Title: Far-from-equilibrium dynamics of a strongly coupled non-Abelian plasma with non-zero charge density or external magnetic field

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Abstract: Using holography, we study the evolution of a spatially homogeneous, far from equilibrium, strongly coupled N=4 supersymmetric Yang-Mills plasma with a non-zero charge density or a background magnetic field. This gauge theory problem corresponds, in the dual gravity description, to an initial value problem in Einstein-Maxwell theory with homogeneous but anisotropic initial conditions. We explore the dependence of the equilibration process on different aspects of the initial departure from equilibrium and, while controlling for these dependencies, examine how the equilibration dynamics are affected by the presence of a non-vanishing charge density or an external magnetic field. The equilibration dynamics are remarkably insensitive to the addition of even large chemical potentials or magnetic fields, the equilibration time is set primarily by the form of the initial departure from equilibrium. For initial deviations from equilibrium which are well localized in scale, we formulate a simple model for equilibration times which agrees quite well with our results.


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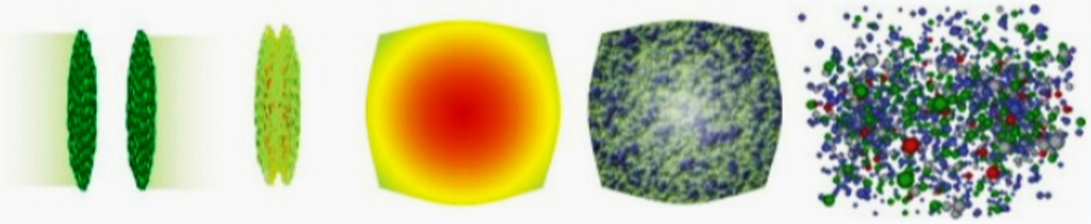
Far-from-equilibrium dynamics of a strongly coupled non-abelian plasma with non-zero charge density or external magnetic field
with Laurence Yaffe
JHEP 07 (2015) 012, arXiv:1503.07148

Outline

- Motivation
- Proxy Field Theory
  - N=4 SYM + EM
- Holographic Implementation
  - Solving Maxwell-Einstein
- Results
  - Neutral Plasma
  - Charged Plasma
  - Magnetized Plasma
- Discussion
  - Simple Model

Motivation

- Want to understand strongly coupled and far-from-equilibrium QCD plasma.



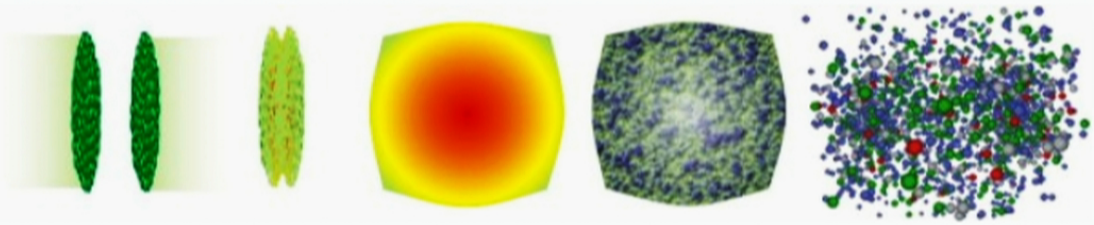
- Gauge/gravity duality (Holography) examines certain strongly coupled gauge theories by considering gravity.

- Numerical techniques allow us to study far-from-equilibrium dynamics.

The problem of a strongly coupled, far-from-equilibrium plasma turns into a calculation of numerical General Relativity.

Motivation

- Heavy Ion Collisions have a small but finite chemical potential and associated baryon density (charge density)
- Large but transient EM fields may play a significant role in Heavy Ion Collisions, despite small coupling



Proxy Field Theory

Our Field Theory

Simplest Gauge/Gravity example is $\mathcal{N} = 4$ SYM $SU(N_c)$ qualitatively good toy theory for QCD

- on 4d minkowski space
- translational inv., x-y rotation inv.
- couple conserved current of $U(1)$ subgroup of $SU(4)_R$ global symmetry to a background Abelian gauge field.

Proxy Field Theory

Trace Anomaly

$$A^{\rm ext}_\alpha(x) \equiv \mu\,\delta^0_\alpha + \tfrac{1}{2}\mathcal{B}\,(x^1\delta^2_\alpha - x^2\delta^1_\alpha)$$

- chemical potential μ (conjugate to charge density in equilibrium) adds a scale, but does not effect the tracelessness of $T^{\alpha\beta}$

- In contrast, the magnetic field effects the microscopic dynamics - generates a non-zero trace anomaly:

$$T^\alpha{}_\alpha = -\tfrac{1}{4}\,\kappa\,\left(F^{\rm ext}_{\mu\nu}\right)^2 = -\tfrac{1}{2}\,\kappa\,\mathcal{B}^2$$

$$\kappa \equiv (N_c^2-1)/(2\pi^2)$$

- implies logarithmic dependence on renormalization point

Proxy Field Theory

Enlarged Theory

$$A_\alpha^{\text{ext}}(x) \equiv \tfrac{1}{2}\mathcal{B}\,(x^1\delta_\alpha^2 - x^2\delta_\alpha^1)$$

For the magnetic case, it is convenient to interpret the trace anomaly by considering the enlarged theory:

$\mathcal{N}=4$ SYM $SU(N_c)$ + EM, with total Action

$$S_{\text{SYM+EM}} = S_{\text{SYM, min. coupled}} + S_{\text{EM}}$$

$$S_{\text{EM}} \equiv -\int d^4x\, \tfrac{1}{4e^2}\, F_{\mu\nu}^2$$

- where the electromagnetic coupling e is arbitrarily weak ➔ EM fields are classical background fields
- SYM fields charged under this $U(1)$ cause a running coupling (four fermions and three complex scalars)

Proxy Field Theory

Scale Dependence

$$\mu\frac{d}{d\mu}e^{-2} \equiv \beta_{1/e^2}(e^{-2}) = -b_0 + O(e^2)$$

$$b_0 \equiv \kappa\left[\tfrac{1}{6}\sum_{\alpha}(q_{\rm f}^{\alpha})^2 + \tfrac{1}{12}\sum_{a}(q_{\rm s}^{a})^2\right]$$

Just like charge renormalization in weakly coupled QED

How much of the stress energy comes from the background gauge field?

Total stress energy well-defined, partitioning is ambiguous

$$T^{\alpha\beta}_{\rm tot} \equiv T^{\alpha\beta}_{\rm EM}(\mu) + \Delta T^{\alpha\beta}_{\rm SYM}(\mu) \qquad T^{\alpha\beta}_{\rm EM}(\mu) \equiv \frac{1}{e^2(\mu)}\left[F^{\alpha\nu}F^{\beta}{}_{\nu} - \tfrac{1}{4}\eta^{\alpha\beta}F^{\mu\nu}F_{\mu\nu}\right]$$

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scale dependence between each piece must cancel...

$$\mu\frac{d}{d\mu}\Delta T^{\alpha\beta}_{\rm SYM}(\mu) = -\mu\frac{d}{d\mu}T^{\alpha\beta}_{\rm EM}(\mu) = b_0\left[F^{\alpha\nu}F^{\beta}{}_{\nu} - \tfrac{1}{4}\eta^{\alpha\beta}F^{\mu\nu}F_{\mu\nu}\right]$$

$$\varepsilon(\mu) \equiv \Delta T^{00}_{\rm SYM}(\mu)/\kappa$$

specialized to zero-temperature, ground state energy density:

$$\varepsilon(\mu) = c_0\,\mathcal{B}^2 - \tfrac{1}{4}\mathcal{B}^2\ln(|\mathcal{B}|/\mu^2)$$

$$\varepsilon_L \equiv \varepsilon(1/L)\,, \qquad \varepsilon_{\mathcal{B}} \equiv \varepsilon(|\mathcal{B}|^{1/2})$$

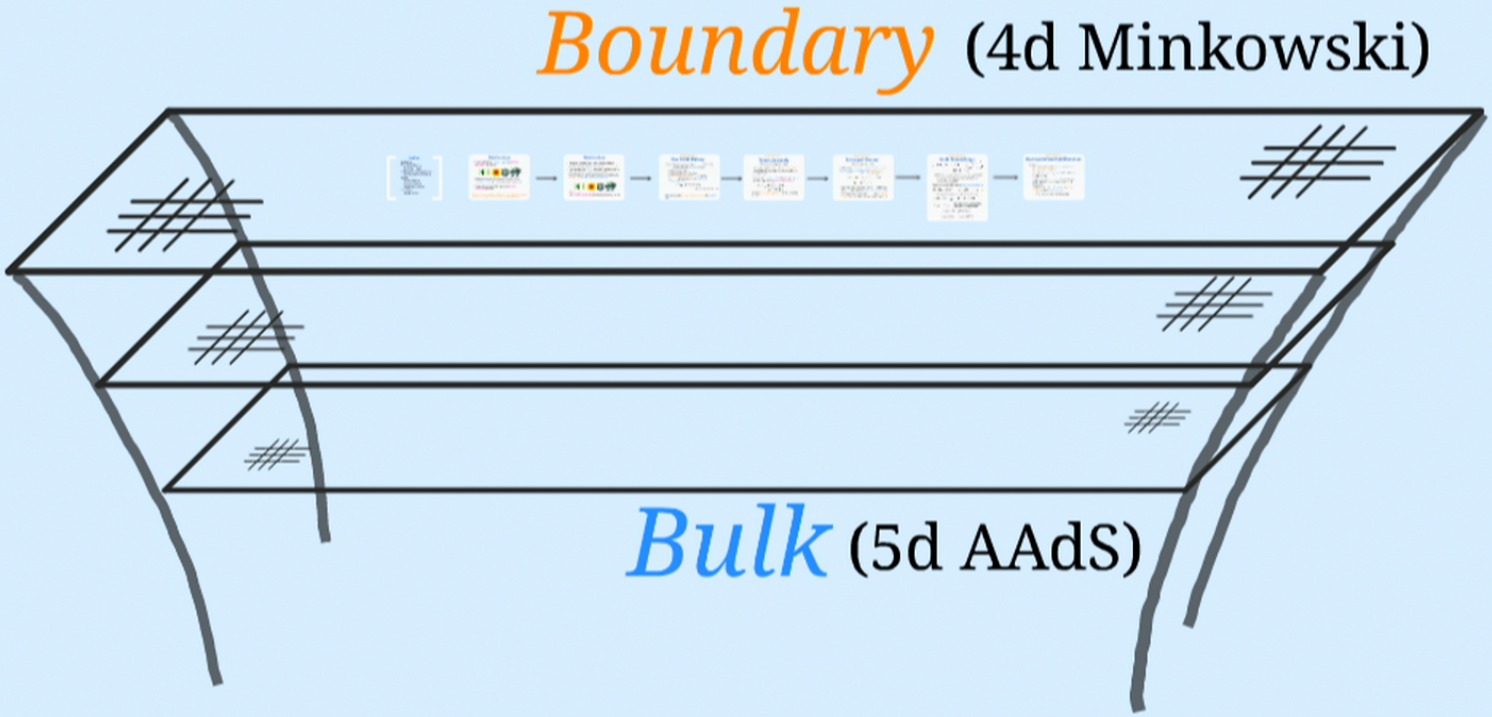
Proxy Field Theory

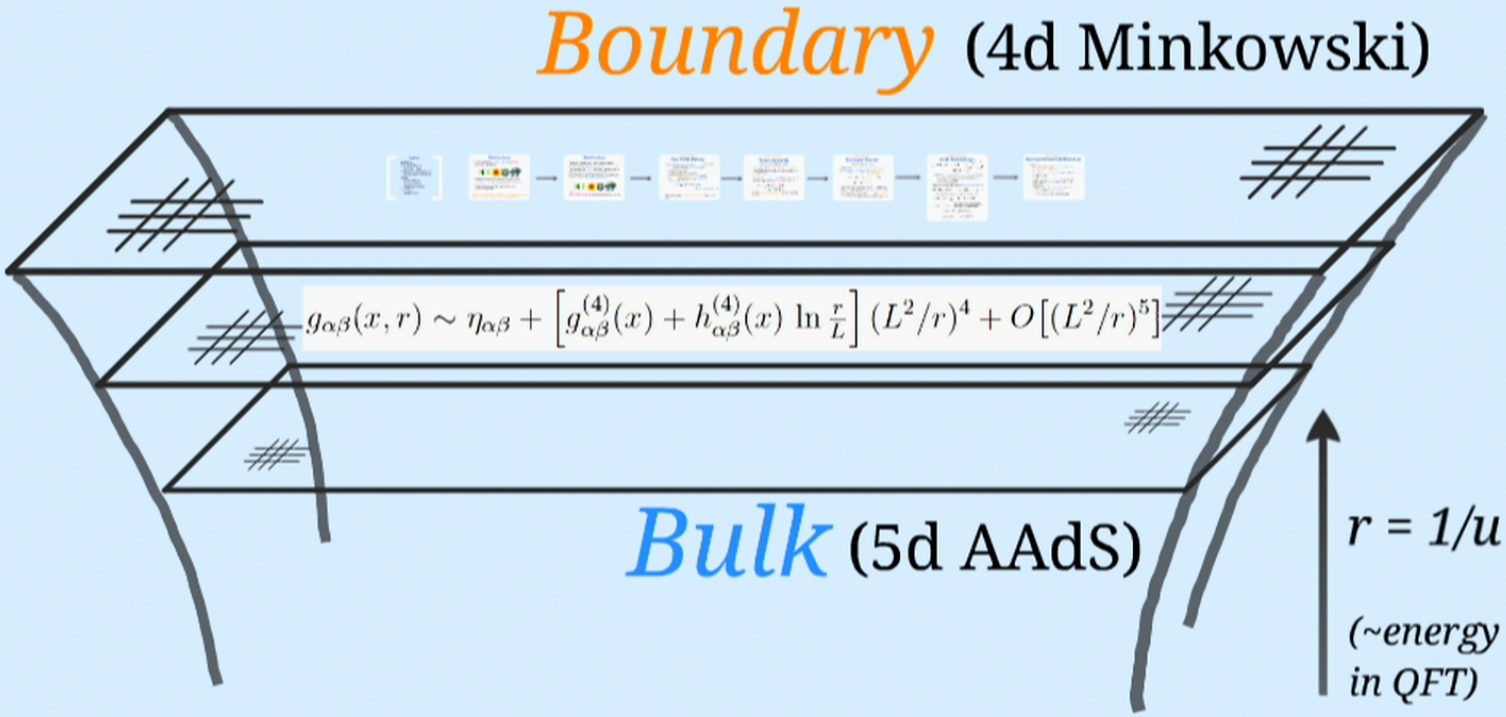
Isotropization/Equilibration

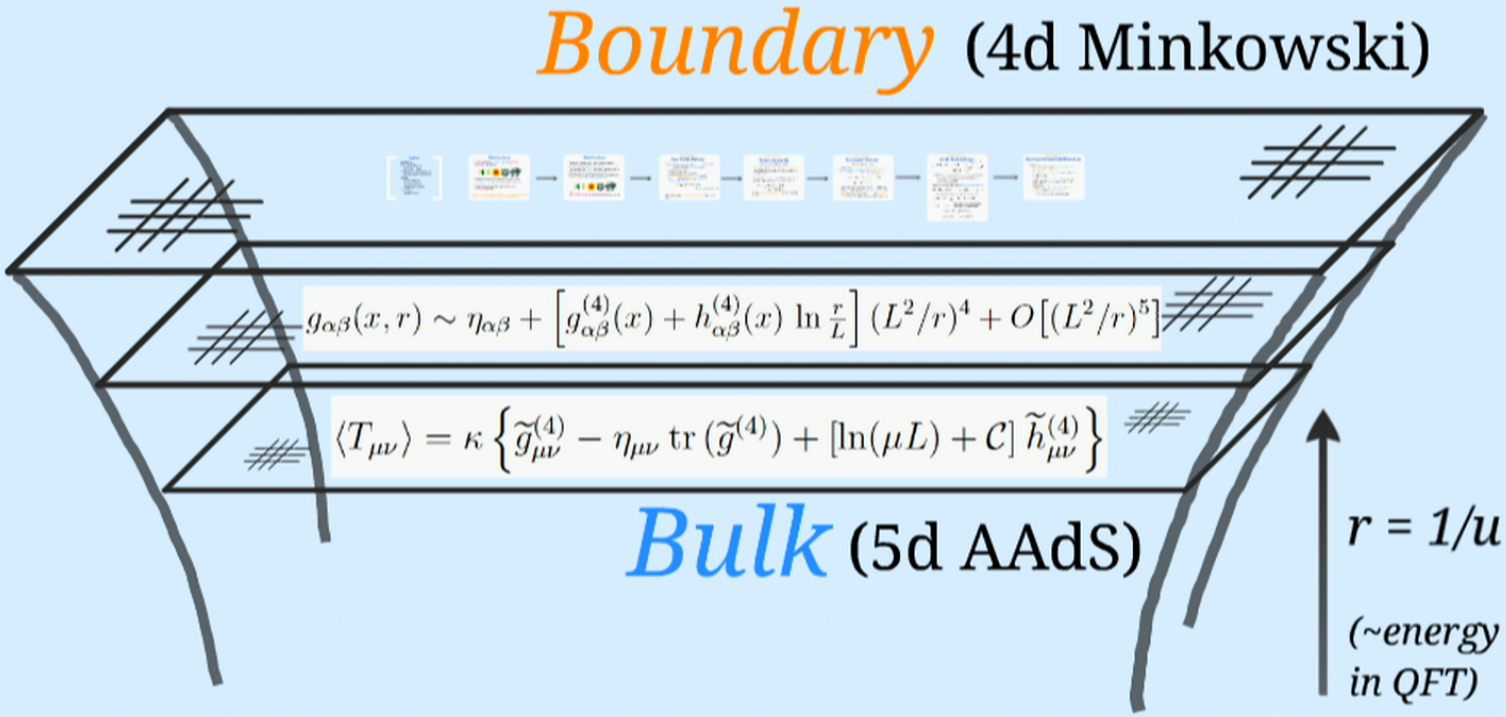
We want:

- to set initial state in some far-from-equilibrium and anisotropic configuration
- to watch the stress-energy tensor evolve in time as the system settles

Boundary (4d Minkowski)


Boundary (4d Minkowski)
Bulk (5d AAdS)


Boundary (4d Minkowski)
$g_{\alpha\beta}(x,r) \sim \eta_{\alpha\beta} + \left[g^{(4)}_{\alpha\beta}(x) + h^{(4)}_{\alpha\beta}(x)\,\ln\frac{r}{L}\right](L^2/r)^4 + O\left[(L^2/r)^5\right]$
Bulk (5d AAdS)
r = 1/u
(~energy in QFT)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$\langle T_{\mu\nu}\rangle = \kappa\left\{\widetilde{g}^{(4)}_{\mu\nu} - \eta_{\mu\nu}\,\mathrm{tr}\,(\widetilde{g}^{(4)}) + [\ln(\mu L) + \mathcal{C}]\,\widetilde{h}^{(4)}_{\mu\nu}\right\}$
Bulk (5d AAdS)
r = 1/u
(~energy in QFT)

Holographic Implementation

Bulk Action

Consistent truncation (top-down construction) for our boundary theory, $S = S_{\rm SYM} + \int d^4x\, j^\alpha(x) A^{\rm ext}_\alpha(x)$ gives Einstein + Maxwell:

$$S_5 \equiv \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-G}\,\left(R - 2\Lambda - L^2\,F_{MN}F^{MN}\right)$$

$$G_5 \equiv \tfrac{\pi}{2}L^3/N_c^2 \qquad \Lambda \equiv -6/L^2$$

- 5d Chern-Simons term could be present, but not relevant if only turning on charge density OR magnetic field

Methods: Setup

Coordinate Choice

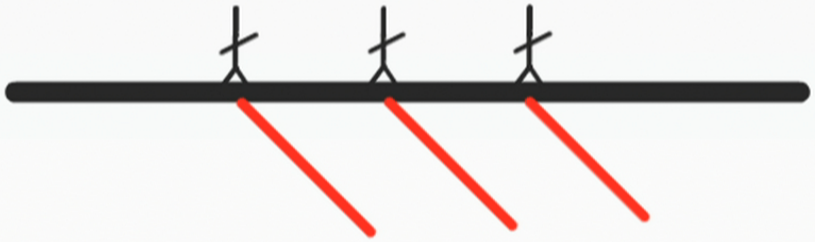
Following *Chesler, Yaffe arXiv:1309.1439*, we chose generalized Eddington Finklestein coordinates:

$$\{v, x, y, z, r\}$$

Boundary coordinates

$$ds^2 = \frac{r^2}{L^2} g_{\mu\nu}(x, r) dx^\mu dx^\nu - 2\omega_\mu(x) dx^\mu dr$$

- coordinates remain regular across future horizon
- infalling radial null geodesics (affine parameter, r)



Ansatz/Symmetries

Since we are interested in homogenous isotropization, we demand the following:

- trans. invariance in x, y, z
- rot. invariance in x-y plane

Bulk Metric

$$ds^2 = -A(v,r)dv^2 + 2drdv + \Sigma(v,r)^2\left(e^{B(v,r)}(dx^2+dy^2) + e^{-2B(v,r)}dz^2\right)$$

- anisotropy isolated in B
- at the boundary, v coincides with boundary time t.
- radial diffeomorphism freedom $r \to r + \lambda(v)$

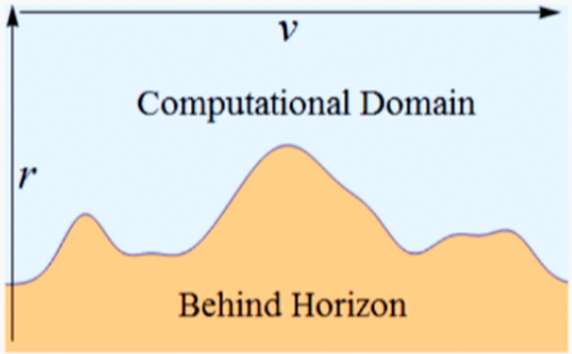
Methods: Solving Radially

Horizon Fixing

Our residual radial gauge, $\lambda(v)$, must be chosen.

$$r \to r + \lambda(v)$$

We use this freedom to "fix" the horizon.



Holographic Implementation

Field Equations

Maxwell's EQs give:

$$F_{xy}(x,r) \equiv \mathcal{B} = const$$

$$F_{rv}(x,r) \equiv \mathcal{E}(v,r) = \rho\, L\, \Sigma^{-3}(v,r)$$

Einstein EQs give:

1) $$\Sigma'' + \tfrac{1}{2}(B')^2\,\Sigma = 0\,,$$

$$A'' - 6(\Sigma'/\Sigma^2)\,d_+\Sigma + \tfrac{3}{2}\,B' d_+ B = +\tfrac{5}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4} + \tfrac{7}{3}\,\mathcal{E}^2 L^2 - 2/L^2$$

$$(d_+B)' + \tfrac{3}{2}(\Sigma'/\Sigma)\,d_+B + \tfrac{3}{2}B'\,(d_+\Sigma)/\Sigma = -\tfrac{2}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4}\,,$$

$$(d_+\Sigma)'/\Sigma + 2(\Sigma'/\Sigma^2)\,d_+\Sigma = -\tfrac{1}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4} - \tfrac{1}{3}\mathcal{E}^2 L^2 + 2/L^2$$

$$d_+(d_+\Sigma) - A'\,(d_+\Sigma) + \tfrac{1}{2}\Sigma\,(d_+B)^2 = 0\,,$$

$$d_+ \equiv \partial_v + A(v,r)\,\partial_r$$

At some initial time, v_0, Given: ε, λ, $B(v_0,r)$ and (ρ or $\mathcal{B}$)
System reduces to nested set of radial ODEs


Methods: Solving Radially
Asymptotic Solutions
Expand each function in a reference expansion, e.g.
• Solve order by order in λ and $\lambda_{\mathrm{eff}}(\lambda)$;
• up is two unknown coefficients in each function - along with residual gauge parameter $\lambda(\nu)$
• enforce Minkowski boundary (the leading term):
• subleading terms cannot be found from asymptotics alone - it's the subleading terms give the correct energy values!
Methods: Solving Radially
Radial Solver Choice
Finite Difference Methods:
• subdivide interval, replace derivatives approx. using local interpolation
• error - for some p method gives $(1/N)^p$ where N is the number of points
• definitely handling singularity in EOMs
Spectral Methods:
• spectral grid, or collocation points, optimized
• derivatives approx. using full domain - global
• error - exponential decrease in number of points
• easy way to handle singularity in EOMs
Methods: Solving Radially
Spectral Methods
• Instead we can expand in terms of Gaussian functions
• Derivatives become matrices
• coefficients of the above simply evaluating R on x_i
Methods: Solving Radially
Spectral Methods (cont)
Tricky part is:
• how to choose collocation points?
• which cardinal functions to use?
• how to handle divergences in differential equations?
For a finite, non-periodic domain, best choice is a Gauss-Lobatto grid, and Chebyshev Polys.
As for the divergences:
• subtract off asymptotic piece or function
• spectral methods removes a row and replaces with boundary condition - remove divergent row, replace with boundary condition for subtracted function

Holographic Implementation

Time Stepping

Once we have $d_+ B$ and B' on a given timeslice v_0,
(as well as our other metric functions)

$$\partial_v B(v_0, r) = d_+ B(v_0, r) - A(v_0, r)\, \partial_r B(v_0, r)$$

The stationarity condition on the horizon gives us $\partial_v \lambda$

Integrate forward, $v_0 \to v_0 + \Delta v$, with a Runge-Kutta 4th order method to find:

$$B(v_0 + \Delta v), \quad \lambda(v_0 + \Delta v)$$

- Restart the entire radial process, now on the new timeslice
- watch system equilibrate

Equilibrium Configurations

Neutral Plasma

$(\rho = 0, \mathcal{B} = 0)$

Equilibrates to Schwarzschild Black Brane:

$$ds^2 = -U(\tilde{r})\, dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - \frac{m\, L^2}{\tilde{r}^2}$$

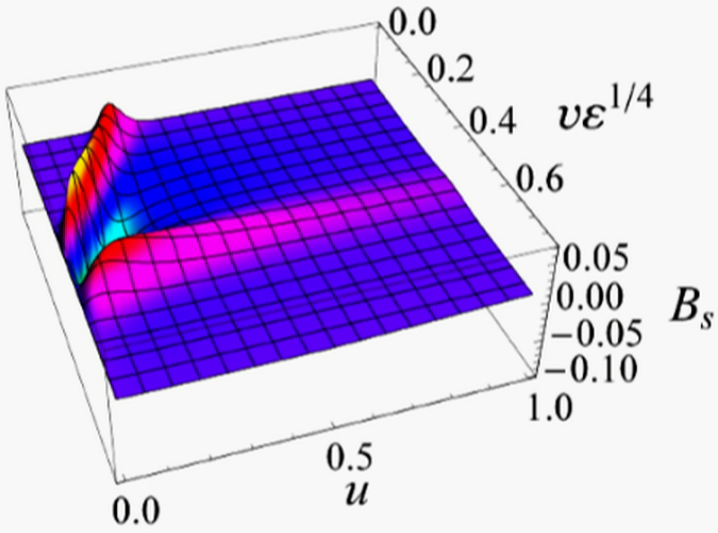
$$\varepsilon = \frac{3}{4} m L^{-4} = \frac{3}{4}(\pi T)^4$$

Results: Neutral Plasma

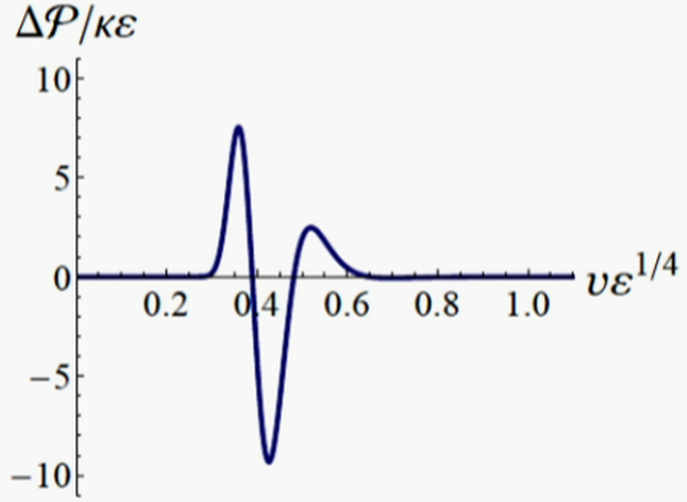
General Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

$$\Delta\mathcal{P} \equiv \tfrac{1}{2}(T^{11} + T^{22}) - T^{33}$$



B_s



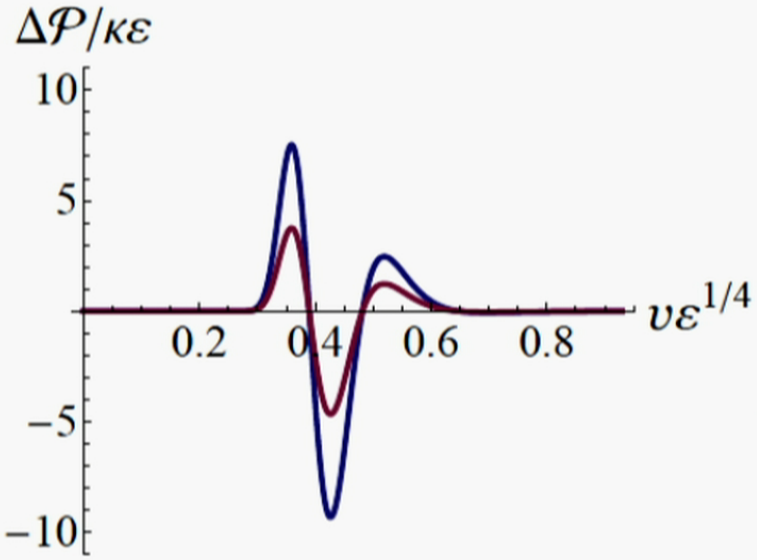
Results: Neutral Plasma

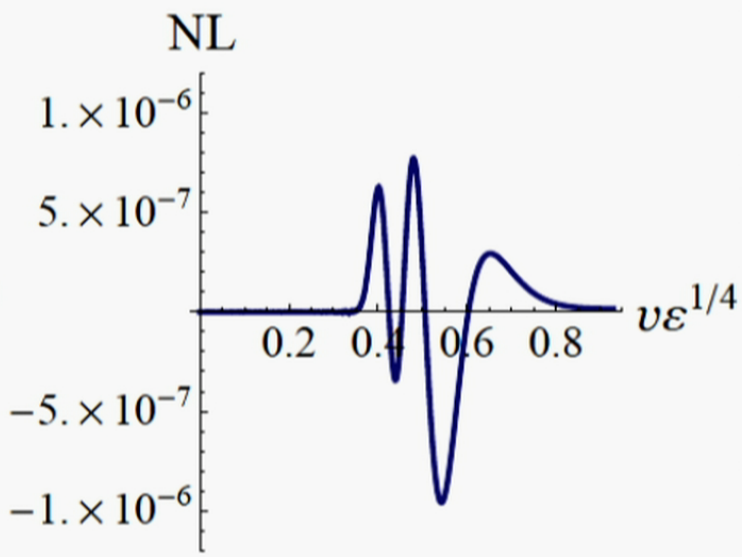
Sensitivity to Gaussian

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

- Amplitude and r_0 interesting

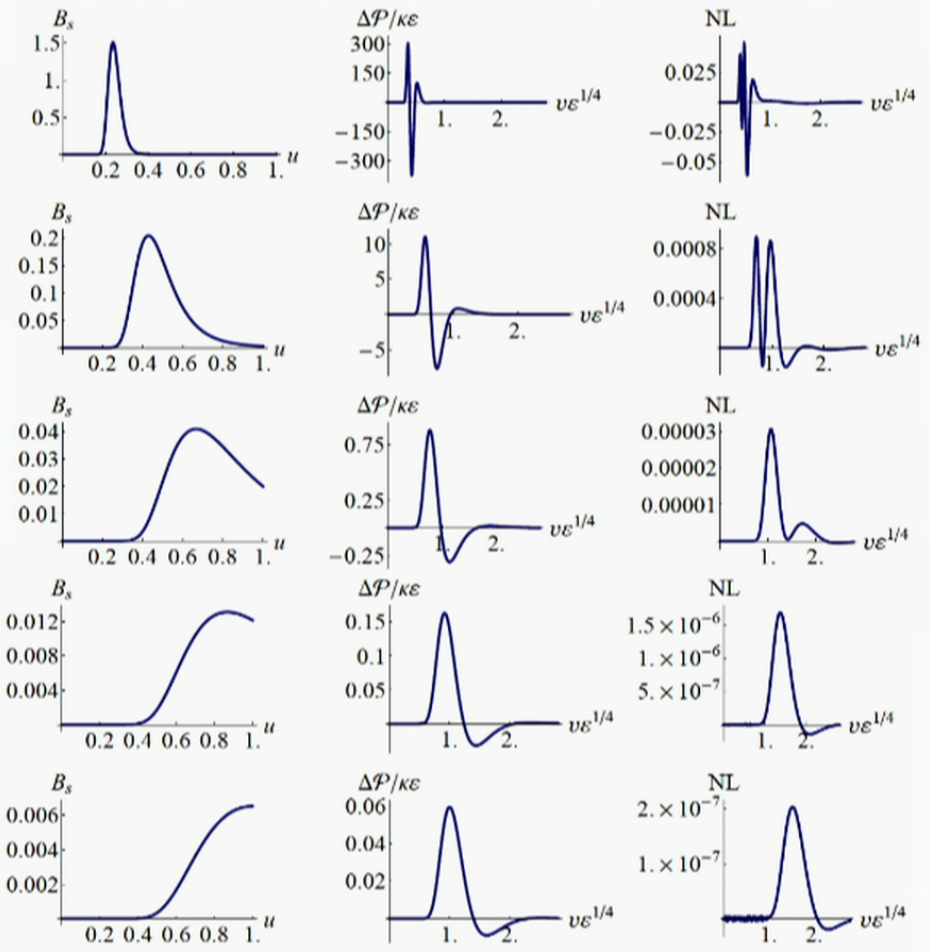
Amplitude test. Double initial Gaussian's Amp, check results.





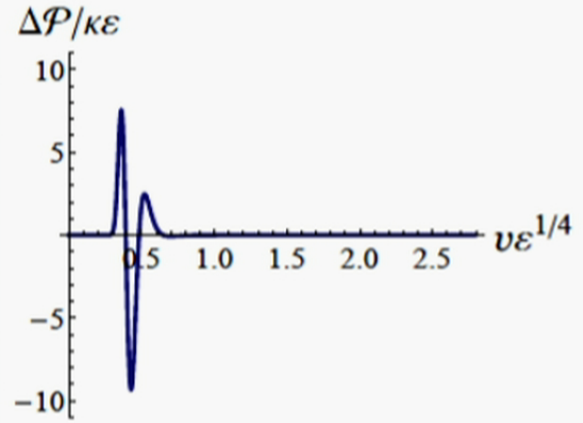
Results: Neutral Plasma

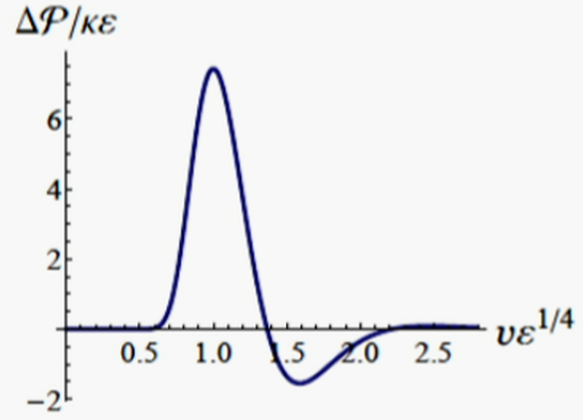
Depth & Non-linearities



Results: Neutral Plasma

Matched Bndry Anisotropy





Charged Plasma

$(\rho \neq 0, \mathcal{B} = 0)$

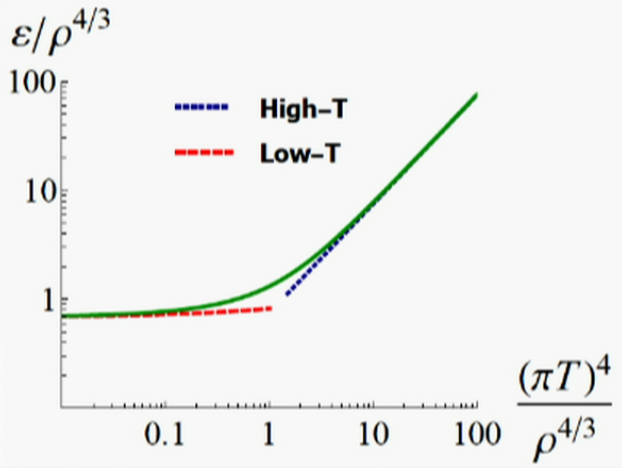
Equilibrates to Reissner-Nordstrom Black Brane

Myers et al. hep-th/9902170

$$ds^2 = -U(\tilde{r})\, dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - m\,\frac{L^2}{\tilde{r}^2} + \tfrac{1}{3}\,(\rho L^3)^2\,\frac{L^4}{\tilde{r}^4}$$

$$(\rho_{\max} L^3)^4 = \tfrac{4}{3}\, m^3 \qquad \mu = \tfrac{1}{2}\,\rho\,(L^2/\tilde{r}_{\rm h})^2$$

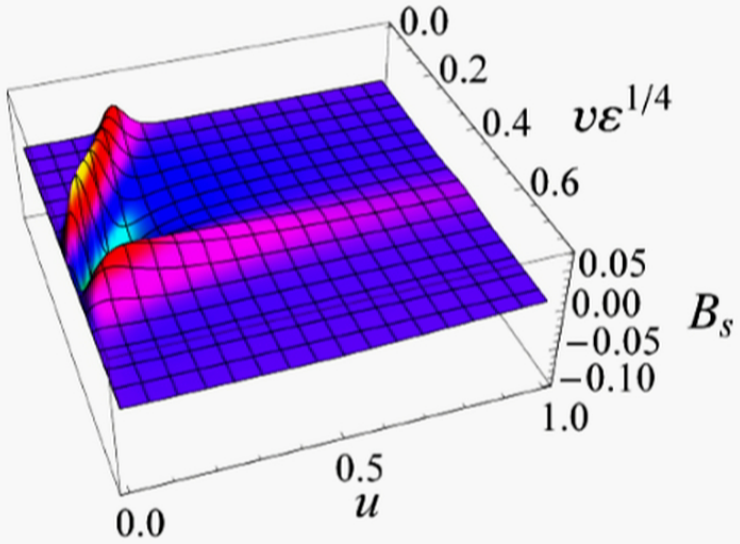


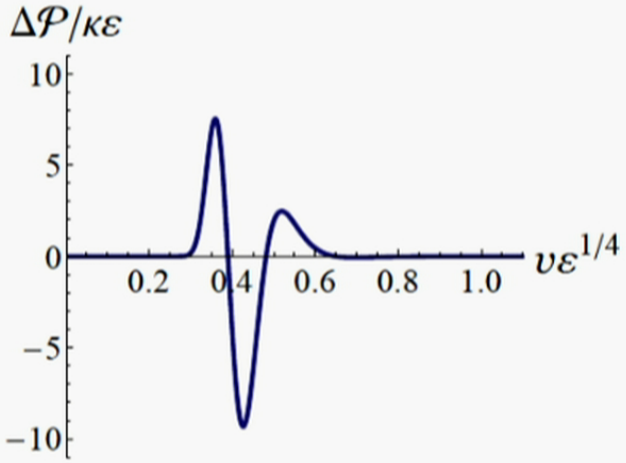
Results: Charged Plasma

Generic Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

$$\Delta\mathcal{P} \equiv \tfrac{1}{2}(T^{11} + T^{22}) - T^{33}$$



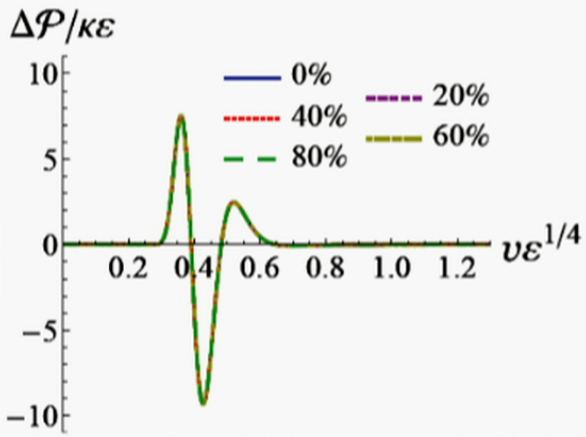


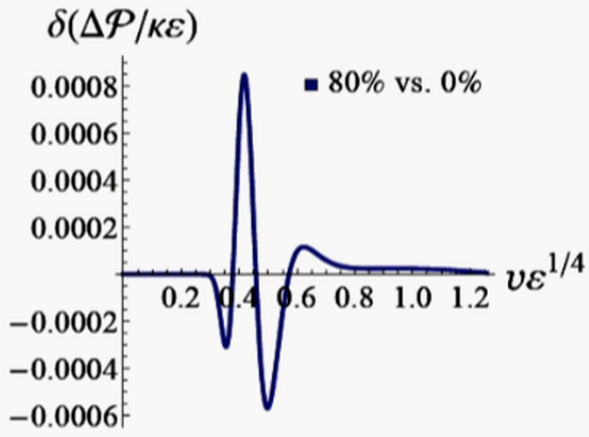
Results: Charged Plasma

Anisotropy

ε held fixed

$(\rho_{\max} L^3)^4 = \frac{4}{3} m^3$





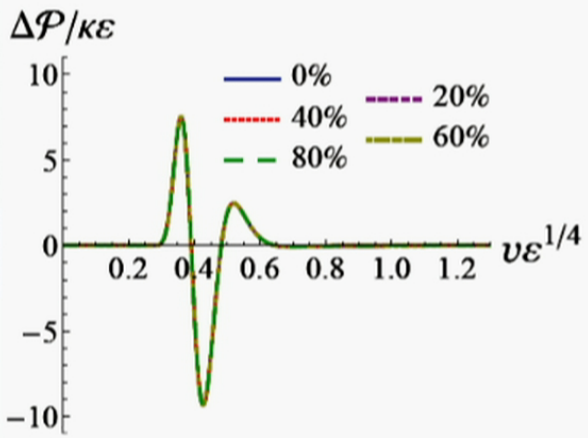
Results: Charged Plasma

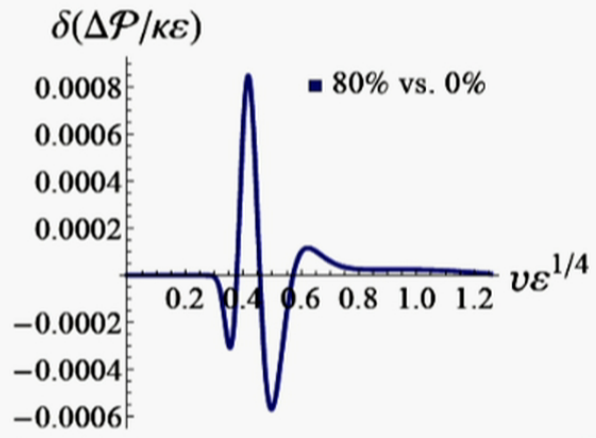
Anisotropy

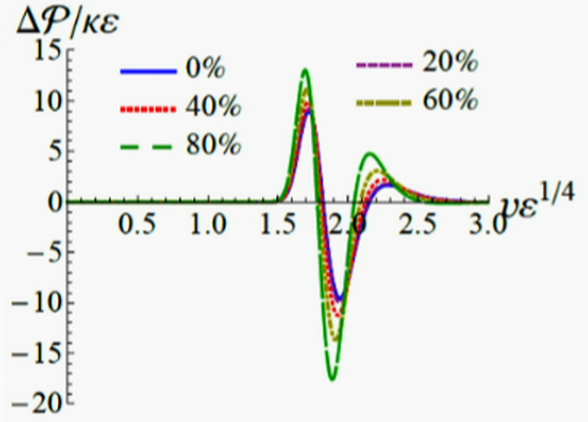
ε held fixed

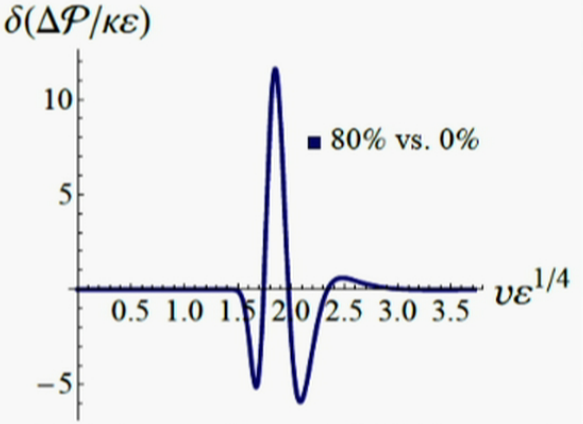
$(\rho_{\max} L^3)^4 = \frac{4}{3} m^3$

$\mu/T = 0,\ 0.34,\ 0.73,\ 1.26,\ 2.21$





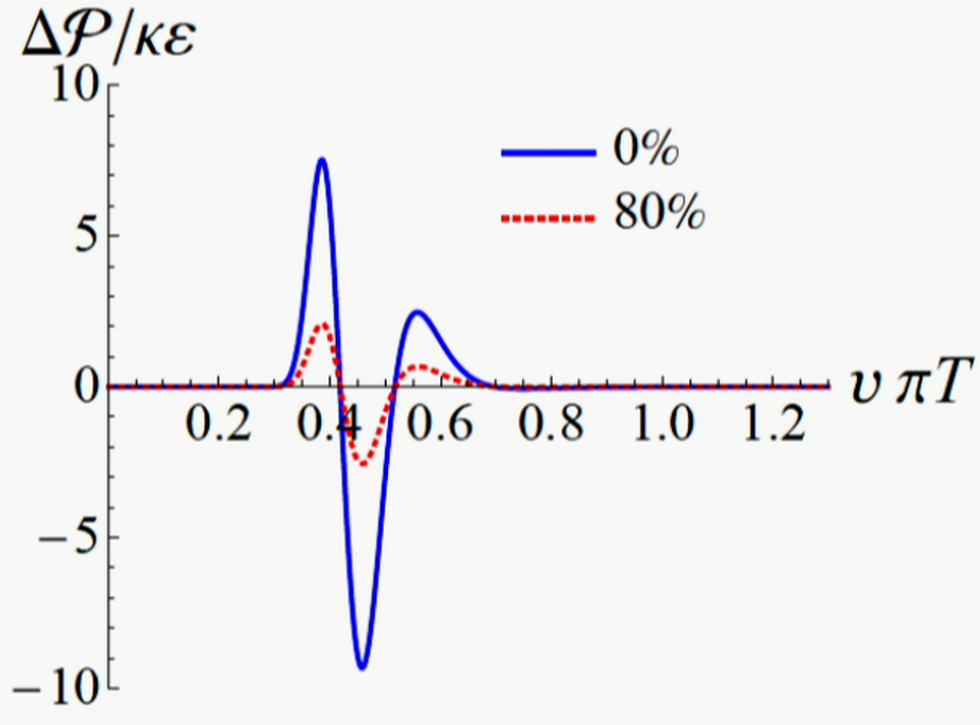




Results: Charged Plasma

Constant Temperature

- if we instead keep T fixed (as opposed to ε), and express time in units of T



Results: Charged Plasma

Quasi-normal Modes

From asymptotically late time of $\Delta\mathcal{P}/\kappa\mathcal{E}$

$$\sim e^{\lambda_I t} cos[\lambda_R t + \phi]$$

Charged ($\rho \neq 0$, $\mathcal{B} = 0$)			
$\rho/\rho_{\max}$	Re $\lambda/\varepsilon^{1/4}$	Im $\lambda/\varepsilon^{1/4}$	Linearized $\lambda/\varepsilon^{1/4}$
0.0	3.35208 ± 0.00004	-2.95144 ± 0.00013	$3.35207 - 2.95150\,i$
0.1	3.34564 ± 0.00016	-2.95468 ± 0.00019	$3.34568 - 2.95460\,i$
0.2	3.32624 ± 0.00020	-2.96429 ± 0.00019	$3.32630 - 2.96444\,i$
0.3	3.29319 ± 0.00028	-2.98266 ± 0.00036	$3.29327 - 2.98287\,i$
0.4	3.24572 ± 0.00007	-3.01376 ± 0.00008	$3.24574 - 3.01377\,i$
0.5	3.18366 ± 0.00016	-3.06498 ± 0.00007	$3.18370 - 3.06491\,i$
0.6	3.11311 ± 0.00002	-3.15177 ± 0.00002	$3.11311 - 3.15176\,i$
0.7	3.07021 ± 0.00006	-3.29402 ± 0.00006	$3.07022 - 3.29399\,i$
0.8	3.11863 ± 0.00265	-3.41376 ± 0.00295	$3.11848 - 3.42004\,i$

Charged Plasma Summary

- charge sensitivity more pronounced with pulses originating closer to horizon
- non-linearity sizes comparable to what was seen for neutral plasma
- low sensitivity to charge density when ε held fixed
- low senitivity to charge density when T held fixed

ack Brane

$dx^i)^2$

$\frac{L^4}{\tilde{r}^4}$

$L^2/\tilde{r}_{\rm h})^2$

$)^4$

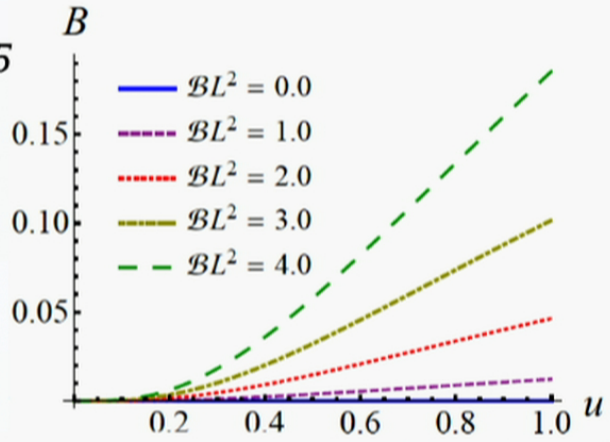
Magnetized Plasma

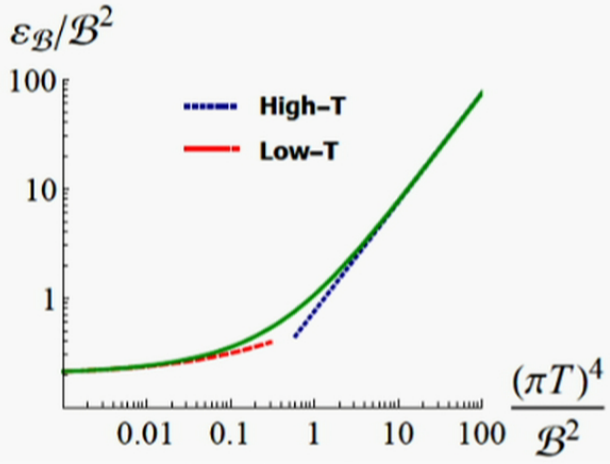
$(\rho = 0, \mathcal{B} \neq 0)$

Equilibrates to Magnetic Black Brane (Numerical Solution)

D'Hoker, Kraus 0908.3875

Anisotopy in the EQ case.
B picks a direction.



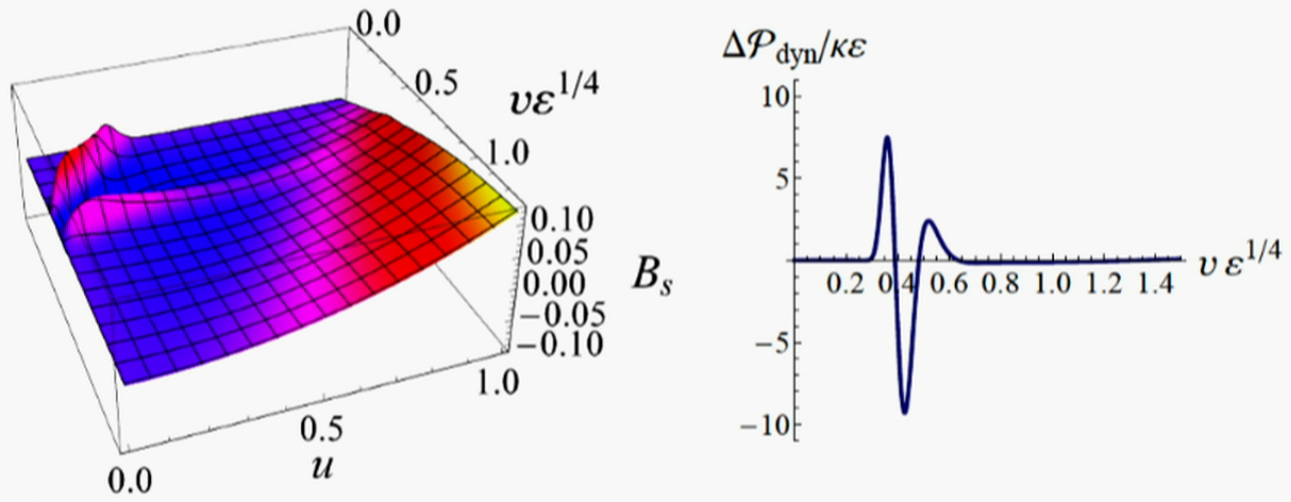


Results: Magnetized Plasma

Generic Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2} + \tfrac{1}{3}\mathcal{B}^2\, r^{-4} \ln r$$

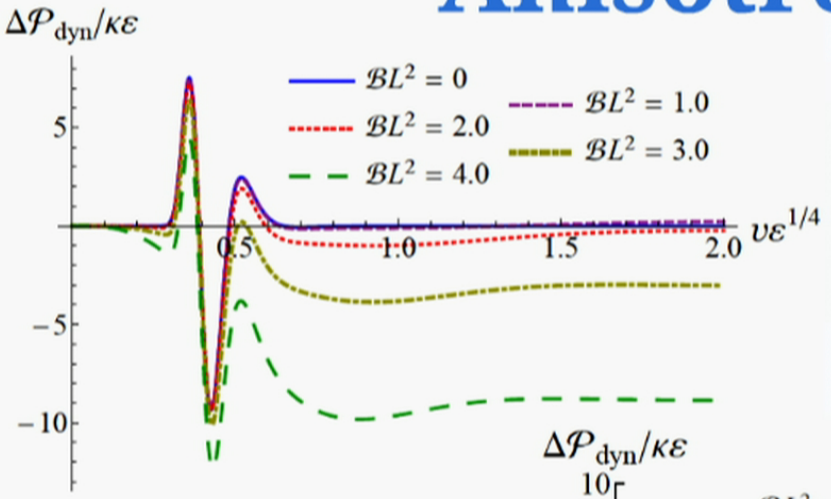
$$\Delta\mathcal{P}_{\rm dyn} \equiv \tfrac{1}{2}(T^{11} + T^{22}) - T^{33} + \tfrac{1}{4}\kappa\,\mathcal{B}^2$$



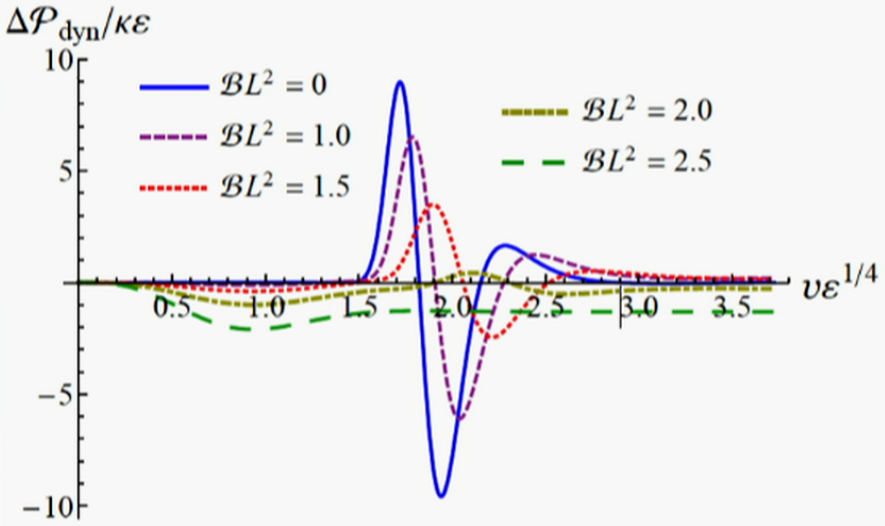
Results: Magnetized Plasma

Anisotropy

ε_L held fixed



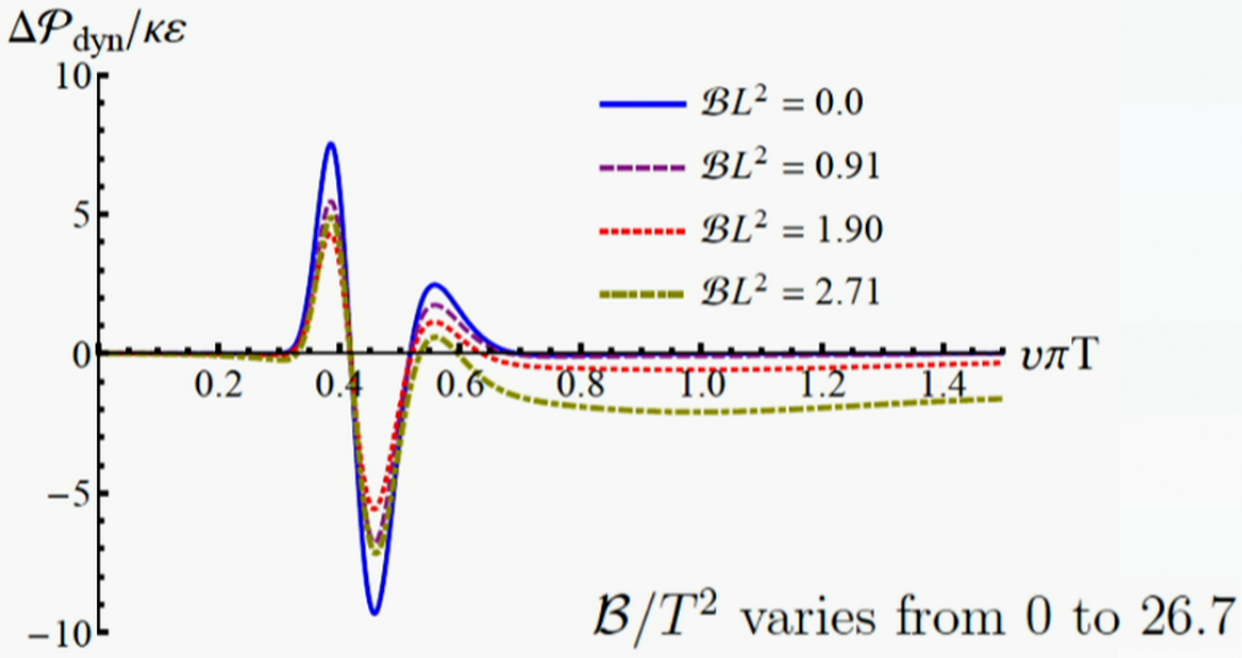
$\mathcal{B}/T^2 = 0,\ 13.0,\ 30.2,\ 30.5,\ 26.3$



Results: Magnetized Plasma

Constant Temperature

- if we instead keep T fixed (as opposed to ε_L), and express time in units of T



$\mathcal{B}/T^2$ varies from 0 to 26.7

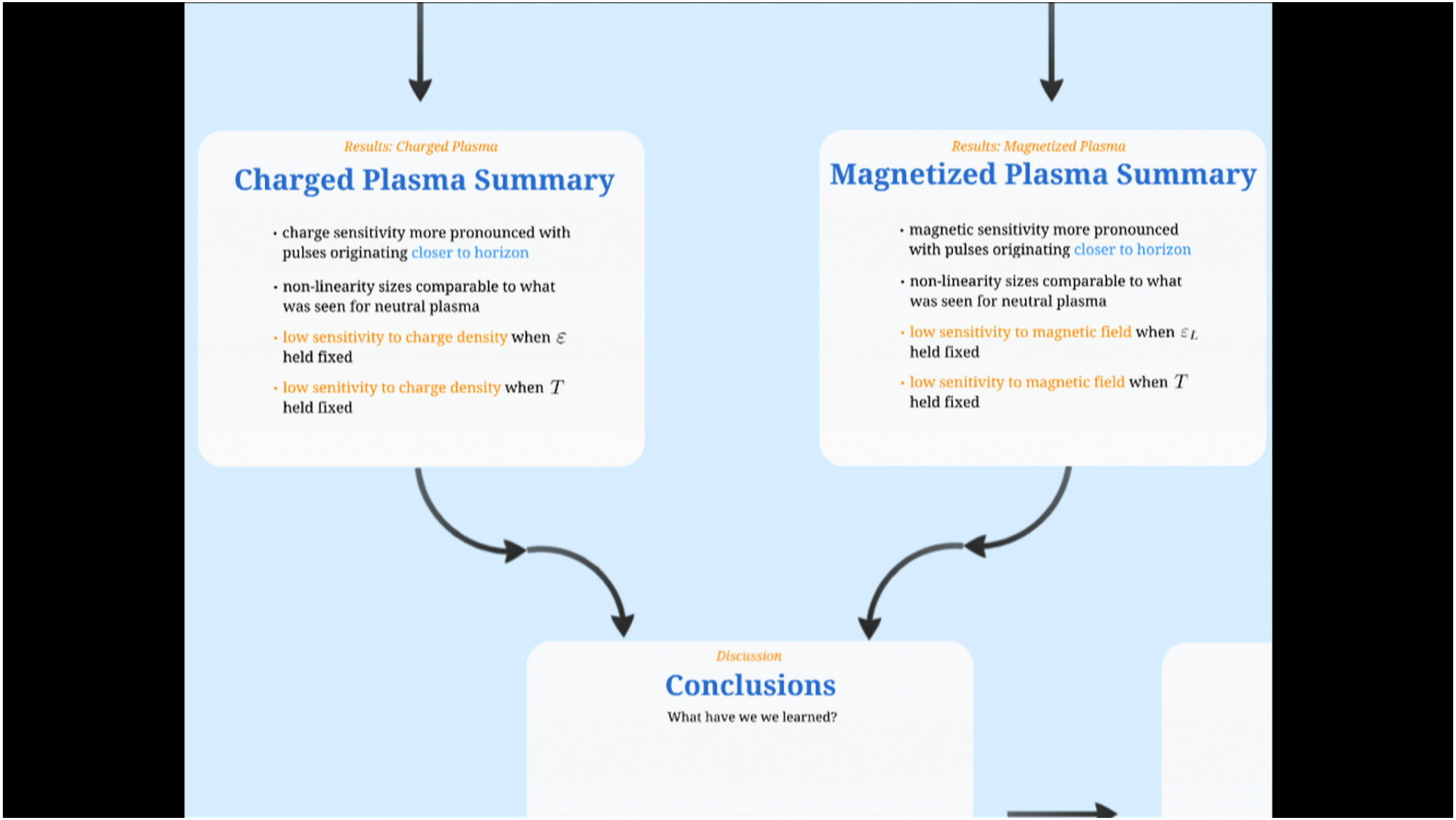
Results: Magnetized Plasma

Quasi-normal Modes

From asymptotically late time of $\Delta\mathcal{P}_{\rm dyn}/\kappa\mathcal{E}$

$$\sim e^{\lambda_I t} cos[\lambda_R t + \phi]$$

Magnetic ($\mathcal{B} \neq 0$, $\rho = 0$)						
$\mathcal{B}/T^2$	$\varepsilon_{\mathcal{B}}/T^4$	$\overline{\mathcal{P}}/\kappa T^4$	$\Delta\mathcal{P}/\kappa T^4$	$\mathrm{Re}\,\lambda/\varepsilon_{\mathcal{B}}^{1/4}$	$\mathrm{Im}\,\lambda/\varepsilon_{\mathcal{B}}^{1/4}$	$\lambda/(\pi T)$
0	73.06	24.35	0	3.3521 ± 0.0001	-2.9514 ± 0.0001	$3.1195 - 2.7466\,i$
0.990	72.98	24.16	−1.13	3.357 ± 0.001	-2.93 ± 0.06	$3.124 - 2.73\,i$
5.344	80.74	22.15	−10.60	3.372 ± 0.002	-2.92 ± 0.06	$3.217 - 2.79\,i$
12.953	125.85	13.98	−15.76	3.264 ± 0.007	-2.78 ± 0.01	$3.480 - 2.96\,i$
17.821	170.16	3.79	−9.36	3.161 ± 0.002	-2.69 ± 0.04	$3.634 - 3.09\,i$
22.836	226.69	−11.35	5.00	3.061 ± 0.008	-2.60 ± 0.03	$3.780 - 3.21\,i$
30.161	328.21	−42.21	39.97	2.94 ± 0.01	-2.49 ± 0.03	$3.98 - 3.38\,i$


Results: Charged Plasma
Charged Plasma Summary
• charge sensitivity more pronounced with pulses originating closer to horizon
• non-linearity sizes comparable to what was seen for neutral plasma
• low sensitivity to charge density when $\mathcal{E}$ held fixed
• low senitivity to charge density when T held fixed
Results: Magnetized Plasma
Magnetized Plasma Summary
• magnetic sensitivity more pronounced with pulses originating closer to horizon
• non-linearity sizes comparable to what was seen for neutral plasma
• low sensitivity to magnetic field when $\mathcal{E}_L$ held fixed
• low senitivity to magnetic field when T held fixed
Discussion
Conclusions
What have we we learned?

Discussion

Conclusions

What have we we learned?

To a good (often extremely good) level of accuracy:

- the pressure anisotropy is a linear functional of the initial anisotropy profile
- the timecourse is insensitive to charge density or magnetic field when the initial pulse profile is held fixed and either:

Discussion

Simple Model

Consider the time of peak anisotropy in the charged case, $t_{\rm peak}$, as a function of equilibrium parameters and initial profile parameters

$$t_{\rm peak}/L = f(\varepsilon L^4, TL, r_0/L, \sigma/L, \mathcal{A})$$

where we've exchanged charge fraction, $\rho/\rho_{\rm max}$, for T

- small non-linearity size: indep of $\mathcal{A}$
- consider small width pulses, or for non-negligible width combine r_0 and σ

$$t_{\rm peak}/L \approx g(\varepsilon L^4, TL, r_{\rm eff}/L) \qquad r_{\rm eff} = r_0 + n\sigma$$

$$t_{\text{peak}}/L \approx g(\varepsilon L^4, TL, r_{\text{eff}}/L) \qquad r_{\text{eff}} = r_0 + n\sigma$$

- our results show small dependence on charge density at fixed energy density - i.e. indep of temp
- also results show small dependence on charge density at fixed temp - i.e. indep of energy density

Main dependence on effective depth of initial pulse

$$t_{\text{peak}}/L \approx h(r_{\text{eff}}/L)$$

functional form limited by scaling relations, we find:

Discussion

Scaling Relations

$$x \to \tilde{x} = \alpha^{-1} x \qquad r \to \tilde{r} = \alpha\gamma^{-2} r$$

$$\begin{aligned}
\widetilde{B}(\widetilde{x},\widetilde{r}) &\equiv B(x(\widetilde{x}), r(\widetilde{r}))\,, \\
\widetilde{\Sigma}(\widetilde{x},\widetilde{r}) &\equiv (\alpha/\gamma)\,\Sigma(x(\widetilde{x}), r(\widetilde{r}))\,, \\
\widetilde{A}(\widetilde{x},\widetilde{r}) &\equiv (\alpha/\gamma)^2 A(x(\widetilde{x}), r(\widetilde{r}))\,,
\end{aligned}$$

$$\widetilde{L} \equiv \gamma^{-1}\, L \qquad \widetilde{\mathcal{B}} \equiv \alpha^2\, \mathcal{B} \qquad \widetilde{\rho} \equiv \alpha^3\, \rho$$

$$\alpha = \gamma$$

scaling by dimension

Discussion

Scaling Relations

$$x \to \tilde{x} = \alpha^{-1} x \qquad r \to \tilde{r} = \alpha\gamma^{-2} r$$

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\widetilde{A}(\widetilde{x},\widetilde{r}) &\equiv (\alpha/\gamma)^2 A(x(\widetilde{x}), r(\widetilde{r}))\,,
\end{aligned}$$

$$\widetilde{L} \equiv \gamma^{-1} L \qquad \widetilde{\mathcal{B}} \equiv \alpha^2 \mathcal{B} \qquad \widetilde{\rho} \equiv \alpha^3 \rho$$

$\alpha = \gamma$

scaling by dimension

$\alpha = 1, \gamma > 0$

AdS radius scaling

$$t_{\text{peak}}/L \approx g(\varepsilon L^4, TL, r_{\text{eff}}/L) \qquad r_{\text{eff}} = r_0 + n\sigma$$

- our results show small dependence on charge density at fixed energy density - i.e. indep of temp
- also results show small dependence on charge density at fixed temp - i.e. indep of energy density

Main dependence on effective depth of initial pulse

$$t_{\text{peak}}/L \approx h(r_{\text{eff}}/L)$$

functional form limited by scaling relations, we find:

$$t_{\text{peak}} \approx CL^2/r_{\text{eff}}$$

Discussion

Simple Model

Same argument goes for our Magnetic results, with the replacements

$$t_{\text{peak}}/L = f(\varepsilon_L L^4, \mathcal{B}L^2, r_0/L, \sigma/L, \mathcal{A})$$

$$t_{\text{peak}}/L \approx g(\varepsilon_L L^4, \mathcal{B}L^2, r_{\text{eff}}/L)$$

$$t_{\text{peak}} \approx CL^2/r_{\text{eff}}$$

For charged *and* magnetized plasmas our model suggests:

$$t_{\text{peak}} \approx CL^2/r_{\text{eff}}$$

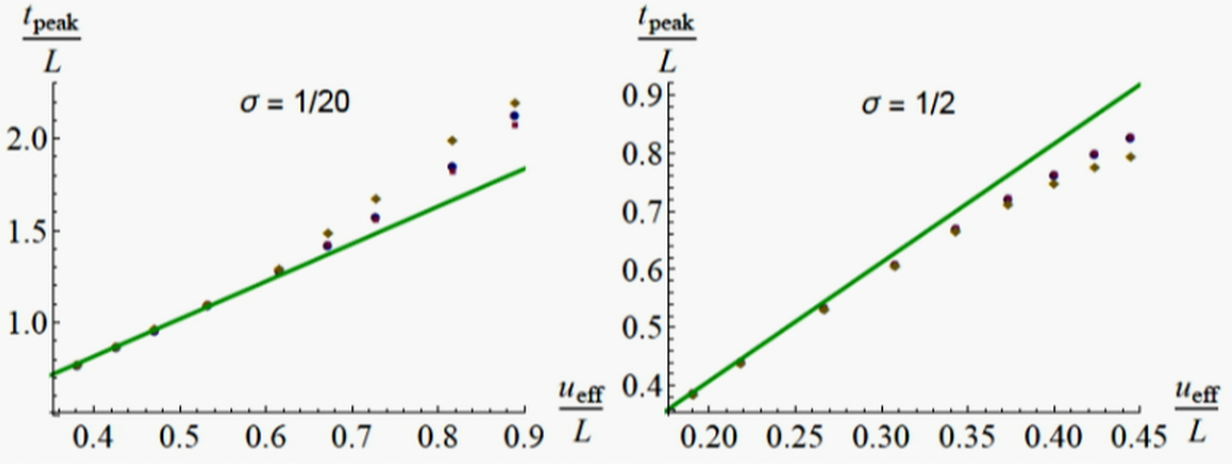
Discussion

Comparison to Model

We test against a sharper width pulse, and wider pulse, in uncharged, charged, and magnetized plasmas, at varying depths.

$$t_{\text{peak}} \approx CL^2/r_{\text{eff}} \qquad r_{\text{eff}} = r_0 + n\sigma$$

We find good agreement using n = 2.5, and $C \approx 2.04$



Conclusions

- developed far-from-equilibrium numerical evolution to include charge density and magnetic fields (including log difficulties).
- explored the surprising amount of linearity exhibited by the evolution.
- found that both a substantially large chemical potential or magnetic field has a very small effect on equilibration times - and developed a simple model.
- We see that changing the balance between fermions and scalars (via chemical potential) has negligible effect, suggesting insensitivity to the precise makeup of the plasma -- small confidence boost in the applicability of N=4 as a proxy for QCD.