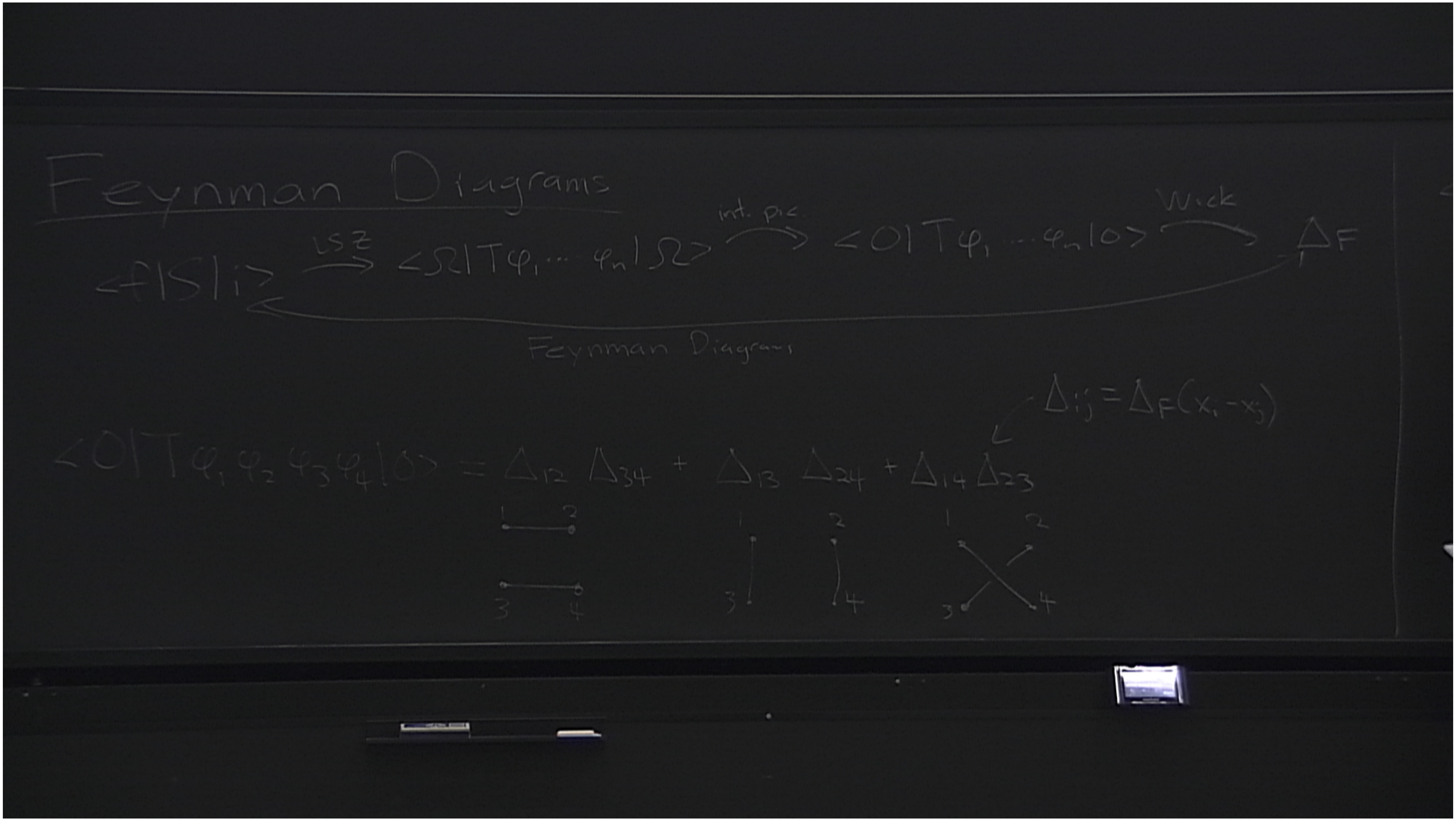
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 5

Date: Oct 19, 2015 09:00 AM

URL: http://pirsa.org/15100040

Abstract:


Feynman Diagrams
$\langle f|S|i\rangle \xrightarrow{LSZ} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xrightarrow{\text{int. pic.}} \langle 0|T\varphi_1\cdots\varphi_n|0\rangle \xrightarrow{\text{Wick}} \Delta_F$
Feynman Diagrams
$\Delta_{ij} = \Delta_F(x_i - x_j)$
$\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$
1 2
3 4
1 2
3 4
1 2
3 4

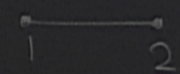
$\varphi_n|0\rangle \xrightarrow{\text{Wick}} \Delta_F$

$\Delta_{ij} = \Delta_F(x_i - x_j)$

$$\langle\Omega|\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[-i\int dt\, H_{int}(t)]|0\rangle}{\langle 0|T \exp[-i\int dt\, H_{int}(t)]|0\rangle}$$

$$H_{int}(t) = \int d^3y \, \frac{\lambda}{4!}\varphi^4(y)$$

Numerator: $\langle 0| T\varphi_1\varphi_2 + T\varphi_1\varphi_2\left(-i\frac{\lambda}{4!}\right)\int d^4y\, \varphi_y^4 |0\rangle$

$\langle 0|T\varphi_1\varphi_2|0\rangle =$ 

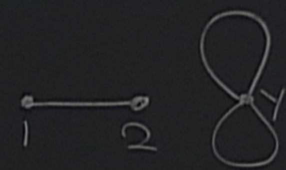
$$\langle 0|T\varphi_1\varphi_2 \, \frac{-i\lambda}{4!}\int d^4y\, \varphi_y^4|0\rangle = 3\, \frac{-i\lambda}{4!}\, \Delta_{12}\int d^4y\, \Delta_{yy}\Delta_{yy} + Q$$

Wick contractions

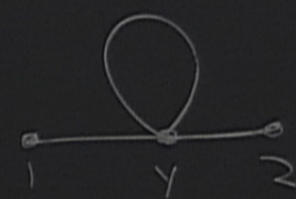
$\varphi_y\varphi_y\varphi_y\varphi_y$ or $\varphi_y\varphi_y\varphi_y\varphi_y$ or $\varphi_y\varphi_y\varphi_y\varphi_y$

$$Q = 12\left(\frac{-i\lambda}{4!}\right)\int d^4y\, \Delta_{1y}\,\Delta_{2y}\,\Delta_{yy}$$

$$\langle 0|\,\varphi_1\varphi_2\,\frac{-i\lambda}{4!}\int d^4y\,\varphi_y\,|0\rangle =$$



$+$



Position

Position Space Feynman Diagrams for n-point functions:

1. For each line: $\underset{x}{\bullet}\!\!\longrightarrow\!\!\!\!\!-\!\!\underset{y}{\bullet} = \Delta_F(x-y)$

2. For vertex: $\times_z = -i\lambda \int d^4 z$

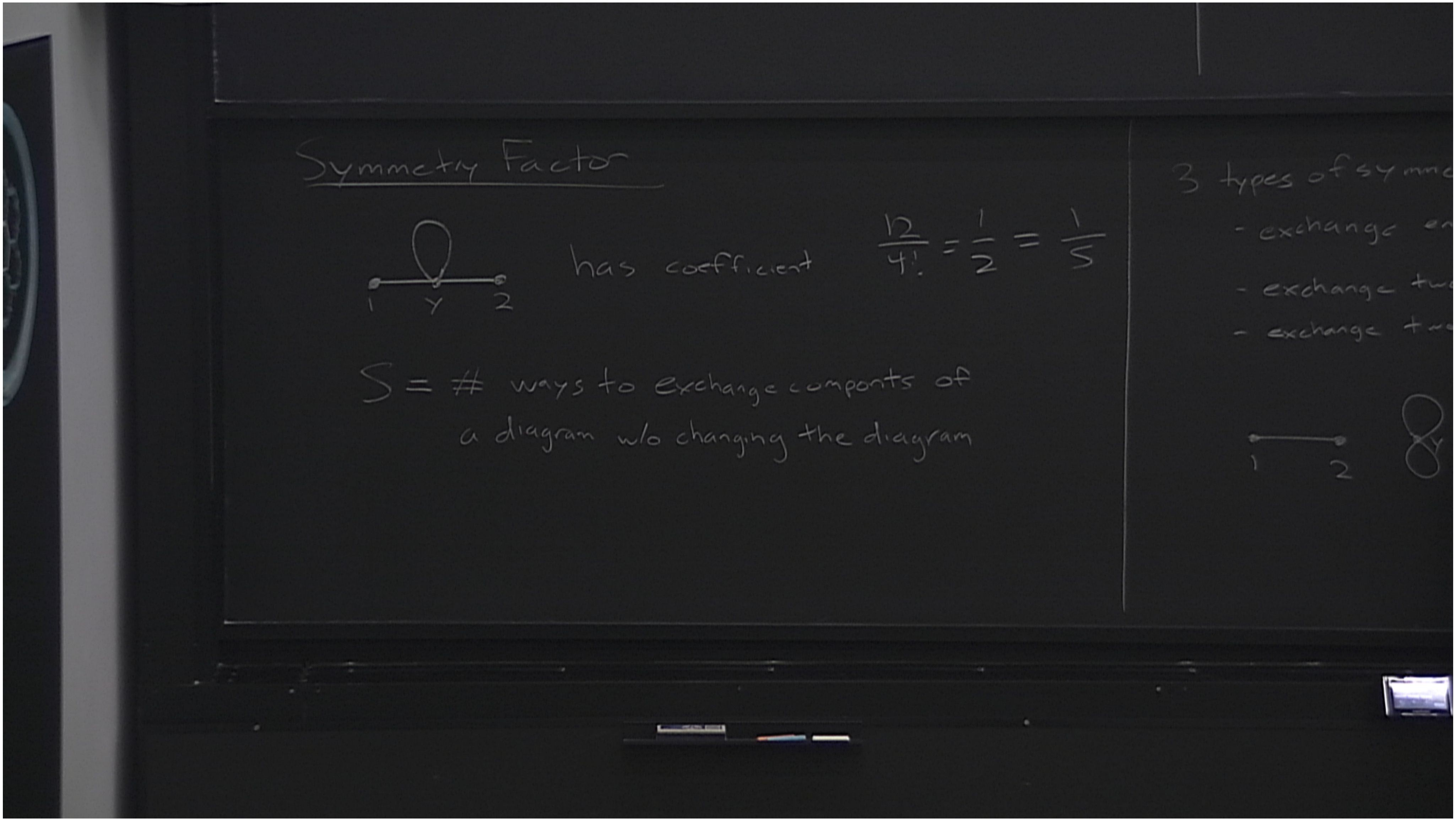
Position Space Feynman Diagrams for n-point functions:

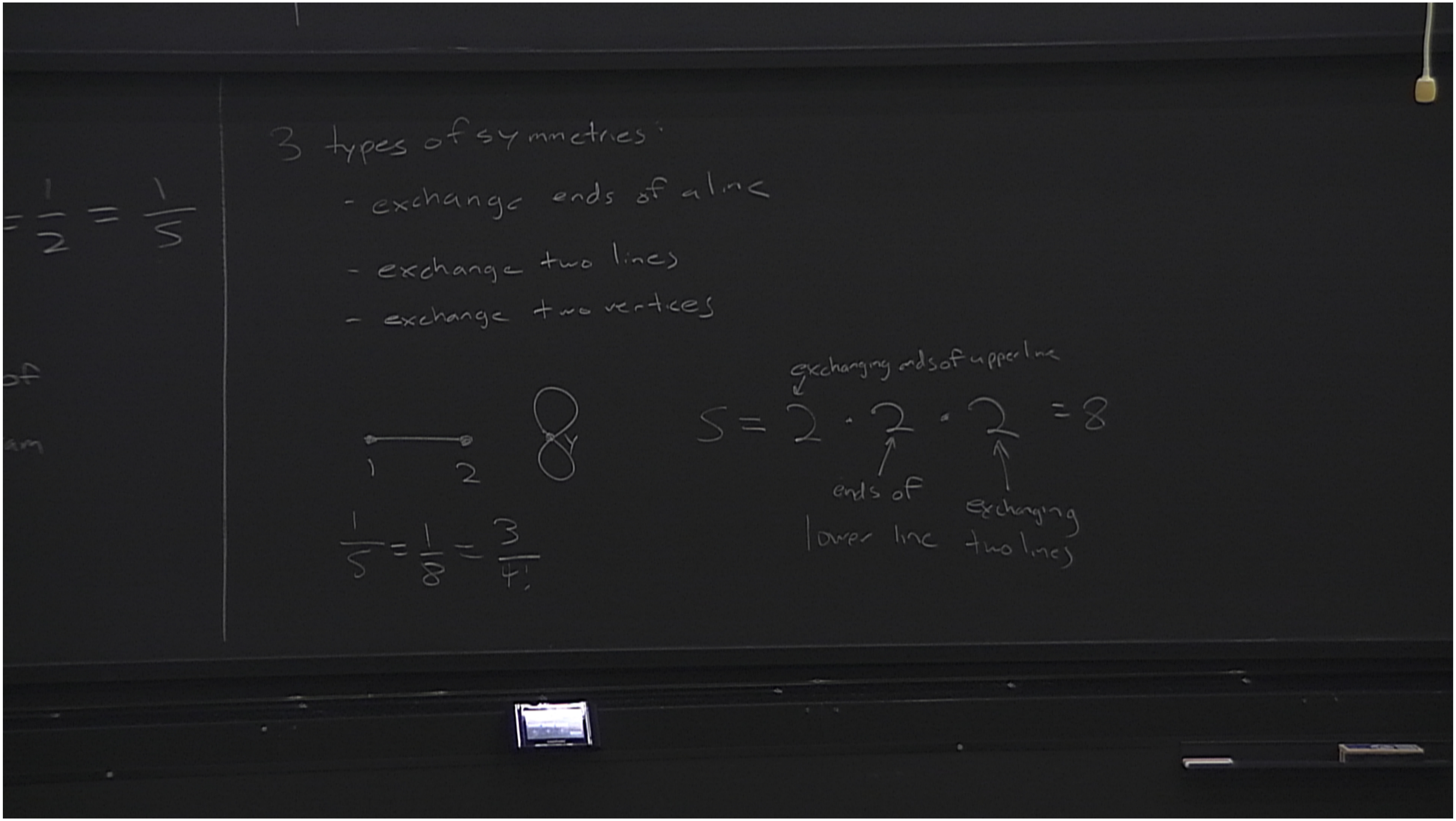
1. For each line: $\underset{x}{\bullet}\!\!-\!\!-\!\!-\!\!\underset{y}{\bullet} \; = \; \Delta_F(x-y)$

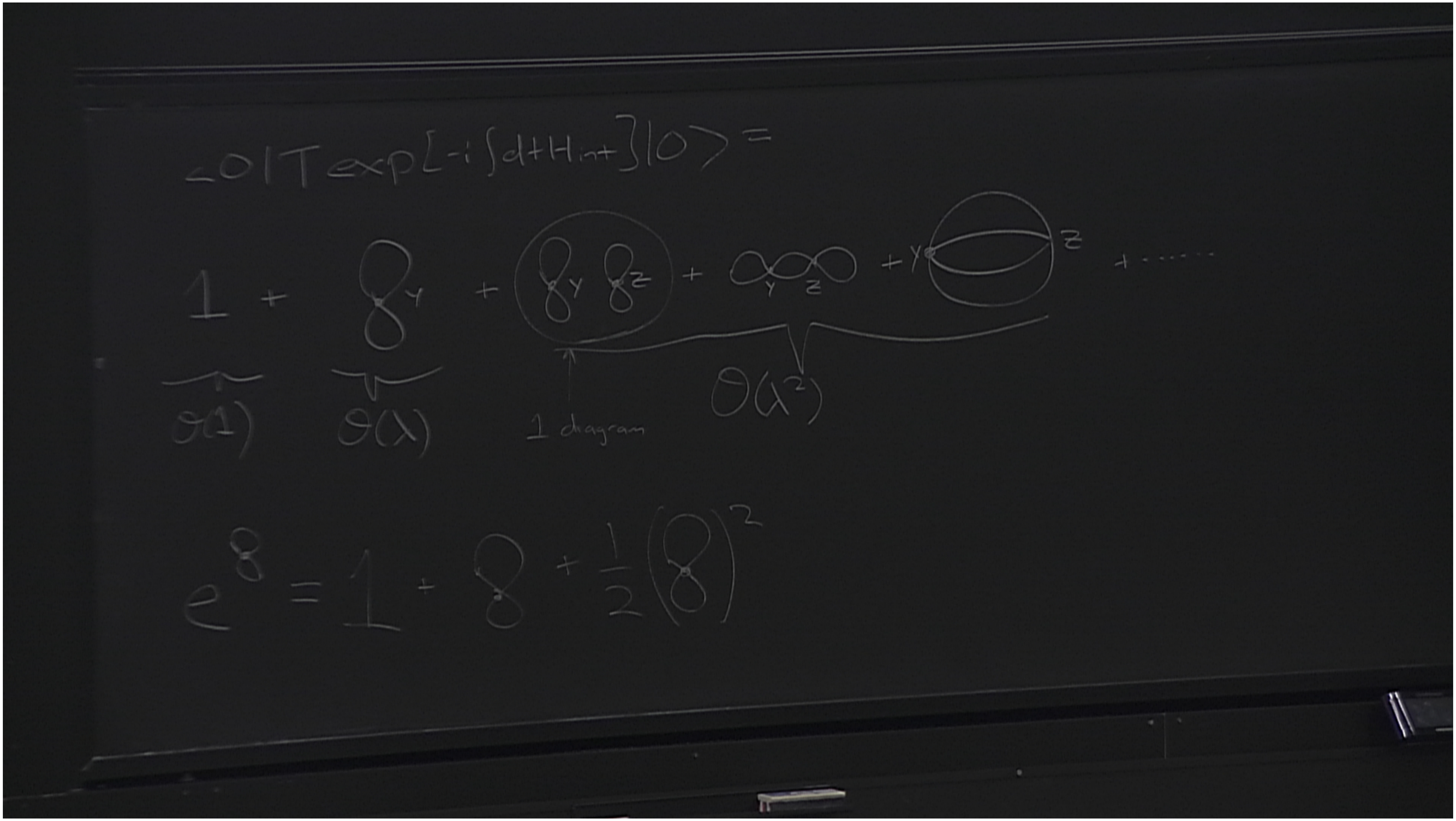
2. For vertex: $\times_z \; = \; -i\lambda \int d^4 z$ (depends on H_{int})

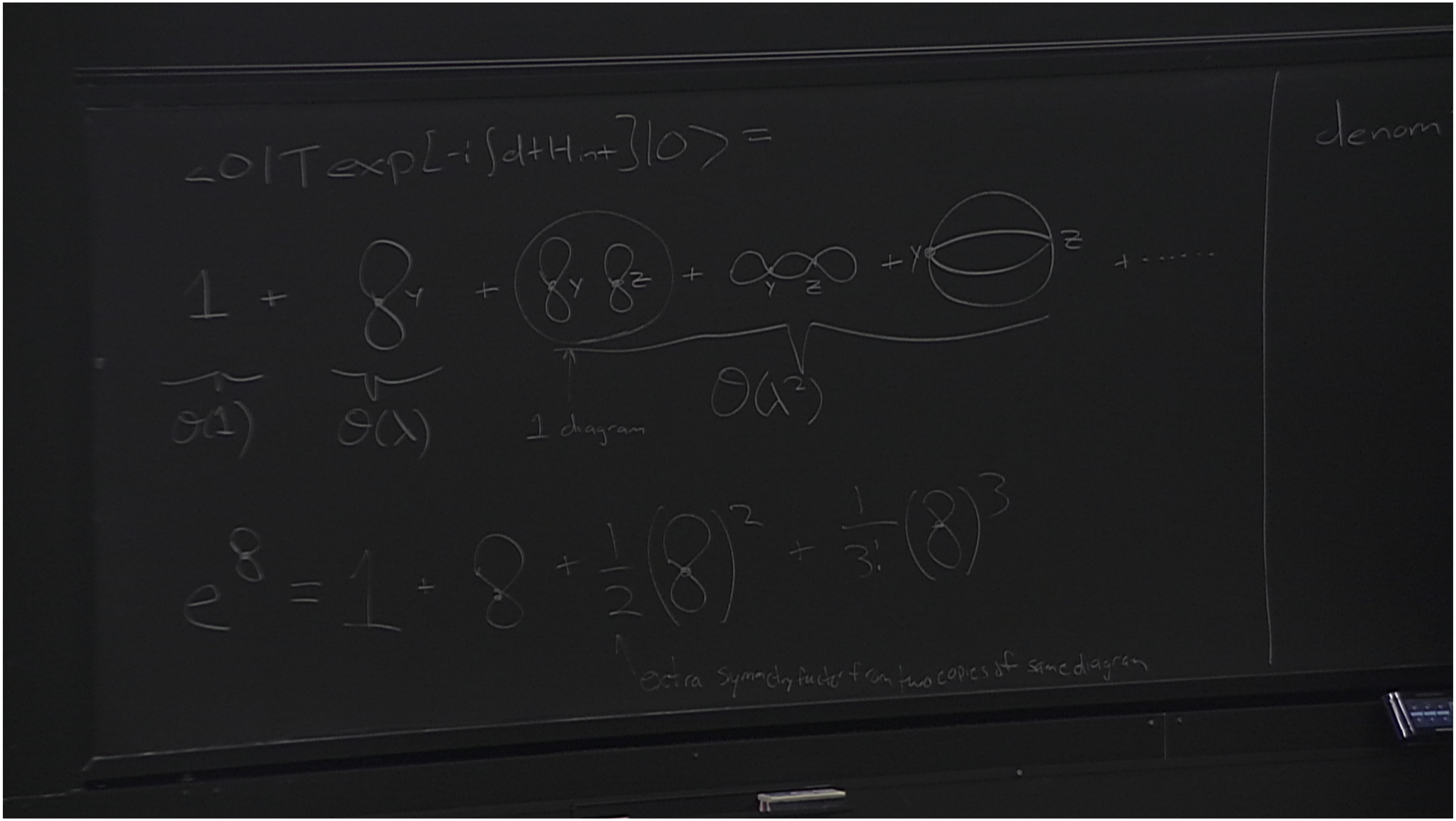
3. For each external point $\underset{x}{\bullet}\!\!-\!\!- \; = 1$ (will change w/ fermions or gauge bosons)

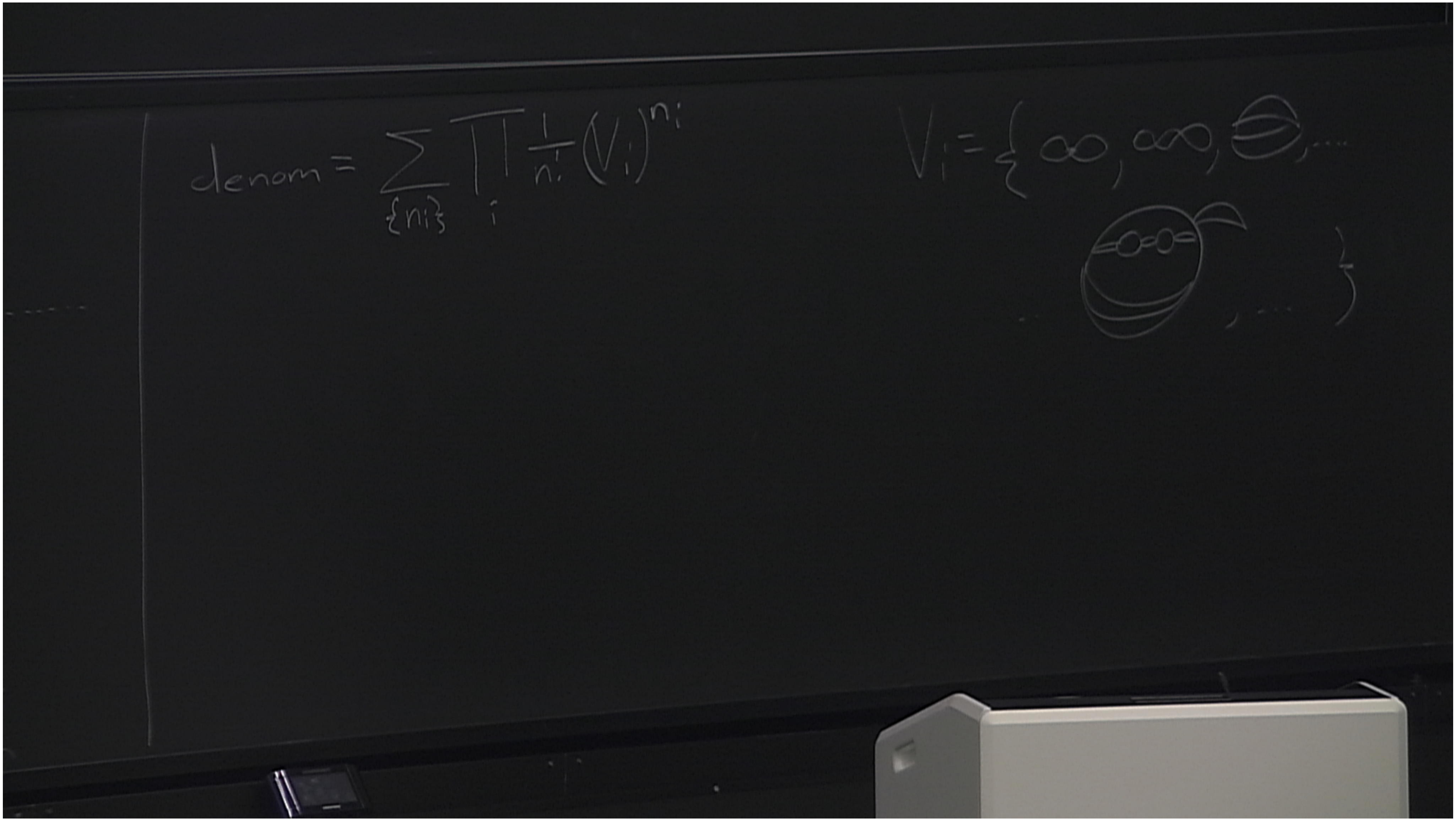
4. Divide by a symmetry factor S.


Symmetry Factor
1 Y 2
has coefficient $\frac{12}{4!} = \frac{1}{2} = \frac{1}{S}$
S = # ways to exchange componts of
a diagram w/o changing the diagram
3 types of sym
- exchange
- exchange
- exchange
1 2

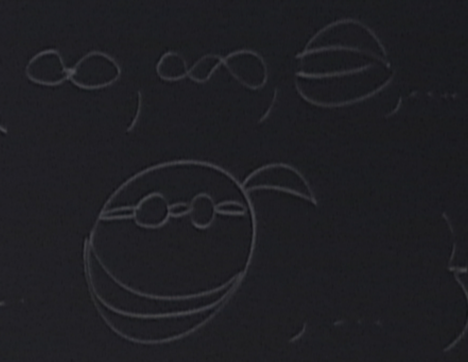

3 types of symmetries:
- exchange ends of a line
- exchange two lines
- exchange two vertices
exchanging ends of upper line
S = 2 · 2 · 2 = 8
ends of lower line
exchanging two lines
1/S = 1/8 = 3/4!
= 1/2 = 1/5


<0|T exp[-i∫dt H_int]|0> =
1 + + + + + ...
y z
O(1)
O(λ)
1 diagram
O(λ²)
e = 1 + + ½ ( )²


$\langle 0| T \exp[-i\int dt\, H_{int}] |0\rangle =$
$1 +$
y
z
$\mathcal{O}(1)$
$\mathcal{O}(\lambda)$
1 diagram
$\mathcal{O}(\lambda^2)$
$e^{8} = 1 + 8 + \frac{1}{2}(8)^2 + \frac{1}{3!}(8)^3$
extra symmetry factor from two copies of same diagram
denom


$\text{denom} = \sum_{\{n_i\}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$
$V_i = \{$

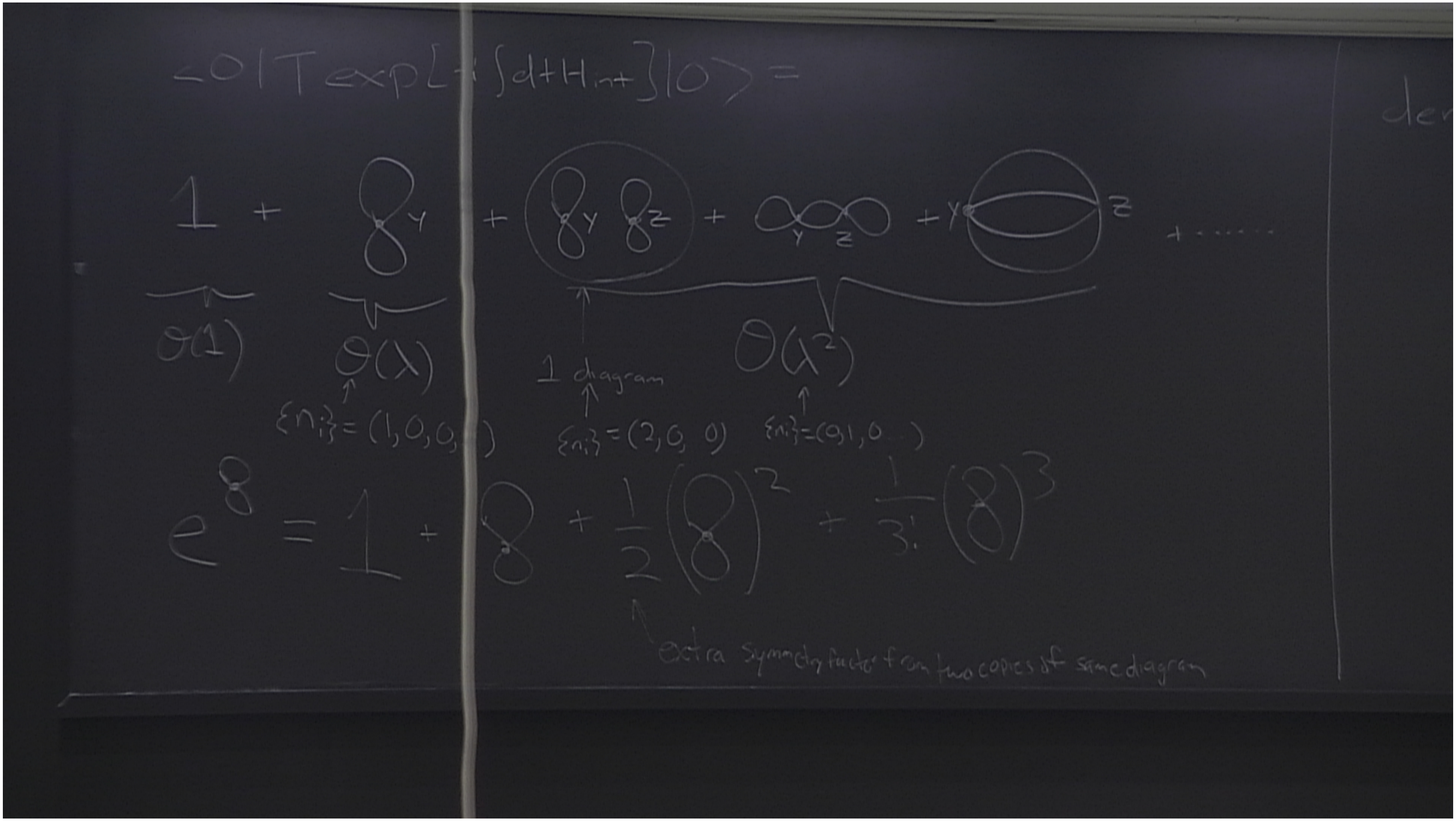
$$\text{denom} = \sum_{\{n_i\}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$$

$V_i = \{$  $\}$

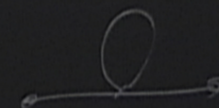
$$= \left(\sum_{n_1} \frac{1}{n_1!} V_1^{n_1} \right) \left(\sum_{n_2} \frac{1}{n_2!} V_2^{n_2} \right) \cdots \cdots$$

$$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i} \right)$$

$$= \prod_i \exp(V_i) = \exp\left(\sum_i V_i\right) = \exp(\infty + \infty\!\infty + \cdots)$$



<0|T exp[-i∫dt H_int]|0> =
O(1)
O(λ)
O(λ²)
1 diagram
{n_i} = (1,0,0,)
{n_i} = (2,0, 0)
{n_i} = (0,1,0 . .)
extra symmetry factor from two copies of same diagram

$\{C_i\} = \{$  ,  , $\ldots \}$

$$\text{numerator} = \sum_j \sum_{\{n_i\}} \prod_i C_j \frac{1}{n_i!}(V_i)^{n_i}$$

$$= \sum_j C_j \sum_{\{n_i\}} \prod_i \frac{1}{n_i!}(V_i)^{n_i}$$

$$= \sum_j C_j \exp\left(\sum_i V_i\right)$$

$\boxed{\text{vacuum diagrams cancel!}}$

$$= \prod_i \exp(V_i) = \exp\left(\sum_i V_i\right) = \exp\left(\ldots \right)$$

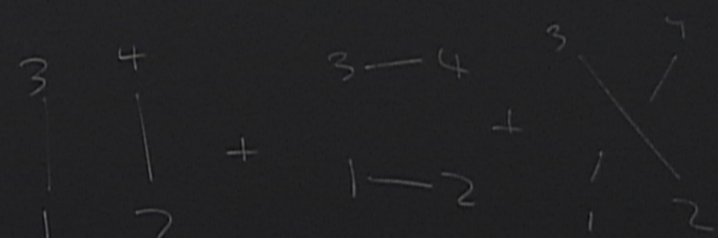
...diagram

$\langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$ = sum of all connected n-point functions w/ n external po...

Scattering Amplitudes $\quad 2 \to 2$

$$\langle f | S | i \rangle = i^4 \prod_i \int dx_i \, e^{i\lambda p_i x_i} (\partial_i^2 + m^2) \langle \Omega | T\varphi_1 \ldots \varphi_4 | \Omega \rangle$$

$$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$$



$$+ \, O(\lambda)$$

$$= \Delta_{13} \Delta_{24}$$

Vacuum diagrams cancel!

$$(\partial_1^2 + m^2)(\partial_3^2 + m^2)\Delta_F(x_1 - x_3) = F(x_1 - x_3)$$

$$\int d^4x_1\, d^4x_3\, e^{i(p_1\cdot x_1 - p_3\cdot x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13}\, d^4\bar{x}_{13}\, e^{i(\bar{p}_{13}\cdot x_{13} + p_{13}\cdot \bar{x}_{13})} F(\bar{x}_{13})$$

$$= \frac{1}{2}\int d^4x_{13}\, e^{i\bar{p}_{13}\cdot x_{13}} \tilde{F}(p_{13})$$

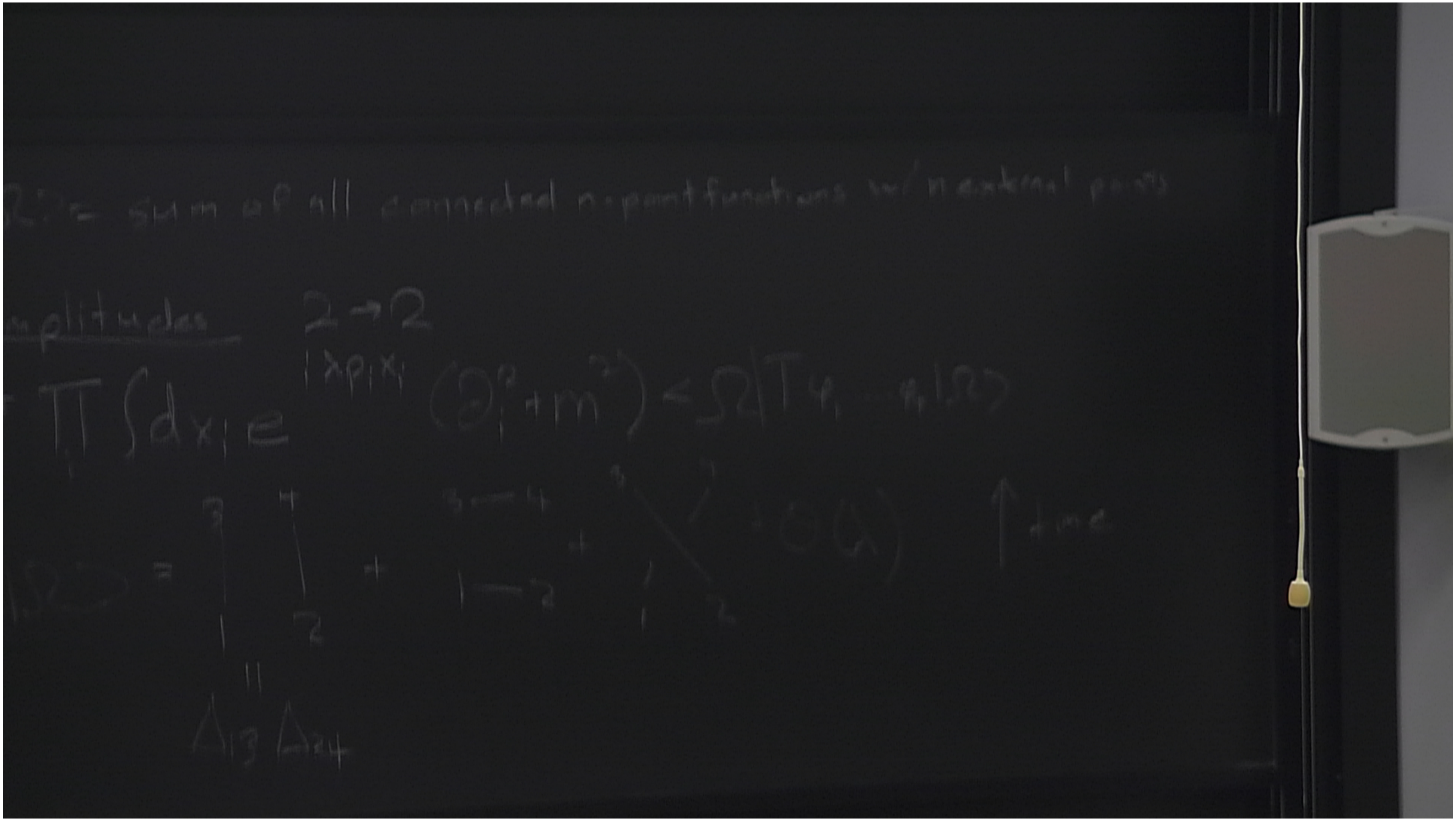
$$= (2\pi)^4 \delta^4(p_1 - p_3)\, \tilde{F}\left(\frac{p_1 + p_3}{2}\right)$$

$$x_{13} = x_1 + x_3$$

$$\bar{x}_{13} = x_1 - x_3$$

$$p_{13} = \frac{p_1 + p_3}{2}$$

$$\bar{p}_{13} = \frac{p_1 - p_3}{2}$$


sum of all connected n-point functions w/ n external points
amplitudes
2 → 2
time

$\Delta_{13}\,\Delta_{24}$

$F(\bar{x}_{13})$

$$\langle f|S|i\rangle_{11} \propto \delta(p_1-p_3)\,\delta(p_2-p_4)$$

same for X'

$$\langle f|S|i\rangle_{=} \propto \delta^{(4)}(p_1+p_2)\,\delta^{(4)}(p_3+p_4)$$

$$p_1^0+p_2^0 \geq m+m > 0$$

$F(\bar{x}_{13})$

$$\langle f|S|i\rangle_{11} \propto \delta(p_1 - p_3)\,\delta(p_2 - p_4)$$

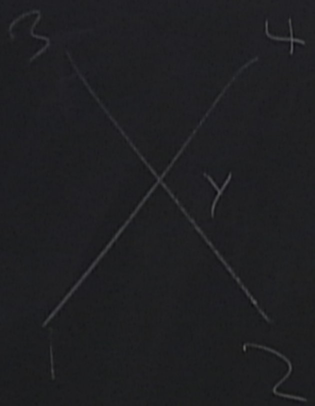
same for

$$\langle f|S|i\rangle_{=} \propto \delta^{(4)}(p_1 + p_2)\,\delta^{(4)}(p_3 + p_4)$$

$$p_1^0 + p_2^0 \geq m + m > 0$$

same for or

only fully connected diagrams contribute to scattering



$$= -i\lambda \int d^4y \, \Delta_{1y}\Delta_{2y}\Delta_{3y}\Delta_{4y}$$

$$(\partial_1^2 + m^2)\,\Delta_{1y} = -i\,\delta^{(4)}(x_1 - y)$$

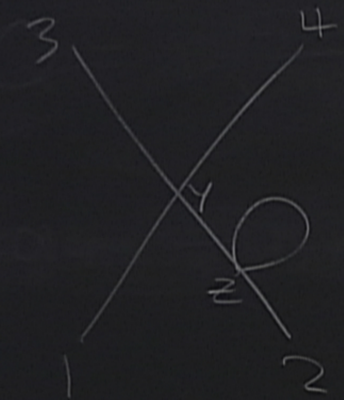
$$\langle f|S|i\rangle_{\times} = -i\lambda \int d^4y\, e^{i(p_1+p_2-p_3-p_4)\cdot y}$$

$$= -i\lambda (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

$$i\mathcal{M} = -i\lambda$$

$$\langle f|i|S\rangle_{cc} = i\mathcal{M}(2\pi)^4\delta^4(\Sigma p)$$

Completely
Connected



$$= \frac{(-i\lambda)^2}{2} \int d^4y \int d^4z \, \Delta_{2z} \Delta_{zz} \Delta_{1y} \Delta_{3y} \Delta_{4y} \Delta_{yz}$$

$$\langle f|S|i\rangle = \frac{(-i\lambda)^2}{2} \int d^4y \, d^4z \, e^{i(p_1 - p_3 - p_4)\cdot y} \, e^{i p_2 z} \, \Delta_{zz} \Delta_{yz}$$

$$\cdots y \, \Delta_{yz}$$

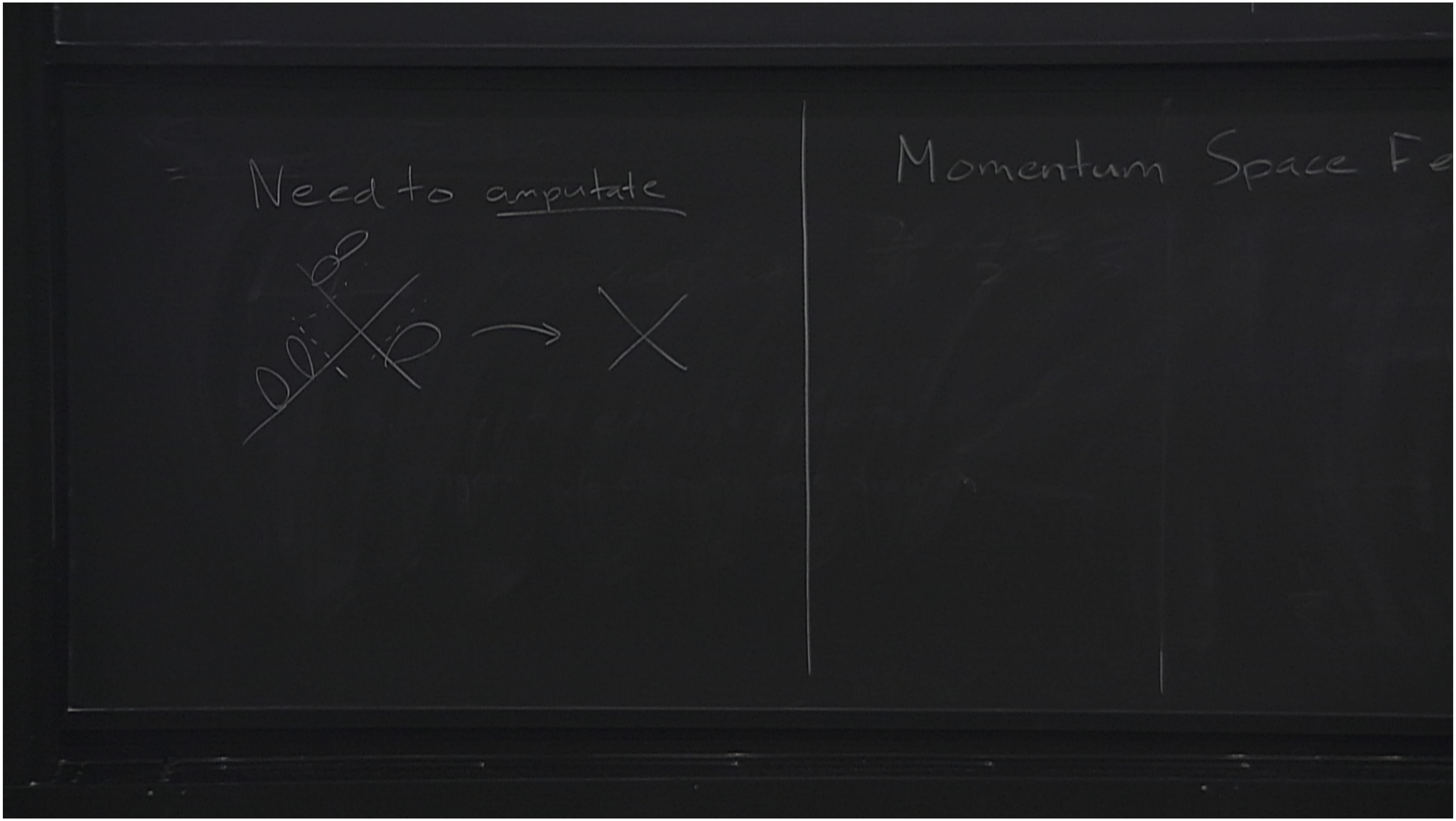
$$\Delta_{zz}\Delta_{yz}$$

$$\Delta_{yz} = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} e^{ik\cdot(y-z)}$$

$$\langle f|S|i\rangle_{x\ell} = \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} \frac{i}{q^2-m^2+i\epsilon}$$

$$(2\pi)^4 \delta^4(p_1+k-p_3-p_4) \cdot (2\pi)^4 \delta^{(4)}(p_2-k)$$

$$\int d^4k \, \delta(p_2-k) \frac{1}{k^2-m^2+i\epsilon} = \frac{1}{p_2^2-m^2} = \frac{1}{0} \, !$$


Need to amputate
Momentum Space Fe

Momentum Space Feynman Rules for Scattering Amplitudes

$i\mathcal{M}$ = sum of all completely connected, amputated diagrams

1 For each internal line $\underset{\vec{p}}{\longrightarrow} = \dfrac{i}{p^2 - m^2 + i\varepsilon}$

2. For each vertex $\times = -i\lambda$ (depends on H_{int})

3. For each external line $\succ\!\!\!- = 1$ (change for fermions & gauge bosons)

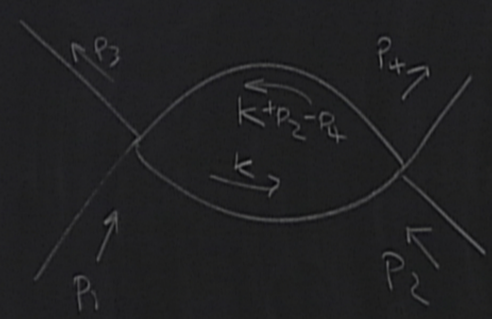
4. Impose momentum conservation at each vertex

vacuum diagrams cancel!

denom

5. Divide by symmetry factor

6. Integrate over any undermined momenta.



$$= \frac{(-i\lambda)^2}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2+i\varepsilon}\,\frac{i}{(k+p_2-p_4)^2-m^2+i\varepsilon}$$