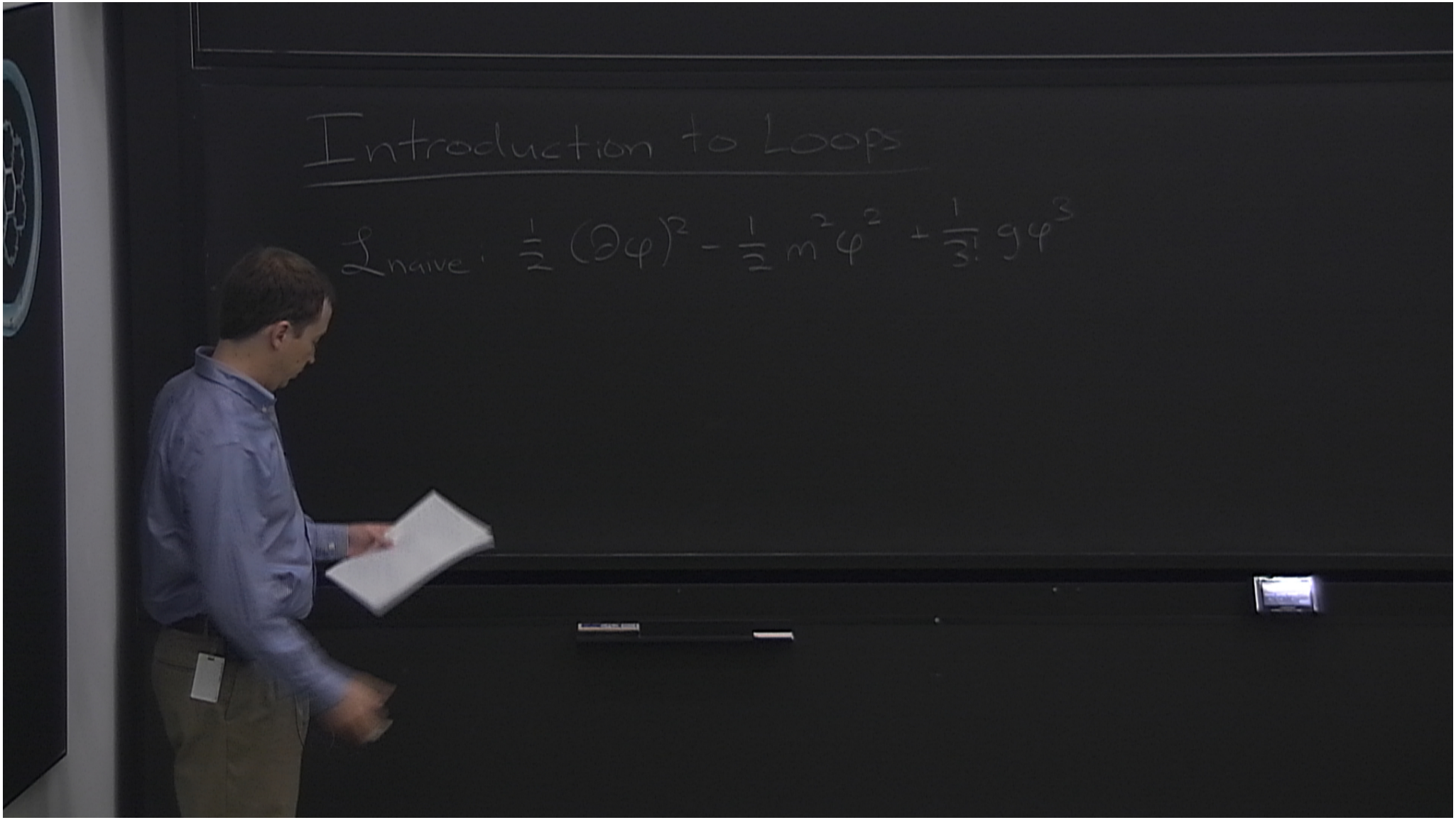
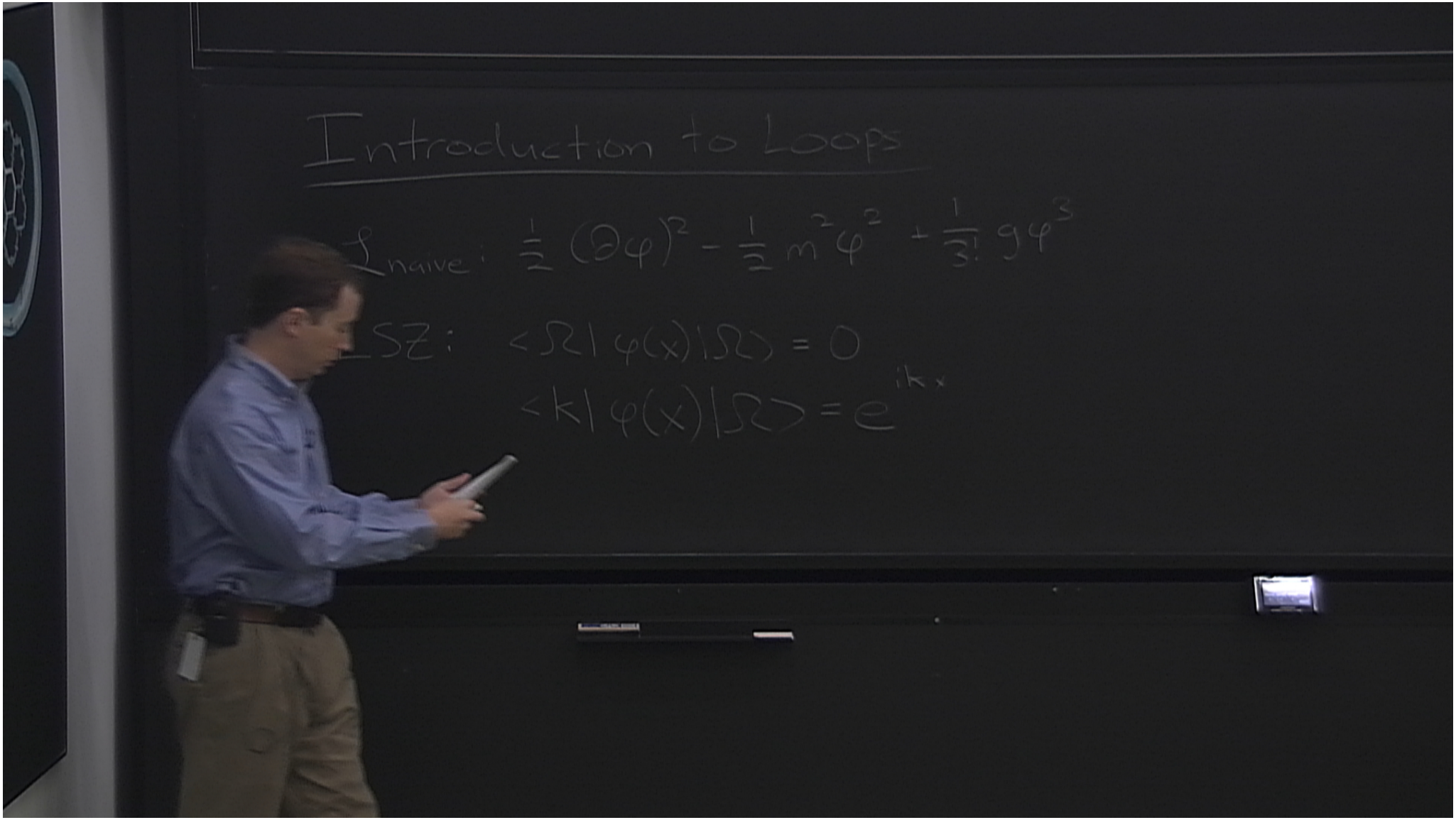
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 7

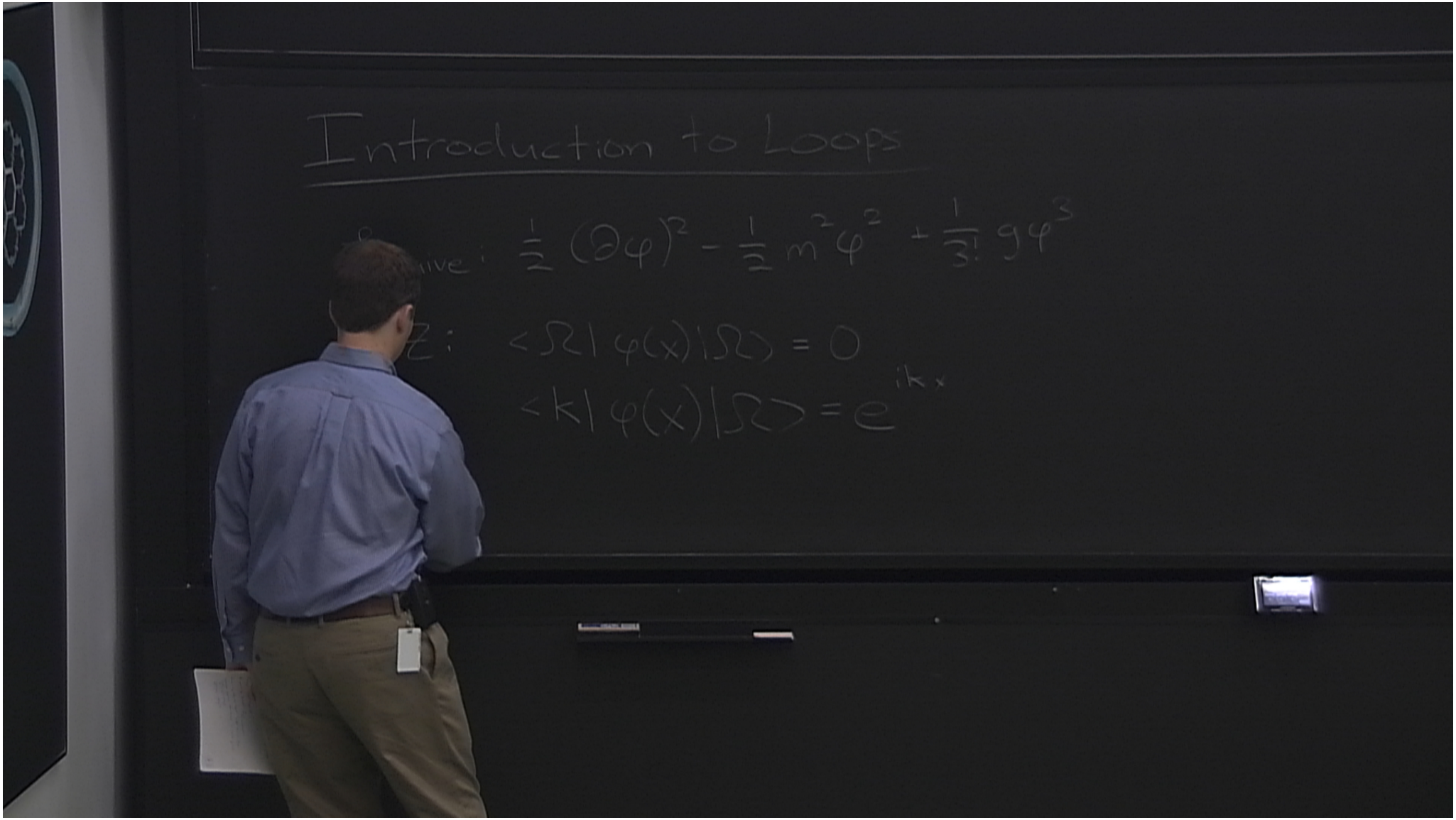
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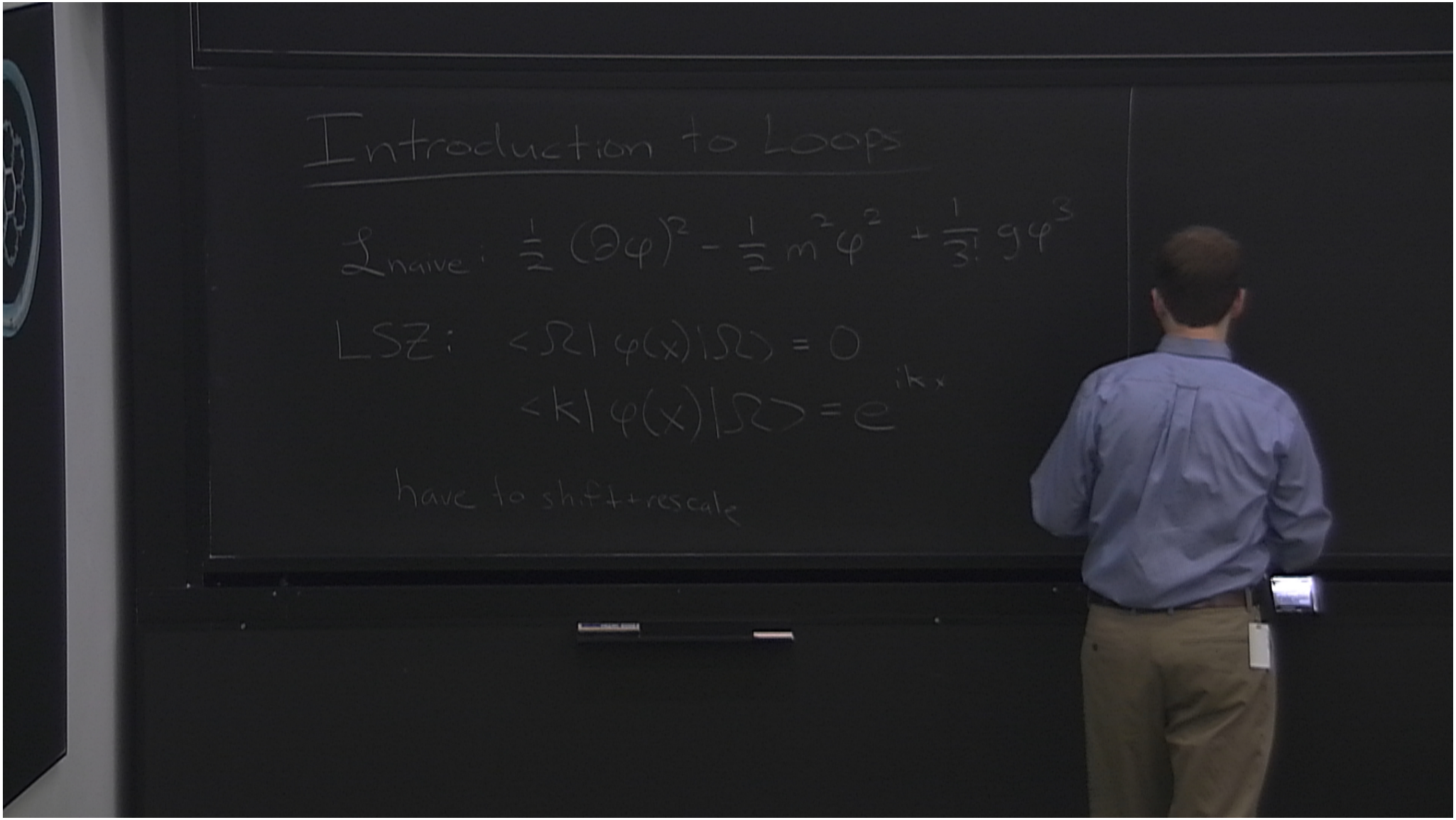
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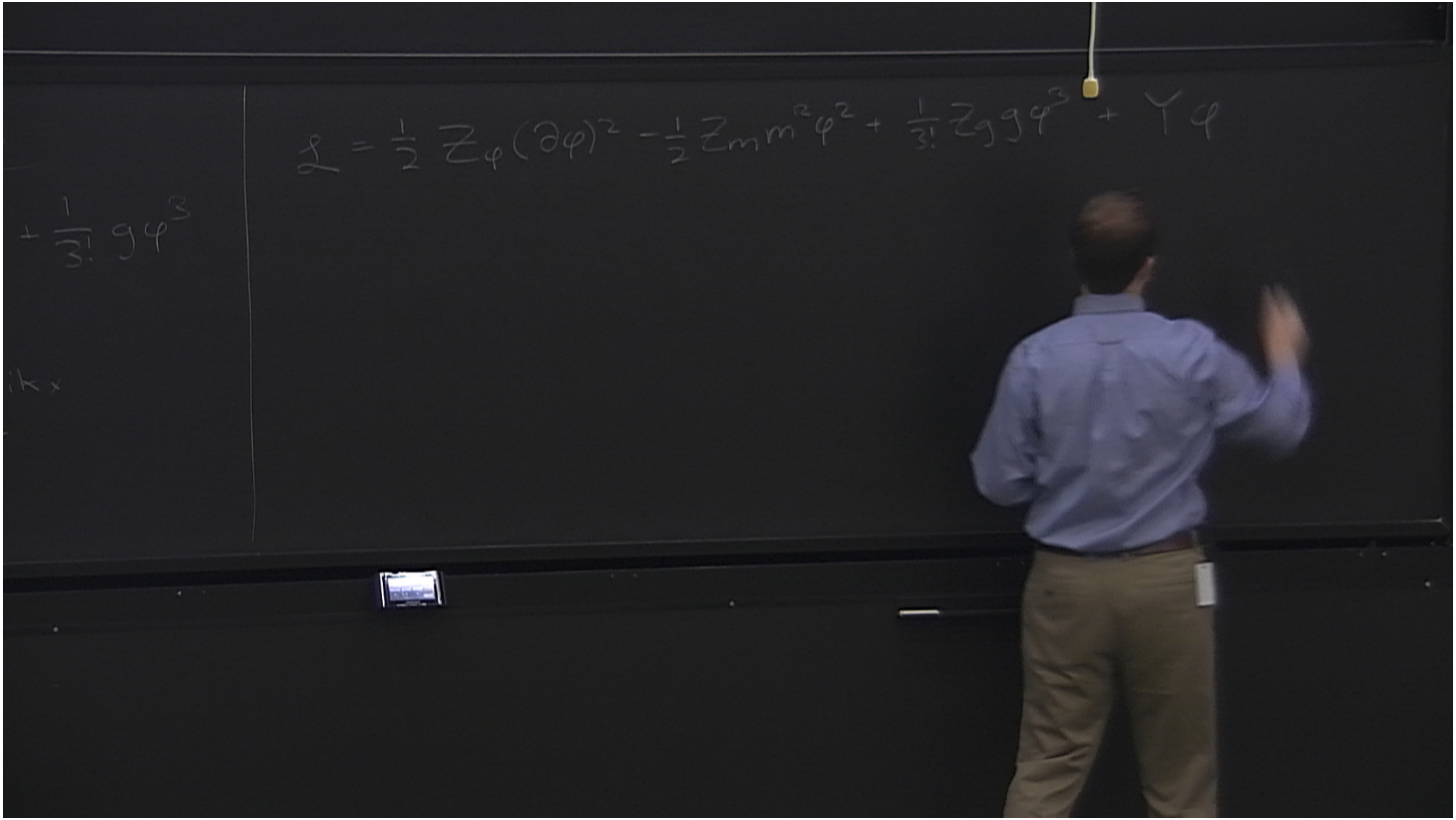
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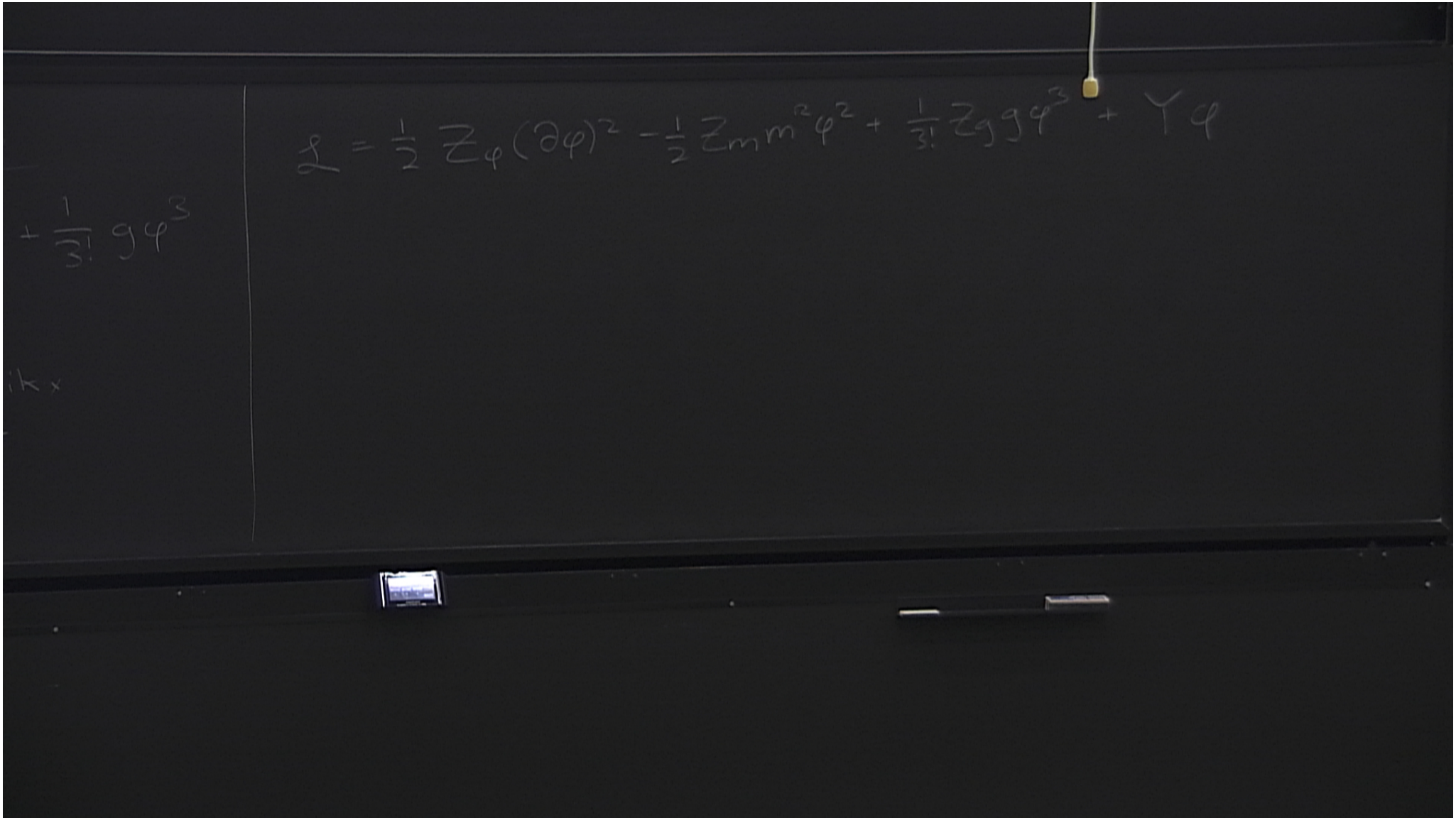

Introduction to Loops
$\mathcal{L}_{naive}: \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$

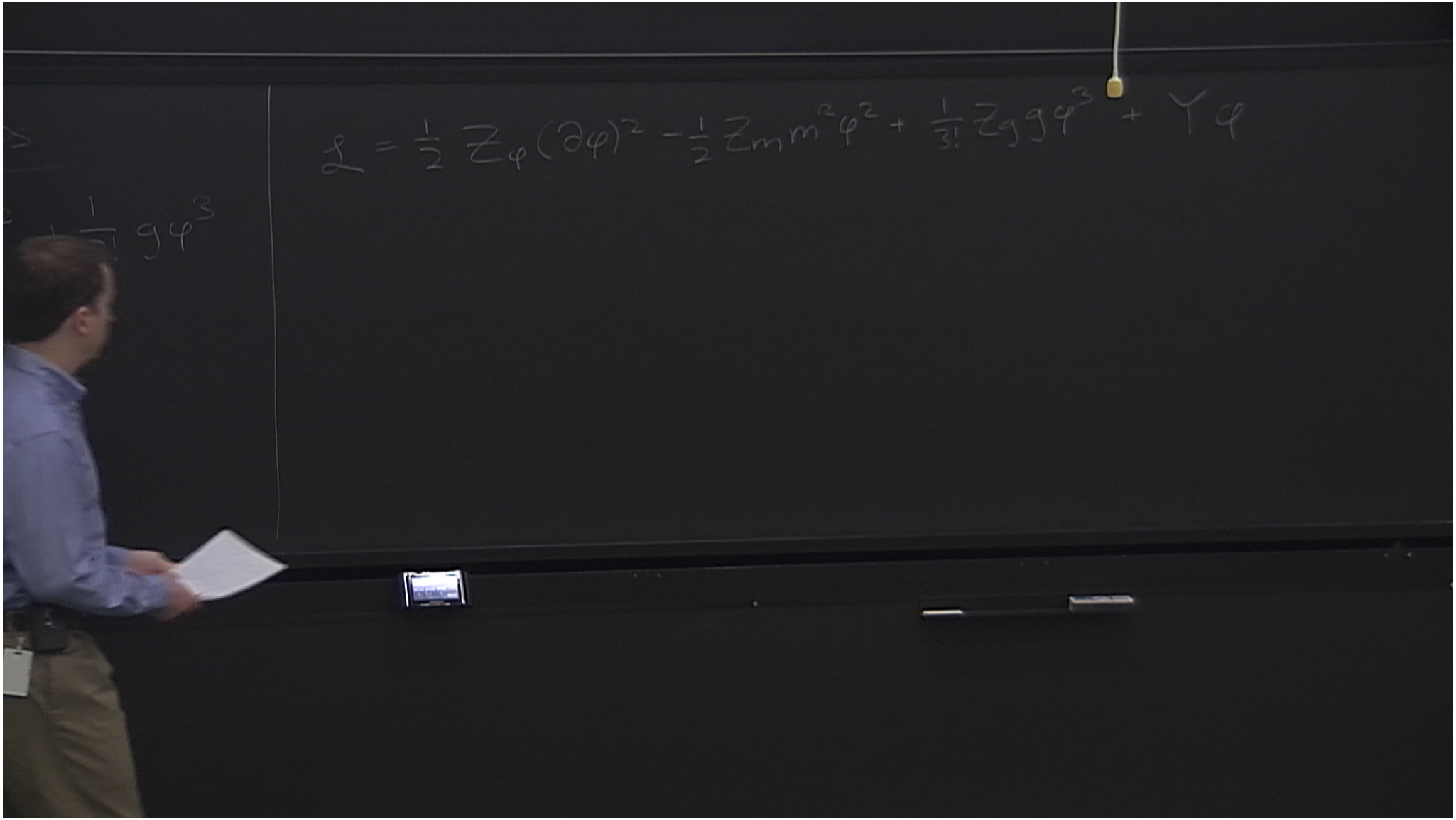

Introduction to Loops
$\mathcal{L}_{naive}: \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$
LSZ: $\langle\Omega|\varphi(x)|\Omega\rangle = 0$
$\langle k|\varphi(x)|\Omega\rangle = e^{ikx}$


Introduction to Loops
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Introduction to Loops
$\mathcal{L}_{naive}: \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$
LSZ: $\langle\Omega|\varphi(x)|\Omega\rangle = 0$
$\langle k|\varphi(x)|\Omega\rangle = e^{ikx}$
have to shift + rescale


$\mathcal{L} = \frac{1}{2} Z_\varphi (\partial\varphi)^2 - \frac{1}{2} Z_m m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y \varphi$
$+ \frac{1}{3!} g\varphi^3$
$ik \cdot x$

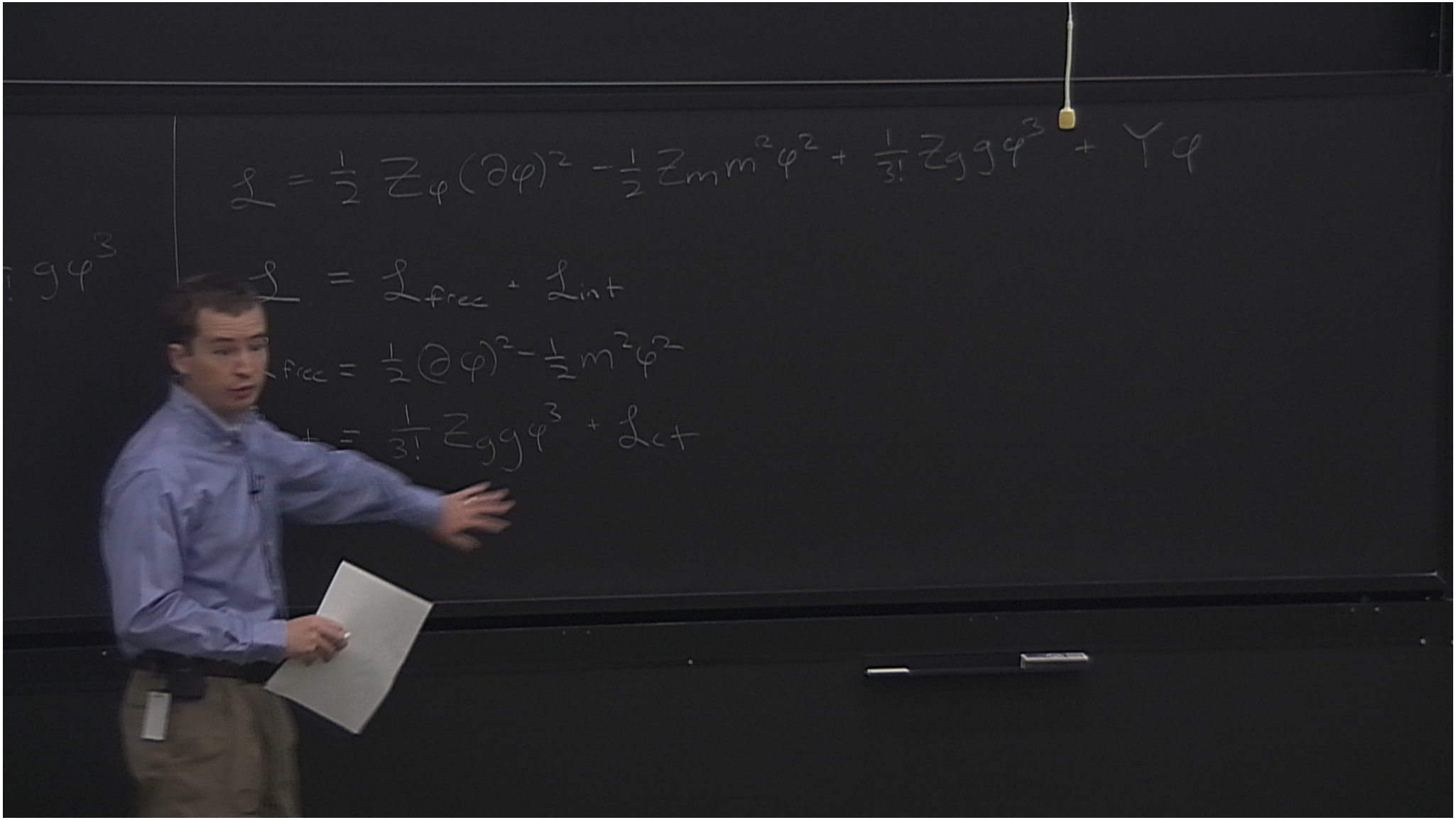

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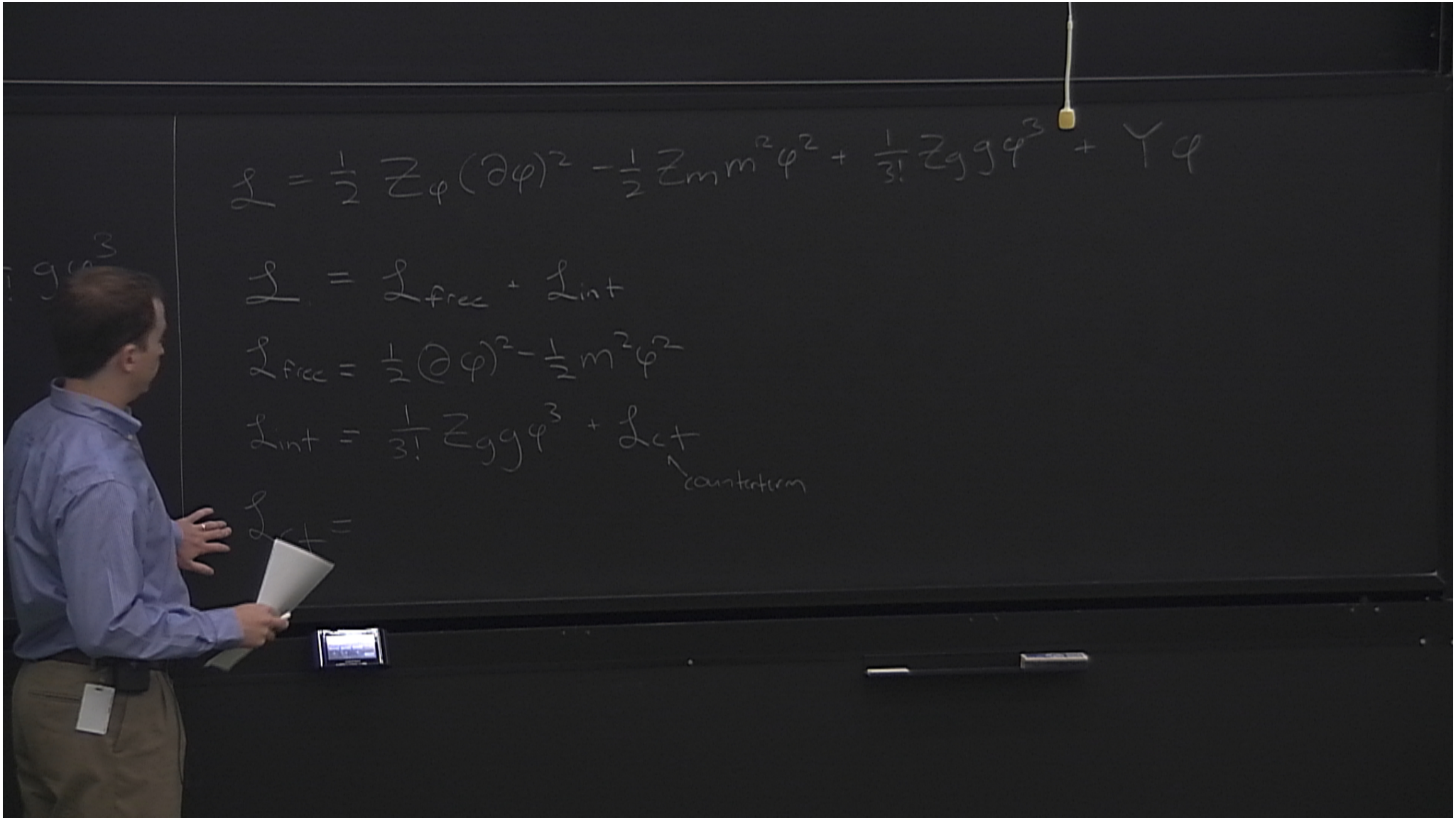

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$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L}_{free} =$$


$\mathcal{L} = \frac{1}{2} Z_\varphi (\partial\varphi)^2 - \frac{1}{2} Z_m m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y\varphi$
$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$
$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2$
$= \frac{1}{3!} Z_g g \varphi^3 + \mathcal{L}_{ct}$


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counterterm
$\mathcal{L}_{ct} =$

$\mathcal{L} = \frac{1}{2} Z_\varphi (\partial\varphi)^2 - \frac{1}{2} Z_m m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y\varphi$

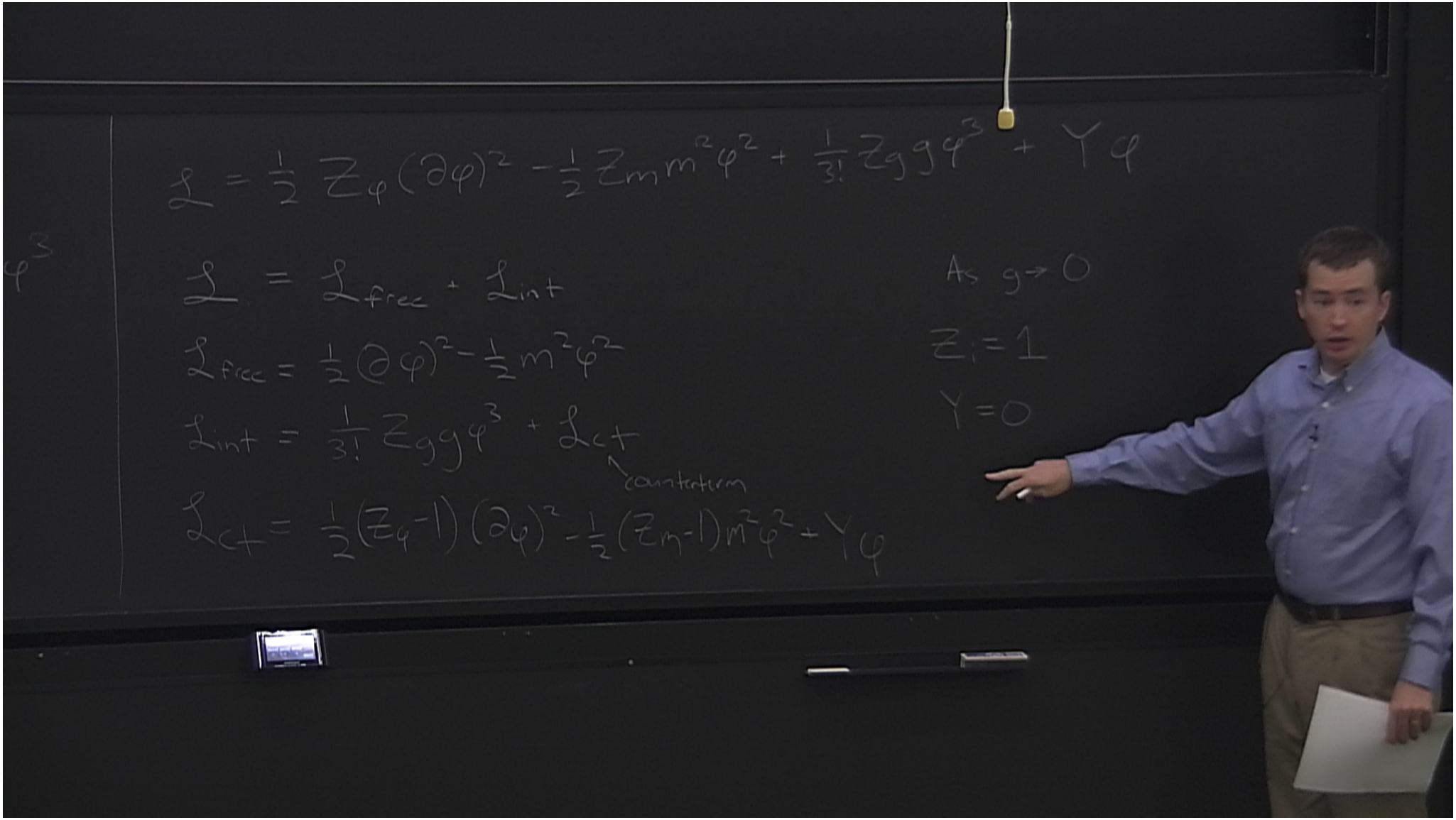
$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$

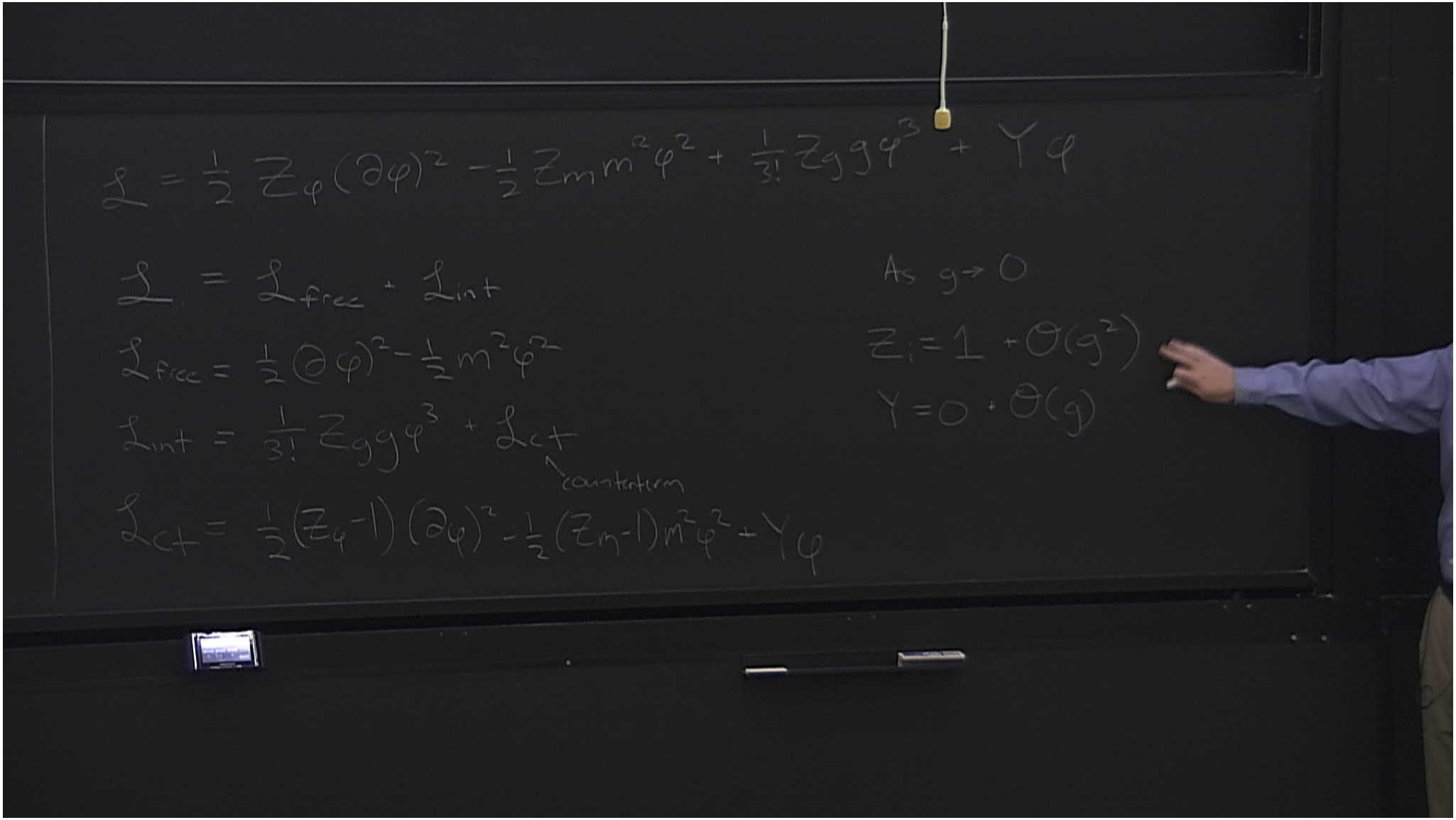
$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2$

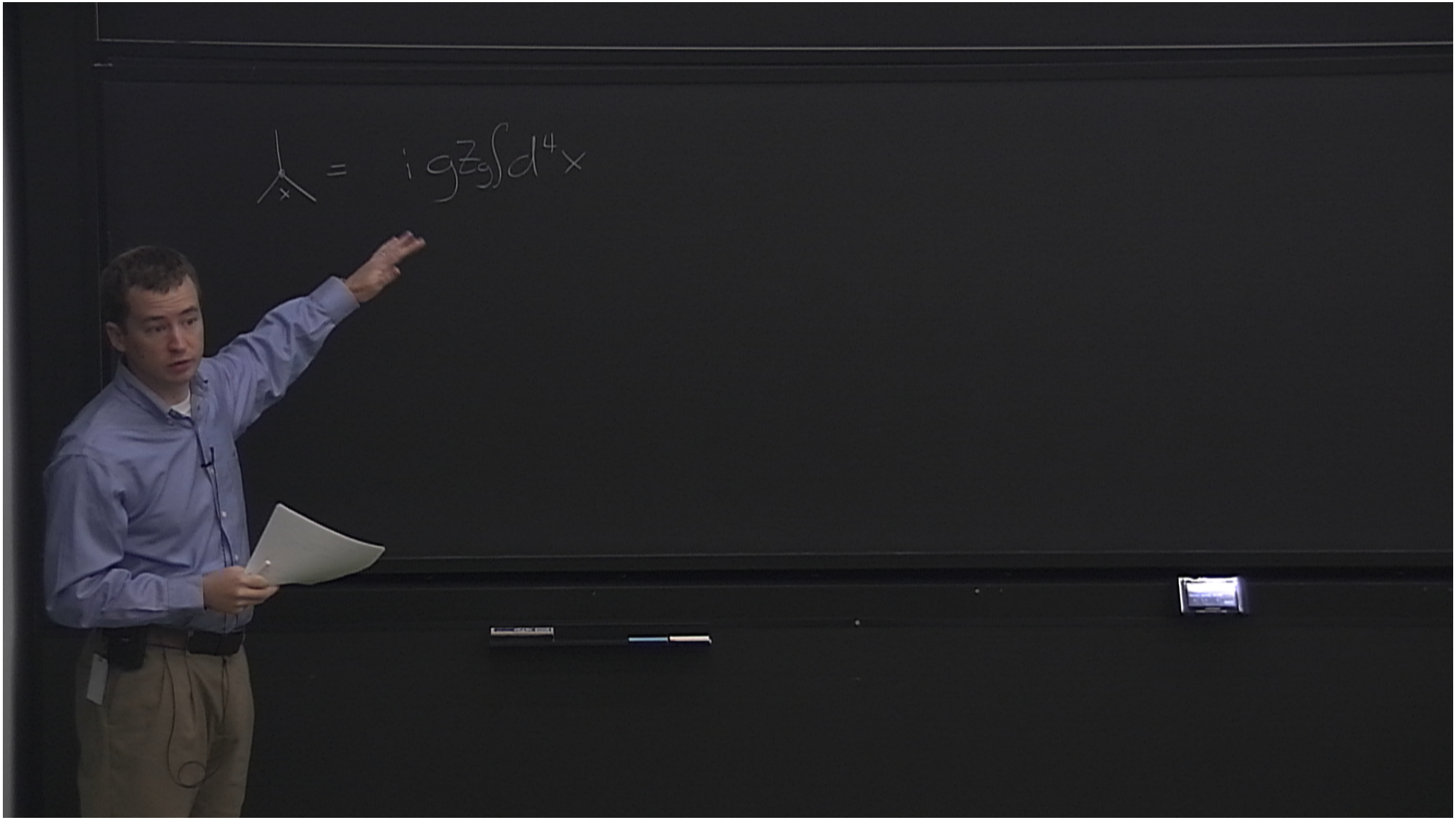
$\mathcal{L}_{int} = \frac{1}{3!} Z_g g \varphi^3 + \mathcal{L}_{ct}$

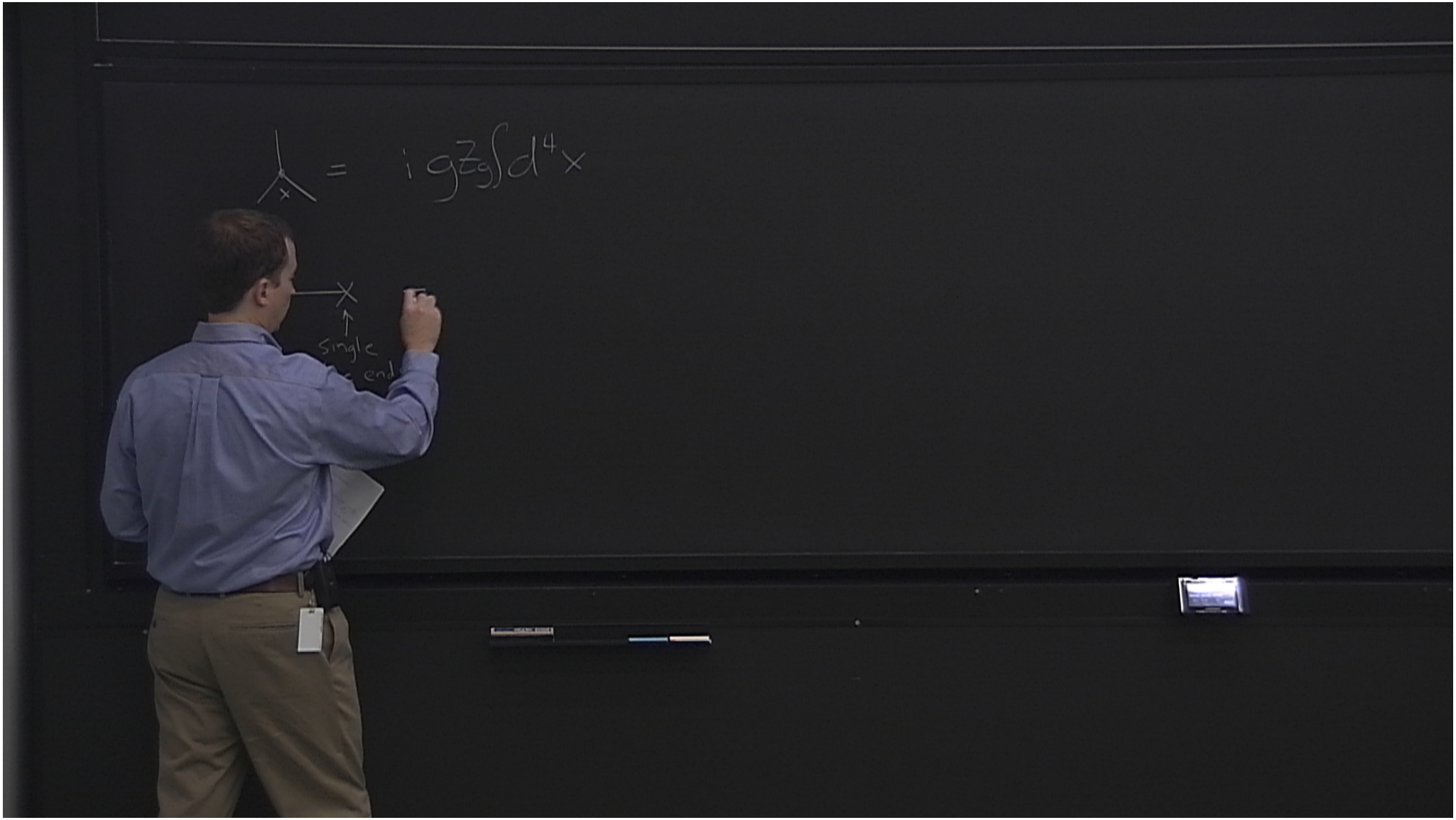
counterterm

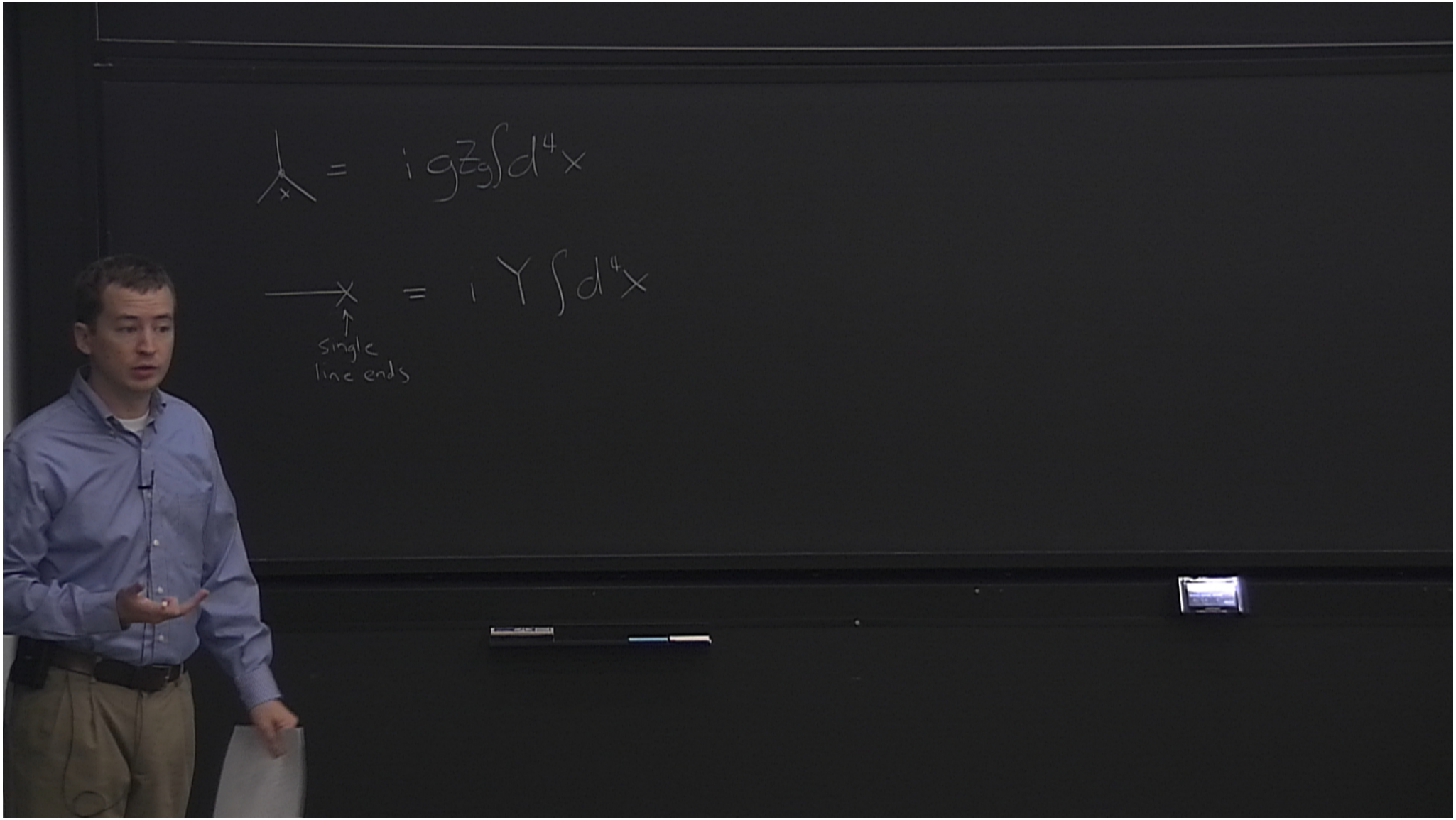
$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1) m^2 \varphi^2 + Y\varphi$

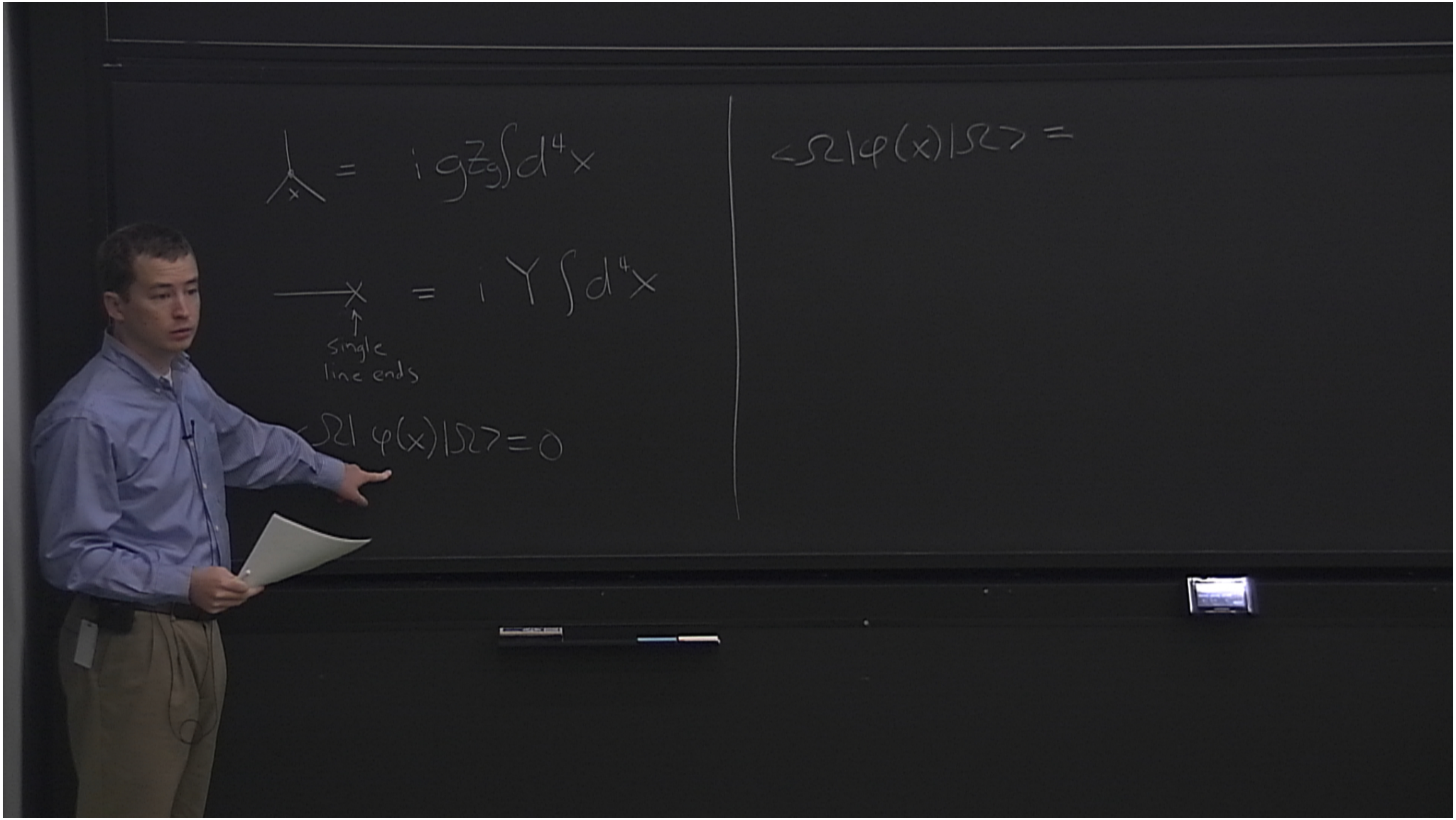

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$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1) m^2 \varphi^2 + Y\varphi$
As $g \to 0$
$Z_i = 1$
$Y = 0$

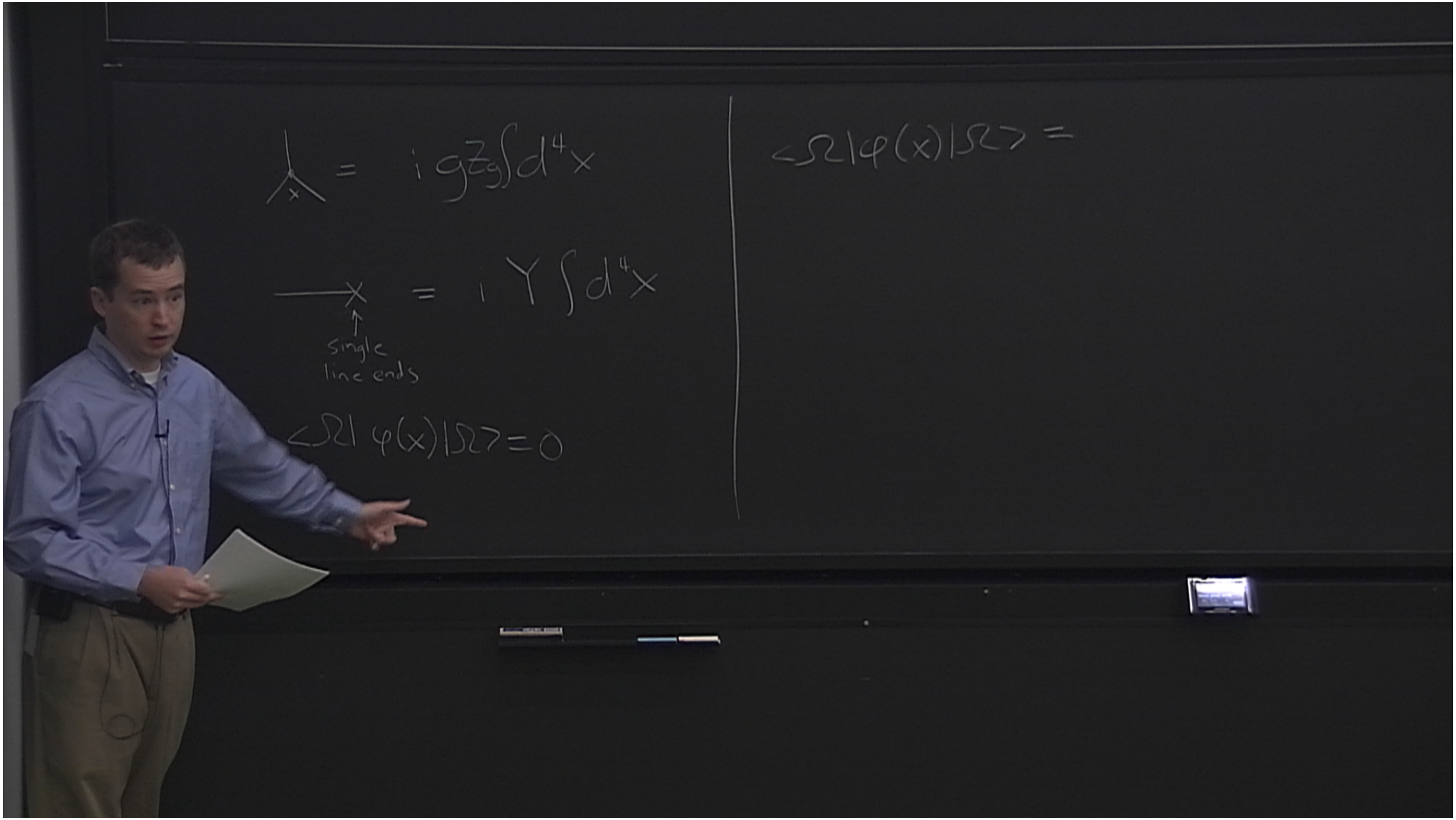

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counterterm
$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1) m^2 \varphi^2 + Y\varphi$
As $g \to 0$
$Z_i = 1 + \mathcal{O}(g^2)$
$Y = 0 + \mathcal{O}(g)$

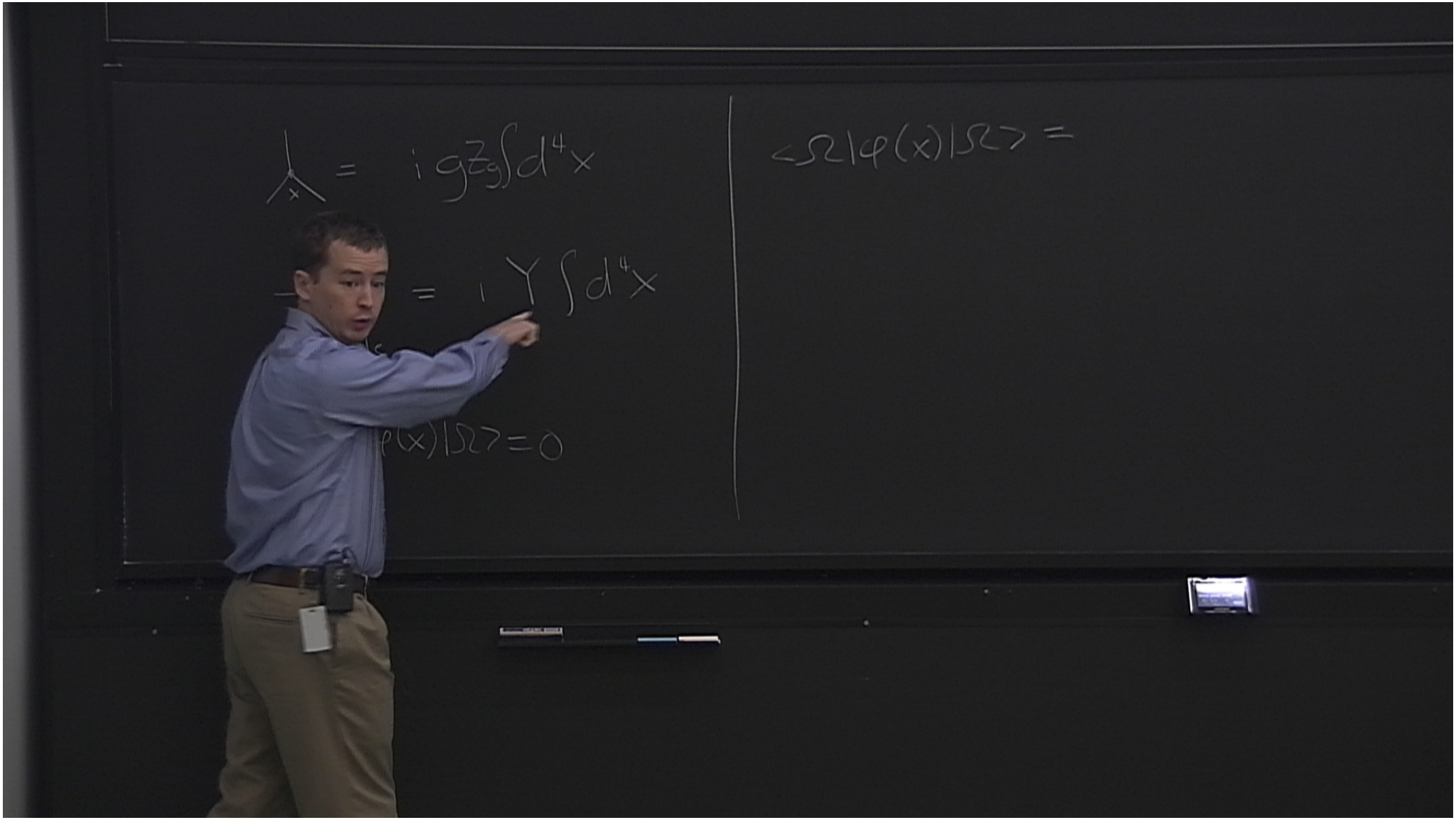

= i g Z_g ∫ d^4 x

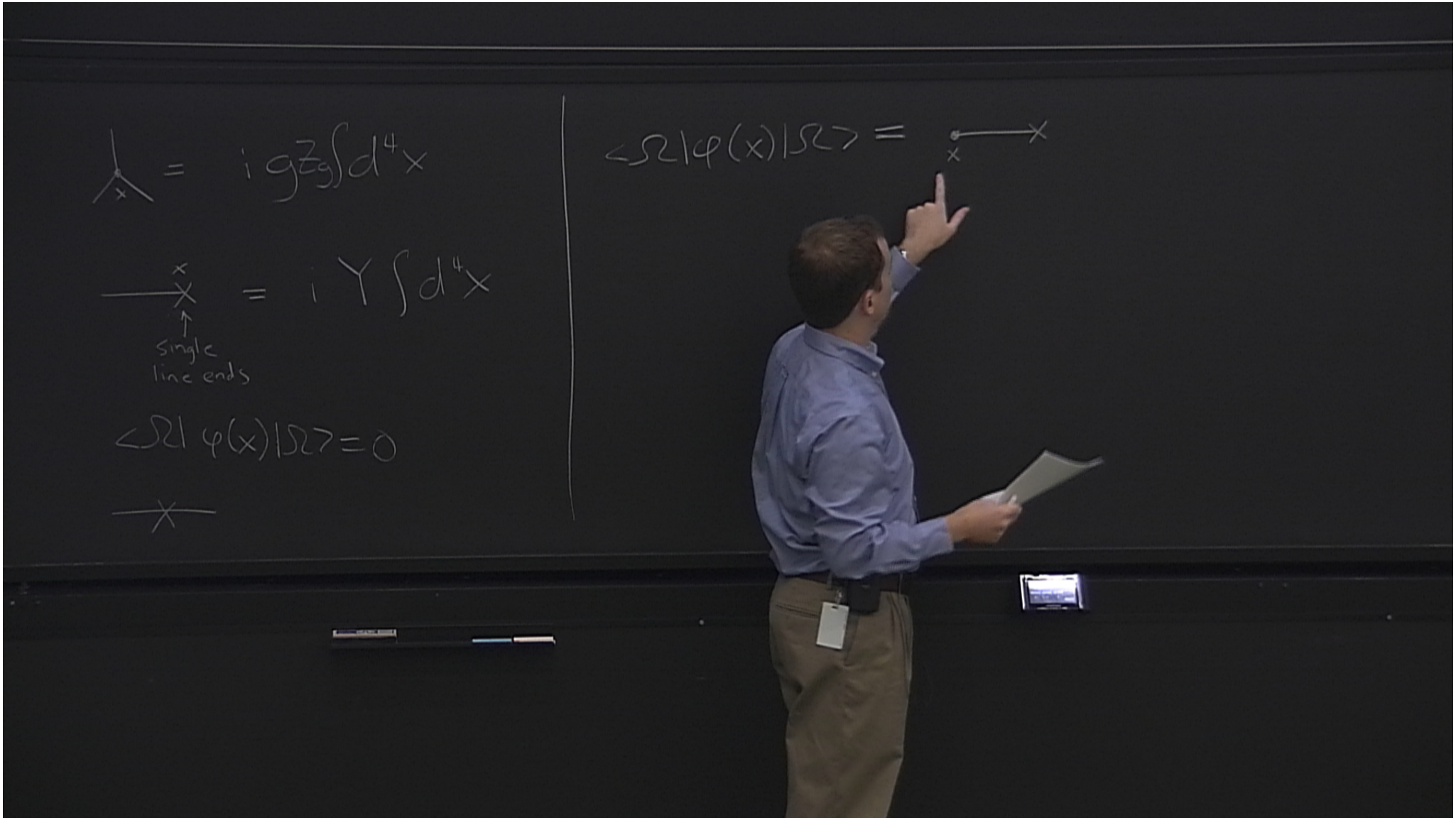

= i g Z_g ∫ d^4 x
single

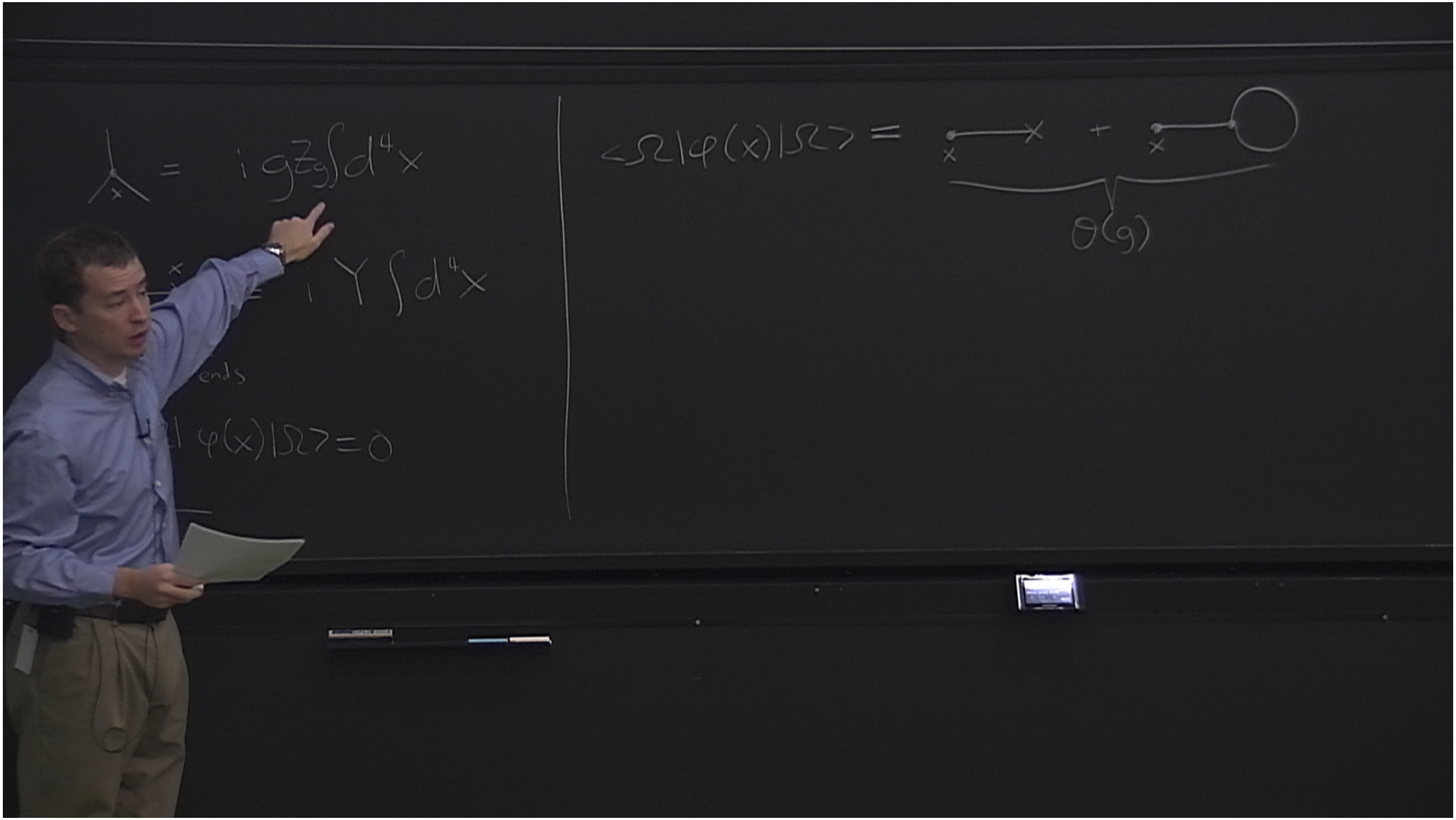

$= i g Z_g \int d^4x$
$= i Y \int d^4x$
single
line ends

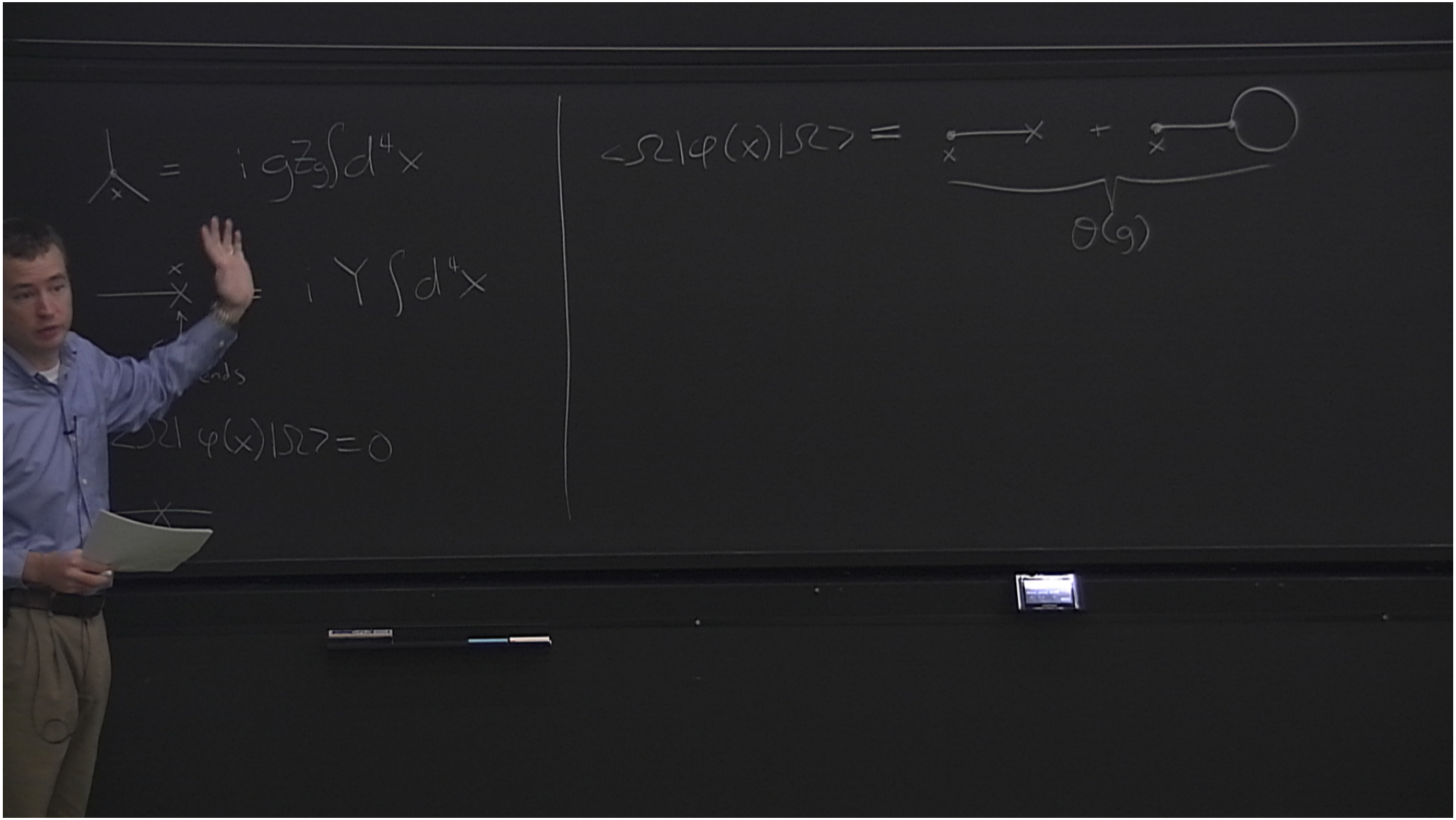

= $ig^3 Z_g \int d^4x$
= $i Y \int d^4x$
single line ends
$\langle \Omega | \varphi(x) | \Omega \rangle = 0$
$\langle \Omega | \varphi(x) | \Omega \rangle =$

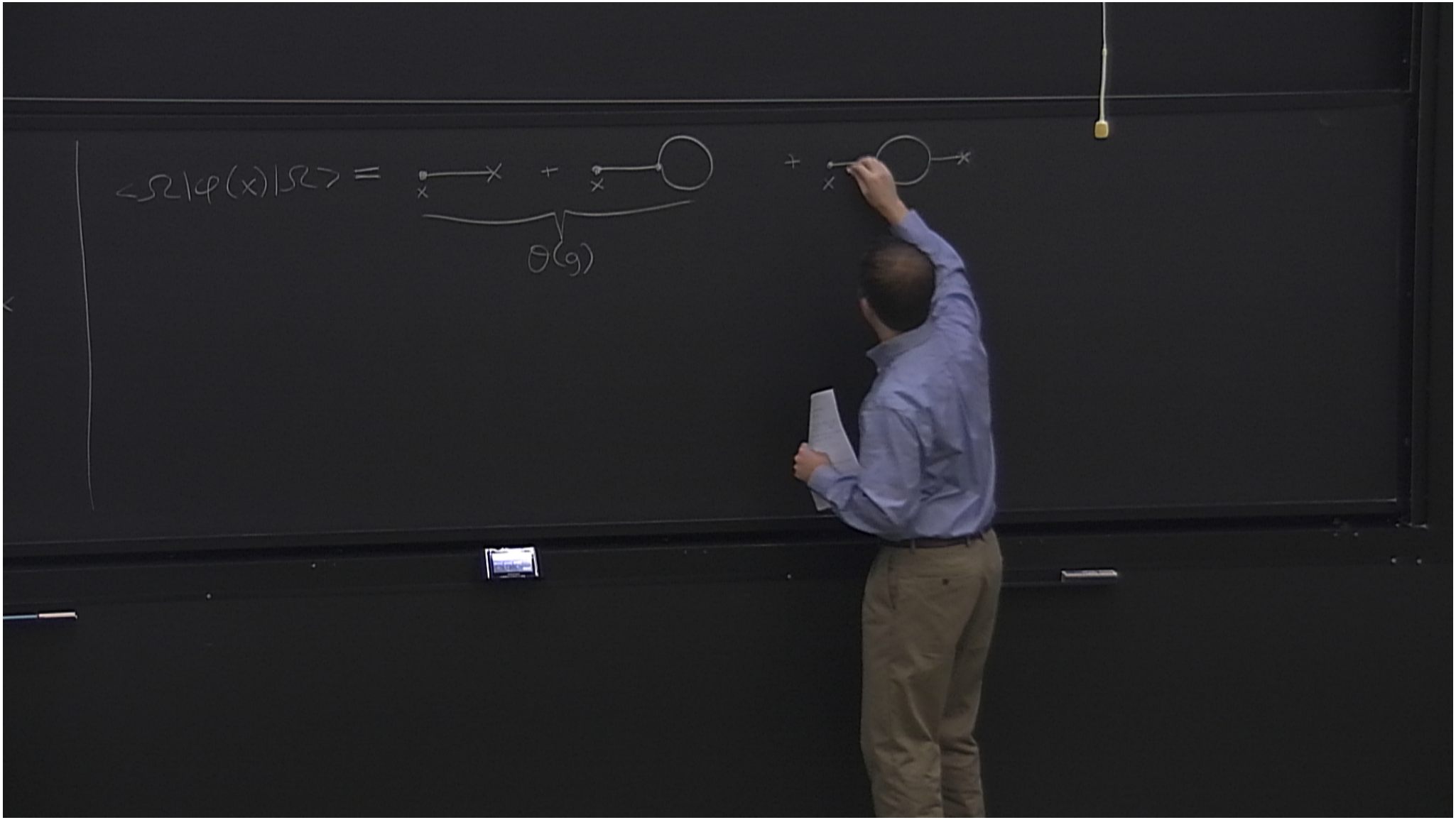

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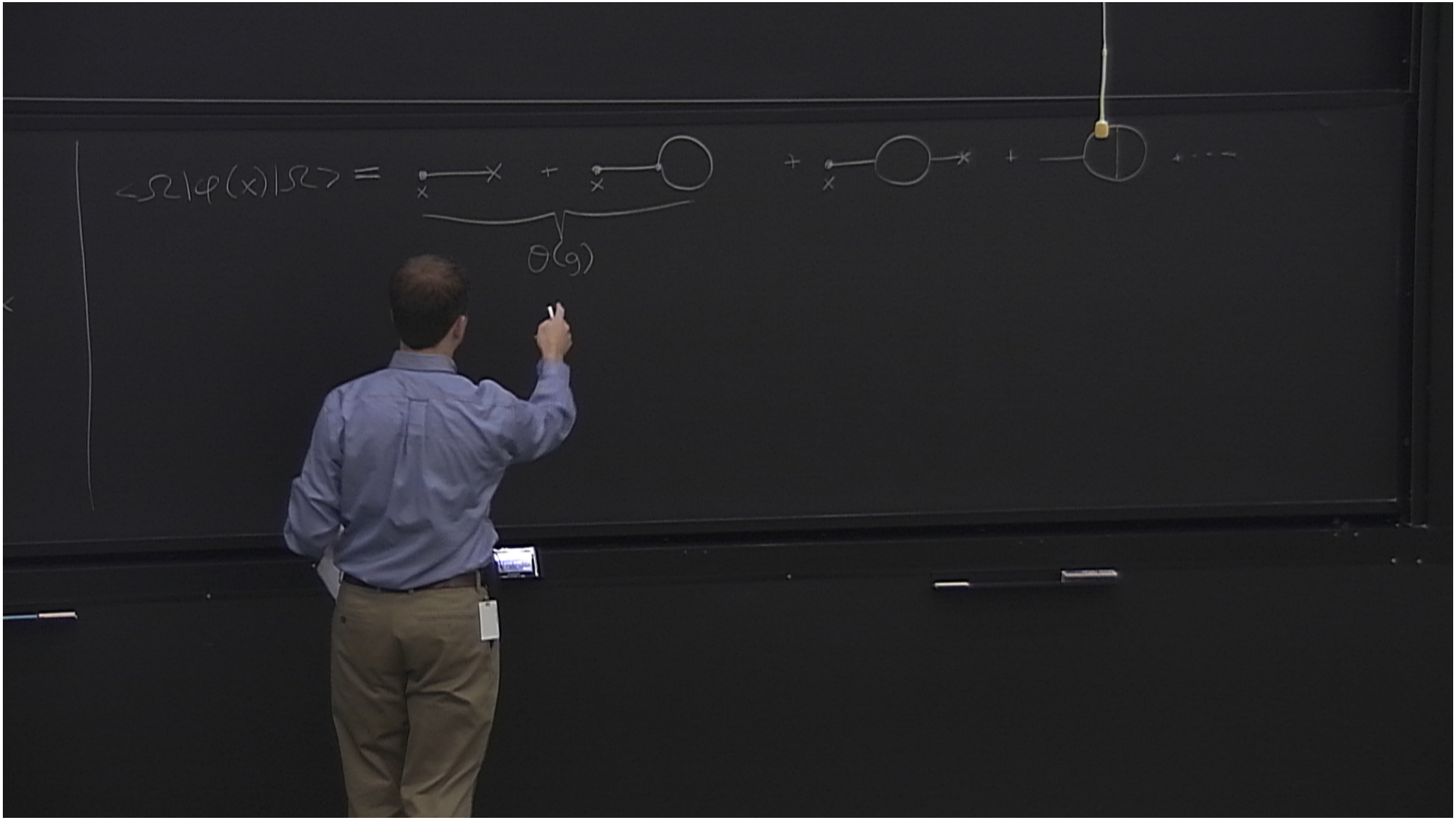

$= i g Z_g \int d^4x$
$= i Y \int d^4x$
$\langle \Omega | \varphi(x) | \Omega \rangle =$
$\varphi(x)|\Omega\rangle = 0$

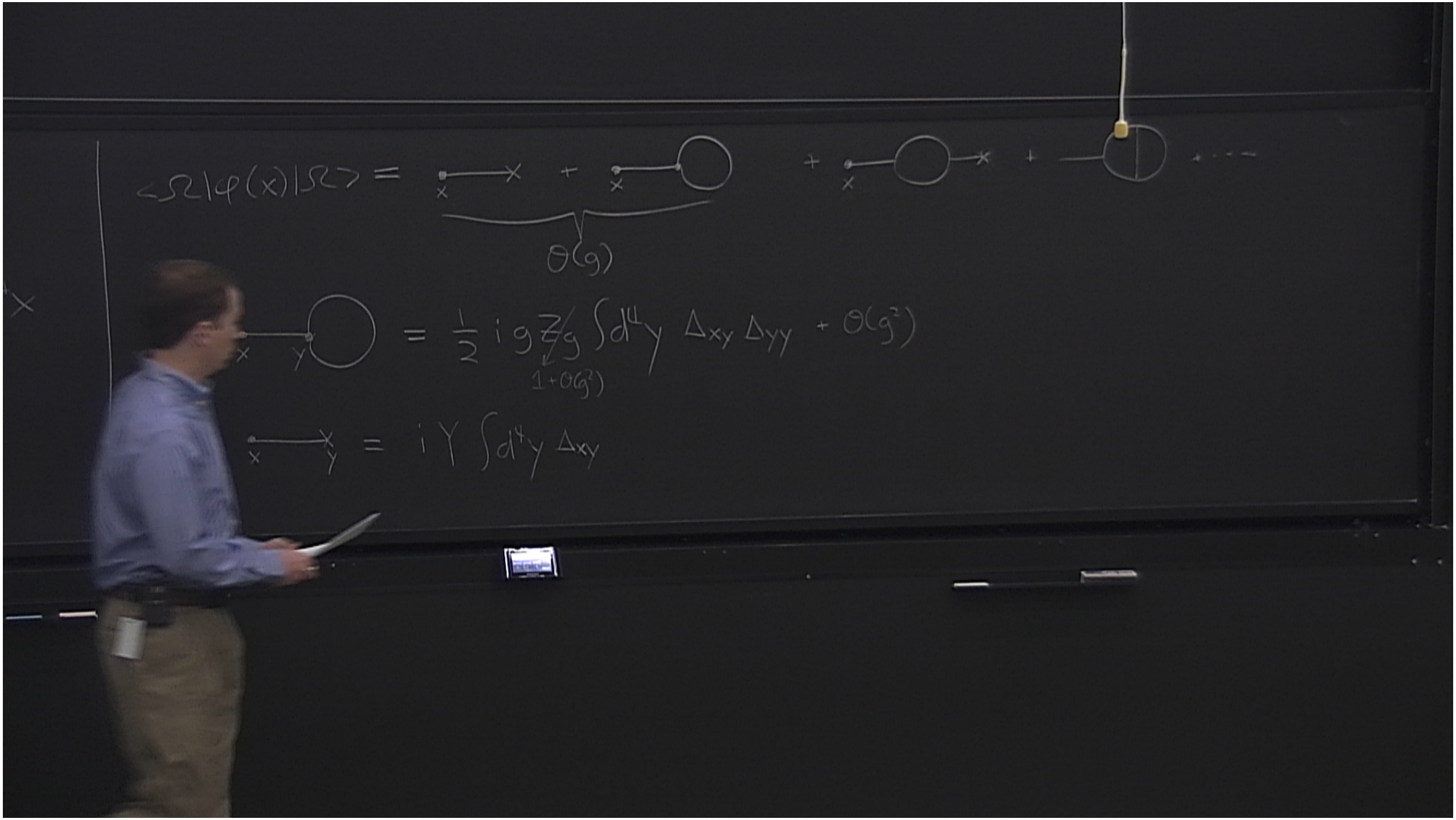

= i g Z g ∫ d⁴x
= i Y ∫ d⁴x
single line ends
⟨Ω| φ(x) |Ω⟩ = 0
⟨Ω| φ(x) |Ω⟩ =

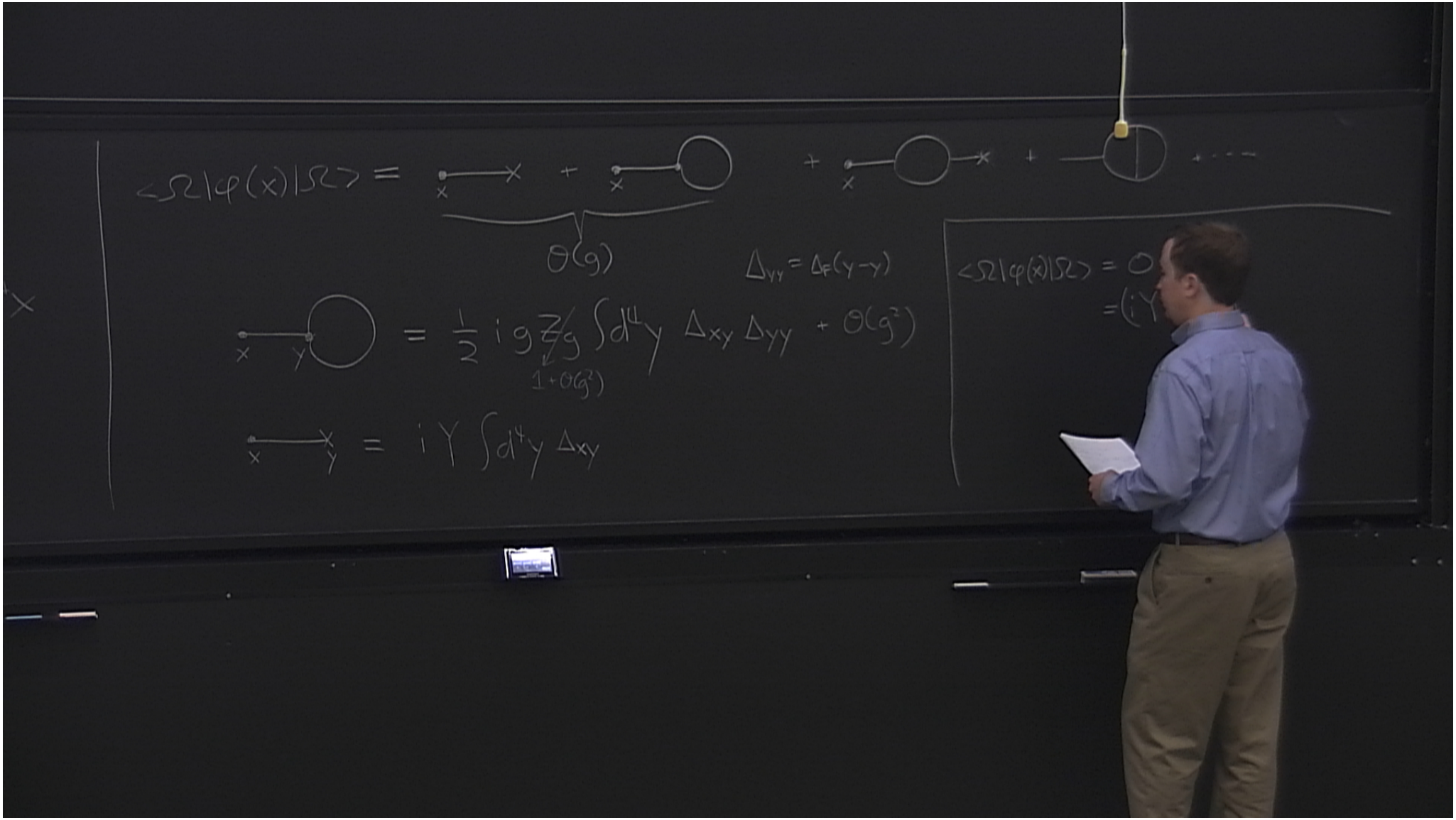

$= i g z_g \int d^4x$
$\langle \Omega | \varphi(x) | \Omega \rangle =$
$\theta(g)$
ends
$\varphi(x)|\Omega\rangle = 0$

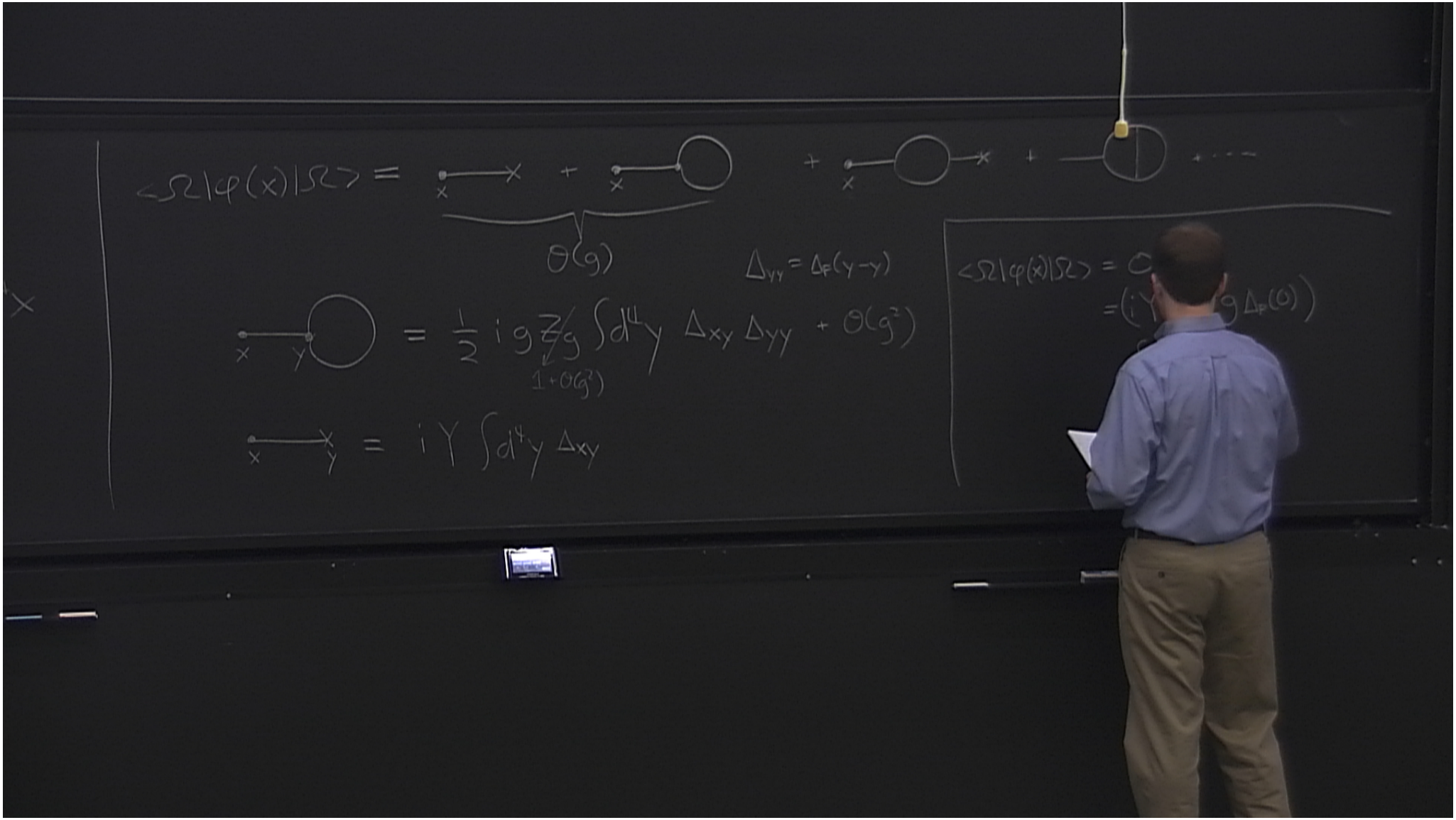

= i g Z_g ∫ d^4 x
= i Y ∫ d^4 x
ends
⟨Ω| φ(x) |Ω⟩ = 0
⟨Ω| φ(x) |Ω⟩ =
+
θ(g)

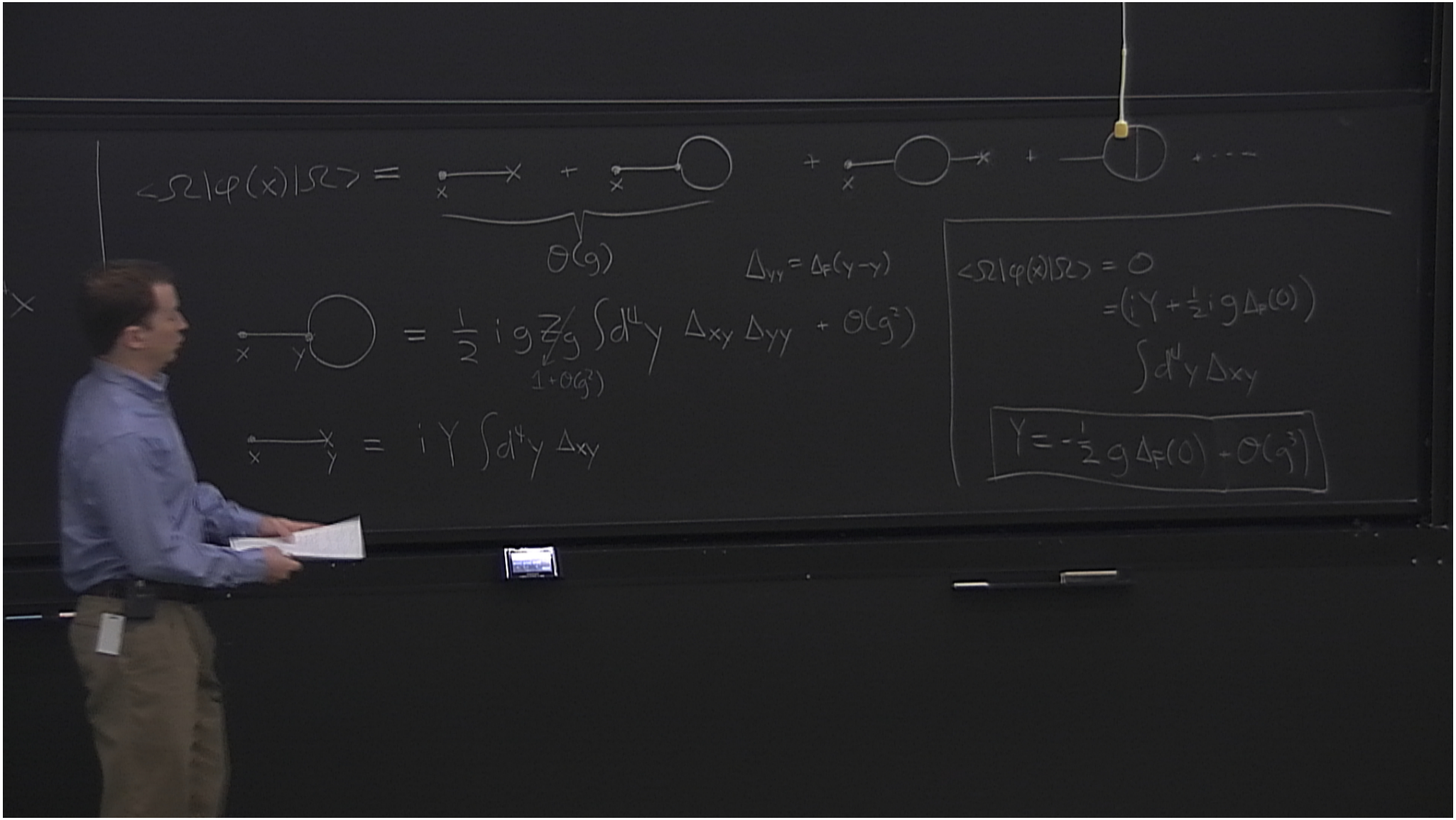

$\langle \Omega | \varphi(x) | \Omega \rangle =$
x
+
x
+
x
$\theta(g)$

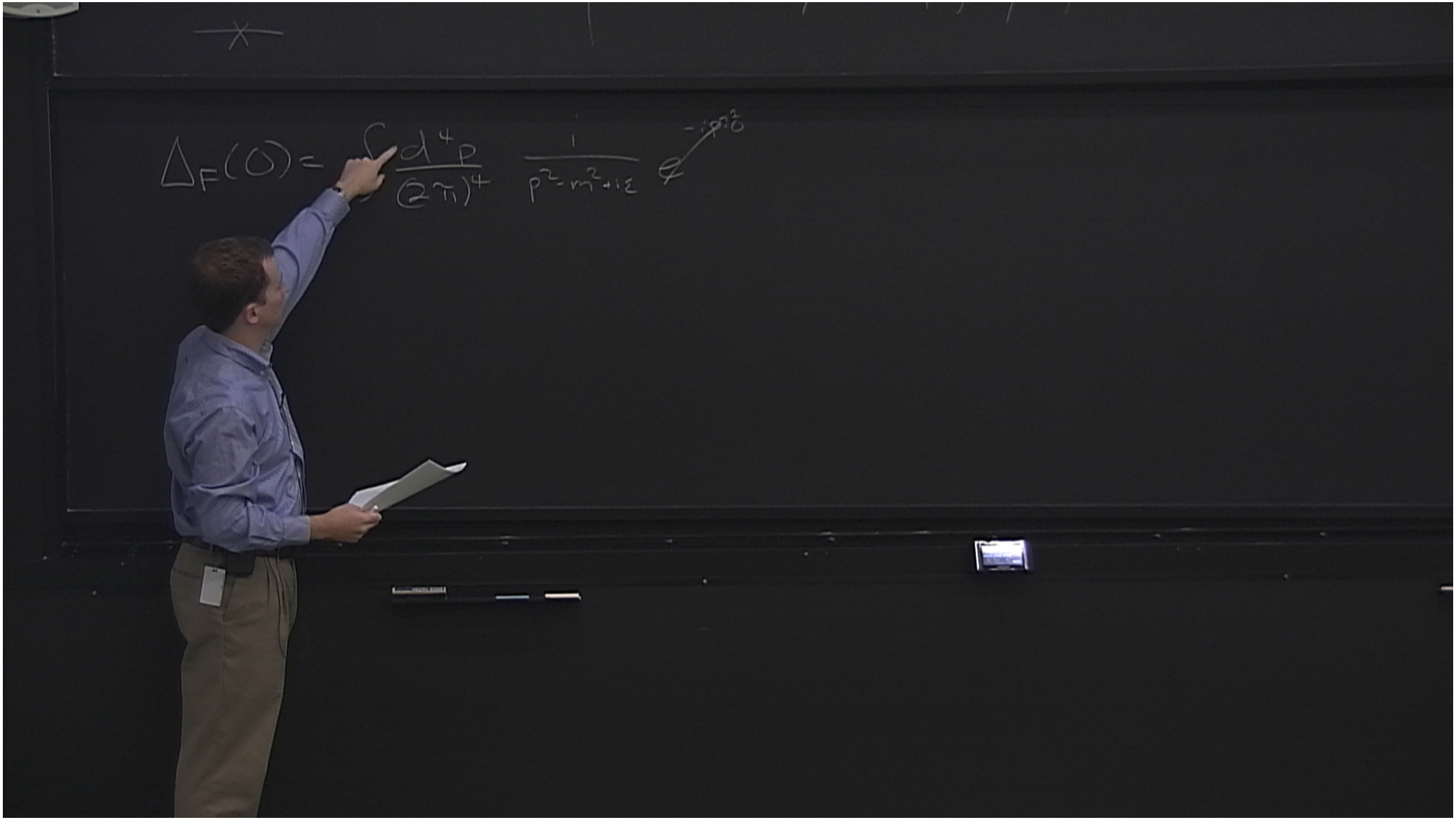

$\langle \Omega | \varphi(x) | \Omega \rangle =$
$\theta(g)$

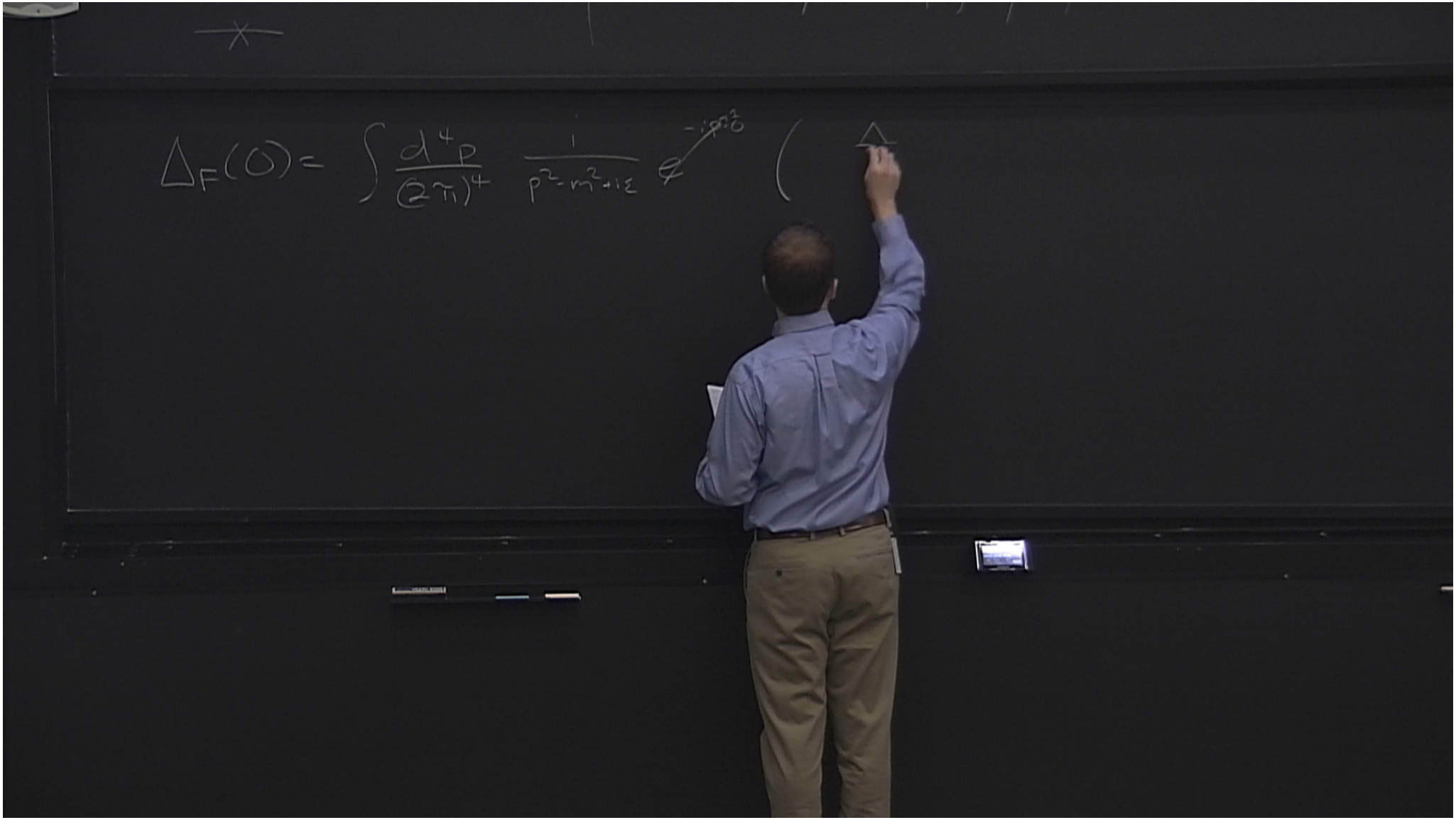

$\langle \Omega | \varphi(x) | \Omega \rangle =$
$\theta(g)$
$= \frac{1}{2} i g \frac{Z_g}{1+O(g^2)} \int d^4y \, \Delta_{xy} \Delta_{yy} + O(g^2)$
$= i Y \int d^4y \, \Delta_{xy}$

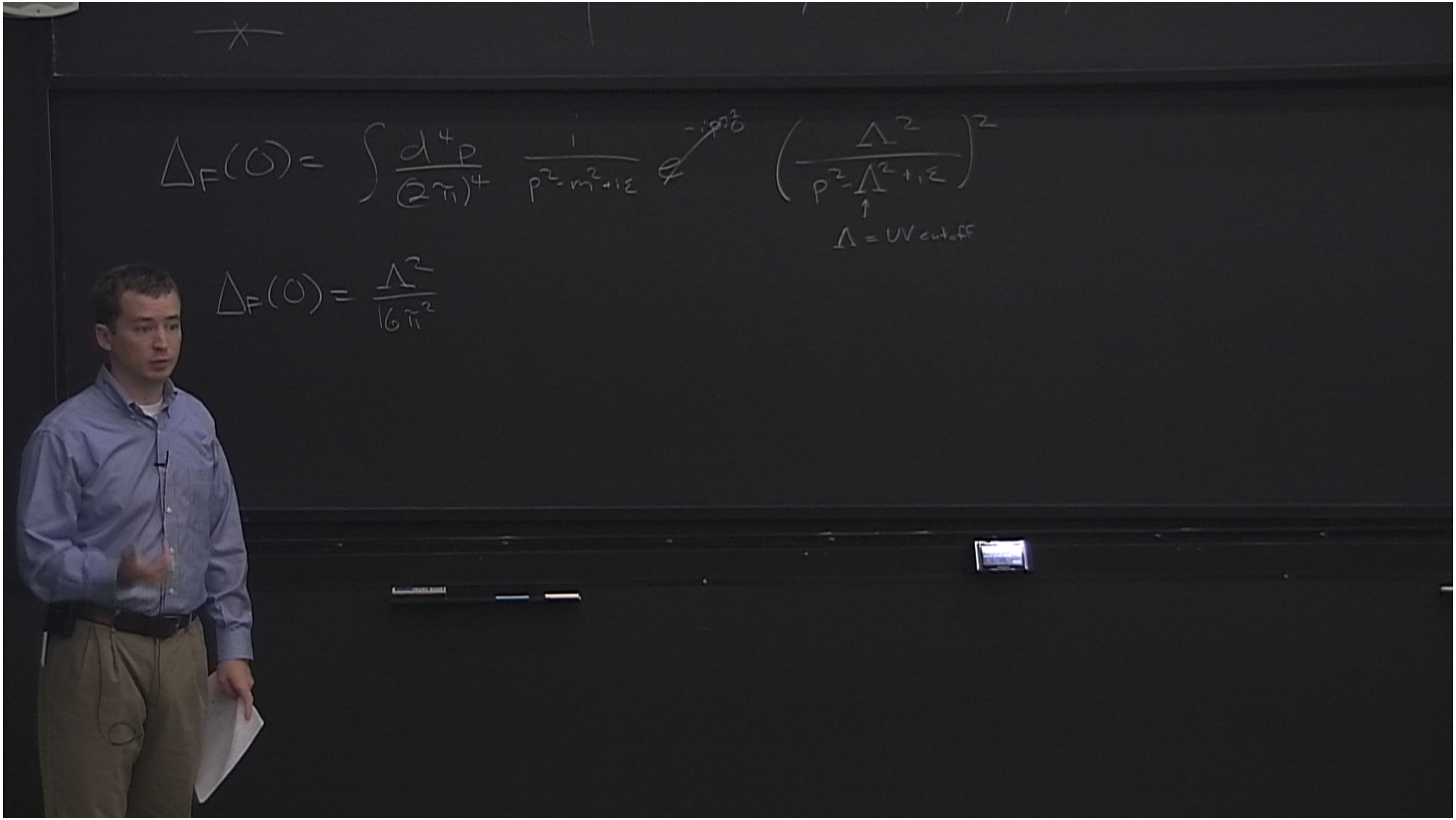

$\langle\Omega|\varphi(x)|\Omega\rangle =$
$\Theta(g)$
$\Delta_{yy} = \Delta_F(y-y)$
$= \frac{1}{2} i g \frac{Z}{g} \int d^4y \, \Delta_{xy} \Delta_{yy} + \Theta(g^2)$
$1+\Theta(g^2)$
$= iY \int d^4y \, \Delta_{xy}$
$\langle\Omega|\varphi(x)|\Omega\rangle = 0$
$= (iY$

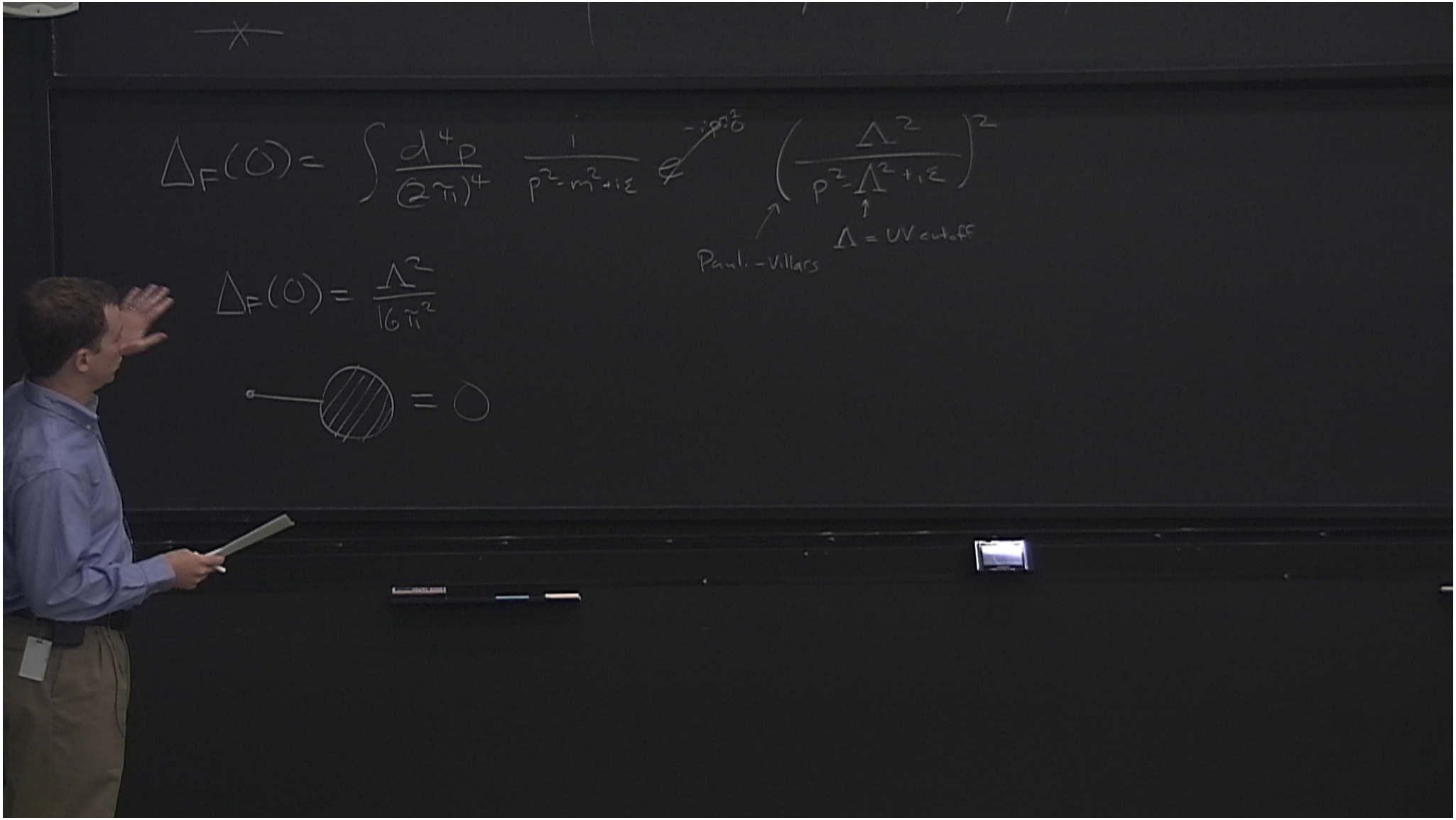

$\langle\Omega|\varphi(x)|\Omega\rangle =$
$\theta(g)$
$\Delta_{yy} = \Delta_F(y-y)$
$= \frac{1}{2} i g \not{Z}_g \int d^4y \, \Delta_{xy} \Delta_{yy} + \mathcal{O}(g^3)$
$1 + \mathcal{O}(g^2)$
$= iY \int d^4y \, \Delta_{xy}$
$\langle\Omega|\varphi(x)|\Omega\rangle = \mathcal{O}$
$= (iY$
$g \Delta_F(0))$

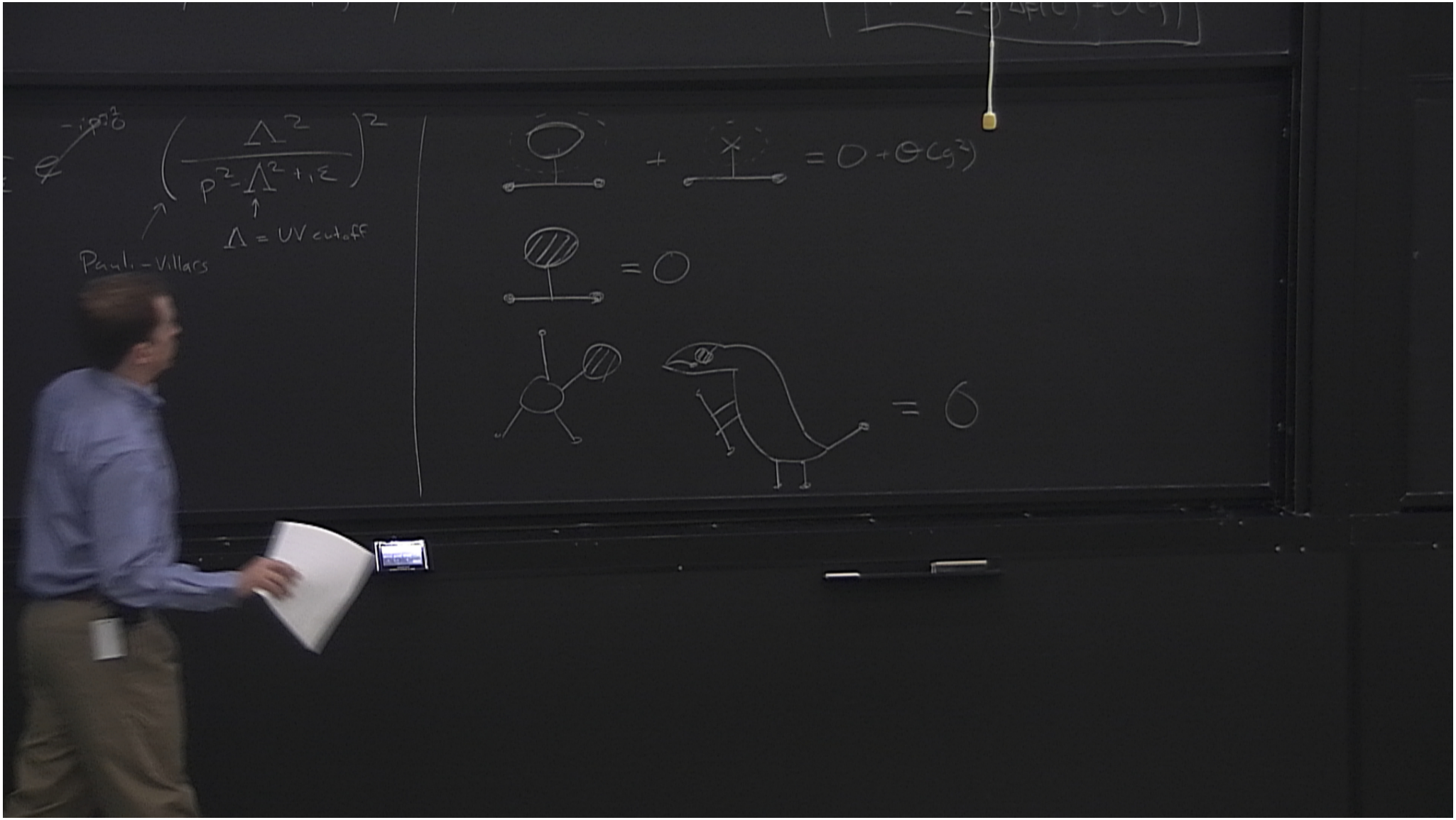

$\langle\Omega|\varphi(x)|\Omega\rangle =$
$\theta(g)$
$\Delta_{yy} = \Delta_F(y-y)$
$= \frac{1}{2} i g Z_g \int d^4y\, \Delta_{xy}\Delta_{yy} + \mathcal{O}(g^3)$
$(1+\mathcal{O}(g^2))$
$= iY \int d^4y\, \Delta_{xy}$
$\langle\Omega|\varphi(x)|\Omega\rangle = 0$
$= (iY + \frac{1}{2} i g \Delta_F(0)) \int d^4y\, \Delta_{xy}$
$Y = -\frac{1}{2} g \Delta_F(0) + \mathcal{O}(g^3)$

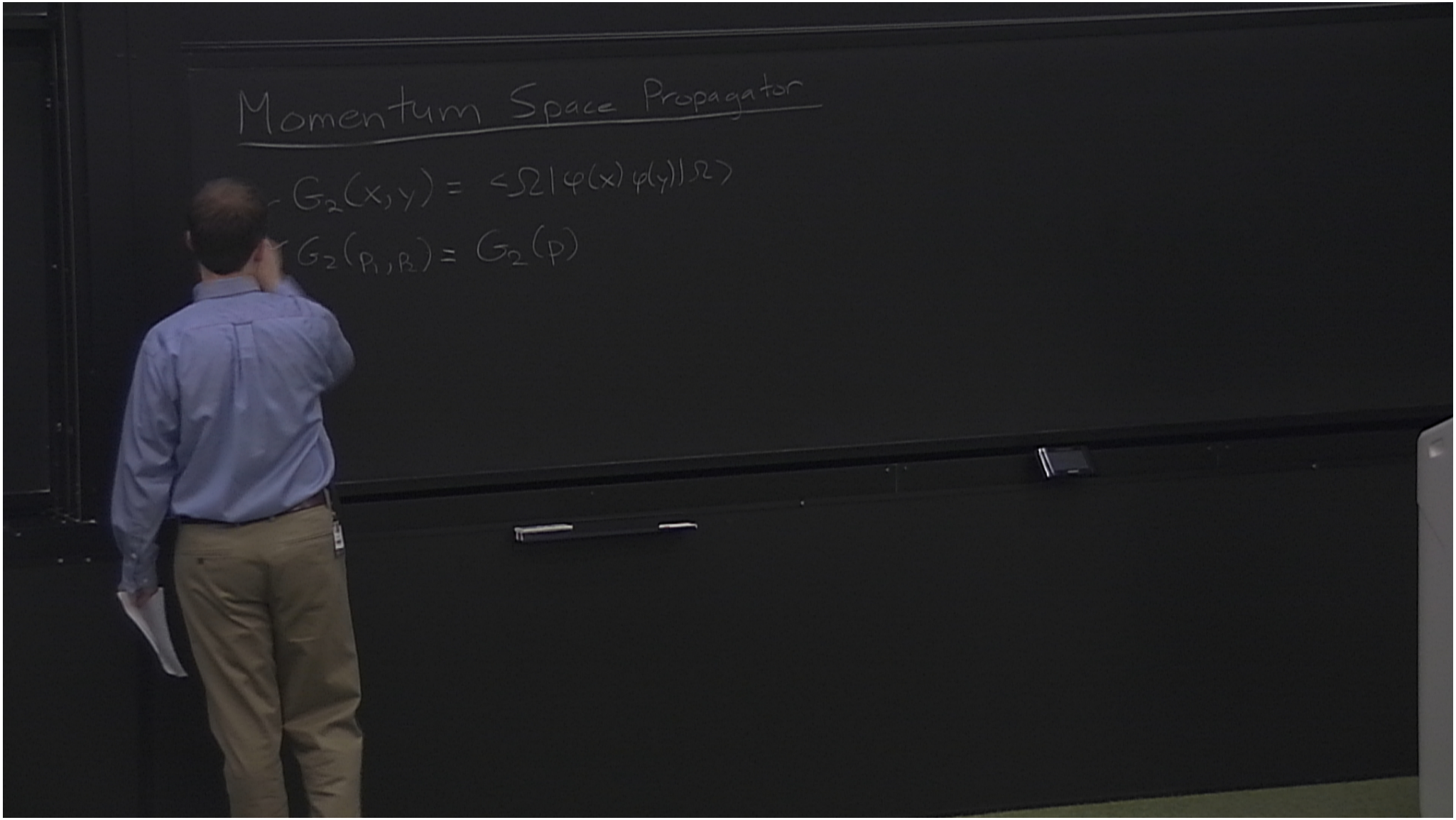

$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot 0}$

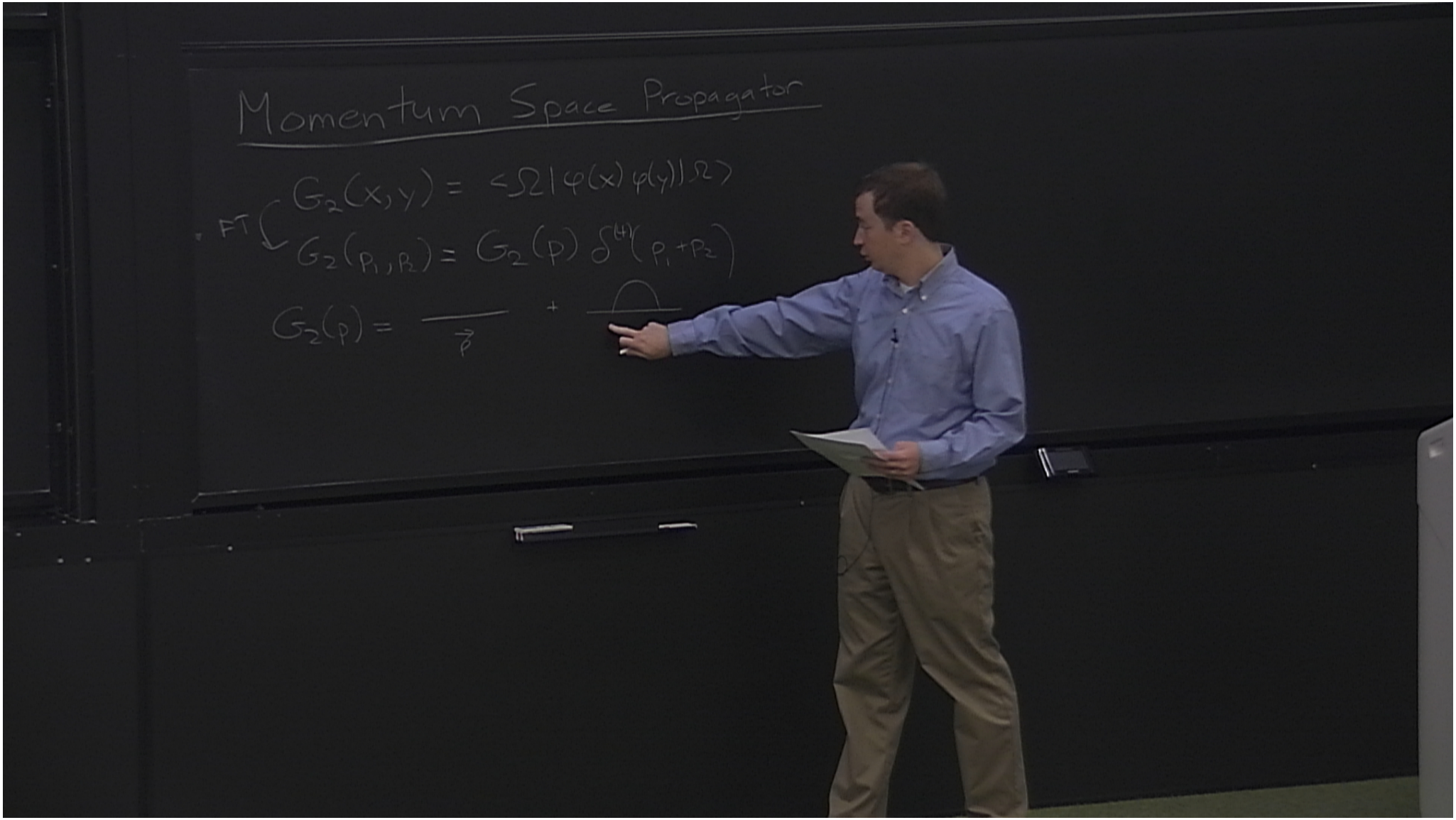

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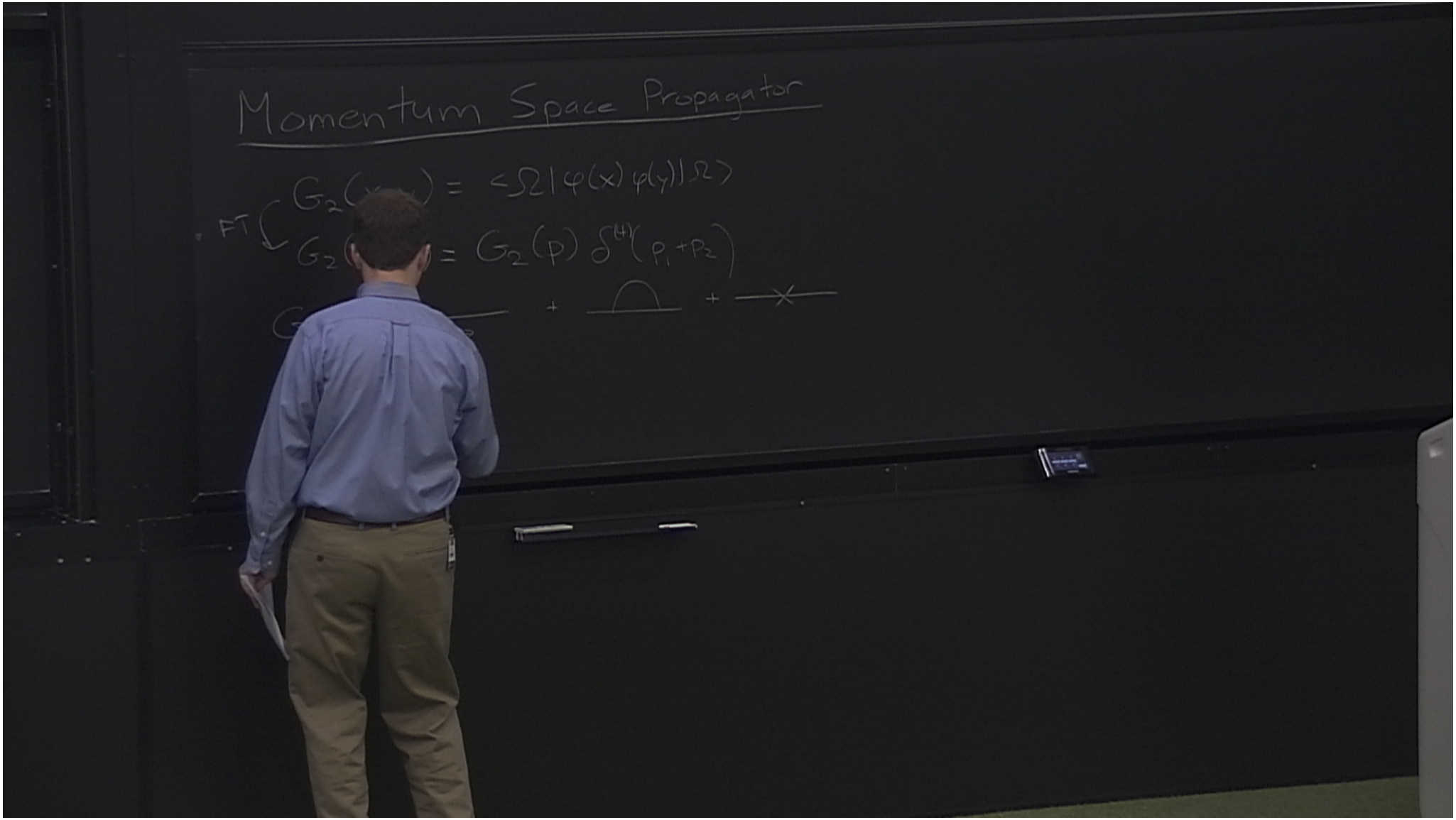

$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot 0} \left(\frac{\Lambda^2}{p^2 - \Lambda^2 + i\varepsilon}\right)^2$
Λ = UV cutoff
$\Delta_F(0) = \frac{\Lambda^2}{16\pi^2}$

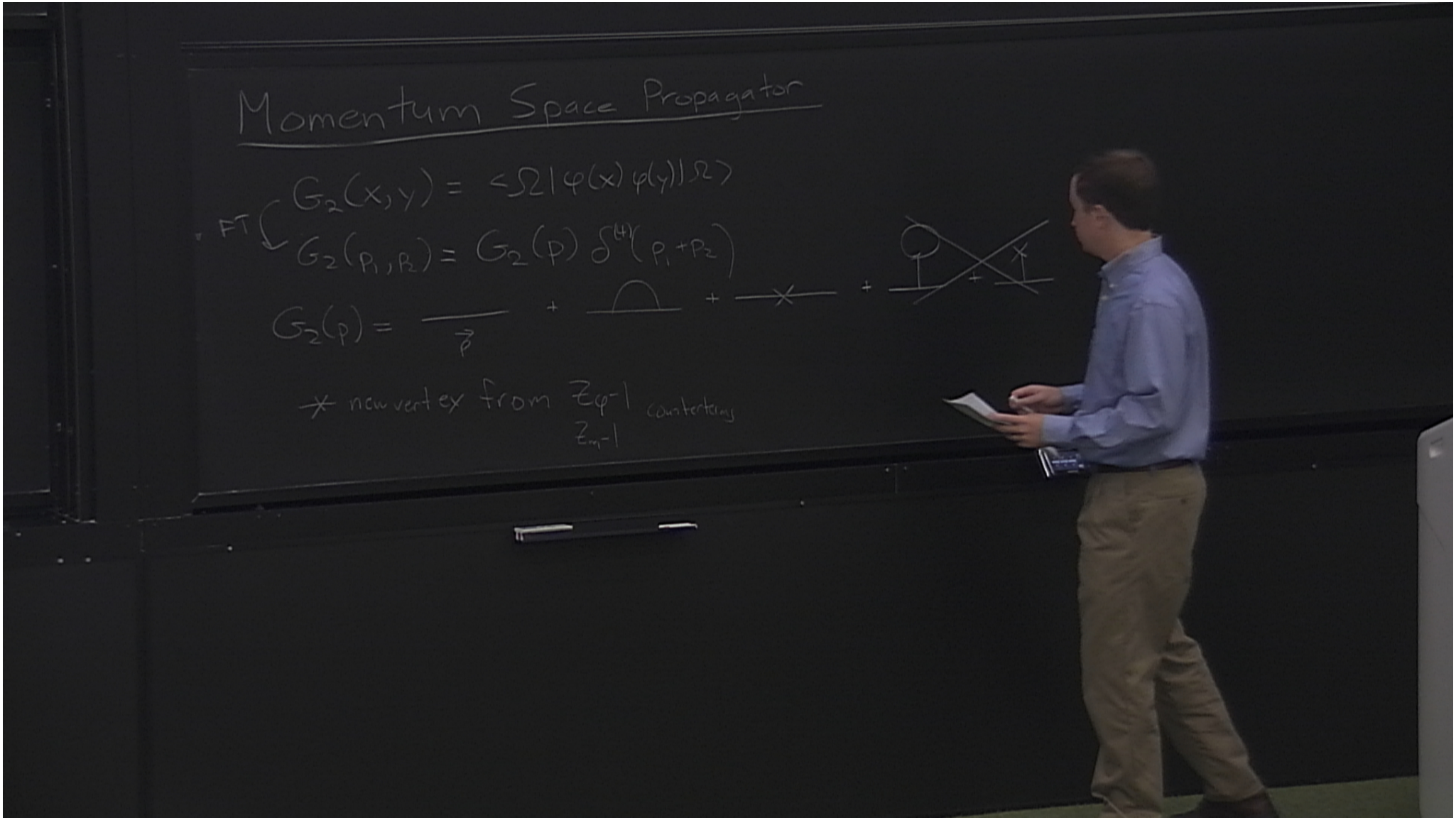

$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot 0} \left(\frac{\Lambda^2}{p^2 - \Lambda^2 + i\varepsilon} \right)^2$
Pauli-Villars
Λ = UV cutoff
$\Delta_F(0) = \frac{\Lambda^2}{16\pi^2}$
$= 0$

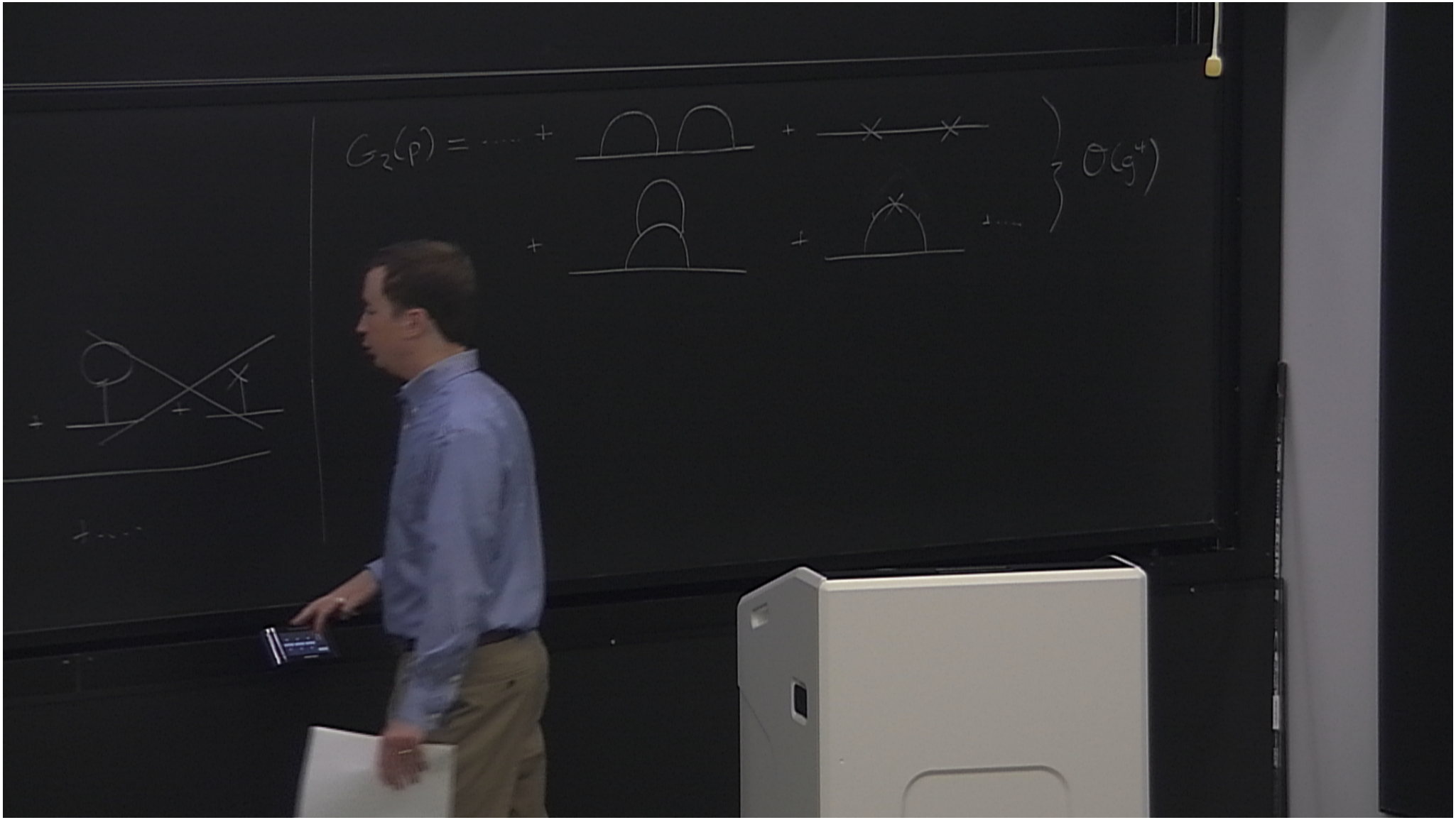

$\left(\frac{\Lambda^2}{p^2-\Lambda^2+i\varepsilon}\right)^2$
Pauli-Villars
Λ = UV cutoff
$= 0 + O(g^4)$
$= 0$
$= 0$

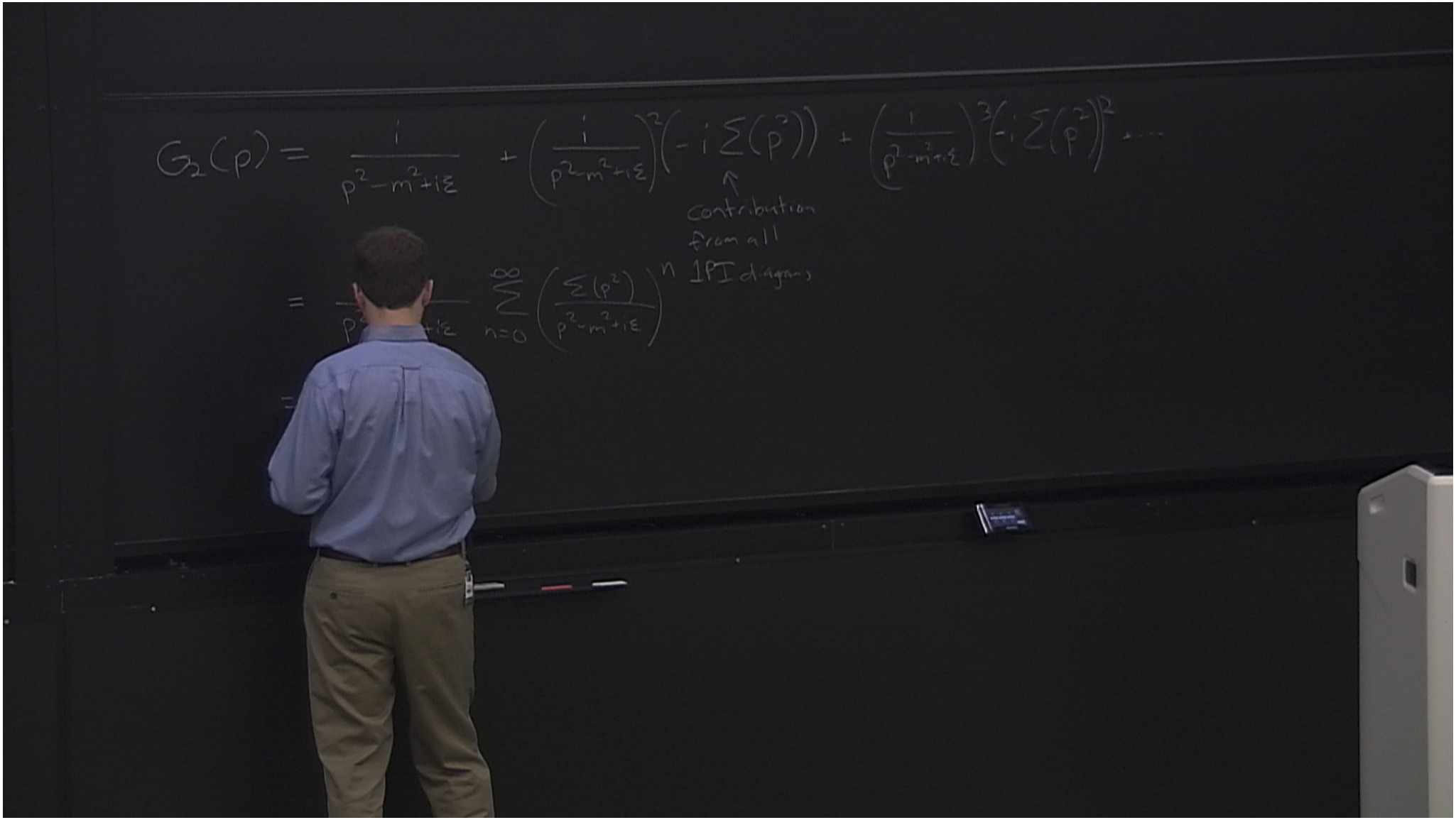

Momentum Space Propagator
$G_2(x,y) = \langle \Omega | \varphi(x)\varphi(y) | \Omega \rangle$
$G_2(p_1,p_2) = G_2(p)$

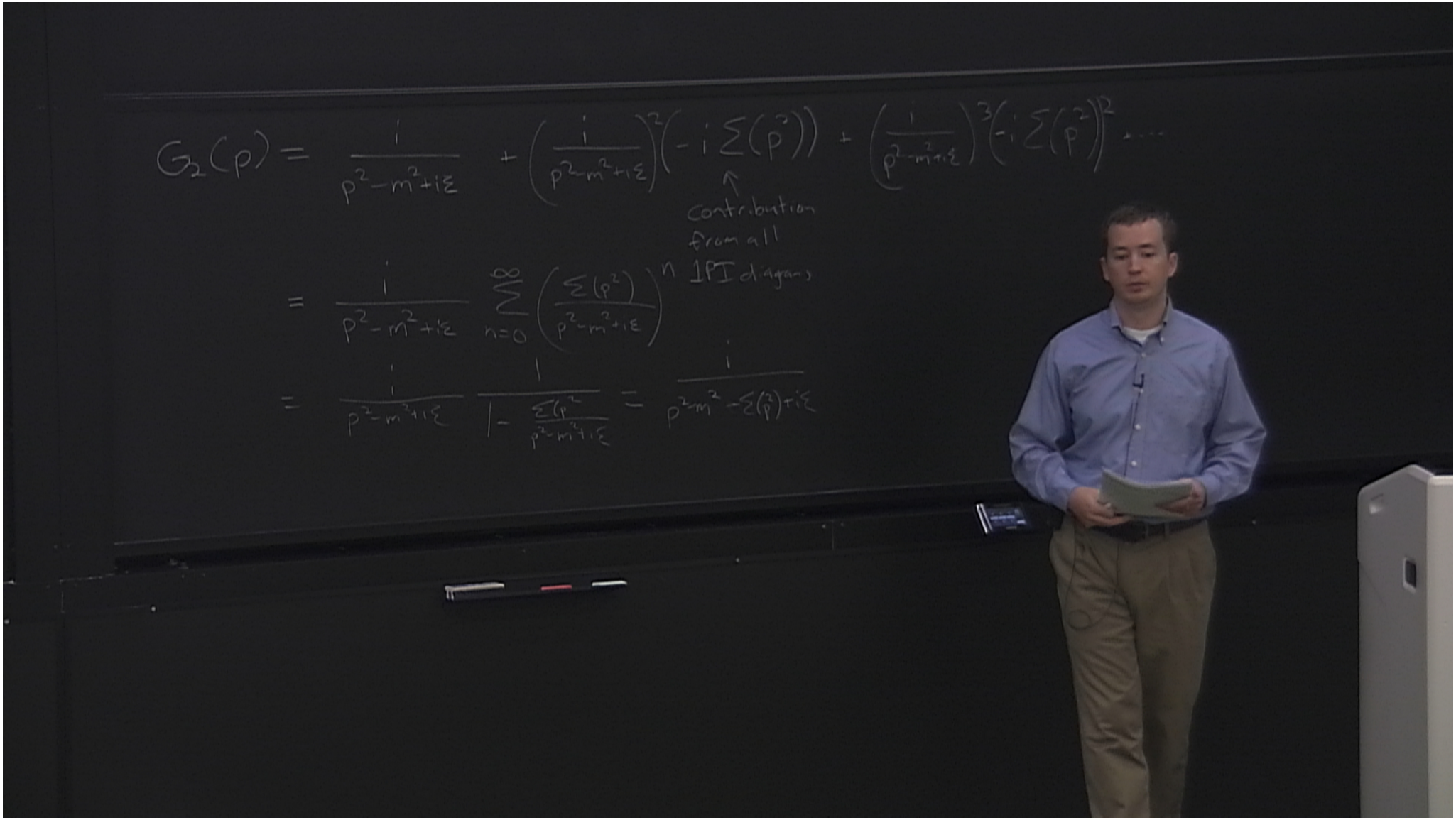

Momentum Space Propagator
FT
$G_2(x,y) = \langle \Omega | \varphi(x)\varphi(y) | \Omega \rangle$
$G_2(p_1,p_2) = G_2(p)\,\delta^{(4)}(p_1+p_2)$
$G_2(p) =$
$\vec{p}$
+

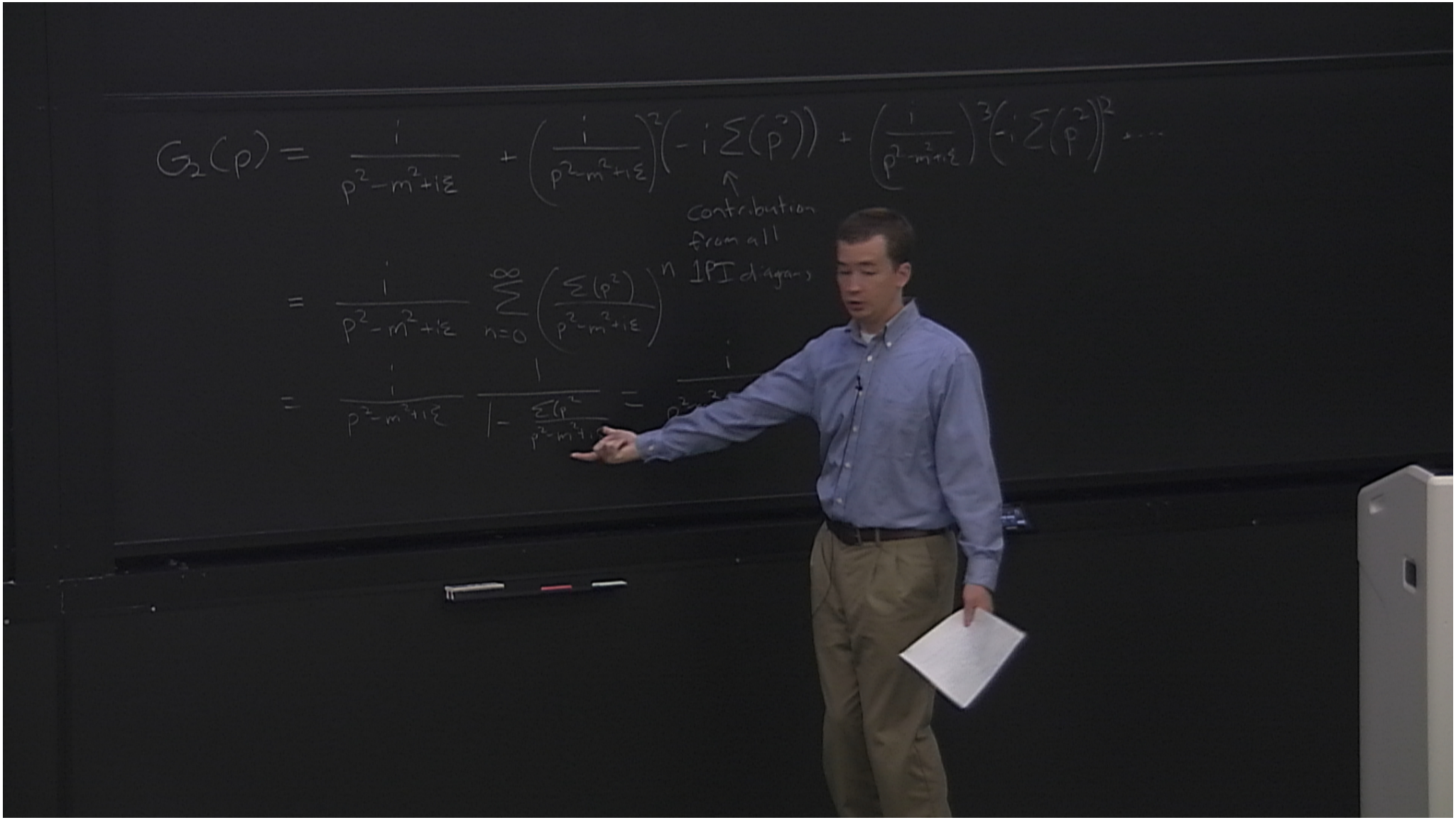

Momentum Space Propagator
$G_2(x-y) = \langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle$
FT
$G_2(p)\,\delta^{(4)}(p_1+p_2)$

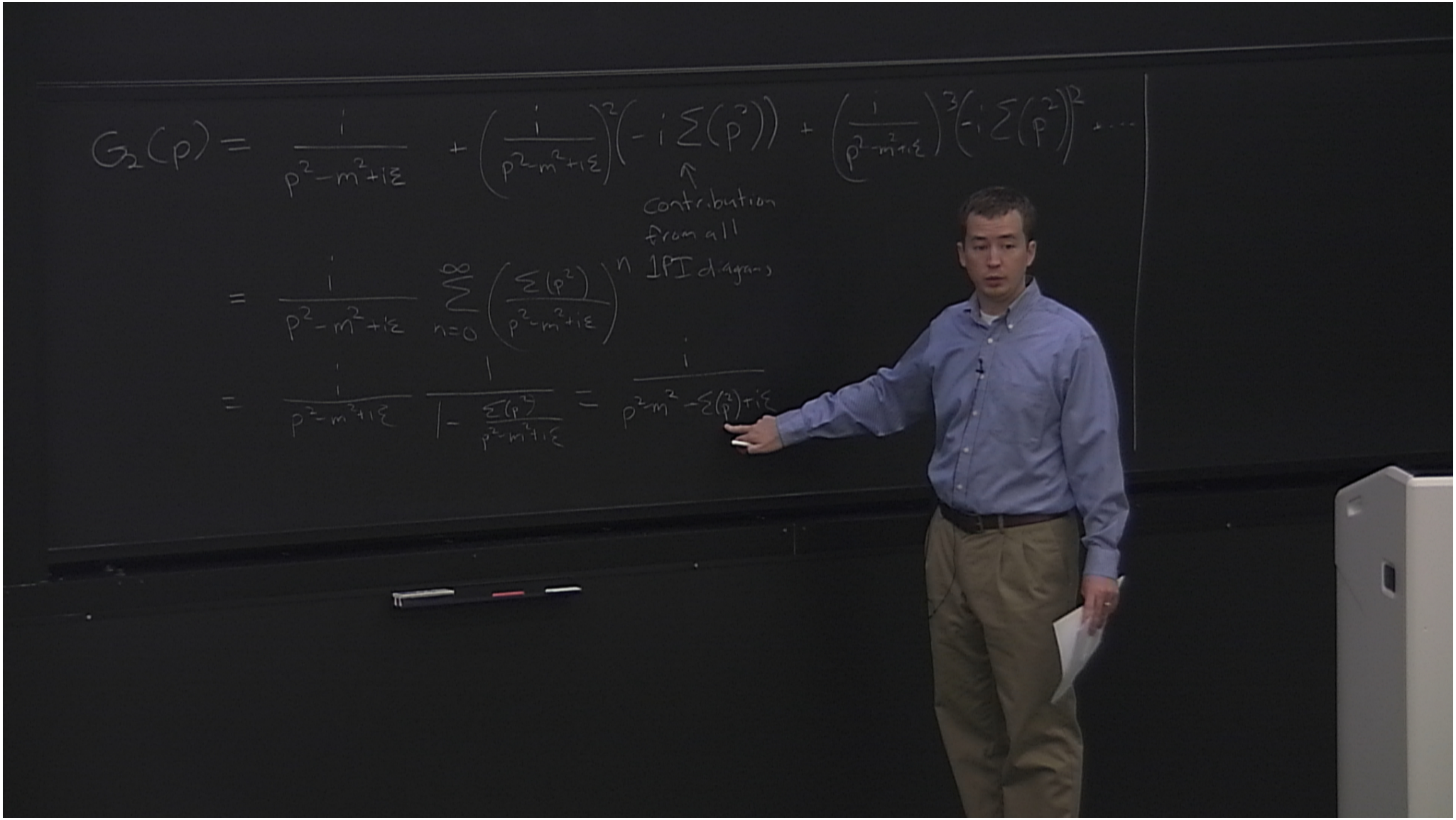

Momentum Space Propagator
$G_2(x,y) = \langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle$
FT
$G_2(p_1,p_2) = G_2(p)\,\delta^{(4)}(p_1+p_2)$
$G_2(p) =$
$\vec{p}$
+
+
+
* new vertex from $Z_\varphi - 1$ counterterms
$Z_{m^2}-1$

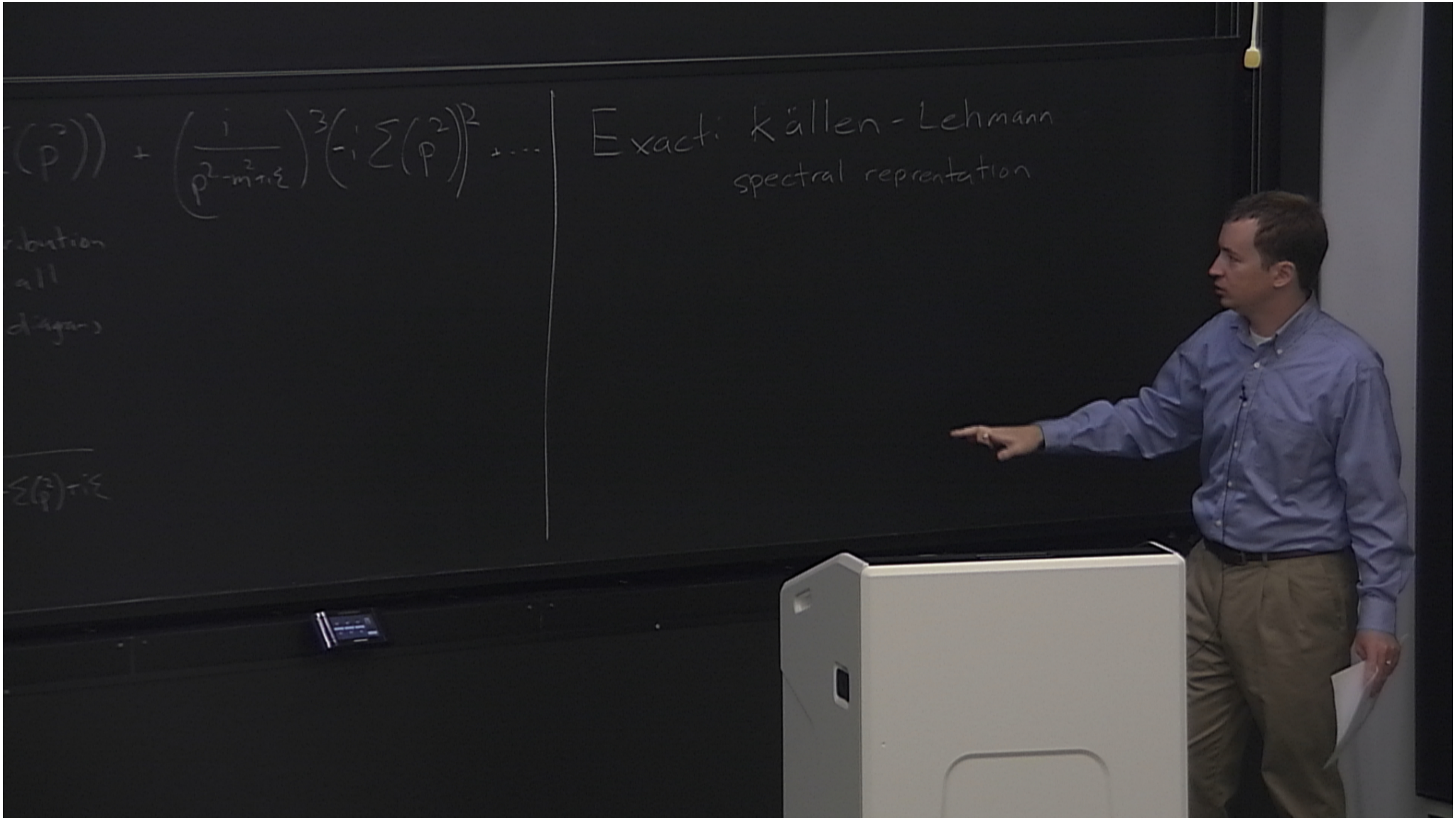

$G_2(p) = \cdots +$
$+ \cdots \Big\} O(g^4)$

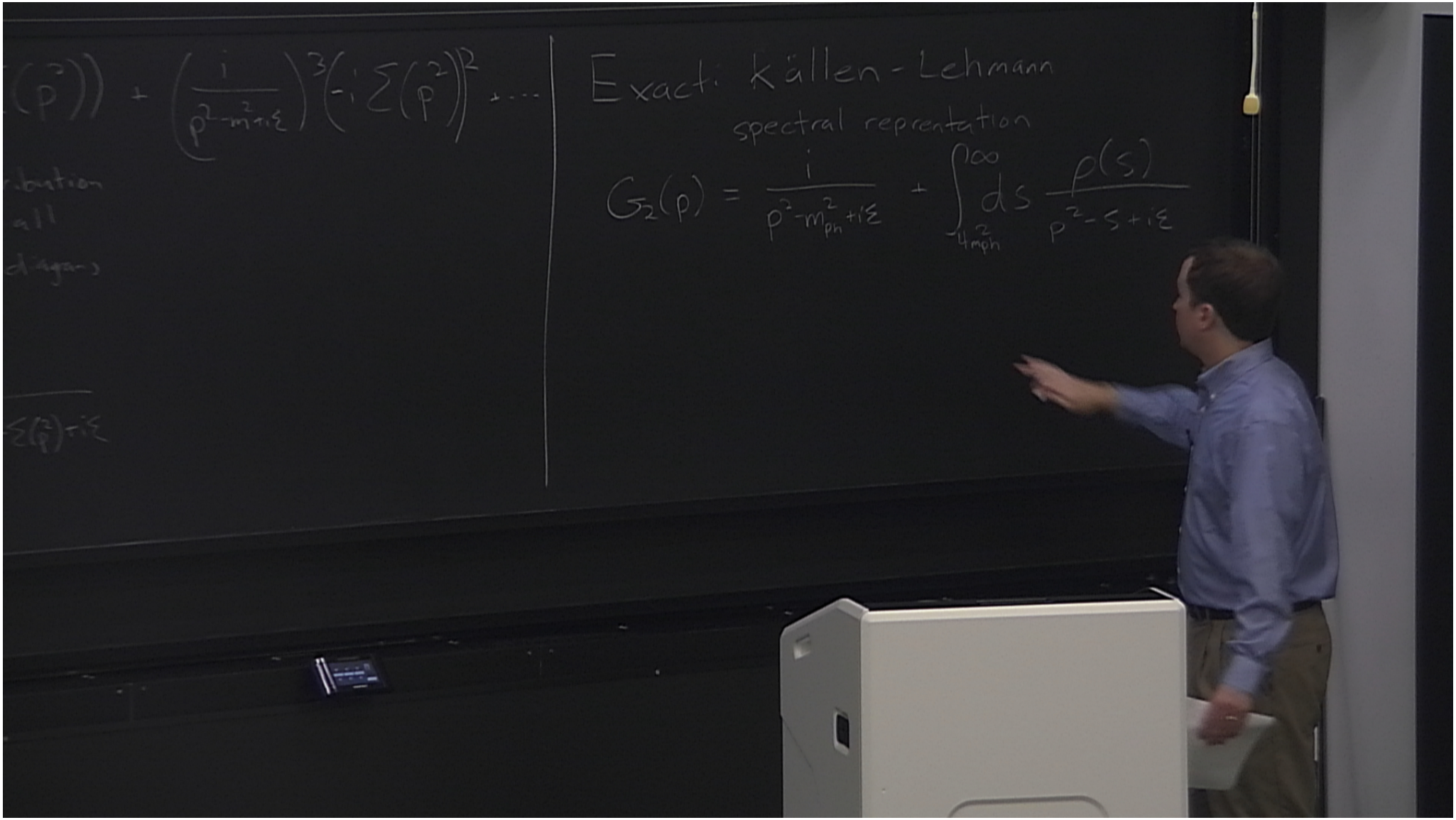

$G_2(p) = \frac{i}{p^2 - m^2 + i\varepsilon} + \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^2 (-i\Sigma(p^2)) + \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^3 (-i\Sigma(p^2))^2 + \cdots$
Contribution from all 1PI diagrams
$= \sum_{n=0}^{\infty} \left(\frac{\Sigma(p^2)}{p^2 - m^2 + i\varepsilon}\right)^n$

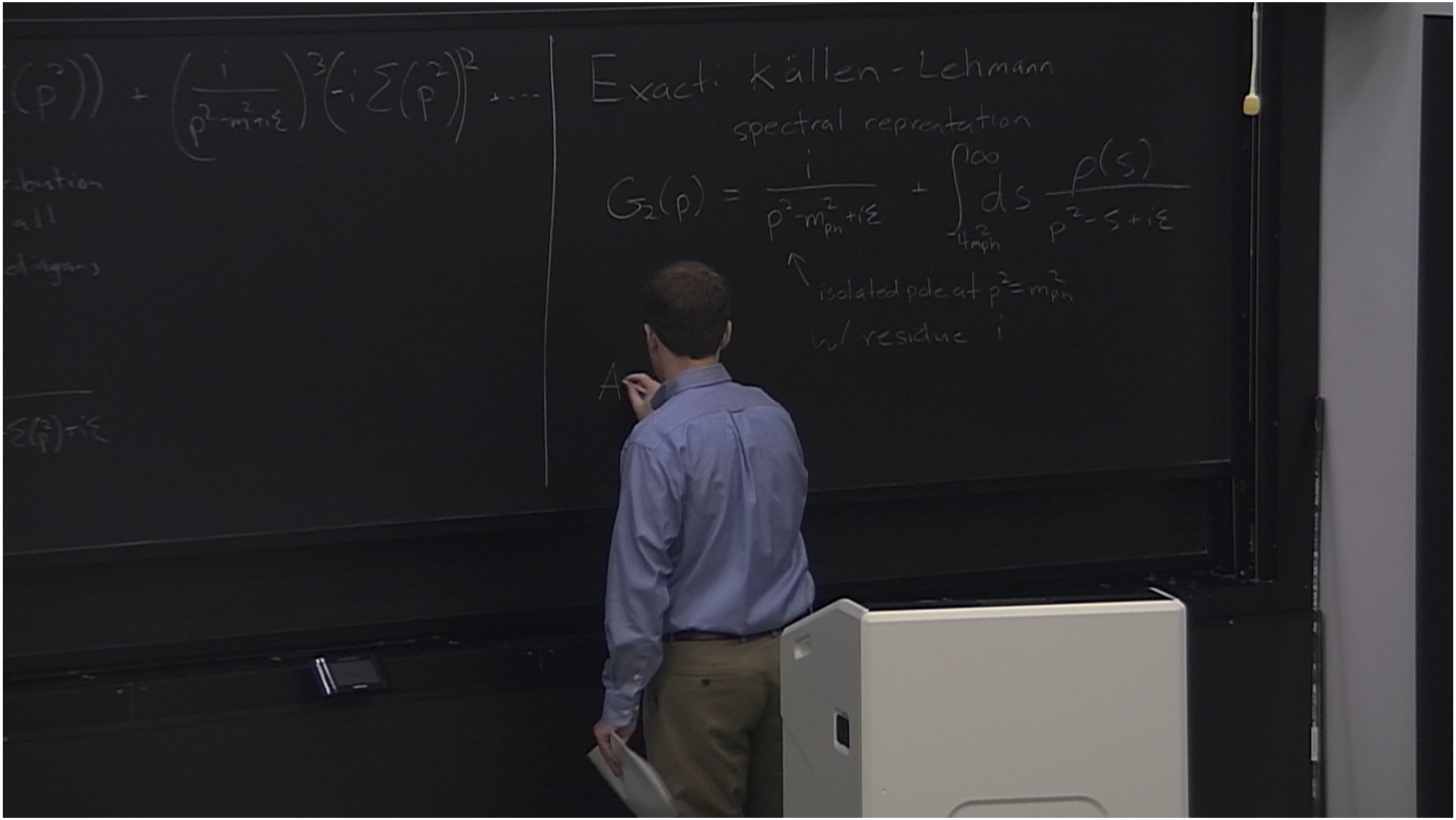

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↑ Contribution from all 1PI diagrams
$= \frac{i}{p^2 - m^2 + i\varepsilon} \sum_{n=0}^{\infty} \left(\frac{\Sigma(p^2)}{p^2 - m^2 + i\varepsilon}\right)^n$
$= \frac{i}{p^2 - m^2 + i\varepsilon} \frac{1}{1 - \frac{\Sigma(p^2)}{p^2 - m^2 + i\varepsilon}} = \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\varepsilon}$

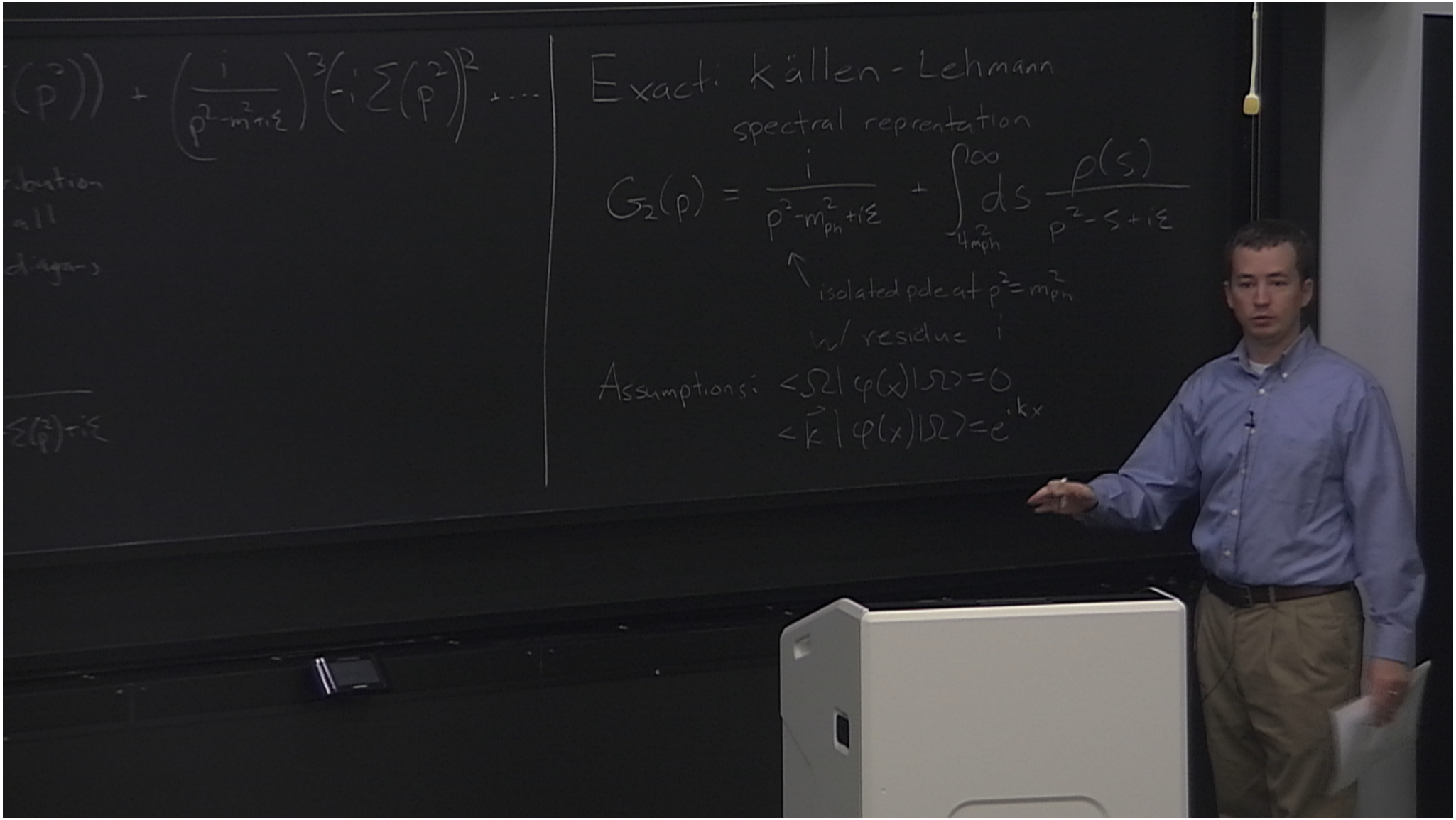

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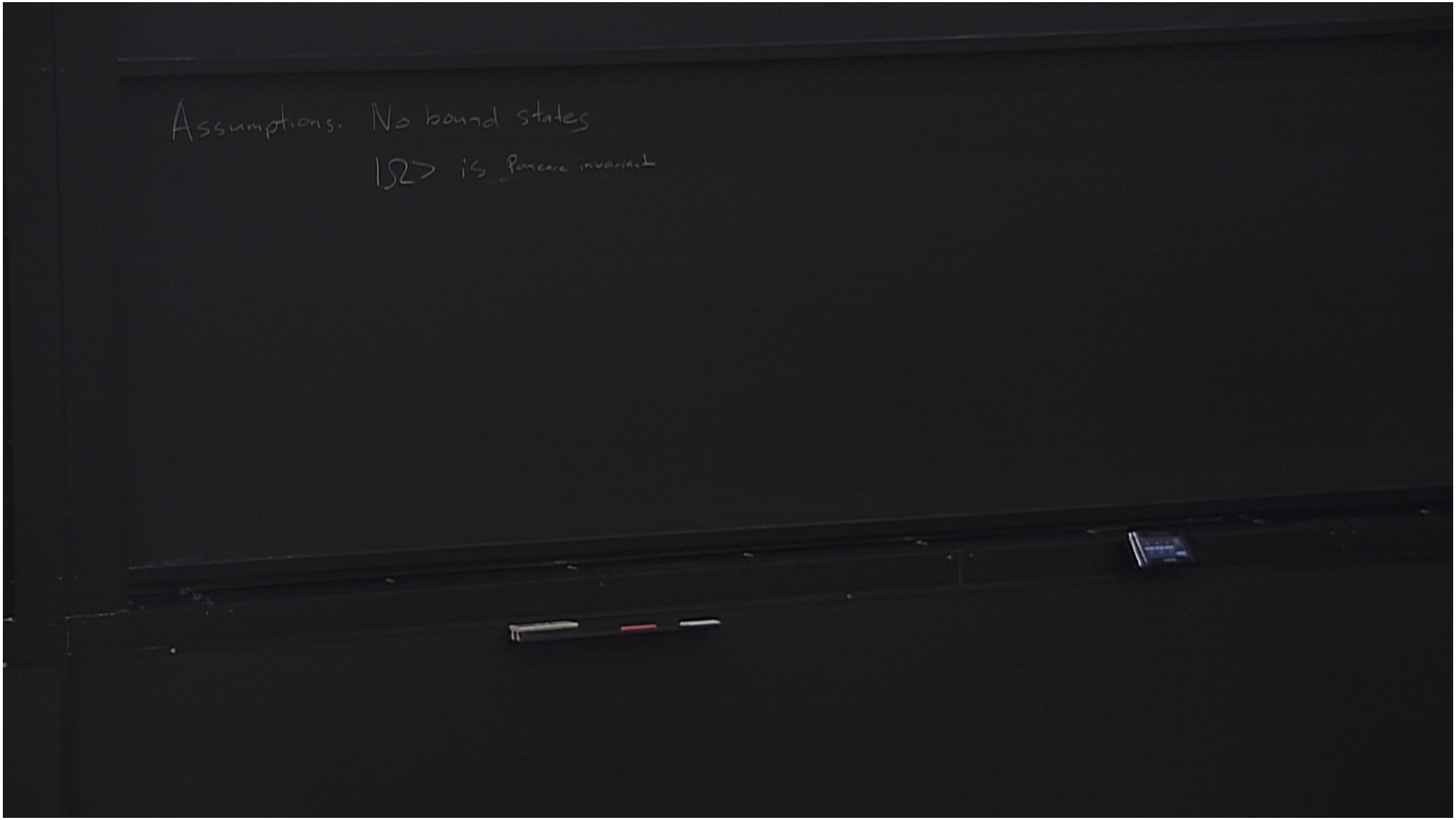

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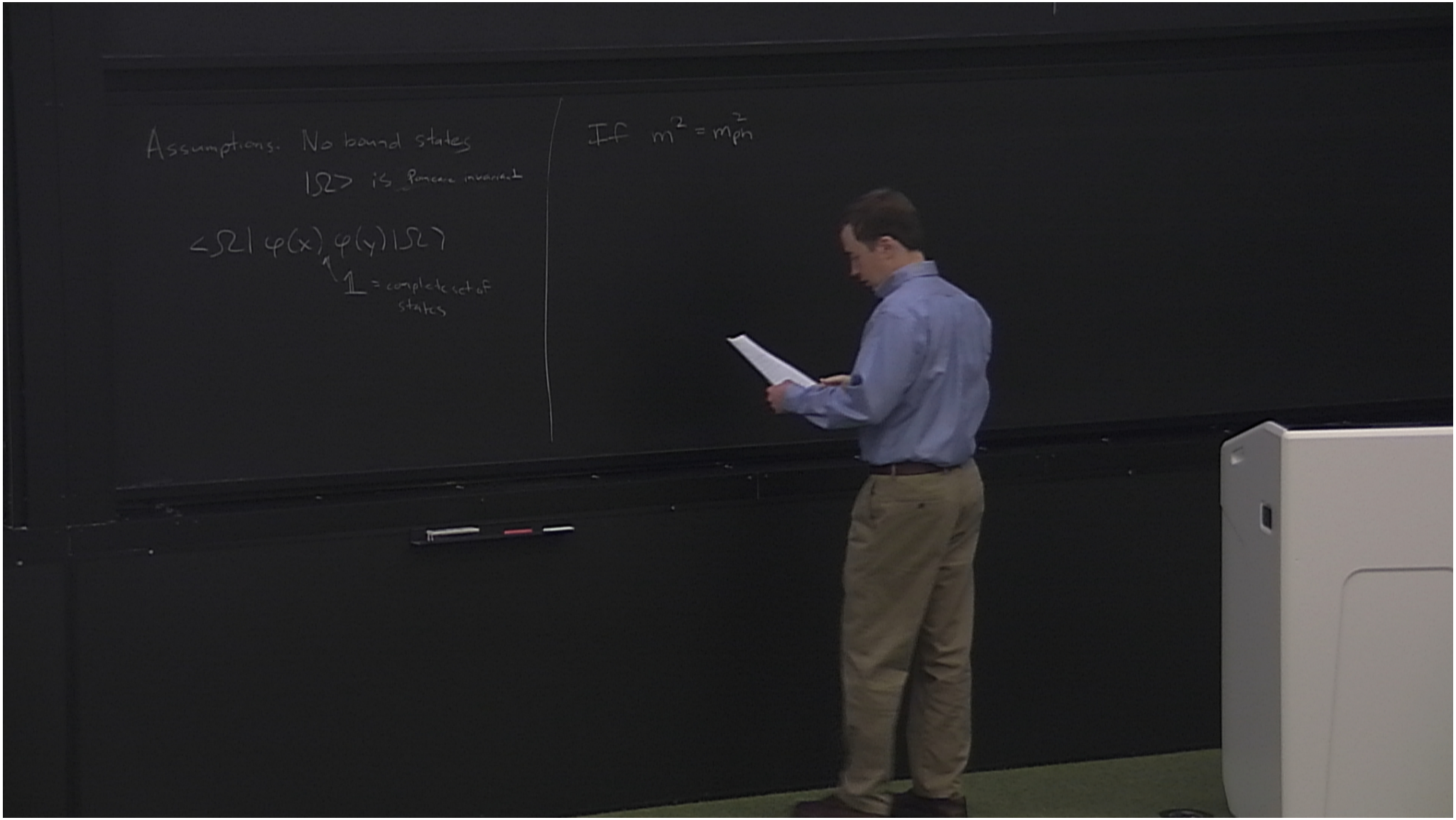

Exact: Källen-Lehmann
spectral representation

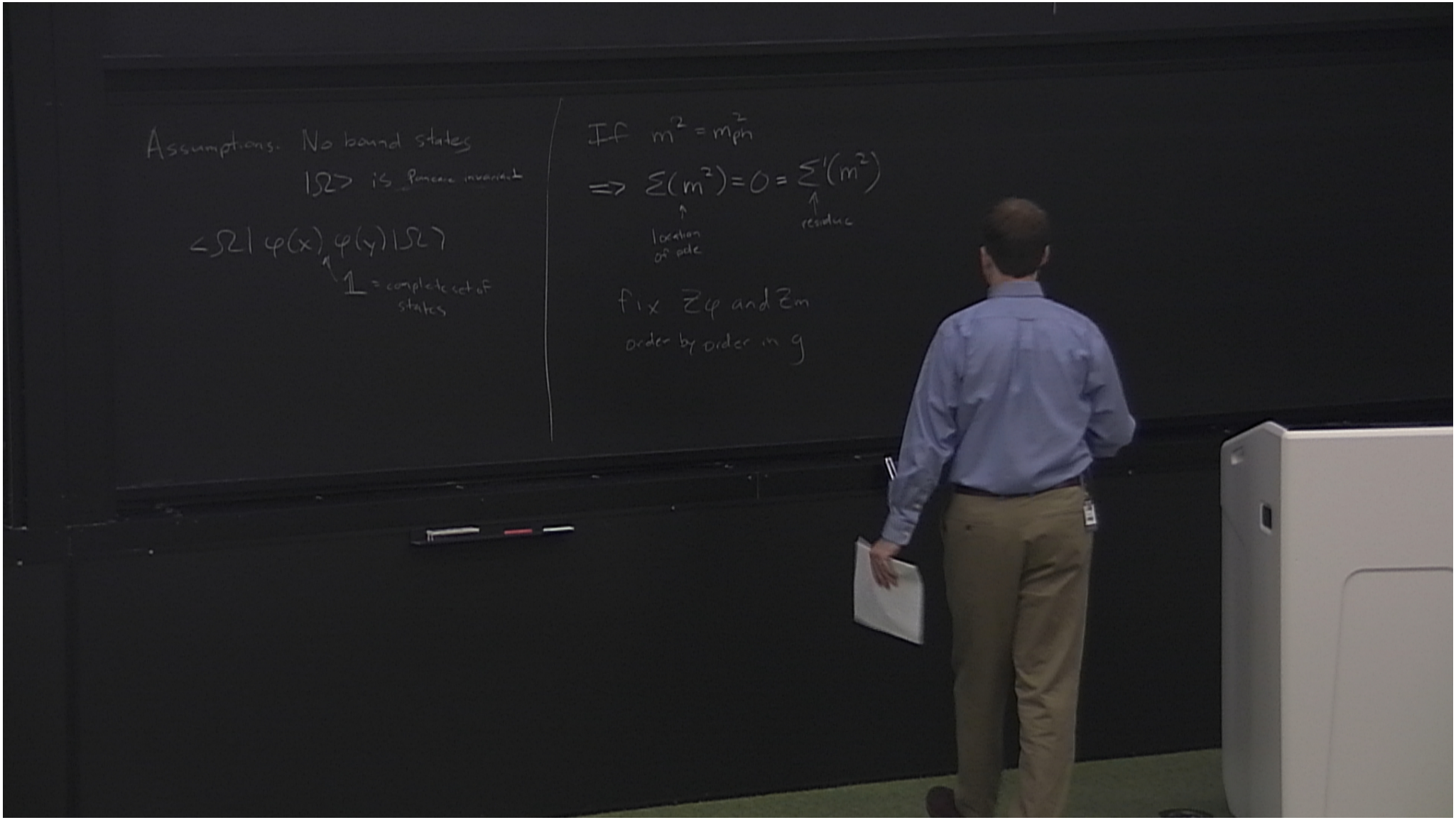

Exact: Källen-Lehmann
spectral representation
$G_2(p) = \frac{i}{p^2 - m_{ph}^2 + i\varepsilon} + \int_{4m_{ph}^2}^{\infty} ds \frac{\rho(s)}{p^2 - s + i\varepsilon}$

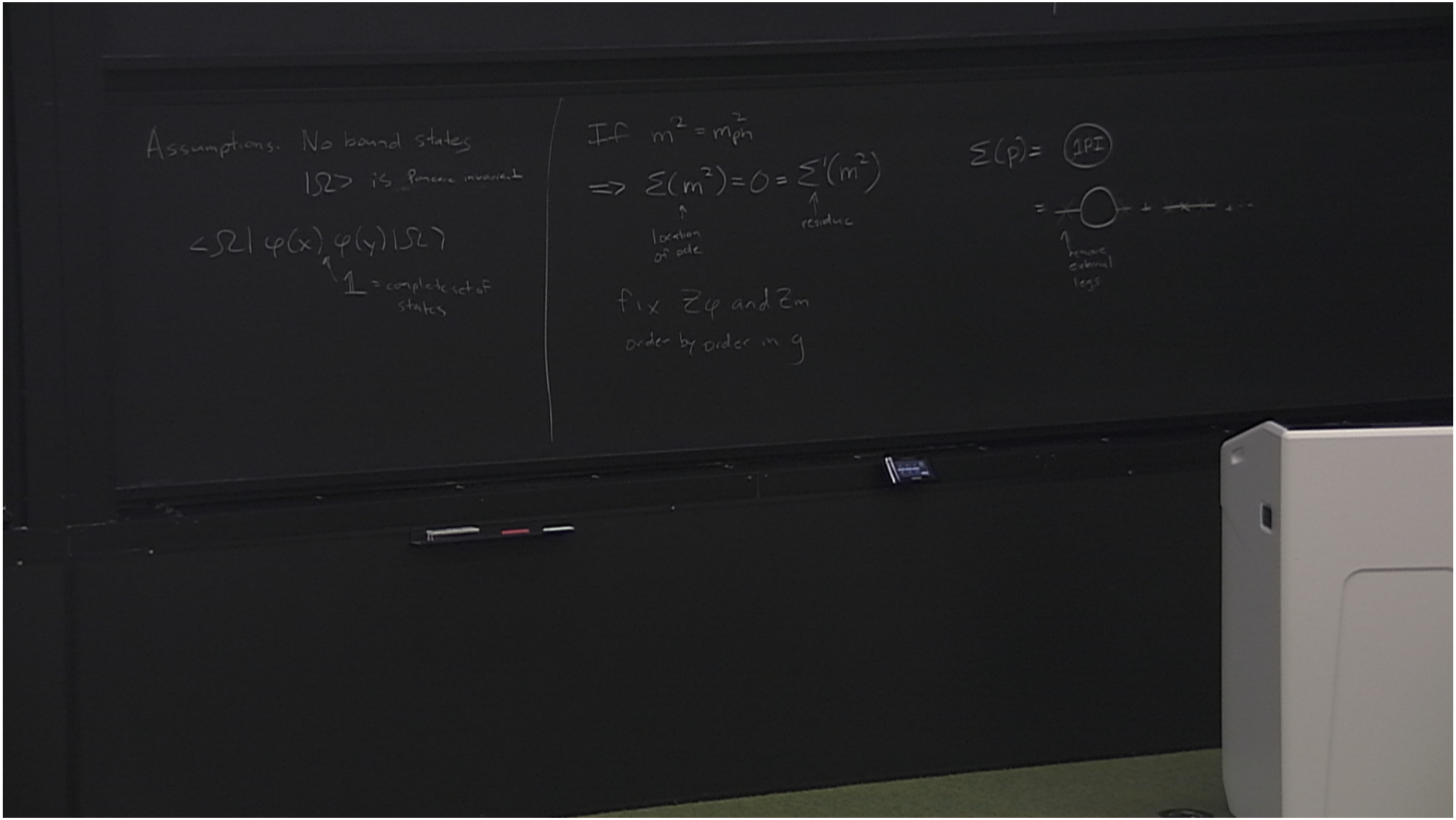

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isolated pole at $p^2 = m_{ph}^2$
w/ residue i

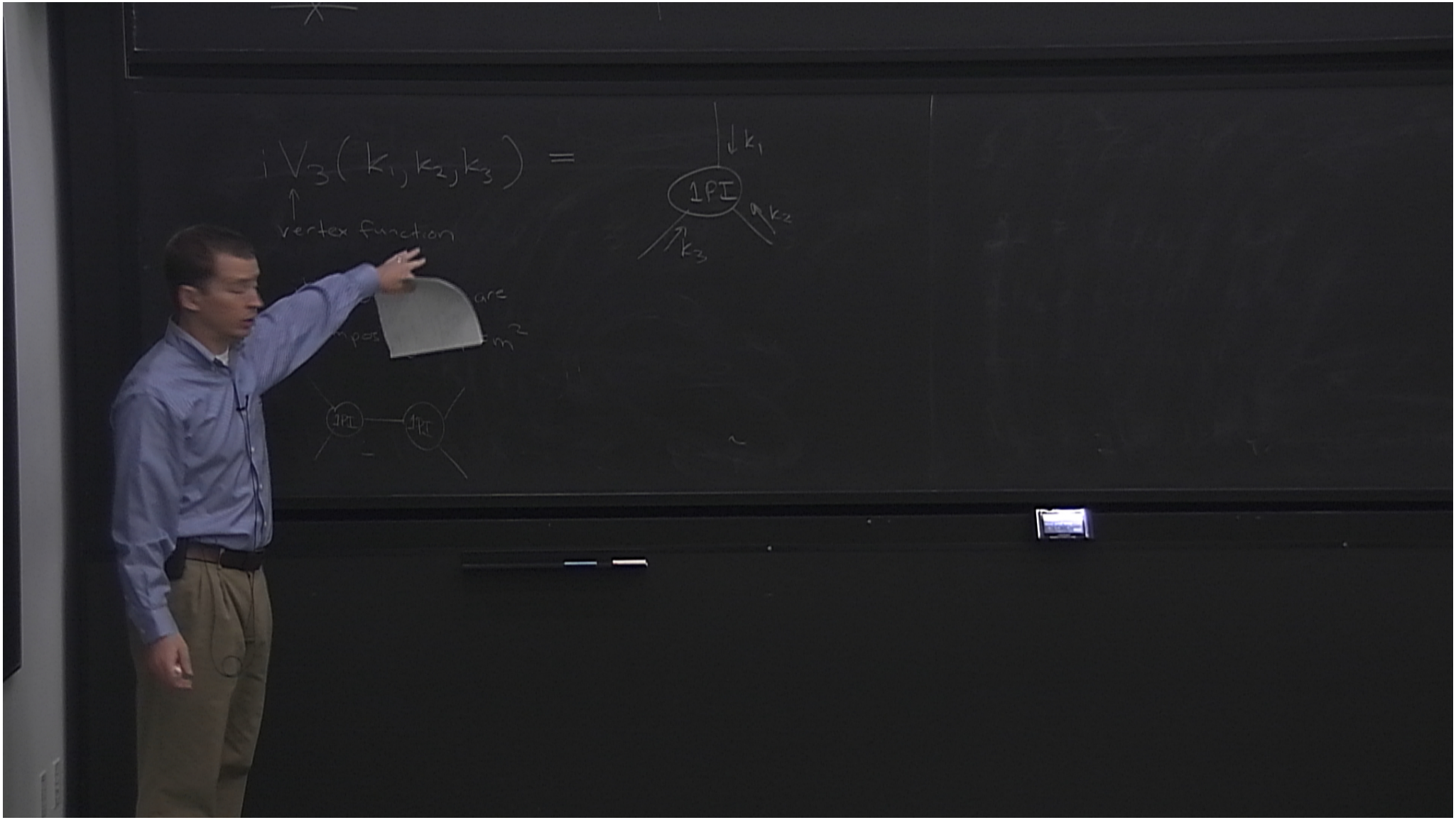

Exact: Källen-Lehmann
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$G_2(p) = \frac{i}{p^2 - m_{ph}^2 + i\varepsilon} + \int_{4m_{ph}^2}^{\infty} ds \frac{\rho(s)}{p^2 - s + i\varepsilon}$
isolated pole at $p^2 = m_{ph}^2$
w/ residue 1
Assumptions: $\langle \Omega | \varphi(x) | \Omega \rangle = 0$
$\langle \vec{k} | \varphi(x) | \Omega \rangle = e^{ikx}$

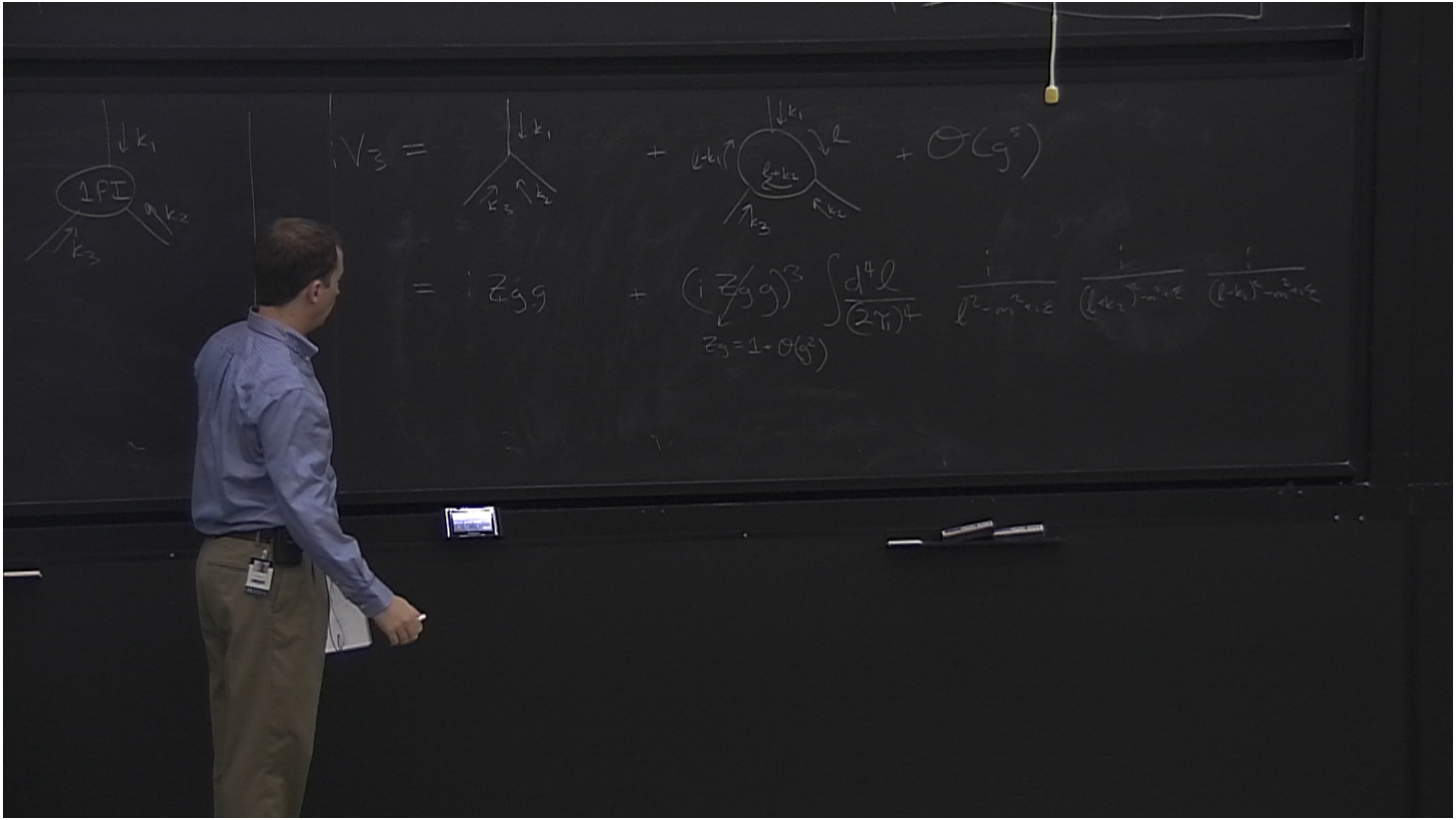

Assumptions. No bound states
|Ω> is Poincare invariant

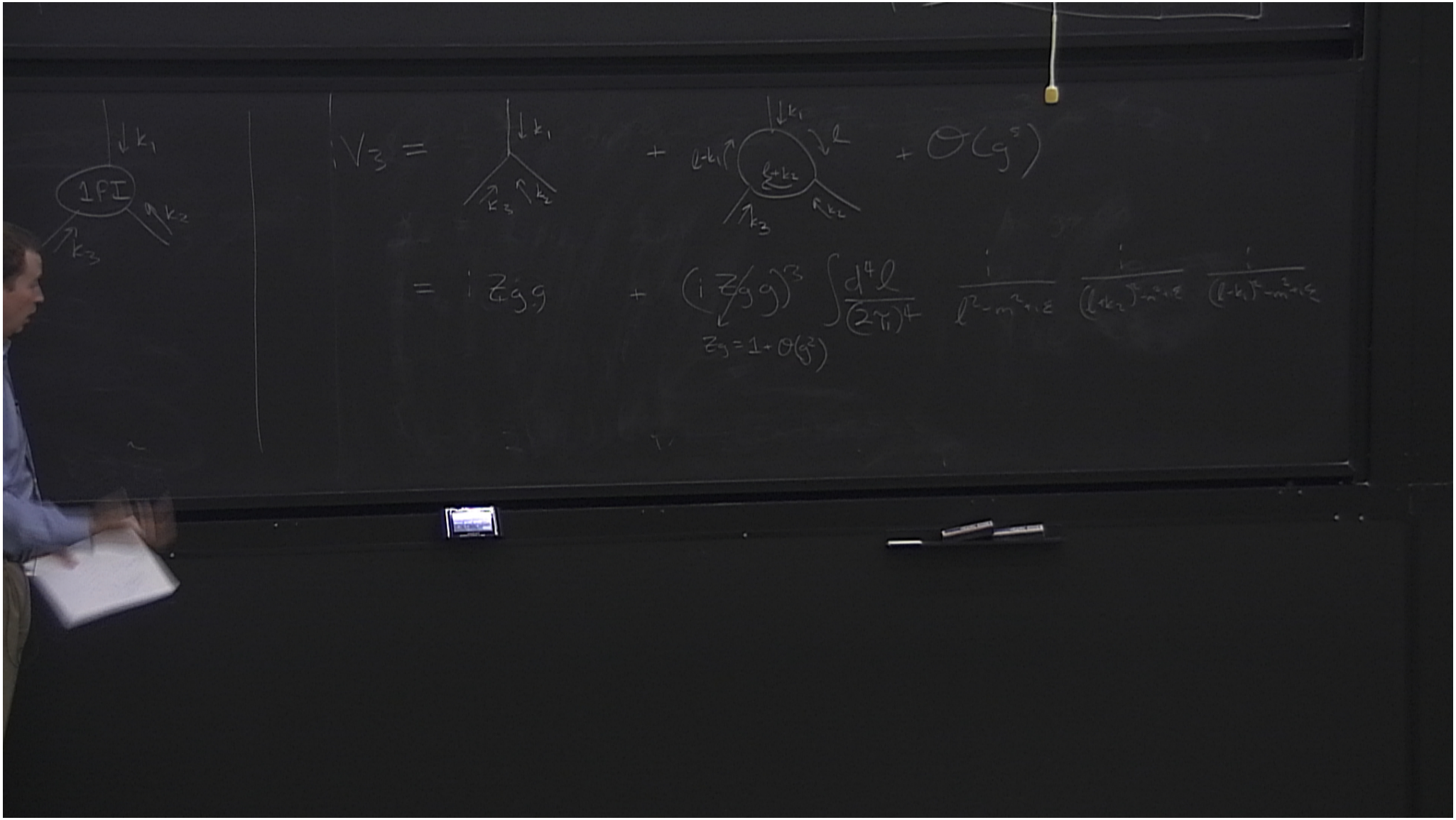

Assumptions. No bound states
|Ω> is Poincare invariant
<Ω| φ(x) φ(y) |Ω>
1 = complete set of states
If $m^2 = m_{ph}^2$

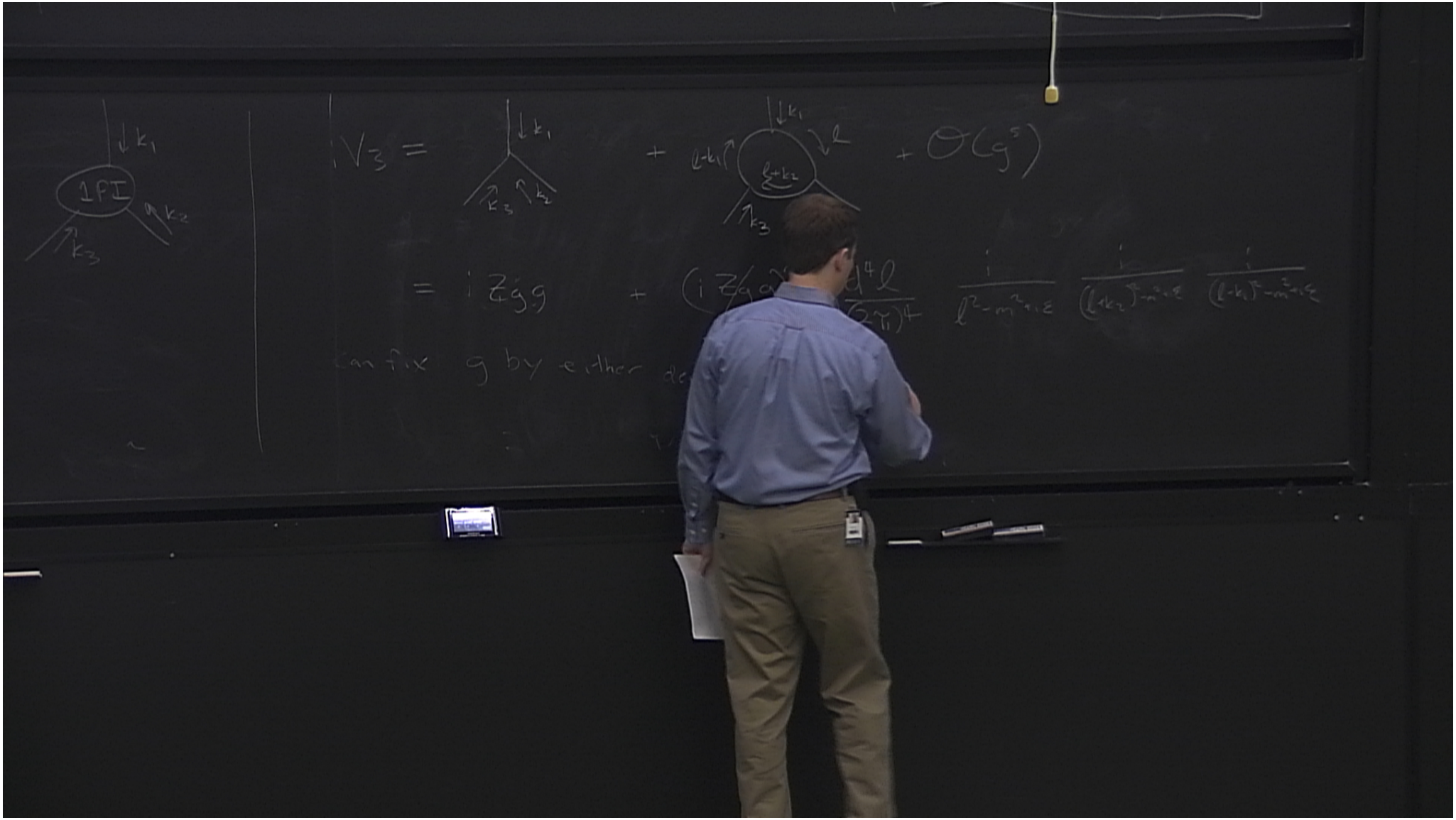

Assumptions. No bound states
|Ω> is Poincare invariant
<Ω| φ(x) φ(y) |Ω>
1 = complete set of states
If $m^2 = m_{ph}^2$
$\Rightarrow \Sigma(m^2) = 0 = \Sigma'(m^2)$
location of pole
residue
fix Z_φ and Z_m
order by order in g

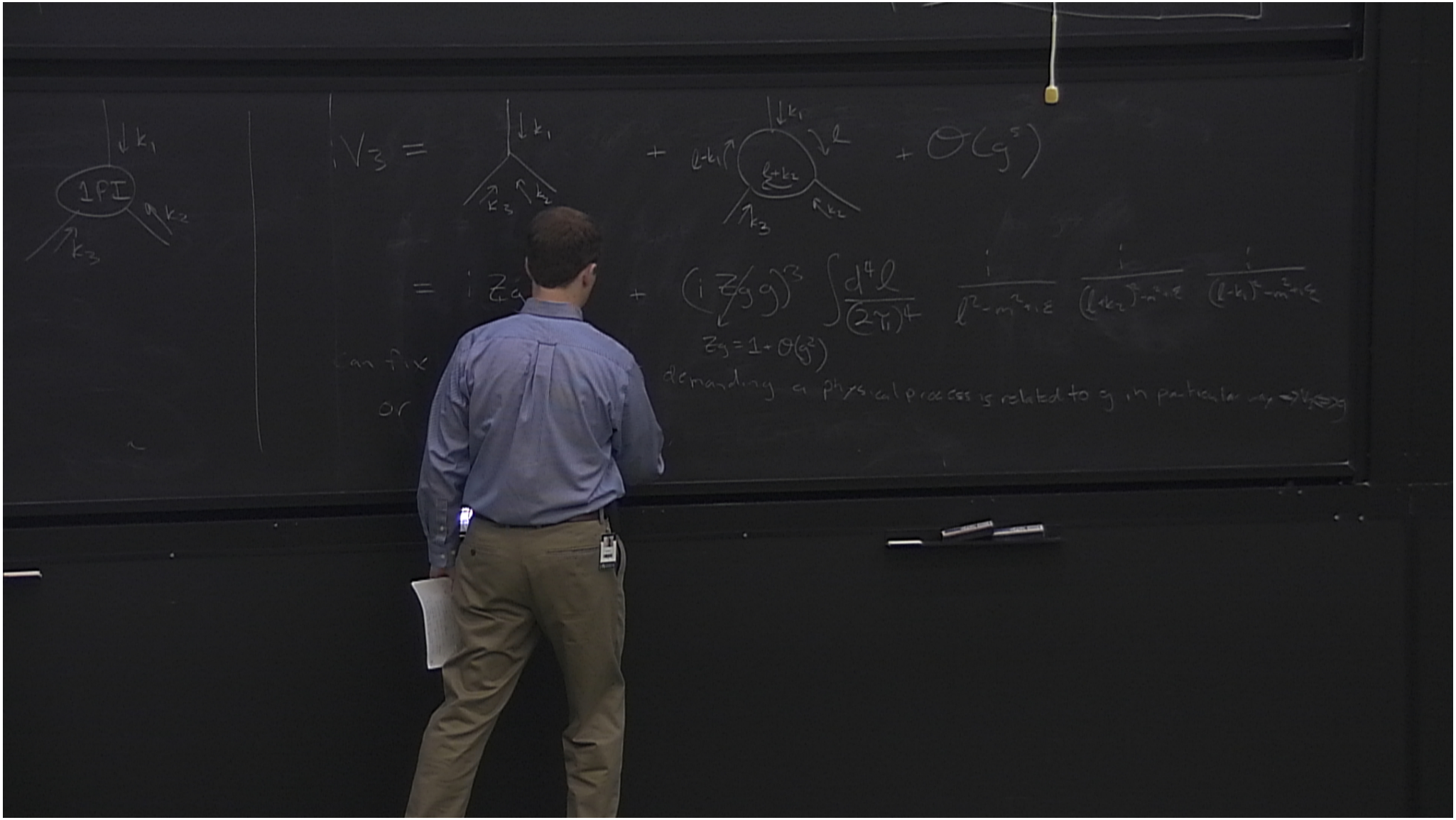

Assumptions. No bound states
|Ω> is Poincare invariant
<Ω| φ(x) φ(y) |Ω>
1 = complete set of states
If m² = m²ph
⇒ Σ(m²) = 0 = Σ'(m²)
location of pole
residue
fix Zφ and Zm
order by order in g
Σ(p) = 1PI
= + + ...
remove external legs

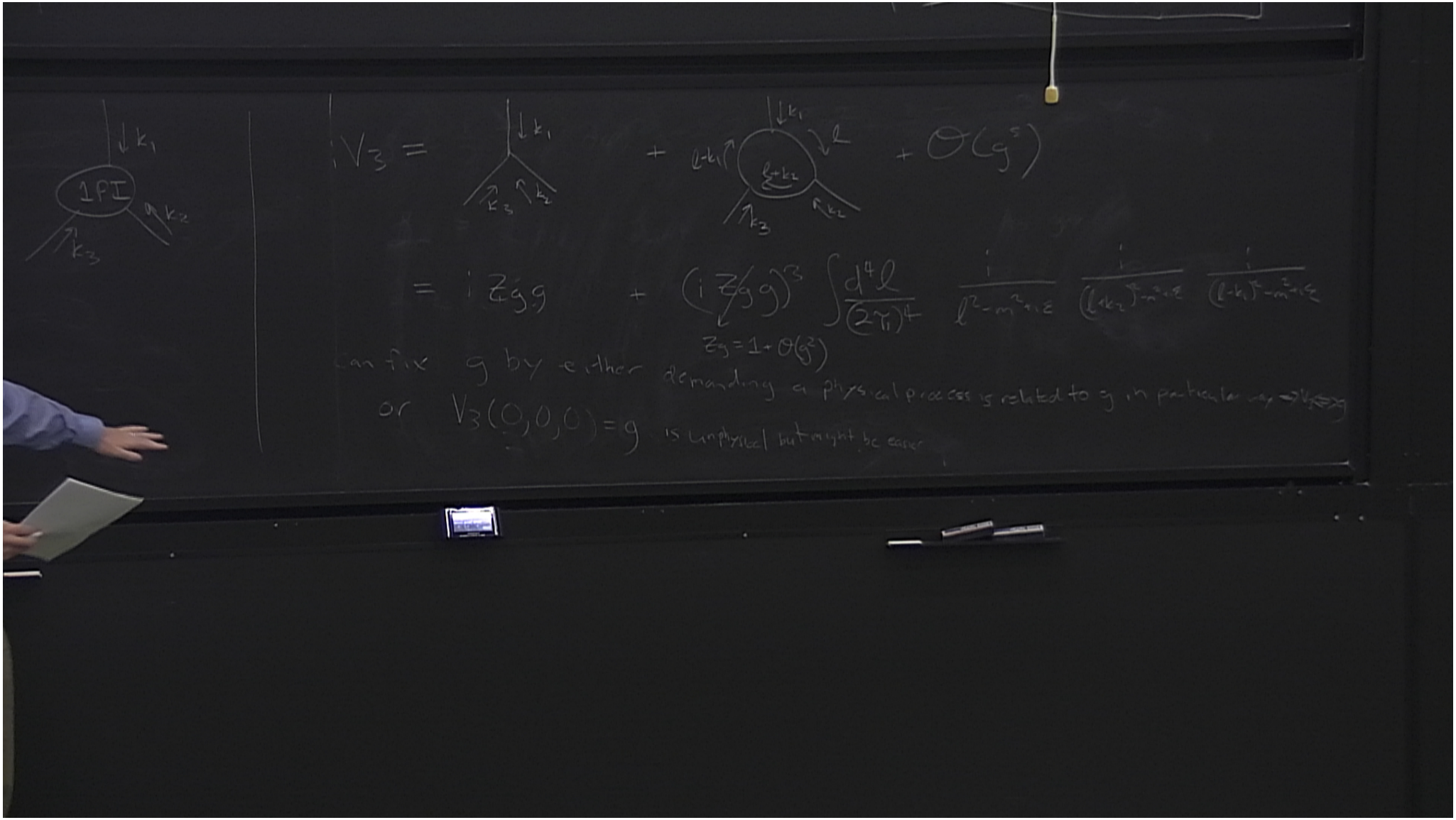

$iV_3(k_1, k_2, k_3) =$
vertex function
1PI
k_1
k_2
k_3
1PI
1PI

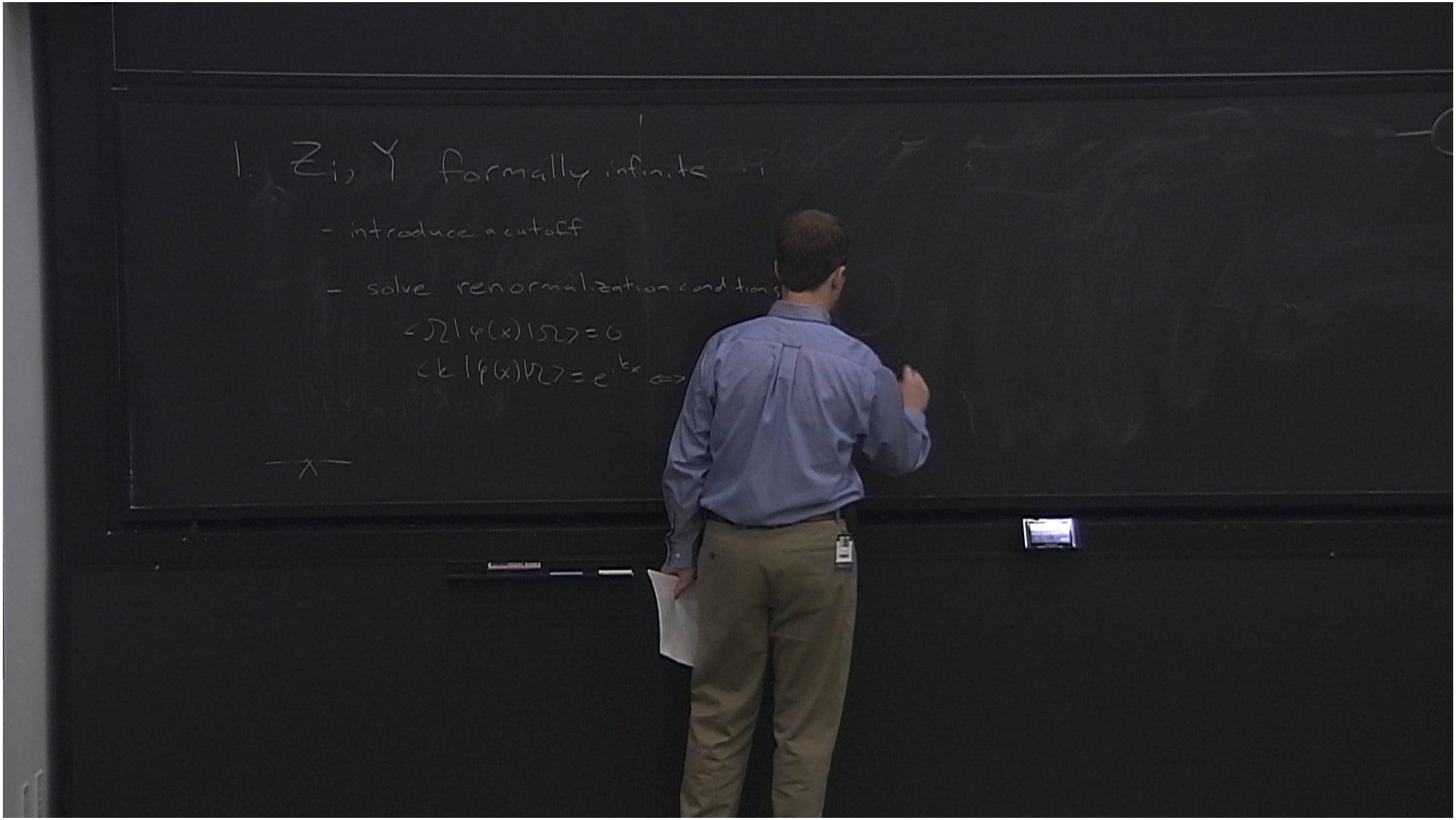

1PI
k_1
k_2
k_3
$iV_3 =$
$+ \mathcal{O}(g^5)$
$= i Z_g g + (i Z_g g)^3 \int \frac{d^4 l}{(2\pi)^4}$
$Z_g = 1 + \mathcal{O}(g^2)$

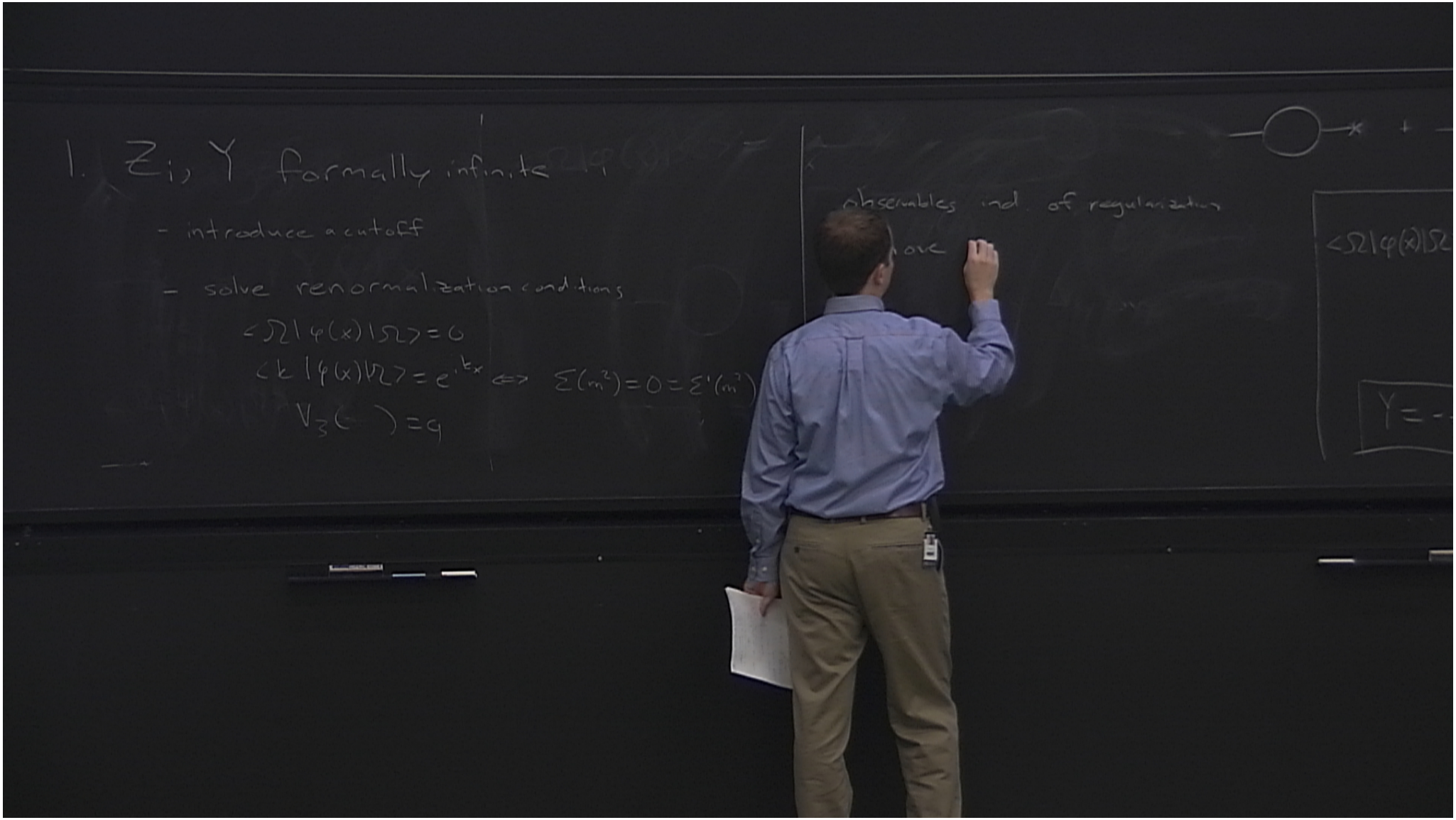

1PI
k_1
k_2
k_3
$iV_3 =$
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$= i Z_g g + (i Z_g g)^3 \int \frac{d^4 l}{(2\pi)^4}$
$Z_g = 1 + \mathcal{O}(g^2)$


1PI
k_1
k_2
k_3
$iV_3 =$
$+ \mathcal{O}(g^5)$
$= iZ_{ggg}$
can fix g by either


1PI
k_1
k_2
k_3
$iV_3 =$
$+ \mathcal{O}(g^5)$
$= iZ_g g$
$+ (iZ_g g)^3 \int \frac{d^4\ell}{(2\pi)^4}$
$Z_g = 1 + \mathcal{O}(g^2)$
demanding a physical process is related to g in particular


1PI
k_1
k_2
k_3
$iV_3 =$
$+ \mathcal{O}(g^5)$
$= iZ_g g + (iZ_g g)^3 \int \frac{d^4l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\varepsilon} \frac{i}{(l-k_2)^2 - m^2 + i\varepsilon} \frac{i}{(l-k_1)^2 - m^2 + i\varepsilon}$
$Z_g = 1 + \mathcal{O}(g^2)$
can fix g by either demanding a physical process is related to g in particular way
or $V_3(0,0,0) = g$ is unphysical but might be easier


1. Z_i, Y formally infinite
- introduce a cutoff
- solve renormalization conditions
$\langle \Omega | \varphi(x) | \Omega \rangle = 0$
$\langle k | \varphi(x) | \Omega \rangle = e^{ikx}$ ⇔


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$V_3(\;) = g$
observables ind of regularization
$\langle \Omega | \varphi(x) | \Omega$
$Y =$

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$V_3(\;) = g$

observables ind. of regularization

remove cutoff

2. There are other choices

