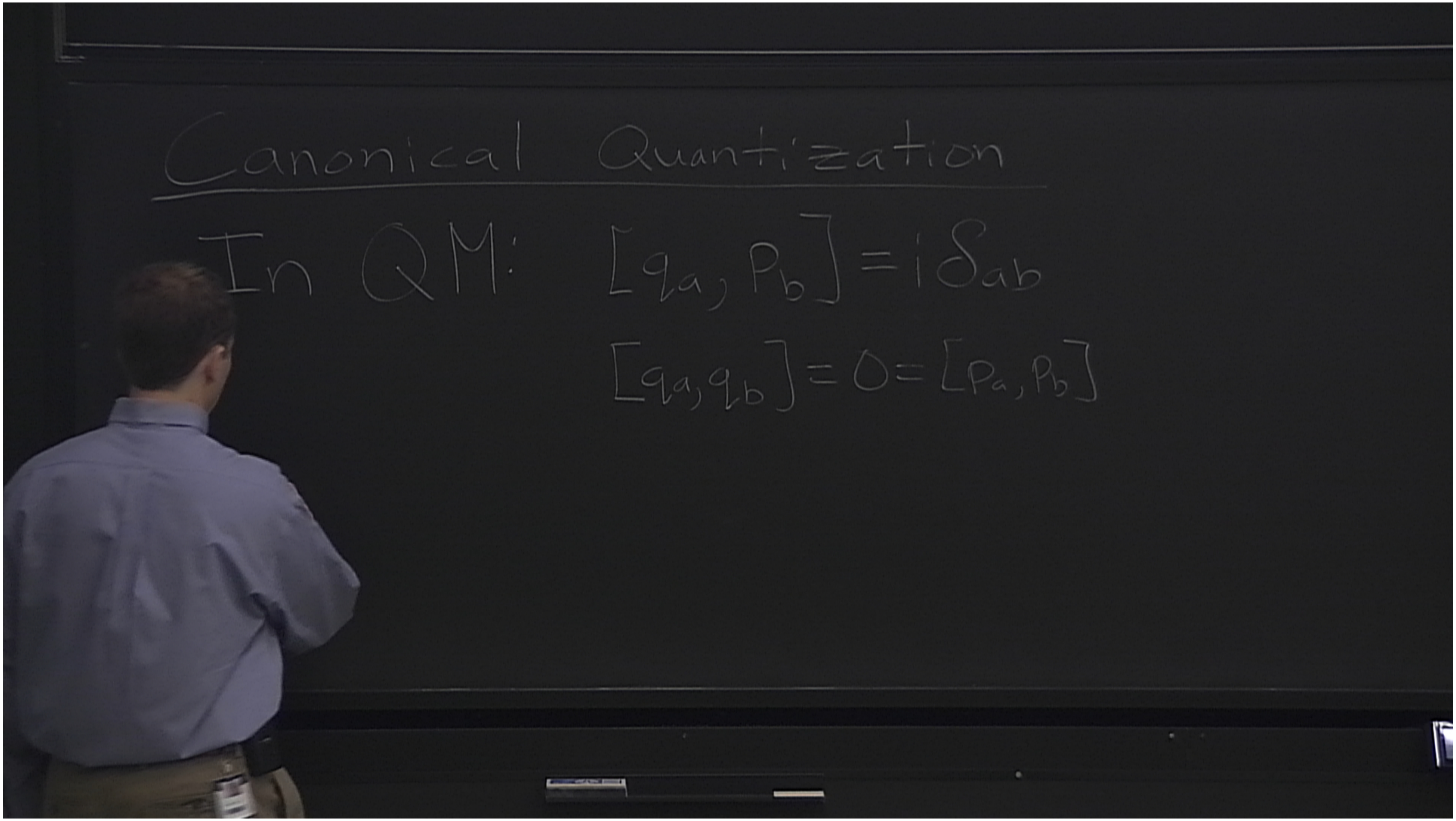
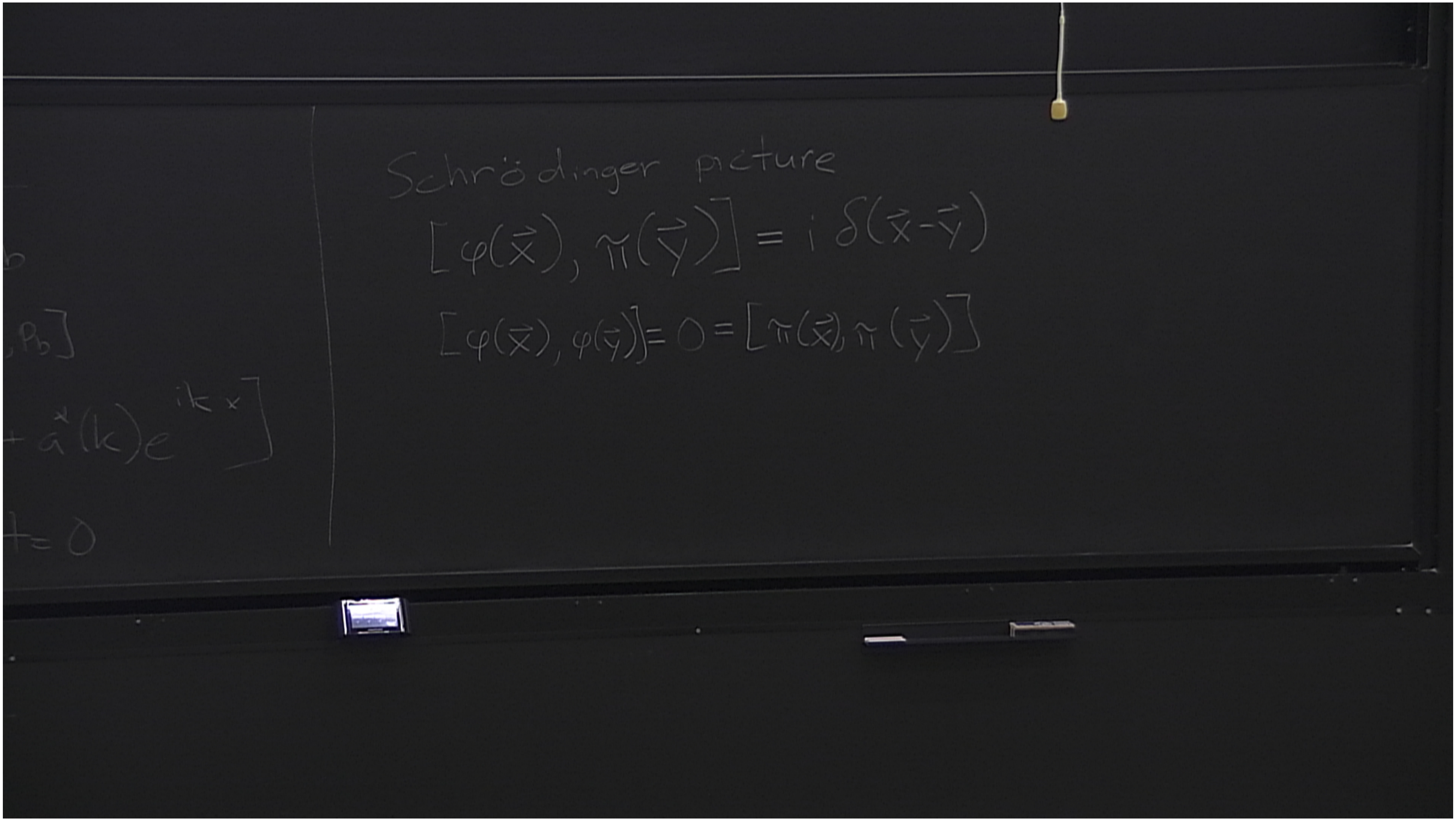
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 2

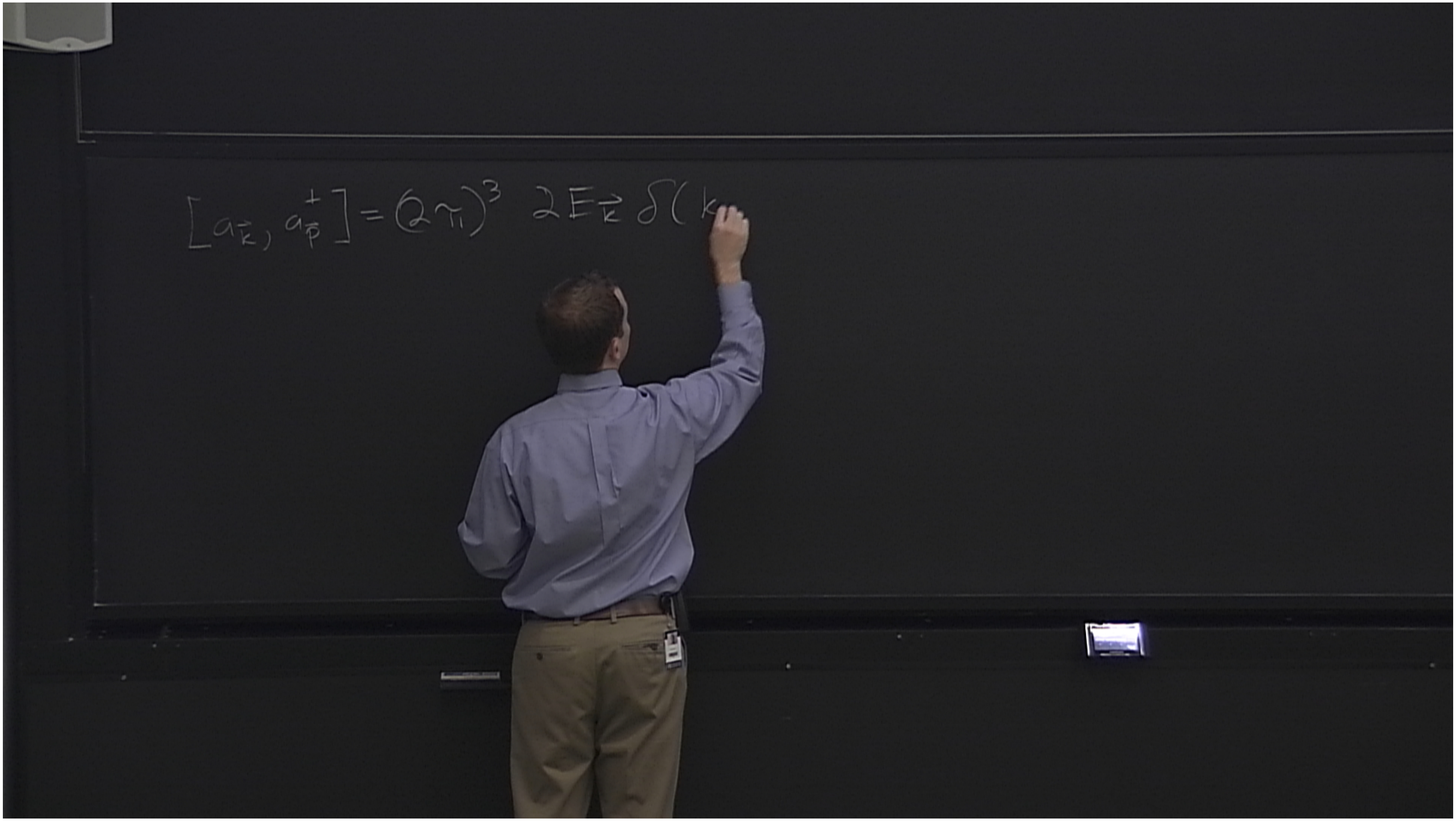
Date: Oct 14, 2015 09:00 AM

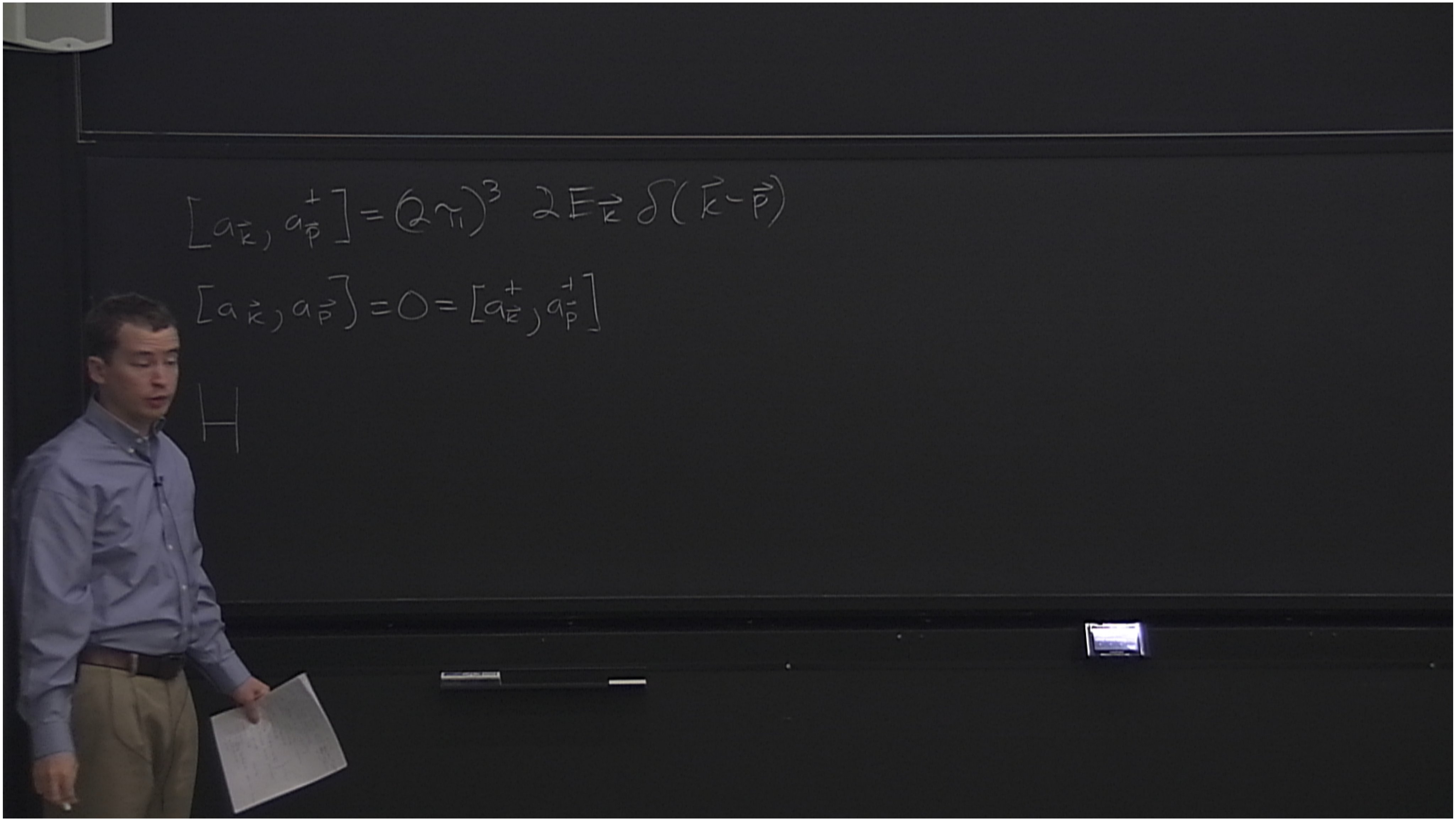
URL: http://pirsa.org/15100037

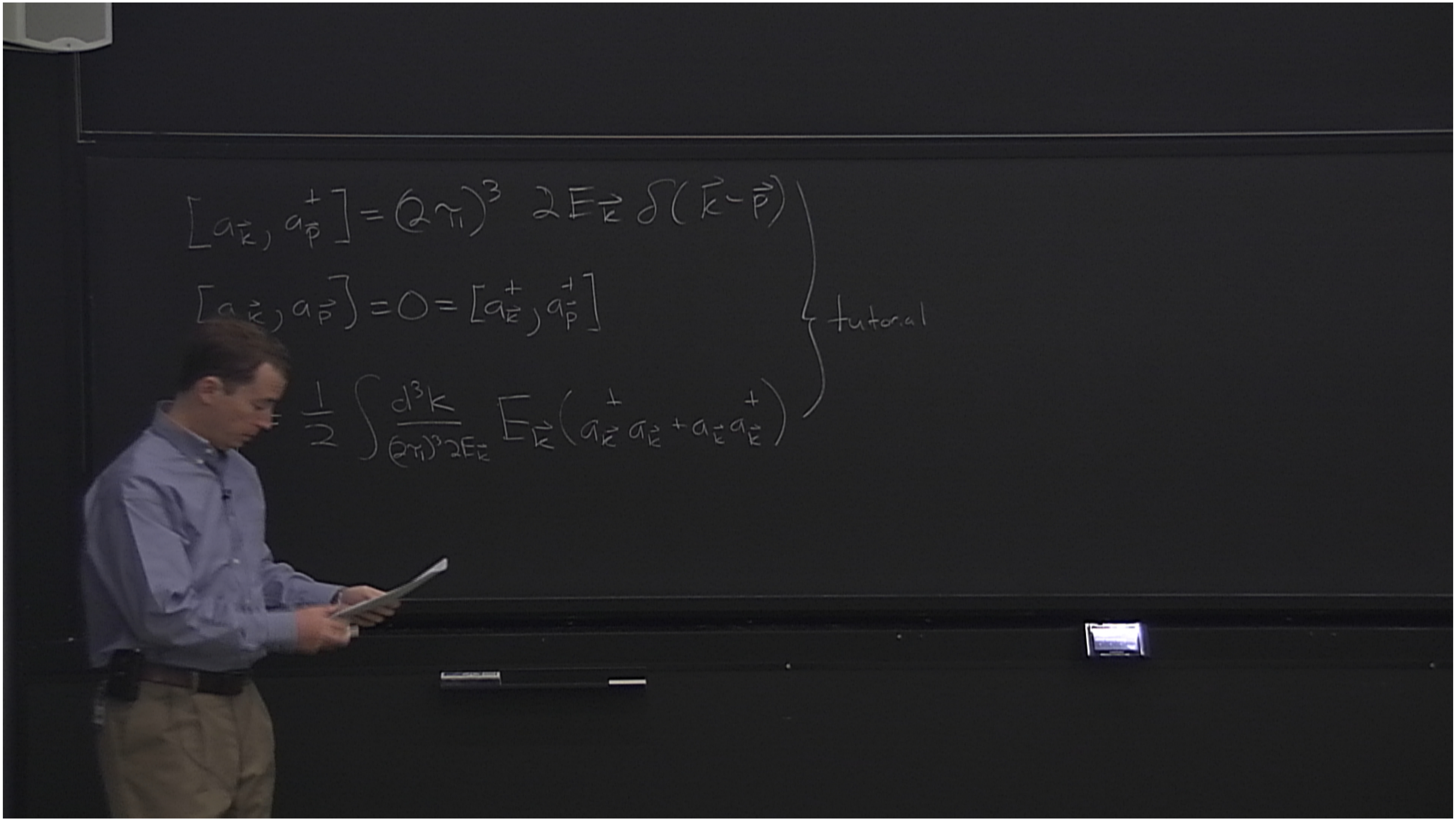
Abstract:

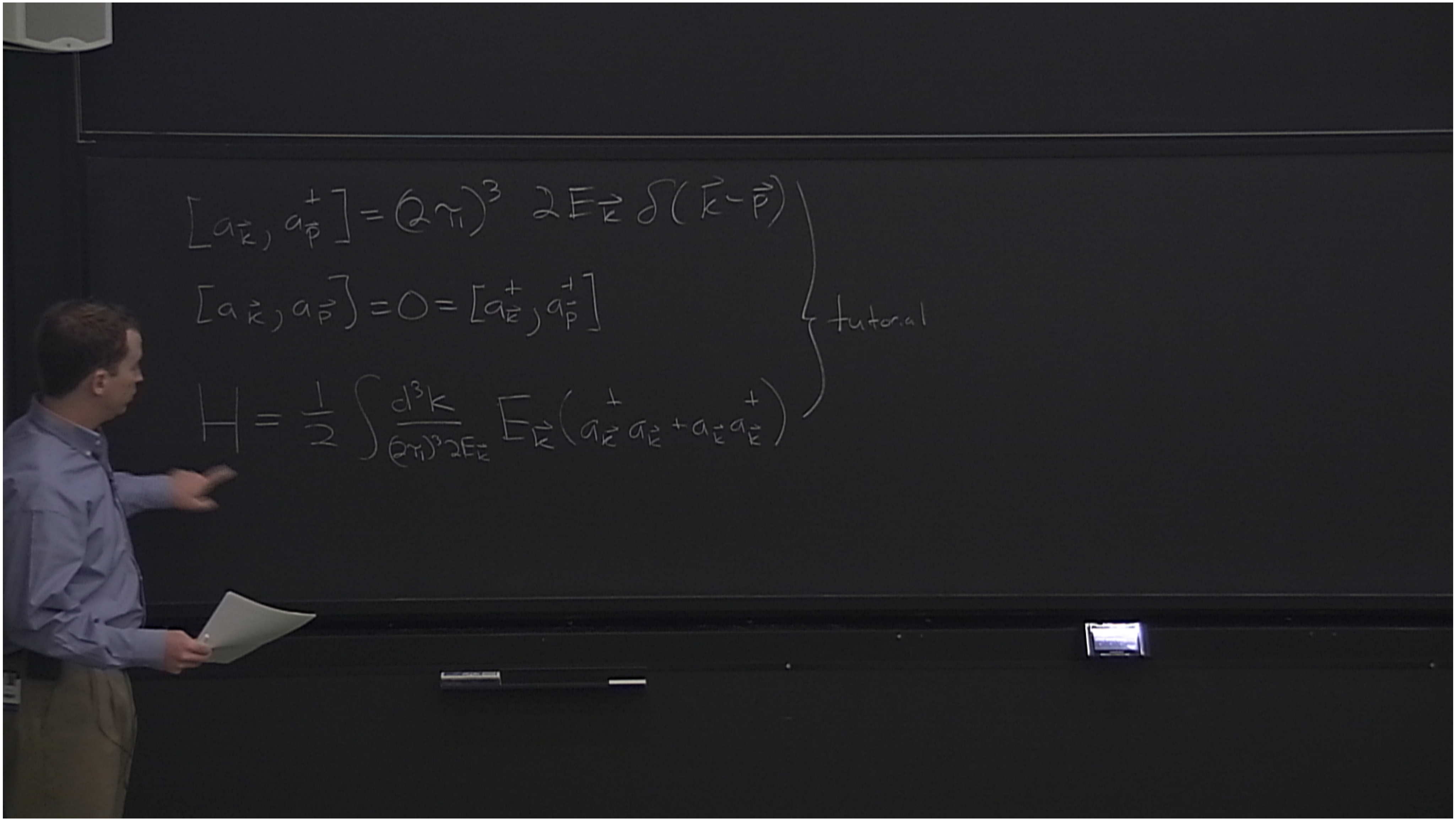

Canonical Quantization
In QM: [q_a, p_b] = iδ_ab
[q_a, q_b] = 0 = [p_a, p_b]

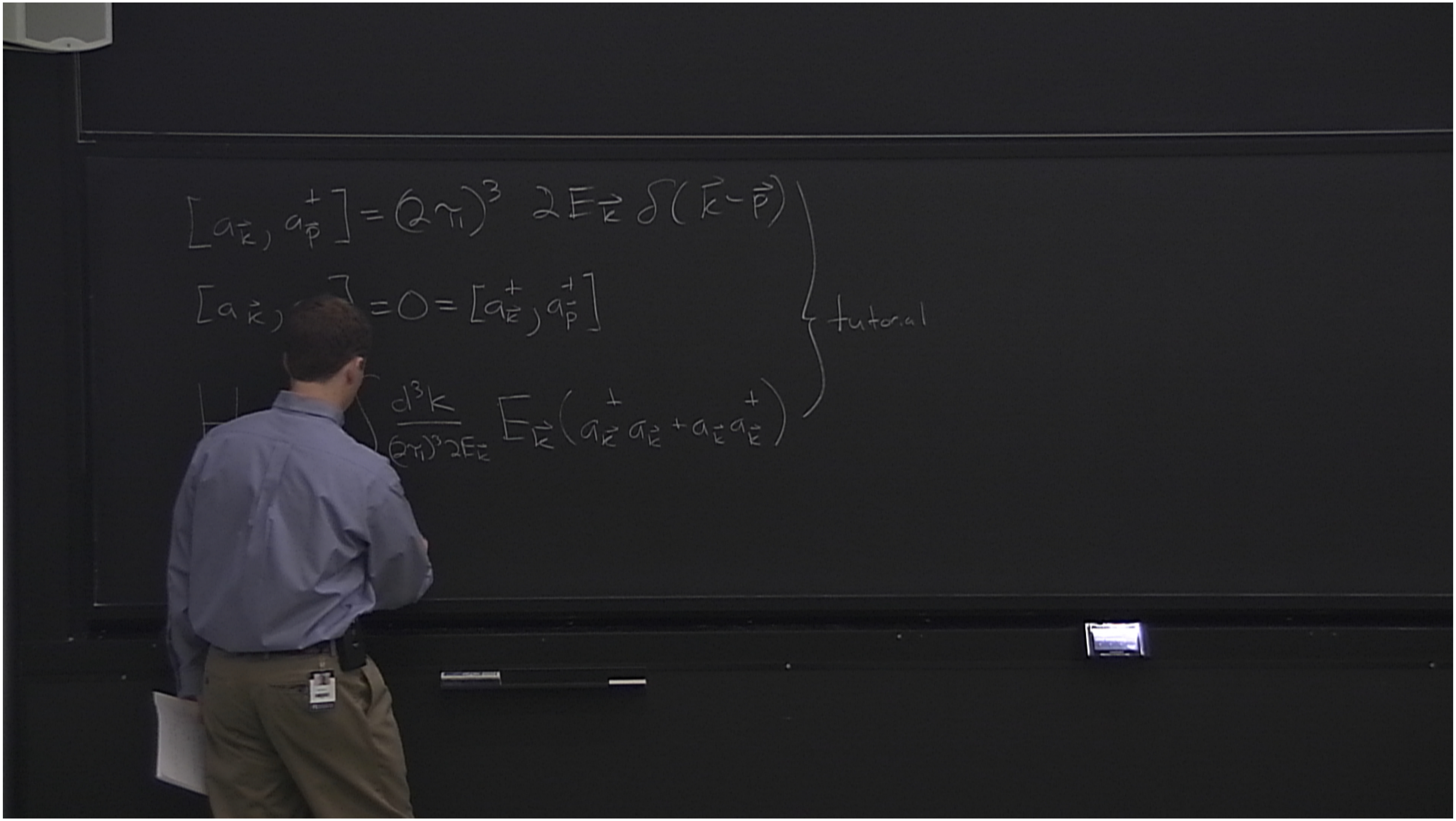

Schrödinger picture
[φ(x⃗), π(y⃗)] = i δ(x⃗ − y⃗)
[φ(x⃗), φ(y⃗)] = 0 = [π(x⃗), π(y⃗)]

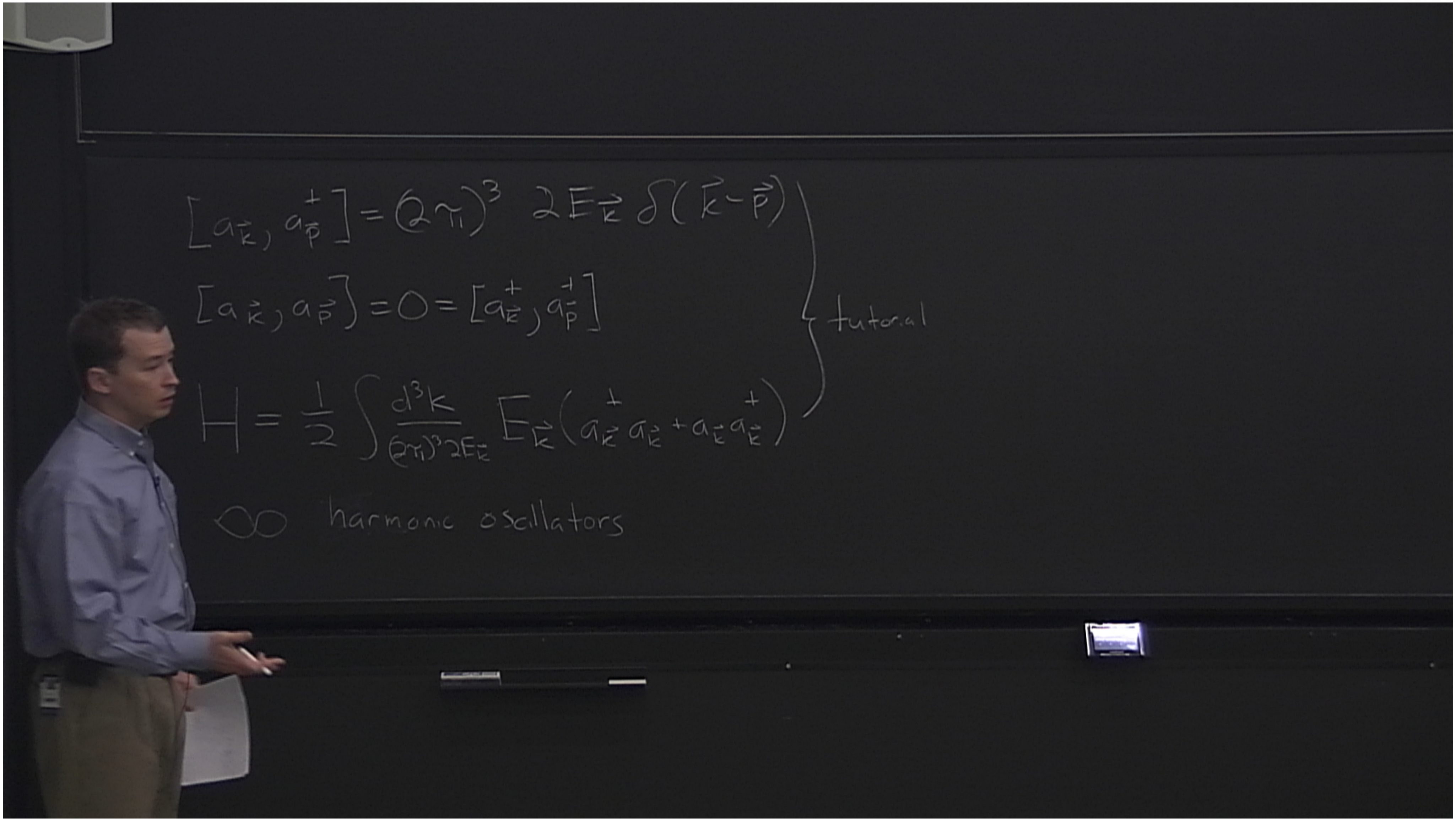

$[a_{\vec{k}}, a^{+}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(k$

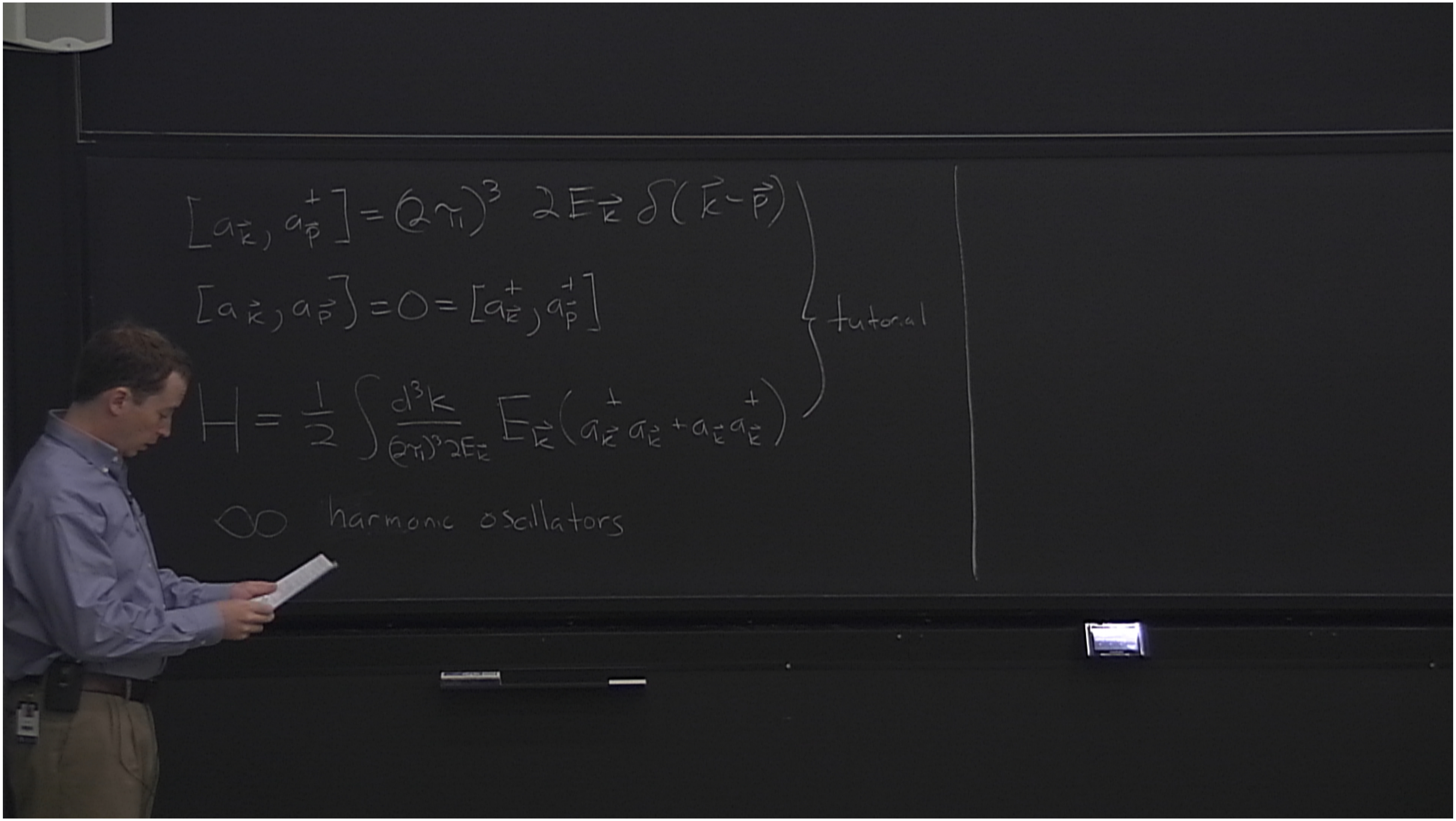

$[a_{\vec{k}}, a^{+}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k} - \vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{+}_{\vec{k}}, a^{+}_{\vec{p}}]$
H

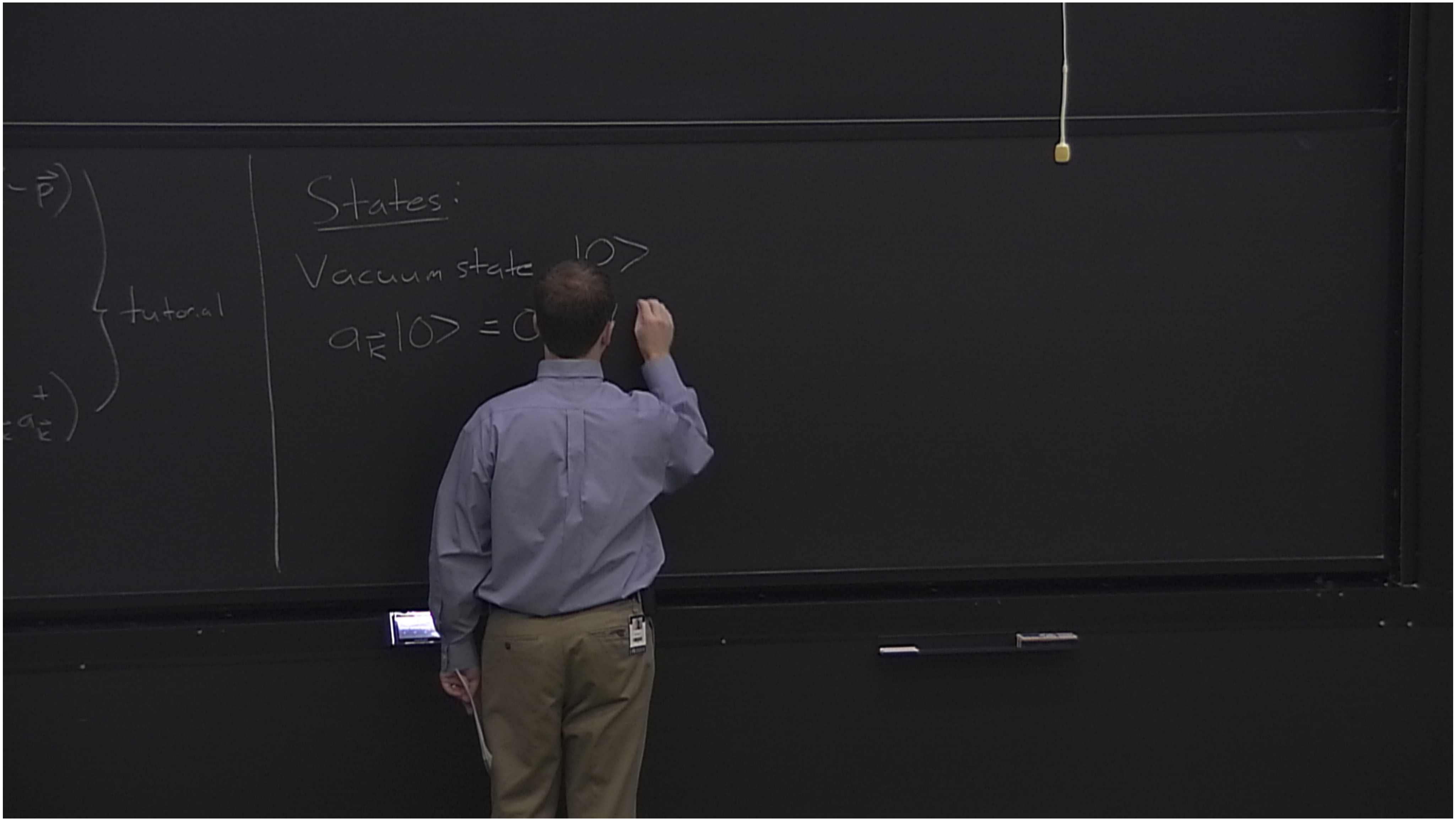

$[a_{\vec{k}}, a_{\vec{p}}^{+}] = (2\pi)^3\, 2E_{\vec{k}}\, \delta(\vec{k}-\vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a_{\vec{k}}^{+}, a_{\vec{p}}^{+}]$
tutorial
$= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} (a_{\vec{k}}^{+} a_{\vec{k}} + a_{\vec{k}} a_{\vec{k}}^{+})$

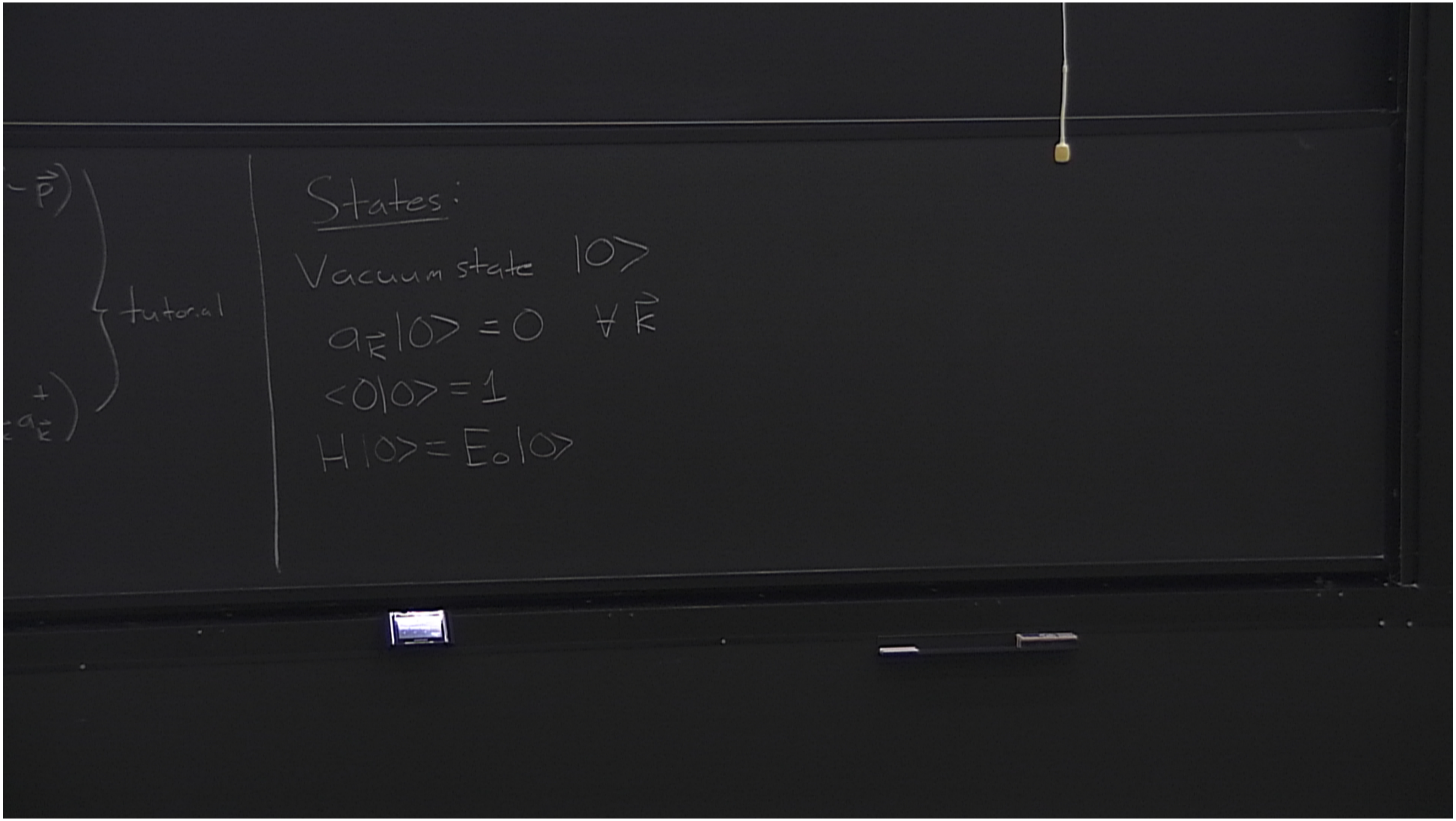

$[a_{\vec{k}}, a^{+}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k} - \vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{+}_{\vec{k}}, a^{+}_{\vec{p}}]$
$H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} (a^{+}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{+}_{\vec{k}})$
tutorial

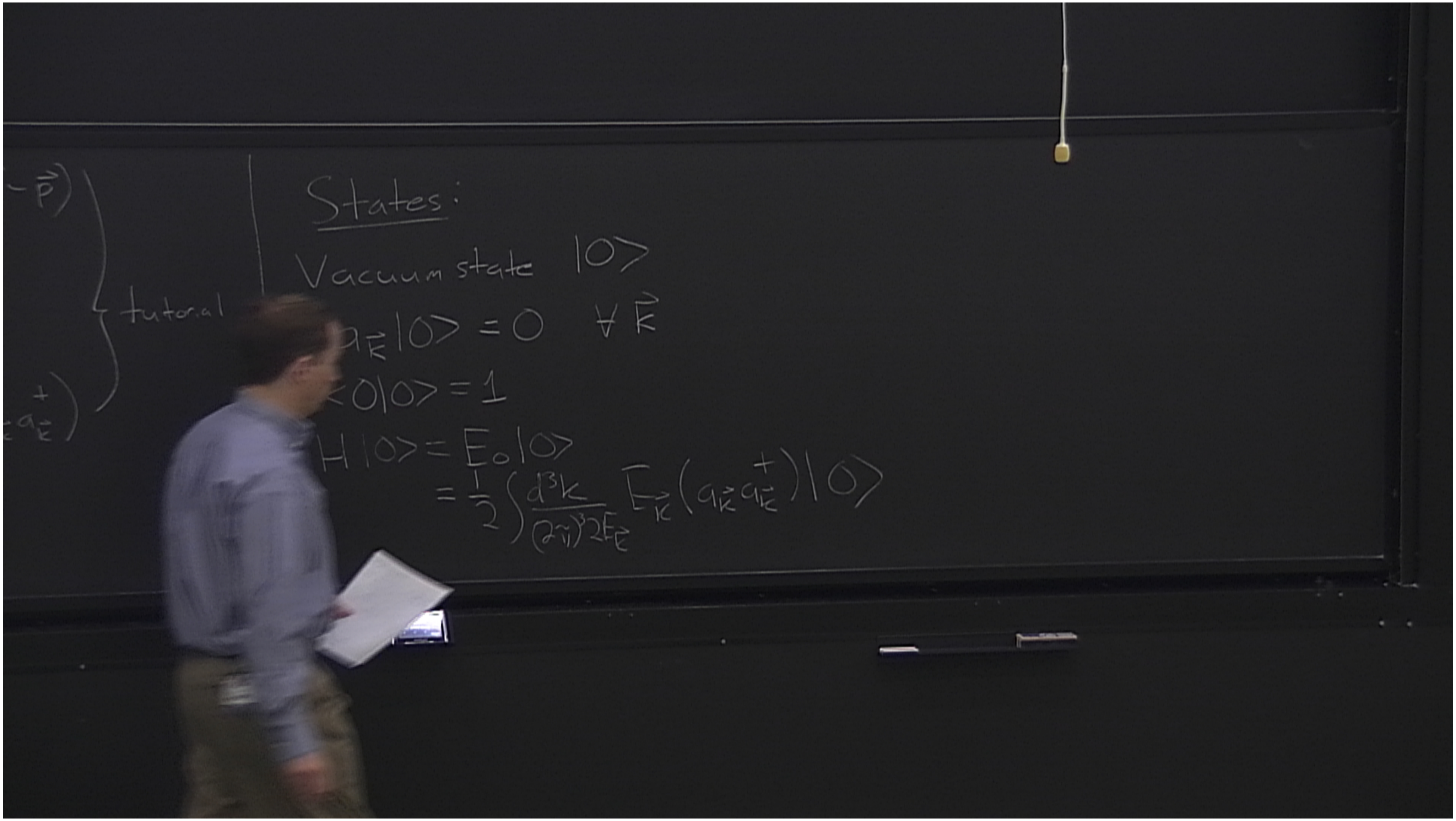

[a_k, a_p^+] = (2π)^3 2E_k δ(k − p)
tutorial

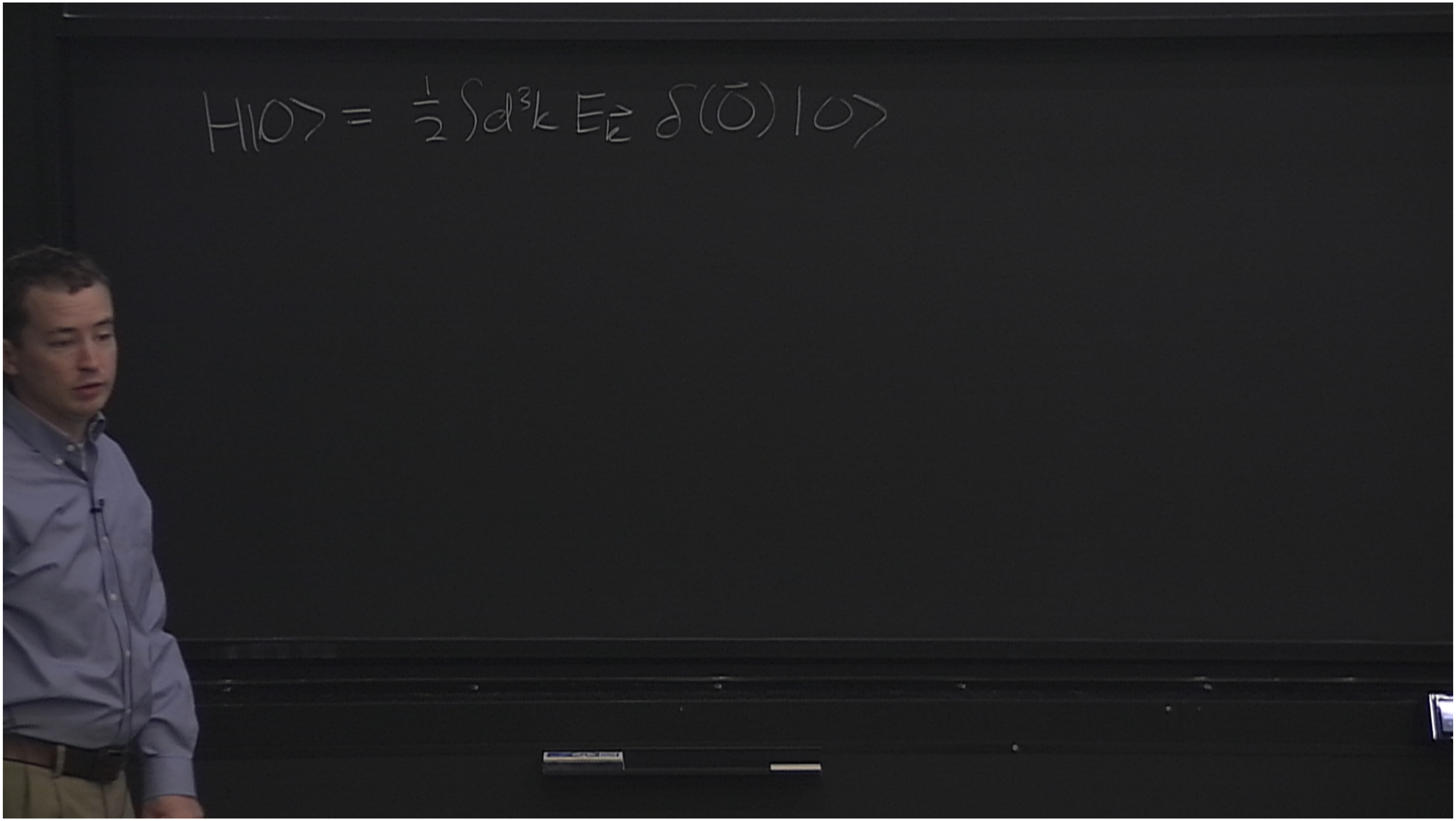

$[a_{\vec{k}}, a^{+}_{\vec{p}}] = (2\pi)^3\, 2E_{\vec{k}}\, \delta(\vec{k}-\vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{+}_{\vec{k}}, a^{+}_{\vec{p}}]$
tutorial
$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} (a^{+}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{+}_{\vec{k}})$
∞ harmonic oscillators

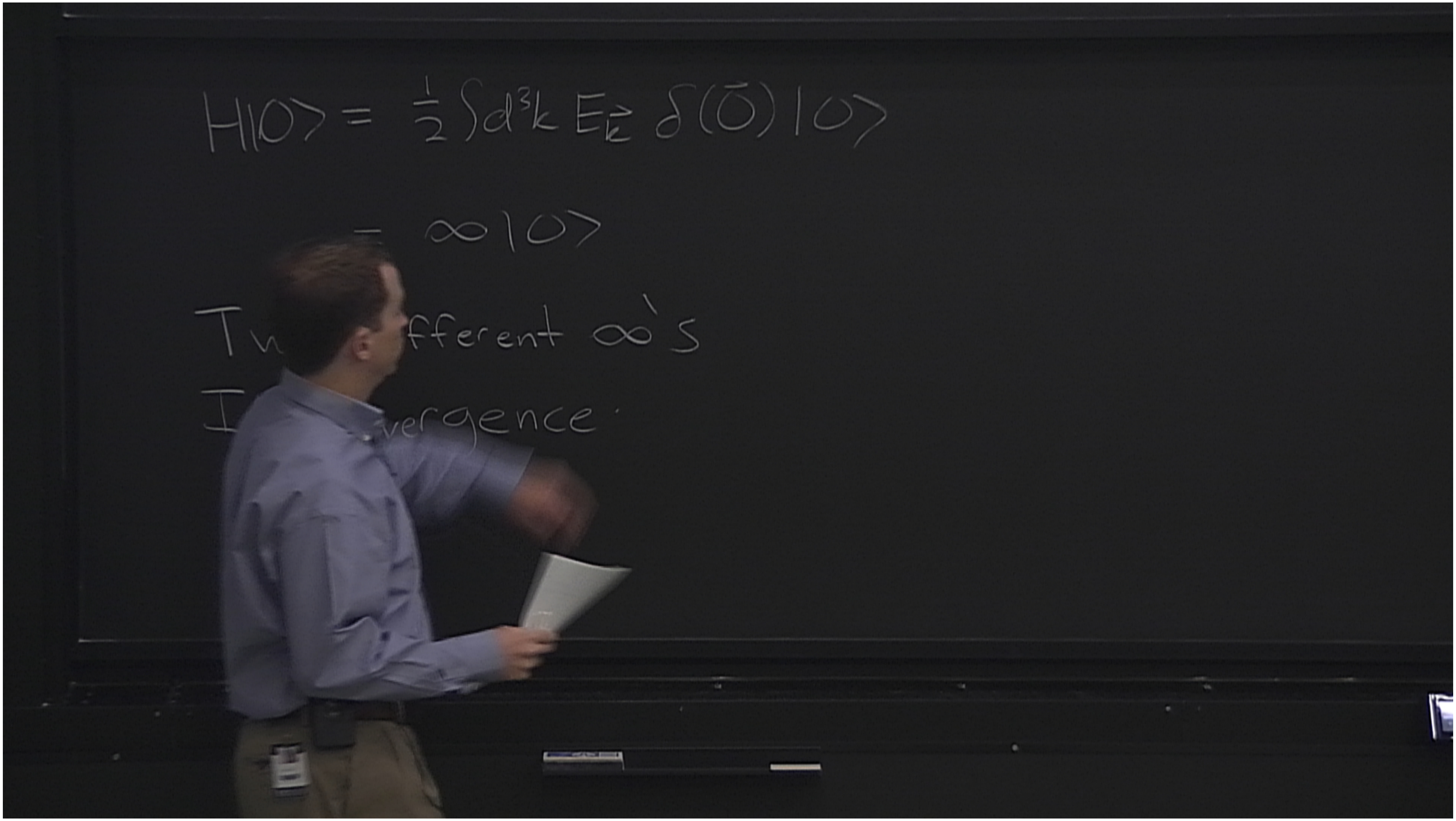

$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3\, 2E_{\vec{k}}\, \delta(\vec{k}-\vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{\dagger}_{\vec{k}}, a^{\dagger}_{\vec{p}}]$
tutorial
$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} (a^{\dagger}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{\dagger}_{\vec{k}})$
∞ harmonic oscillators

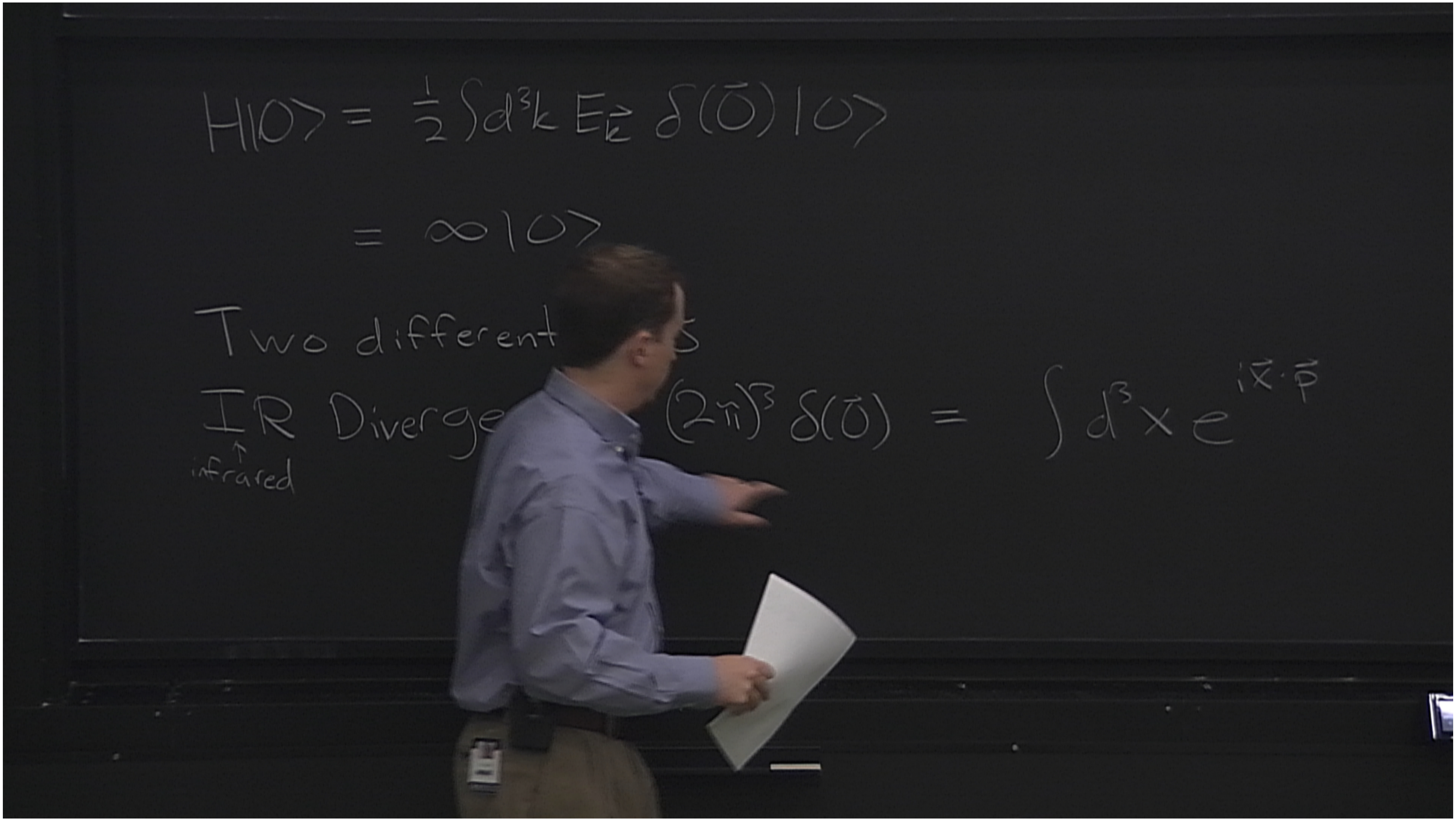

States:
Vacuum state |0>
a_k |0> = 0
tutorial

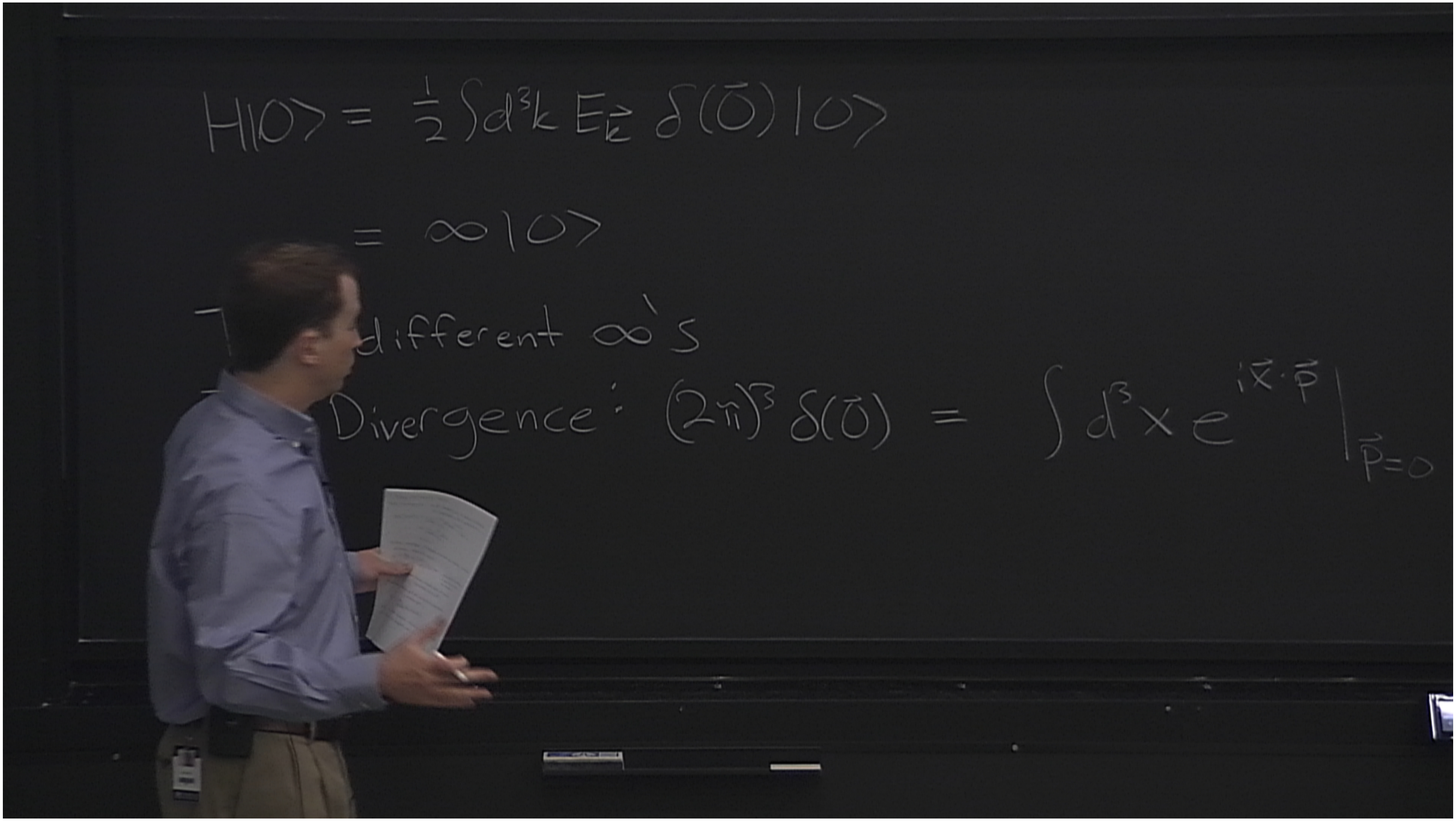

tutorial
States:
Vacuum state |0>
a_k |0> = 0 ∀ k
<0|0> = 1
H |0> = E_0 |0>

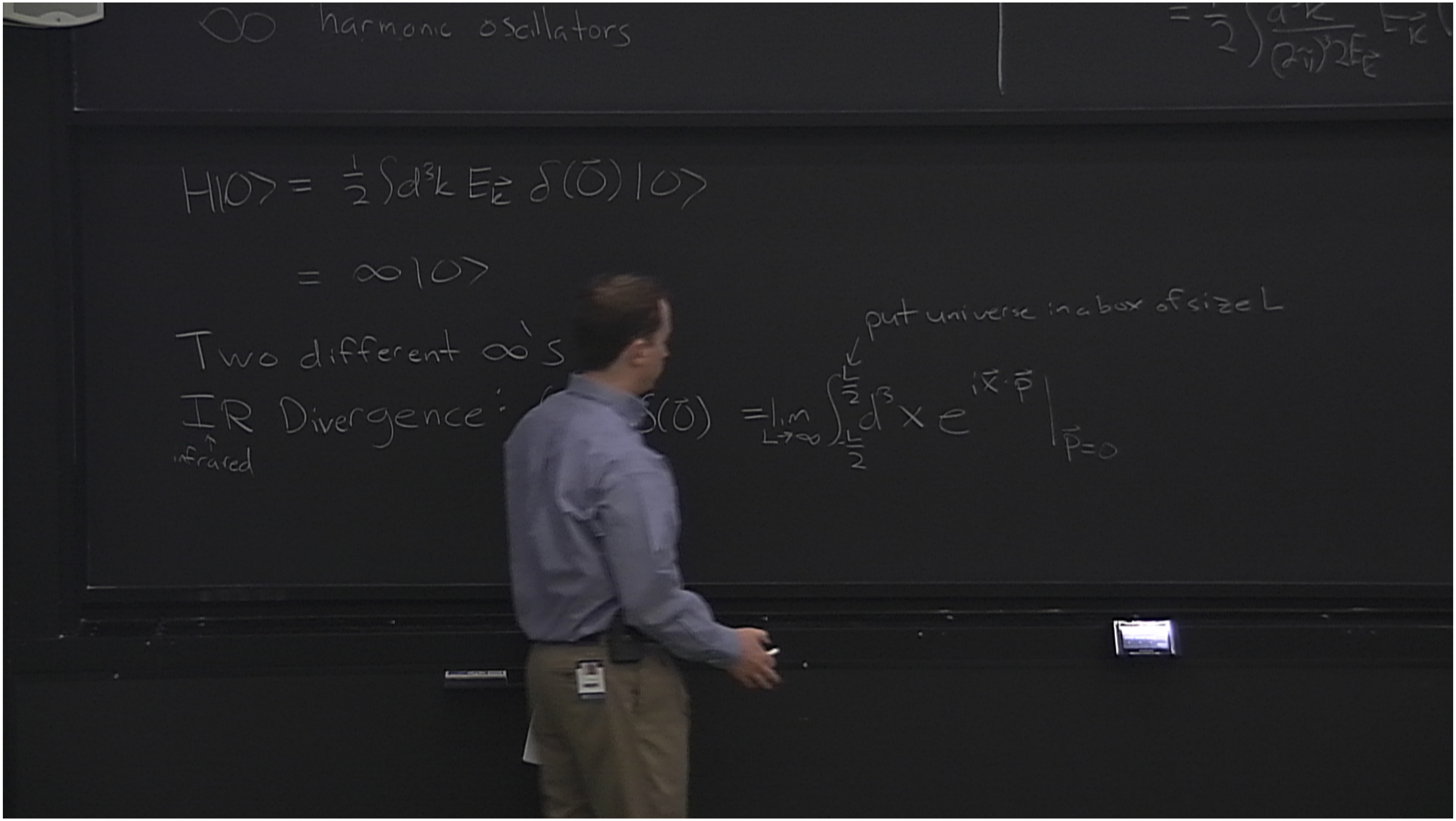

States:
Vacuum state |0>
a_k |0> = 0 ∀ k
<0|0> = 1
H |0> = E_0 |0>
= 1/2 ∫ d^3k / (2π)^3 2E_k E_k (a_k a_k^†) |0>
tutorial

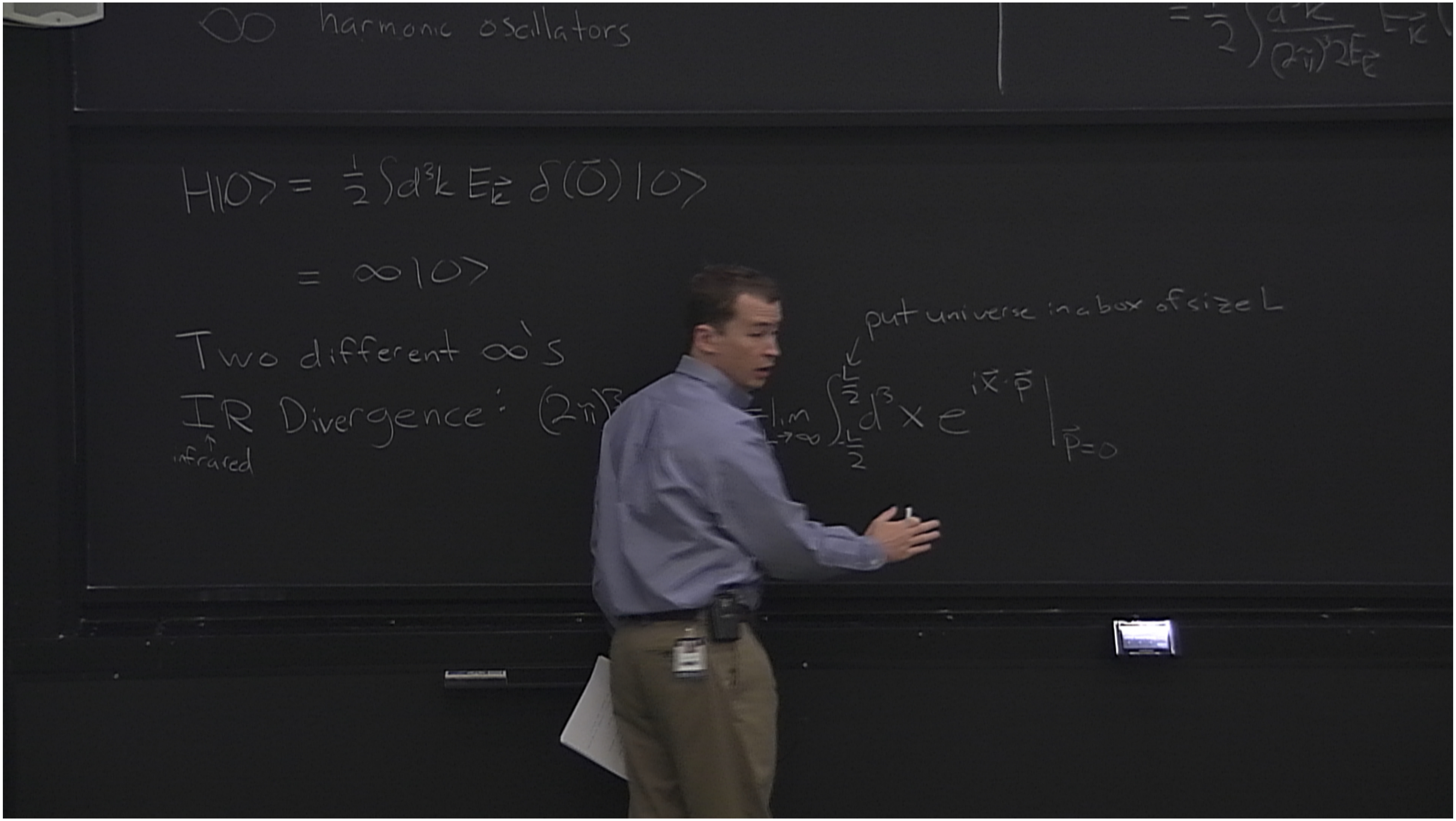

$H|0\rangle = \frac{1}{2}\int d^3k\, E_{\vec{k}}\, \delta(\vec{0})\,|0\rangle$


H|0> = 1/2 ∫d^3k E_k δ(0)|0>
= ∞|0>
fferent ∞'s
vergence

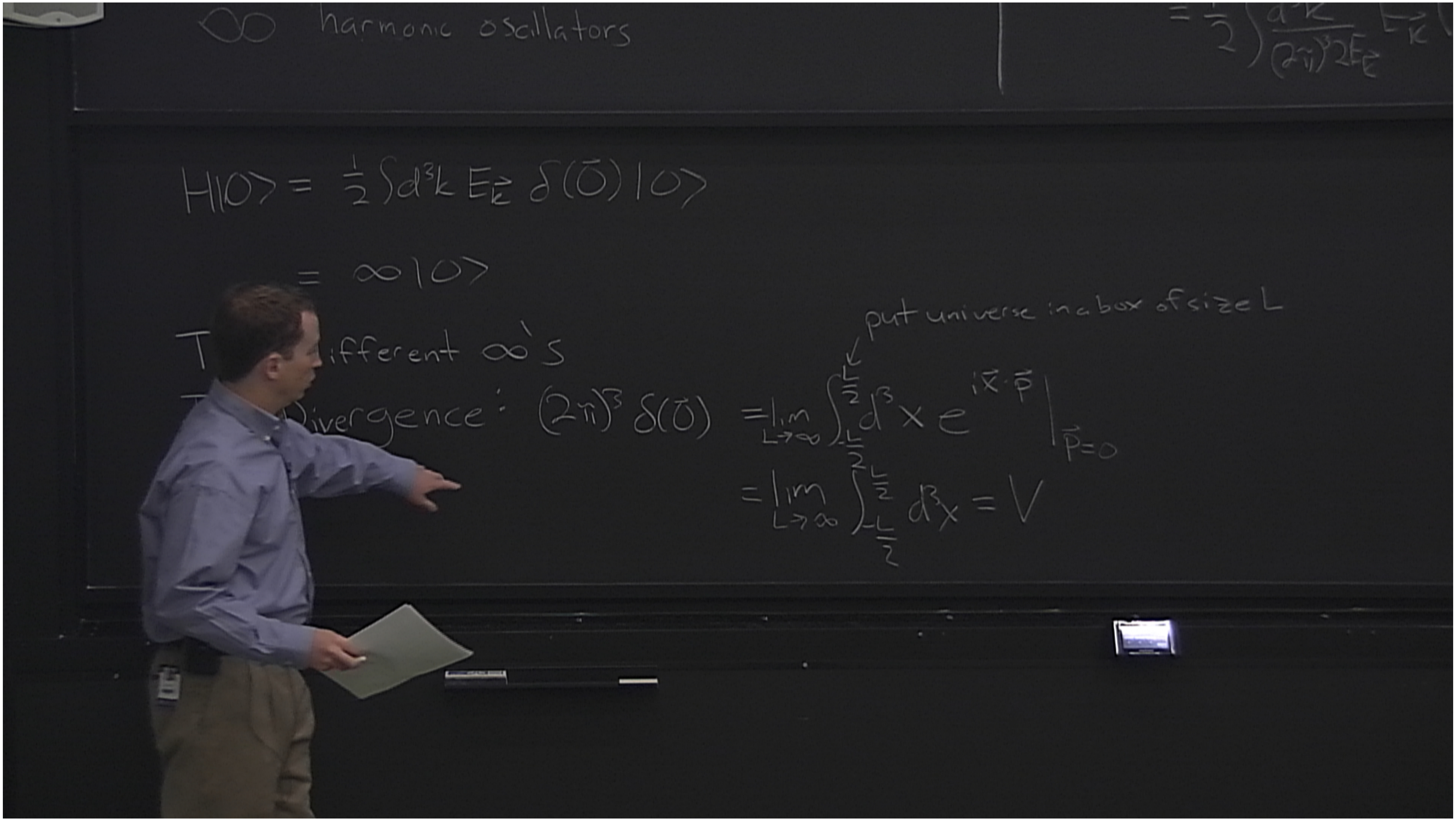

H|0> = ½ ∫d³k E_k δ(0)|0>
= ∞|0>
Two different
IR Diverge
(2π)³ δ(0) = ∫d³x e^(ix·p)
infrared

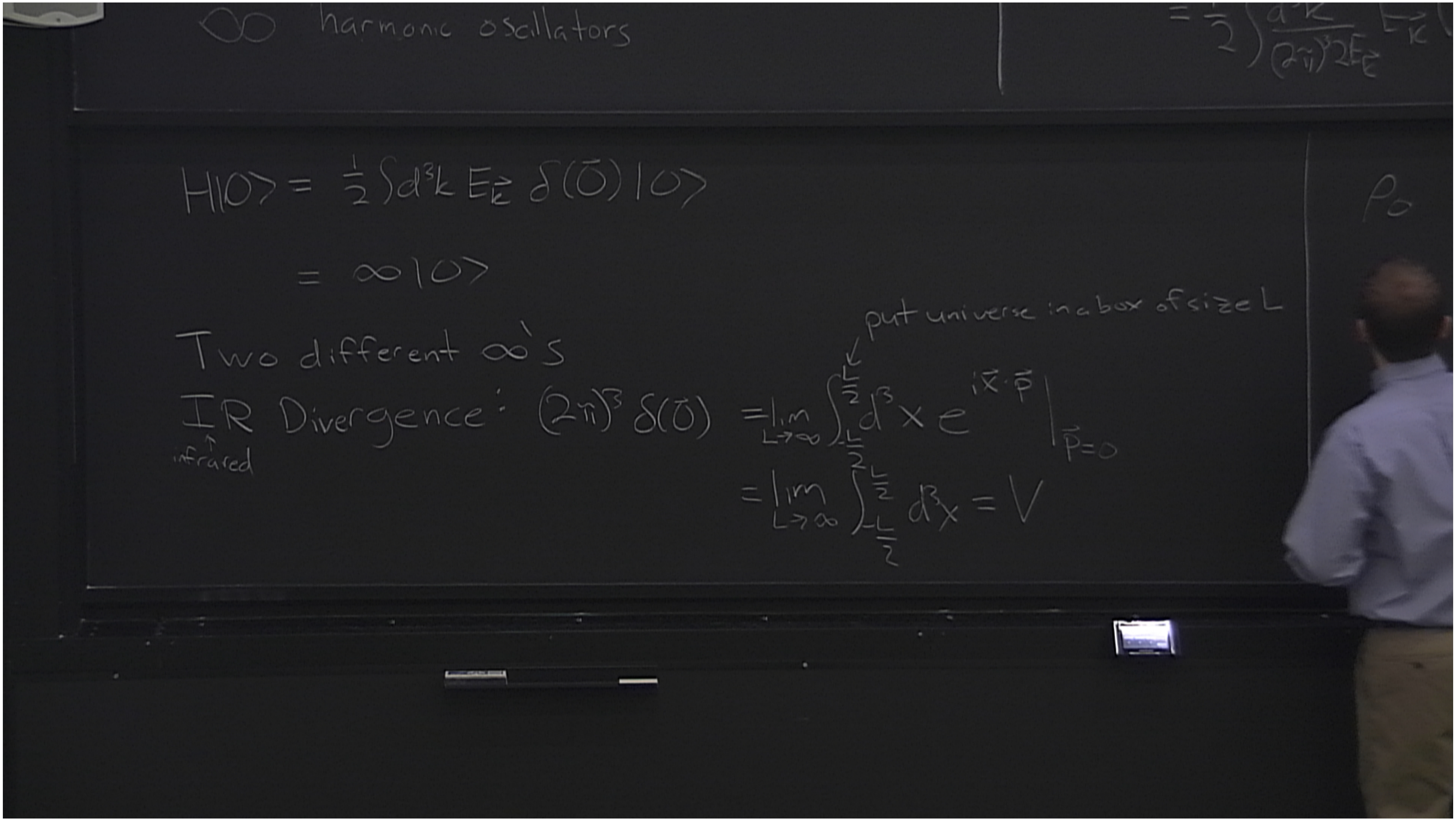

H|0> = ½ ∫d³k E_k δ(0̄)|0>
= ∞|0>
different ∞'s
Divergence: (2π)³ δ(0̄) = ∫d³x e^{ix⃗·p⃗} |_{p⃗=0}

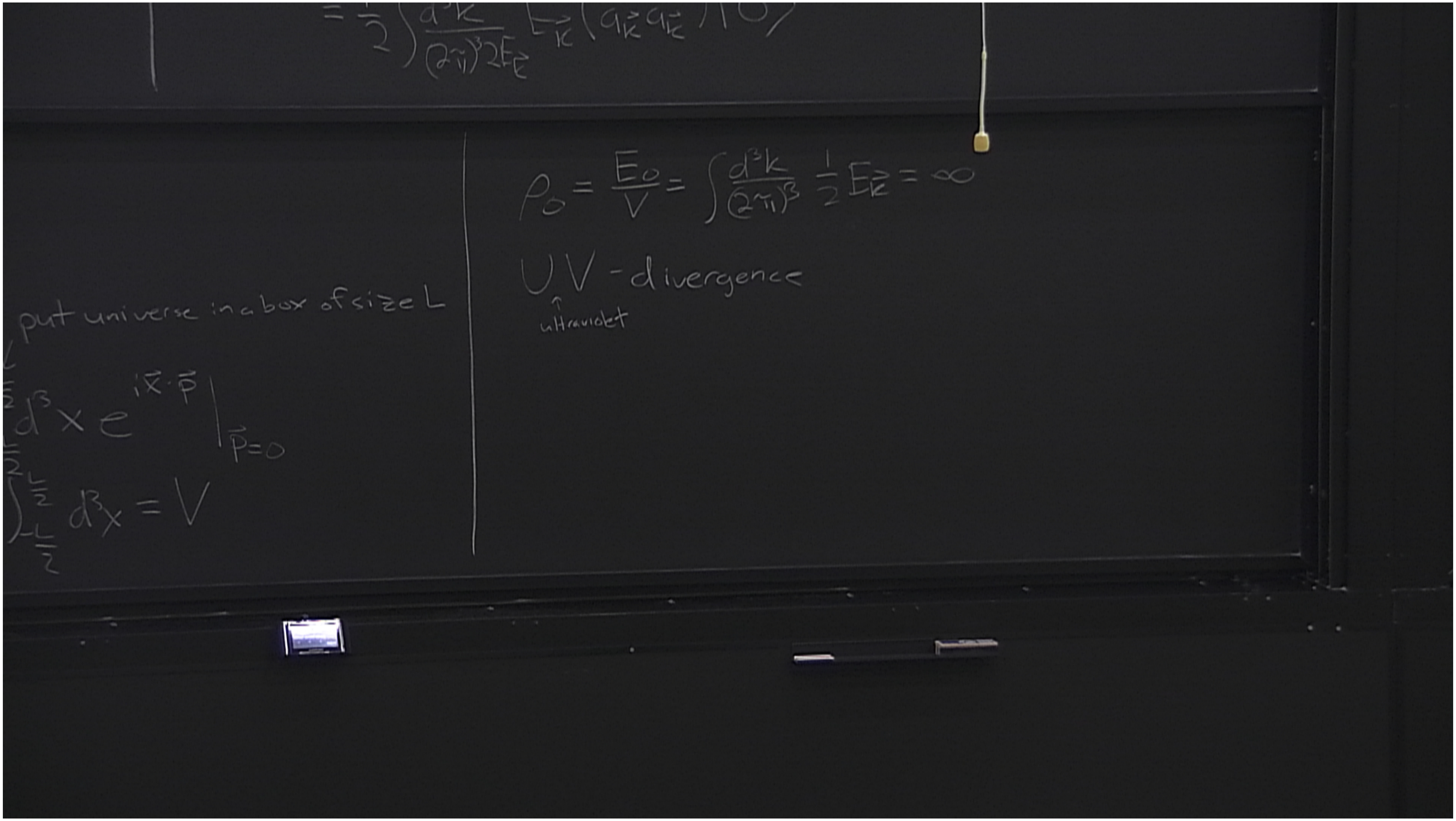

∞ harmonic oscillators
$H|0\rangle = \frac{1}{2}\int d^3k\, E_{\vec{k}}\, \delta(\vec{0})|0\rangle$
$= \infty|0\rangle$
Two different ∞'s
put universe in a box of size L
IR Divergence:
$\delta(\vec{0}) = \lim_{L\to\infty}\int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x\, e^{i\vec{x}\cdot\vec{p}}\Big|_{\vec{p}=0}$
infrared

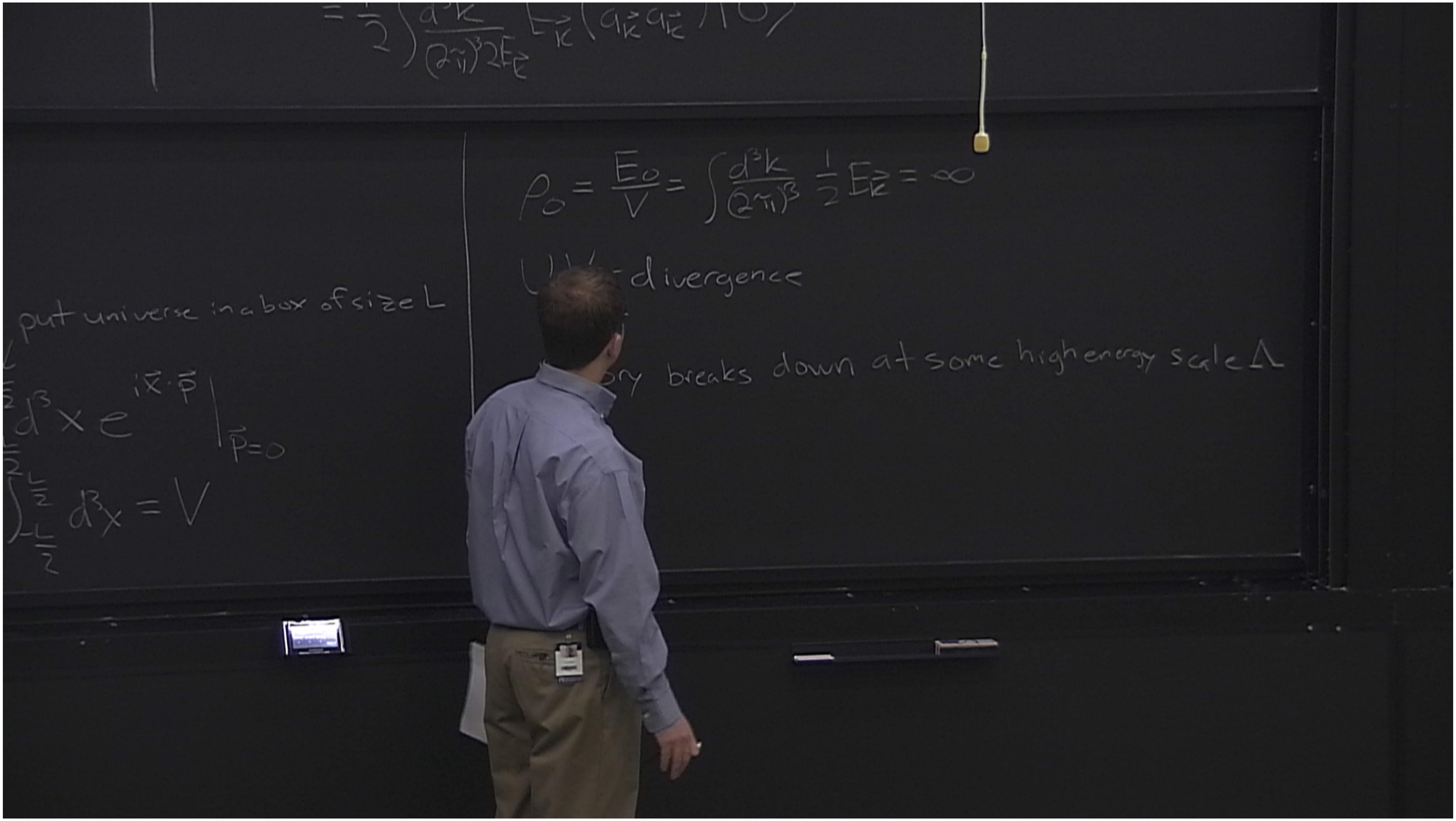


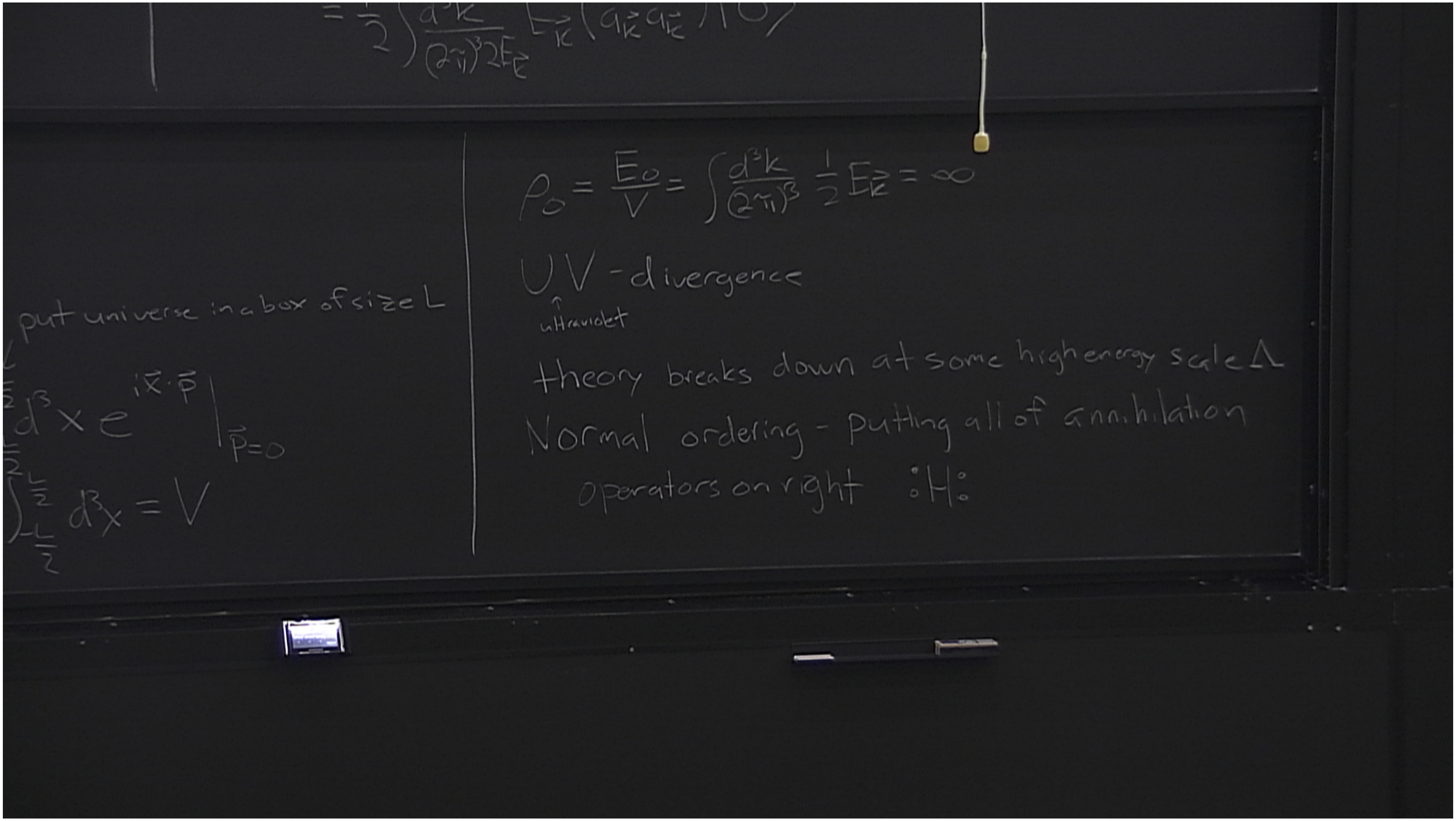
∞ harmonic oscillators
H|0> = ½ ∫d³k E_k δ(0) |0>
= ∞ |0>
Two different ∞'s
IR Divergence: (2π)³
infrared
put universe in a box of size L


∞ harmonic oscillators
H|0> = ½ ∫d³k E_k δ(0)|0>
= ∞|0>
ifferent ∞'s
ivergence: (2π)³ δ(0) = lim L→∞ ∫ d³x e^(ix·p) |p=0
= lim L→∞ ∫ d³x = V
put universe in a box of size L

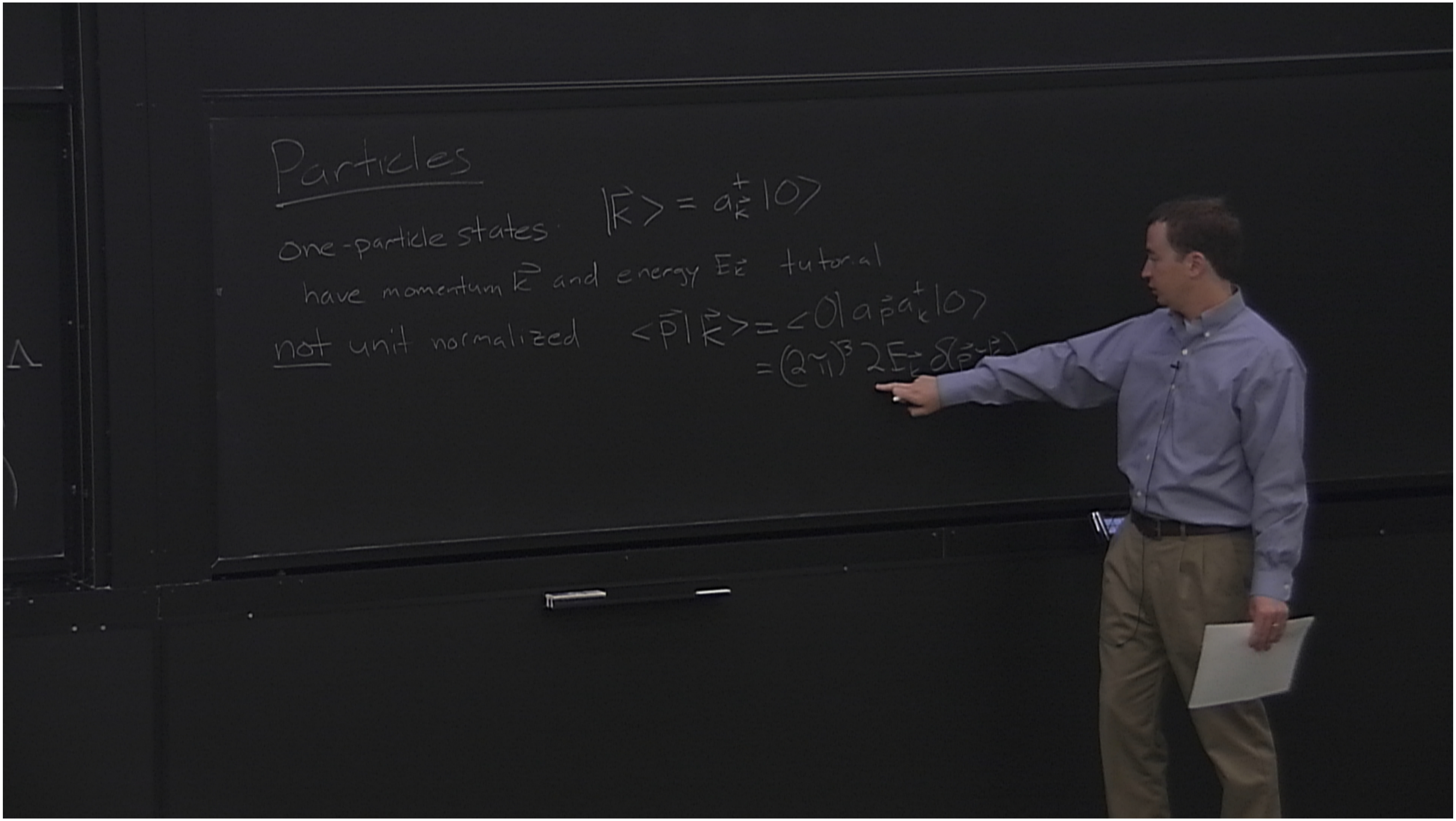

∞ harmonic oscillators
$= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}}$
$H|0\rangle = \frac{1}{2}\int d^3k\, E_{\vec{k}}\, \delta(\vec{0})|0\rangle$
$= \infty|0\rangle$
Two different ∞'s
IR Divergence: $(2\pi)^3\delta(\vec{0}) = \lim_{L\to\infty}\int_{-L/2}^{L/2} d^3x\, e^{i\vec{x}\cdot\vec{p}}\Big|_{\vec{p}=0}$
infrared
put universe in a box of size L
$= \lim_{L\to\infty}\int_{-L/2}^{L/2} d^3x = V$
ρ_0

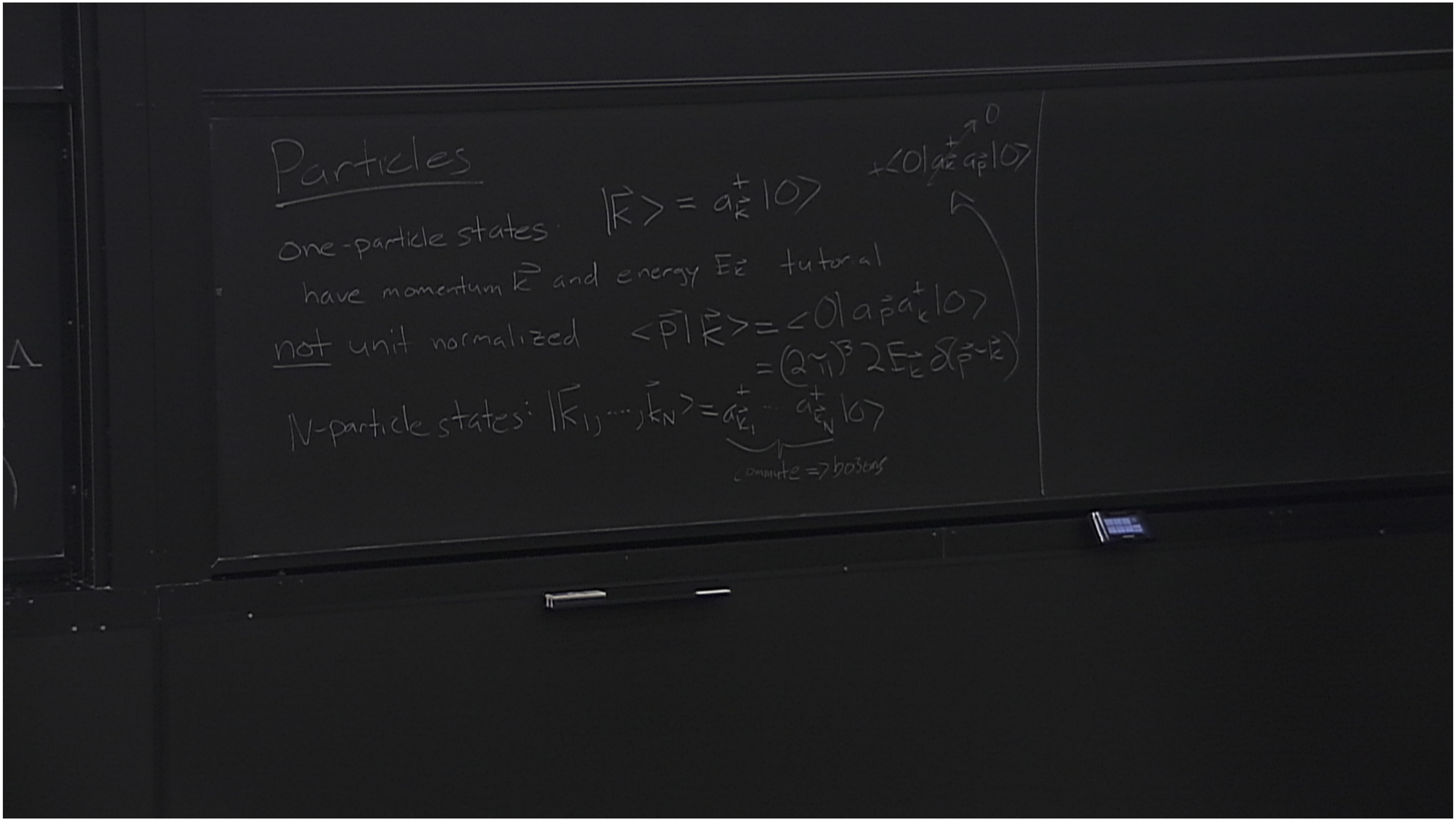

$\rho_0 = \frac{E_0}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$
UV - divergence
ultraviolet
put universe in a box of size L
$\int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V$

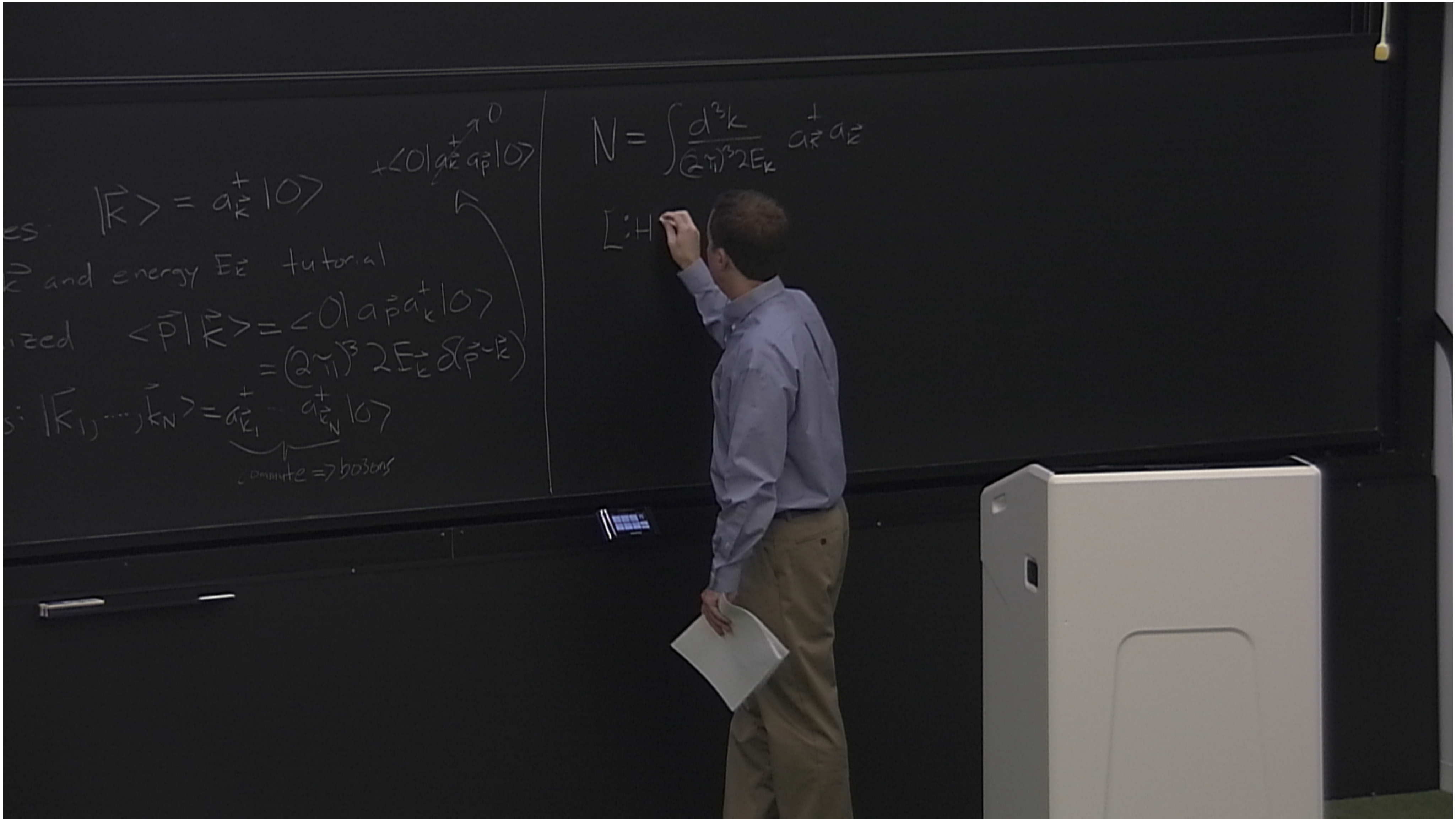

ρ_0 = E_0/V = ∫ d^3k/(2π)^3 1/2 E_k = ∞
UV-divergence
put universe in a box of size L
breaks down at some high energy scale Λ
∫ d^3x = V

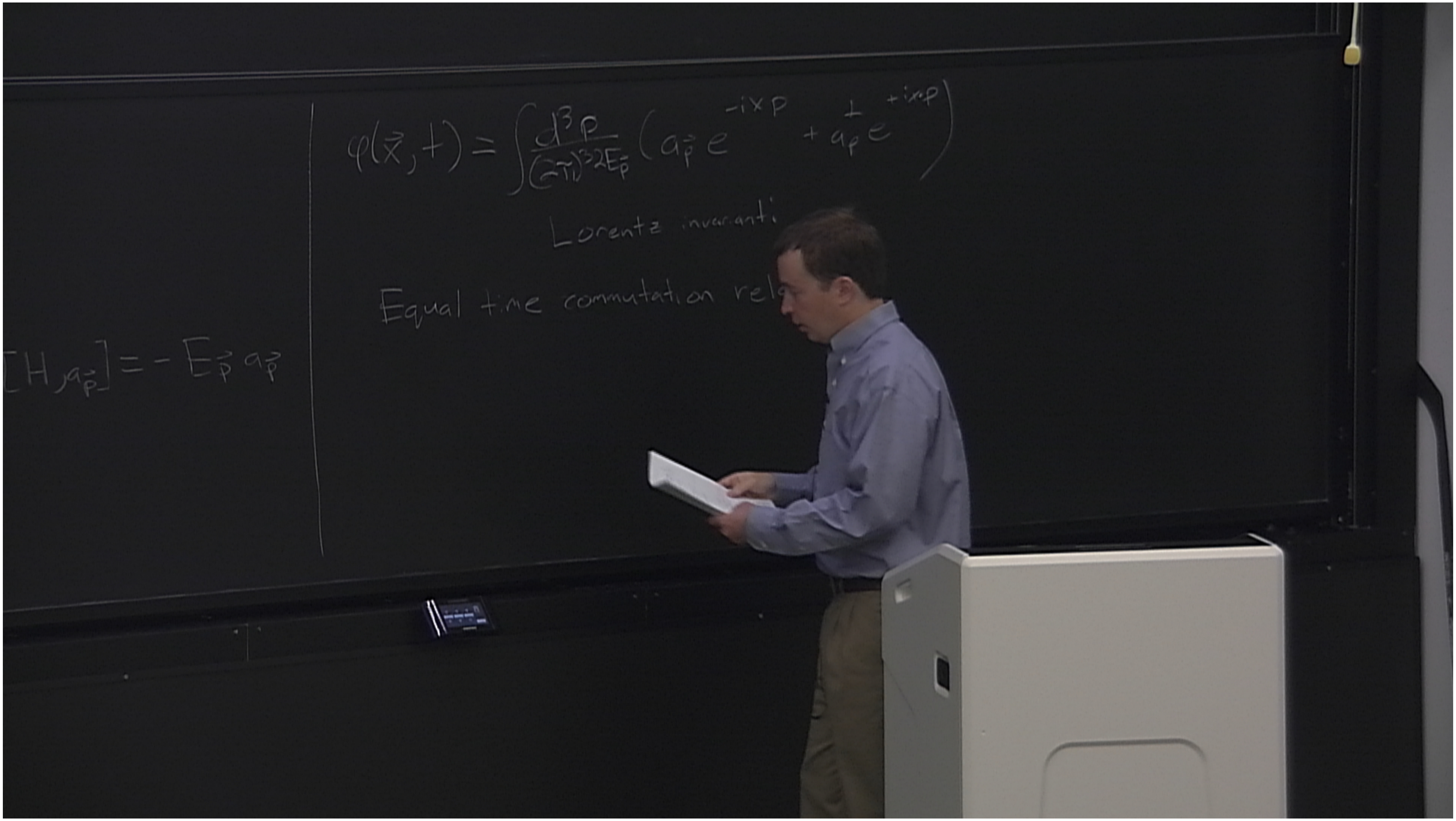


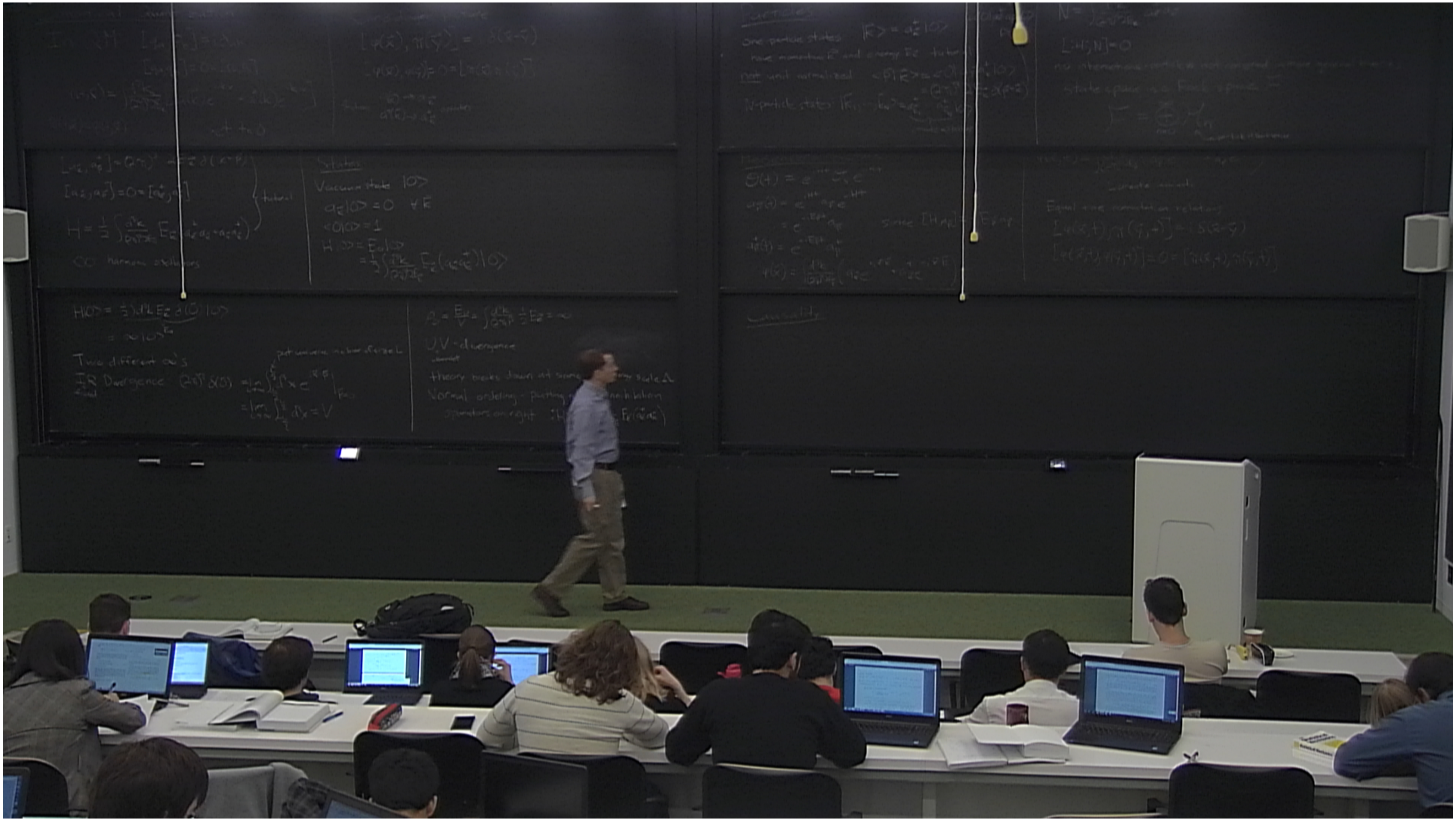
$\rho_0 = \frac{E_0}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$
UV - divergence
ultraviolet
put universe in a box of size L
$\int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V$
theory breaks down at some high energy scale Λ
Normal ordering - putting all of annihilation operators on right $:H:$

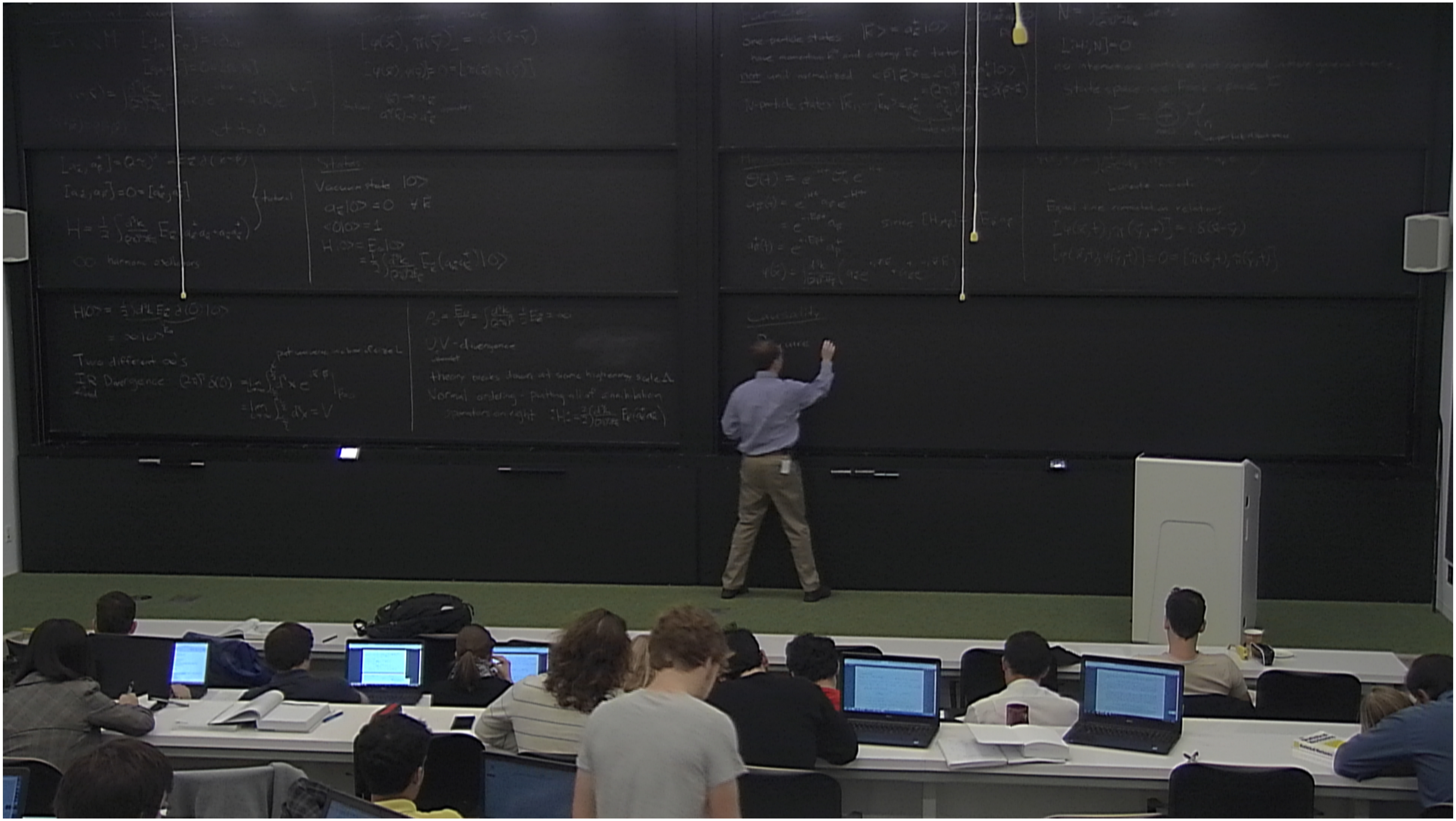

Particles
one-particle states: |k⃗> = a†k⃗ |0>
have momentum k⃗ and energy Ek⃗ tutorial
not unit normalized <p⃗|k⃗> = <0|ap⃗ a†k⃗|0>
= (2π)³ 2Ek⃗ δ(p⃗−k⃗)

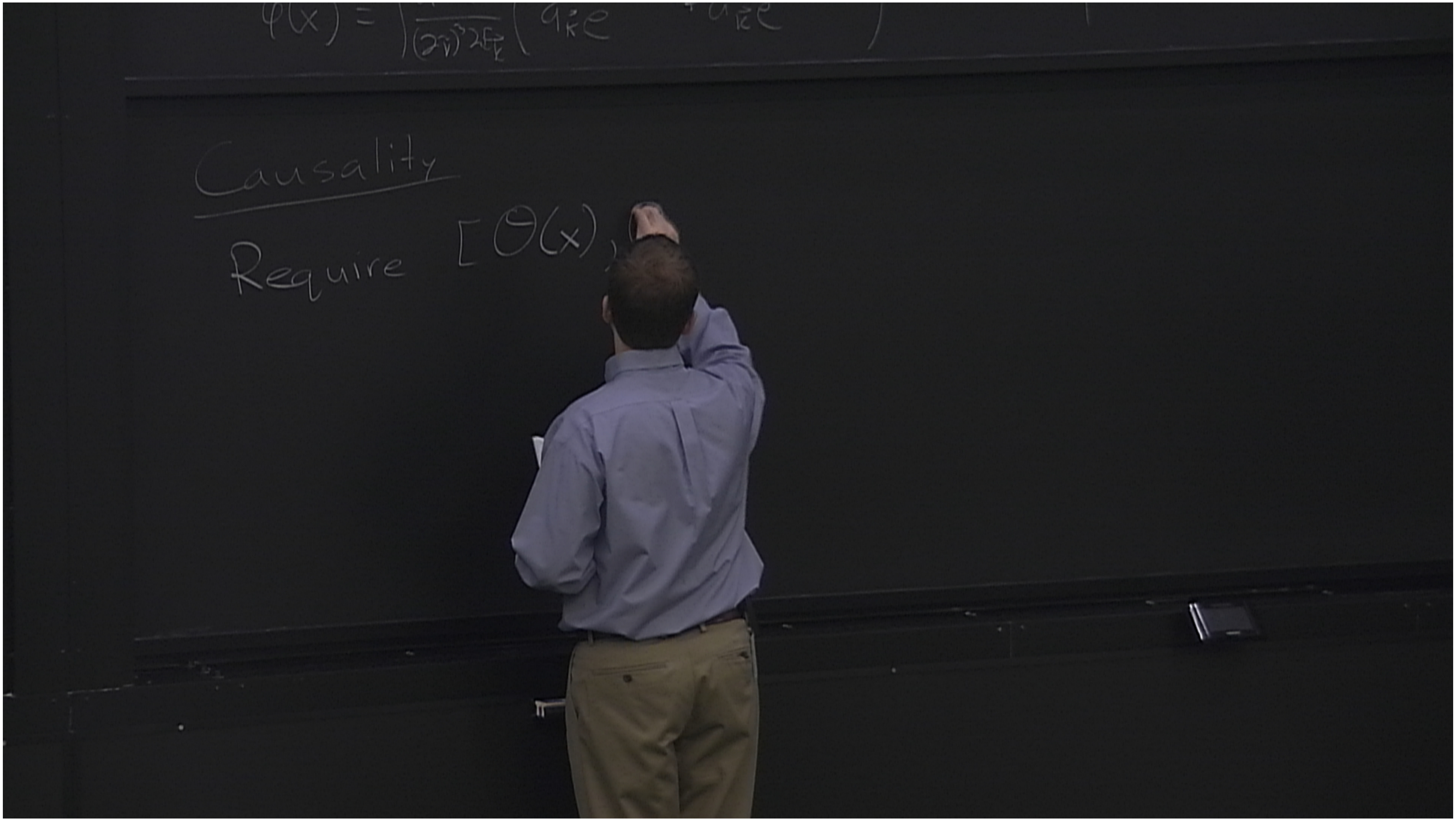

Particles
one-particle states: |k⃗> = a†k⃗ |0>
have momentum k⃗ and energy Ek⃗   tutorial
not unit normalized   <p⃗|k⃗> = <0| ap⃗ a†k⃗ |0>
= (2π)³ 2Ek⃗ δ(p⃗−k⃗)
+ <0| a†k⃗ ap⃗ |0> → 0
N-particle states: |k⃗1, ..., k⃗N> = a†k⃗1 ... a†k⃗N |0>
commute ⇒ bosons

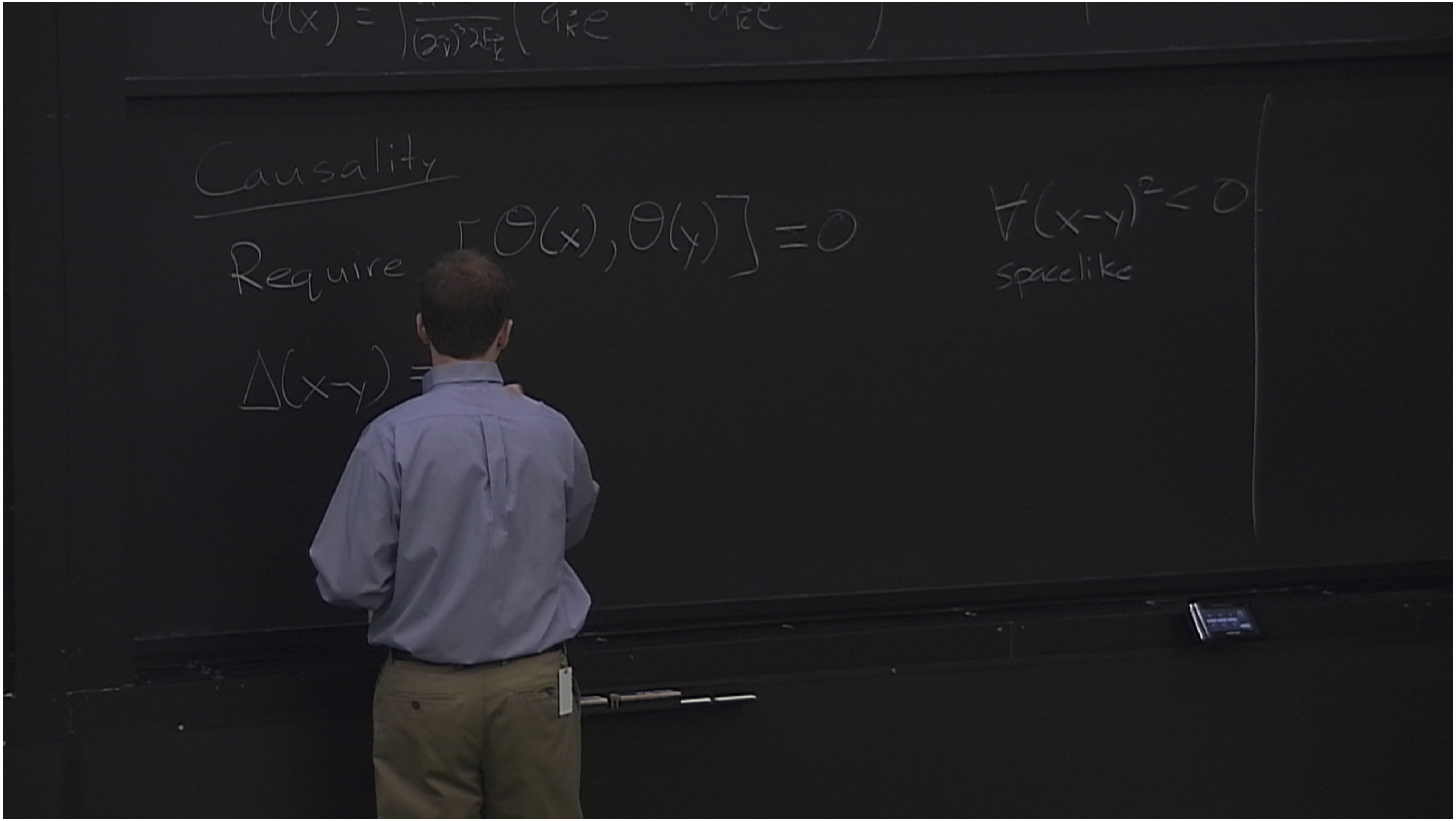

N = ∫ d³k / (2π)³2E_k a†_k a_k
|k> = a†_k |0>
and energy E_k tutorial
<p|k> = <0|a_p a†_k|0> + <0|a†_k a_p|0>
= (2π)³ 2E_k δ(p−k)
|k_1, ..., k_N> = a†_k1 ... a†_kN |0>
commute => bosons

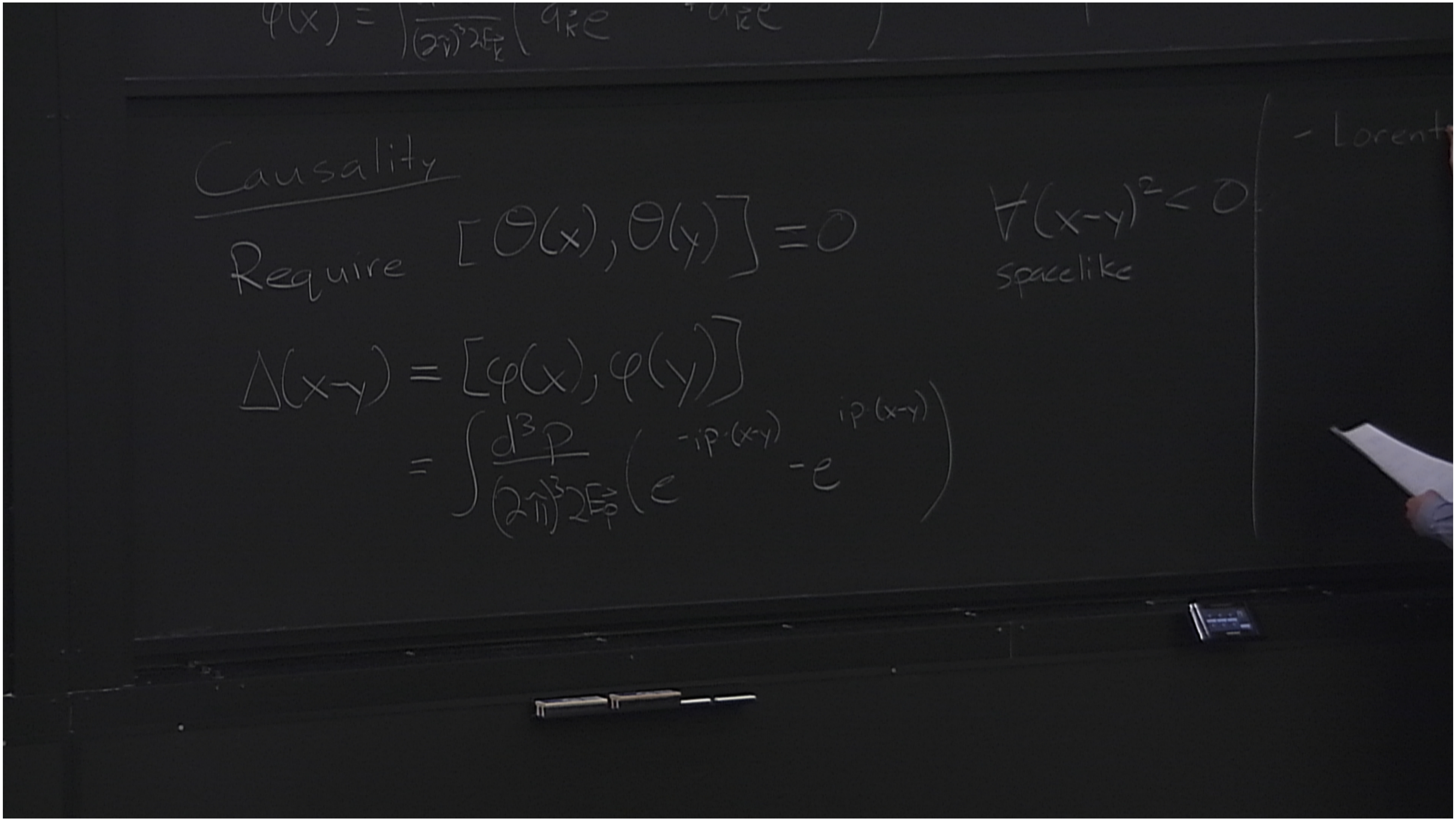

$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(a_{\vec{p}} e^{-ixp} + a_{\vec{p}}^{\dagger} e^{+ixp} \right)$
Lorentz invariant
Equal time commutation rel
$[H, a_{\vec{p}}] = -E_{\vec{p}} a_{\vec{p}}$

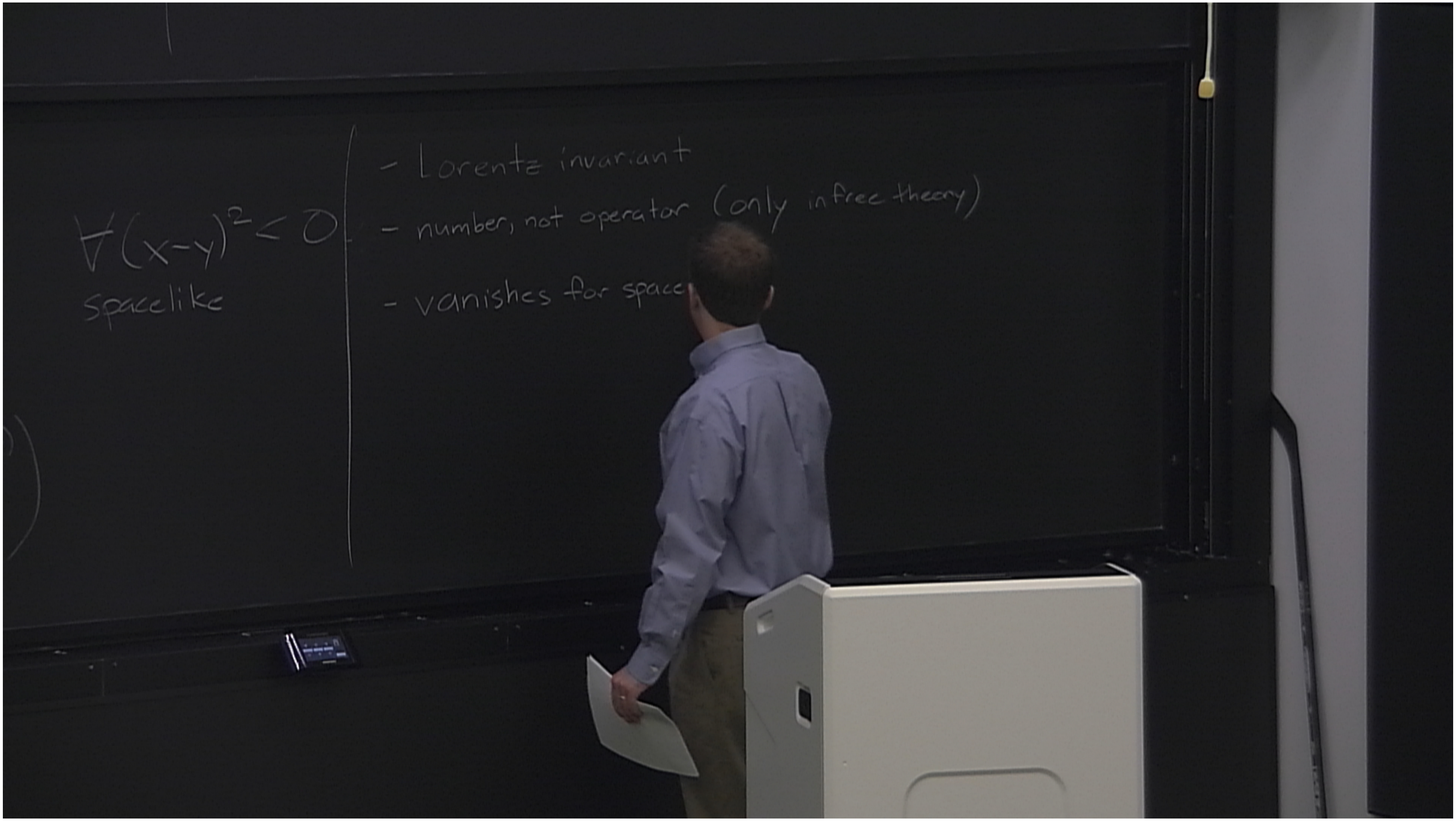


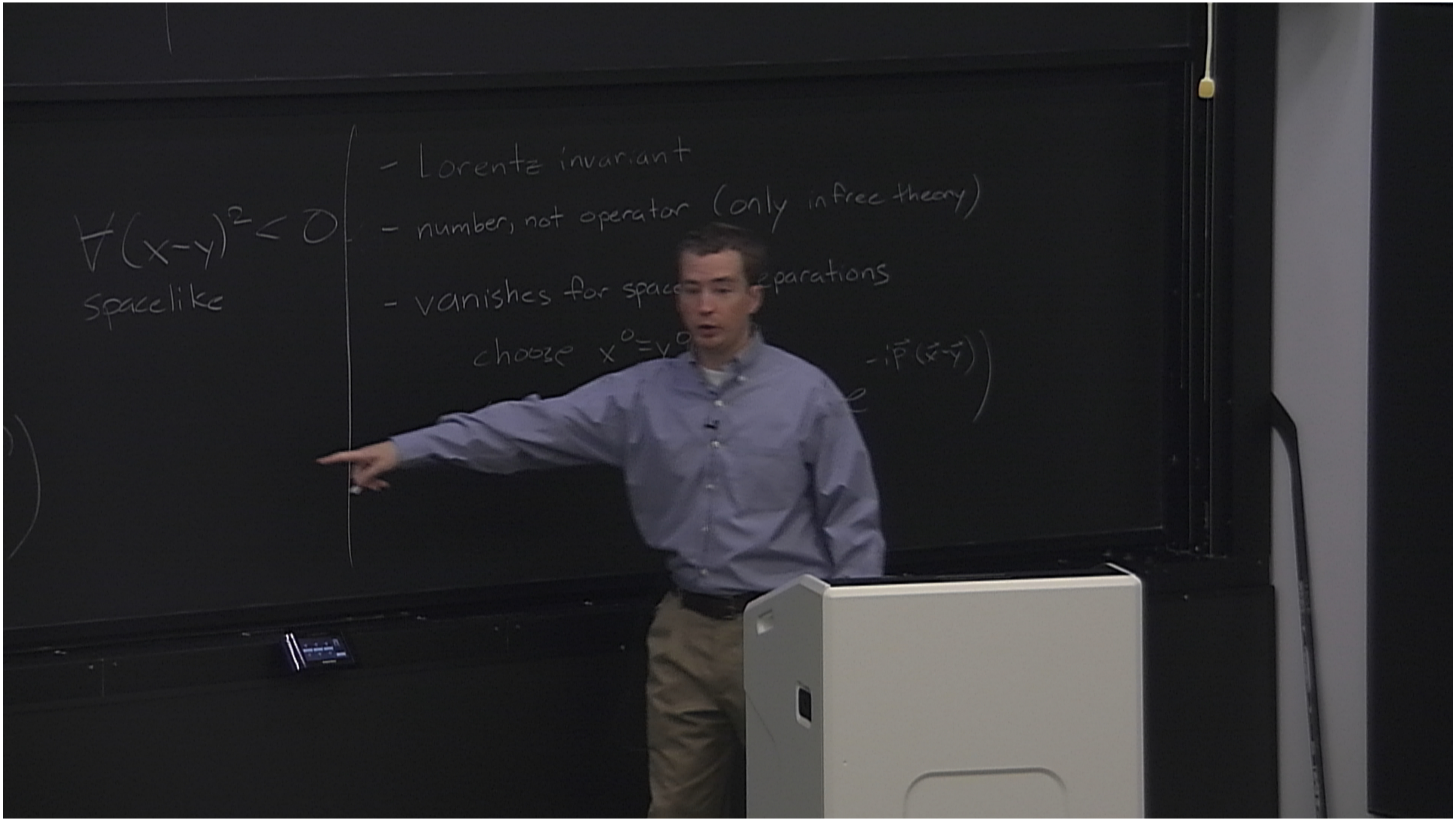


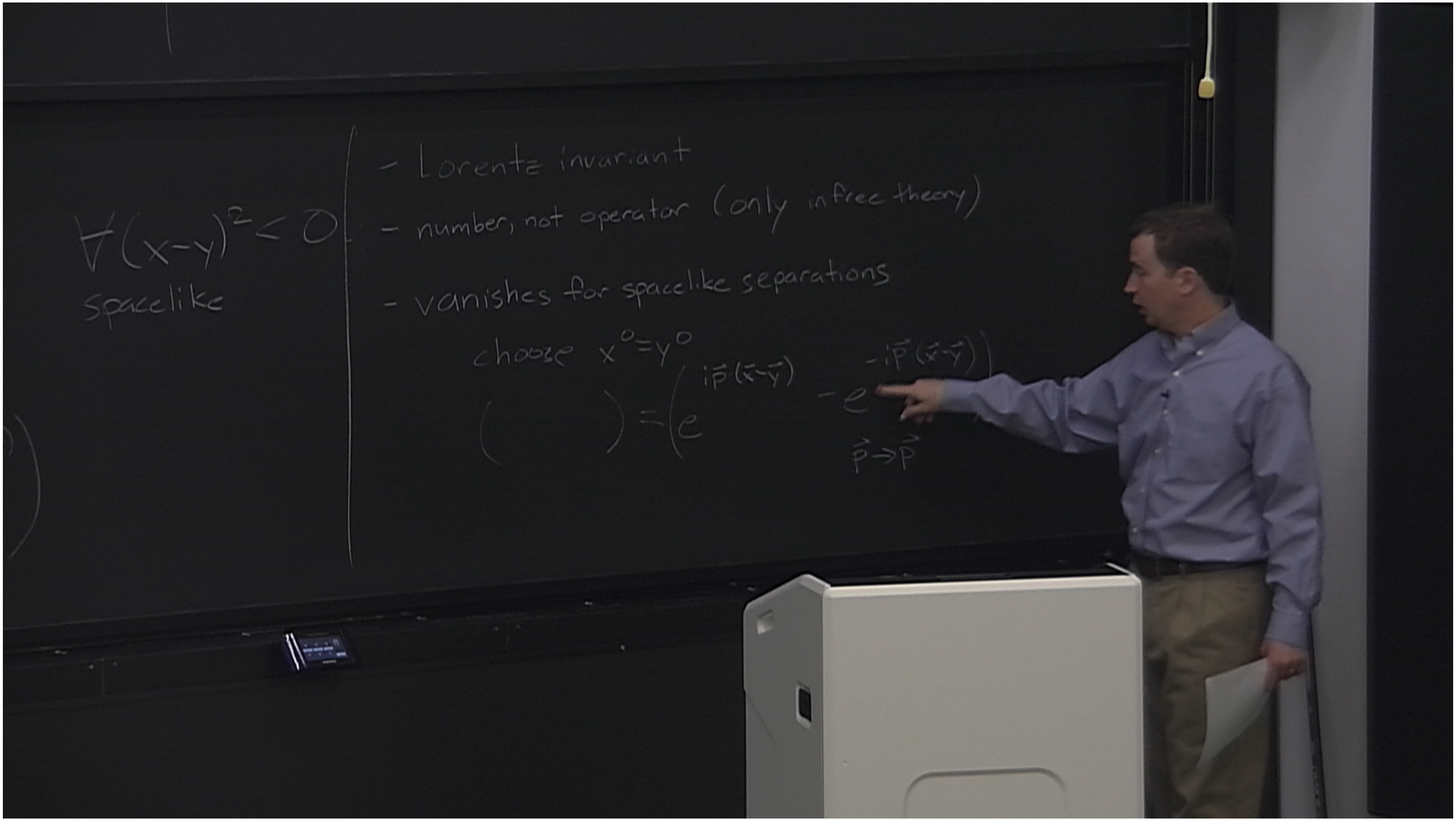

Causality
Require

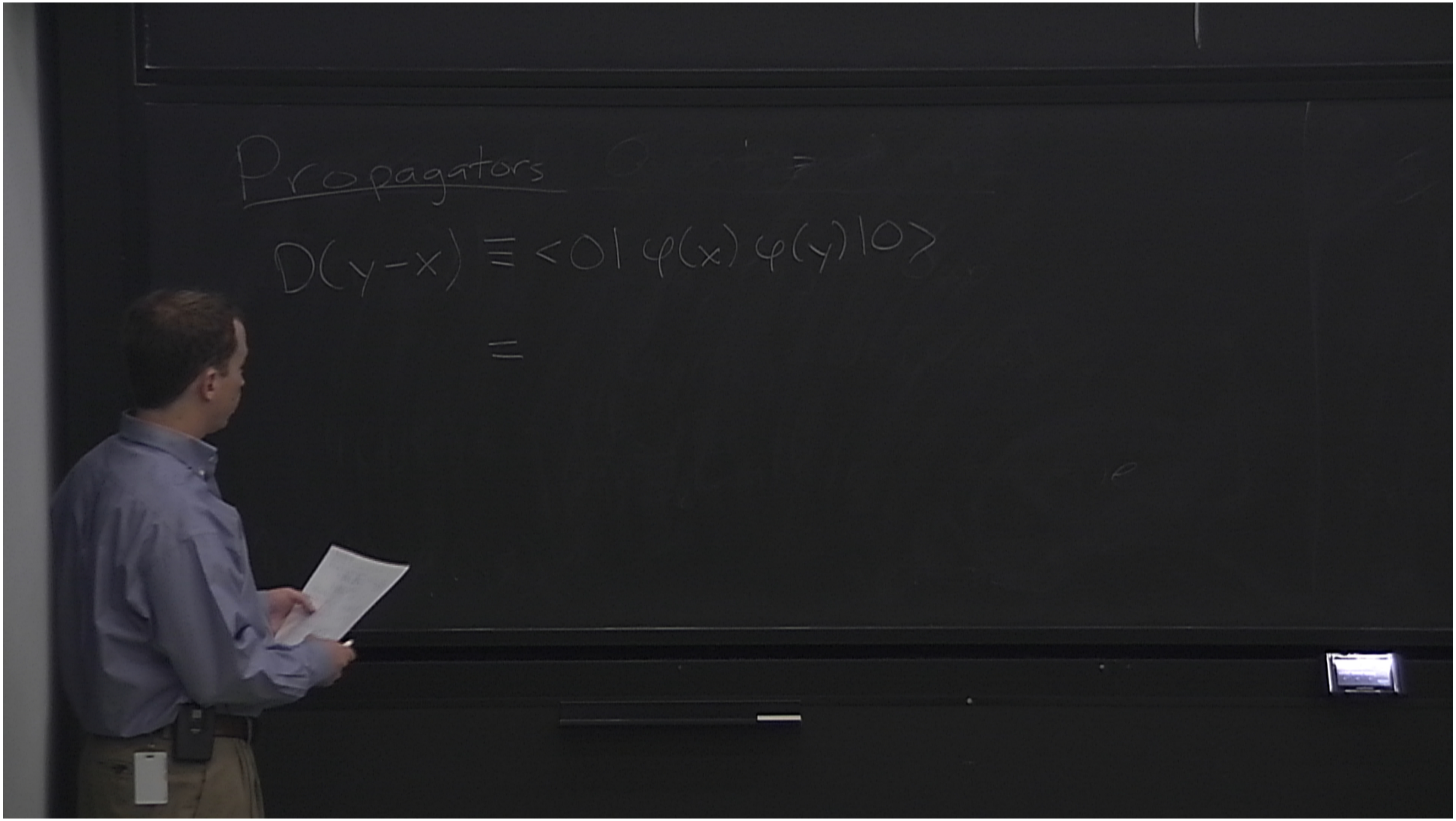

Causality
Require [θ(x), θ(y)] = 0
∀ (x-y)² < 0
spacelike
Δ(x-y) =

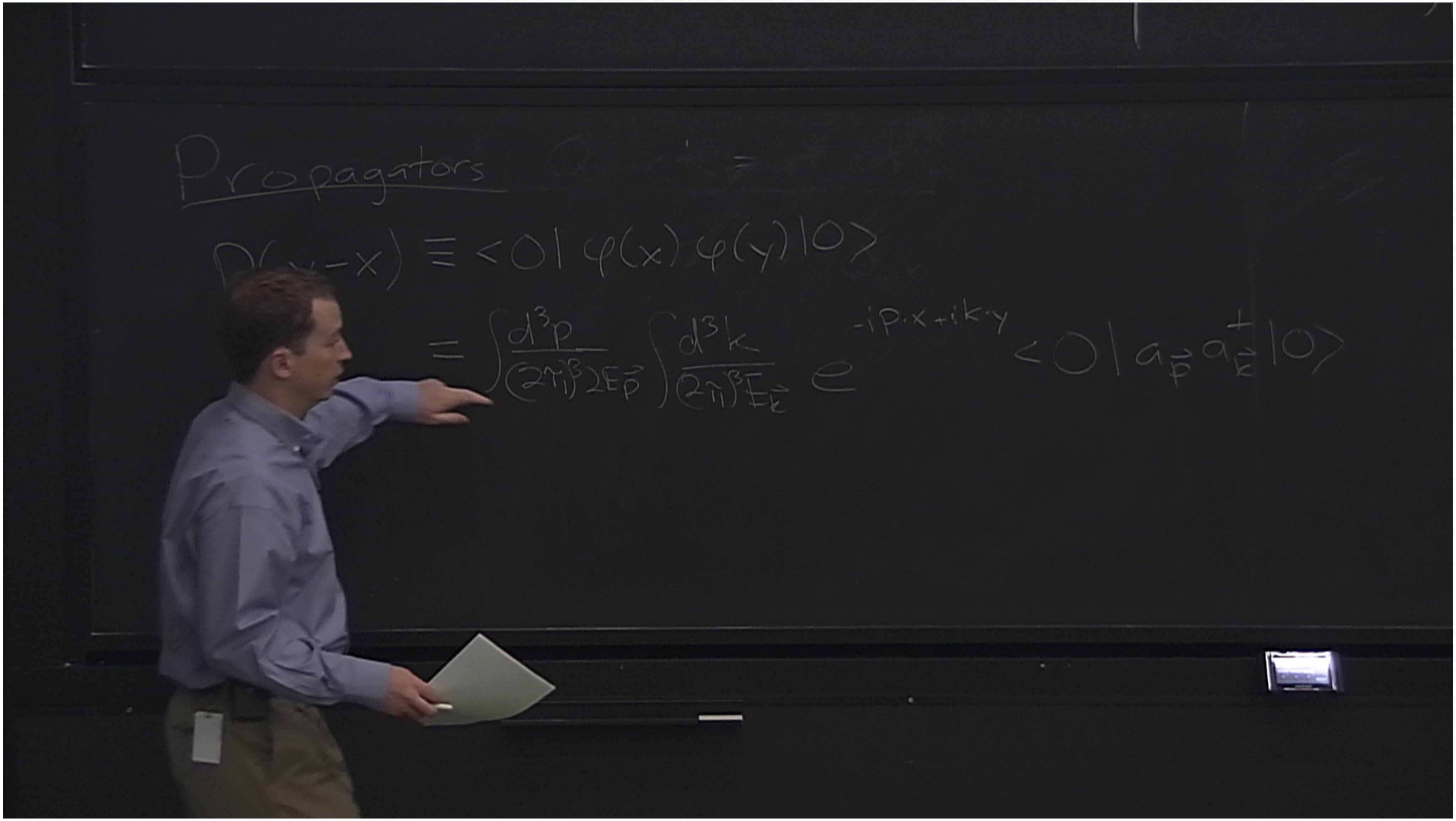

Causality
Require [O(x), O(y)] = 0
∀ (x−y)² < 0
spacelike
Δ(x−y) = [φ(x), φ(y)]
= ∫ d³p / (2π)³ 2E_p (e^{−ip·(x−y)} − e^{ip·(x−y)})

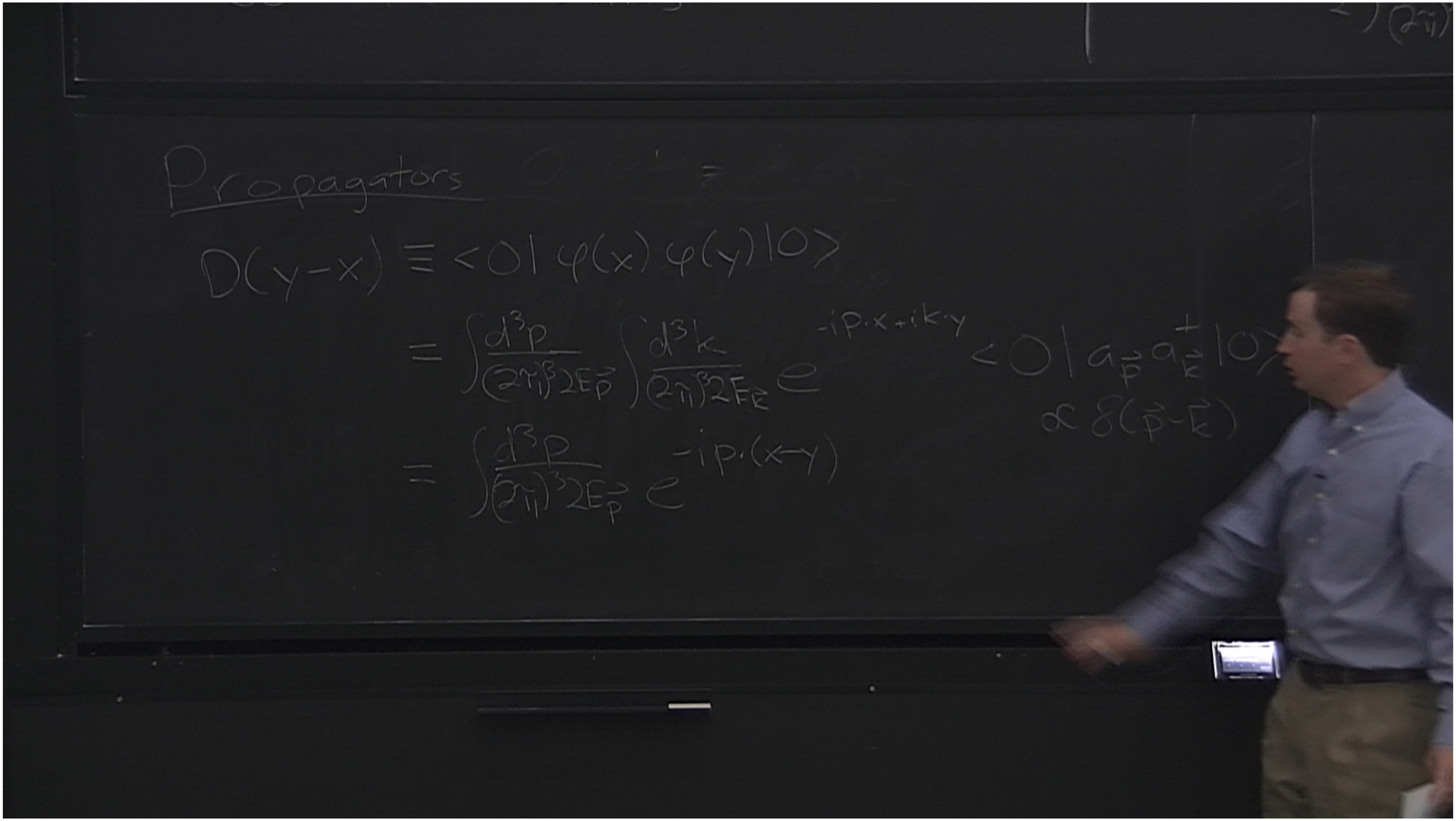

- Lorentz invariant
- number, not operator (only in free theory)
∀(x-y)^2 < 0
spacelike
- vanishes for space

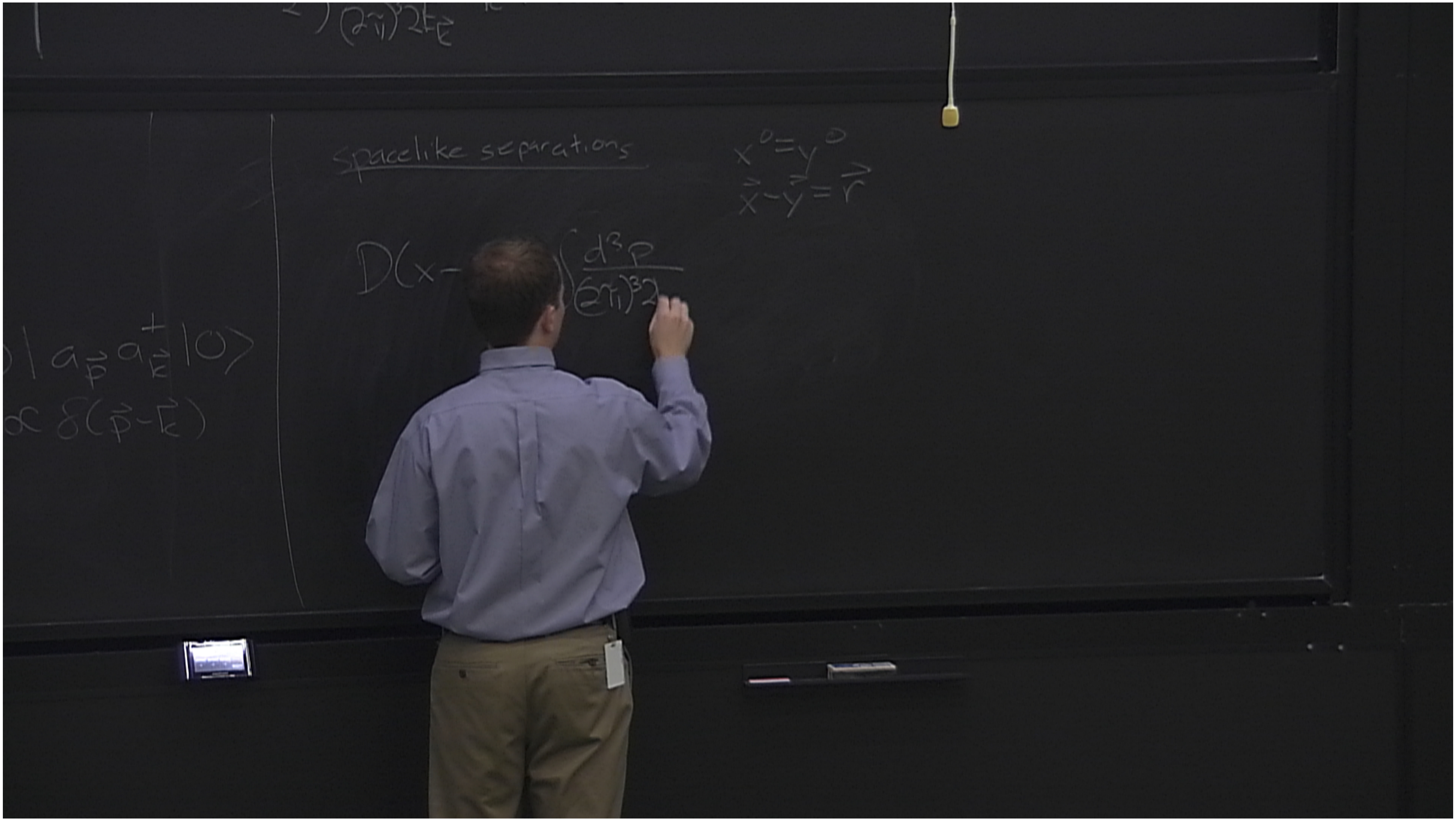

- Lorentz invariant
∀(x−y)² < 0
spacelike
- number, not operator (only in free theory)
- vanishes for
choose

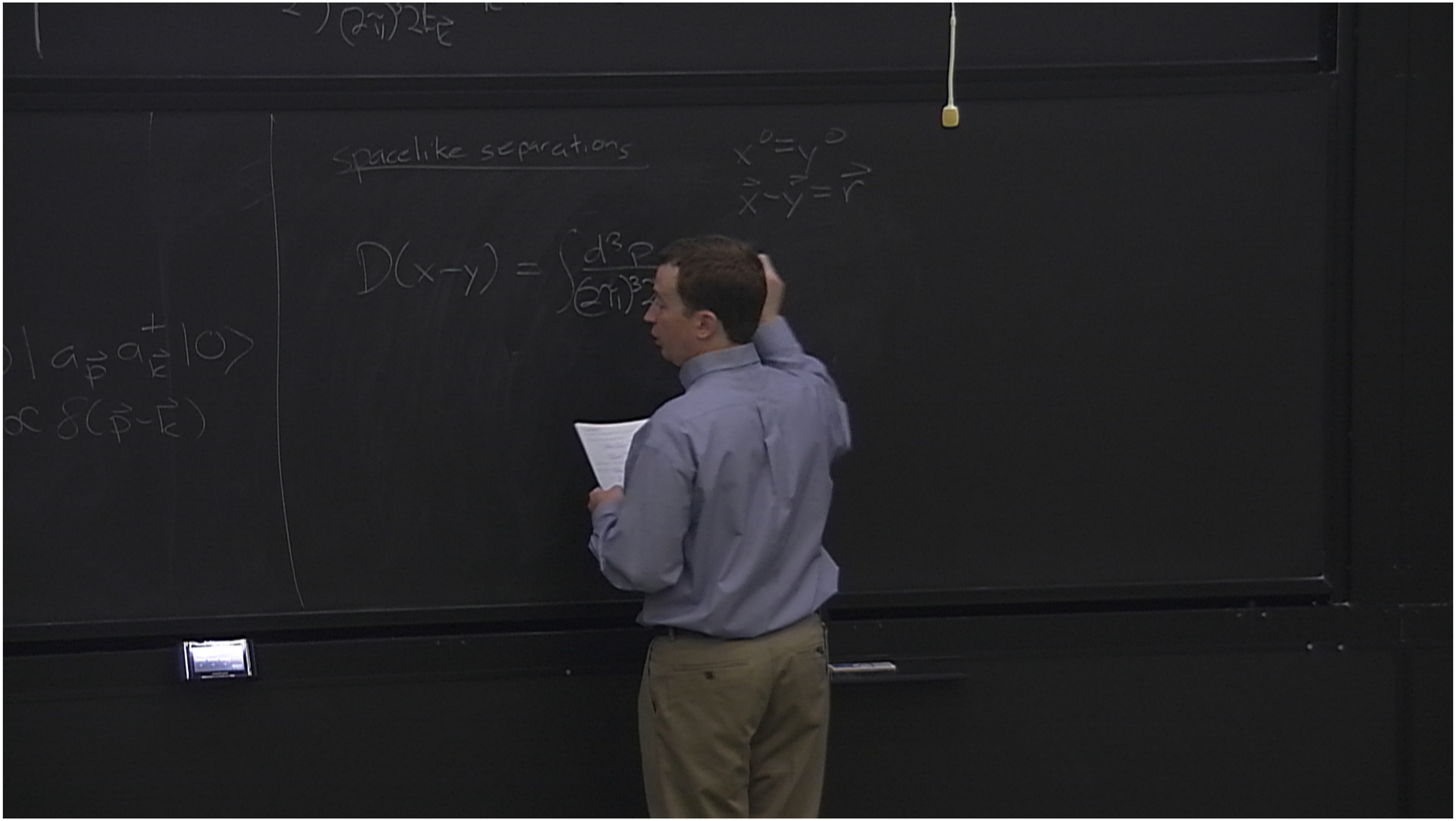

∀(x−y)² < 0
spacelike
- Lorentz invariant
- number, not operator (only in free theory)
- vanishes for spacelike separations
choose x⁰=y⁰
( ) = (e^{ip⃗(x⃗−y⃗)} − e^{−ip⃗(x⃗−y⃗)})
p⃗→p⃗

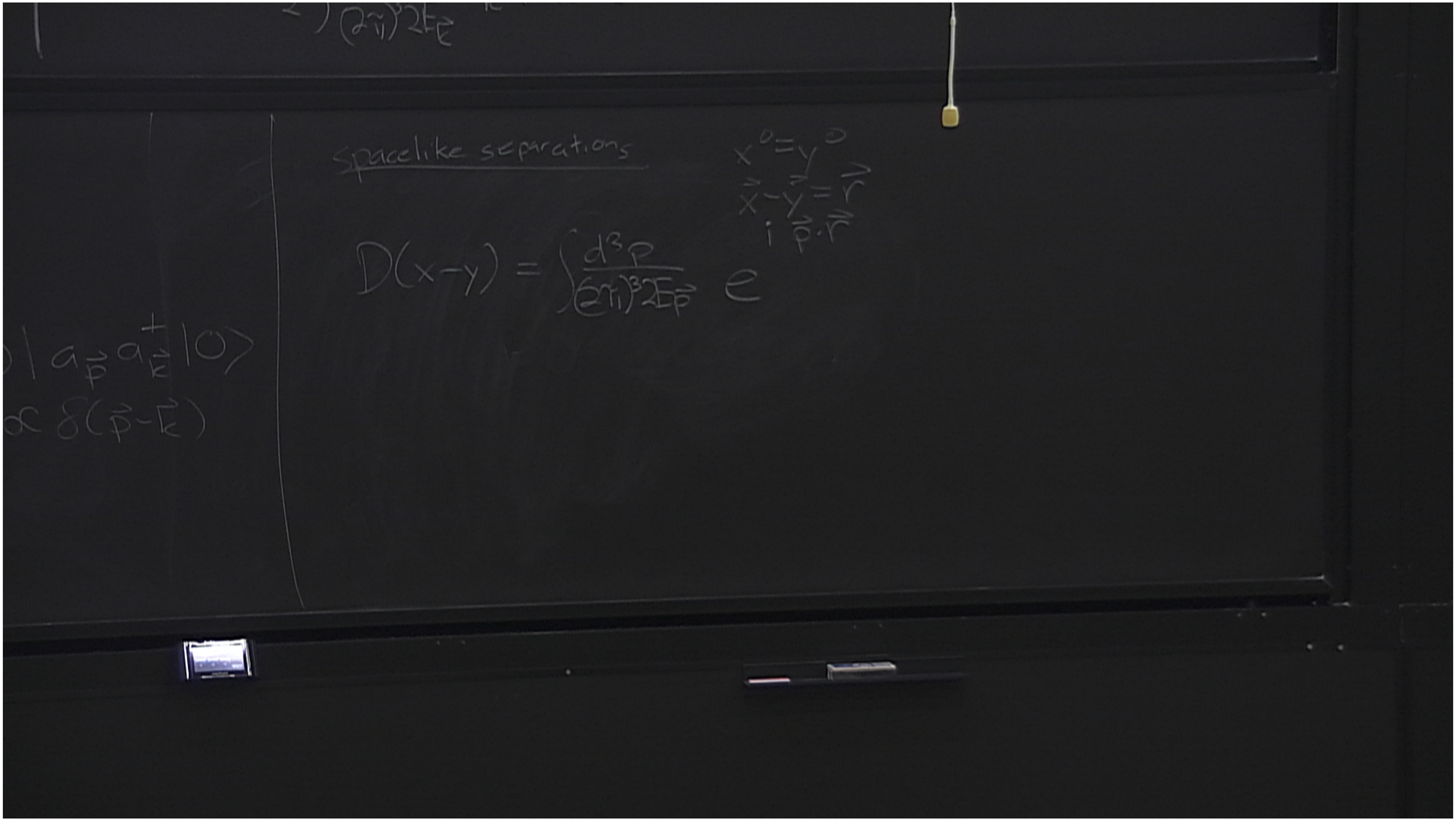

Propagators
D(y-x) ≡ <0| φ(x) φ(y) |0>
=

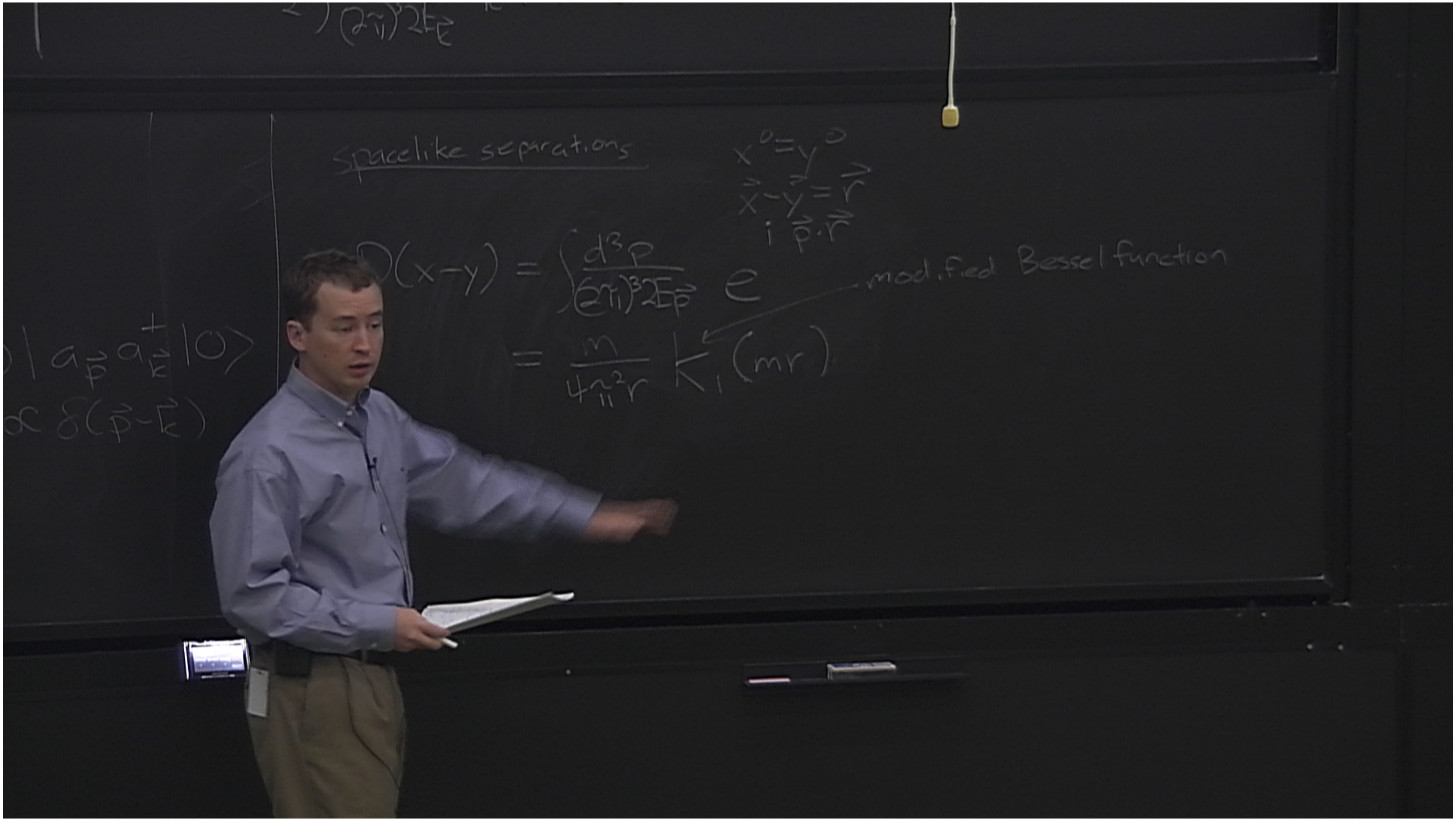

Propagators
D(y−x) ≡ <0| φ(x) φ(y) |0>
= ∫ d³p / (2π)³ 2E_p ∫ d³k / (2π)³ E_k e^(−ip·x + ik·y) <0| a_p a_k† |0>

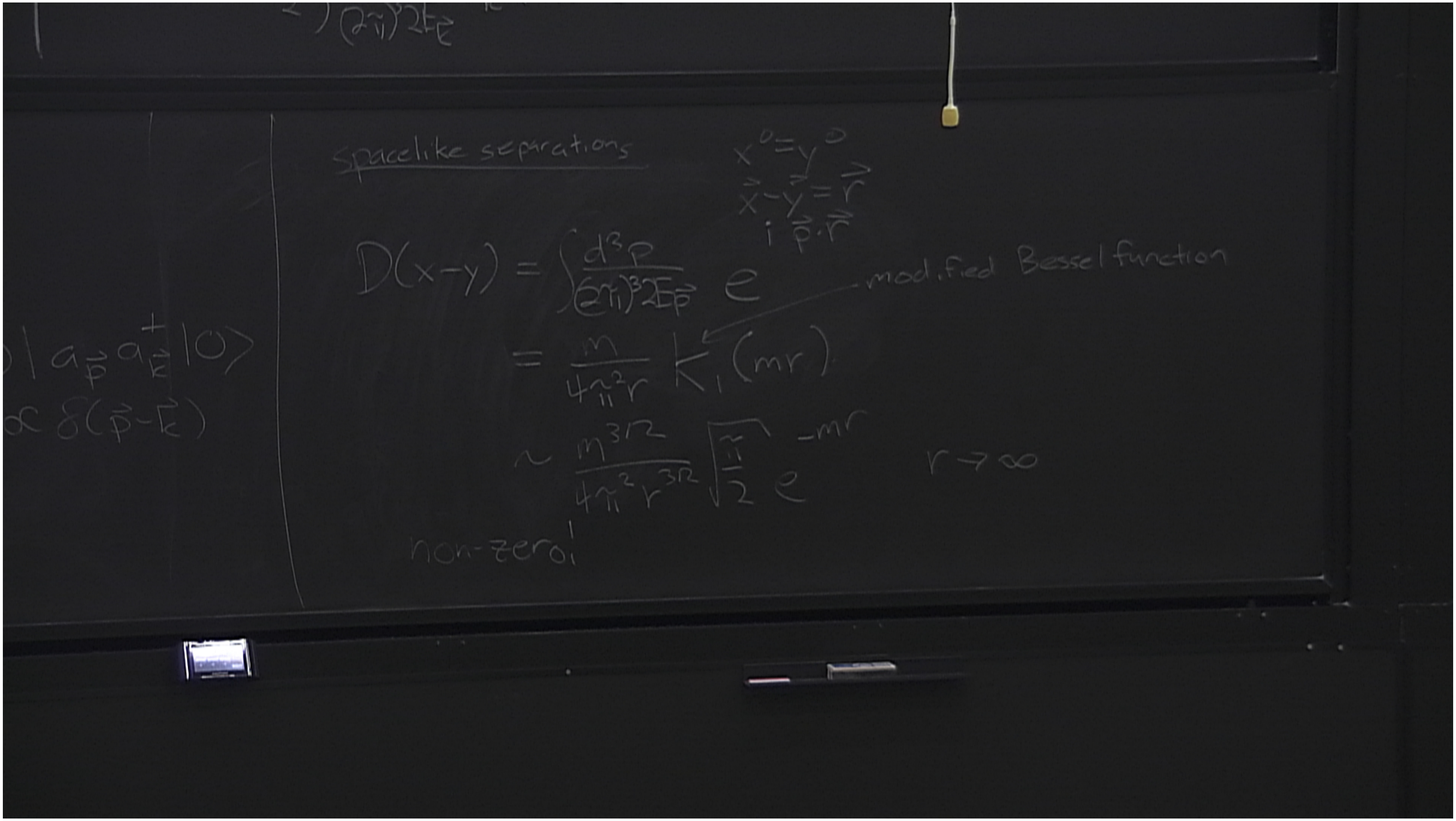

Propagators
$D(y-x) \equiv \langle 0|\varphi(x)\varphi(y)|0\rangle$
$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} e^{-ip\cdot x + ik\cdot y} \langle 0|a_{\vec{p}} a^\dagger_{\vec{k}}|0\rangle$
$\propto \delta(\vec{p}-\vec{k})$
$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip\cdot(x-y)}$

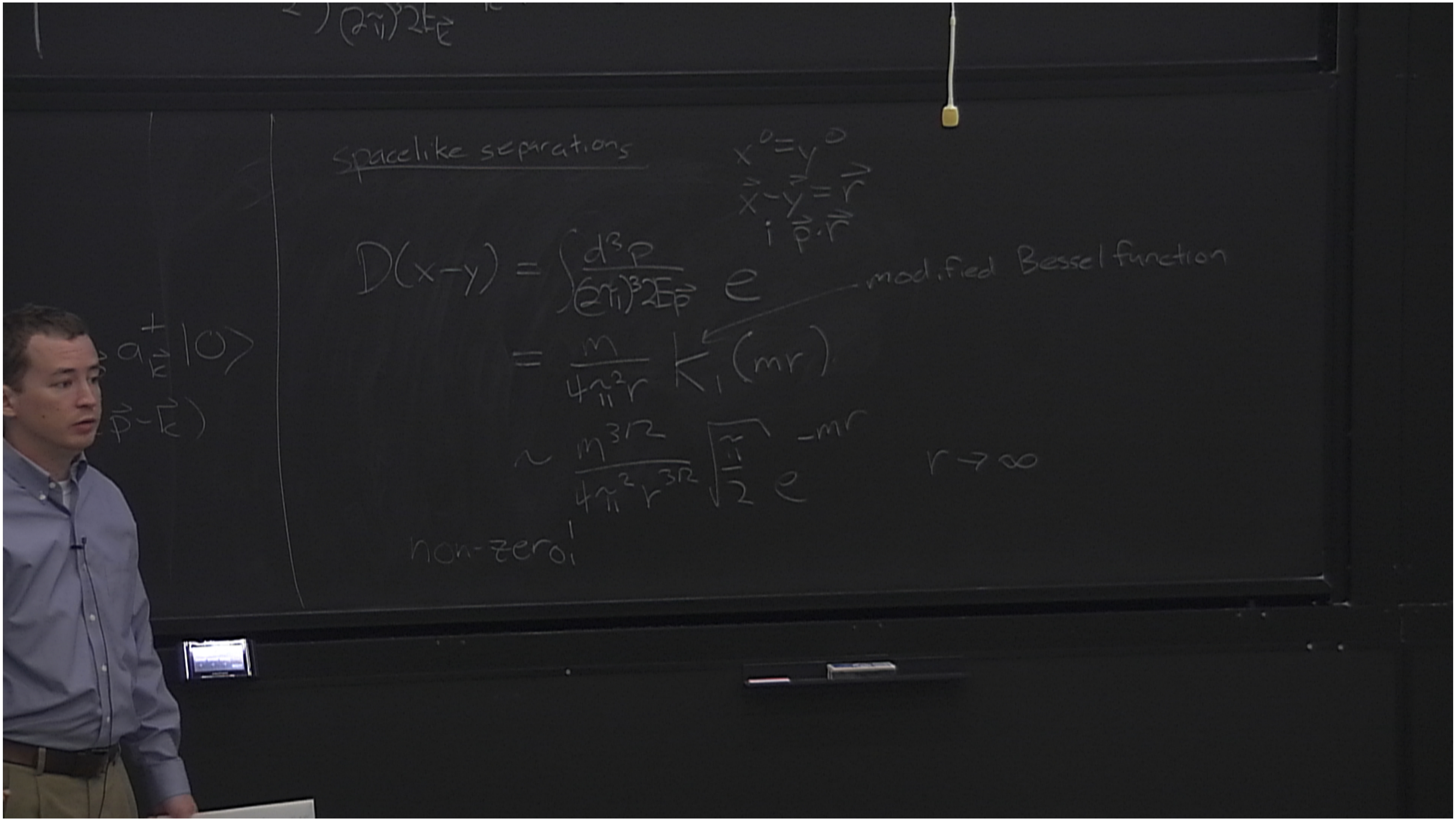

spacelike separations
x⁰=y⁰
x⃗−y⃗=r⃗

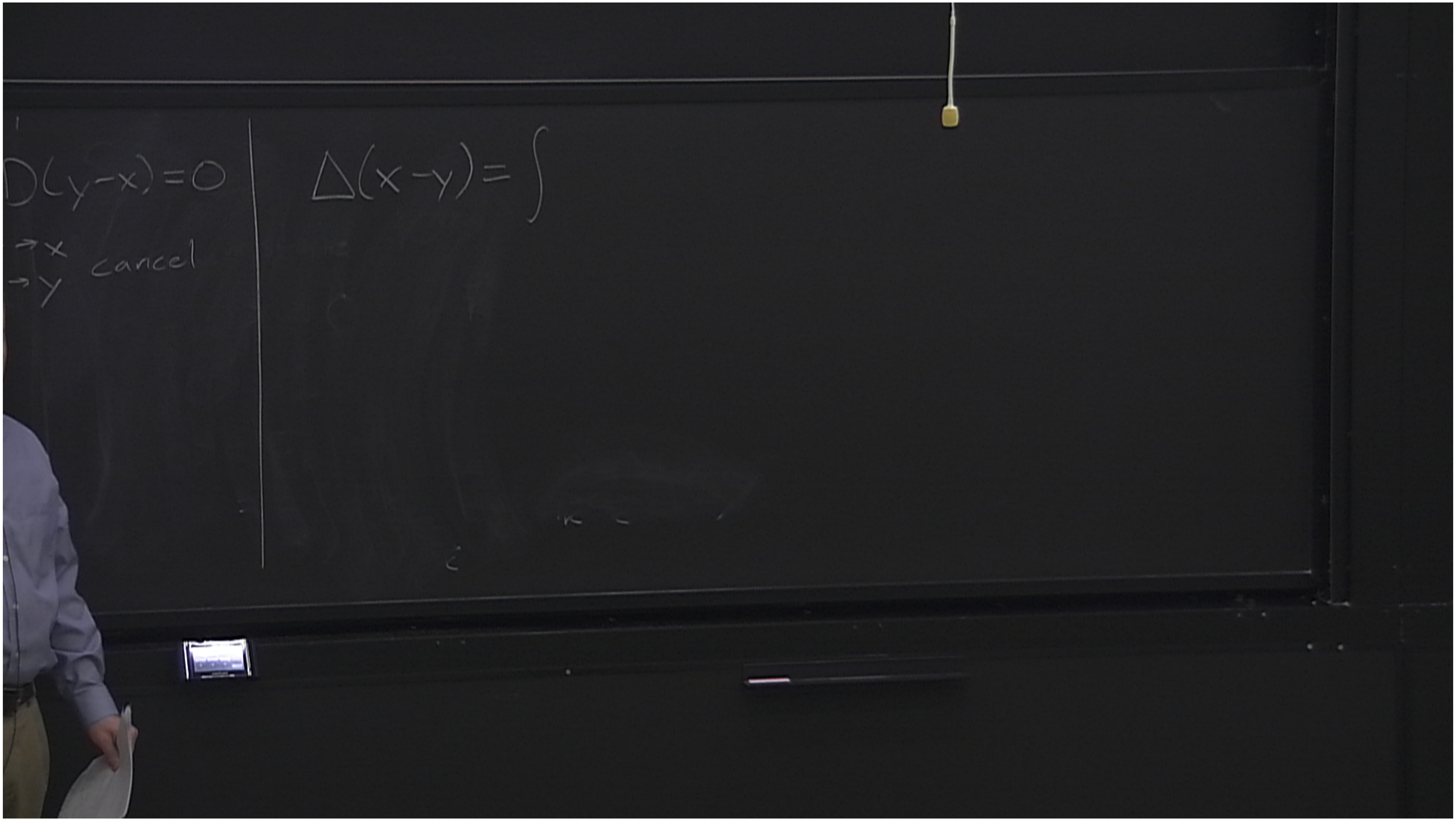

spacelike separations
$x^0 = y^0$
$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2}$
$| a_{\vec{p}} a^{\dagger}_{\vec{k}} |0\rangle$
$\propto \delta(\vec{p} - \vec{k})$

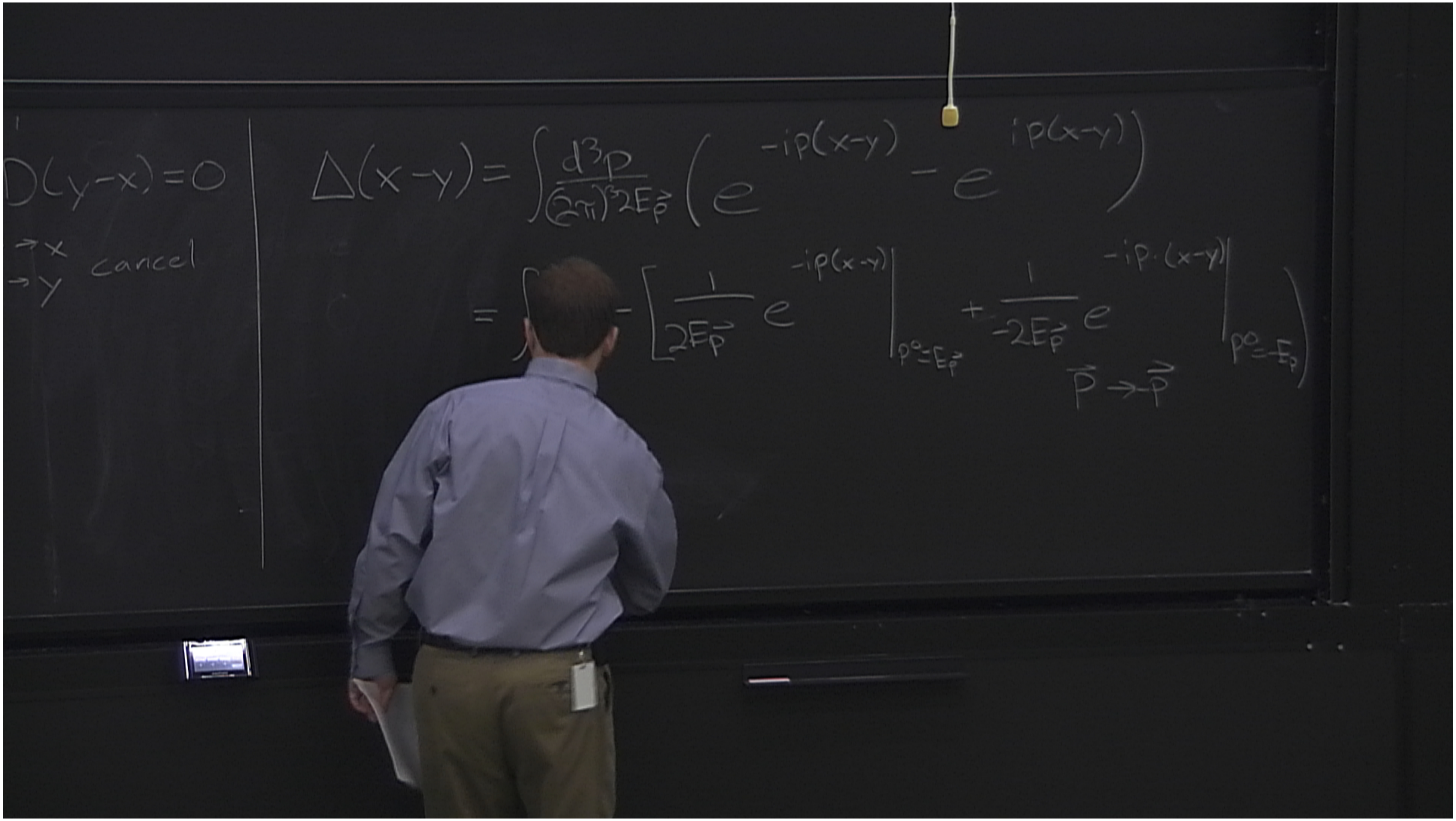

spacelike separations
x⁰ = y⁰
x⃗ − y⃗ = r⃗
D(x−y) = ∫ d³p / (2π)³ 2E_p⃗ e^(i p⃗·r⃗)
|0⟩
∝ δ(p⃗ − k⃗)

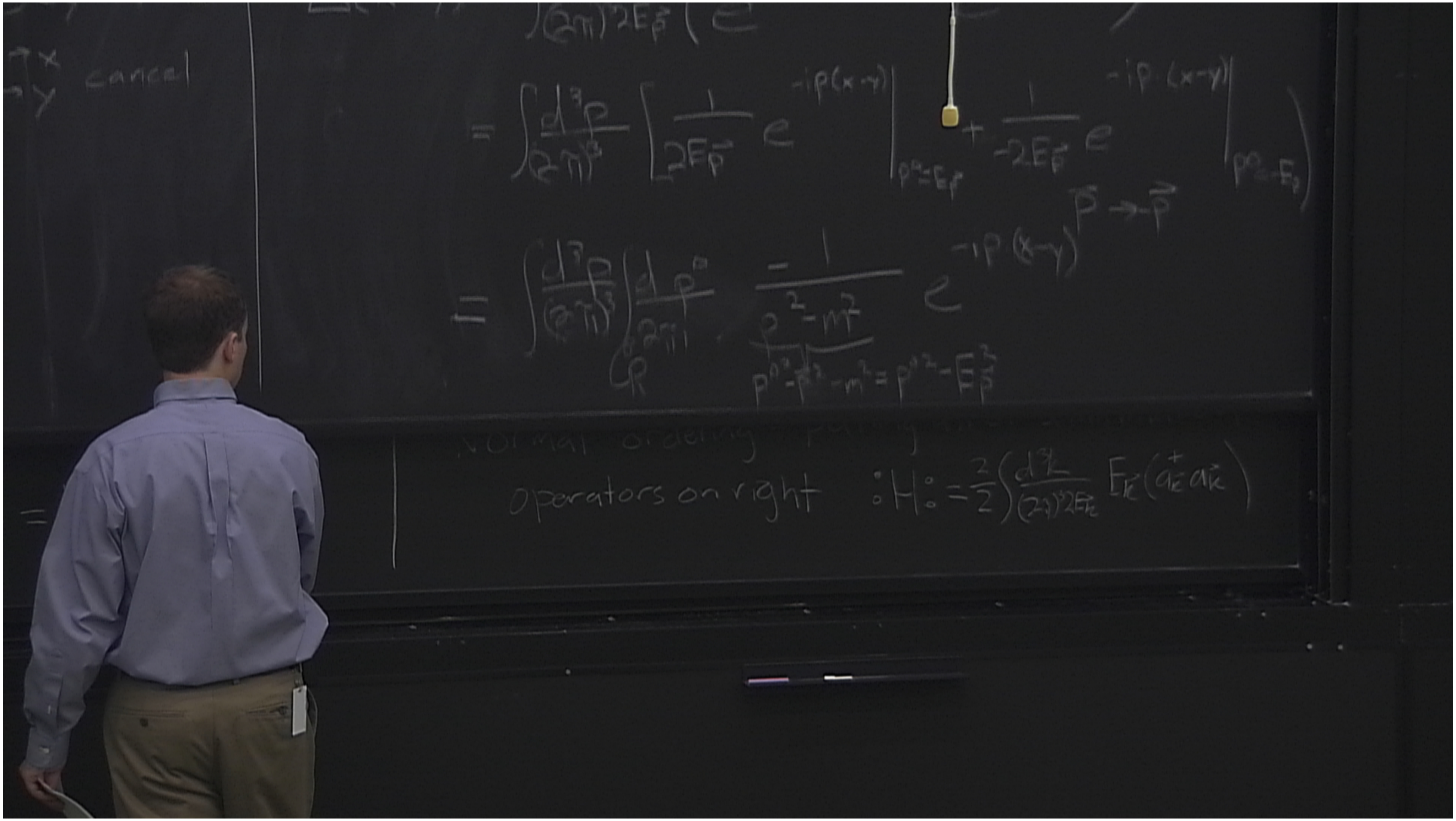

spacelike separations
$x^0 = y^0$
$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$
$= \frac{m}{4\pi^2 r} K_1(mr)$
modified Bessel function
$\propto \delta(\vec{p}-\vec{k})$

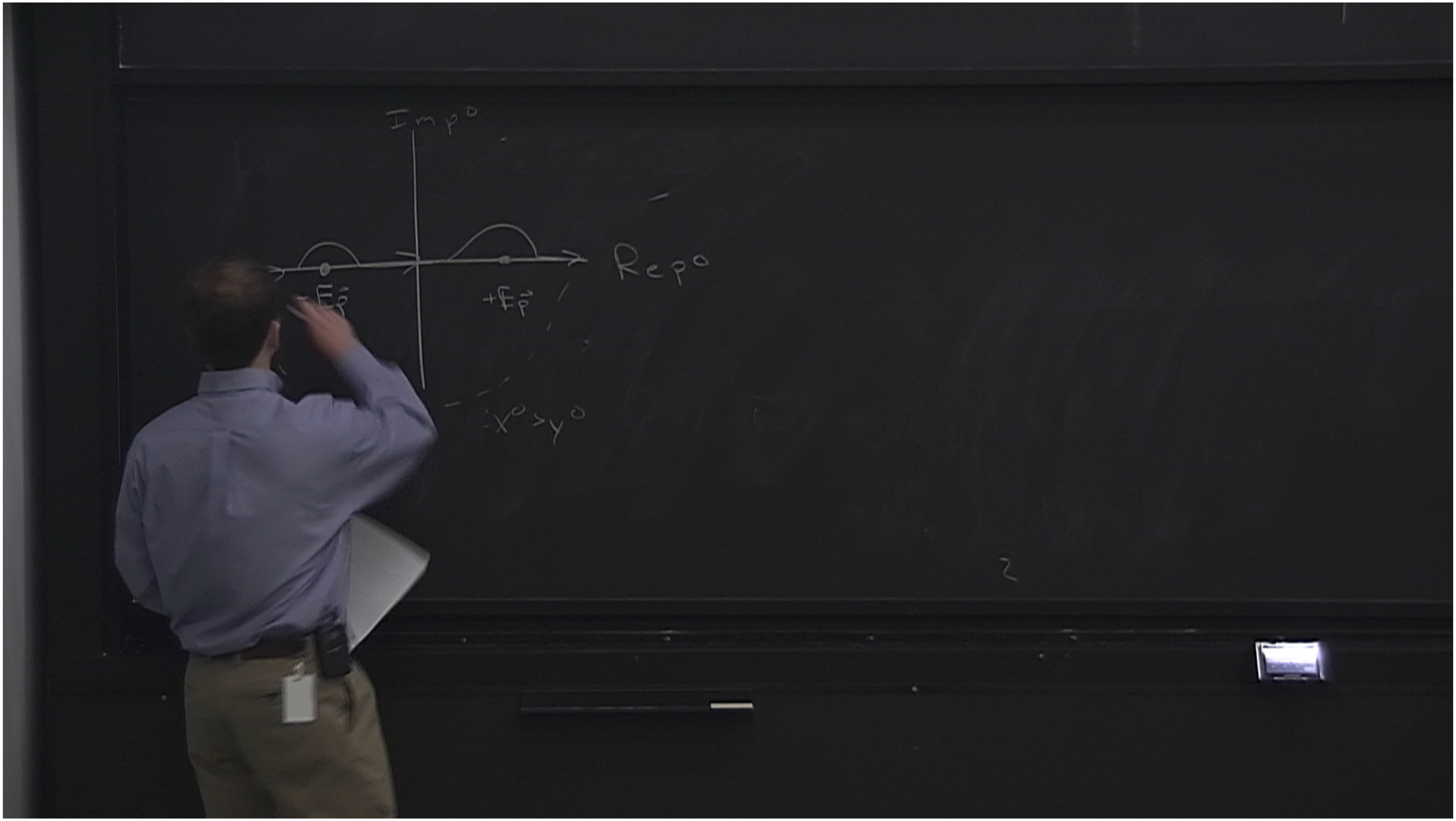

spacelike separations
$x^0 = y^0$
$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$
modified Bessel function
$= \frac{m}{4\pi^2 r} K_1(mr)$
$\sim \frac{m^{3/2}}{4\pi^2 r^{3/2}} \sqrt{\frac{\pi}{2}} e^{-mr}$
$r \to \infty$
non-zero!
$\propto \delta(\vec{p} - \vec{k})$

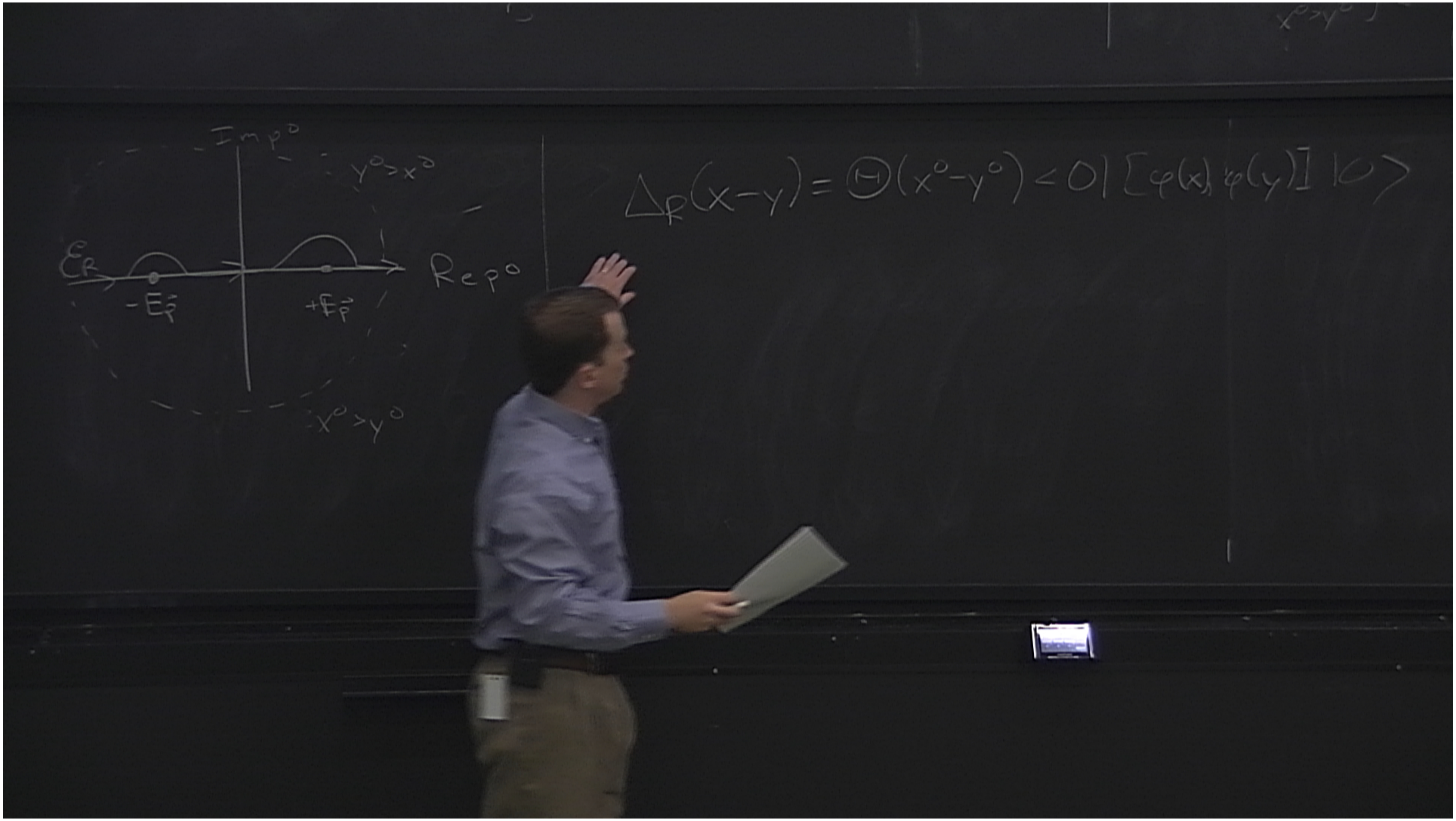

spacelike separations
$x^0 = y^0$
$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i \vec{p}\cdot\vec{r}}$
modified Bessel function
$= \frac{m}{4\pi^2 r} K_1(mr)$
$\sim \frac{m^{3/2}}{4\pi^2 r^{3/2}} \sqrt{\frac{\pi}{2}} e^{-mr}$
$r \to \infty$
non-zero!


D(y−x)=0
→x cancel
→y
Δ(x−y)= ∫

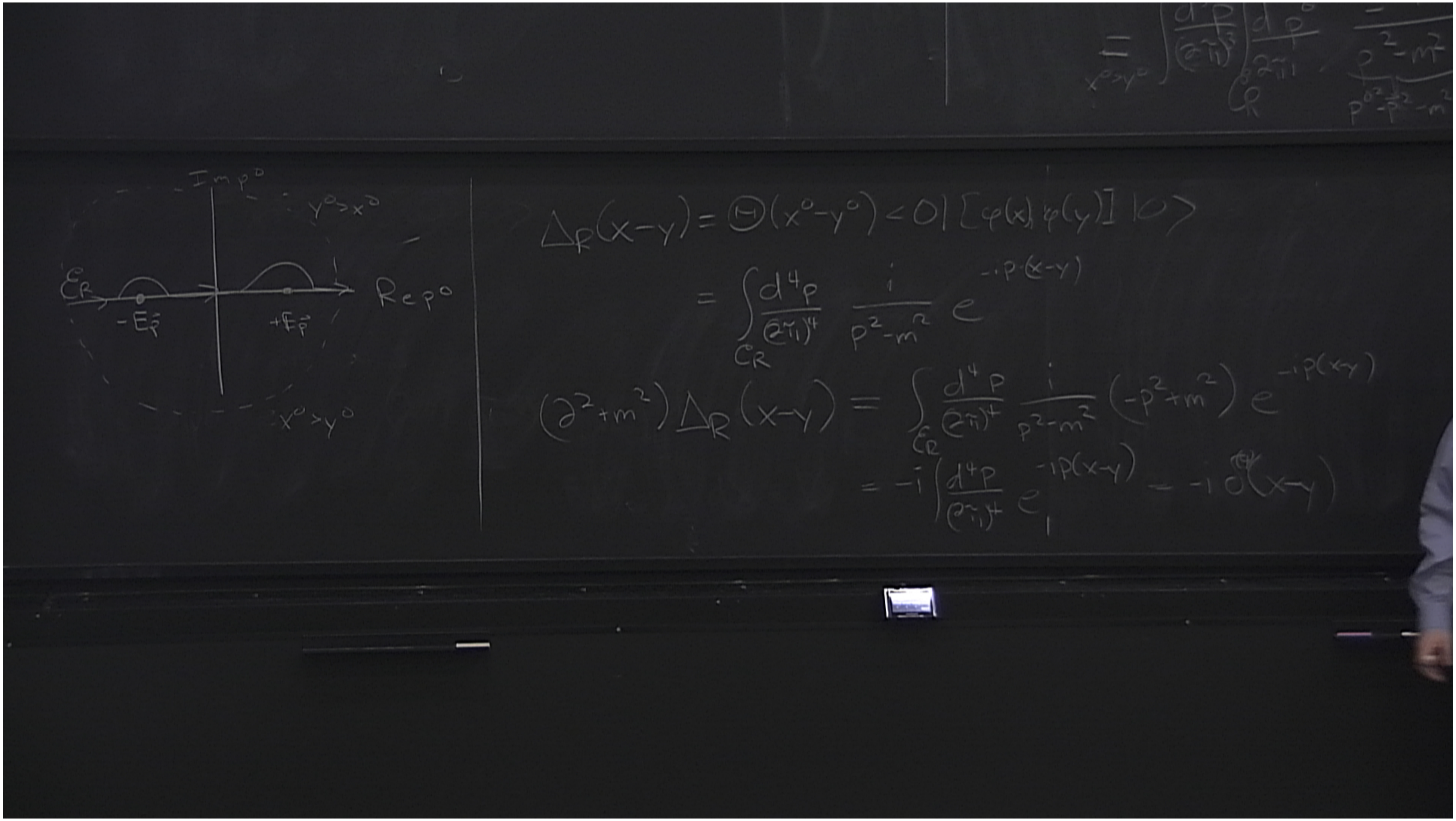

D(y−x)=0
→x
→y cancel
Δ(x−y) = ∫ d³p / (2π)³2E_p (e^{−ip(x−y)} − e^{ip(x−y)})
p→−p

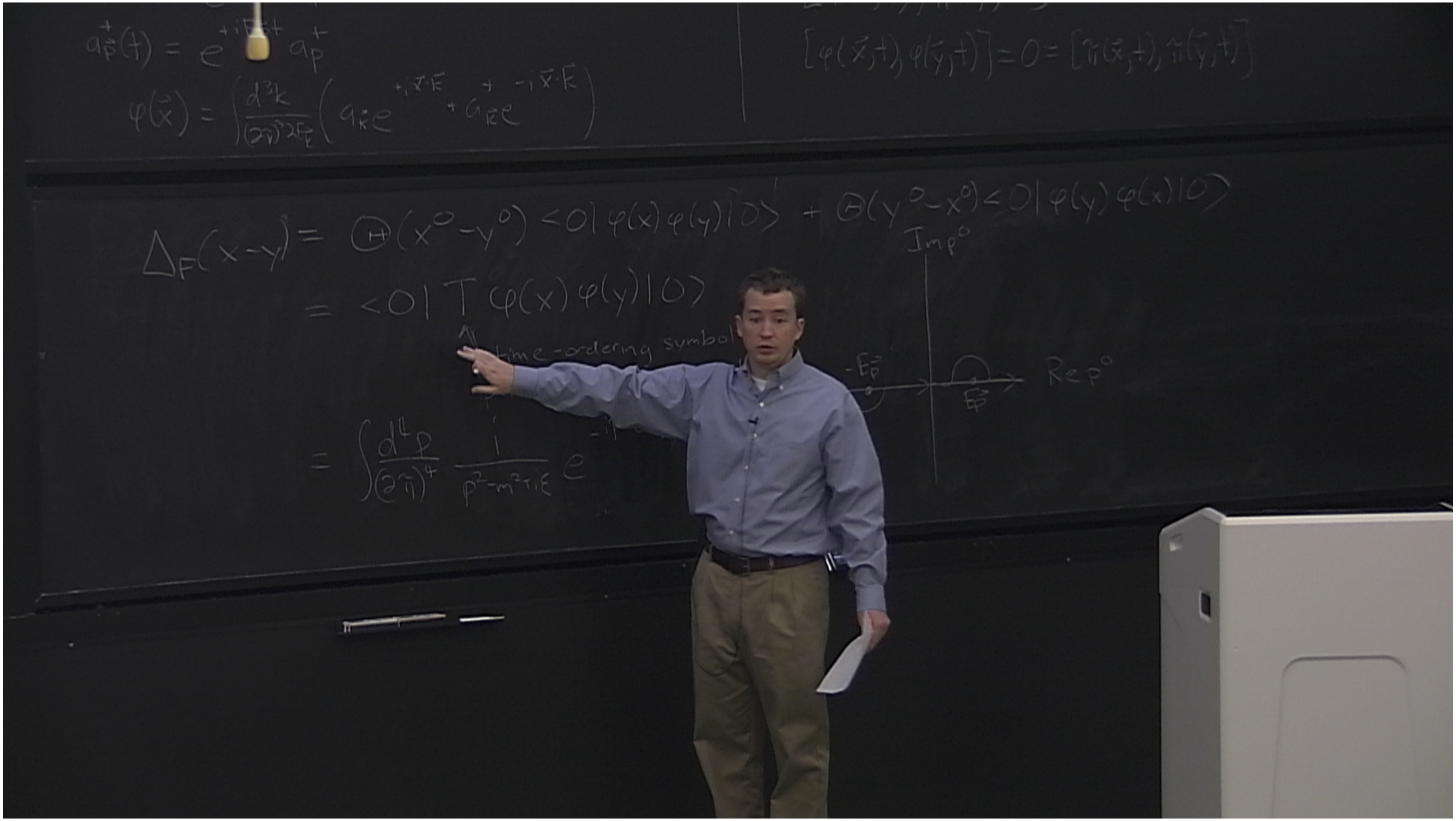

cancel
operators on right


Im p0
Re p0
$+E_{\vec p}$
$x^0 > y^0$



Im p0
y0 > x0
Re p0
-E_p
+E_p
x0 > y0
Δ_R(x−y) = Θ(x0−y0) <0| [φ(x), φ(y)] |0>


Im p°
y° > x°
Re p°
C_R
−E_p
+E_p
x° > y°
Δ_R(x−y) = Θ(x°−y°) <0| [φ(x), φ(y)] |0>
= ∫_{C_R} d⁴p/(2π)⁴ i/(p²−m²) e^{−ip·(x−y)}
(∂²+m²) Δ_R(x−y) = ∫_{C_R} d⁴p/(2π)⁴ i/(p²−m²) (−p²+m²) e^{−ip(x−y)}
= −i ∫ d⁴p/(2π)⁴ e^{−ip(x−y)} = −i δ⁴(x−y)


time-ordering symbol
Im p°
Re p°