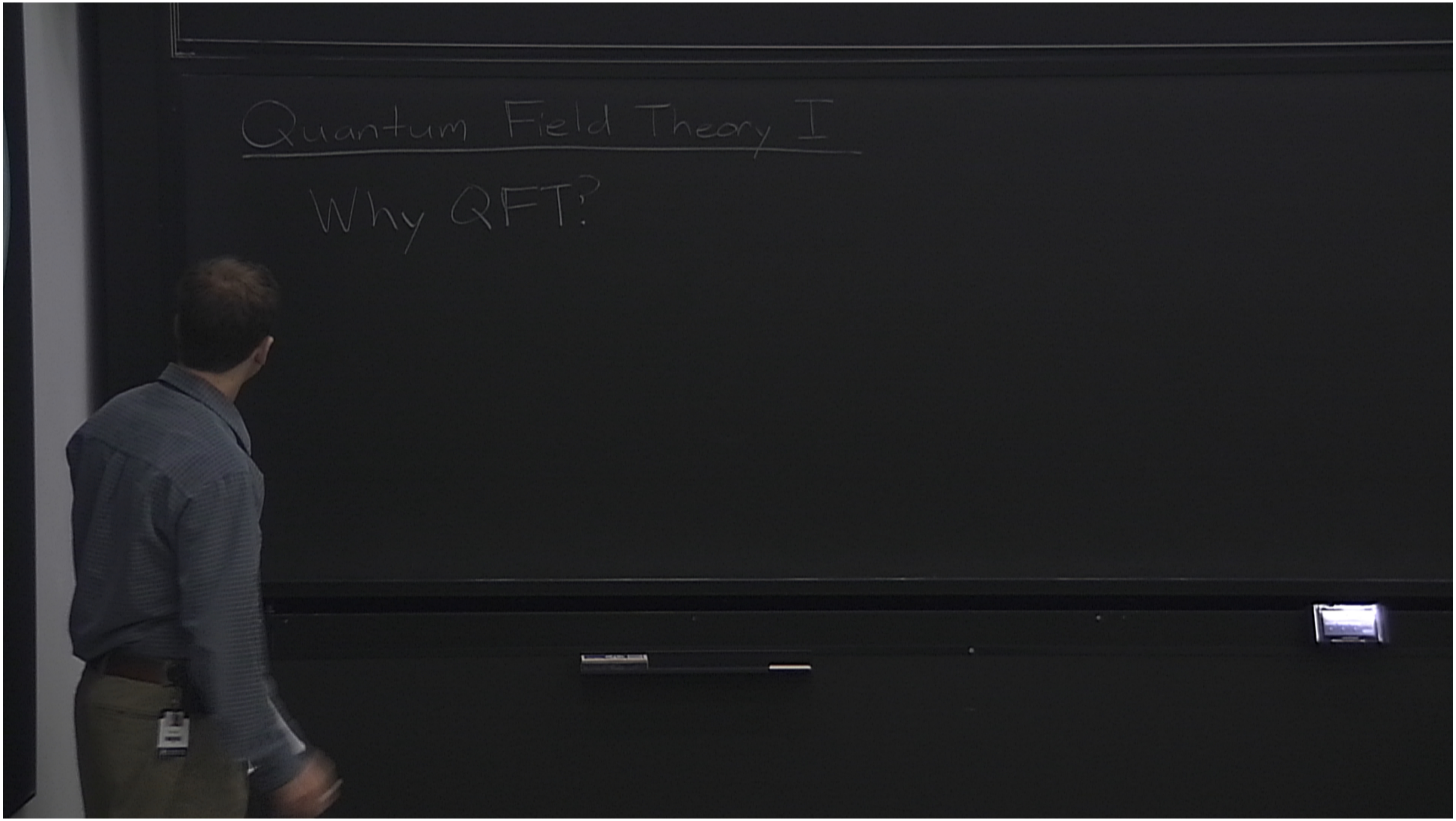
Title: PSI 2015/2016 Quantum Field Theory I - Daniel Wohns and Tibra Ali - Lecture 1

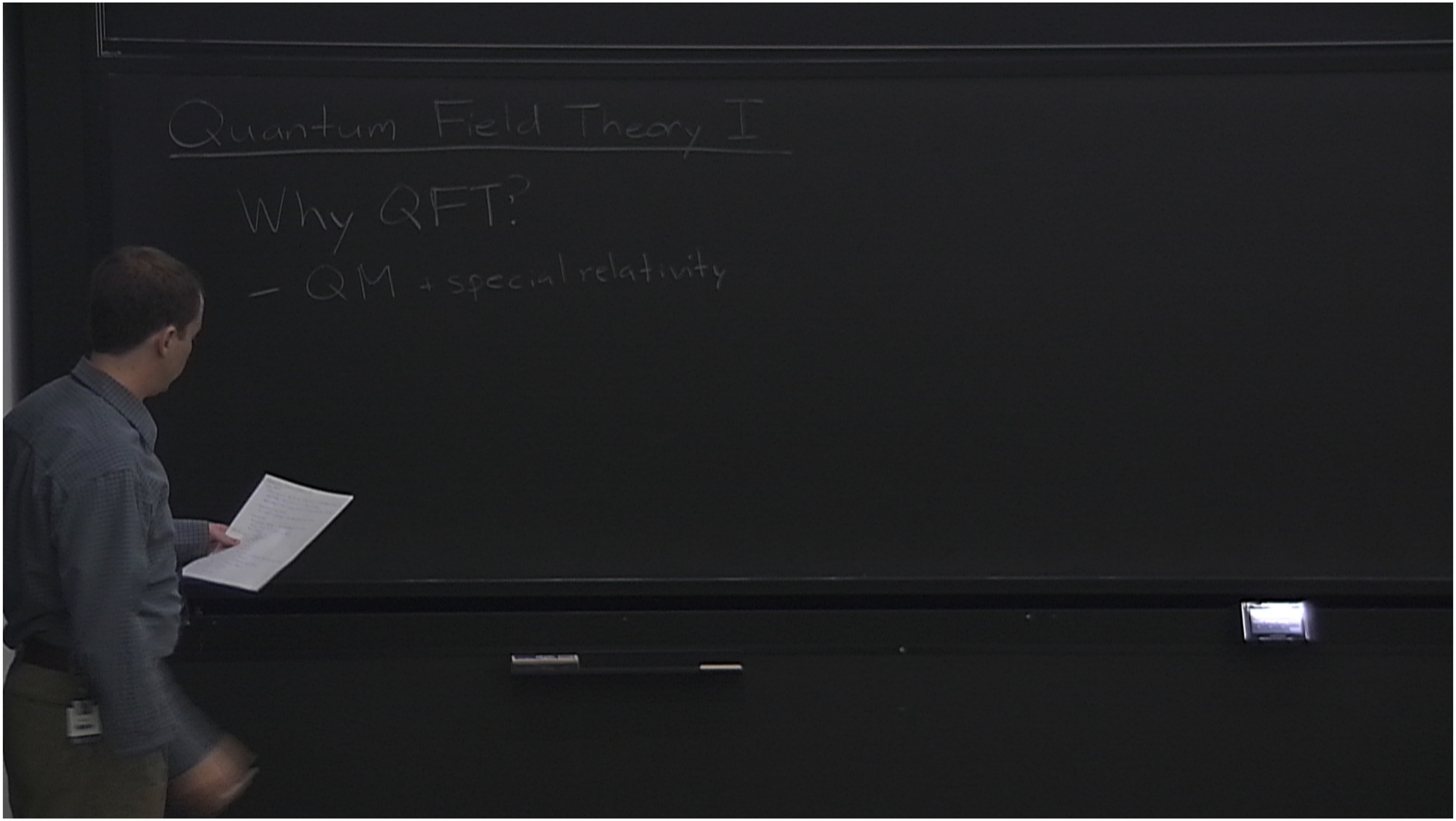
Date: Oct 13, 2015 09:00 AM

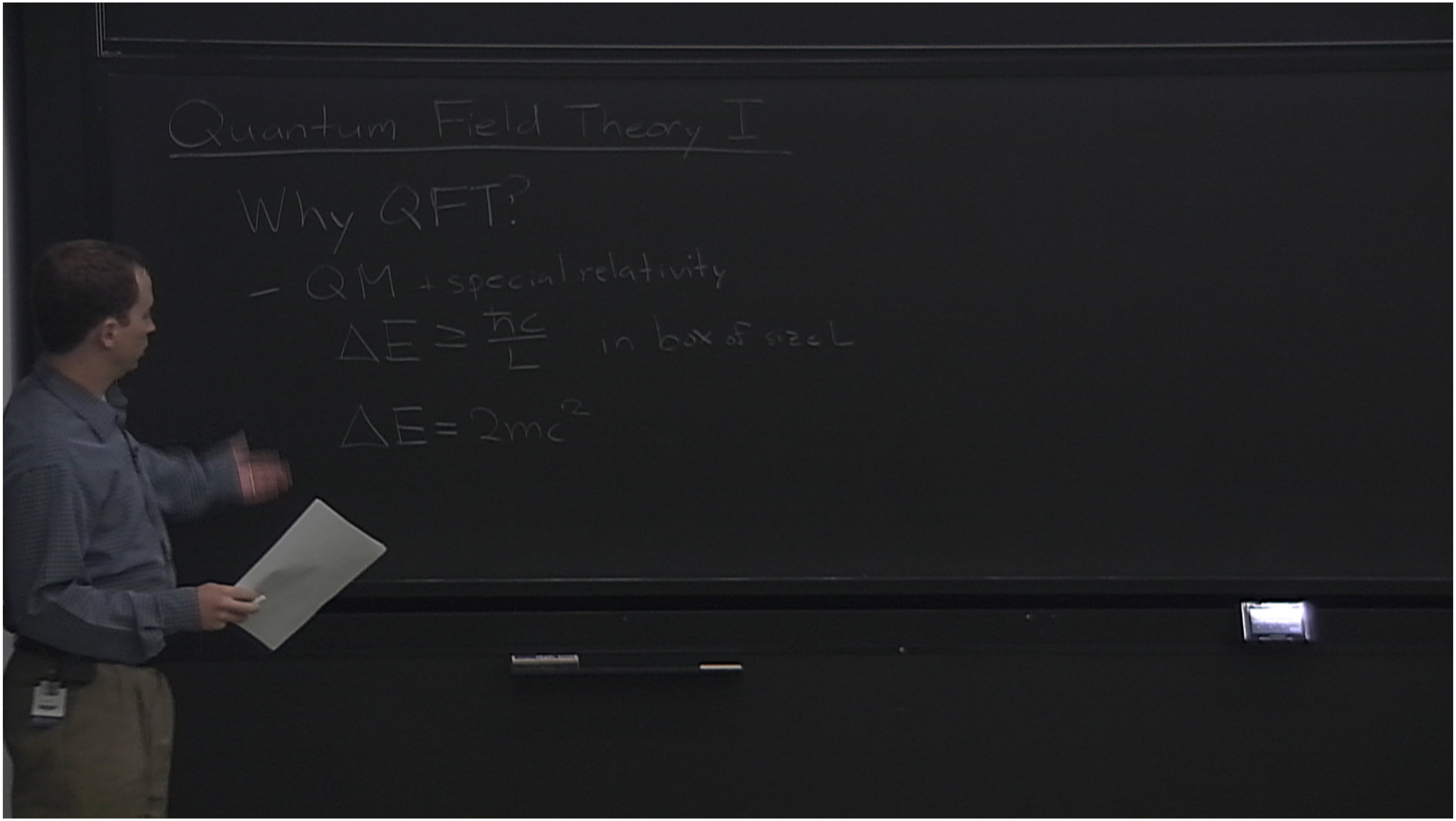
URL: http://pirsa.org/15100036

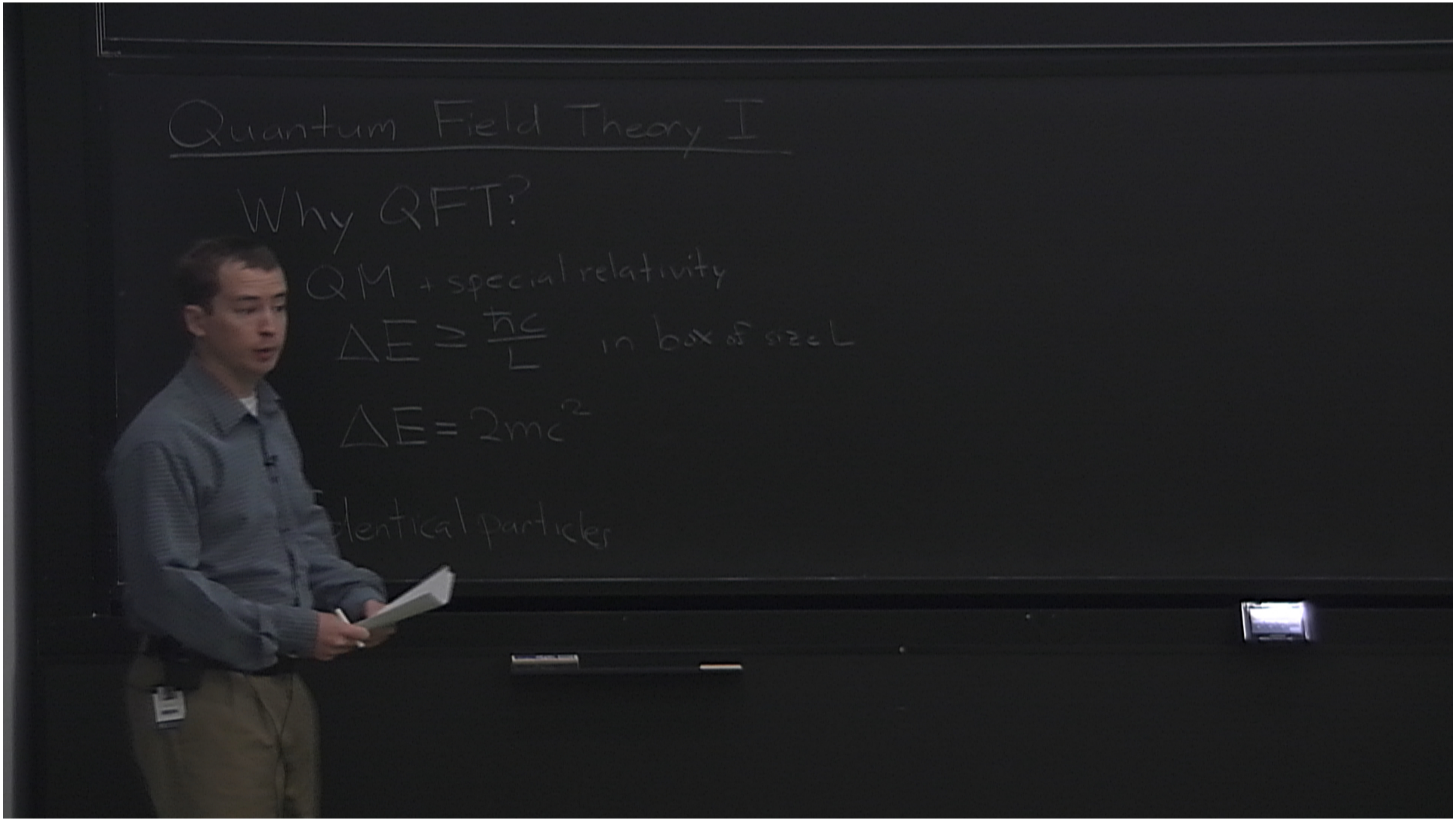
Abstract:


Quantum Field Theory I


Quantum Field Theory I
Why QFT?


Quantum Field Theory I
Why QFT?
– QM + special relativity


Quantum Field Theory I
Why QFT?
– QM + special relativity
$\Delta E \geq \frac{\hbar c}{L}$ in box of size L
$\Delta E = 2mc^2$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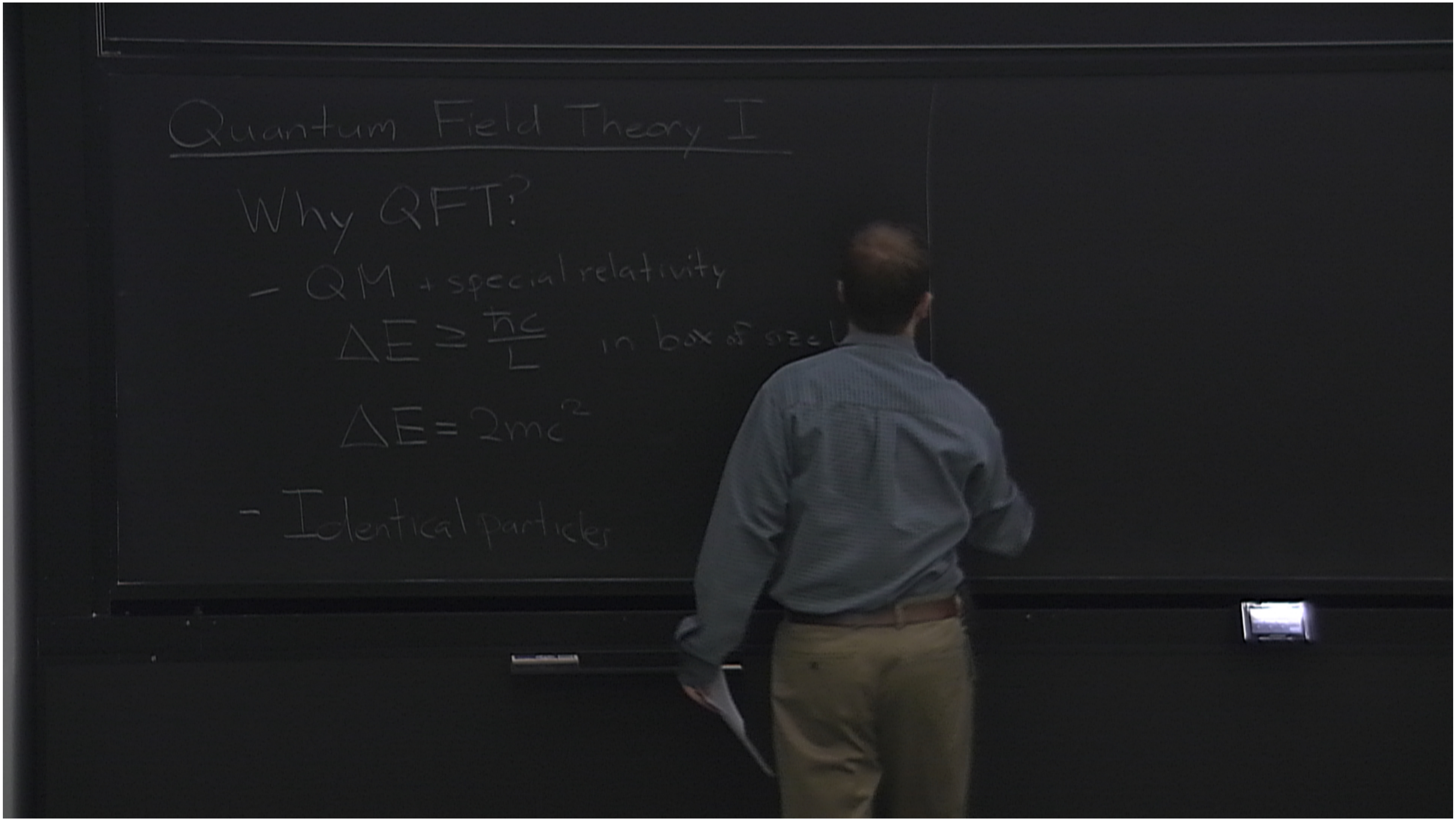
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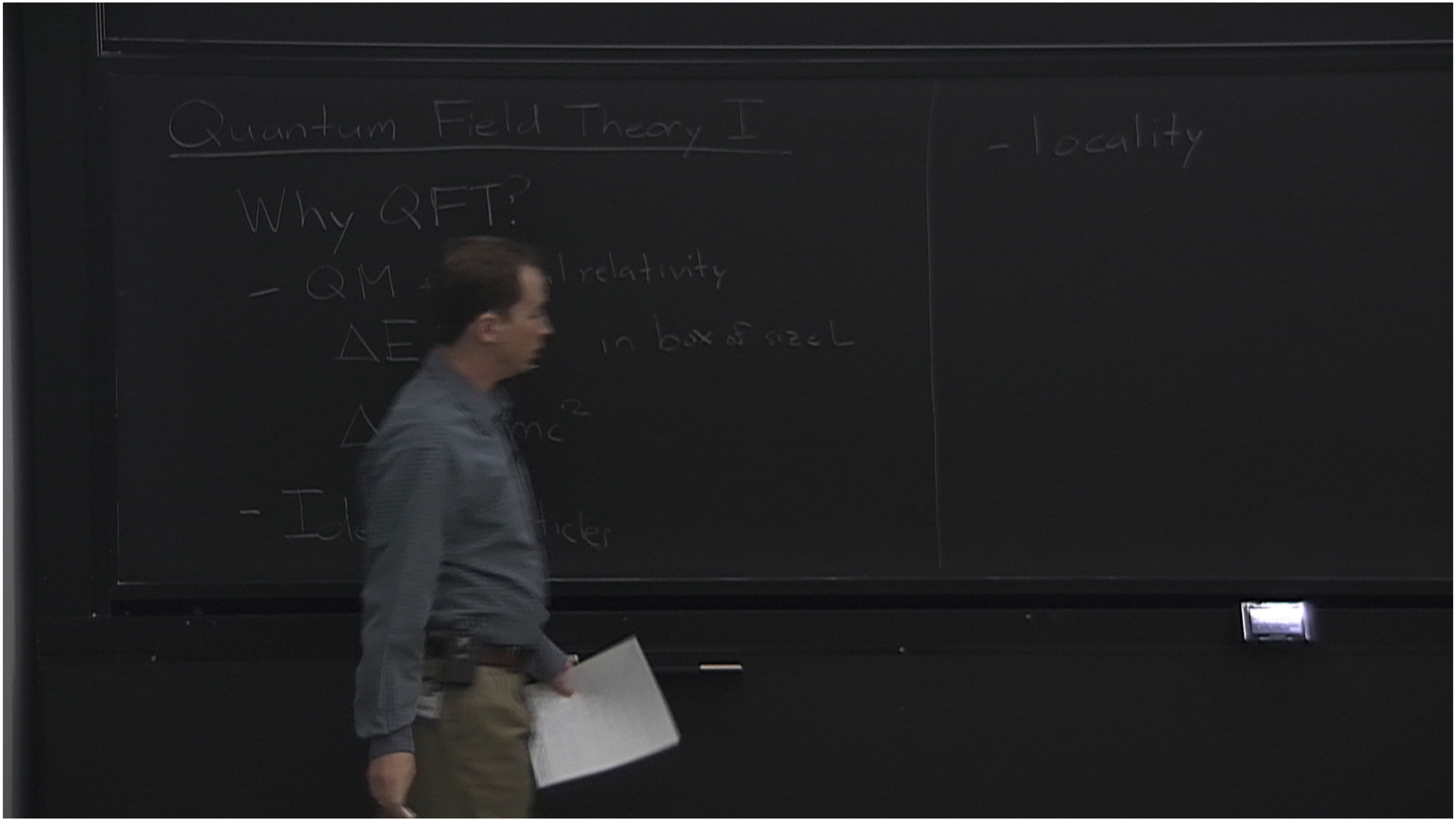
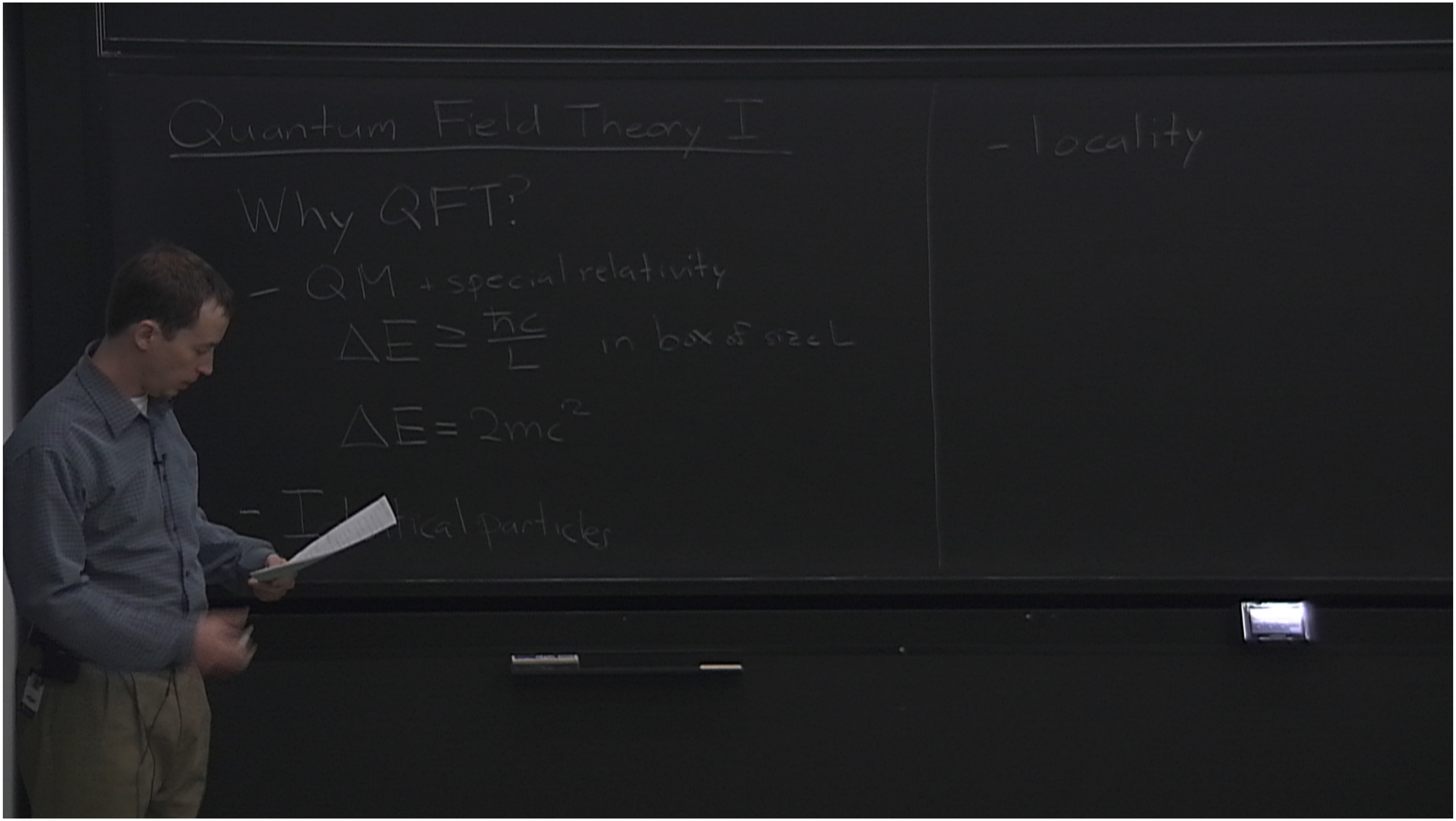
- QM + special relativity
ΔE ≥ ħc/L in box of size
ΔE = 2mc²
- Identical particles


Quantum Field Theory I
Why QFT?
relativity
in box of size L
- locality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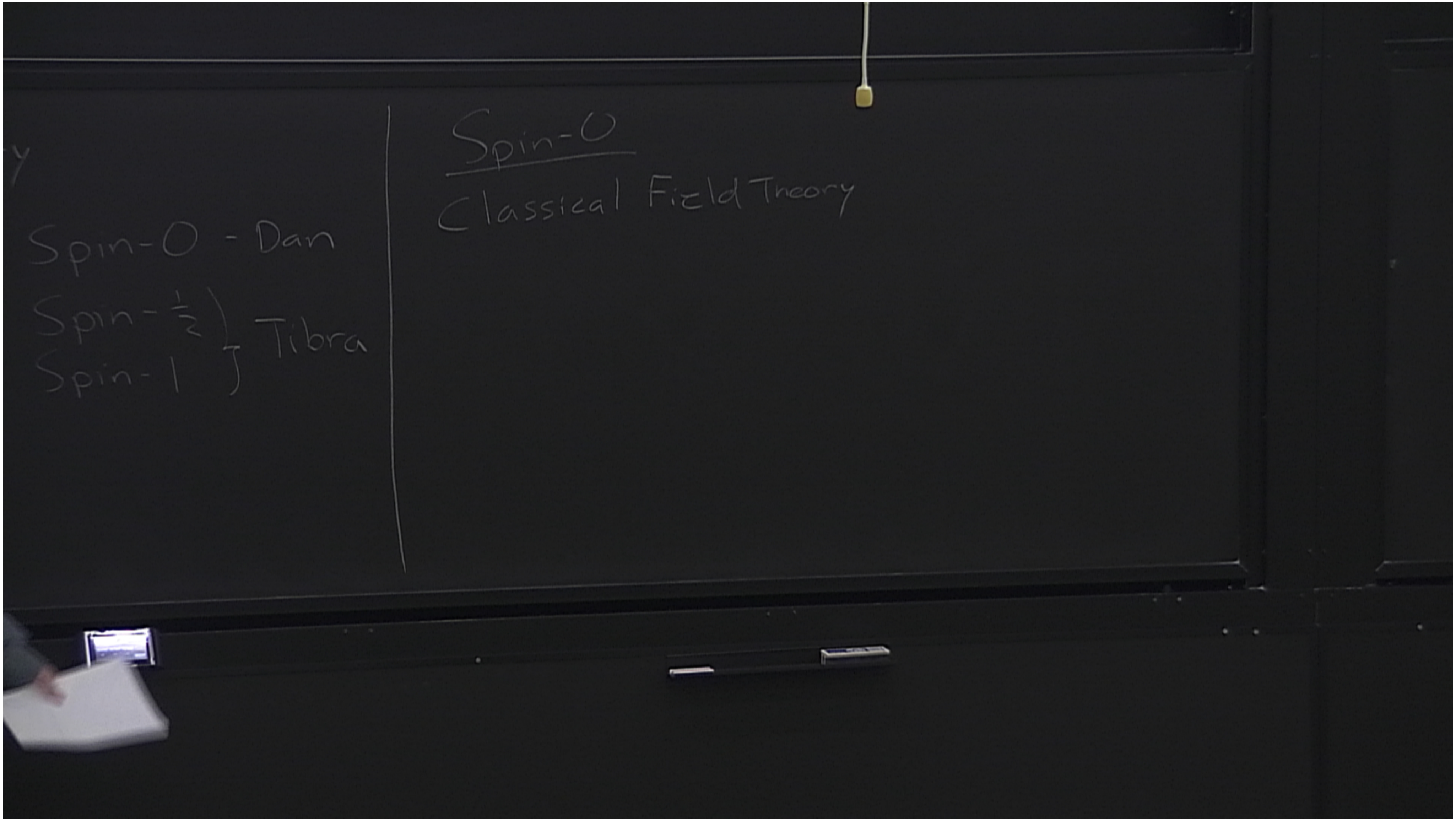
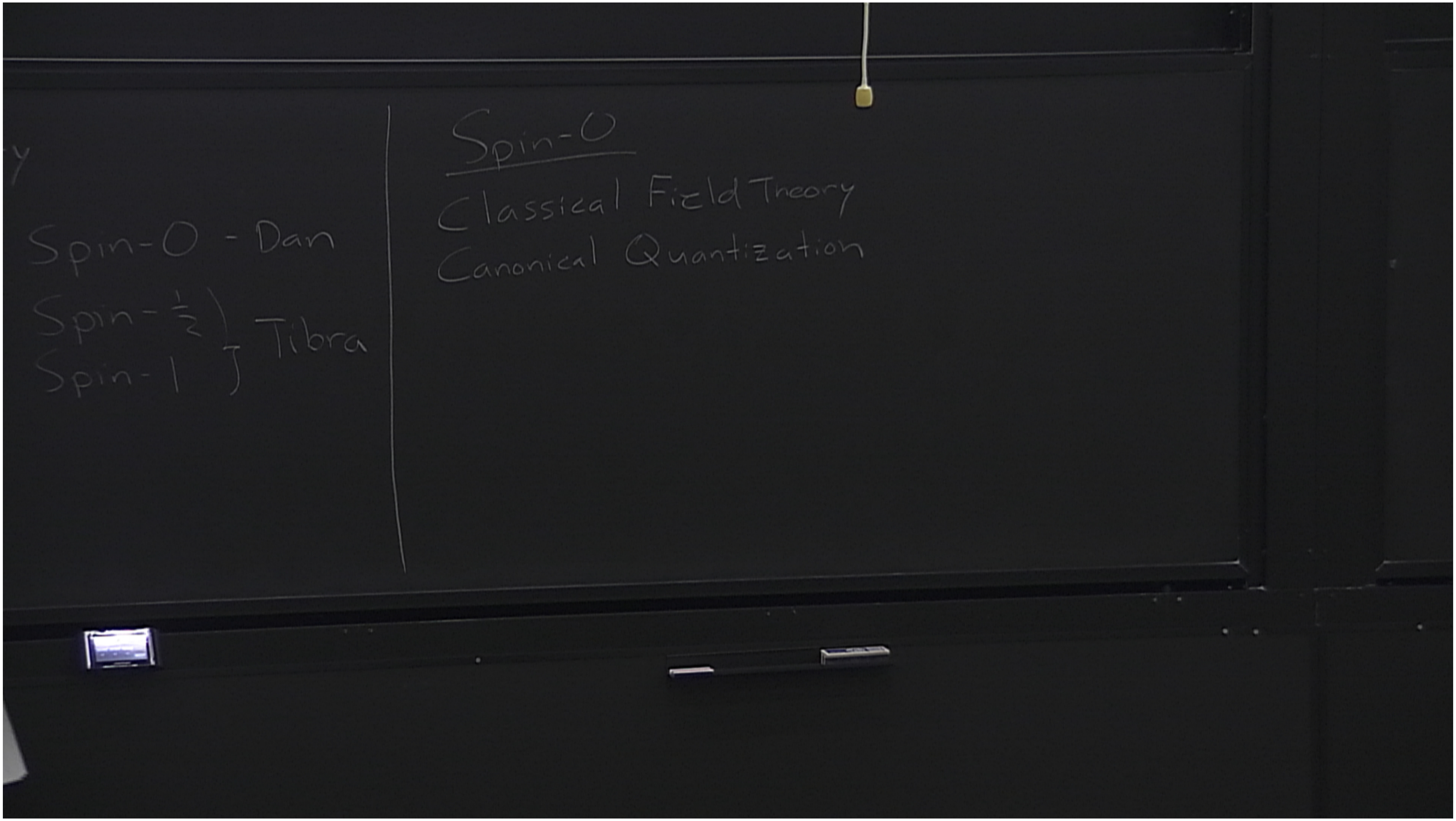
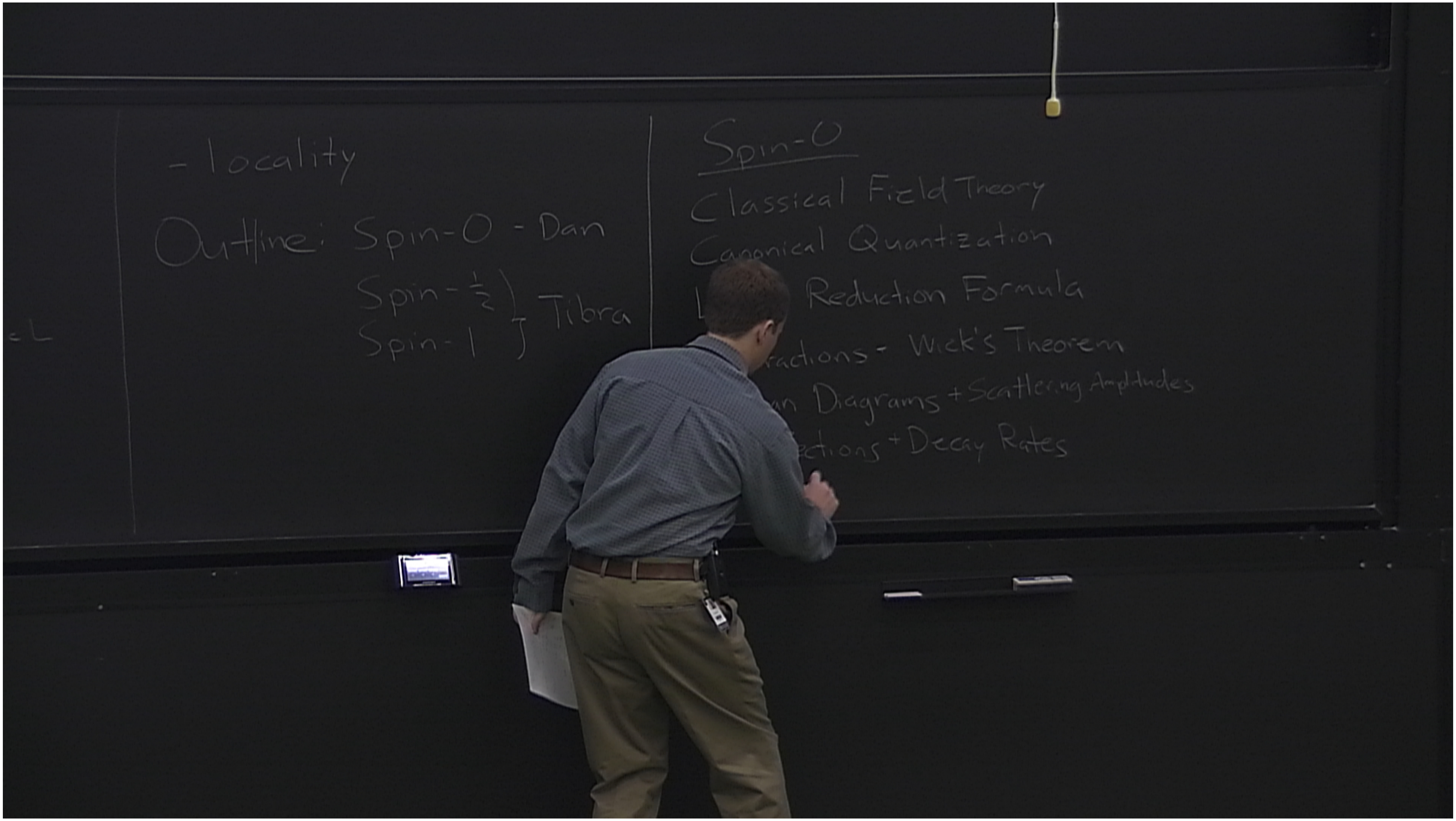
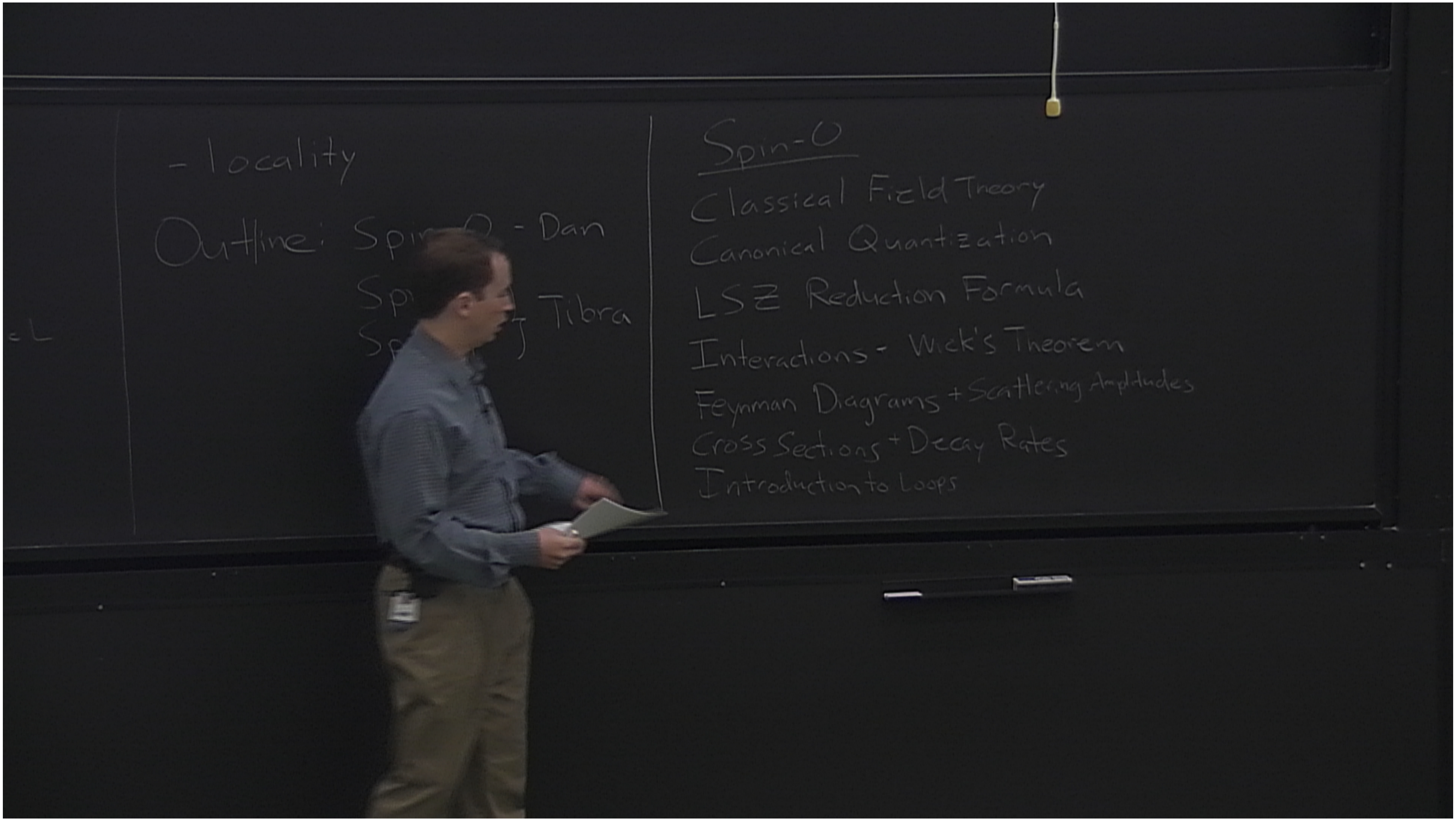
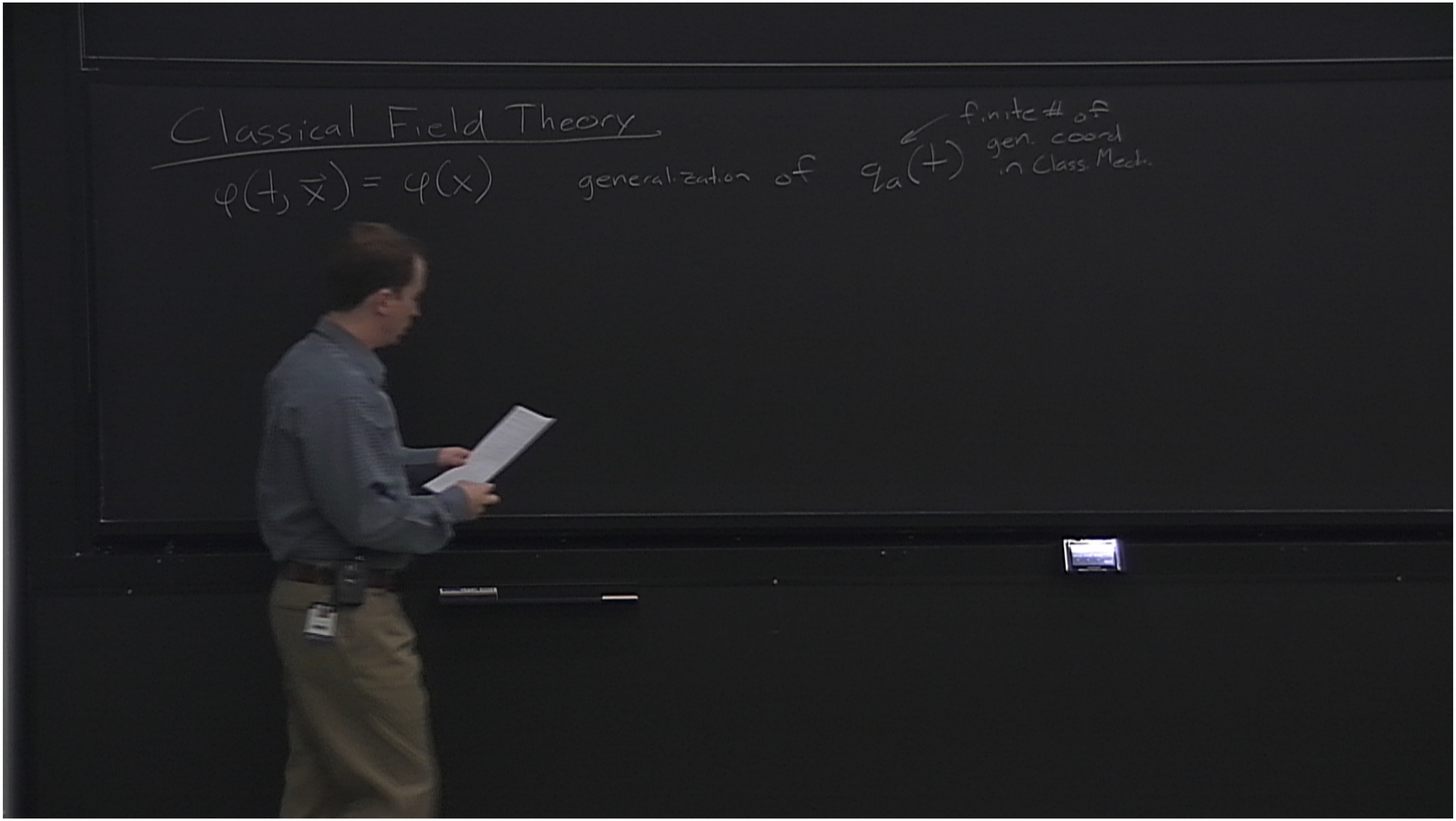
- QM + special relativity
ΔE ≥ ħc/L in box of size L
ΔE = 2mc²
- locality

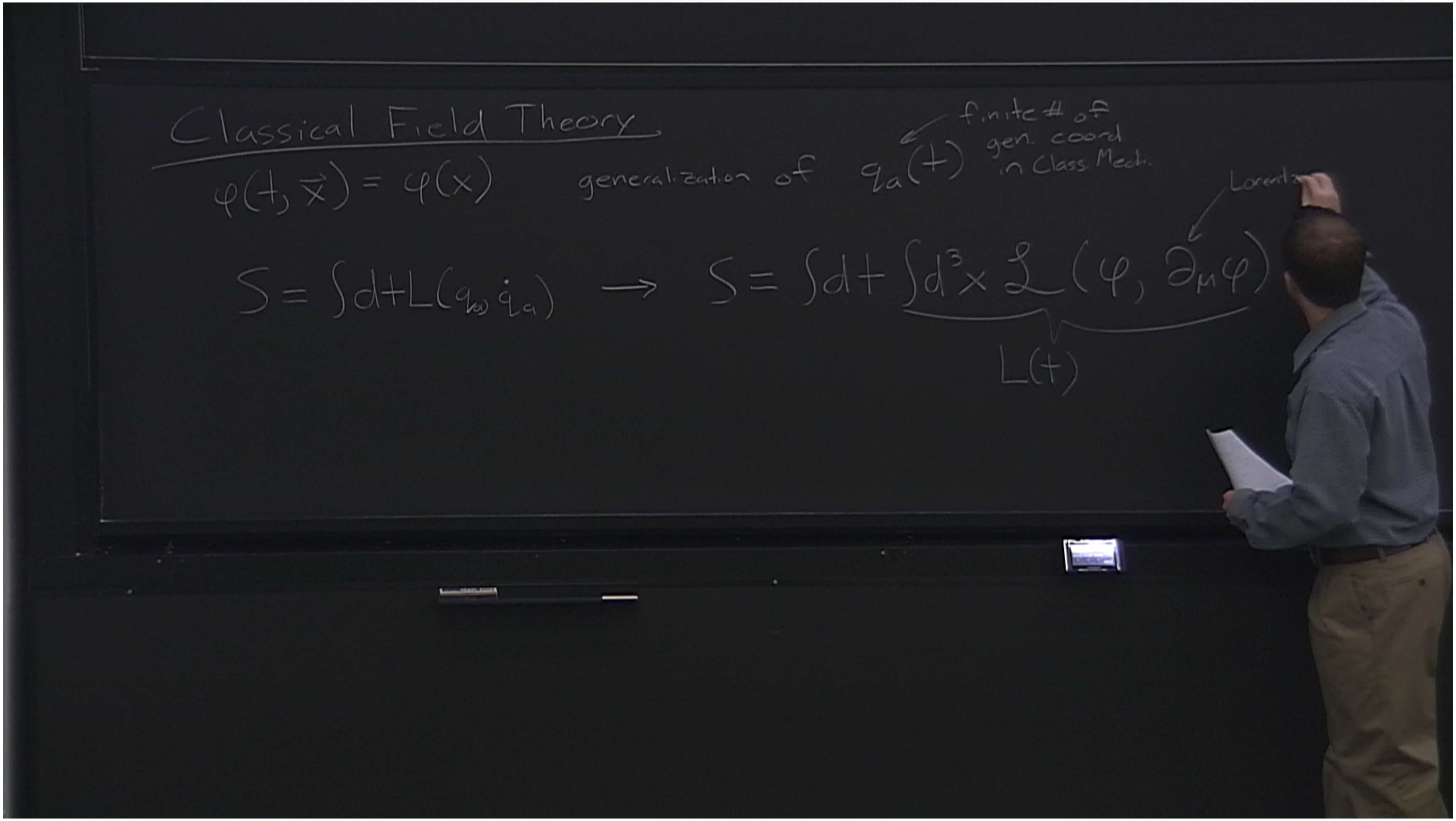

Spin-0
Classical Field Theory
Spin-0 - Dan
Spin-1/2
Spin-1
Tibra

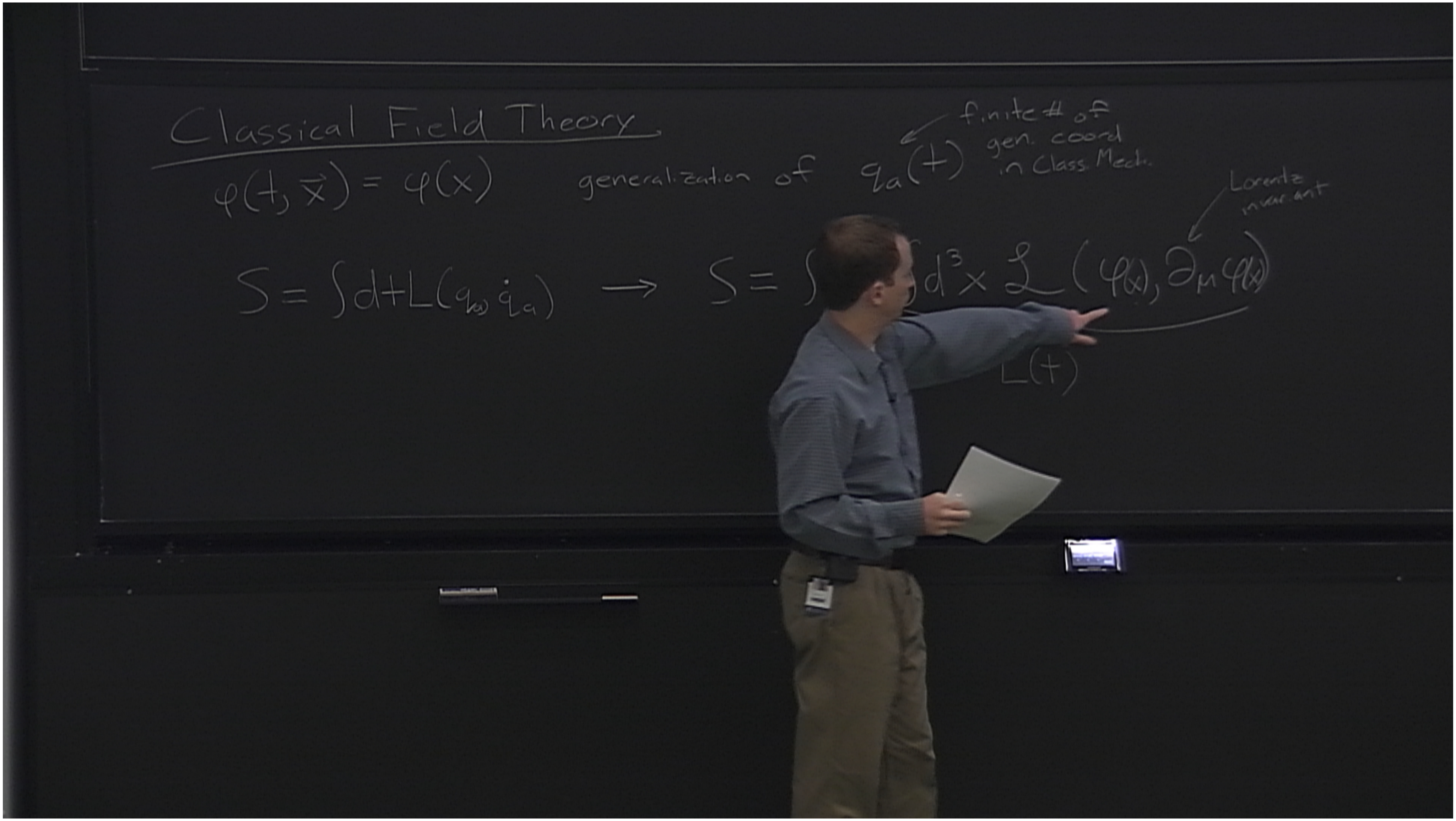

Spin-0 - Dan
Spin-½
Spin-1
Tibra
Spin-0
Classical Field Theory
Canonical Quantization


- locality
Outline: Spin-0 - Dan
Spin-½
Spin-1
Tibra
Spin-0
Classical Field Theory
Canonical Quantization
Reduction Formula
Wick's Theorem
Diagrams + Scattering Amplitudes
Decay Rates


- locality
Outline:
- Dan
Tibra
Spin-0
Classical Field Theory
Canonical Quantization
LSZ Reduction Formula
Interactions - Wick's Theorem
Feynman Diagrams + Scattering Amplitudes
Cross Sections + Decay Rates
Introduction to Loops


Classical Field Theory
φ(t, x⃗) = φ(x)
generalization of
q_a(t)
finite # of
gen. coord
in Class. Mech.


Classical Field Theory
$\varphi(t, \vec{x}) = \varphi(x)$
generalization of $q_a(t)$
finite # of gen. coord in Class. Mech.
$S = \int dt\, L(q_a, \dot{q}_a) \longrightarrow S = \int dt \int d^3x\, \mathcal{L}(\varphi, \partial_\mu \varphi)$
$L(t)$


Classical Field Theory
φ(t, x⃗) = φ(x)
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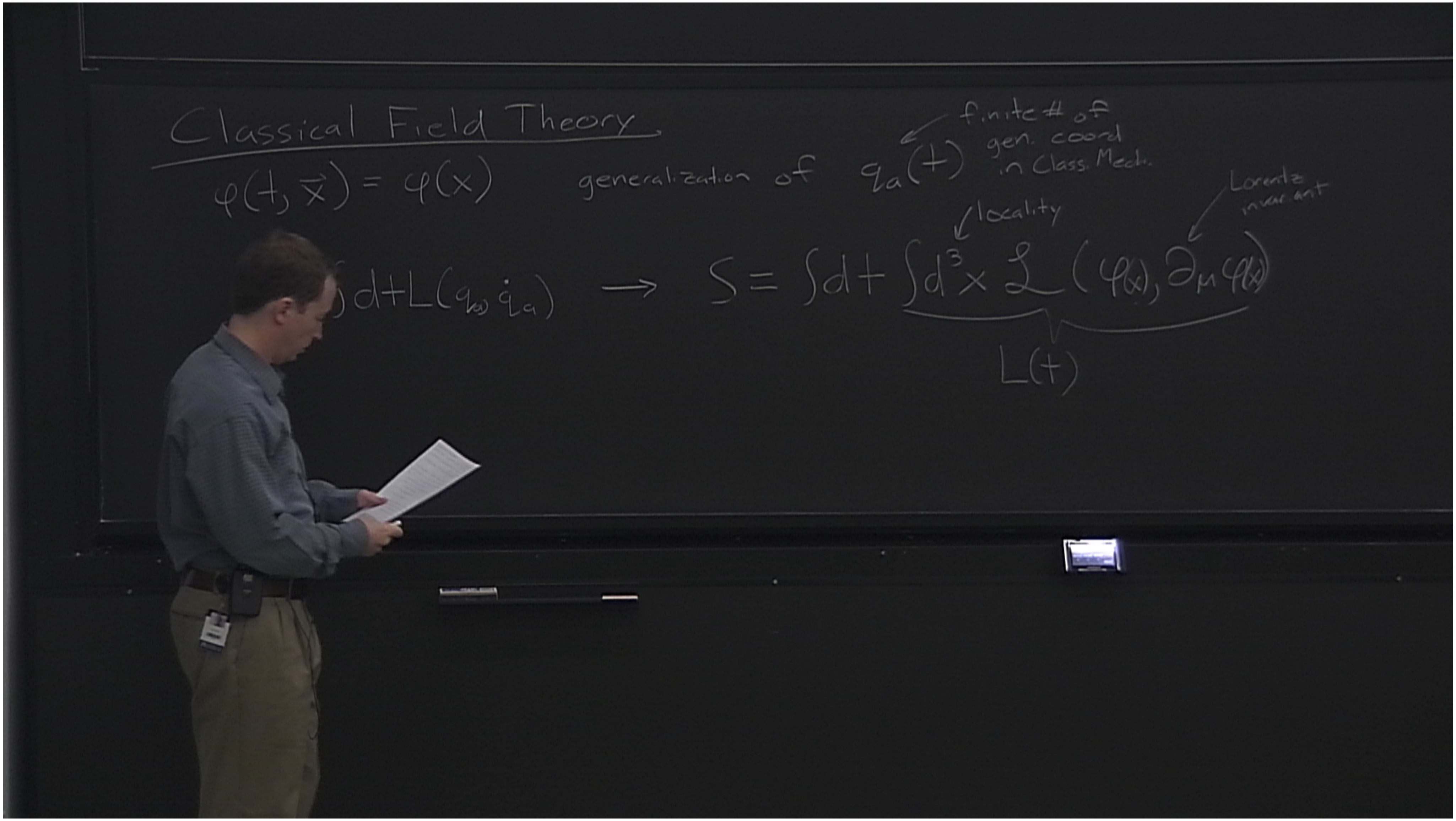
finite # of gen. coord in Class. Mech.
Lorentz invariant
S = ∫dt L(q_a, q̇_a) → S = ∫ d³x ℒ(φ(x), ∂_μ φ(x))
L(t)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φ(t, x⃗) = φ(x)
generalization of
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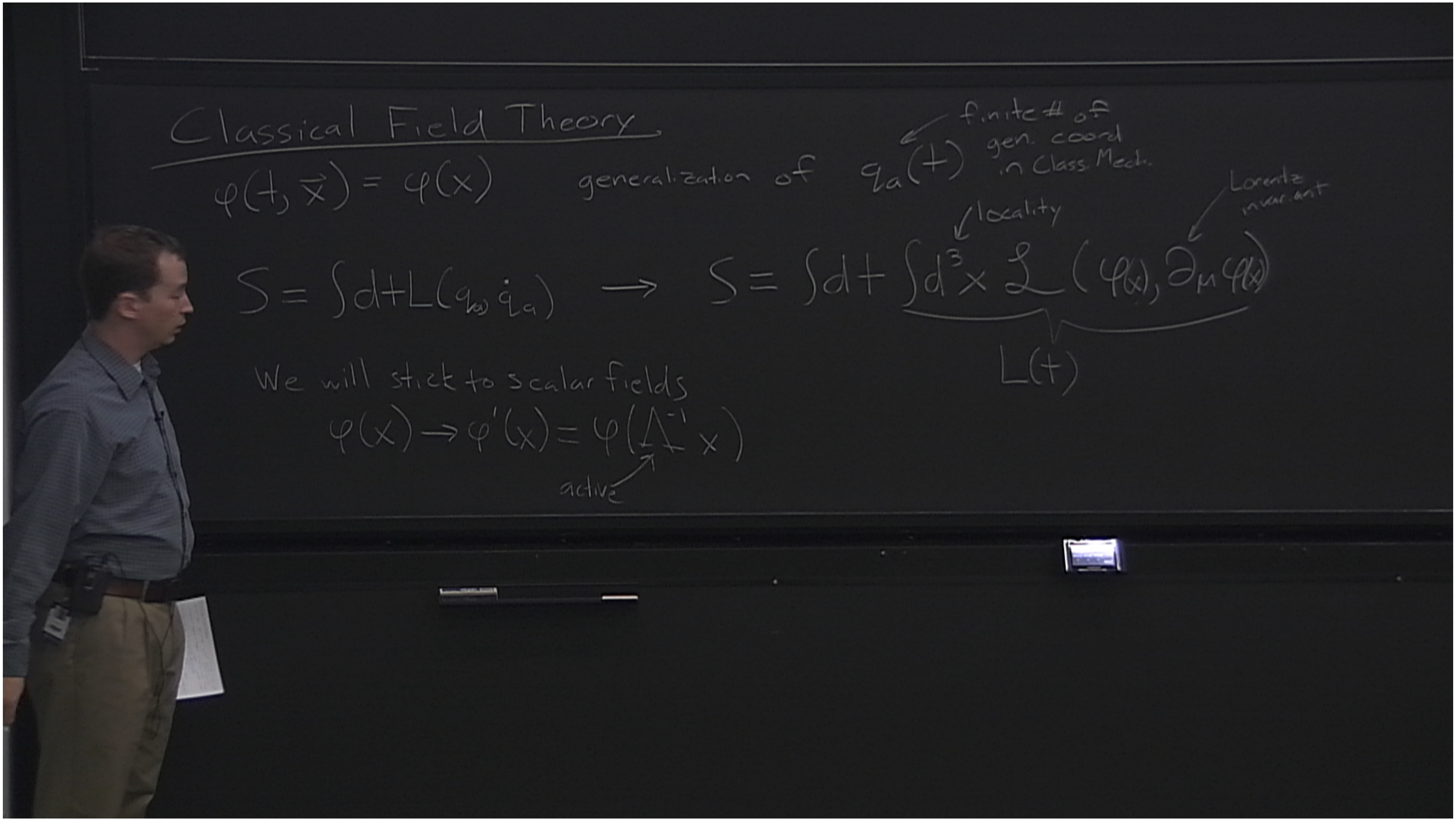
locality
Lorentz invariant
∫dt L(q_a, q̇_a) → S = ∫dt ∫d³x ℒ(φ(x), ∂_μ φ(x))
L(t)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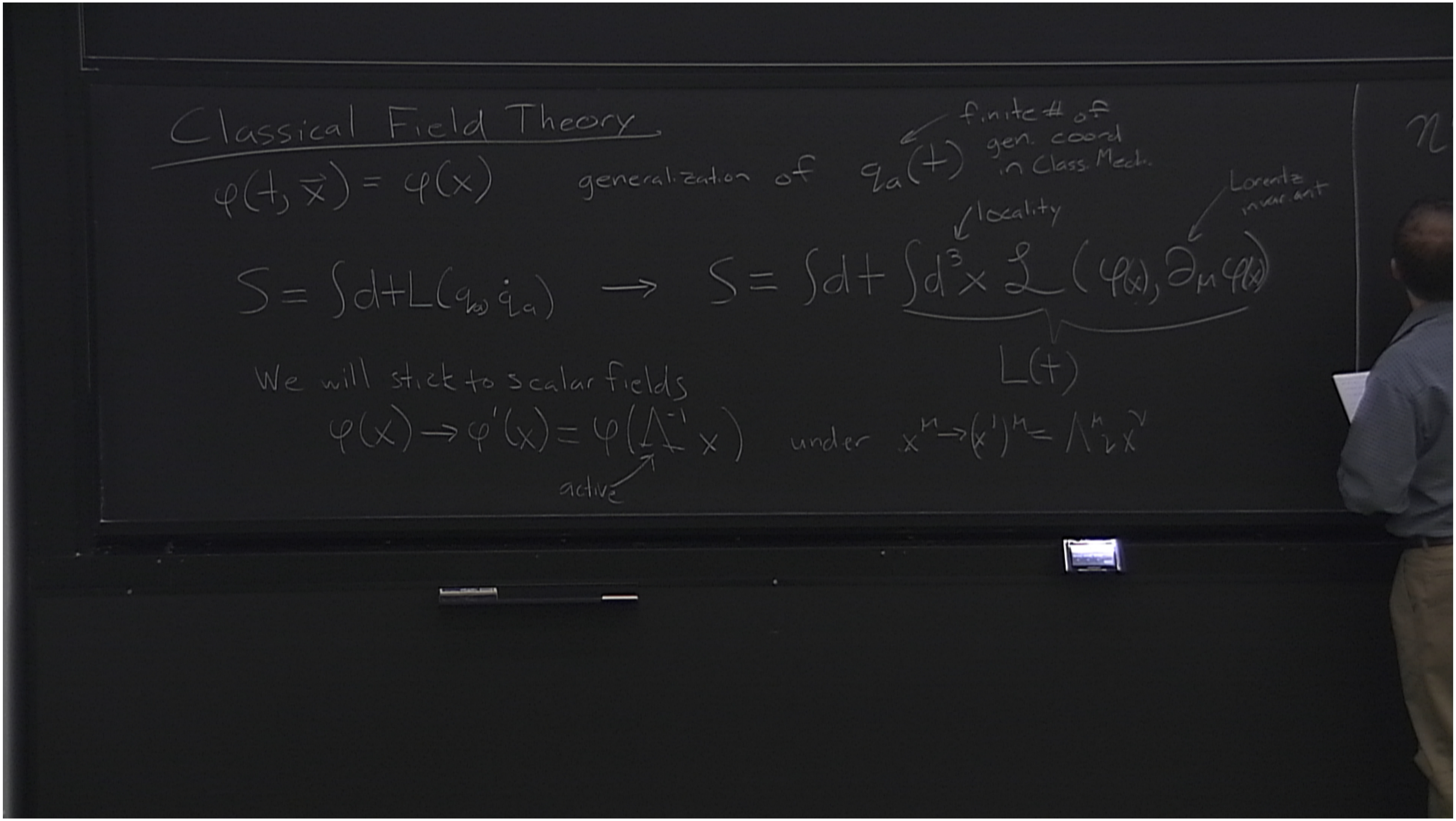
locality
Lorentz invariant
$S = \int dt\, L(q_a, \dot{q}_a) \rightarrow S = \int dt \int d^3x\, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
$L(t)$
We will stick to scalar fields
$\varphi(x) \rightarrow \varphi'(x) = \varphi(\Lambda^{-1}x)$
active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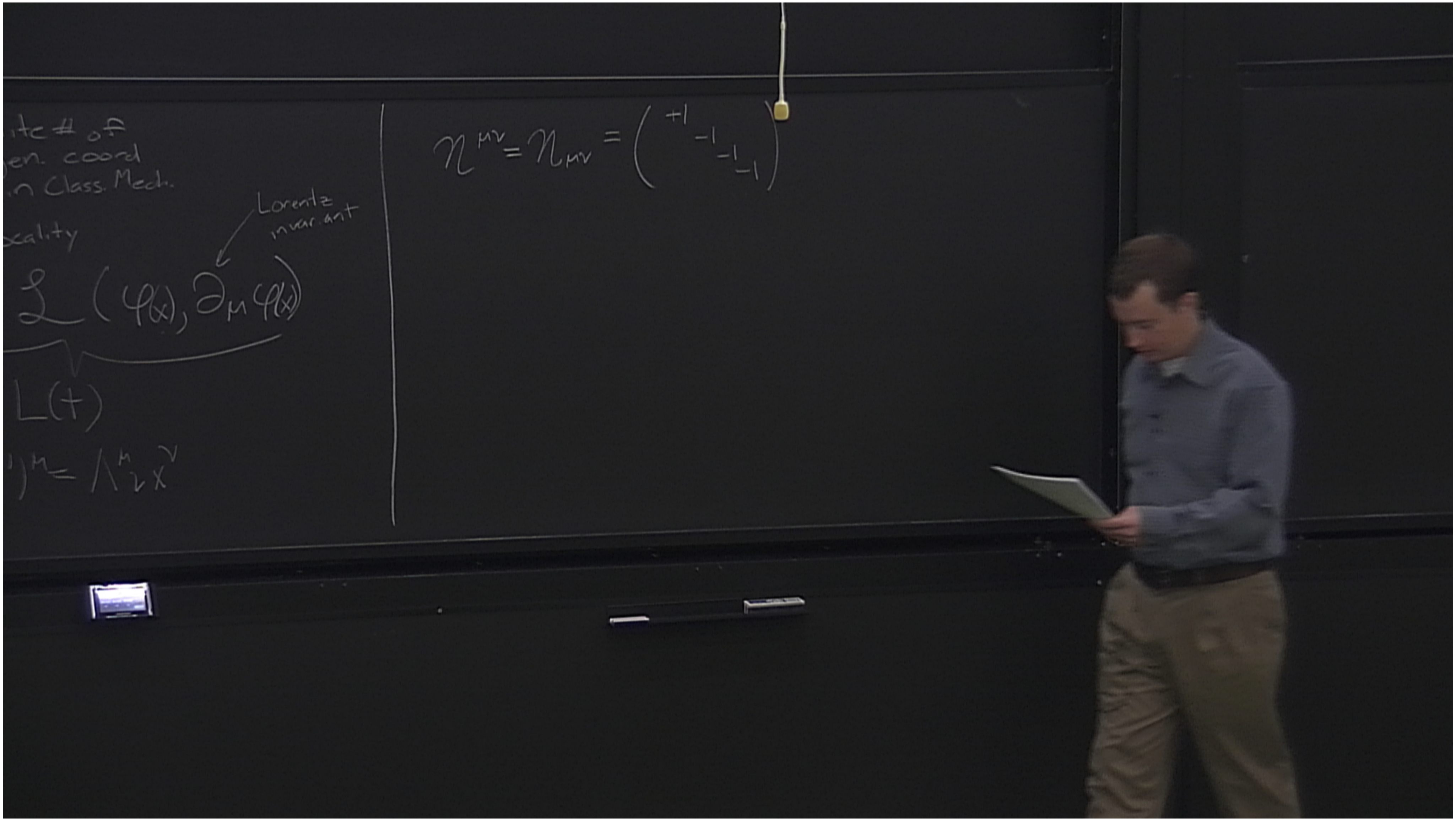
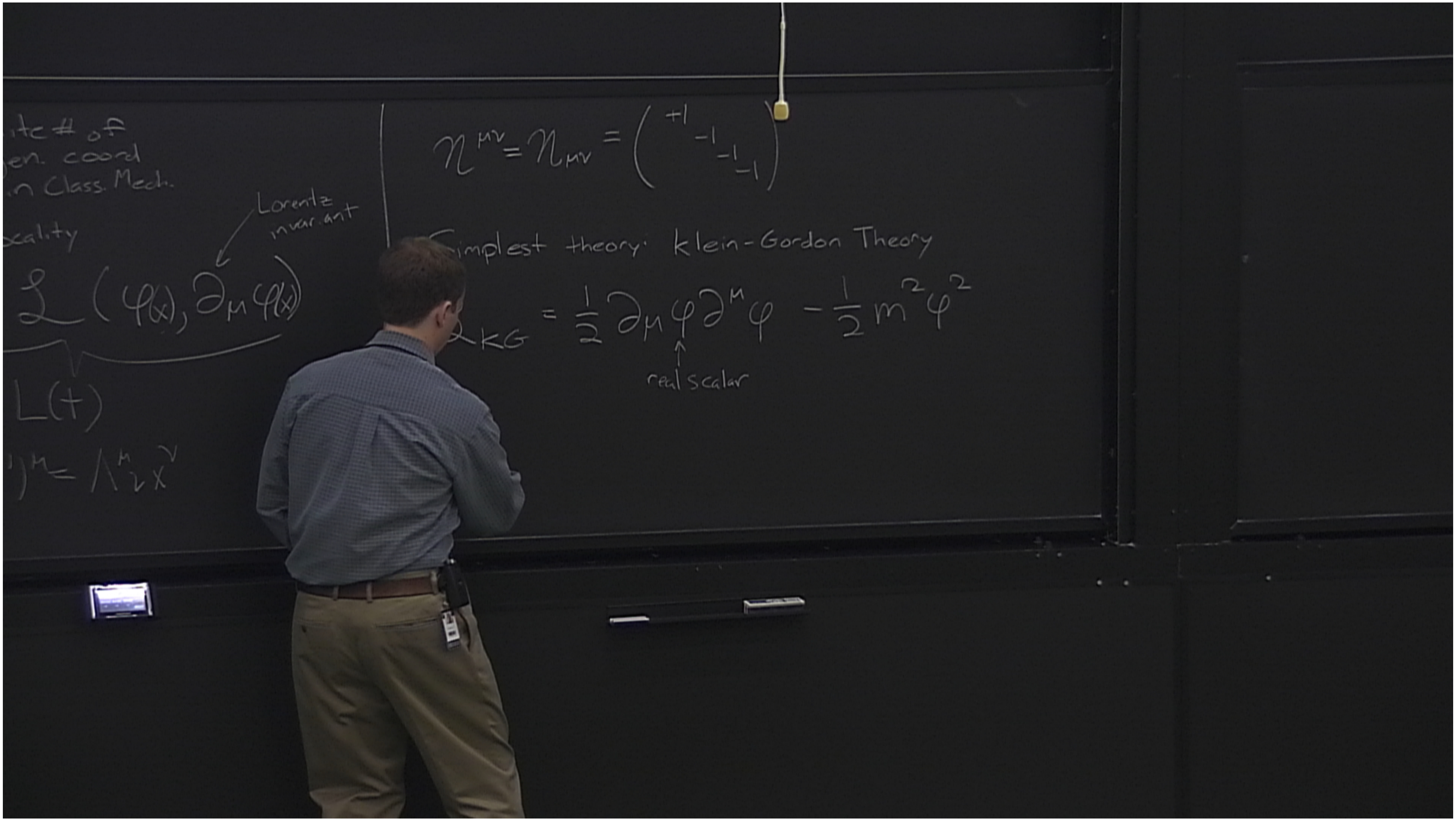
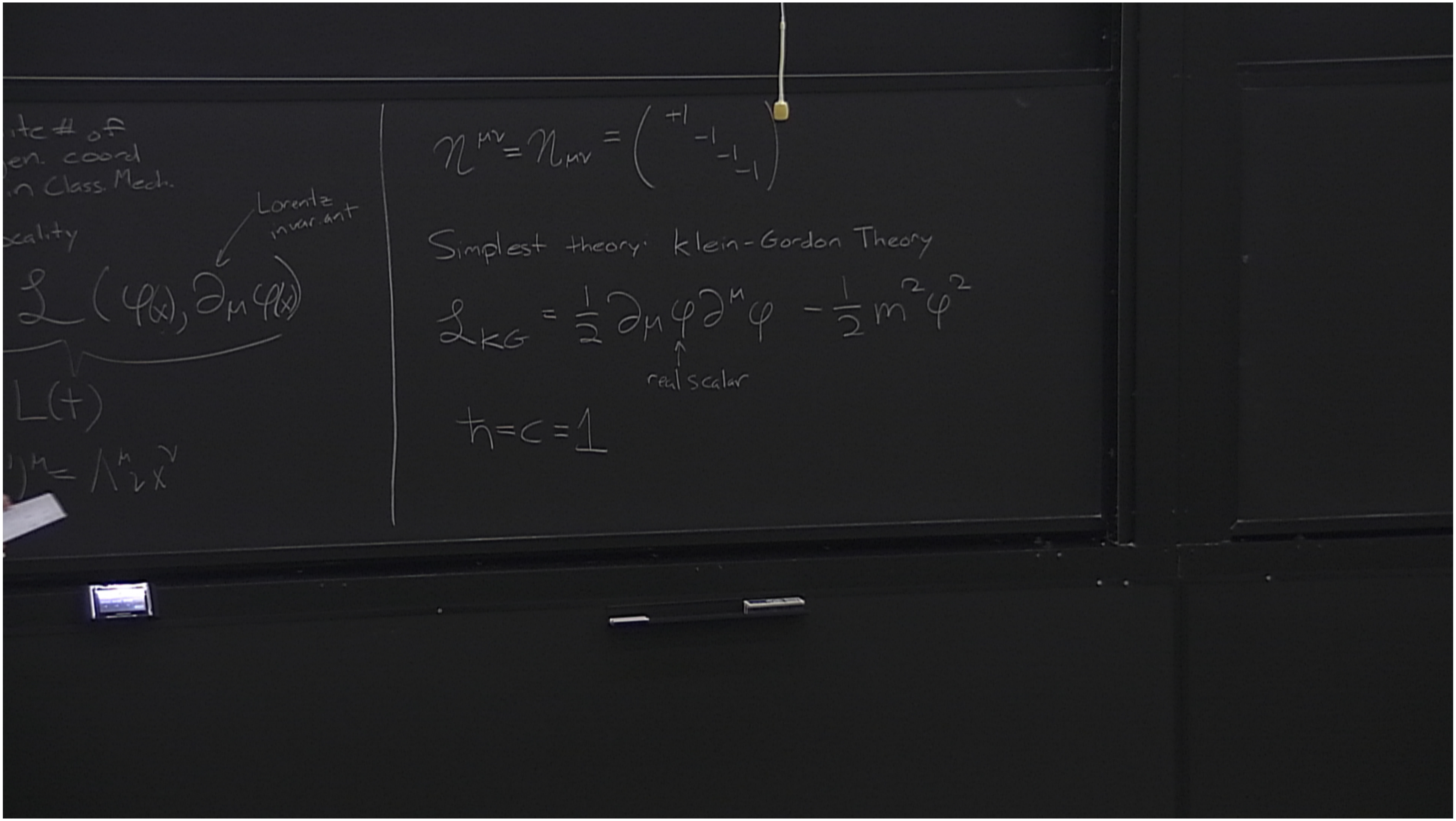
locality
Lorentz invariant
$L(t)$
We will stick to scalar fields
$\varphi(x) \rightarrow \varphi'(x) = \varphi(\Lambda^{-1} x)$ under $x^\mu \rightarrow (x')^\mu = \Lambda^\mu{}_\nu x^\nu$
active


gen. coord
in Class. Mech.
Lorentz
invariant
$\mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
$L(t)$
$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$


gen. coord
in Class. Mech.
Lorentz invariant
Simplest theory: klein-Gordon Theory
real scalar



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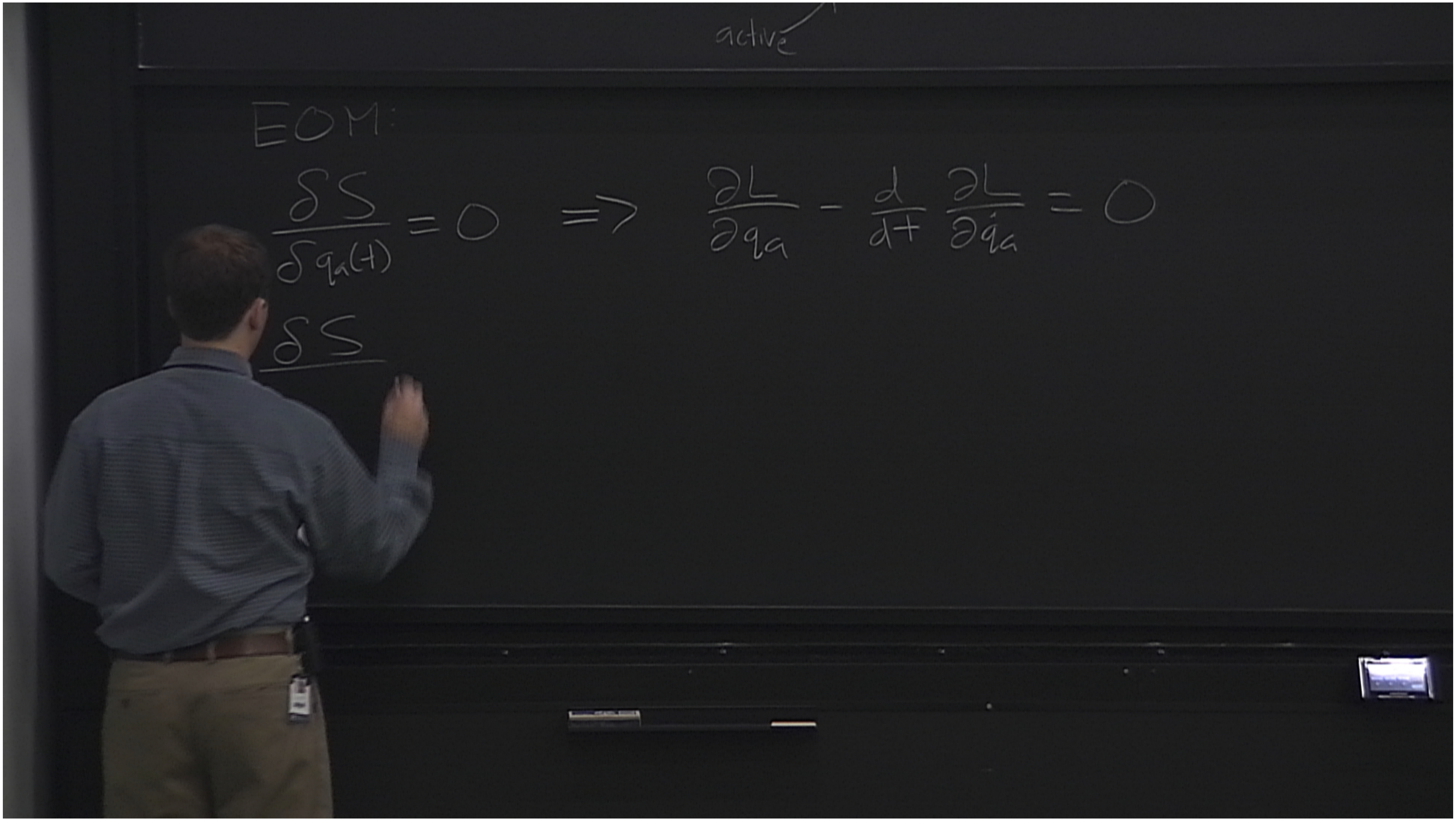
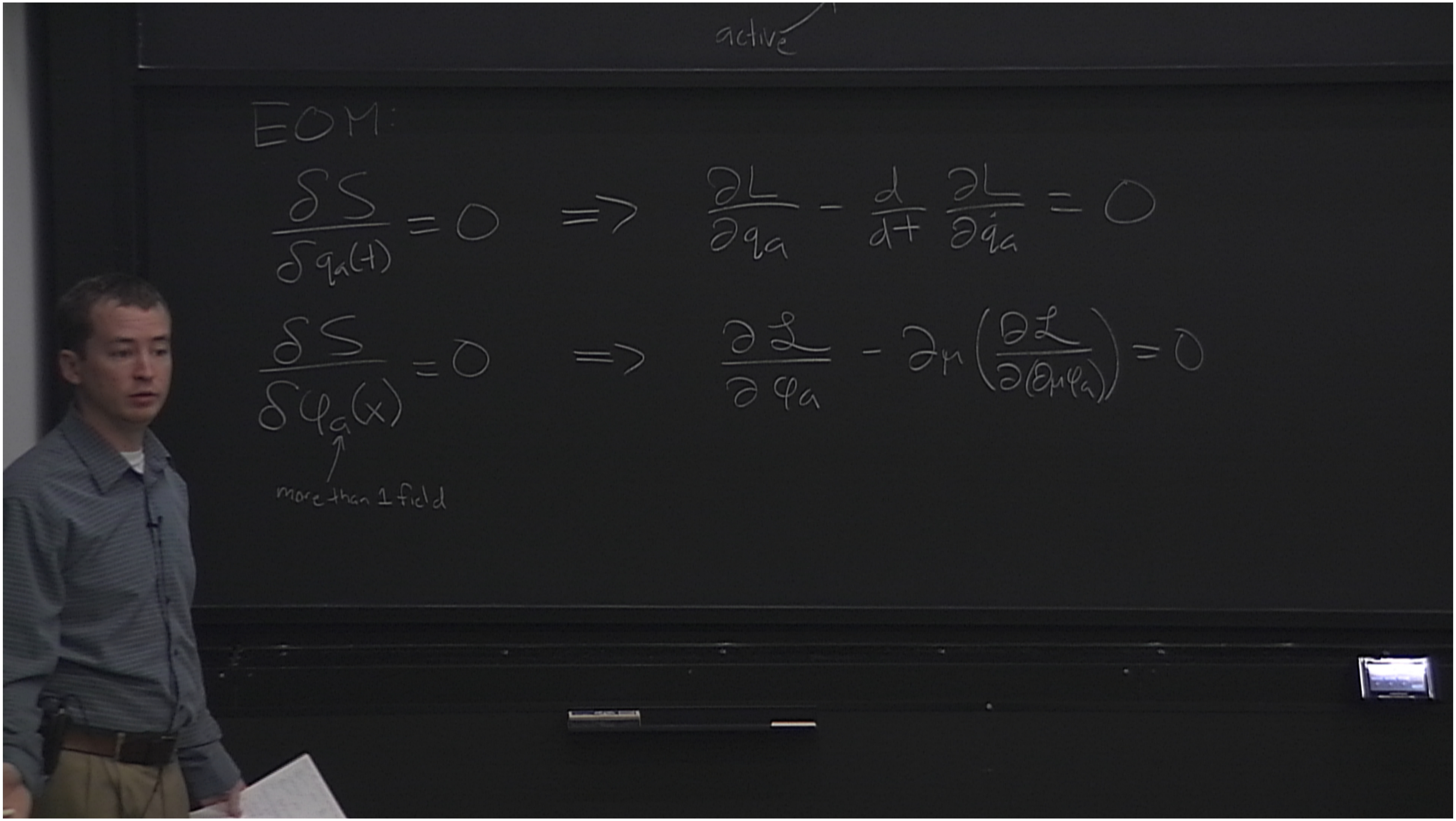
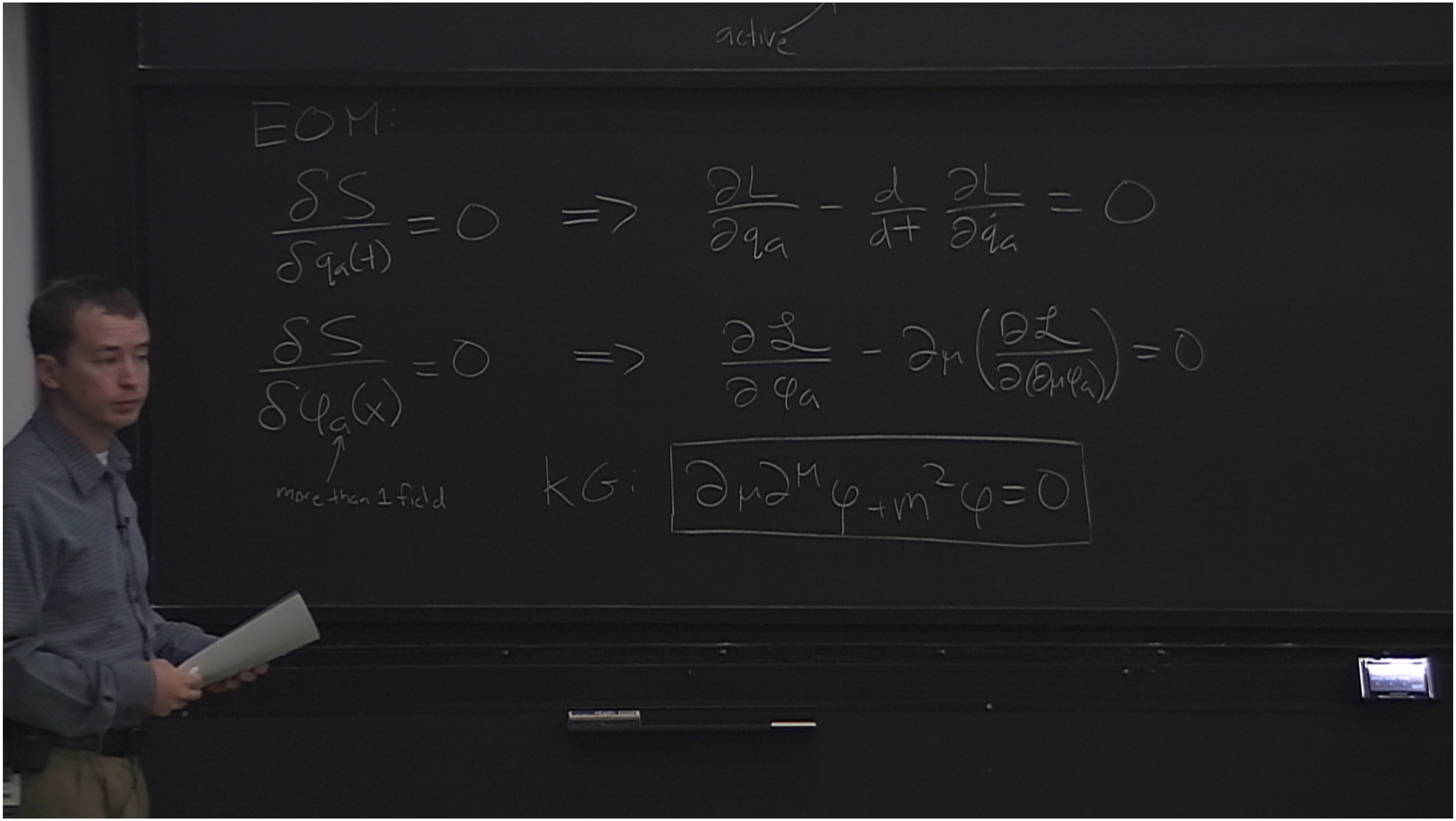
cality
$\mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
$L(t)$
$\Lambda^\mu{}_\nu x^\nu$
$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
Simplest theory: Klein-Gordon Theory
$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$
real scalar
$\hbar = c = 1$


active
EOM:
$\frac{\delta S}{\delta q_a(t)} = 0 \implies \frac{\partial L}{\partial q_a} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_a} = 0$
$\frac{\delta S}{}$


active
EOM:
$\frac{\delta S}{\delta q_a(t)} = 0 \implies \frac{\partial L}{\partial q_a} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_a} = 0$
$\frac{\delta S}{\delta \varphi_a(x)} = 0 \implies \frac{\partial \mathcal{L}}{\partial \varphi_a} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)}\right) = 0$
more than 1 field


active
EOM:
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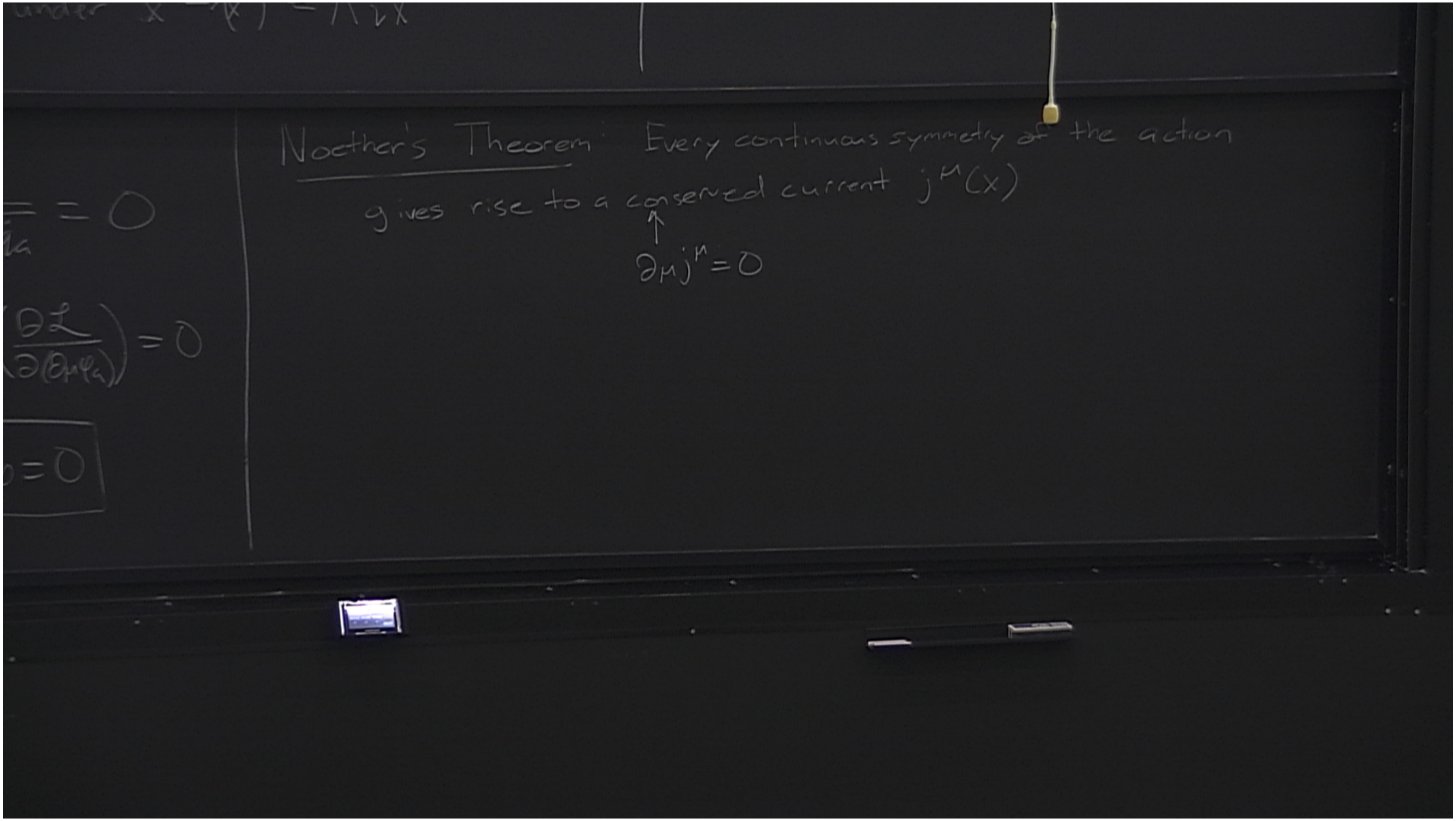
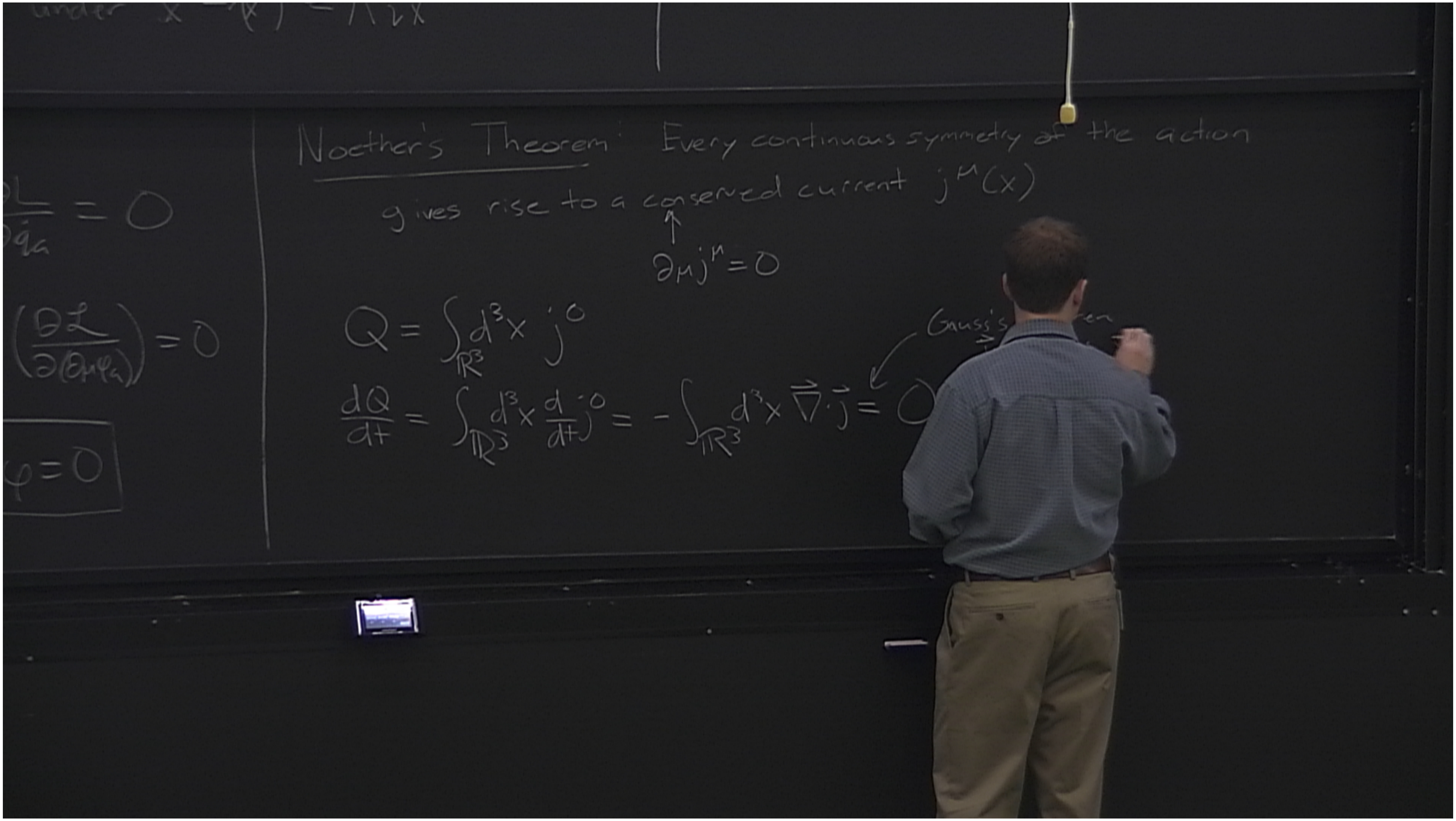
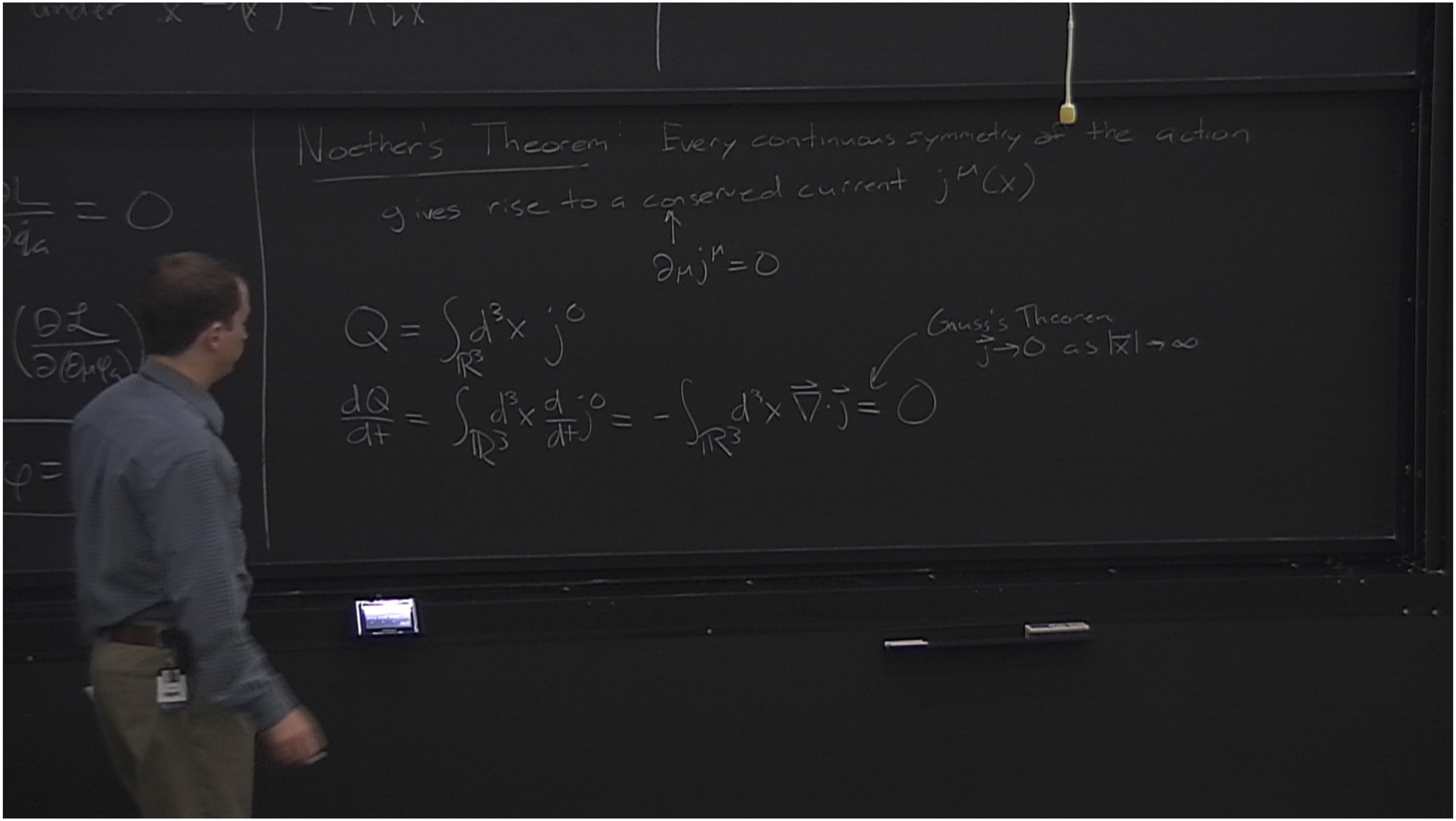
more than 1 field
KG: $\partial_\mu \partial^\mu \varphi + m^2 \varphi = 0$


Noether's Theorem: Every continuous symmetry of the action
gives rise to a conserved current $j^\mu(x)$
$\partial_\mu j^\mu = 0$


Noether's Theorem: Every continuous symmetry of the action
gives rise to a conserved current $j^\mu(x)$
$\partial_\mu j^\mu = 0$
$Q = \int_{\mathbb{R}^3} d^3x\, j^0$
$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \frac{d}{dt} j^0 = -\int_{\mathbb{R}^3} d^3x\, \vec{\nabla}\cdot\vec{j} = 0$
Gauss's


Noether's Theorem: Every continuous symmetry of the action
gives rise to a conserved current $j^\mu(x)$
$\partial_\mu j^\mu = 0$
$Q = \int_{\mathbb{R}^3} d^3x \, j^0$
Gauss's Theorem
$\vec{j} \to 0$ as $|\vec{x}| \to \infty$
$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \frac{d}{dt} j^0 = -\int_{\mathbb{R}^3} d^3x \, \vec{\nabla} \cdot \vec{j} = 0$

$\frac{\partial L}{\partial \dot{q}_a} = 0$

$\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)}\right) = 0$

$\varphi = 0$

Noether's Theorem: Every continuous symmetry of the action gives rise to a conserved current $j^\mu(x)$

$\uparrow$

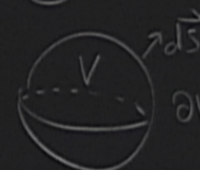
$\partial_\mu j^\mu = 0$

$$Q = \int_{\mathbb{R}^3} d^3x \, j^0$$

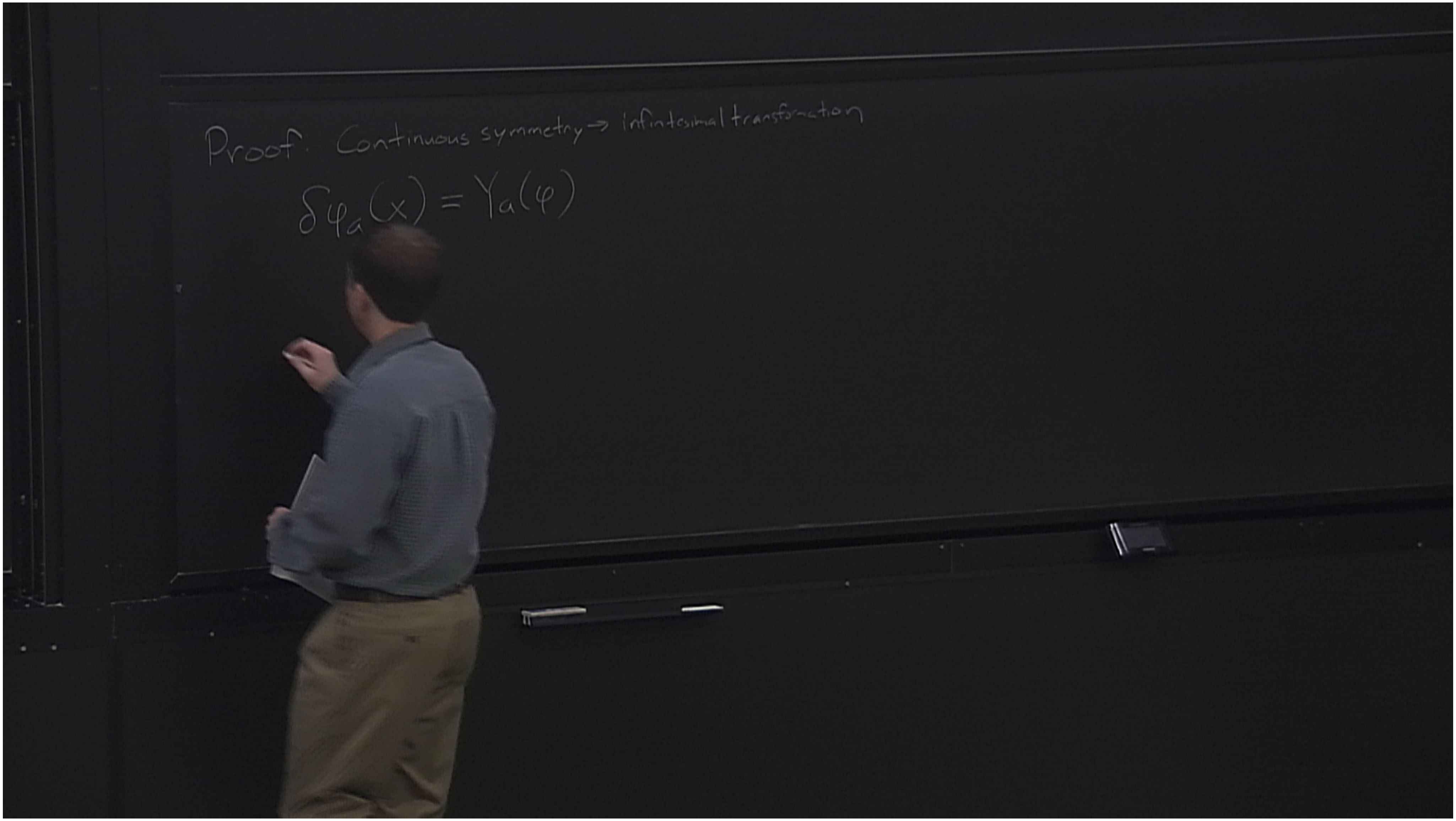
Gauss's Theorem
$\vec{j} \to 0$ as $|\vec{x}| \to \infty$

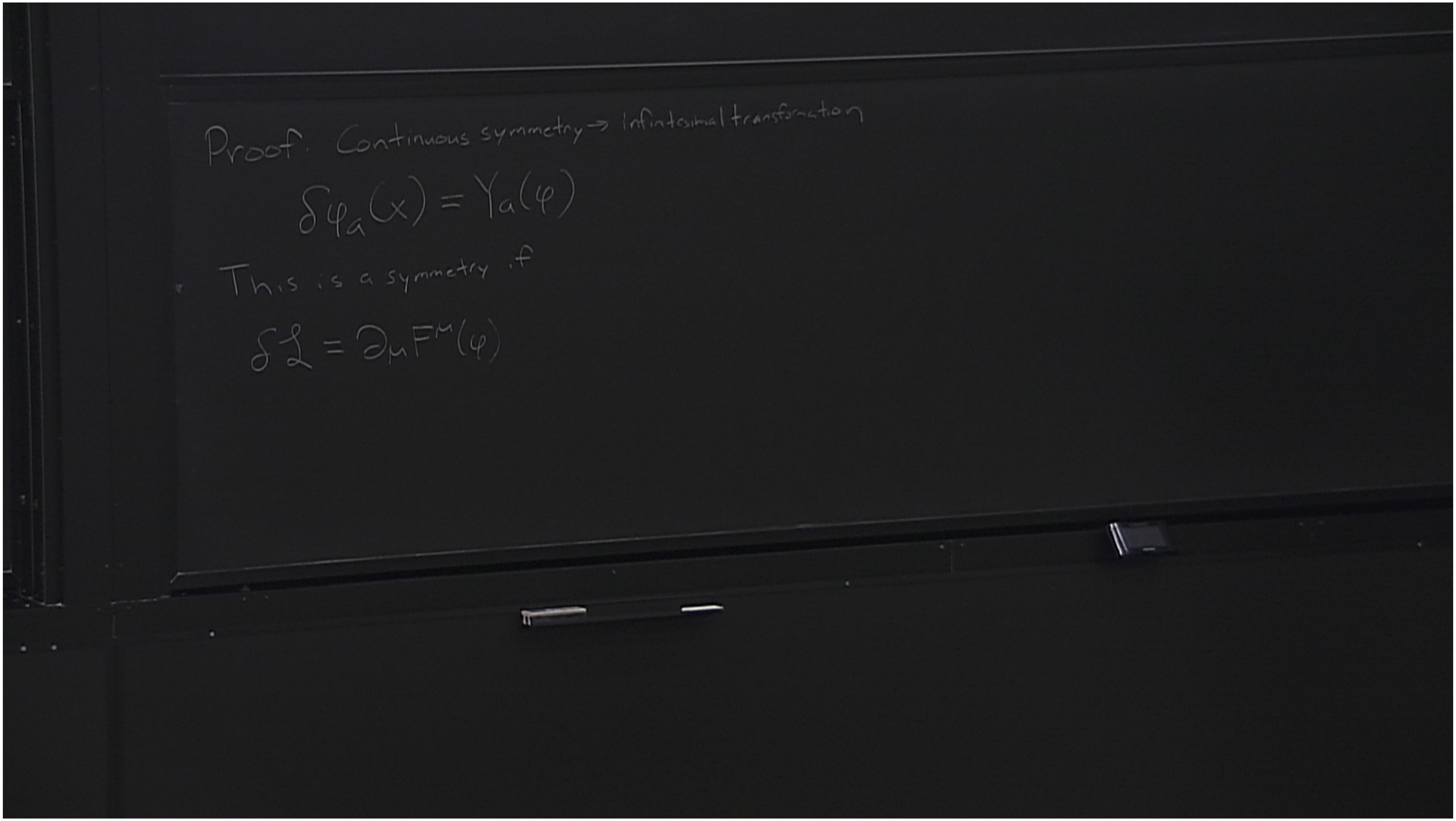
$$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \, \frac{d}{dt} j^0 = -\int_{\mathbb{R}^3} d^3x \, \vec{\nabla} \cdot \vec{j} = 0$$

$$Q_V = \int_V d^3x \, j^0 \qquad \frac{dQ_V}{dt} = -\int_A \vec{j} \cdot d\vec{S}$$



$\partial V = A$


Proof: Continuous symmetry → infintesimal transformation
$\delta\varphi_a(x) = Y_a(\varphi)$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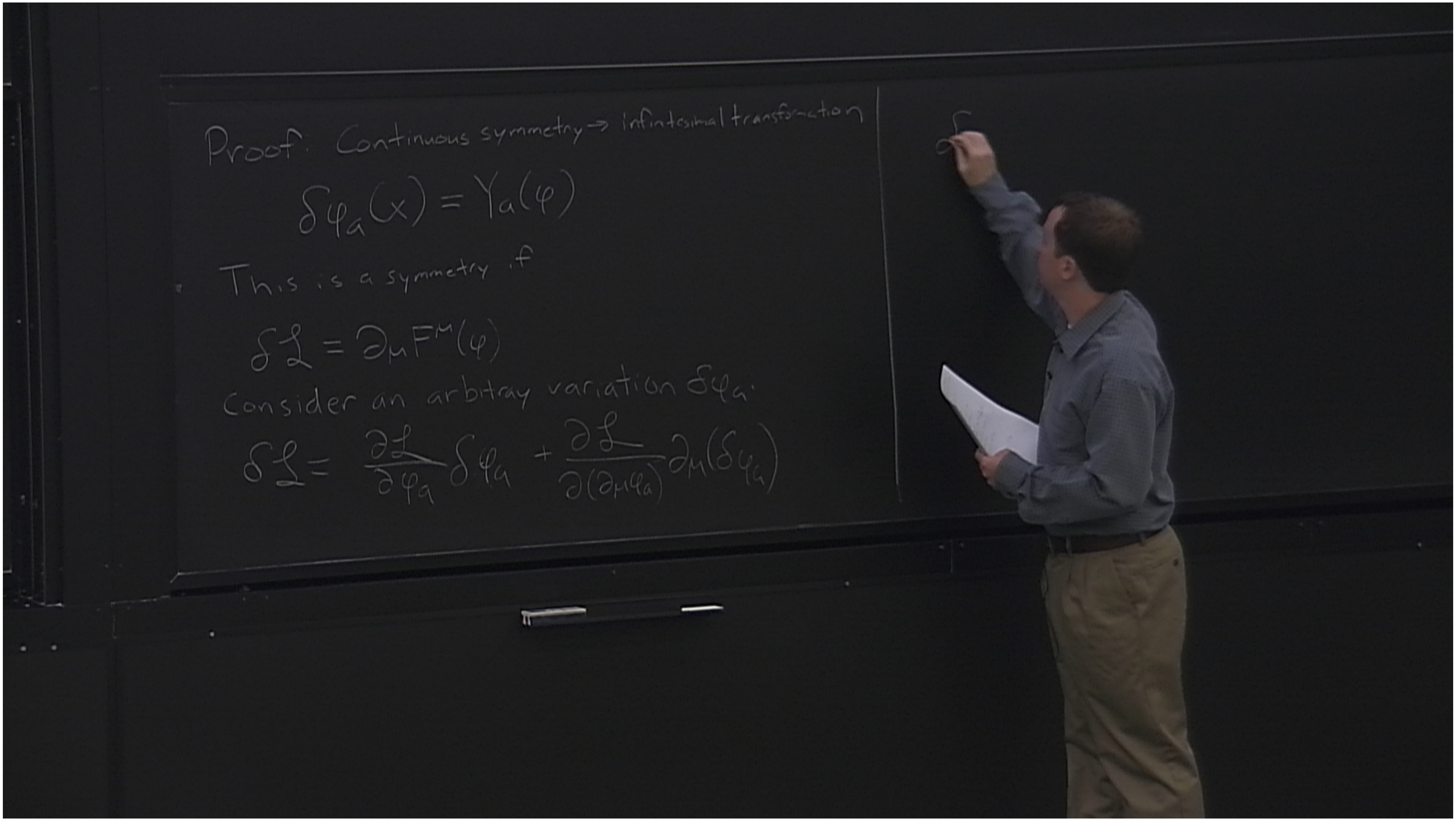
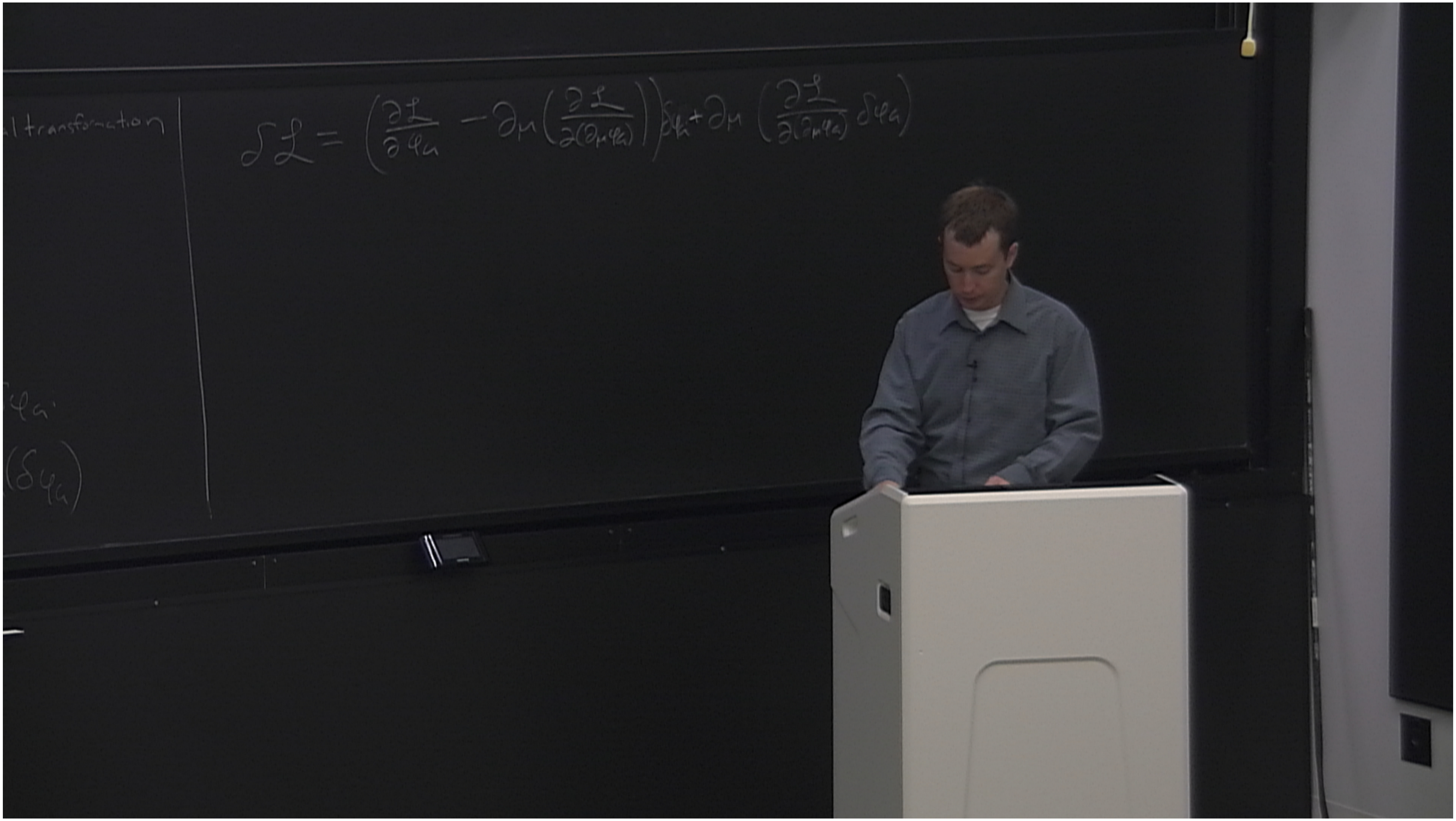
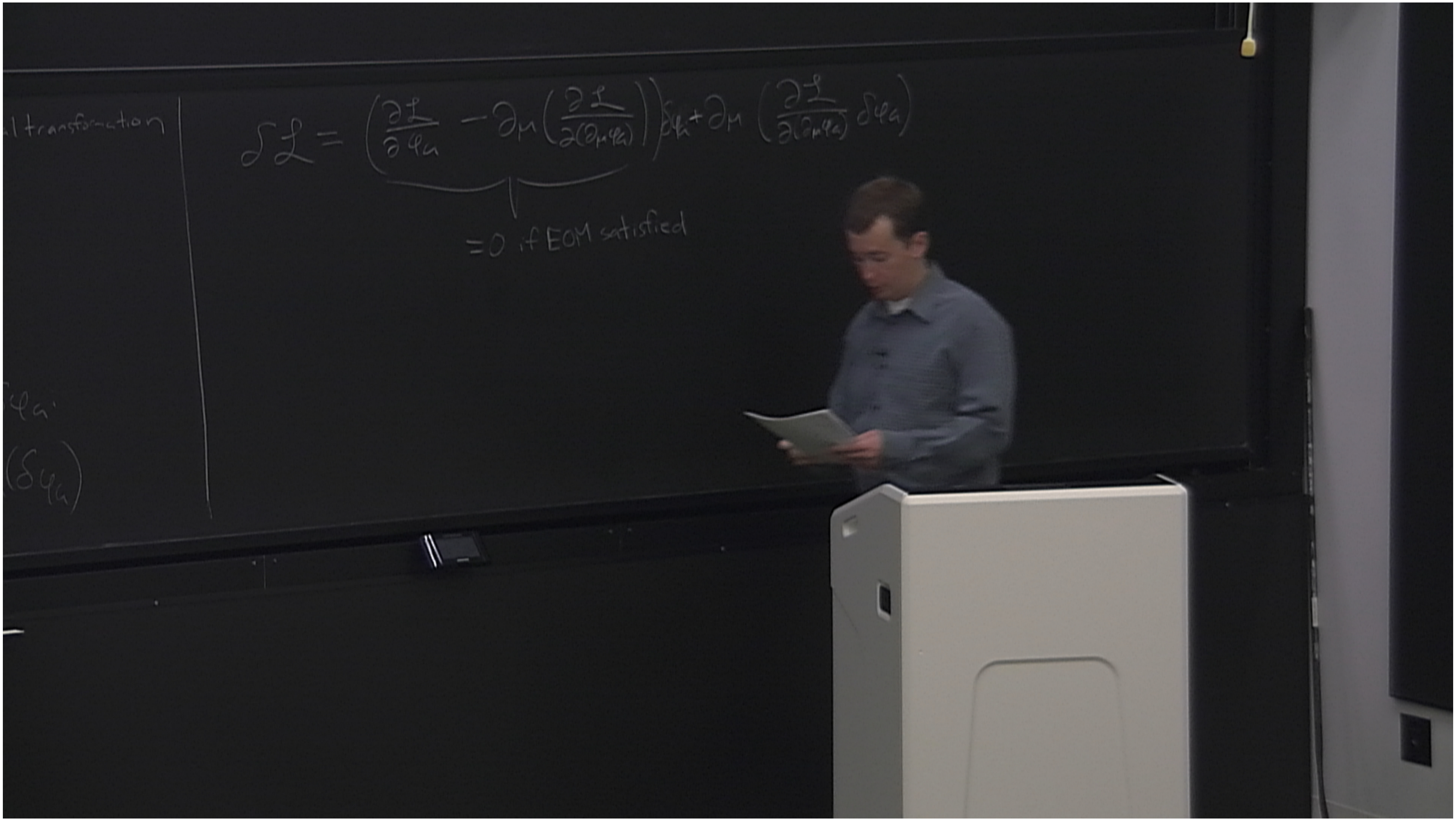
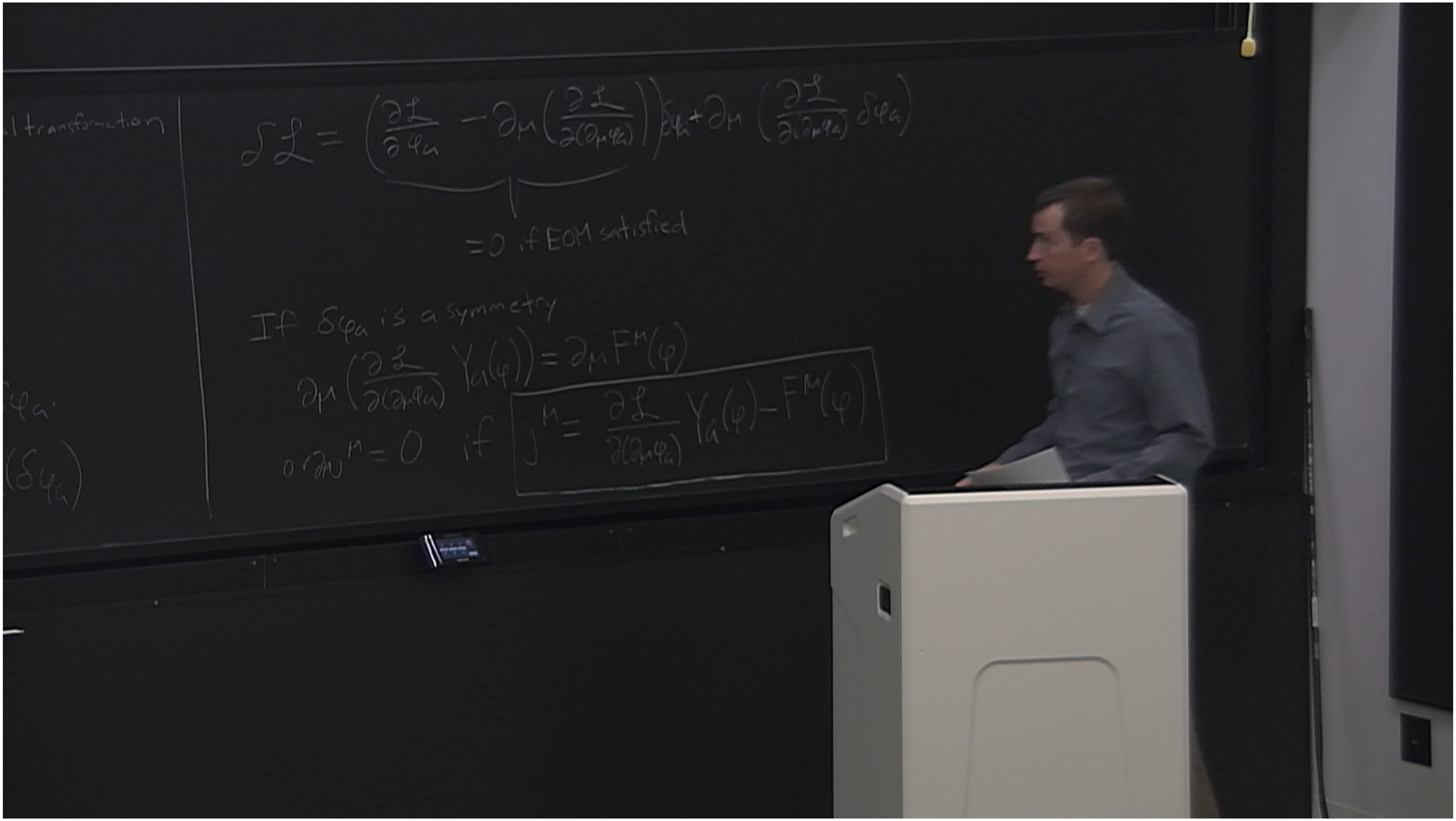
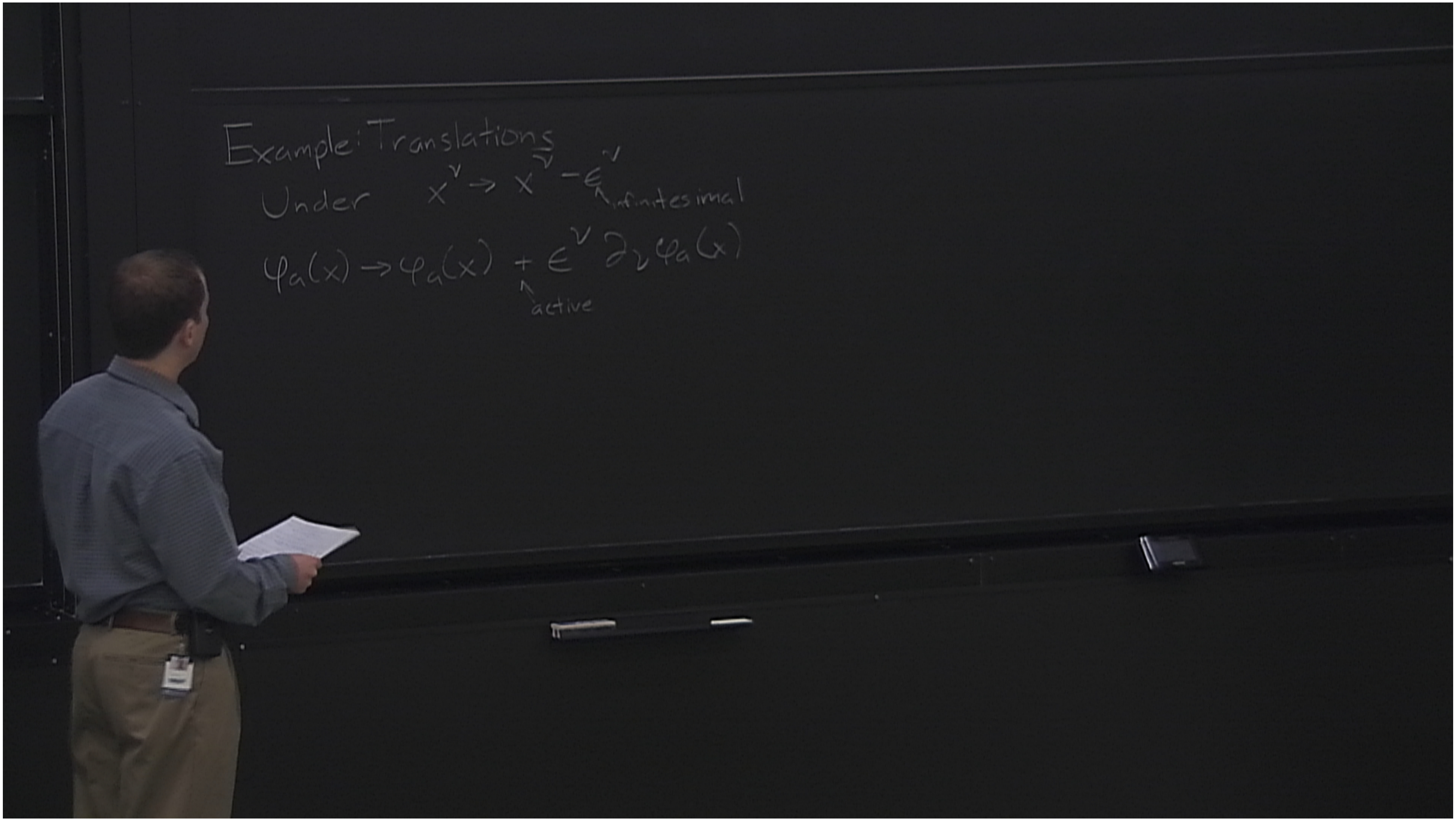
$\delta\varphi_a(x) = Y_a(\varphi)$
This is a symmetry if
$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$

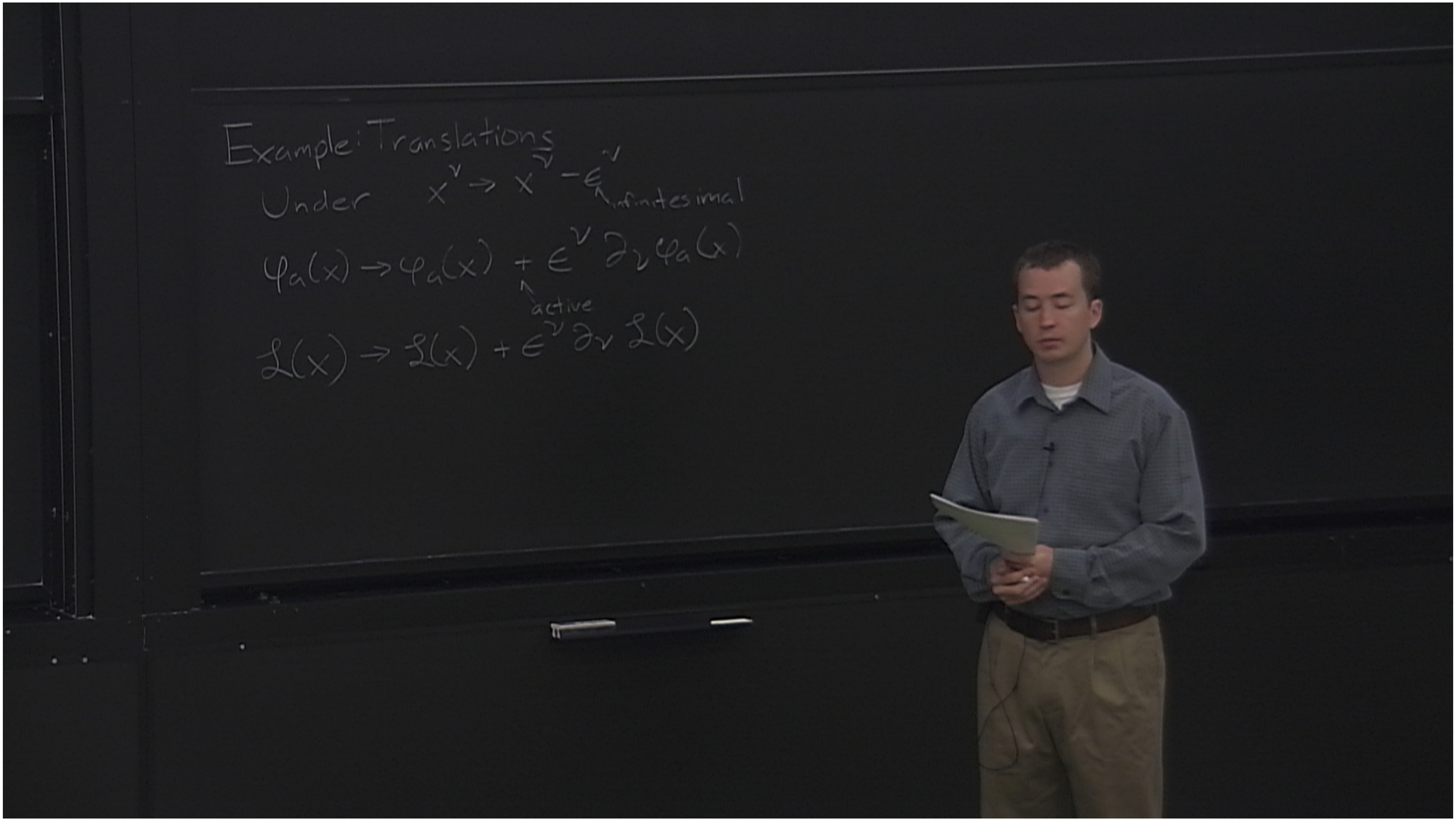

Proof: Continuous symmetry → infintesimal transformation
$\delta\varphi_a(x) = Y_a(\varphi)$
This is a symmetry if
$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$
Consider an arbitray variation $\delta\varphi_a$.
$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_a}\delta\varphi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)}\partial_\mu(\delta\varphi_a)$

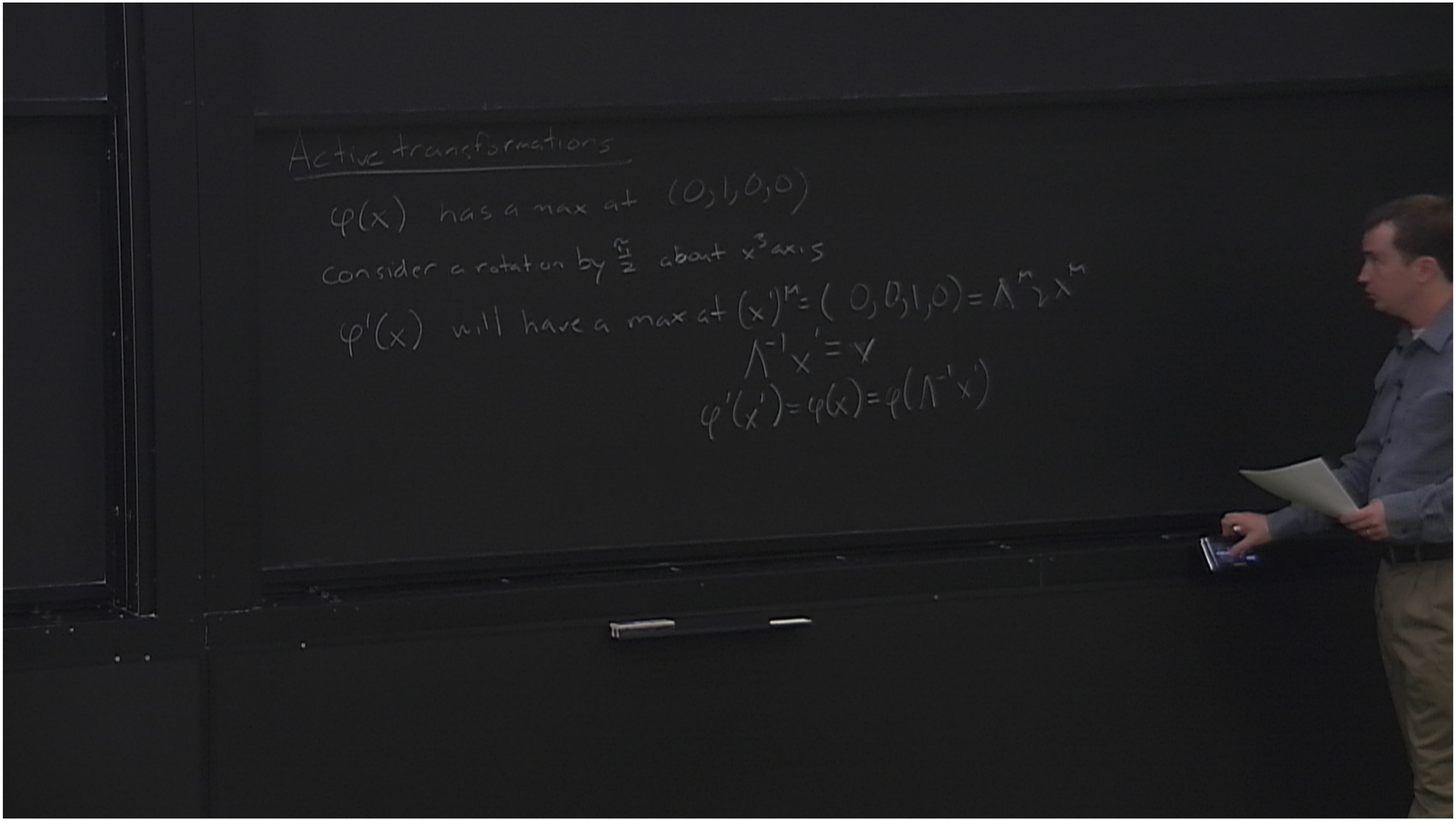

transformation
δL = (∂L/∂φa − ∂μ(∂L/∂(∂μφa)))δφa + ∂μ(∂L/∂(∂μφa) δφa)

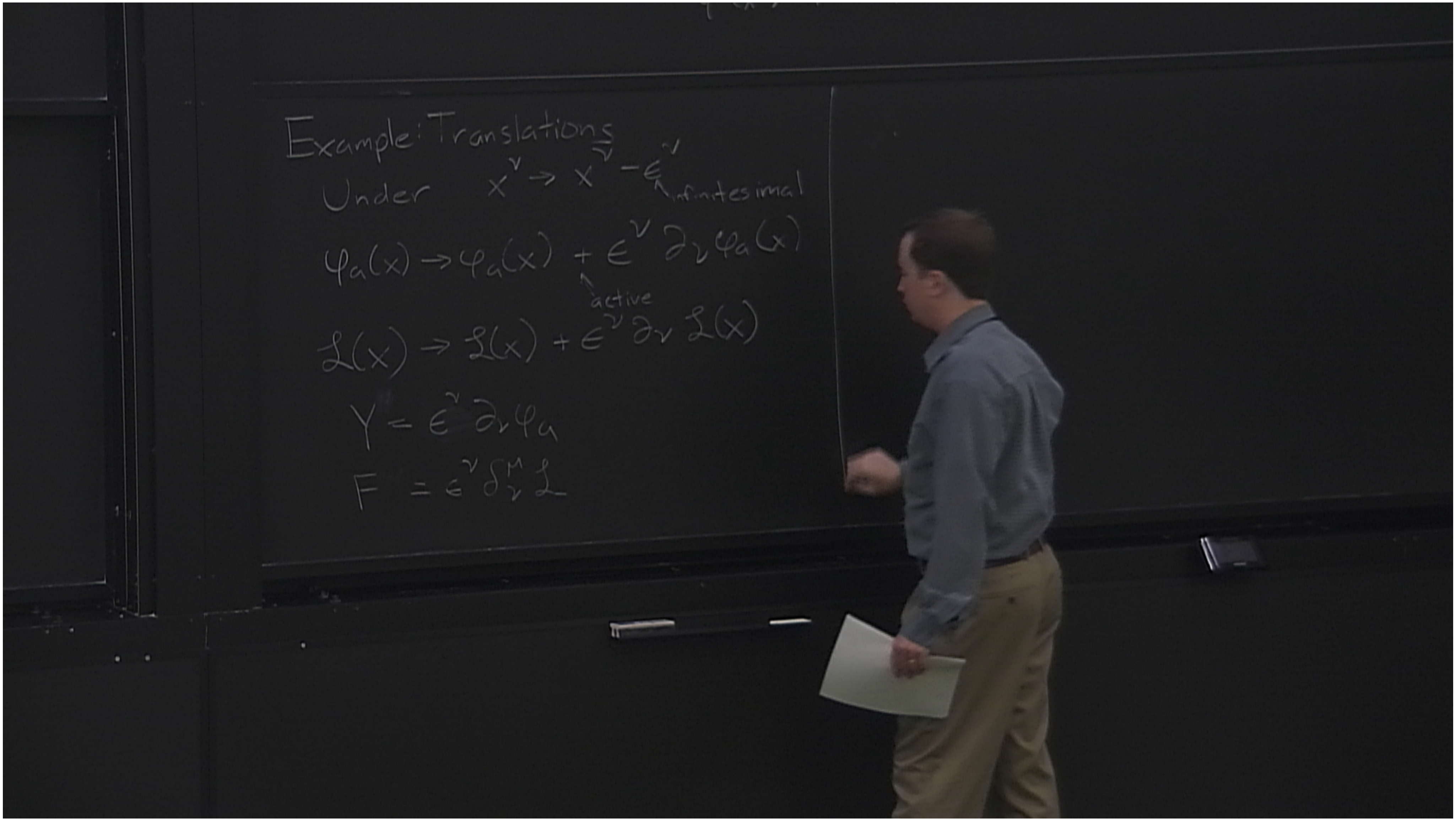

$\delta \mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial \varphi_a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \right) \right) \delta\varphi_a + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \delta\varphi_a \right)$
=0 if EOM satisfied

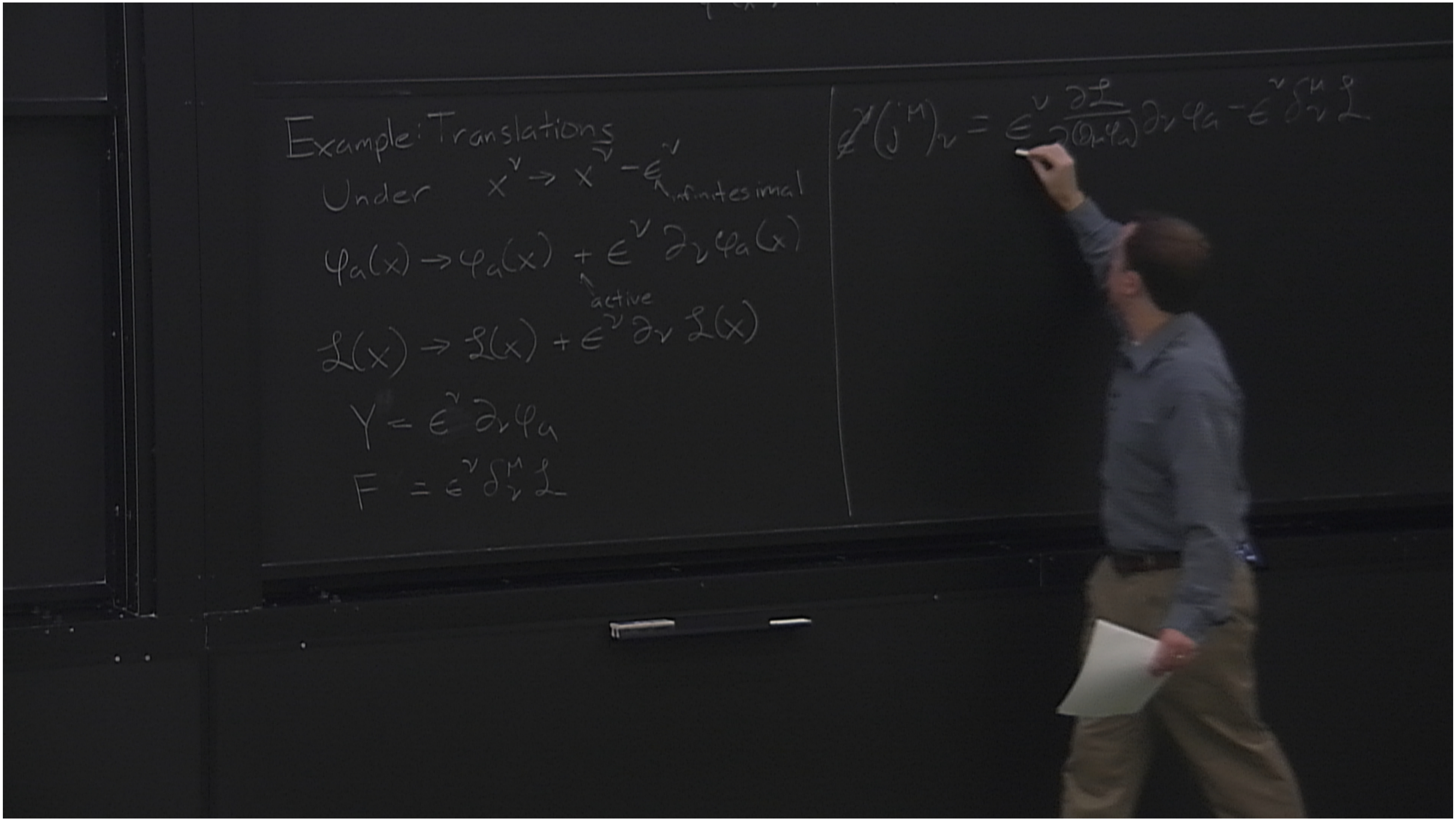

=0 if EOM satisfied
If δφa is a symmetry

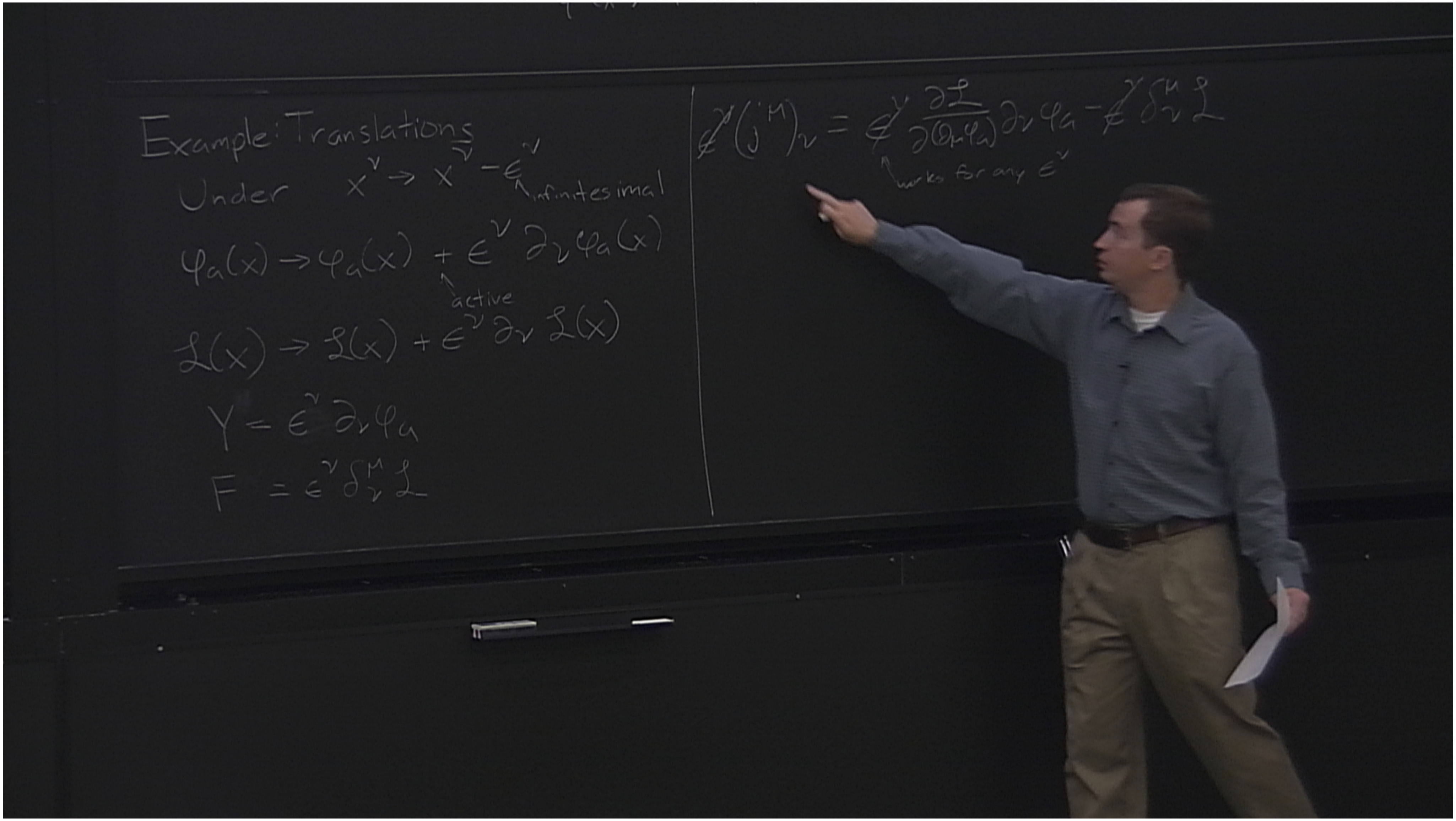

Example: Translations
Under $x^\nu \to x^\nu - \epsilon^\nu$
infintesimal
$\varphi_a(x) \to \varphi_a(x) + \epsilon^\nu \partial_\nu \varphi_a(x)$
active


Example: Translations
Under x^ν → x^ν − ε^ν
infinitesimal
φ_a(x) → φ_a(x) + ε^ν ∂_ν φ_a(x)
active
ℒ(x) → ℒ(x) + ε^ν ∂_ν ℒ(x)


Active transformations
φ(x) has a max at (0,1,0,0)
consider a rotation by π/2 about x^3 axis
φ'(x) will have a max at (x')^μ = (0,0,1,0) = Λ^μ_ν x^ν
Λ^-1 x' = x
φ'(x') = φ(x) = φ(Λ^-1 x')


Example: Translations
Under $x^\nu \to x^\nu - \epsilon^\nu$
infinitesimal
$\varphi_a(x) \to \varphi_a(x) + \epsilon^\nu \partial_\nu \varphi_a(x)$
active
$\mathcal{L}(x) \to \mathcal{L}(x) + \epsilon^\nu \partial_\nu \mathcal{L}(x)$
$Y = \epsilon^\nu \partial_\nu \varphi_a$
$F = \epsilon^\nu \delta^\mu_\nu \mathcal{L}$


Example: Translations
Under $x^\nu \to x^\nu - \epsilon^\nu$
infinitesimal
$\varphi_a(x) \to \varphi_a(x) + \epsilon^\nu \partial_\nu \varphi_a(x)$
active
$\mathcal{L}(x) \to \mathcal{L}(x) + \epsilon^\nu \partial_\nu \mathcal{L}(x)$
$Y = \epsilon^\nu \partial_\nu \varphi_a$
$F = \epsilon^\nu \delta^\mu_\nu \mathcal{L}$
$\epsilon^\nu (j^\mu)_\nu = \epsilon^\nu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \partial_\nu \varphi_a - \epsilon^\nu \delta^\mu_\nu \mathcal{L}$


Example: Translations
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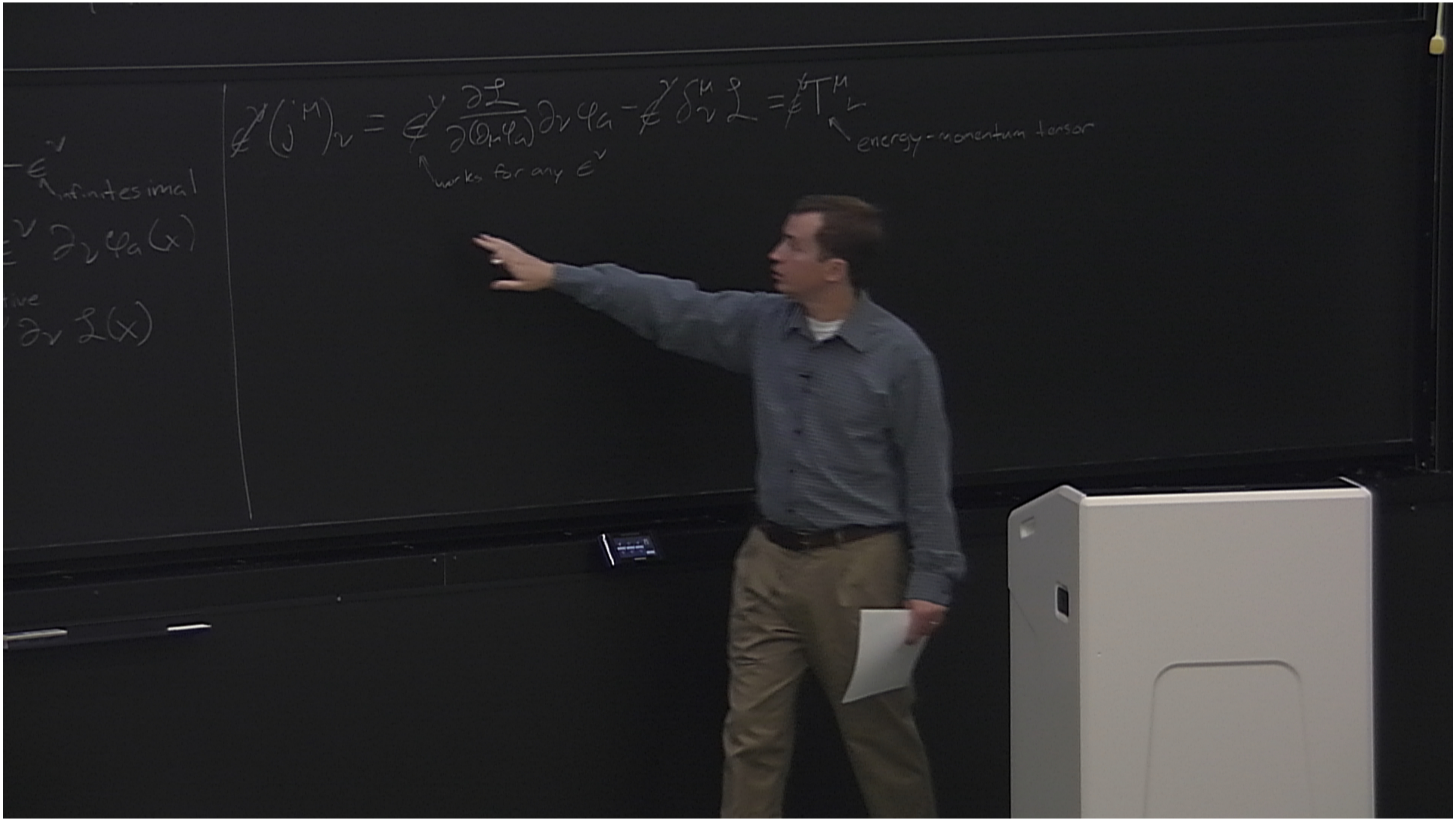
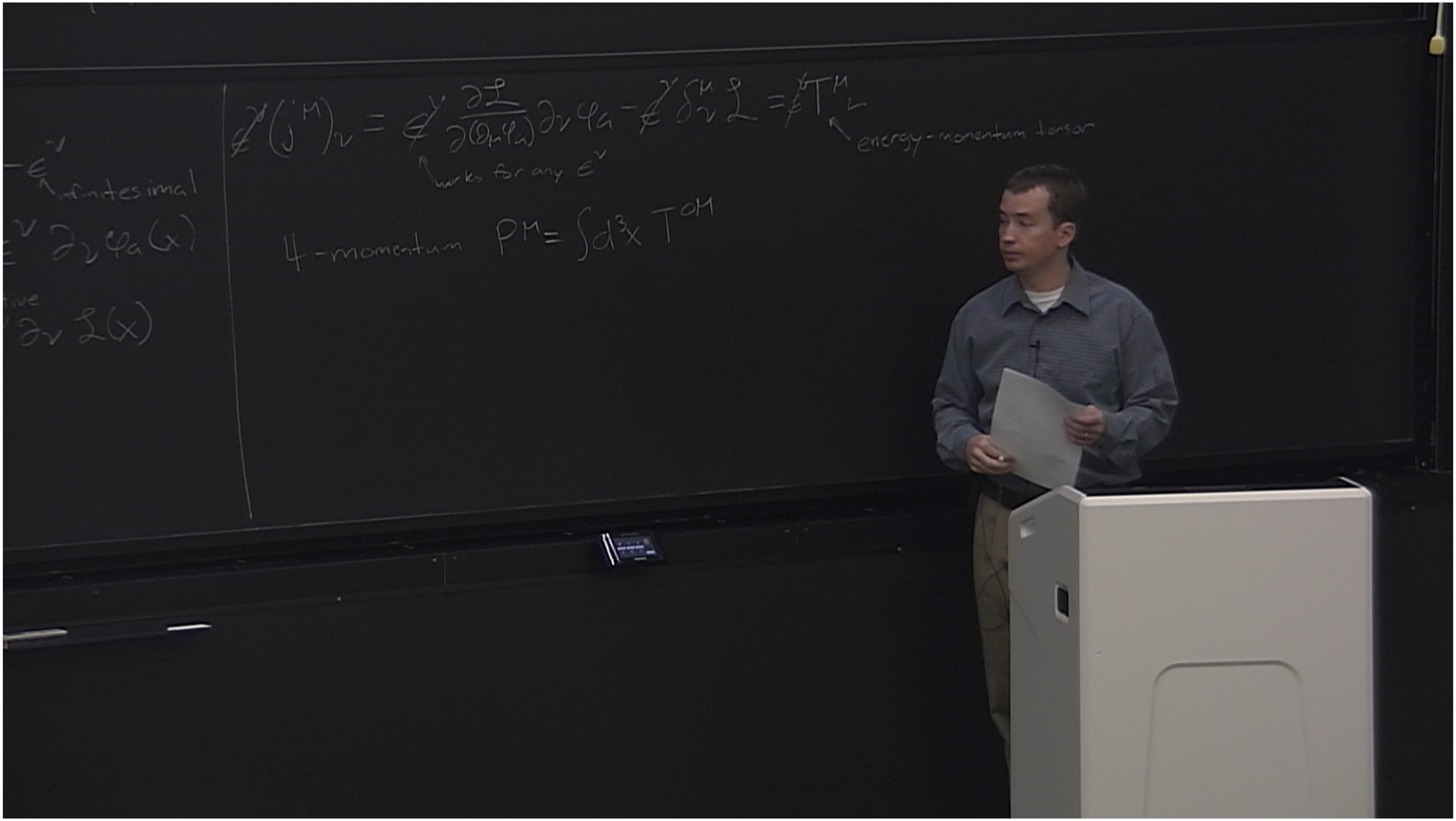
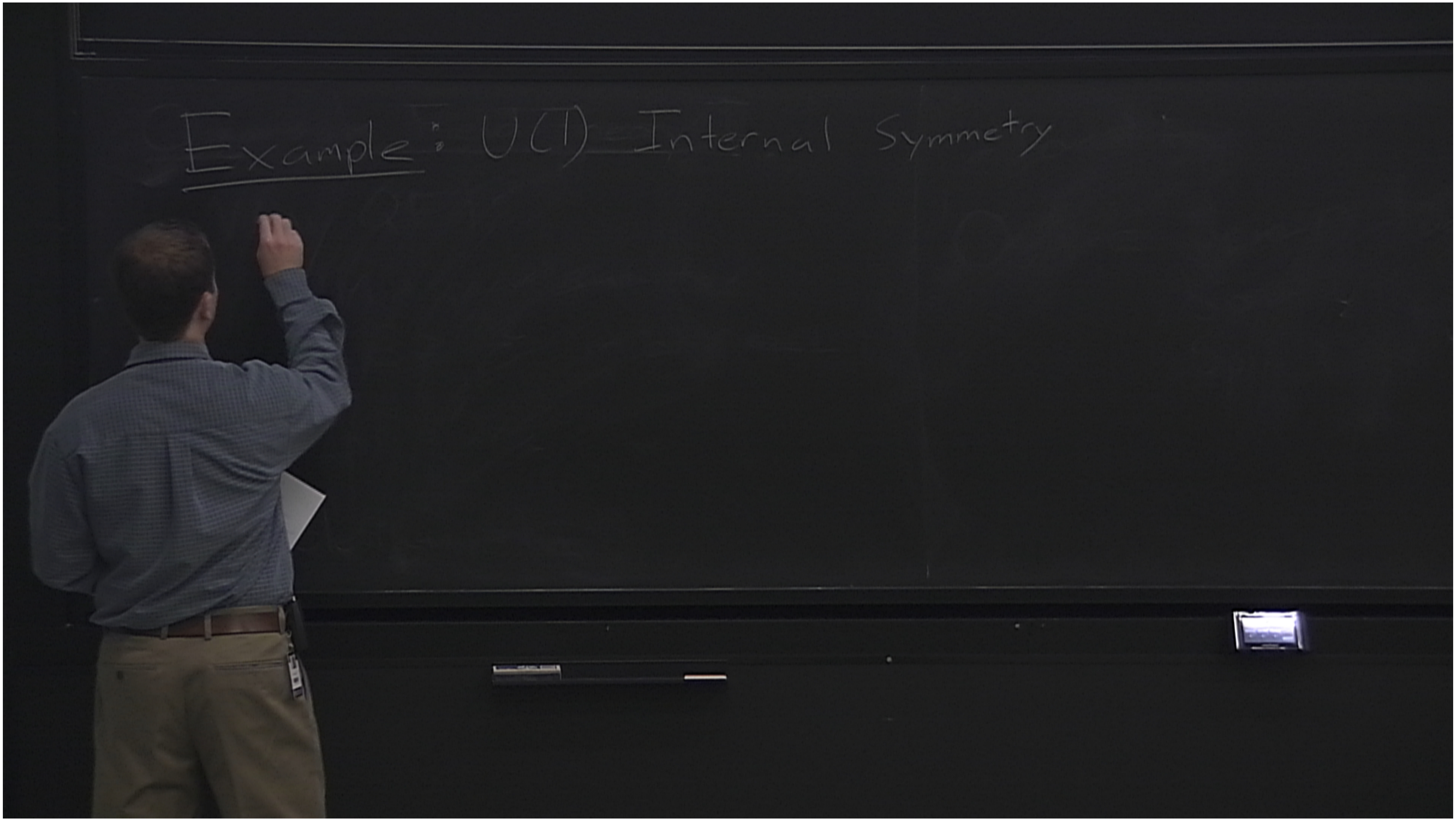
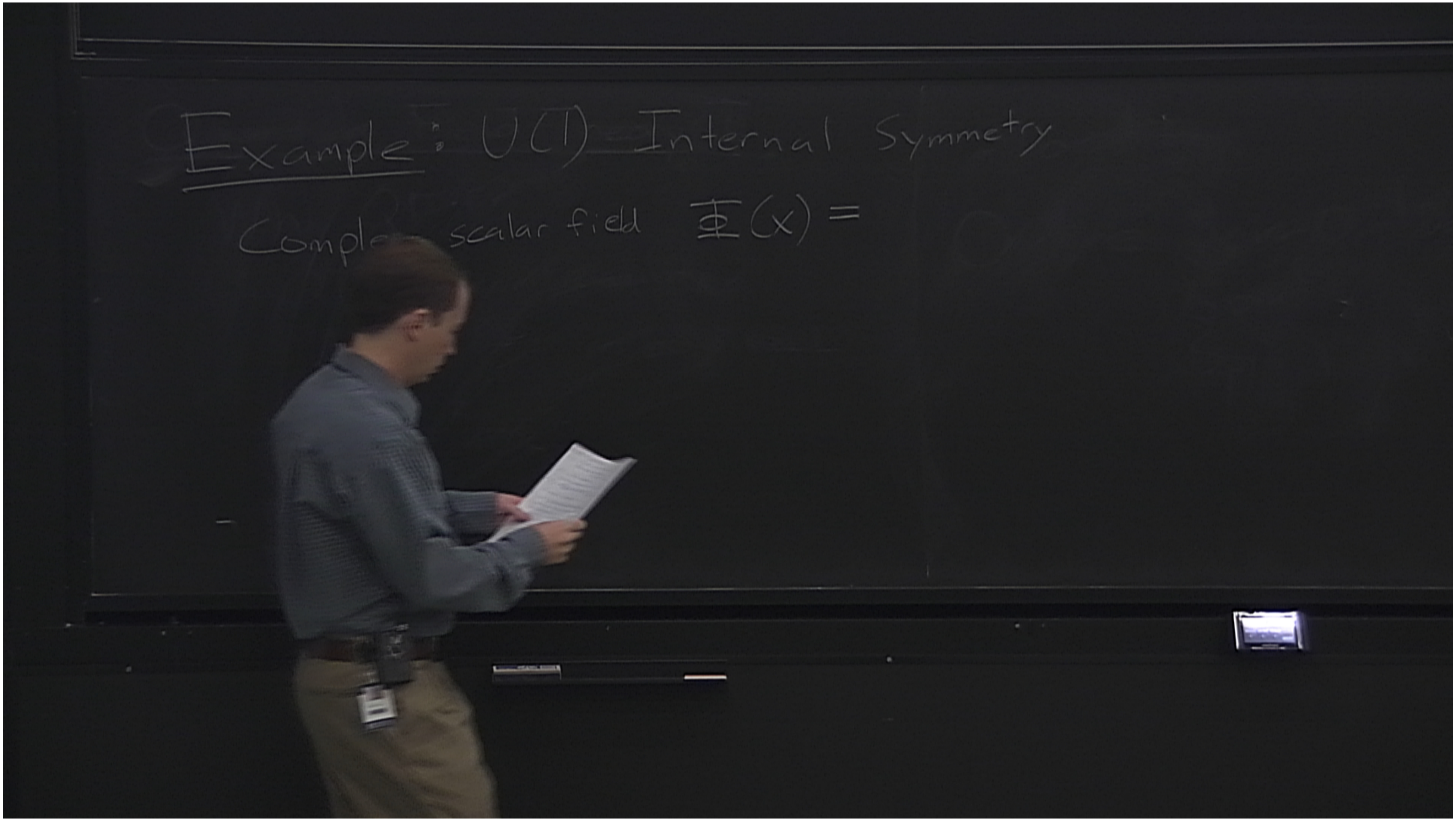
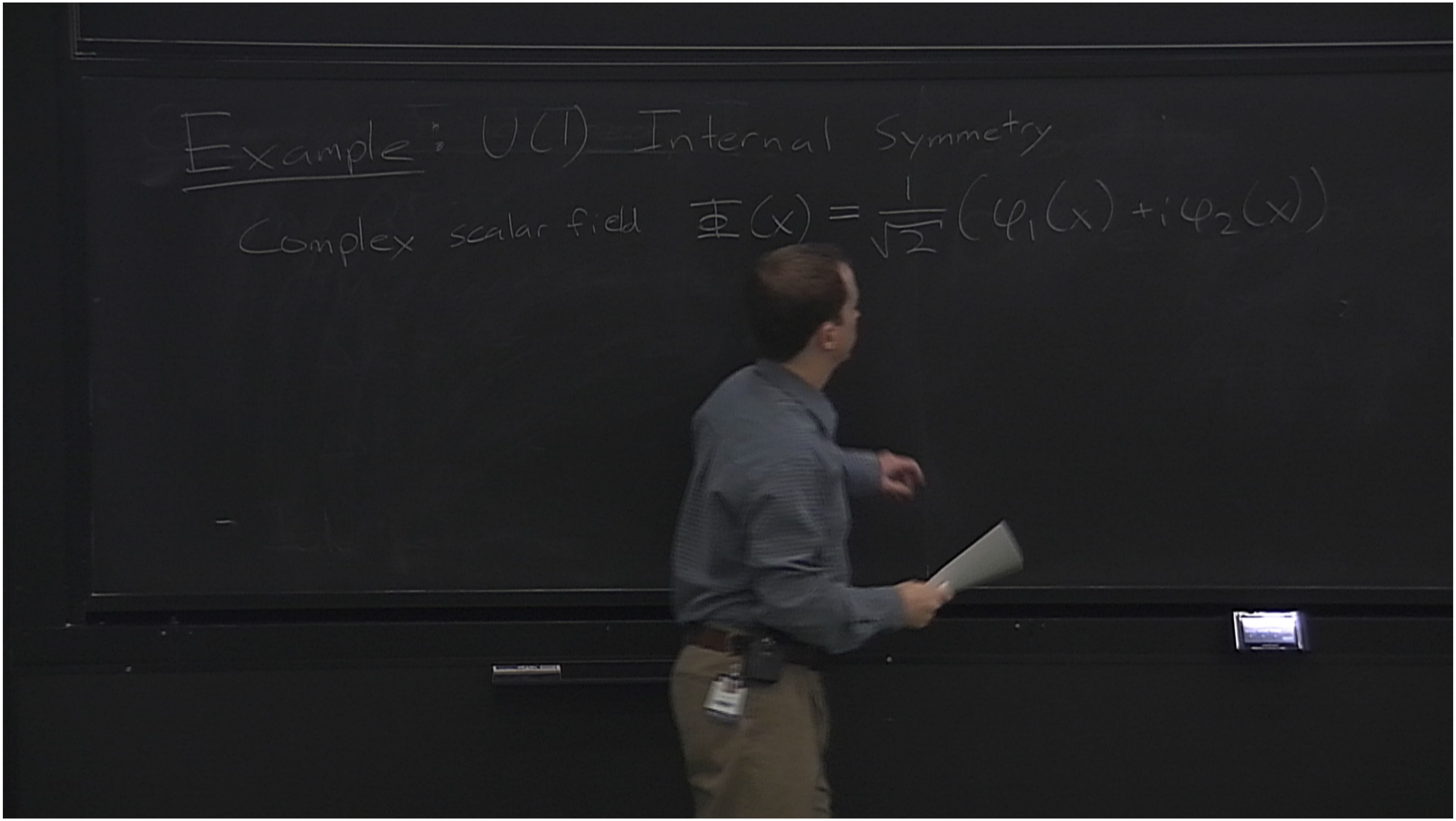
$Y = \epsilon^\nu \partial_\nu \varphi_a$
$F = \epsilon^\nu \delta^\mu_\nu \mathcal{L}$
$\epsilon^\nu (j^\mu)_\nu = \epsilon^\nu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \partial_\nu \varphi_a - \epsilon^\nu \delta^\mu_\nu \mathcal{L}$
works for any ϵ^ν

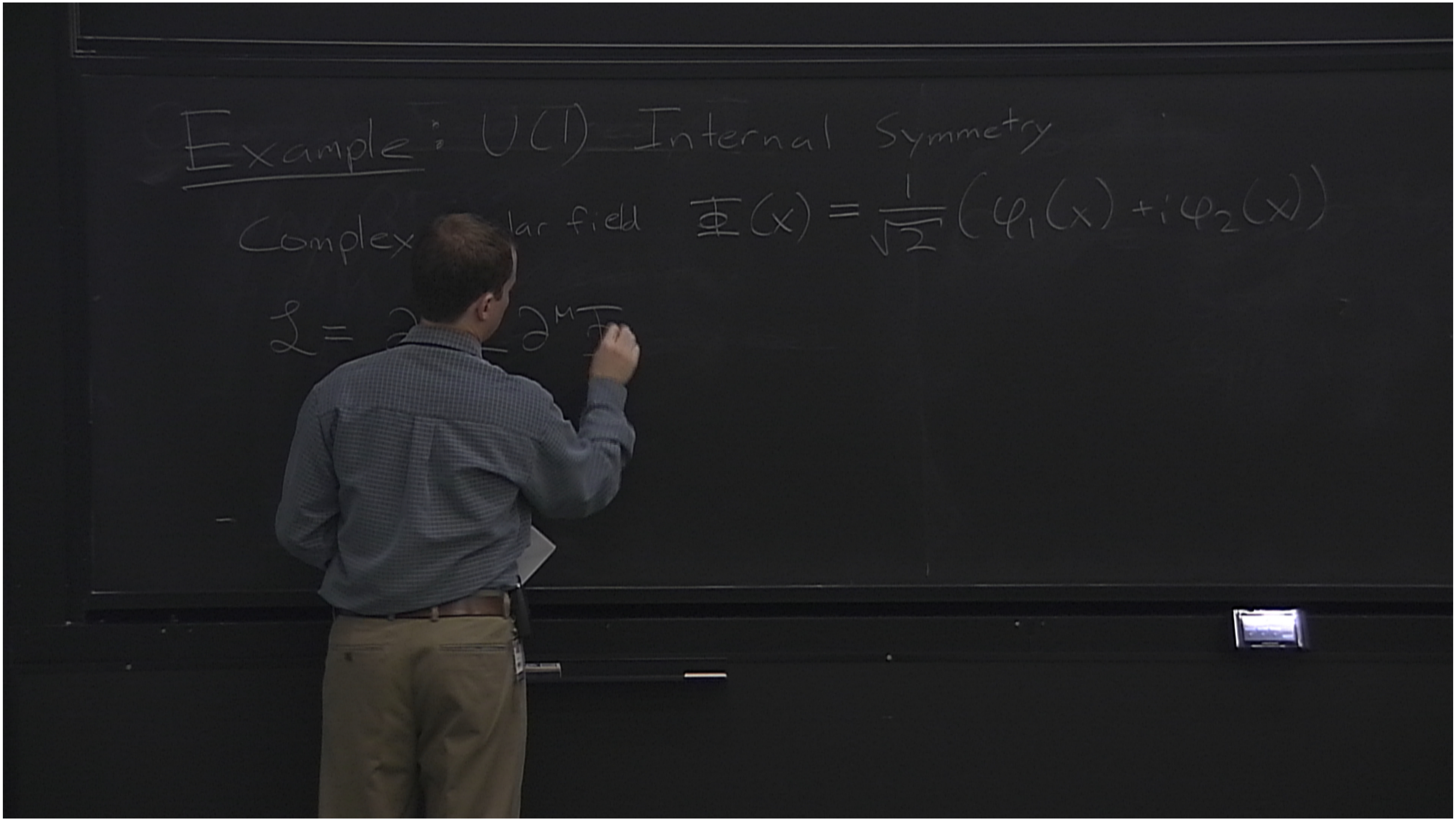

energy-momentum tensor
works for any ϵ^ν
infinitesimal

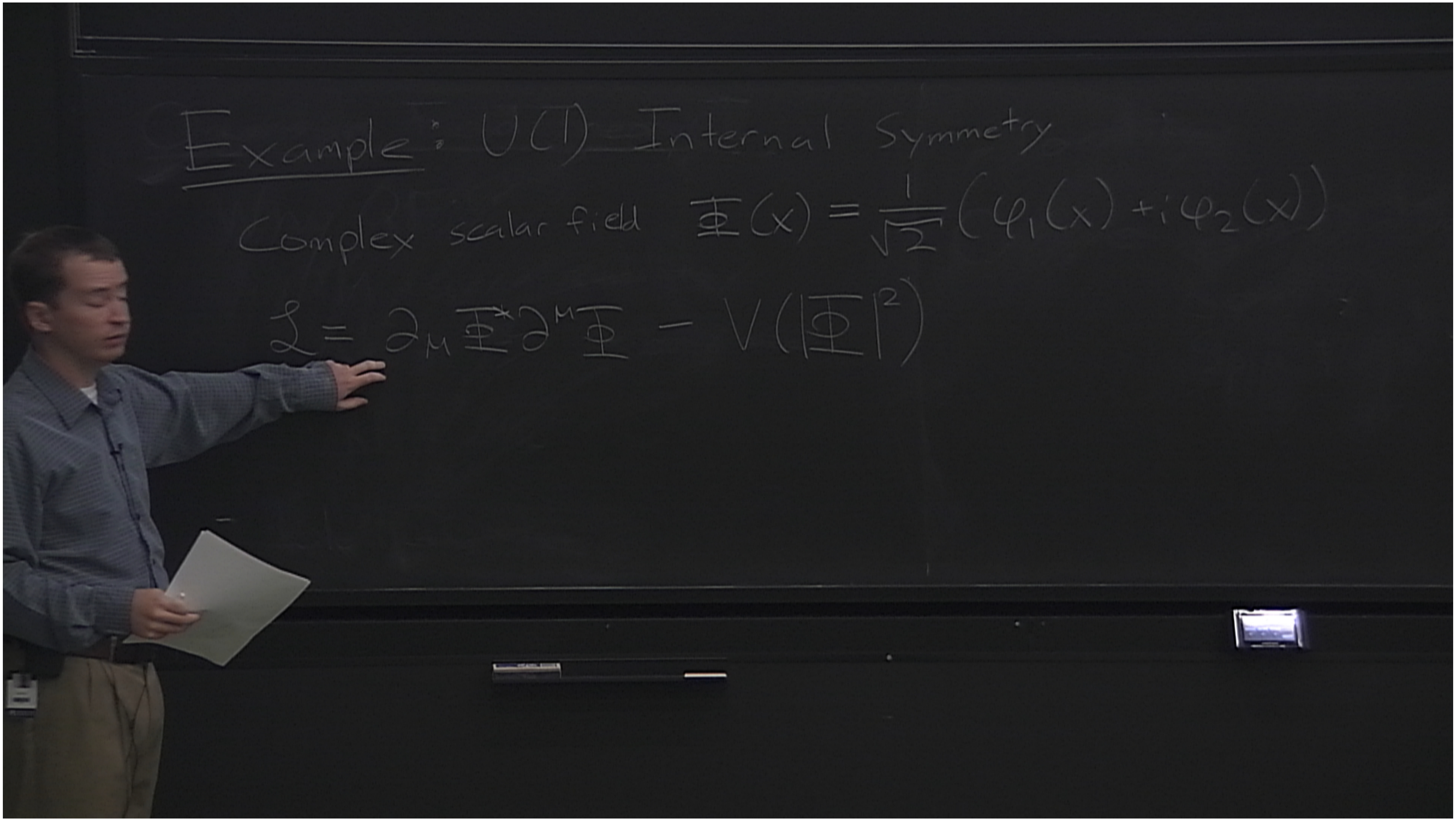

infinitesimal
works for any ε^ν
energy-momentum tensor
4-momentum P^μ = ∫d³x T^0μ

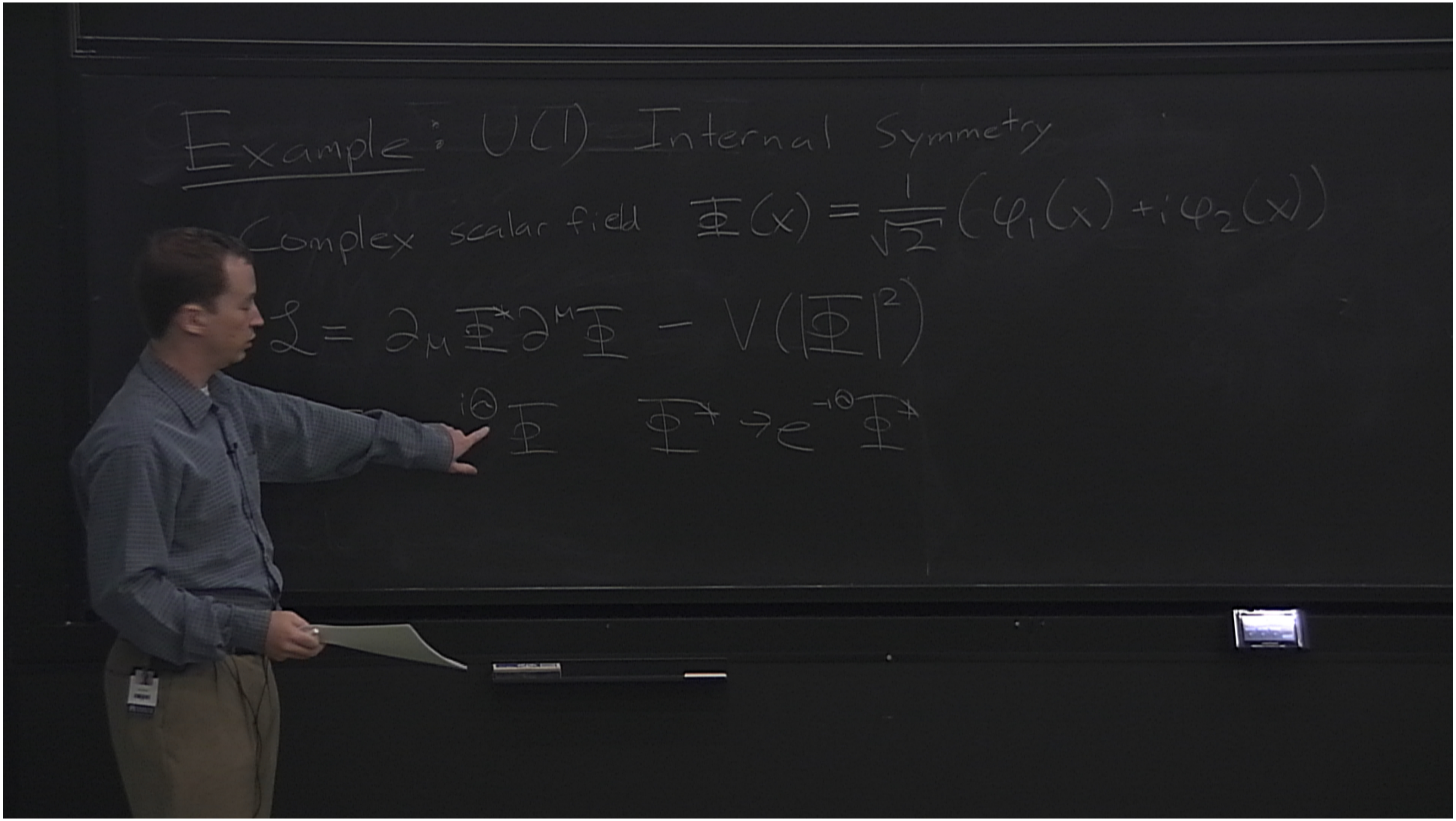

Example: U(1) Internal Symmetry


Example: U(1) Internal Symmetry
Compl scalar field Φ(x) =


Example: U(1) Internal Symmetry
Complex scalar field $\Phi(x) = \frac{1}{\sqrt{2}}(\varphi_1(x) + i\varphi_2(x))$


Example: U(1) Internal Symmetry
Complex lar field Φ(x) = 1/√2 (φ1(x) + iφ2(x))
ℒ = ∂ ∂μ Φ


Example: U(1) Internal Symmetry
Complex scalar field $\Phi(x) = \frac{1}{\sqrt{2}}(\varphi_1(x) + i\varphi_2(x))$
$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$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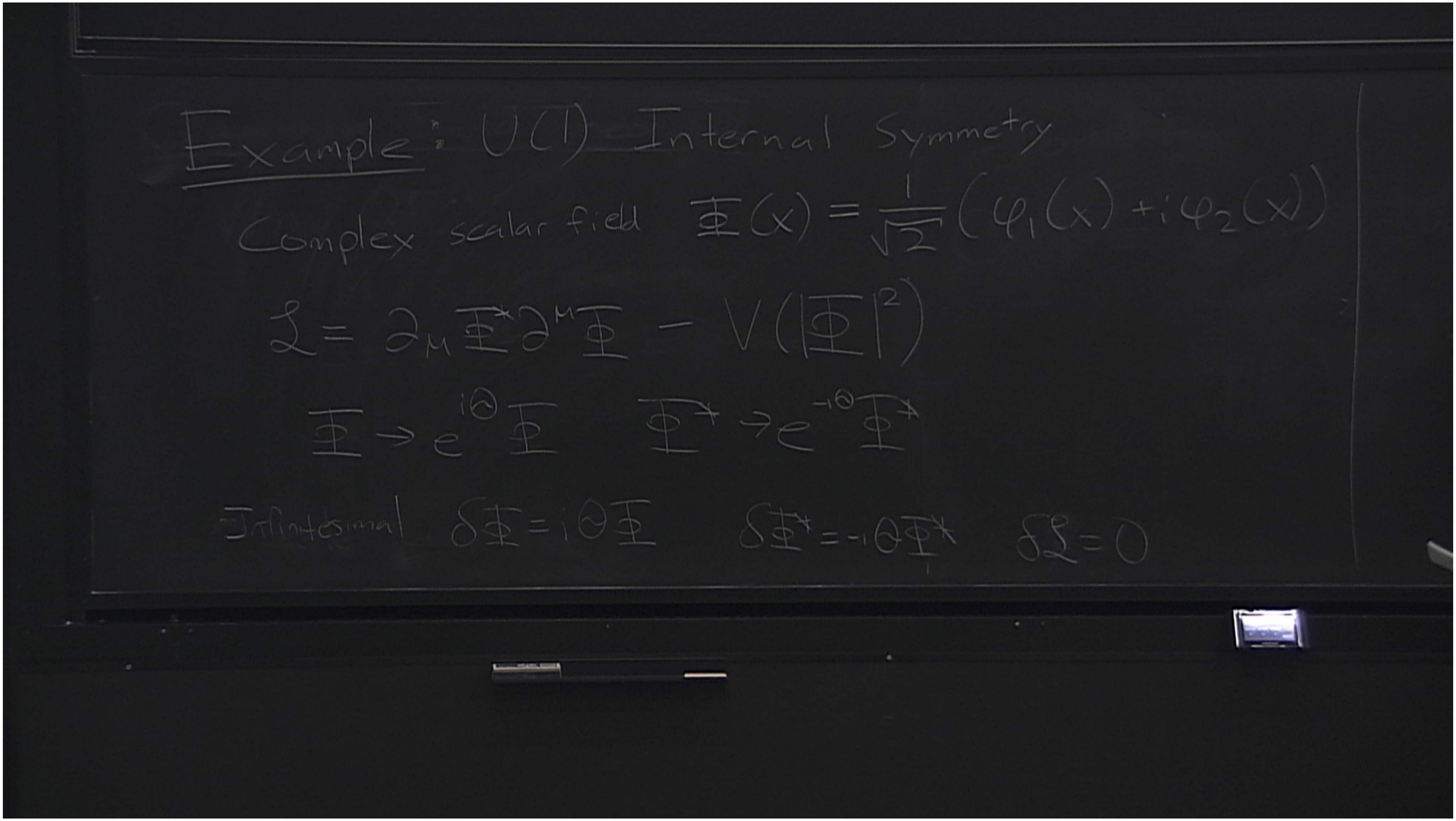
$e^{i\theta}\Phi \quad \Phi^* \to e^{-i\theta}\Phi^*$

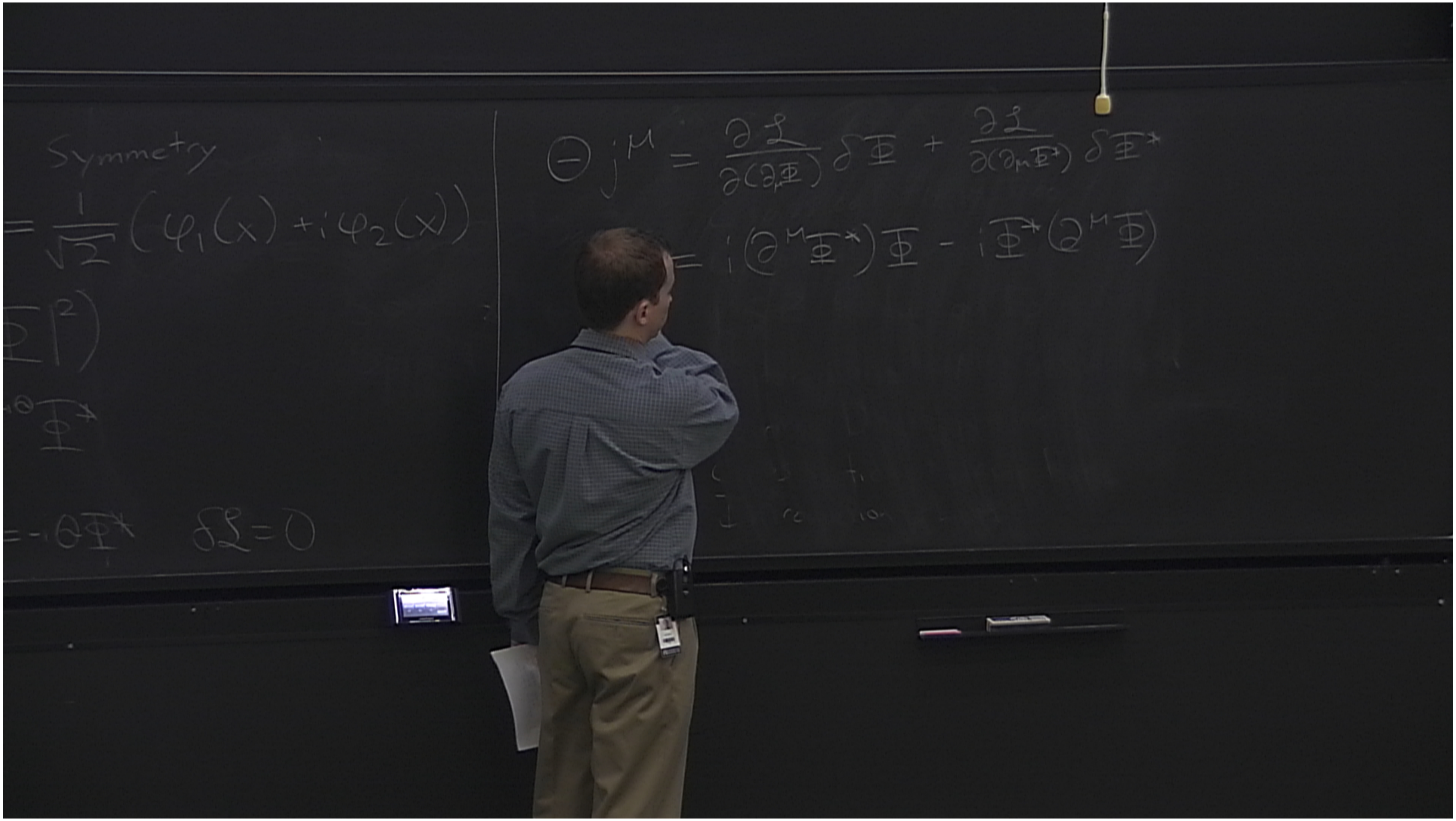

Example: U(1) Internal Symmetry
Complex scalar field $\Phi(x) = \frac{1}{\sqrt{2}}(\varphi_1(x) + i\varphi_2(x))$
$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$
$\Phi \to e^{i\theta}\Phi$ $\Phi^* \to e^{-i\theta}\Phi^*$
Infinitesimal $\delta\Phi = i\theta\Phi$ $\delta\Phi^* = -i\theta\Phi^*$ $\delta\mathcal{L} = 0$

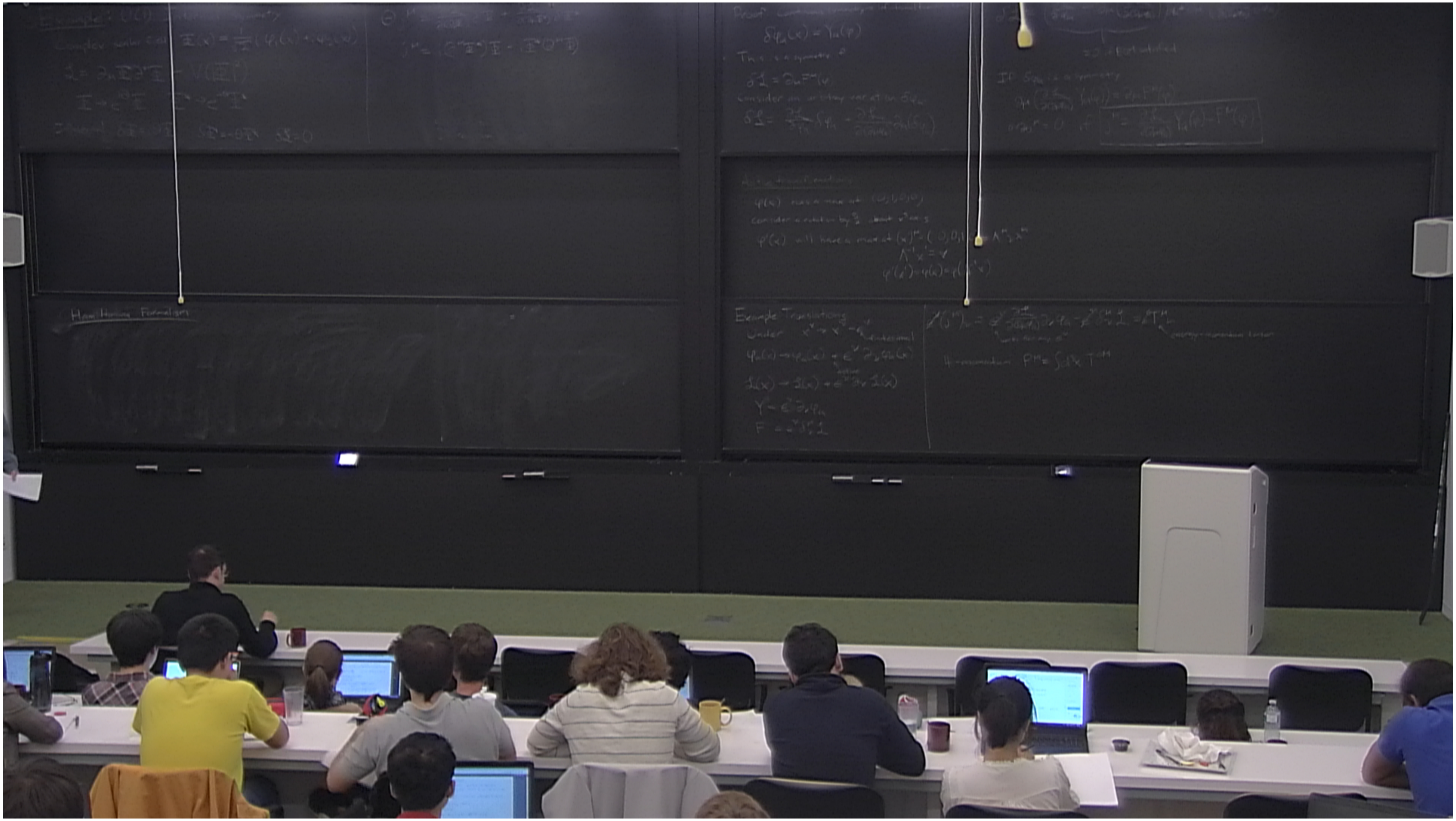
$+ i\varphi_2(x))$

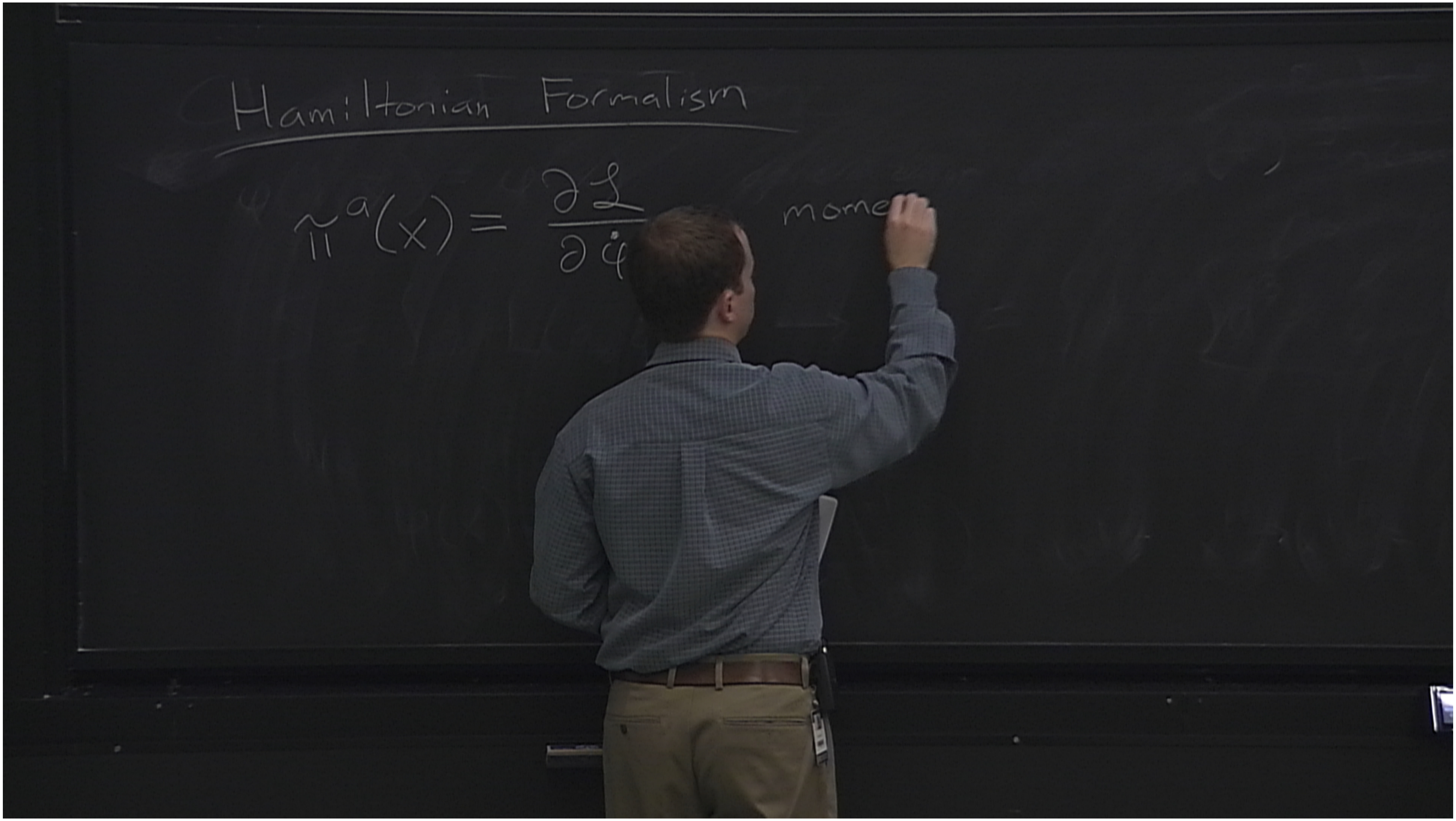
$$\ominus \quad j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)}\delta\Phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^*)}\delta\Phi^*$$

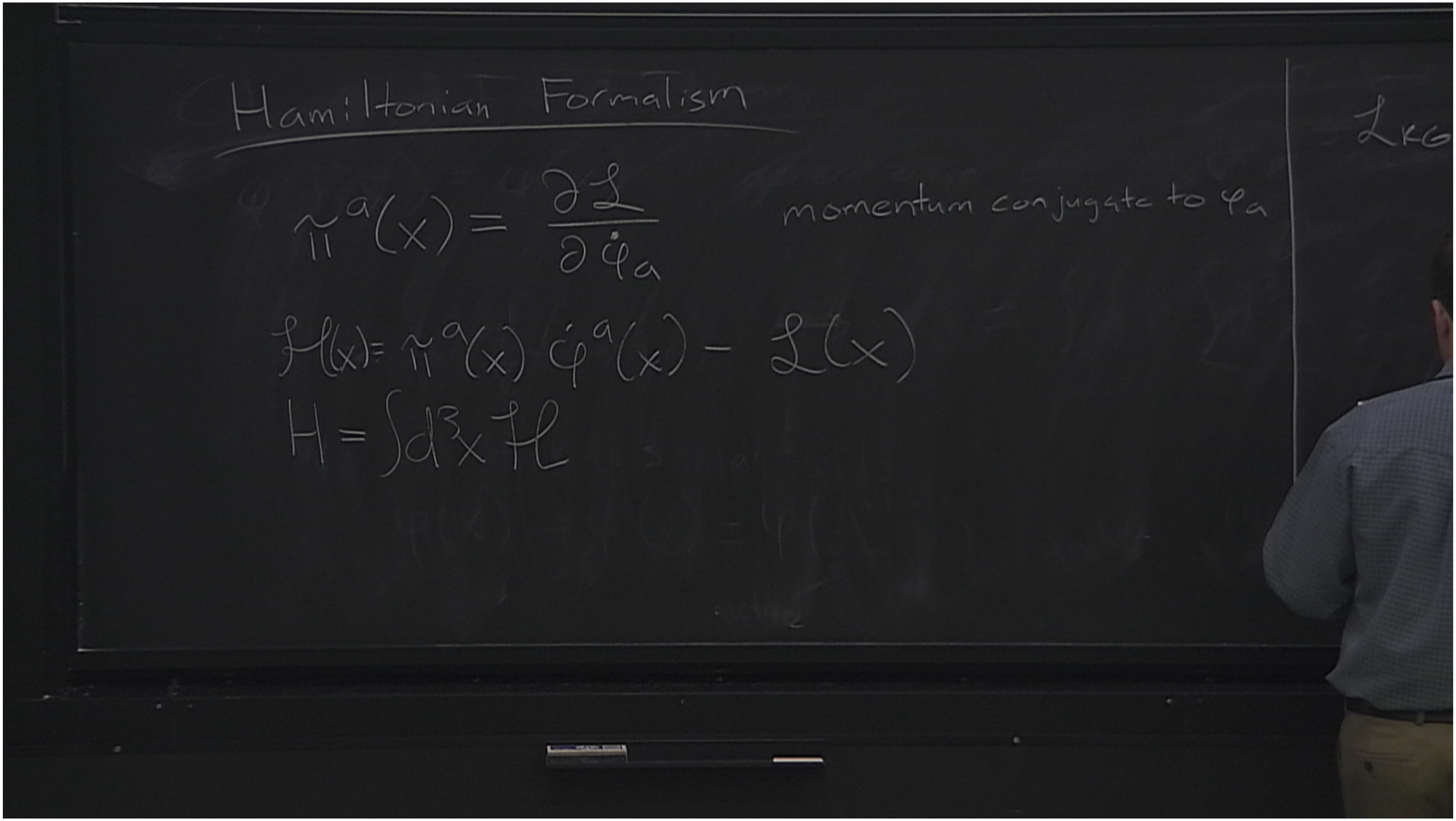
$$j^\mu = i(\partial^\mu \Phi^*)\Phi - i\Phi^*(\partial^\mu \Phi)$$

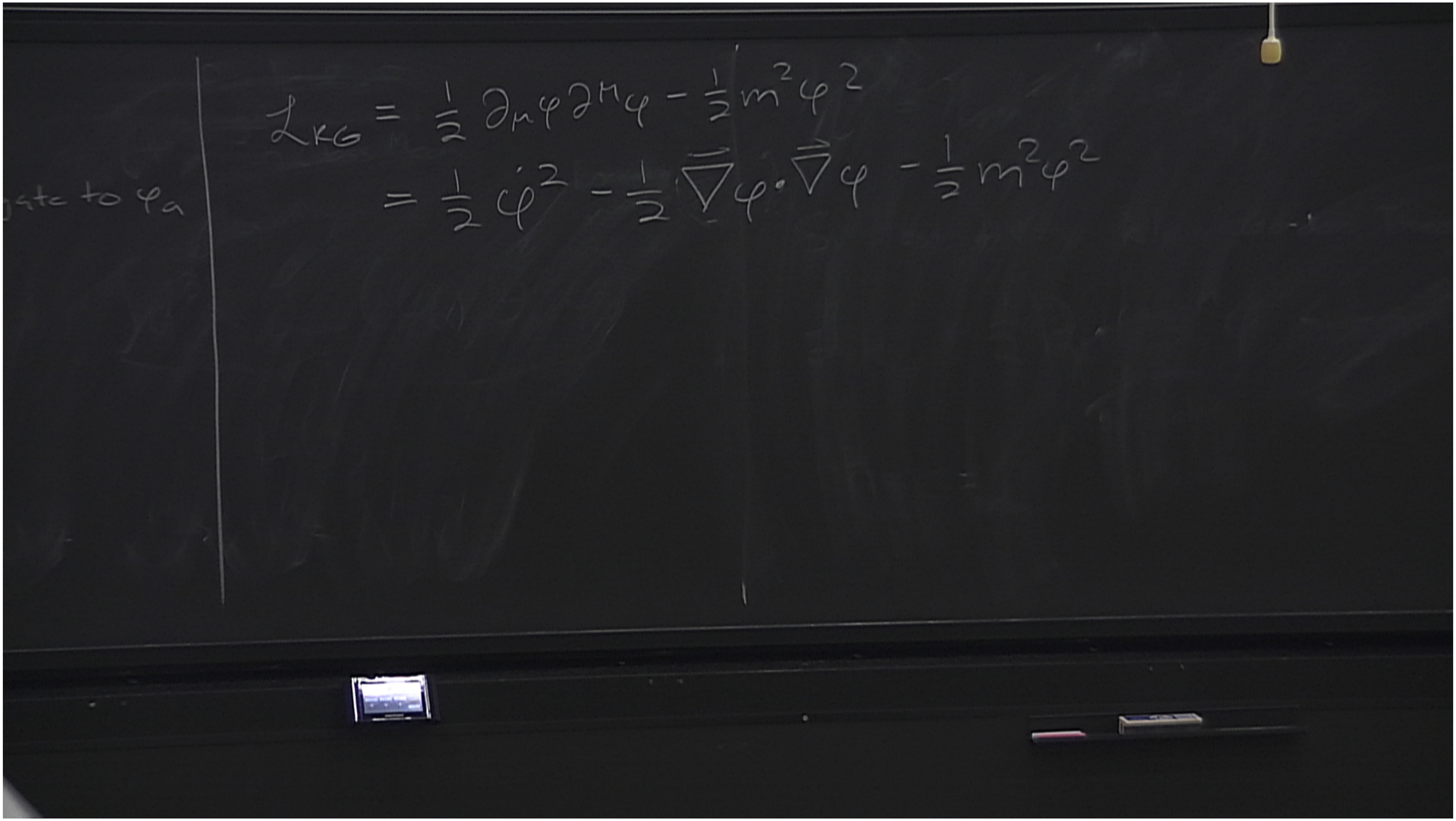
$= 0$


Symmetry
δℒ = 0

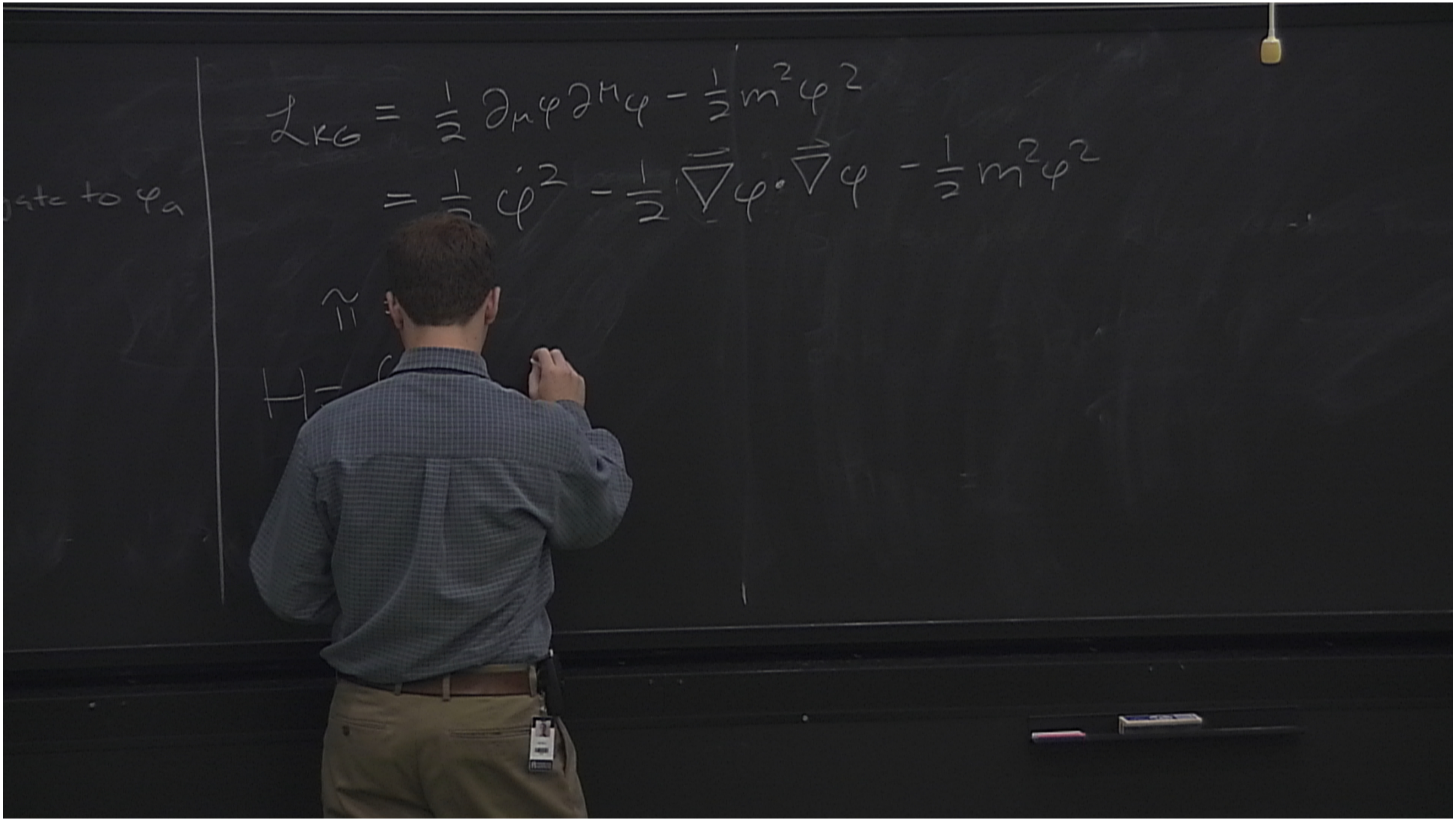



Hamiltonian Formalism
$\tilde{\pi}^a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$
mome


Hamiltonian Formalism
$\pi^a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_a}$
momentum conjugate to φ_a
$\mathcal{H}(x) = \pi^a(x)\,\dot{\varphi}^a(x) - \mathcal{L}(x)$
$H = \int d^3x\, \mathcal{H}$
$\mathcal{L}_{KG}$



ate to φ_a
$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi - \frac{1}{2}m^2\varphi^2$



to φ_a
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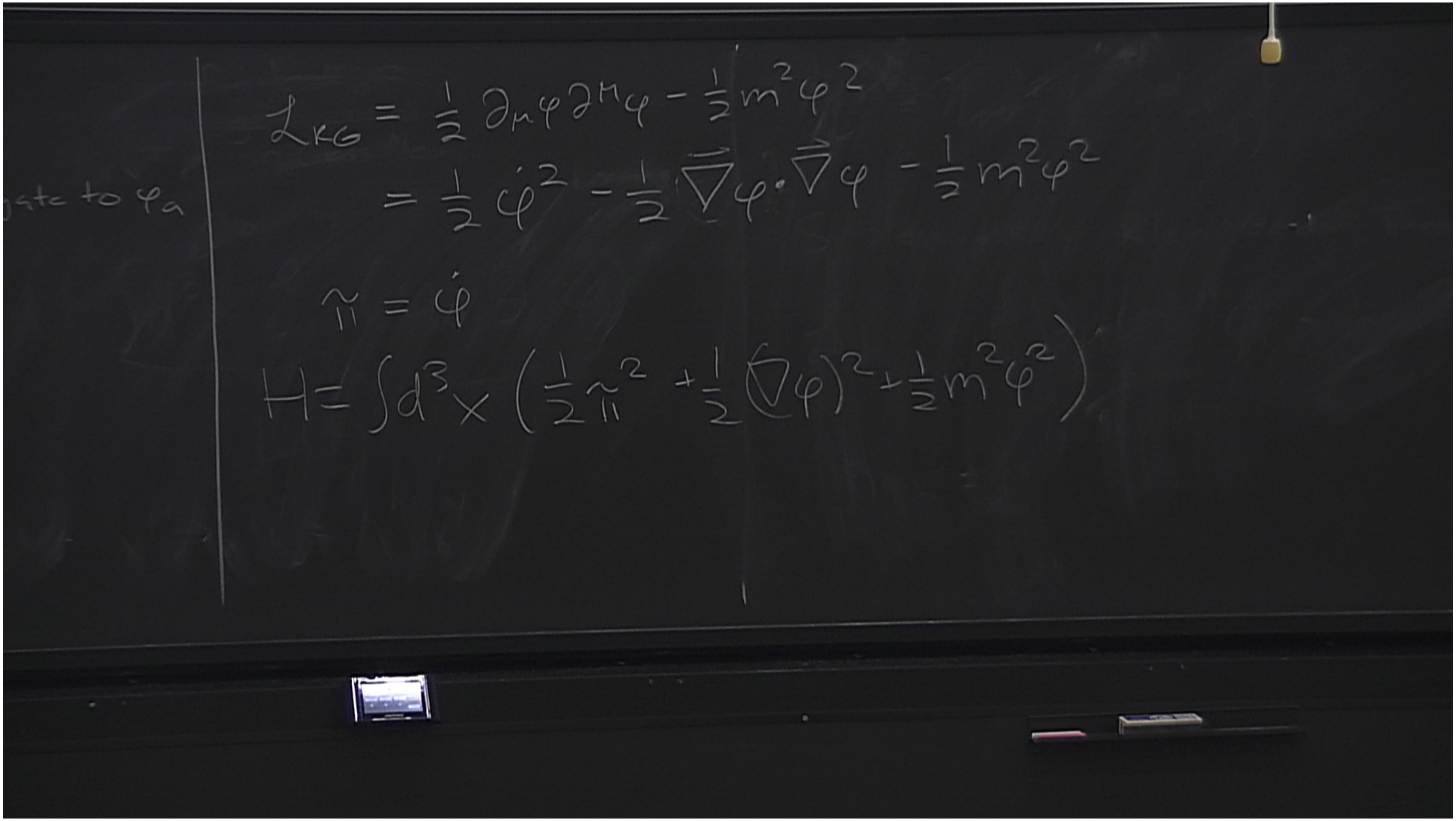
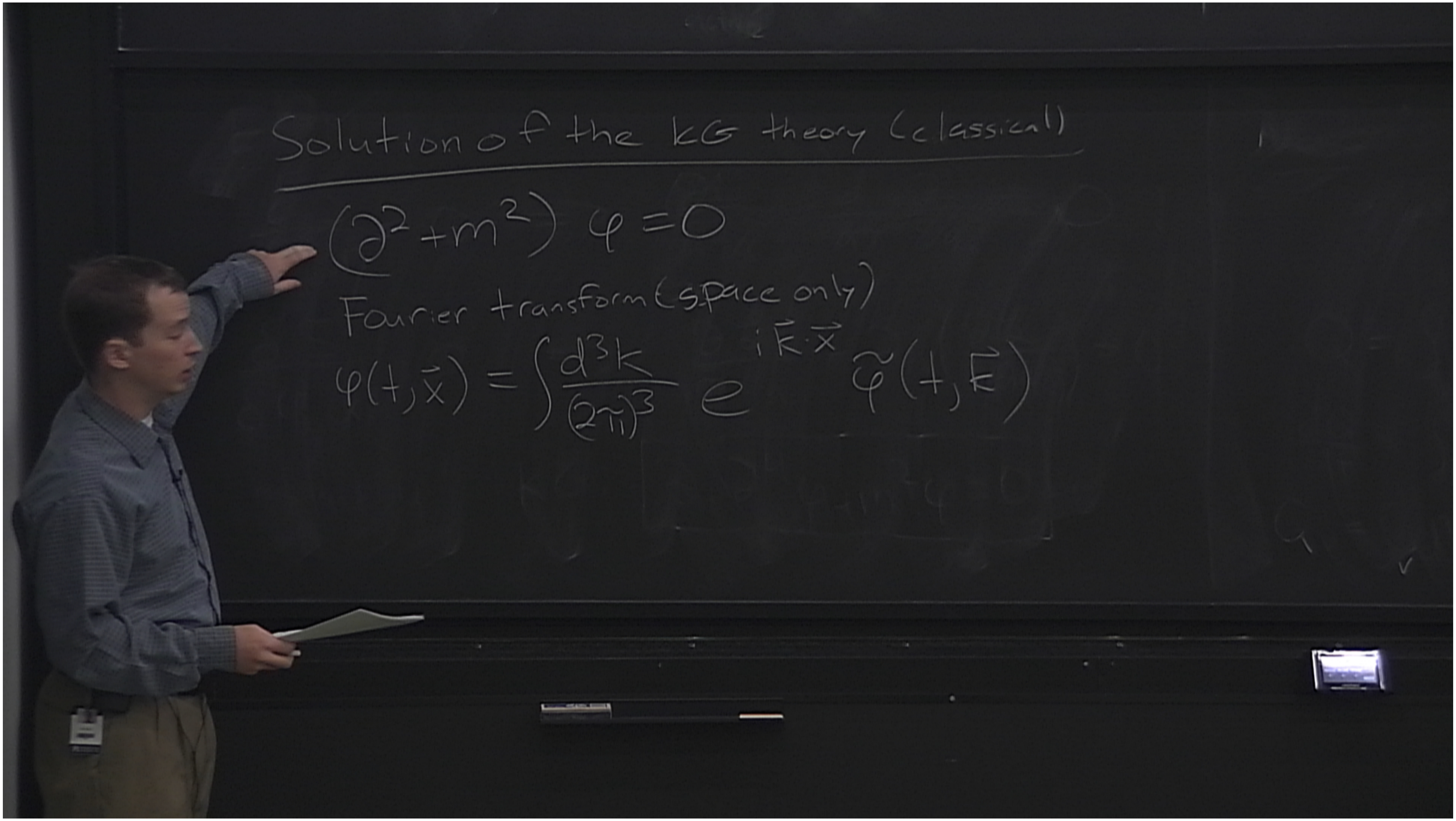
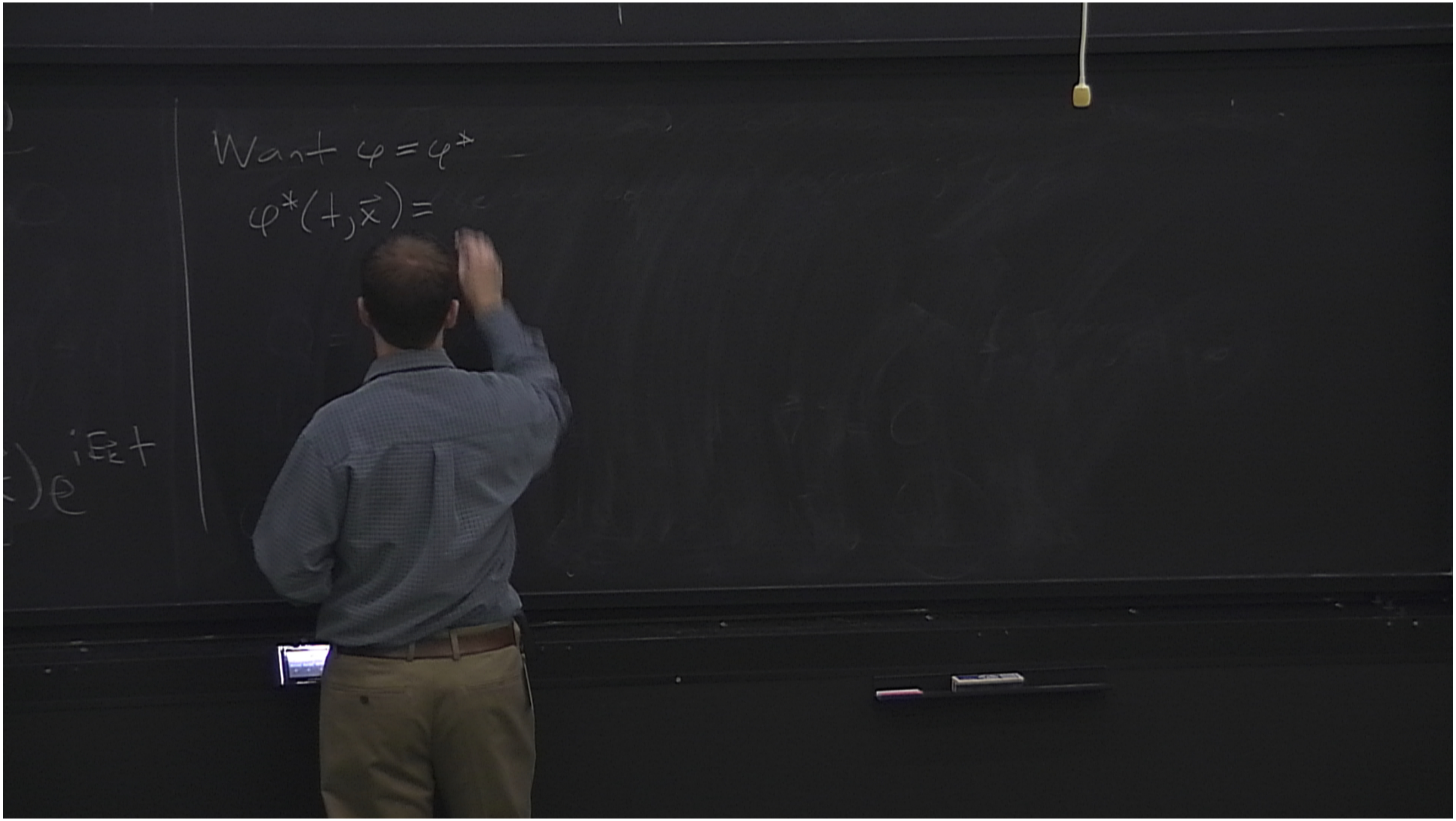
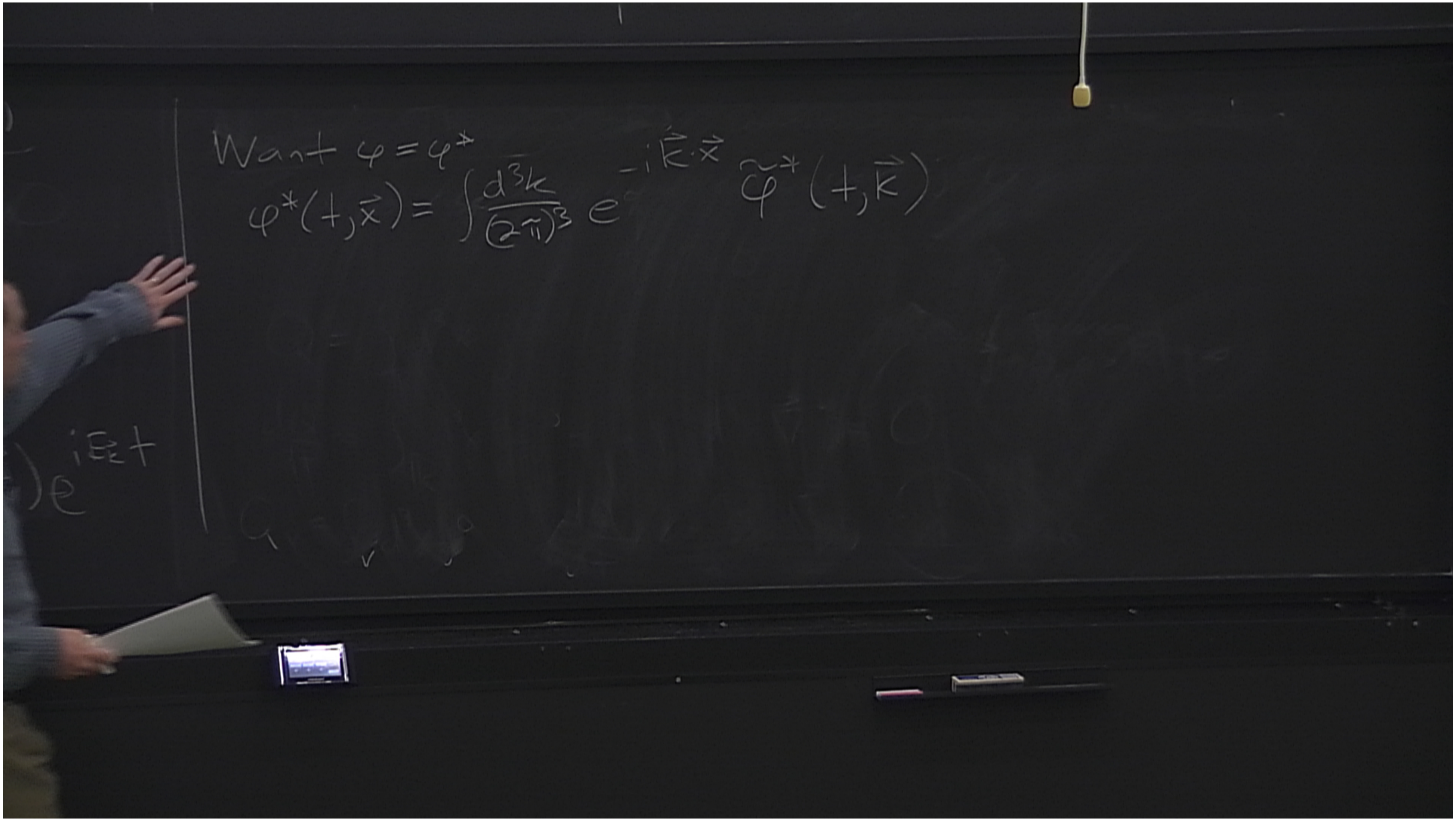
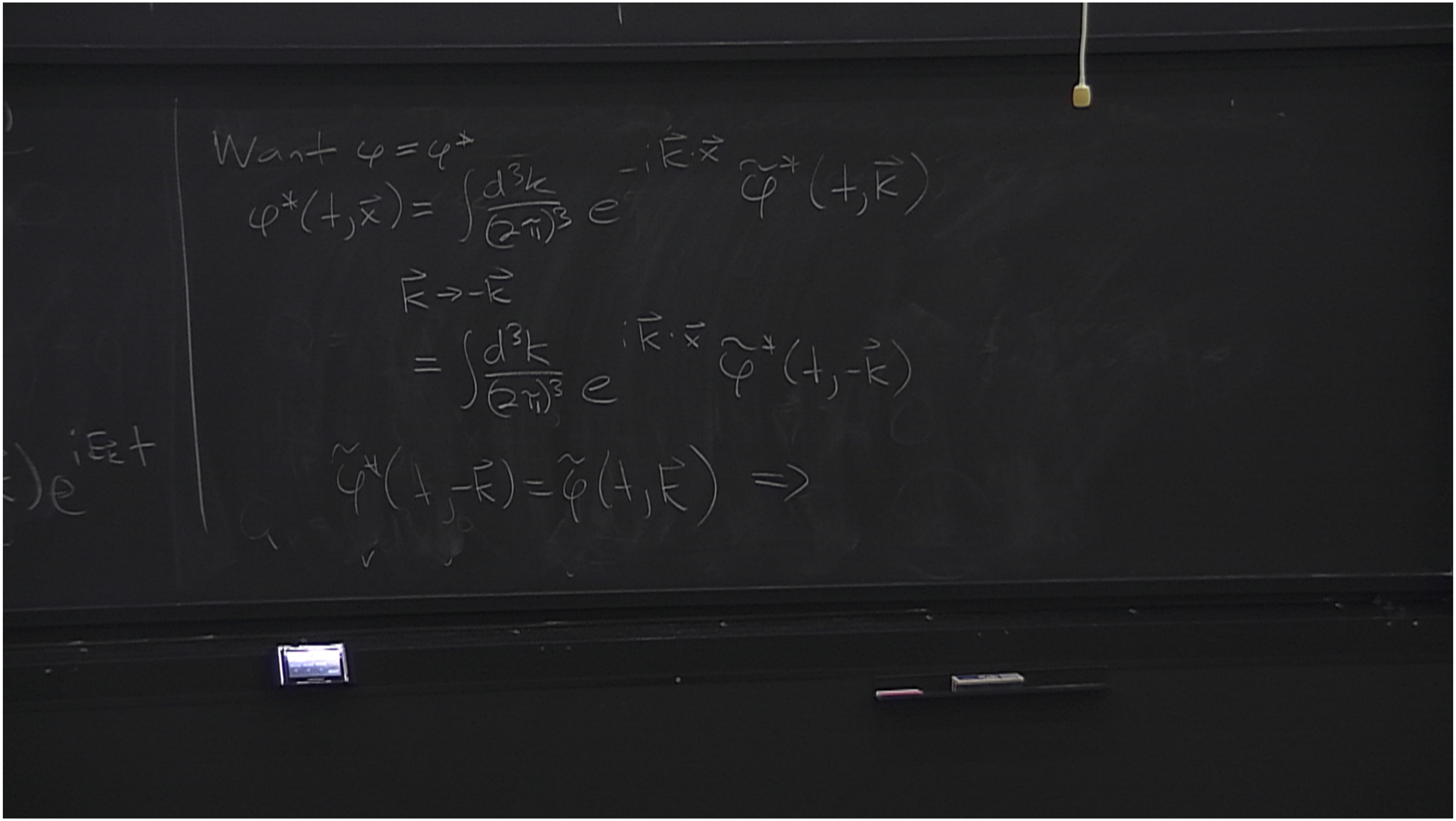
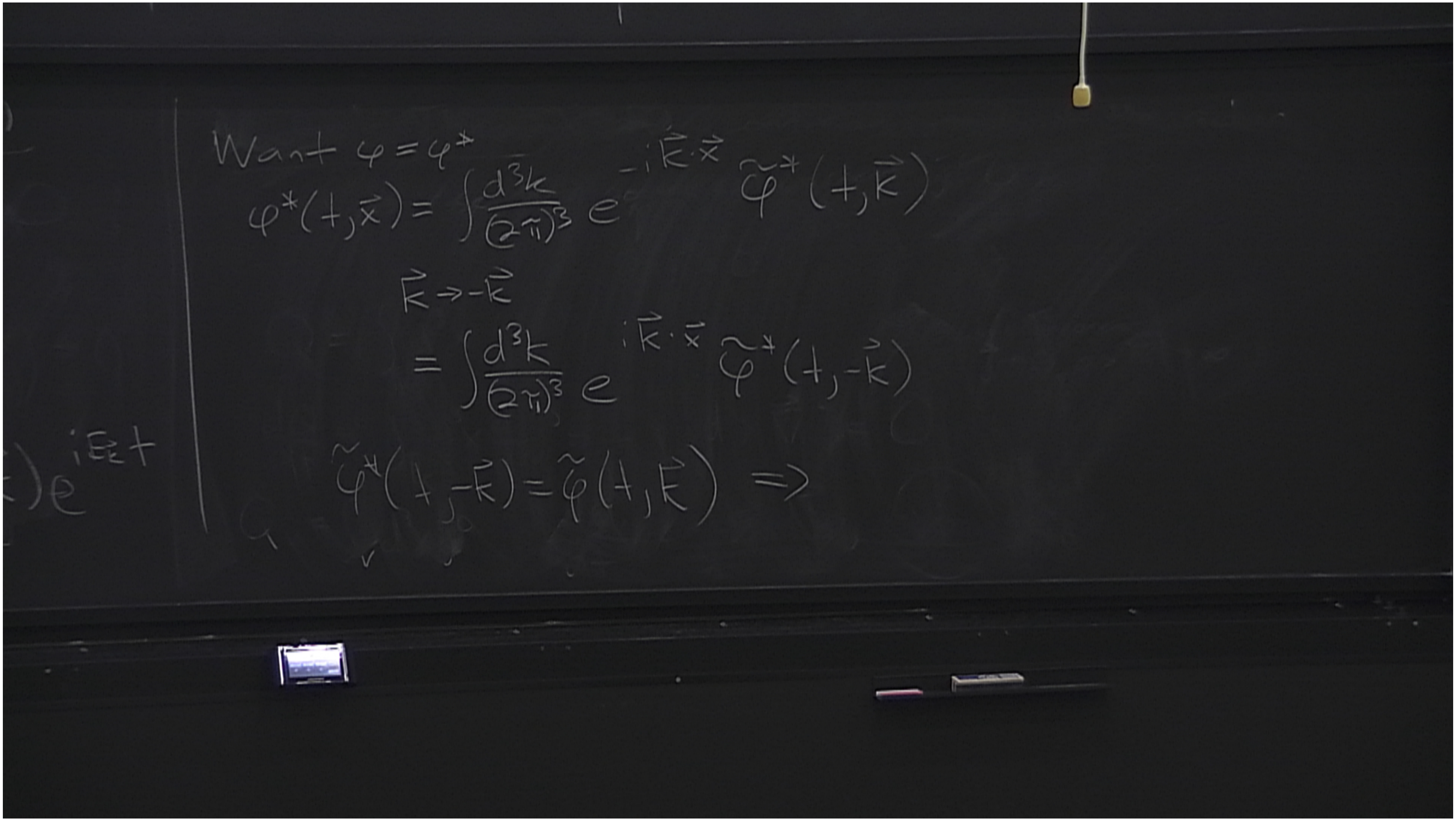

to φ_a
$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi - \frac{1}{2}m^2\varphi^2$
$\pi = \dot{\varphi}$
$H = \int d^3x \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2\right)$


Solution of the KG theory (classical)
$(\partial^2 + m^2)\,\varphi = 0$
Fourier transform (space only)
$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \tilde{\varphi}(t,\vec{k})$


Want φ = φ*
φ*(t, x⃗) =


Want $\varphi = \varphi^*$
$\varphi^*(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k})$
$e^{iE_k t}$


Want $\varphi = \varphi^*$
$\varphi^*(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k})$
$\vec{k} \to -\vec{k}$
$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,-\vec{k})$
$\tilde{\varphi}^*(t,-\vec{k}) = \tilde{\varphi}(t,\vec{k}) \Rightarrow$
$e^{iE_k t}$



Want $\varphi = \varphi^*$
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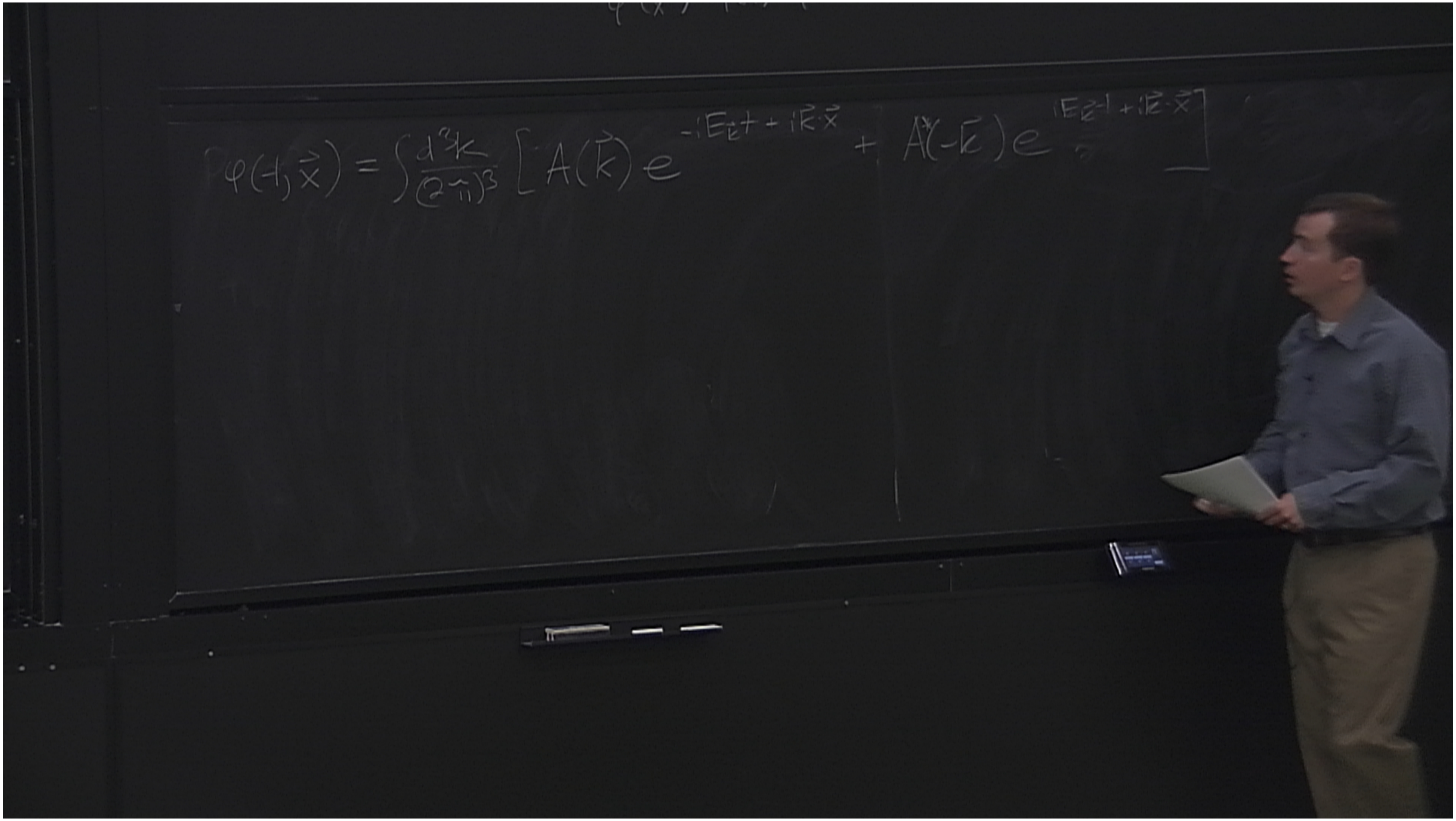
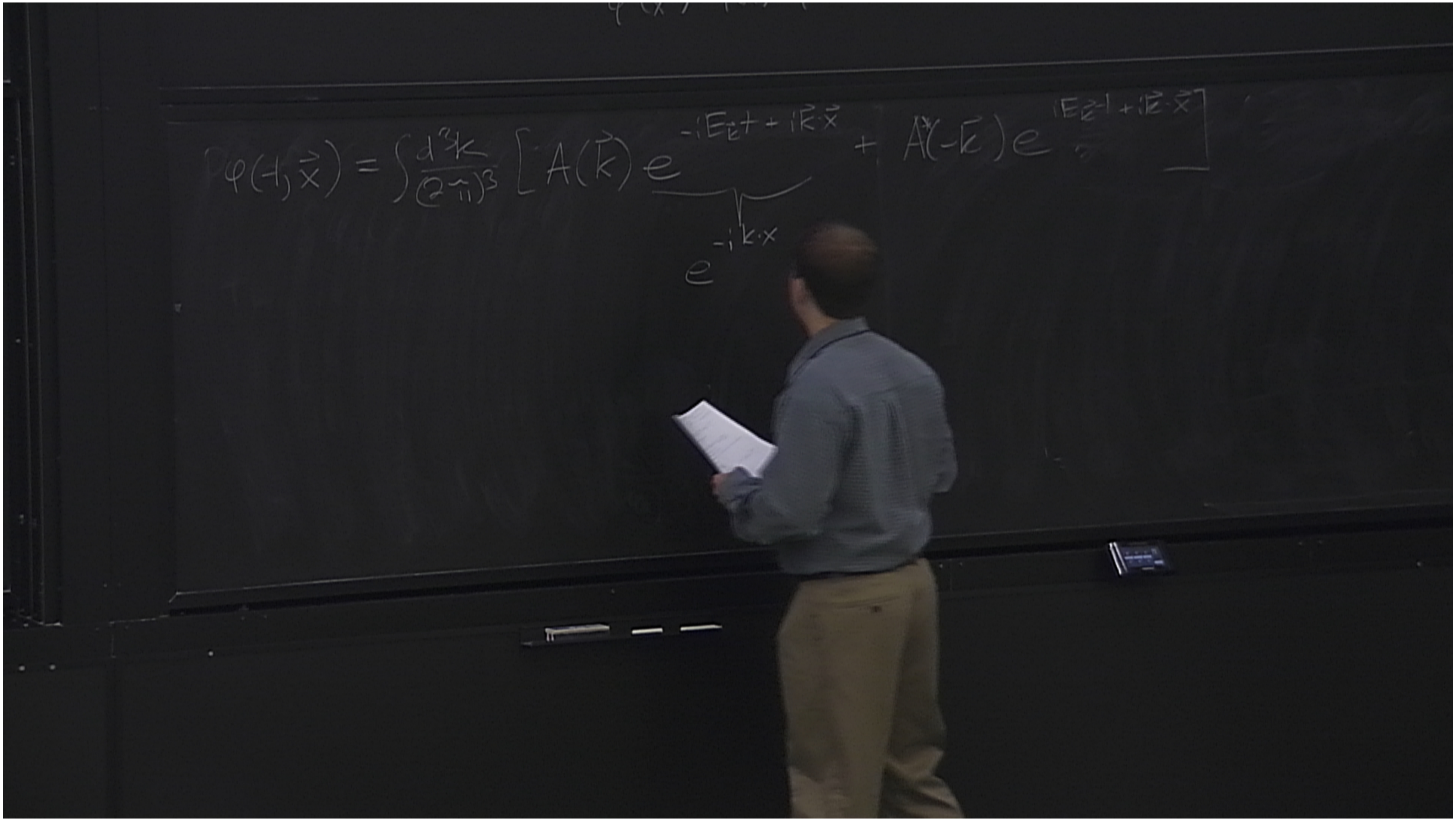
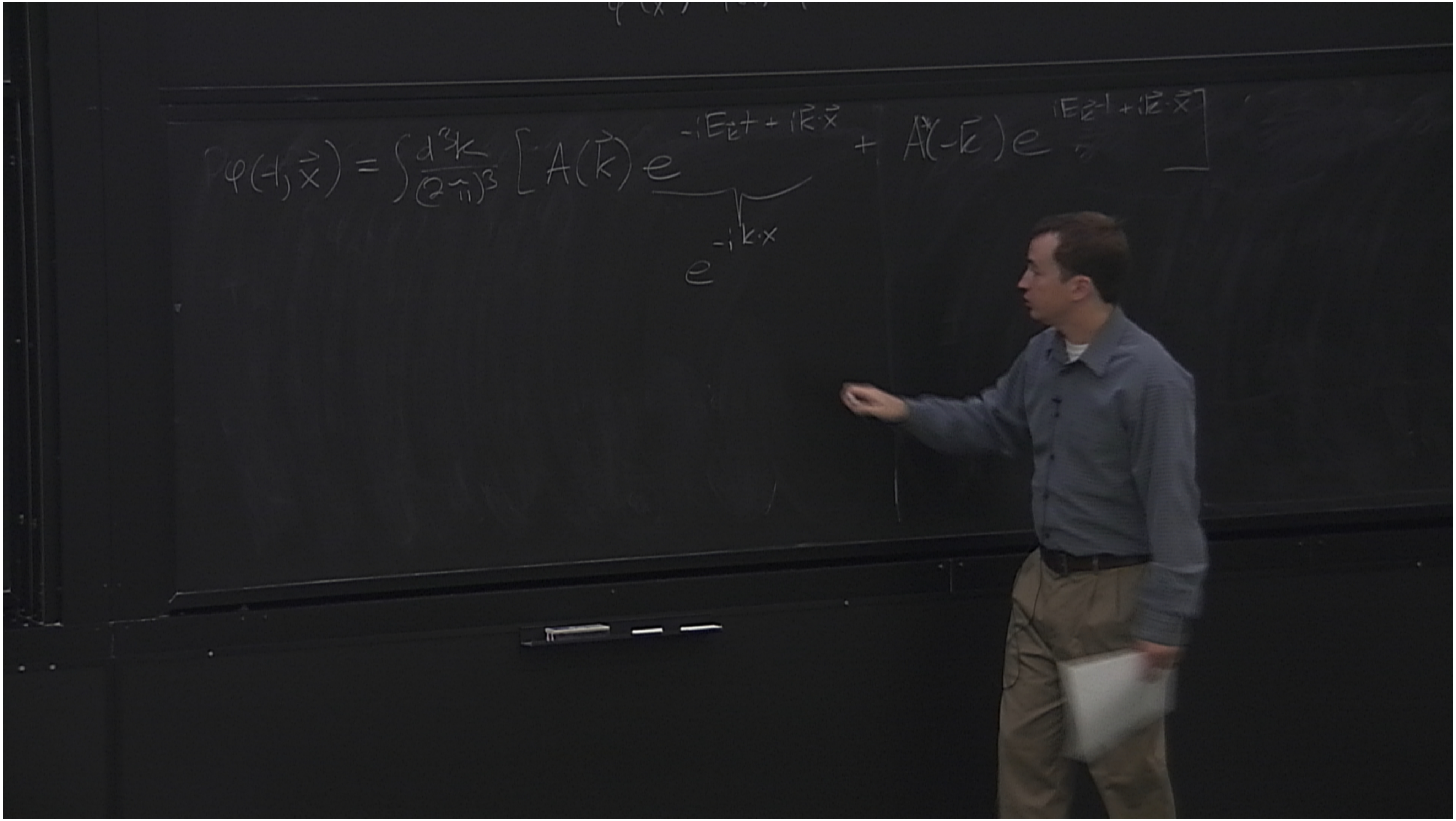
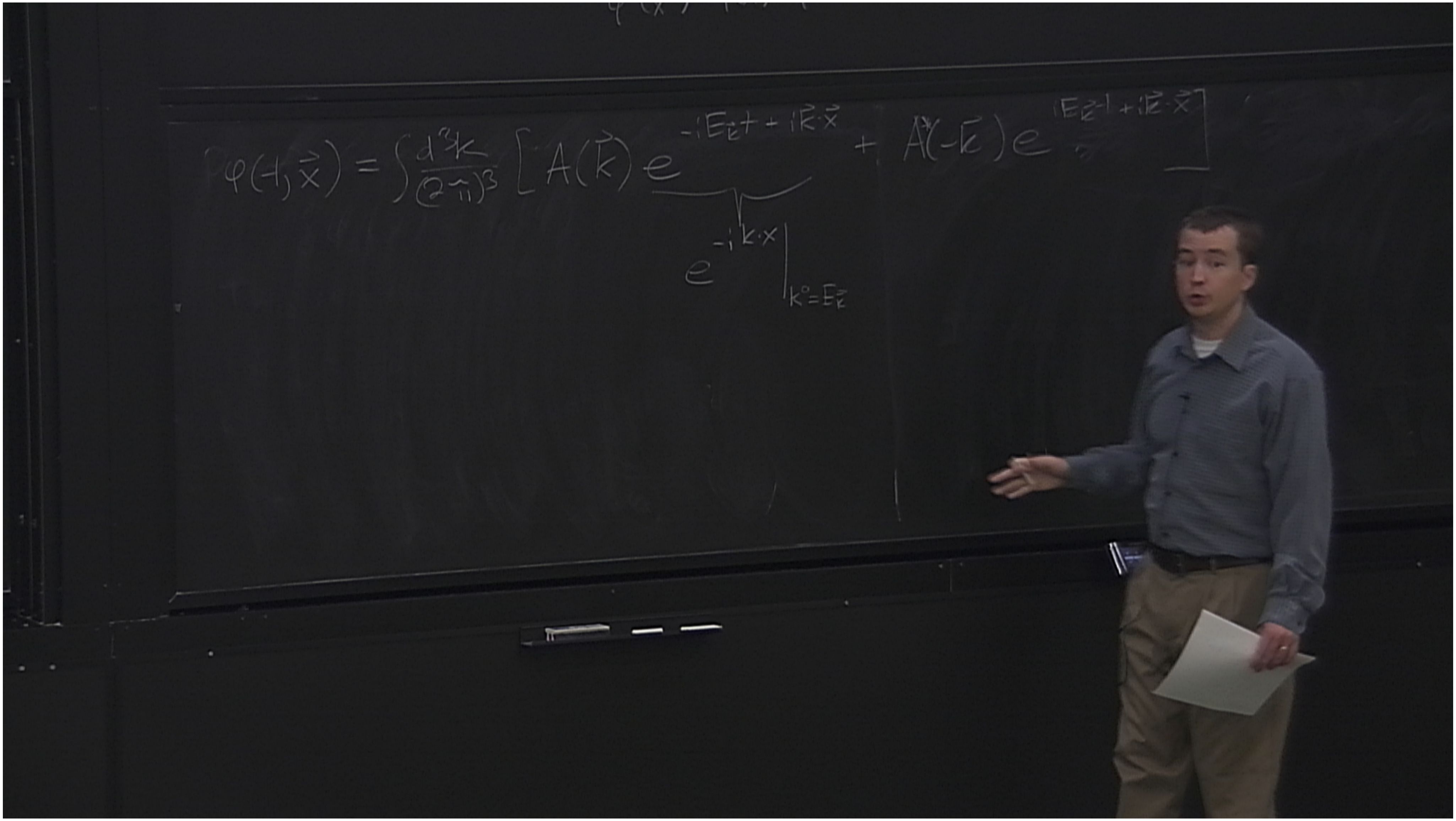
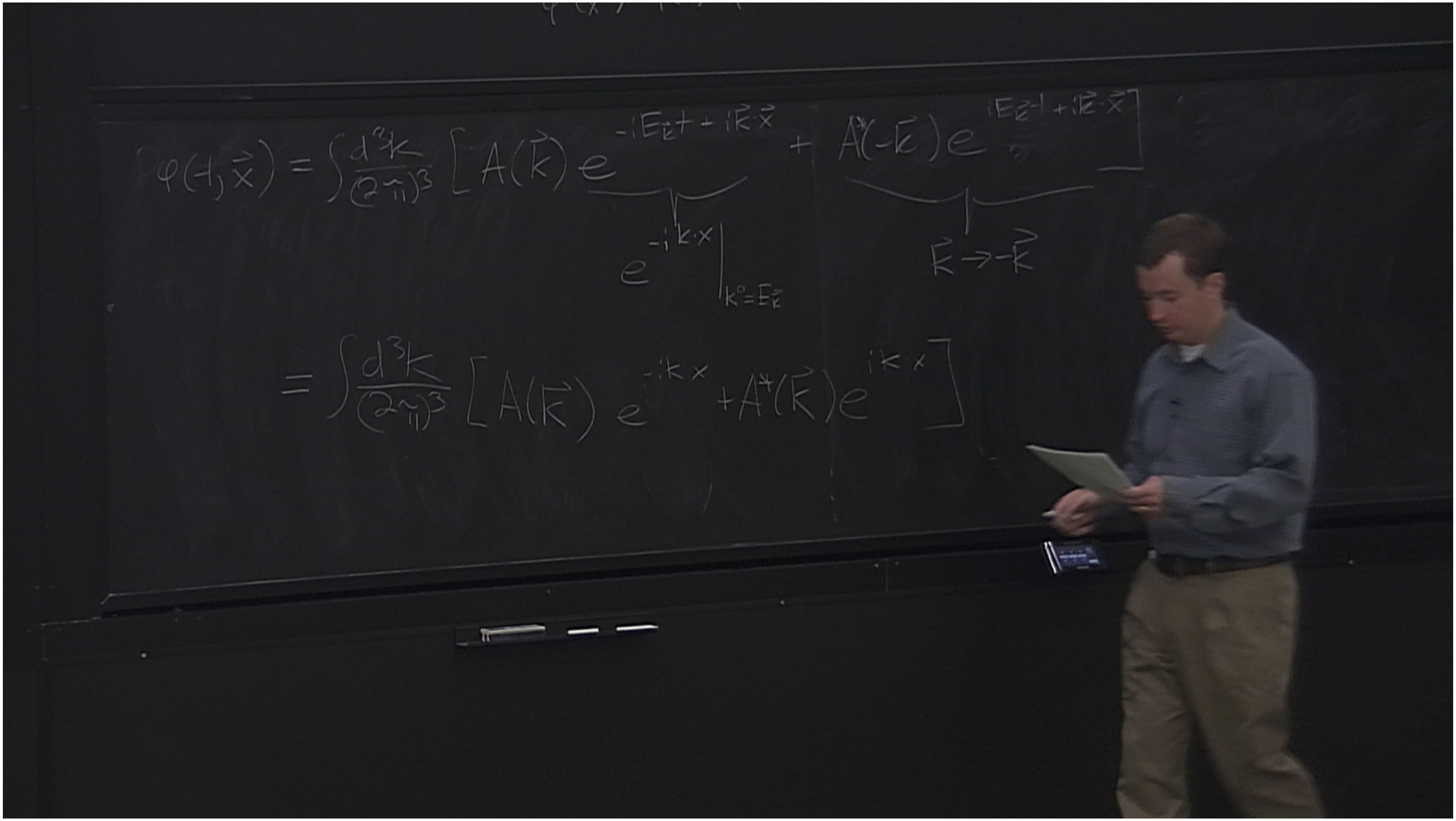
$\tilde{\varphi}^*(t,-\vec{k}) = \tilde{\varphi}(t,\vec{k}) \Rightarrow$

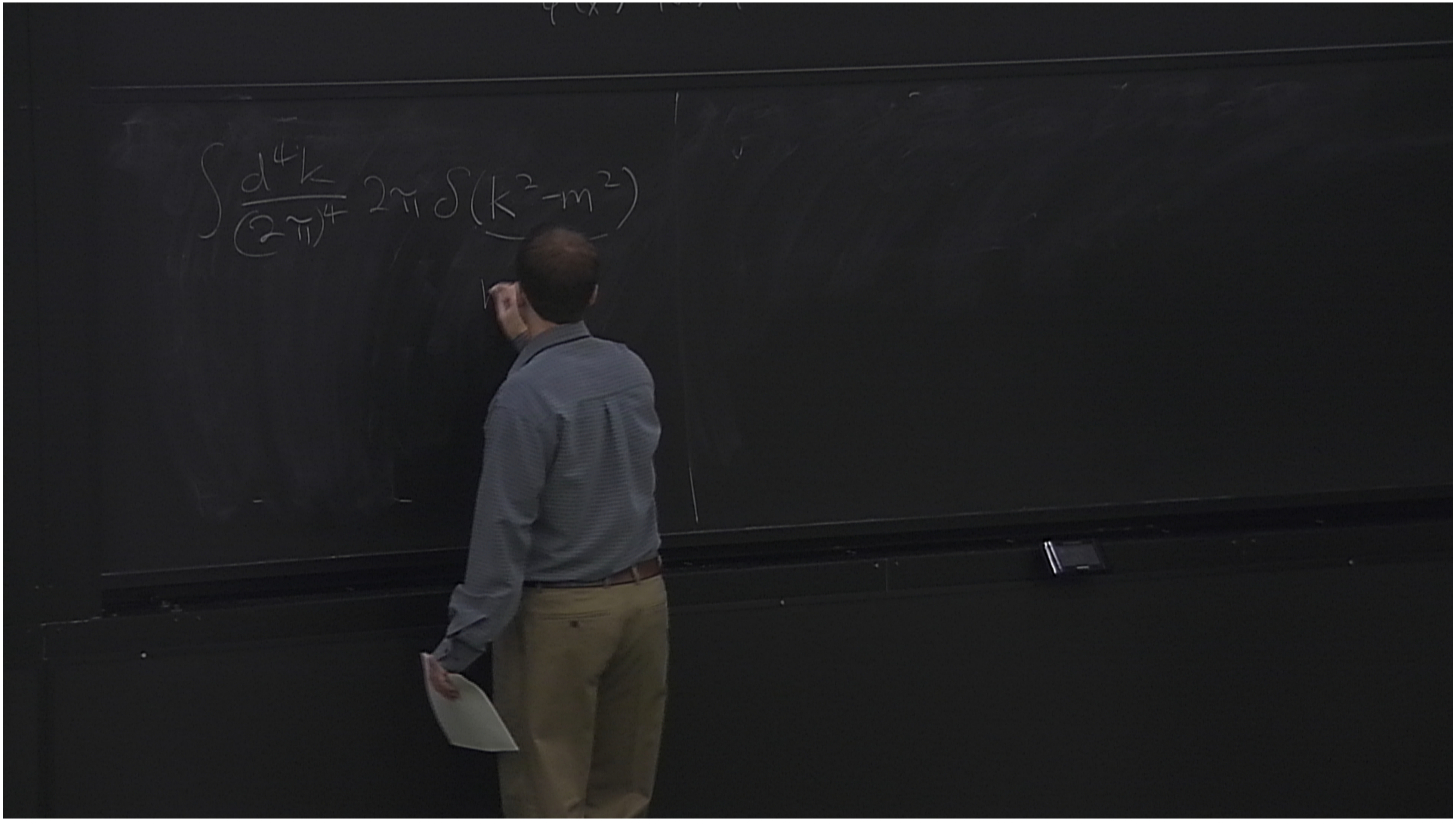

φ(t, x⃗) = ∫ d³k/(2π)³ [A(k⃗) e^(−iE_k t + i k⃗·x⃗) + A*(−k⃗) e^(iE_k t + i k⃗·x⃗)]

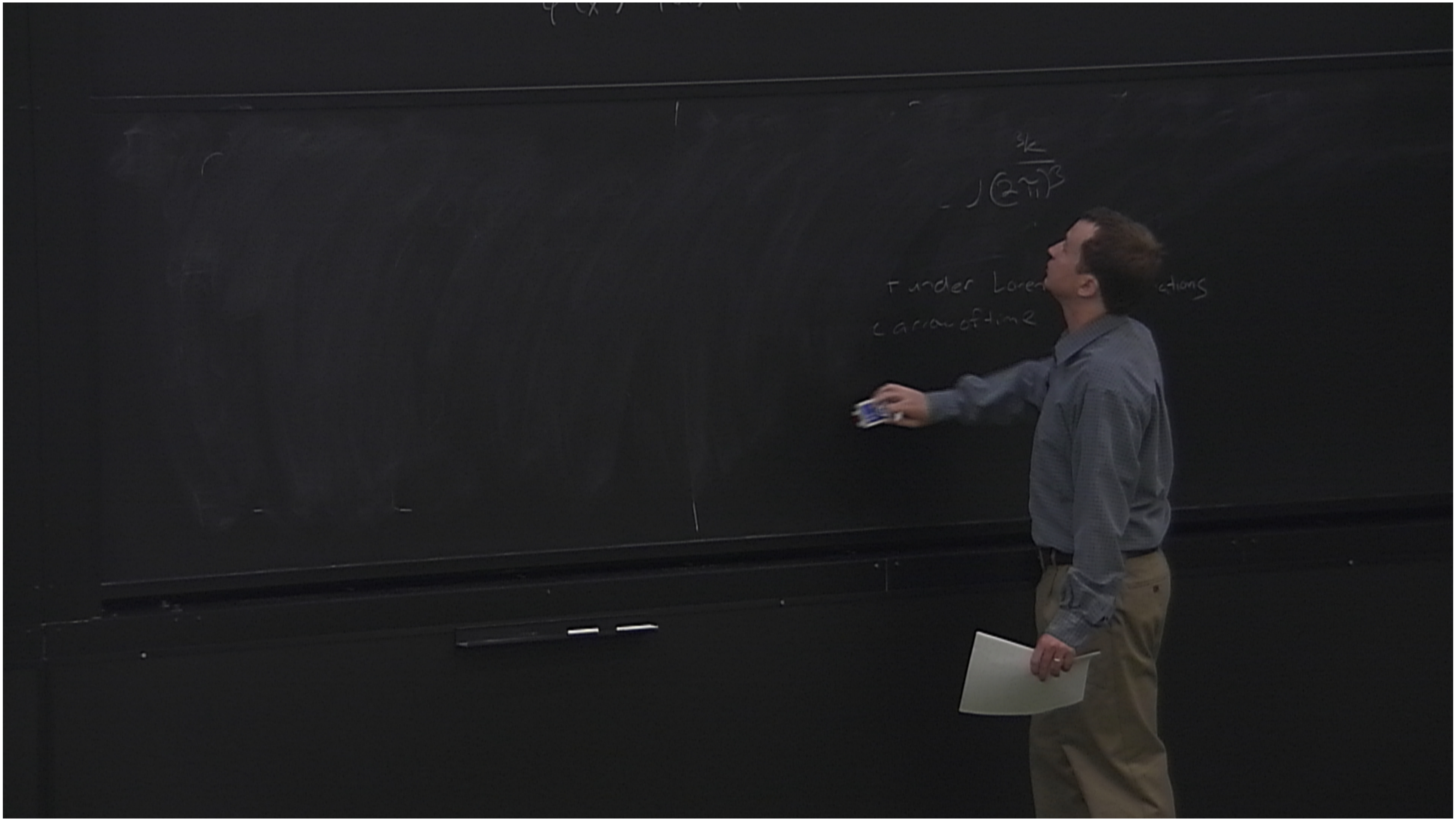

φ(t, x⃗) = ∫ d³k/(2π)³ [A(k⃗) e^{-iE_k t + i k⃗·x⃗} + A*(-k⃗) e^{iE_k t + i k⃗·x⃗}]
e^{-ik·x}

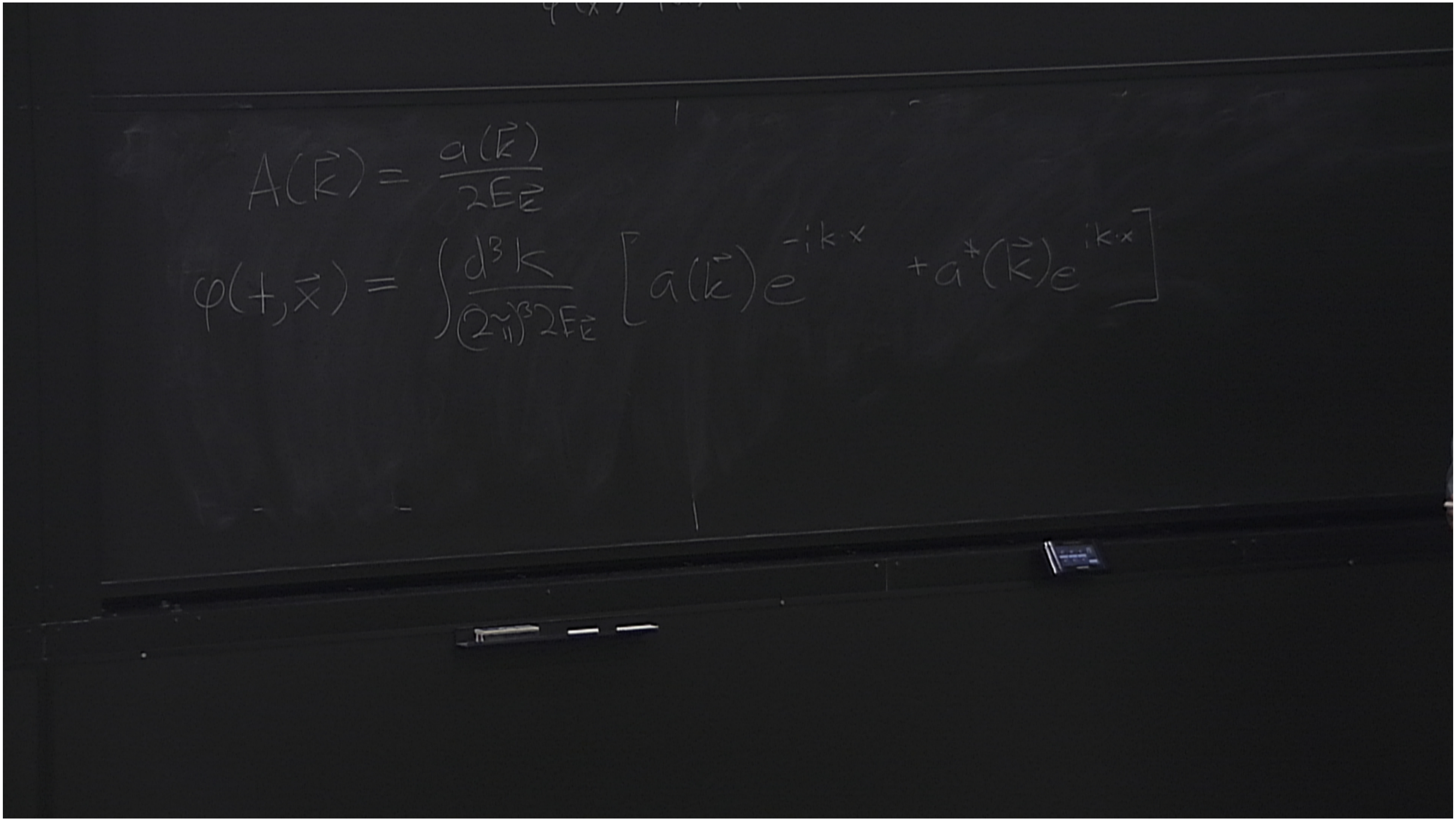

φ(t, x⃗) = ∫ d³k/(2π)³ [A(k⃗) e^{-iE_k t + i k⃗·x⃗} + A*(-k⃗) e^{iE_k t + i k⃗·x⃗}]
e^{-ik·x}


φ(t, x⃗) = ∫ d³k/(2π)³ [A(k⃗) e^(−iE_k t + ik⃗·x⃗) + A*(−k⃗) e^(iE_k t + ik⃗·x⃗)]
e^(−ik·x) |_(k⁰=E_k)


$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} + A^*(-\vec{k}) e^{iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} \right]$
$e^{-ik\cdot x}\Big|_{k^0 = E_{\vec{k}}}$
$\vec{k} \to -\vec{k}$
$= \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-ik\cdot x} + A^*(\vec{k}) e^{ik\cdot x} \right]$


$\int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^2 - m^2)$




$A(\vec{k}) = \frac{a(\vec{k})}{2E_{\vec{k}}}$
$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a(\vec{k}) e^{-ik\cdot x} + a^*(\vec{k}) e^{ik\cdot x} \right]$