Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 4

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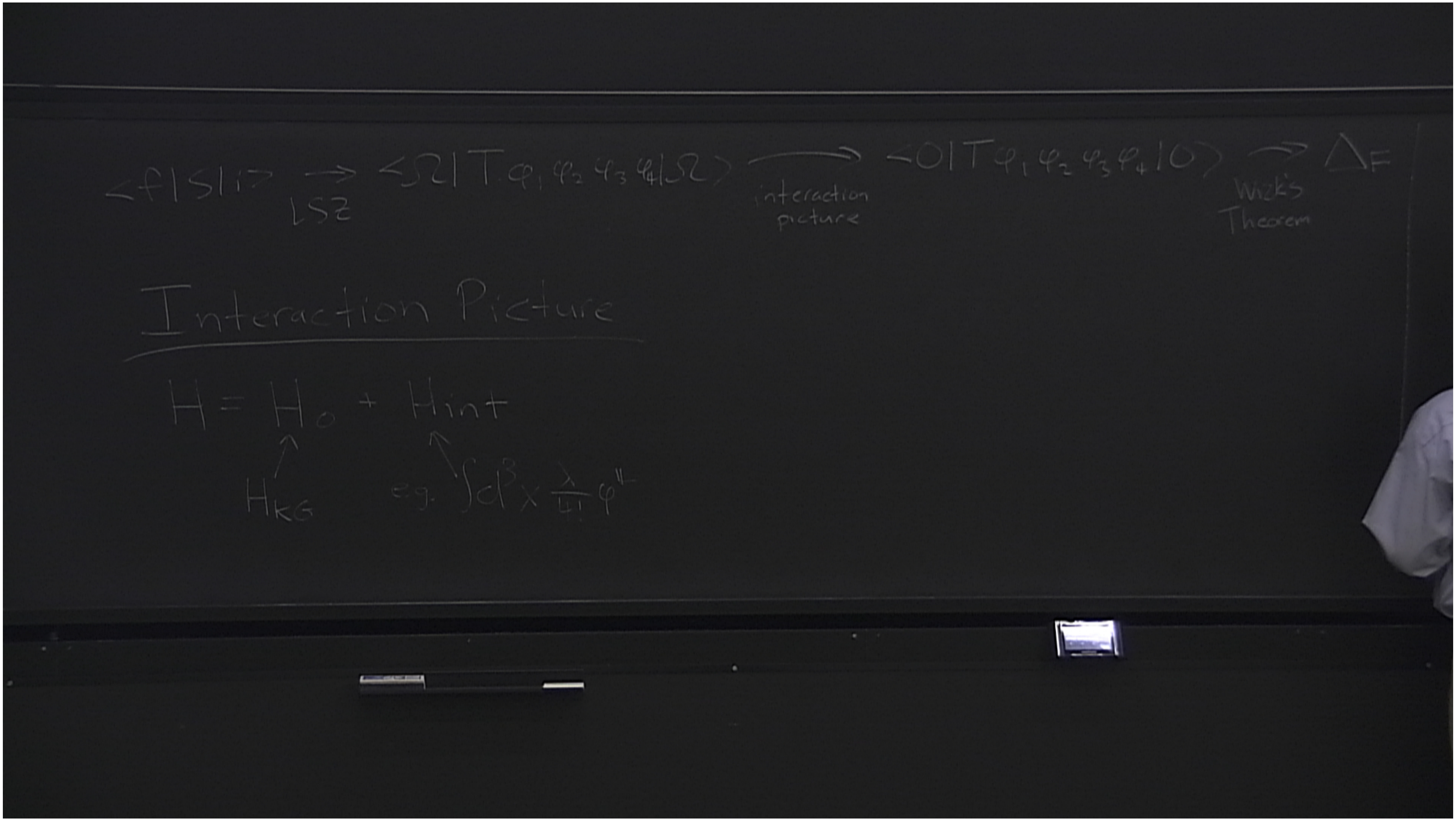
Abstract:



$\langle f|S|i\rangle \xrightarrow[\text{LSZ}]{} \langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle \xrightarrow[\text{interaction picture}]{} \langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle \xrightarrow[\text{Wick's Theorem}]{} \Delta_F$
Interaction Picture
$H = H_0 + H_{int}$
H_{KG}
e.g. $\int d^3x \frac{\lambda}{4!}\varphi^4$

Schrödinger: $i\frac{d}{dt}|\psi\rangle_S = H|\psi\rangle_S$

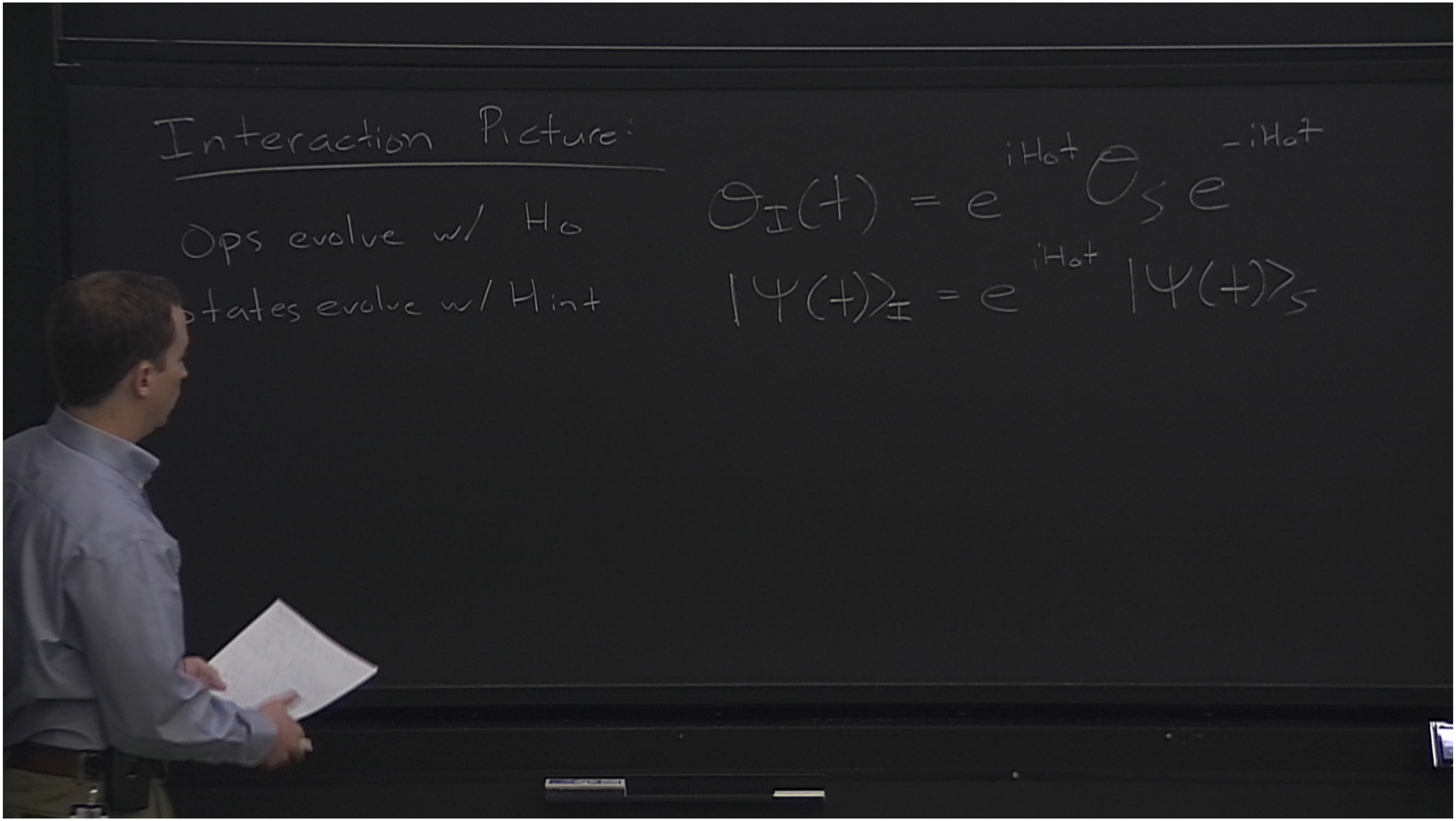
Heisenberg picture: $O_H(t) = e^{iHt} O_S e^{-iHt}$

$$= U^\dagger(t,0)\, O_S\, U(t,0)$$

$$|\psi\rangle_H = e^{iHt}|\psi\rangle_S$$

$$= U^\dagger(t,0)|\psi\rangle_S$$

$$U(t,t') = e^{-iH(t-t')}$$


Interaction Picture:
Ops evolve w/ H_0
states evolve w/ H_{int}
$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$
$|\psi(t)\rangle_I = e^{iH_0 t} |\psi(t)\rangle_S$

Interaction Picture:

Ops evolve w/ H_0

states evolve w/ H_{int}

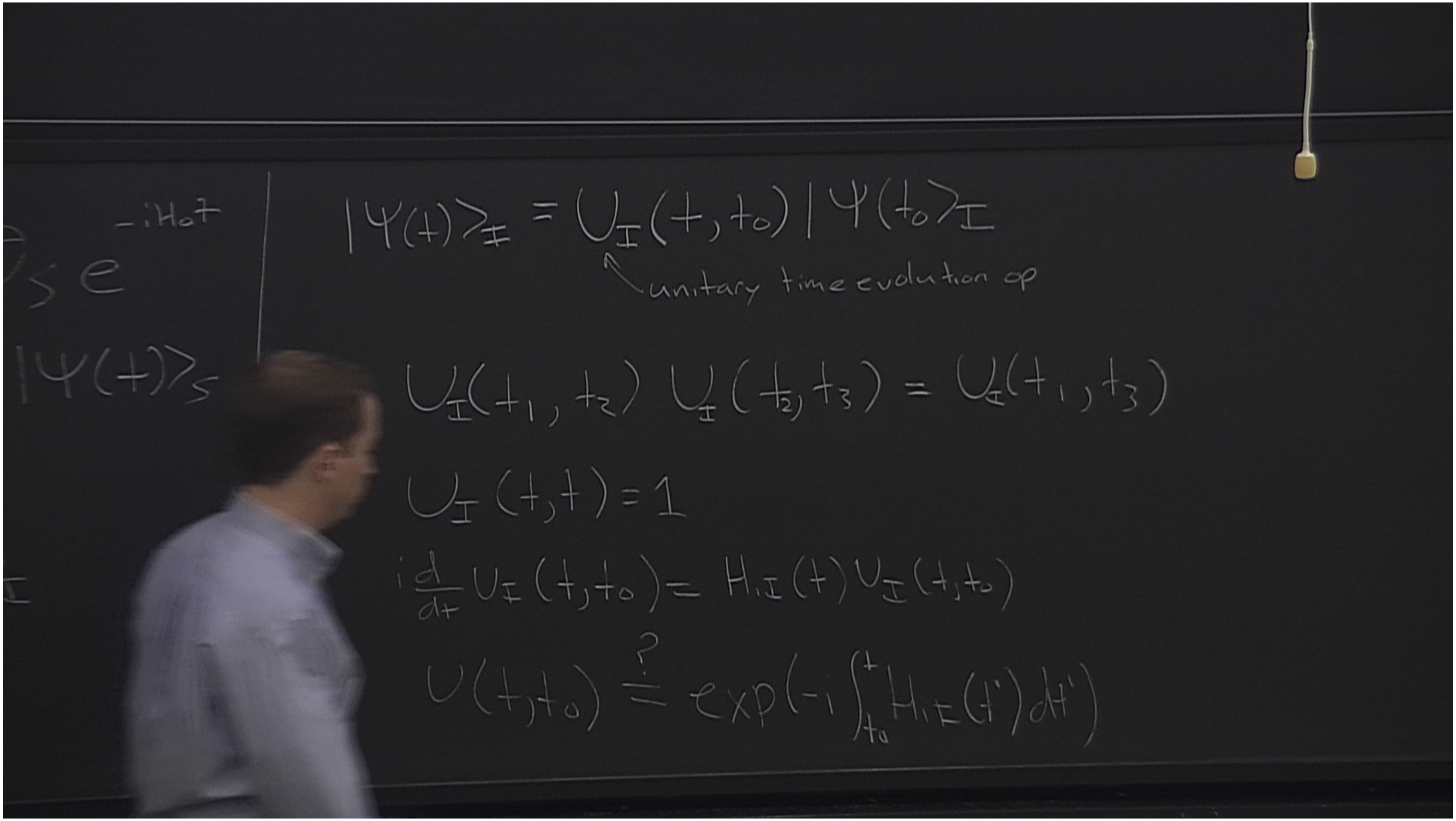
$$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$$

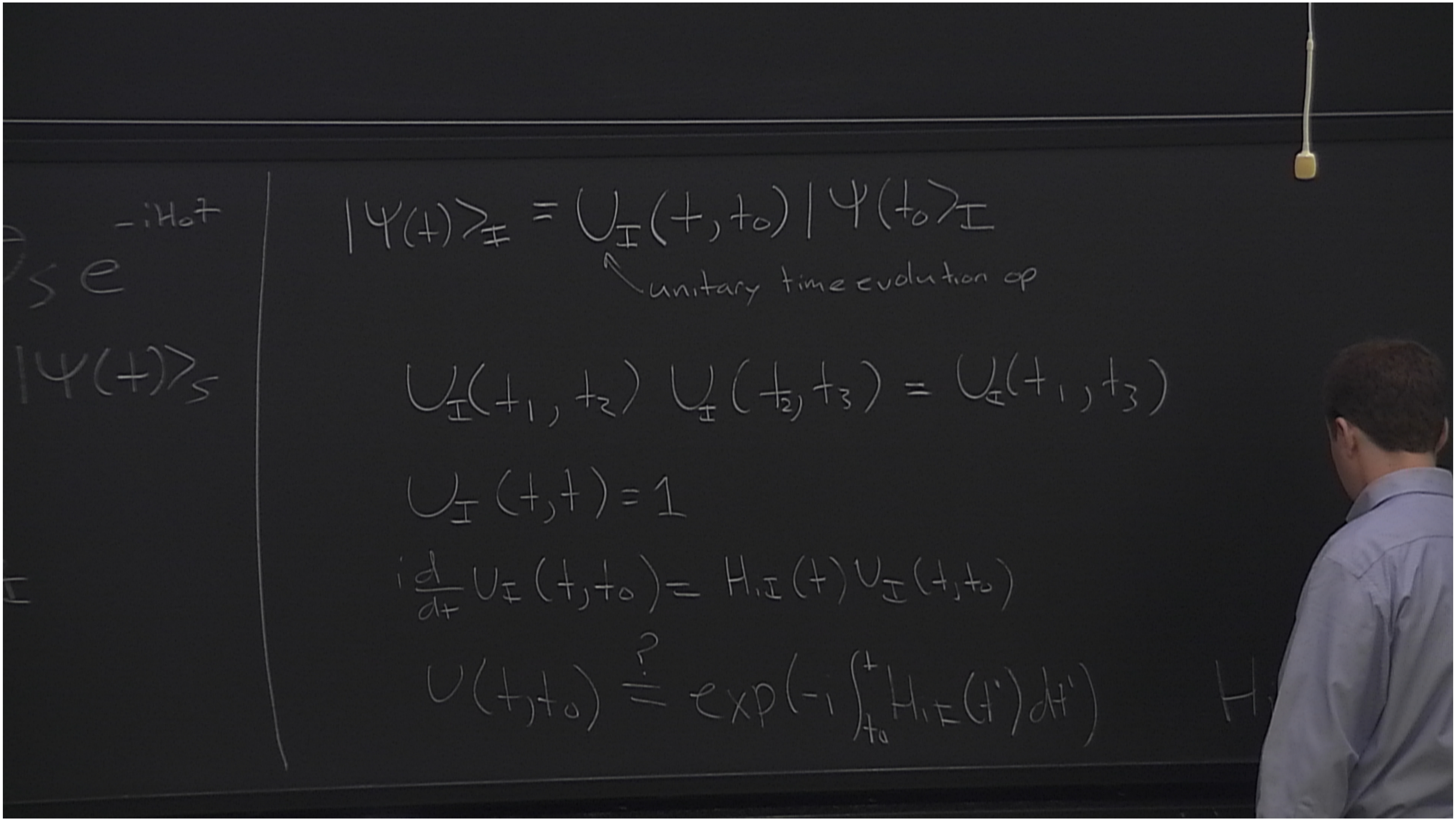
$$|\psi(t)\rangle_I = e^{iH_0 t} |\psi(t)\rangle_S$$

$$i\frac{d}{dt}|\psi\rangle_S = H_S|\psi\rangle_S$$

$$i\frac{d}{dt}\left(e^{-iH_0 t}|\psi\rangle_I\right) = (H_0 + H_{int})_S\, e^{-iH_0 t}|\psi\rangle_I$$

$$i\frac{d}{dt}|\psi\rangle_I = \underbrace{e^{iH_0 t}(H_{int})_S\, e^{-iH_0 t}}_{H_{iI}}|\psi\rangle_I$$


e^{-iH_0t}
$|\Psi(t)\rangle_S$
$|\Psi(t)\rangle_I = U_I(t,t_0)|\Psi(t_0)\rangle_I$
unitary time evolution op
$U_I(t_1,t_2)\,U_I(t_2,t_3) = U_I(t_1,t_3)$
$U_I(t,t) = 1$
$i\frac{d}{dt}U_I(t,t_0) = H_{1,I}(t)\,U_I(t,t_0)$
$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_{1,I}(t')\,dt'\right)$


$e^{-iH_0 t}$
$|\Psi(t)\rangle_S$
$|\Psi(t)\rangle_I = U_I(t,t_0)\,|\Psi(t_0)\rangle_I$
unitary time evolution op
$U_I(t_1,t_2)\,U_I(t_2,t_3) = U_I(t_1,t_3)$
$U_I(t,t) = 1$
$i\frac{d}{dt}U_I(t,t_0) = H_{1I}(t)\,U_I(t,t_0)$
$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_{1I}(t')\,dt'\right)$

$$|\Psi(t)\rangle_I = U_I(t,t_0)\,|\Psi(t_0)\rangle_I$$

unitary time evolution op

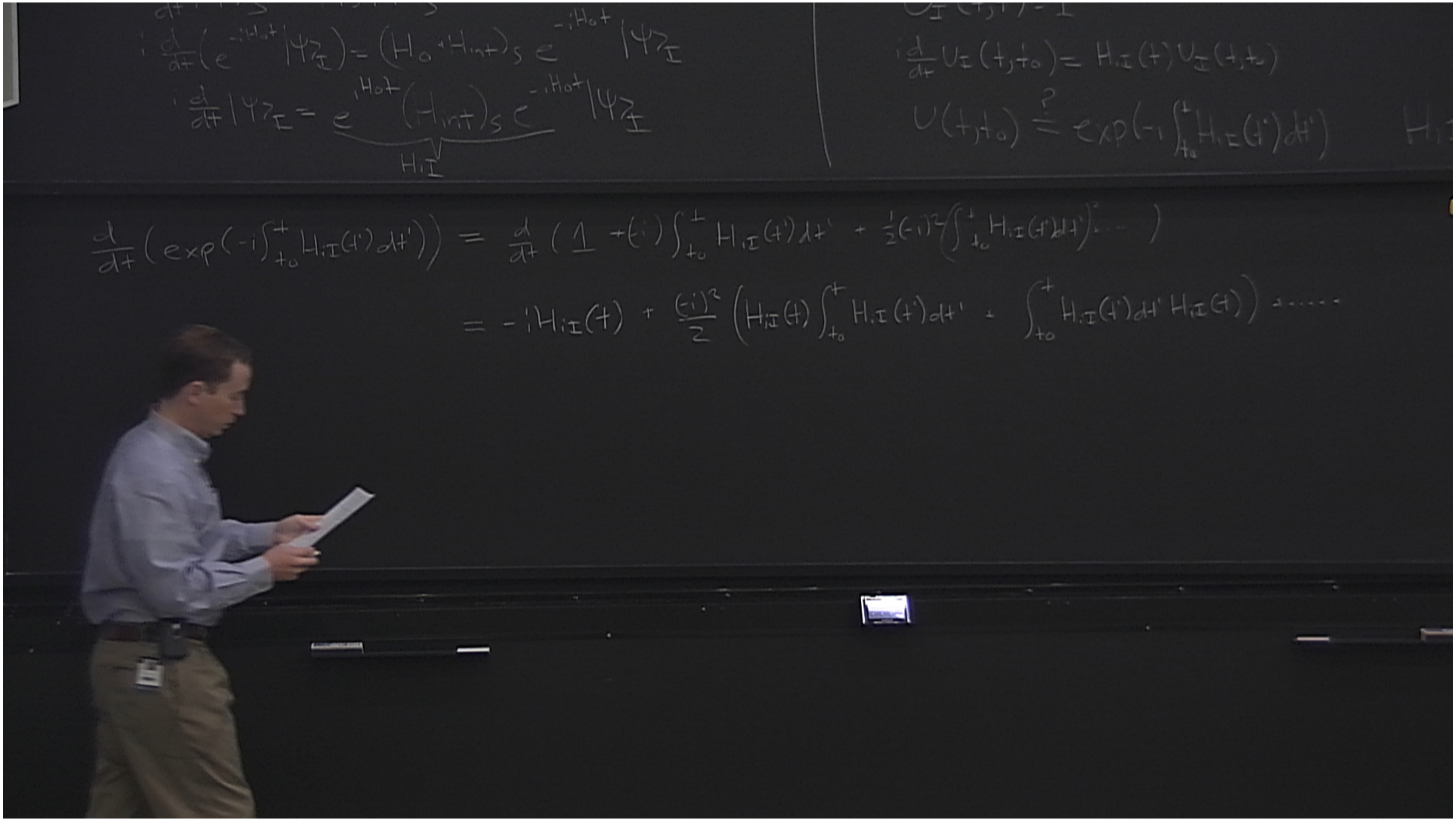
$$U_I(t_1,t_2)\,U_I(t_2,t_3) = U_I(t_1,t_3)$$

$$U_I(t,t) = 1$$

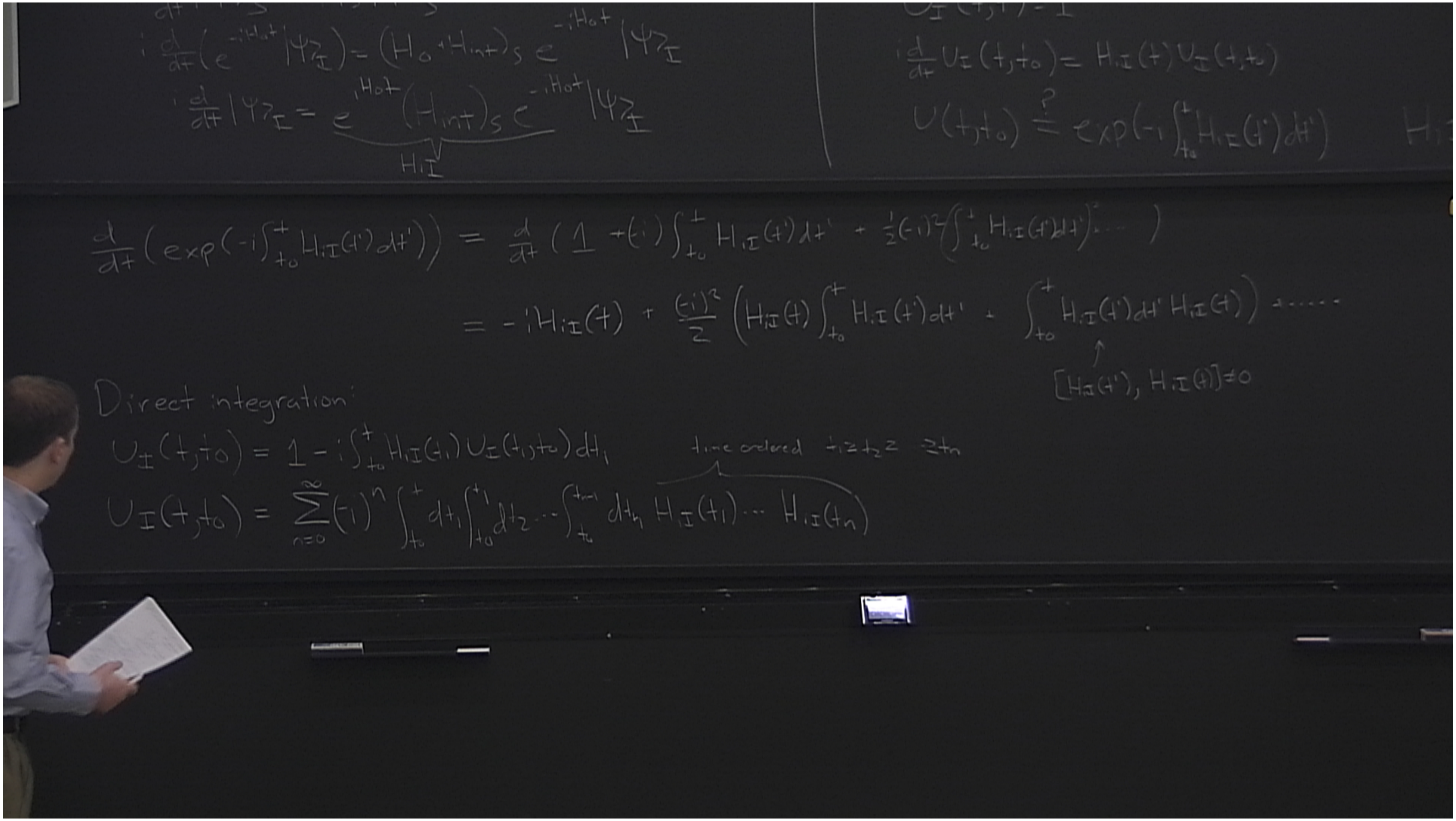
$$i\frac{d}{dt}U_I(t,t_0) = H_{iI}(t)\,U_I(t,t_0)$$

$$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_{iI}(t')\,dt'\right)$$

H_{iI} is an op!



$i\frac{d}{dt}(e^{-iH_0 t}|\psi\rangle_I) = (H_0 + H_{int})_S\, e^{-iH_0 t}|\psi\rangle_I$
$i\frac{d}{dt}|\psi\rangle_I = \underbrace{e^{iH_0 t}(H_{int})_S\, e^{-iH_0 t}}_{H_I}|\psi\rangle_I$
$i\frac{d}{dt}U_I(t,t_0) = H_I(t)\,U_I(t,t_0)$
$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_I(t')\,dt'\right)$
$\frac{d}{dt}\left(\exp\left(-i\int_{t_0}^{t} H_I(t')\,dt'\right)\right) = \frac{d}{dt}\left(1 + (-i)\int_{t_0}^{t} H_I(t')\,dt' + \frac{1}{2}(-i)^2\left(\int_{t_0}^{t} H_I(t')\,dt'\right)^2 + \cdots\right)$
$= -iH_I(t) + \frac{(-i)^2}{2}\left(H_I(t)\int_{t_0}^{t} H_I(t')\,dt' + \int_{t_0}^{t} H_I(t')\,dt'\, H_I(t)\right) + \cdots$


$i\frac{d}{dt}\left(e^{-iH_0t}|\psi\rangle_I\right) = (H_0+H_{int})_S\, e^{-iH_0t}|\psi\rangle_I$
$i\frac{d}{dt}|\psi\rangle_I = \underbrace{e^{iH_0t}(H_{int})_S\, e^{-iH_0t}}_{H_{iI}}|\psi\rangle_I$
$i\frac{d}{dt}U_I(t,t_0) = H_{iI}(t)\,U_I(t,t_0)$
$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_{iI}(t')\,dt'\right)$
$\frac{d}{dt}\left(\exp\left(-i\int_{t_0}^{t} H_{iI}(t')\,dt'\right)\right) = \frac{d}{dt}\left(1 + (-i)\int_{t_0}^{t} H_{iI}(t')\,dt' + \frac{1}{2}(-i)^2\left(\int_{t_0}^{t} H_{iI}(t')\,dt'\right)^2 \cdots\right)$
$= -iH_{iI}(t) + \frac{(-i)^2}{2}\left(H_{iI}(t)\int_{t_0}^{t} H_{iI}(t')\,dt' + \int_{t_0}^{t} H_{iI}(t')\,dt'\,H_{iI}(t)\right) + \cdots\cdots$
$[H_{iI}(t'), H_{iI}(t)] \neq 0$
Direct integration:
$U_I(t,t_0) = 1 - i\int_{t_0}^{t} H_{iI}(t_1)\,U_I(t_1,t_0)\,dt_1$
time ordered $t_1 \geq t_2 \geq \ldots \geq t_n$
$U_I(t,t_0) = \sum_{n=0}^{\infty}(-i)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n\, H_{iI}(t_1)\cdots H_{iI}(t_n)$

$$U_I(t,t) = 1$$

$$i\frac{d}{dt}\left(e^{-iH_0t}|\psi\rangle_I\right) = (H_0 + H_{int})_S\, e^{-iH_0t}|\psi\rangle_I$$

$$i\frac{d}{dt}|\psi\rangle_I = \underbrace{e^{iH_0t}(H_{int})_S\, e^{-iH_0t}}_{H_{1I}}|\psi\rangle_I$$

$$i\frac{d}{dt}U_I(t,t_0) = H_{1I}(t)\,U_I(t,t_0)$$

$$U(t,t_0) \stackrel{?}{=} \exp\left(-i\int_{t_0}^{t} H_{1I}(t')\,dt'\right)$$

$$\frac{d}{dt}\left(\exp\left(-i\int_{t_0}^{t} H_{1I}(t')\,dt'\right)\right) = \frac{d}{dt}\left(1 + (-i)\int_{t_0}^{t} H_{1I}(t')\,dt' + \frac{1}{2}(-i)^2\left(\int_{t_0}^{t} H_{1I}(t')\,dt'\right)^2 + \cdots\right)$$

$$= -iH_{1I}(t) + \frac{(-i)^2}{2}\left(H_{1I}(t)\int_{t_0}^{t} H_{1I}(t')\,dt' + \int_{t_0}^{t} H_{1I}(t')\,dt'\,H_{1I}(t)\right) + \cdots\cdots$$

$$[H_{1I}(t'), H_{1I}(t)] \neq 0$$

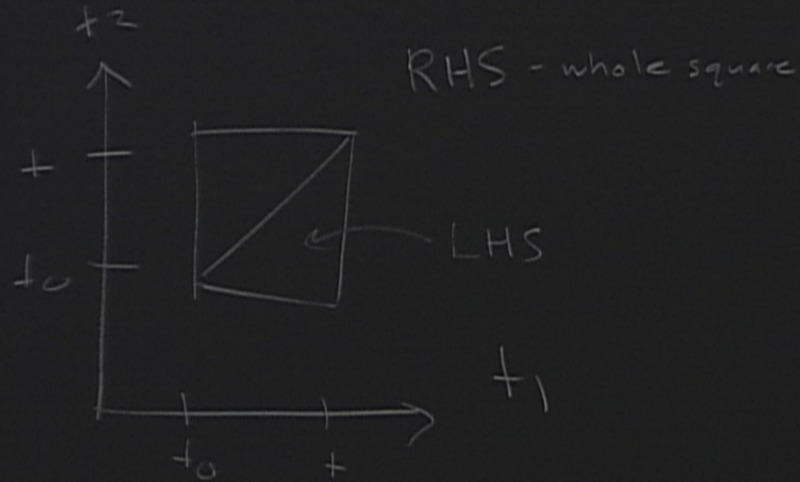
Direct integration:

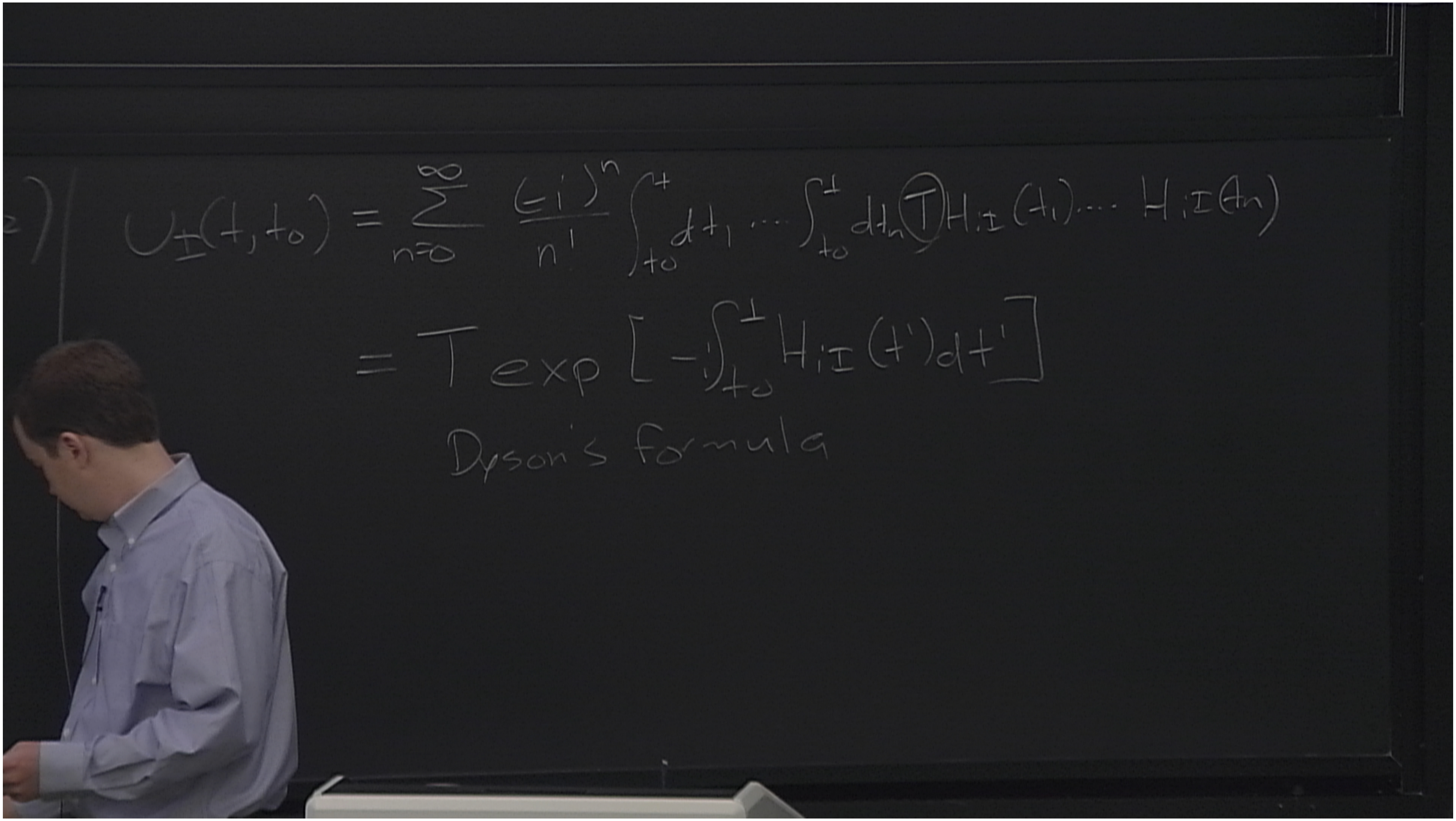
$$U_I(t,t_0) = 1 - i\int_{t_0}^{t} H_{1I}(t_1)\,U_I(t_1,t_0)\,dt_1$$

time ordered $t_1 \geq t_2 \geq \cdots \geq t_n$

$$U_I(t,t_0) = \sum_{n=0}^{\infty}(-i)^n\int_{t_0}^{t}dt_1\int_{t_0}^{t_1}dt_2\cdots\int_{t_0}^{t_{n-1}}dt_n\; H_{1I}(t_1)\cdots H_{1I}(t_n)$$

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, H_{iI}(t_1) H_{iI}(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \; T \, H_{iI}(t_1) H_{iI}(t_2))$$




$U_I(t,t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} dt_n \, T \, H_{iI}(t_1) \cdots H_{iI}(t_n)$
$= T \exp\left[-i \int_{t_0}^{t} H_{iI}(t') dt' \right]$
Dyson's formula

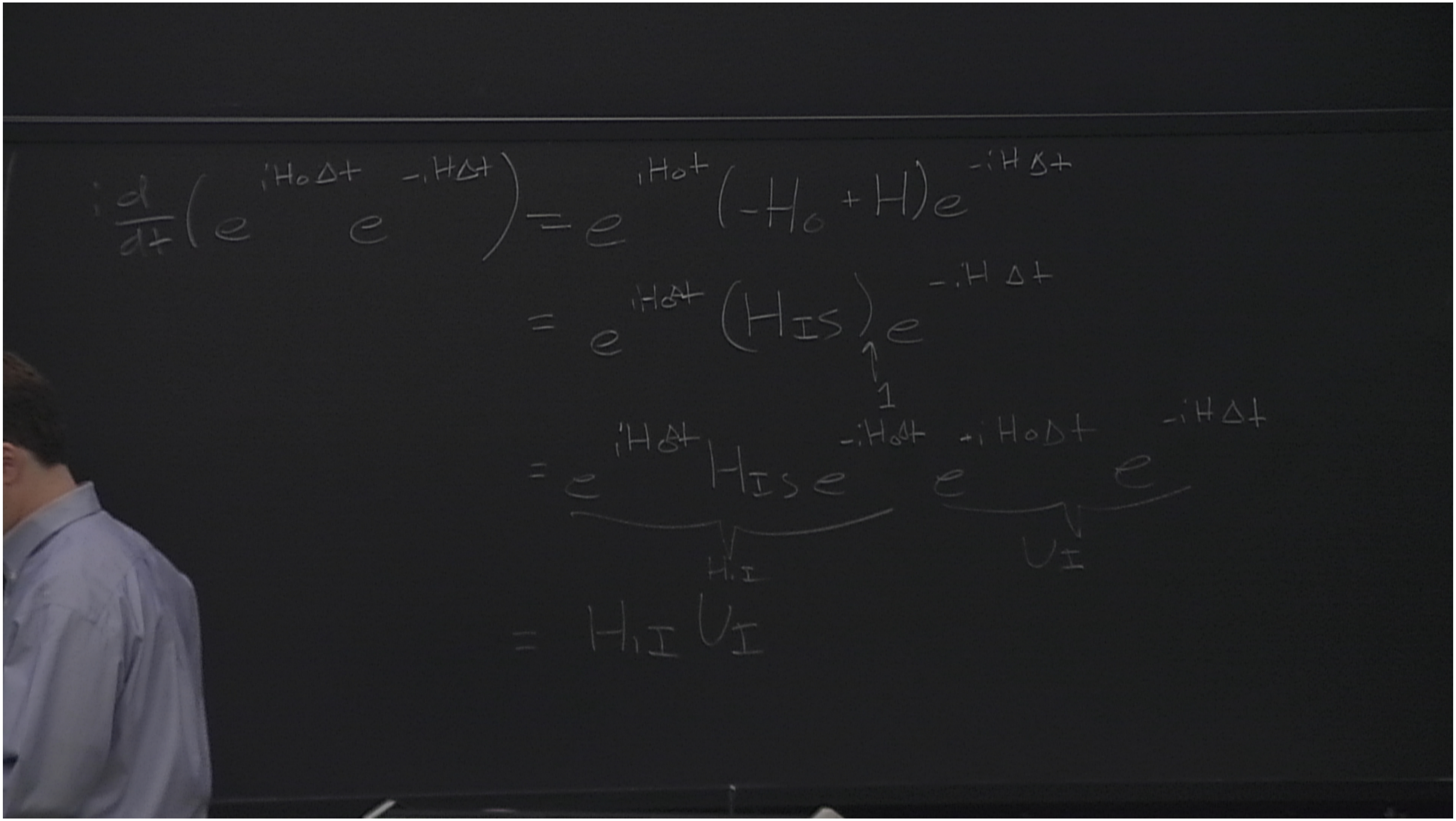
Field in interaction picture:

$$\varphi(t_0,\vec{x}) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\varphi_H(t,\vec{x}) = e^{iH(t-t_0)}\, \varphi(t_0,\vec{x})\, e^{-iH\underbrace{(t-t_0)}_{\Delta t}}$$

$1 \qquad 1$

$$= \underbrace{e^{iH\Delta t}\, e^{-iH_0\Delta t}}_{U_I^\dagger(t,t_0)}\; \underbrace{e^{iH_0\Delta t}\, \varphi(t_0,\vec{x})\, e^{-iH_0\Delta t}}_{\varphi_I}\; \underbrace{e^{iH_0\Delta t}\, e^{-iH\Delta t}}_{U_I(t,t_0)}$$



$i\frac{d}{dt}\left(e^{iH_0\Delta t}e^{-iH\Delta t}\right) = e^{iH_0 t}(-H_0+H)e^{-iH\Delta t}$
$= e^{iH_0\Delta t}(H_{IS})e^{-iH\Delta t}$
1
$= \underbrace{e^{iH_0\Delta t}H_{IS}e^{-iH_0\Delta t}}_{H_{1I}}\underbrace{e^{+iH_0\Delta t}e^{-iH\Delta t}}_{U_I}$
$= H_{1I}U_I$

$U_I^\dagger(t,t_0)$ φ_I $U_I(t,t_0)$

$$H_I(t) = \int d^3x \, \frac{\lambda}{4!} \varphi_I^4 \quad \text{iff} \quad H_{int} = \int d^3x \, \frac{\lambda}{4!} \varphi^4$$

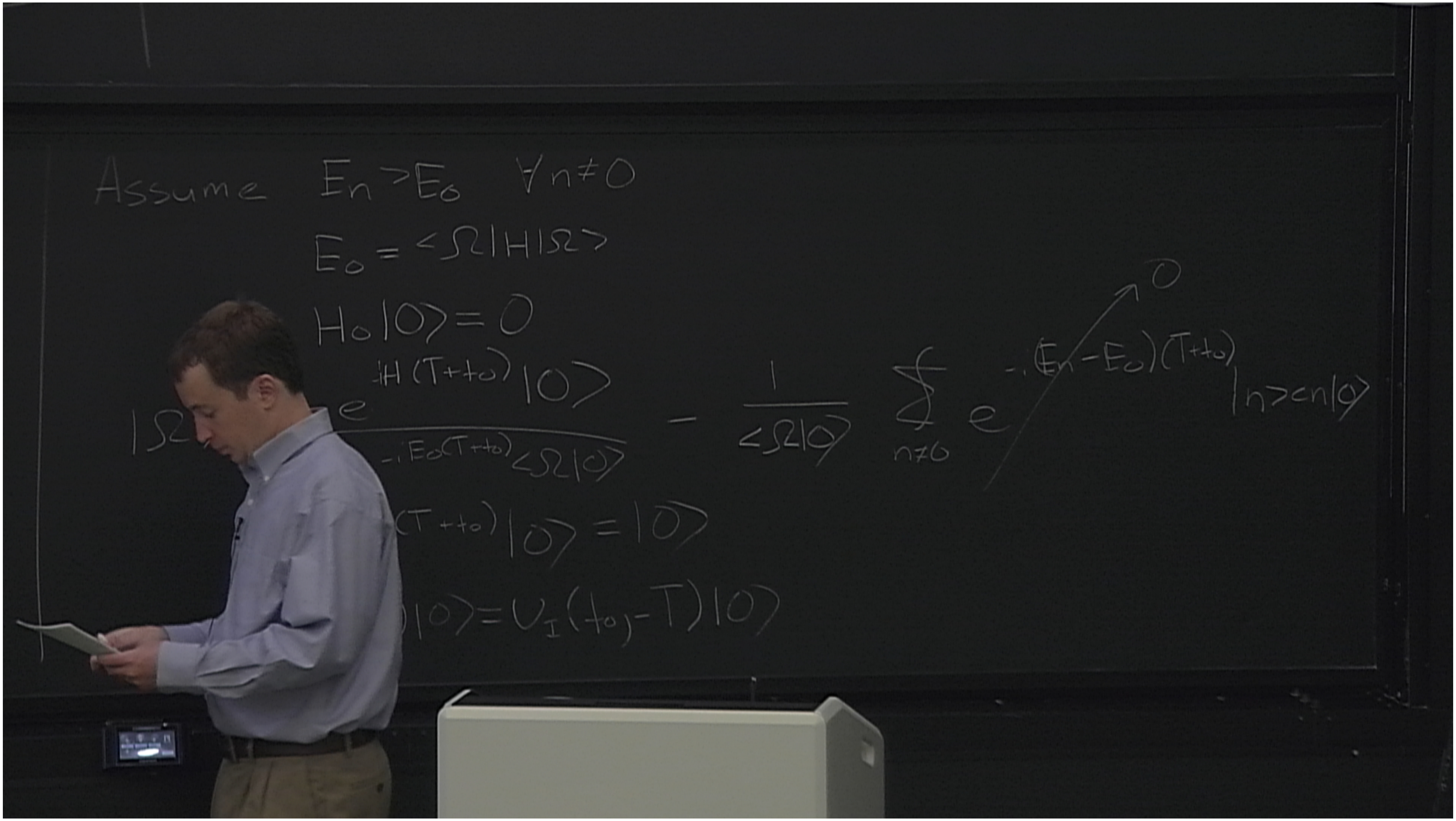
Interacting Vacuum $|\Omega\rangle$

$$e^{iH(T+t_0)} |0\rangle = e^{iH(T+t_0)} \mathbb{1} |0\rangle$$

Complete set of states

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_{n \neq 0} |n\rangle\langle n|$$

other states


Assume $E_n > E_0 \quad \forall n \neq 0$
$E_0 = \langle \Omega | H | \Omega \rangle$
$H_0 |0\rangle = 0$
$e^{-iH(T+t_0)}|0\rangle$
$e^{-iE_0(T+t_0)}\langle \Omega | 0 \rangle$
$- \frac{1}{\langle \Omega | 0 \rangle} \sum_{n \neq 0} e^{-i(E_n - E_0)(T+t_0)} |n\rangle\langle n|0\rangle$
$\rightarrow 0$
$(T+t_0)|0\rangle = |0\rangle$
$|0\rangle = U_I(t_0, -T)|0\rangle$

Assume $E_n > E_0 \quad \forall n \neq 0$

$$E_0 = \langle \Omega | H | \Omega \rangle$$

$$H_0 |0\rangle = 0$$

$$|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{e^{-iH(T+t_0)}|0\rangle}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle} - \frac{1}{\langle\Omega|0\rangle}\sum_{n\neq 0} \cancelto{0}{e^{-i(E_n-E_0)(T+t_0)}}|n\rangle\langle n|0\rangle$$

$$e^{iH_0(T+t_0)}|0\rangle = |0\rangle$$

$$U(t_0,T)|0\rangle = U_I(t_0,-T)|0\rangle$$

H_I

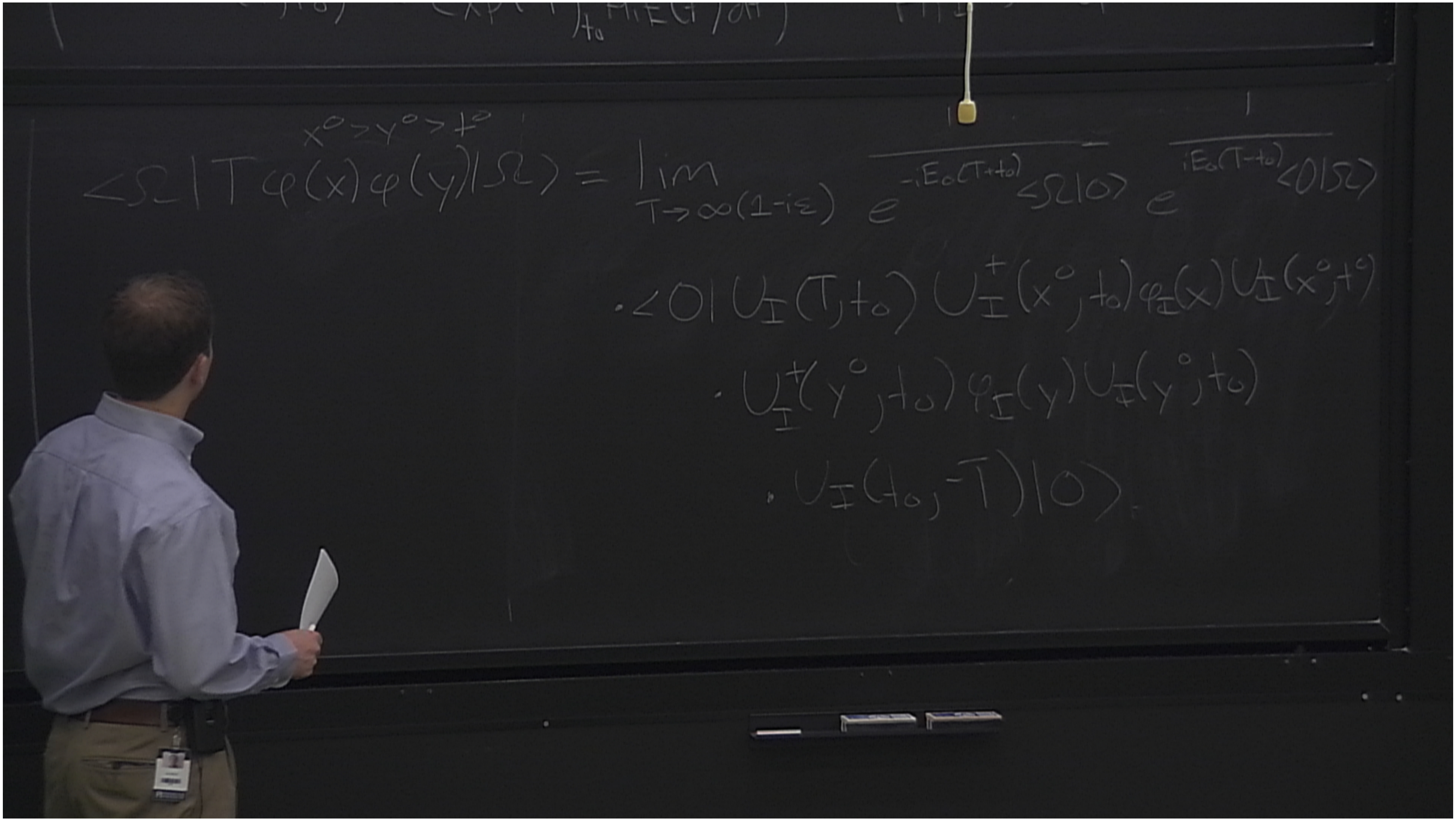
$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{U_I(t_0,-T)|0\rangle}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle}$$

$$\langle\Omega| = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|U_I(T,t_0)}{e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle}$$

H_{iI}

$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{U_I(t_0,-T)|0\rangle}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle}$$

$$\langle\Omega| = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|U_I(T,t_0)}{e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle}$$


$x^0 > y^0 > t^0$
$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{1}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle}\,\frac{1}{e^{iE_0(T-t_0)}\langle 0|\Omega\rangle}$
$\cdot\langle 0|U_I(T,t_0)\,U_I^\dagger(x^0,t_0)\,\varphi_I(x)\,U_I(x^0,t^0)$
$\cdot U_I^\dagger(y^0,t_0)\,\varphi_I(y)\,U_I(y^0,t_0)$
$\cdot U_I(t_0,-T)|0\rangle.$

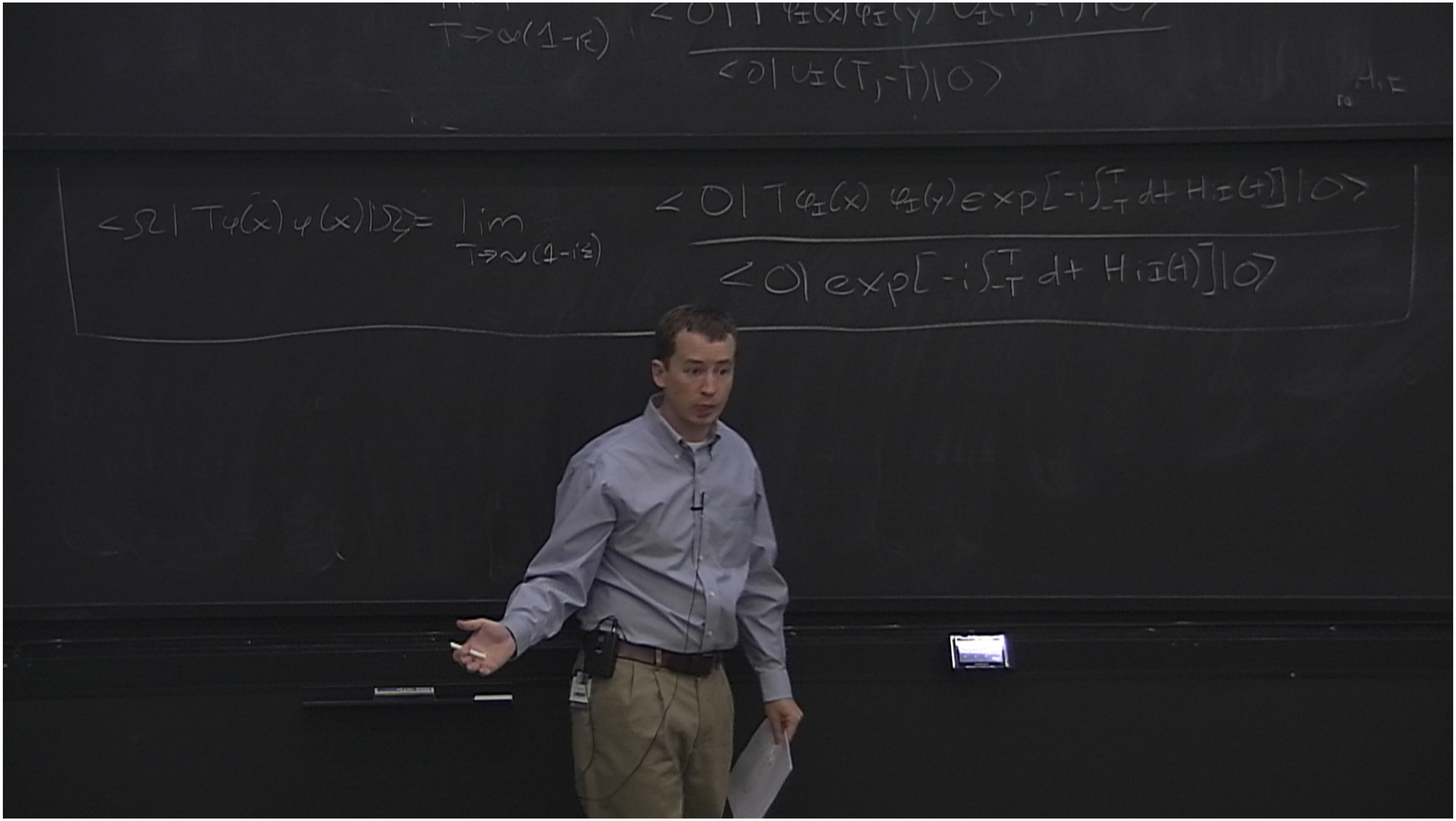
$T \to \infty(1-i\varepsilon)$ $\quad e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle$

$\cdot\, U_I(y, t_0)$

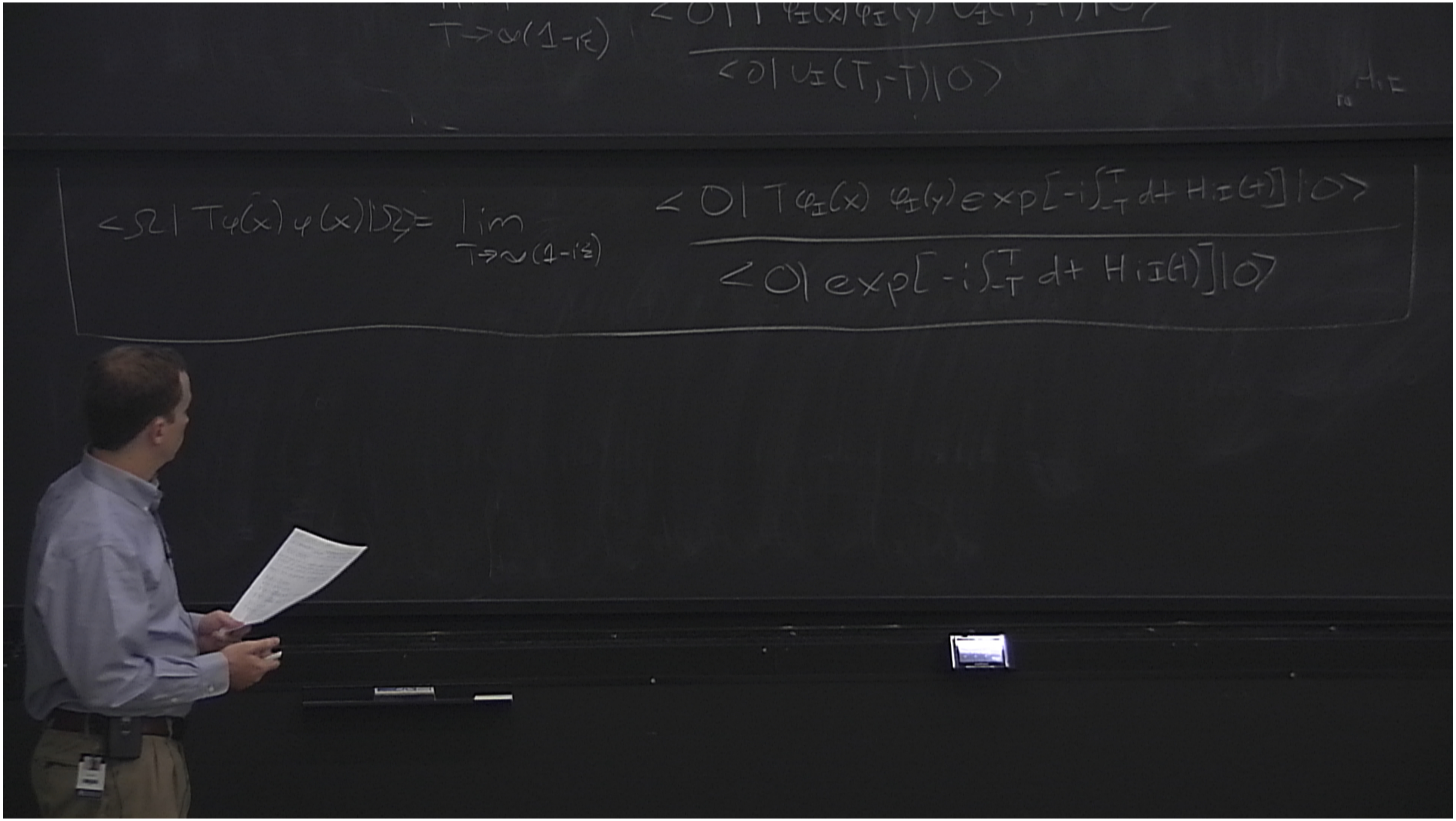
$\cdot\, U_I(t_0, -T$

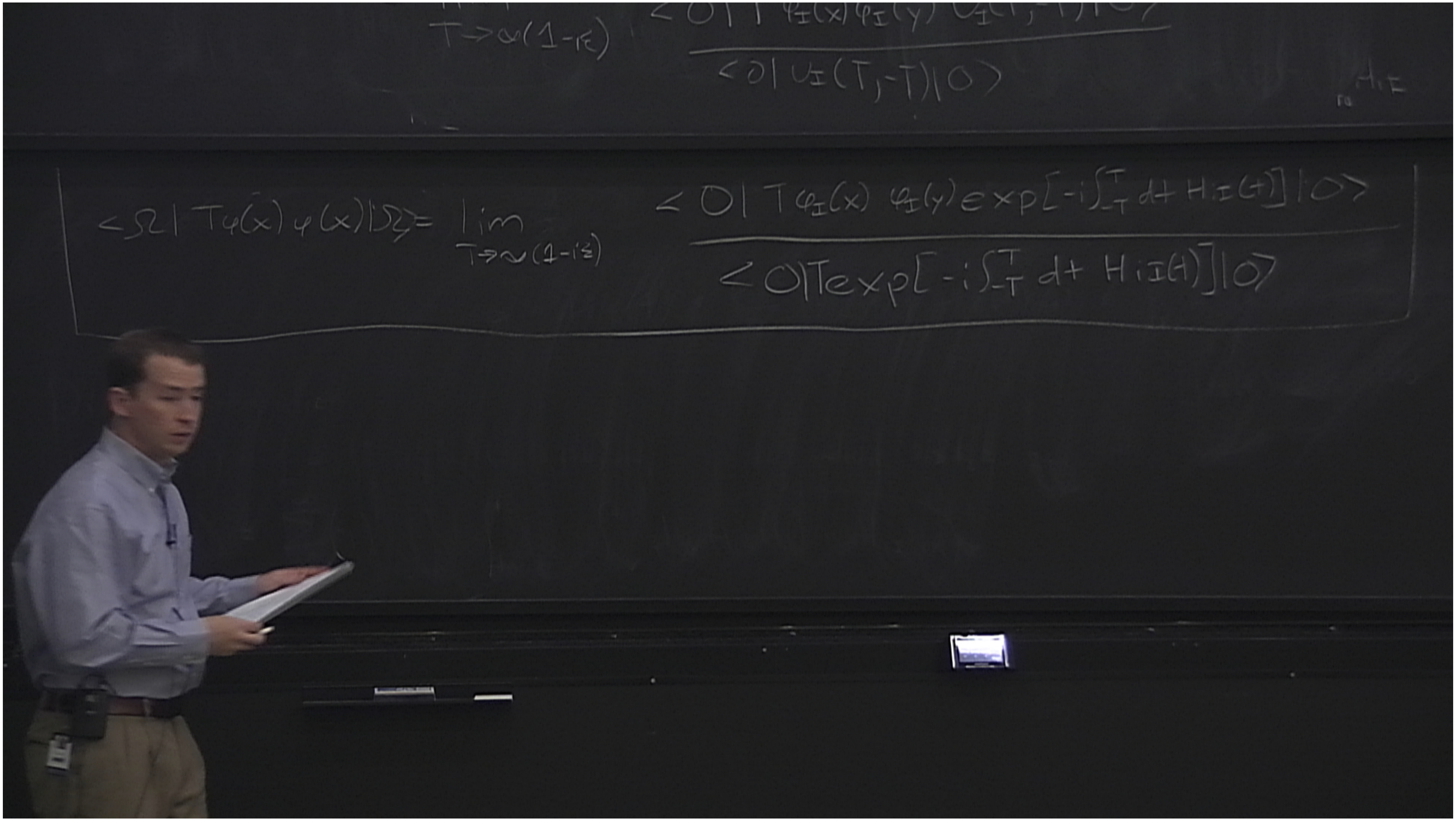
$$1 = \langle\Omega|\Omega\rangle = \frac{1}{|\langle\Omega|0\rangle|^2 e^{-2iE_0T}} \langle 0|\, U_I(T,t_0)\, U_I(t_0,-T)|0\rangle$$

$$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|\, U_I(T,x^0)\varphi_I(x)\, U_I(x^0,y^0)\, \varphi_I(y) U(y^0,-T)|0\rangle}{\langle 0|\, U_I(T,-T)|\,0\rangle}$$



$T \to \infty(1-i\epsilon)$
$\dfrac{\langle 0 | T\, \varphi_I(x)\varphi_I(y)\, U_I(T,-T) | 0 \rangle}{\langle 0 | U_I(T,-T) | 0 \rangle}$
$\langle \Omega | T \varphi(x) \varphi(x) | \Omega \rangle = \lim_{T \to \infty(1-i\epsilon)} \dfrac{\langle 0 | T \varphi_I(x)\, \varphi_I(y) \exp[-i\int_{-T}^{T} dt\, H_I(t)] | 0 \rangle}{\langle 0 | \exp[-i\int_{-T}^{T} dt\, H_I(t)] | 0 \rangle}$


$T \to \infty(1-i\epsilon)$
$\langle 0 | U_I(T,-T) | 0 \rangle$
$\langle \Omega | T \varphi(x) \varphi(x) | \Omega \rangle = \lim_{T \to \infty(1-i\epsilon)} \frac{\langle 0 | T \varphi_I(x) \varphi_I(y) \exp[-i\int_{-T}^{T} dt\, H_{iI}(t)] | 0 \rangle}{\langle 0 | \exp[-i\int_{-T}^{T} dt\, H_{iI}(t)] | 0 \rangle}$



$\langle\Omega|T\varphi(x)\varphi(x)|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\varphi_I(x)\varphi_I(y)\exp[-i\int_{-T}^{T}dt\,H_I(t)]|0\rangle}{\langle 0|T\exp[-i\int_{-T}^{T}dt\,H_I(t)]|0\rangle}$
$\langle 0|U_I(T,-T)|0\rangle$

$$\lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\,\varphi_I(x)\varphi_I(y)\,U_I(T,-T)|0\rangle}{\langle 0|U_I(T,-T)|0\rangle}$$

$$\boxed{\langle\Omega|T\varphi(x)\varphi(x)|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\varphi_I(x)\,\varphi_I(y)\exp\left[-i\int_{-T}^{T}dt\,H_{iI}(t)\right]|0\rangle}{\langle 0|T\exp\left[-i\int_{-T}^{T}dt\,H_{iI}(t)\right]|0\rangle}}$$

Exact expression

drop I's from now on

$$\langle 0|T\,\varphi_1\varphi_2\cdots\varphi_n|0\rangle$$

Wick's Theorem

$$\varphi(x) = \varphi^+(x) + \varphi^-(x)$$

$$\varphi^+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}} e^{-ip\cdot x}$$ positive frequency

$$\varphi^-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a^\dagger_{\vec{p}} e^{ipx}$$ neg. freq.

For $x^0 > y^0$

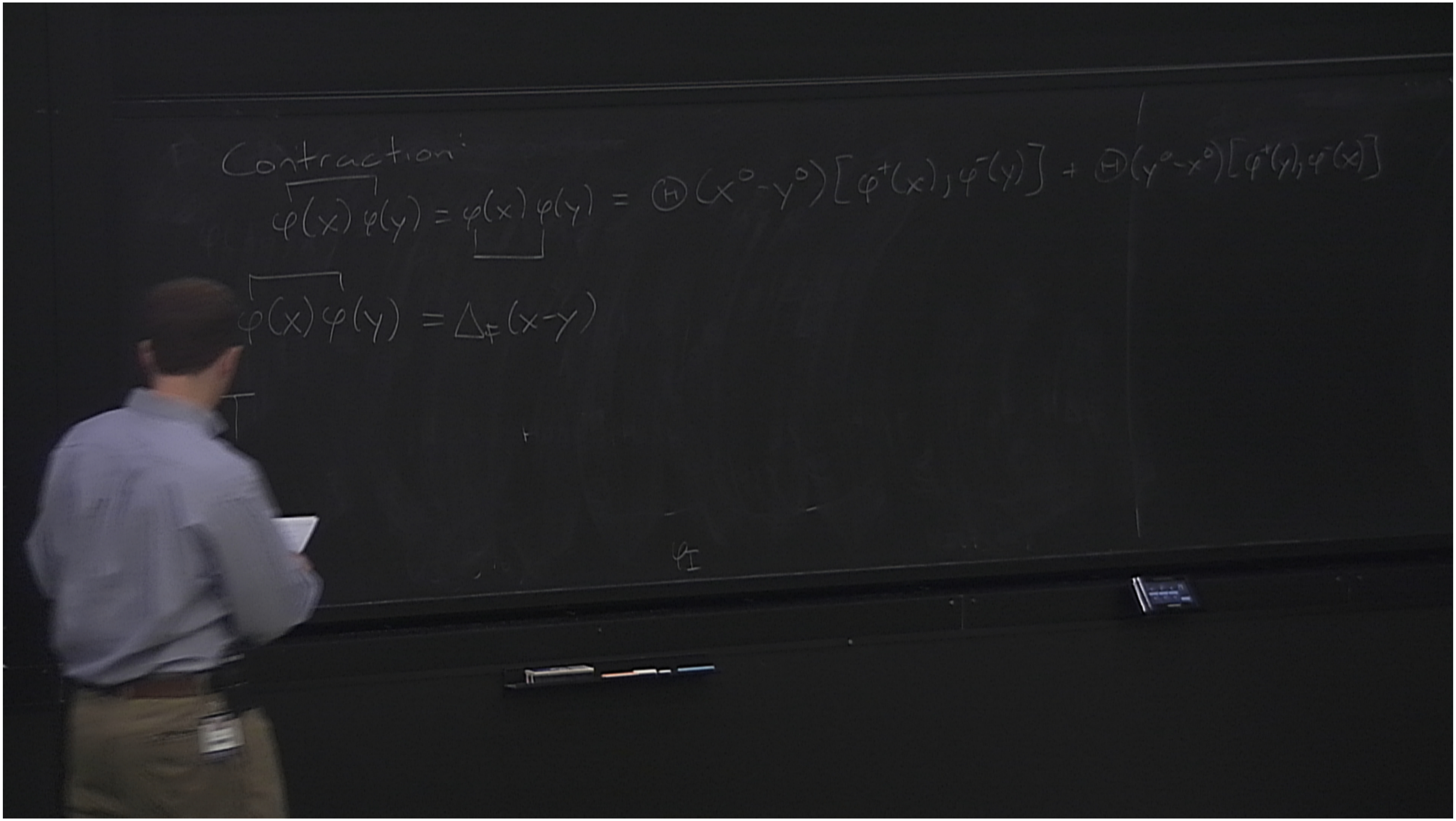
$$T\varphi(x)\varphi(y) = \varphi(x)\varphi(y)$$

$$= \left(\varphi^+(x) + \varphi^-(x)\right)\left(\varphi^+(y) + \varphi^-(y)\right)$$

$$= \varphi^+(x)\varphi^+(y) + \varphi^-(y)\varphi^+(x) + [\varphi^+(x), \varphi^-(y)]$$

$$+ \varphi^-(x)\varphi^+(y) + \varphi^-(x)\varphi^-(y)$$

For $y^0 > x^0$ same except commutator would be $[\varphi^+(y), \varphi^-(x)]$


Contraction:
$\varphi(x)\varphi(y) = \varphi(x)\varphi(y) = \Theta(x^0-y^0)[\varphi^+(x),\varphi^-(y)] + \Theta(y^0-x^0)[\varphi^+(y),\varphi^-(x)]$
$\varphi(x)\varphi(y) = \Delta_F(x-y)$
φ_I

Contraction:

$$\varphi(x)\varphi(y) = \varphi(x)\varphi(y) = \Theta(x^0-y^0)\left[\varphi^+(x), \varphi^-(y)\right] + \Theta(y^0-x^0)\left[\varphi^+(y), \varphi^-(x)\right]$$

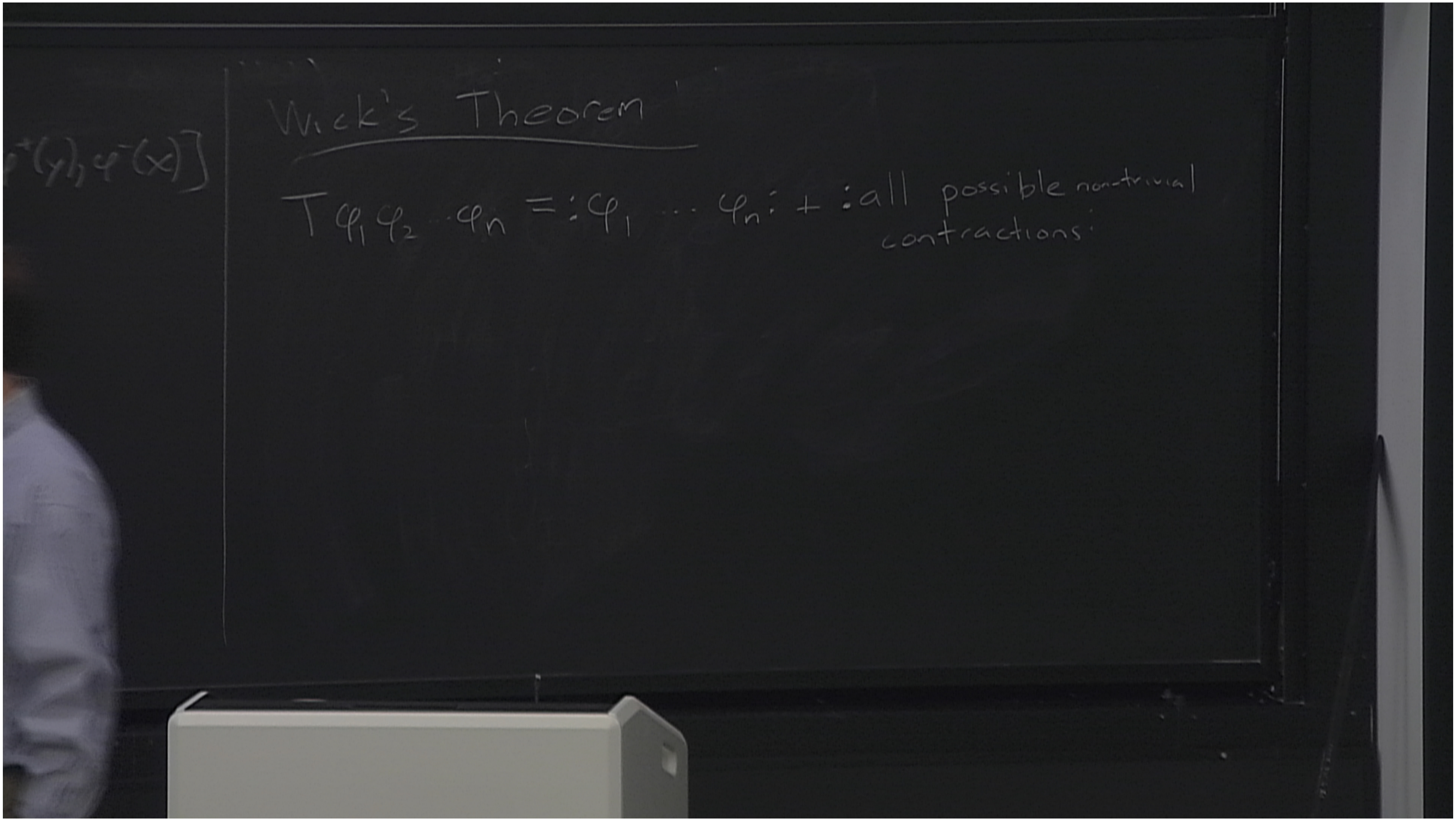
$$\varphi(x)\varphi(y) = \Delta_F(x-y)$$

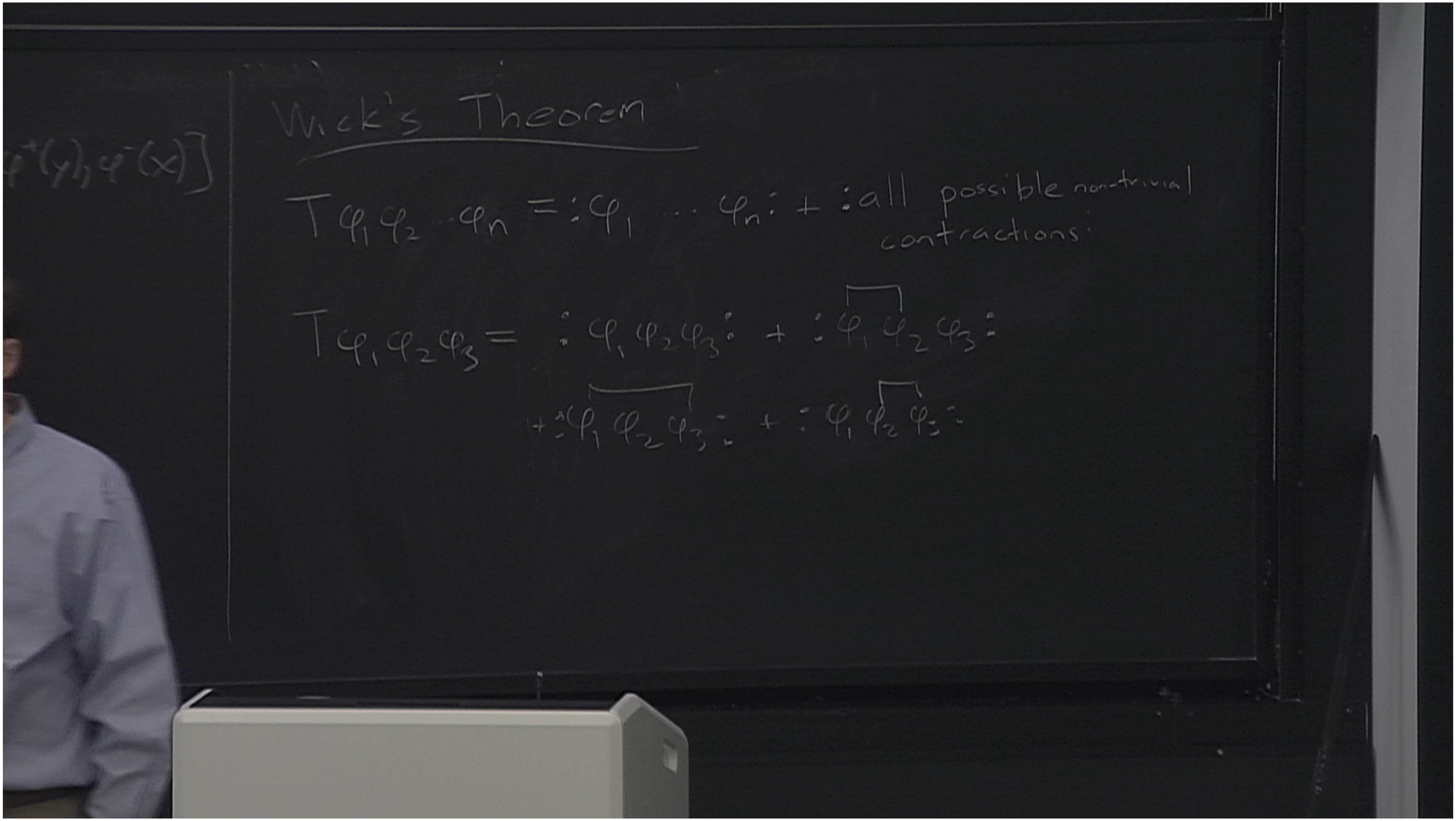
$$\varphi(x)\varphi(y)$$

$$||$$

$$T\varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \Delta_F(x-y)$$

φ_I


$\varphi^+(y), \varphi^-(x)]$
Wick's Theorem
$T\varphi_1\varphi_2\cdots\varphi_n = :\varphi_1\cdots\varphi_n: + :$all possible non-trivial contractions:


Wick's Theorem
$T\varphi_1\varphi_2\cdots\varphi_n = :\varphi_1\cdots\varphi_n: + :$all possible non-trivial contractions$:$
$T\varphi_1\varphi_2\varphi_3 = :\varphi_1\varphi_2\varphi_3: + :\varphi_1\varphi_2\varphi_3:$
$+ :\varphi_1\varphi_2\varphi_3: + :\varphi_1\varphi_2\varphi_3:$

φ_I

Assume Wick's Theorem for $n=m-1$

WLOG $\quad x_1^0 \geq x_2^0 \cdots \geq x_m^0$

$$T\varphi_1 \cdots \varphi_m = \varphi_1 T\varphi_2 \cdots \varphi_m$$

$$= (\varphi_1^+ + \varphi_1^-)\left(:\varphi_2 \cdots \varphi_m: + :\text{all contractions w/o } \varphi_1: \right)$$

φ_1^- can go inside $::$

φ_I

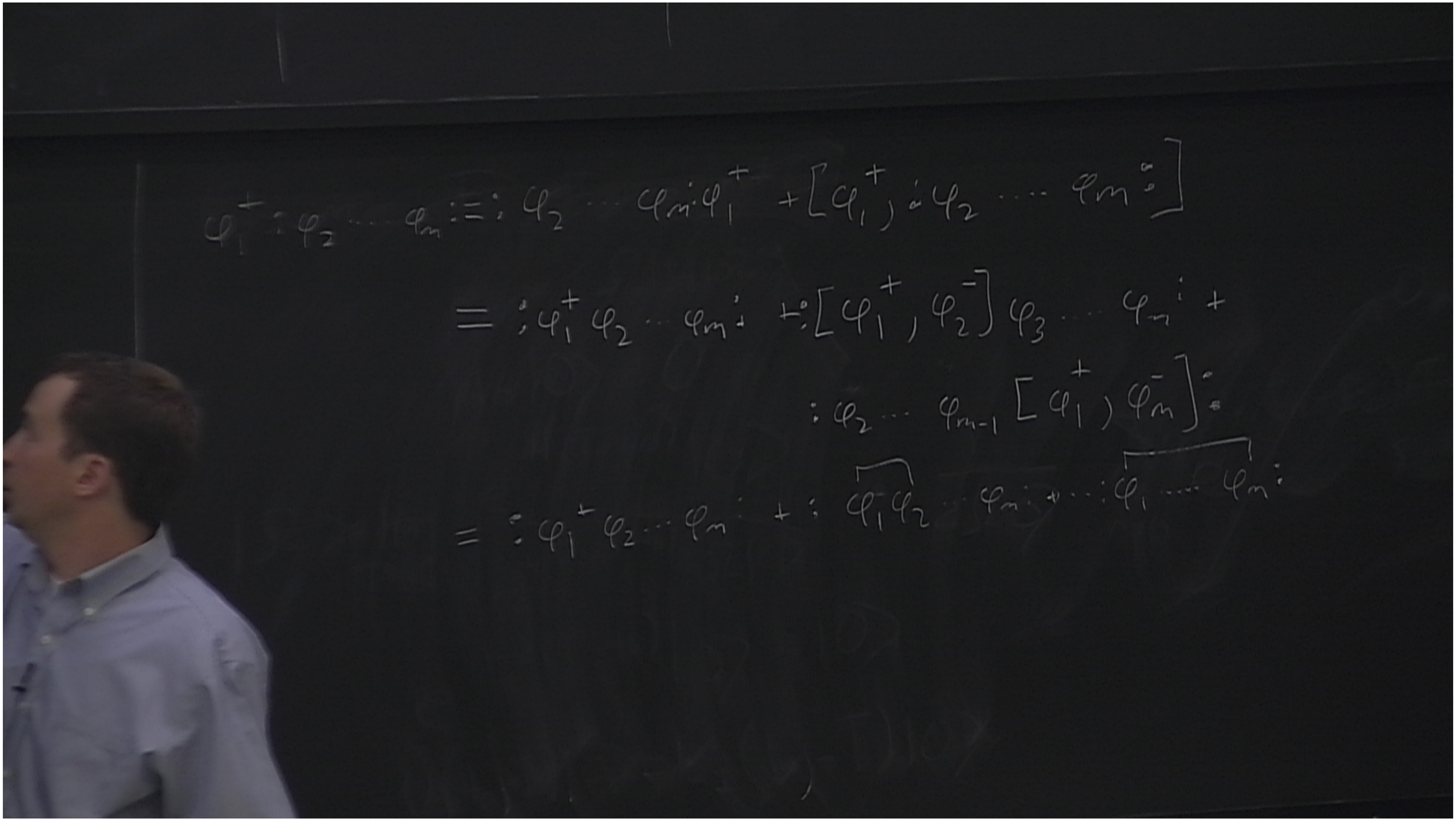
Assume Wick's Theorem for $n=m-1$

WLOG $x_1^0 \geq x_2^0 \cdots \geq x_m^0$

$$T\varphi_1 \cdots \varphi_m = \varphi_1 T\varphi_2 \cdots \varphi_m$$

$$= (\varphi_1^+ + \varphi_1^-)\left(:\varphi_2 \cdots \varphi_m: + :\text{all contractions w/o } \varphi_1: \right)$$

φ_1^- can go inside $::$



$\varphi_1^+ :\varphi_2 \cdots \varphi_m: = :\varphi_2 \cdots \varphi_m \varphi_1^+: + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$
$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-]\varphi_3 \cdots \varphi_m: +$
$:\varphi_2 \cdots \varphi_{m-1} [\varphi_1^+, \varphi_m^-]:$
$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_m: + \cdots :\overbrace{\varphi_1 \cdots \varphi_m}:$

ctions

$$\varphi_1^+ :\varphi_2 \cdots \varphi_n: \;=\; :\varphi_2 \cdots \varphi_n \varphi_1^+: + \left[\varphi_1^+ , :\varphi_2 \cdots \varphi_n:\right]$$

$$= :\varphi_1^+ \varphi_2 \cdots \varphi_n: + :[\varphi_1^+ , \varphi_2^-]\,\varphi_3 \cdots \varphi_n: + \cdots + :\varphi_2 \cdots \varphi_{n-1} [\varphi_1^+ , \varphi_n^-]:$$

$$= :\varphi_1^+ \varphi_2 \cdots \varphi_n: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_n: + \cdots + :\overbrace{\varphi_1 \cdots \varphi_n}:$$

$$\langle 0| :\varphi_1 \cdots \varphi_n: |0\rangle$$

$$\langle 0| T \varphi_1 \varphi_2 \varphi_3 \varphi_4 |0\rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$$

$$\Delta_{12} = \Delta_F(x_1 - x_2)$$

$$= :\varphi_1^+ \varphi_2 \cdots \varphi_n: + :[\varphi_1^+, \varphi_2^-]\,\varphi_3 \cdots \varphi_n: +$$

$$:\varphi_2 \cdots \varphi_{n-1}\,[\varphi_1^+, \varphi_n^-]:$$

$$= :\varphi_1^+ \varphi_2 \cdots \varphi_n: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_n: + \cdots :\overbrace{\varphi_1 \cdots \varphi_n}:$$

$$\langle 0| :\varphi_1 \cdots \varphi_n: |0\rangle = 0$$

$$\langle 0| T \varphi_1 \varphi_2 \varphi_3 \varphi_4 |0\rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$$

$$\Delta_{12} = \Delta_F(x_1 - x_2)$$