Title: Integrability in AdS3/ CFT2 and the Conformal Field Theory of the Higgs branch

Date: Oct 06, 2015 02:00 PM

URL: http://pirsa.org/15100029

Abstract: <p>I will discuss the Higgs-branch CFT2 dual to string theory on AdS3 x S3 x T4. States localised near the small instanton singularity can be described in terms of vector multiplet variables. This theory has a planar, weak-coupling limit, in which anomalous dimensions of single-trace composite operators can be calculated. At one loop, the calculation reduces to finding the spectrum of a spin chain with nearest-neighbour interactions. This CFT2 spin chain matches precisely the one that was previously found as the weak-coupling limit of the integrable system describing the AdS3 side of the duality. In particual I will show that the one-loop dilatation operator in a non-trivial compact subsector corresponds to an integrable spin-chain Hamiltonian. This provides the first direct evidence of integrability on the CFT2 side of the correspondence.

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Integrability in AdS_3/CFT_2 and the Conformal Field Theory of the Higgs branch

Olof Ohlsson Sax

Oct 6, 2015

Based on 1410.0866 with A. Sfondrini and B. Stefański

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Outline

Integrability in AdS_3/CFT_2
and the Conformal Field Theory of the Higgs branch

1. Introduction: AdS_3/CFT_2 and integrability
2. 2D gauge theory for the D1-D5 system
   - UV Lagrangian
   - Coulomb and Higgs branch in the UV and IR
3. Conformal Field Theory of the Higgs branch
   - IR Lagrangian
   - ADHM sigma model and small instantons
   - Effective action at the origin of the Higgs branch
4. Spin chain picture and one-loop dilatation operator
5. Summary and outlook

AdS_3/CFT_2

AdS_3 backgrounds preserving 16 supersymmetries:

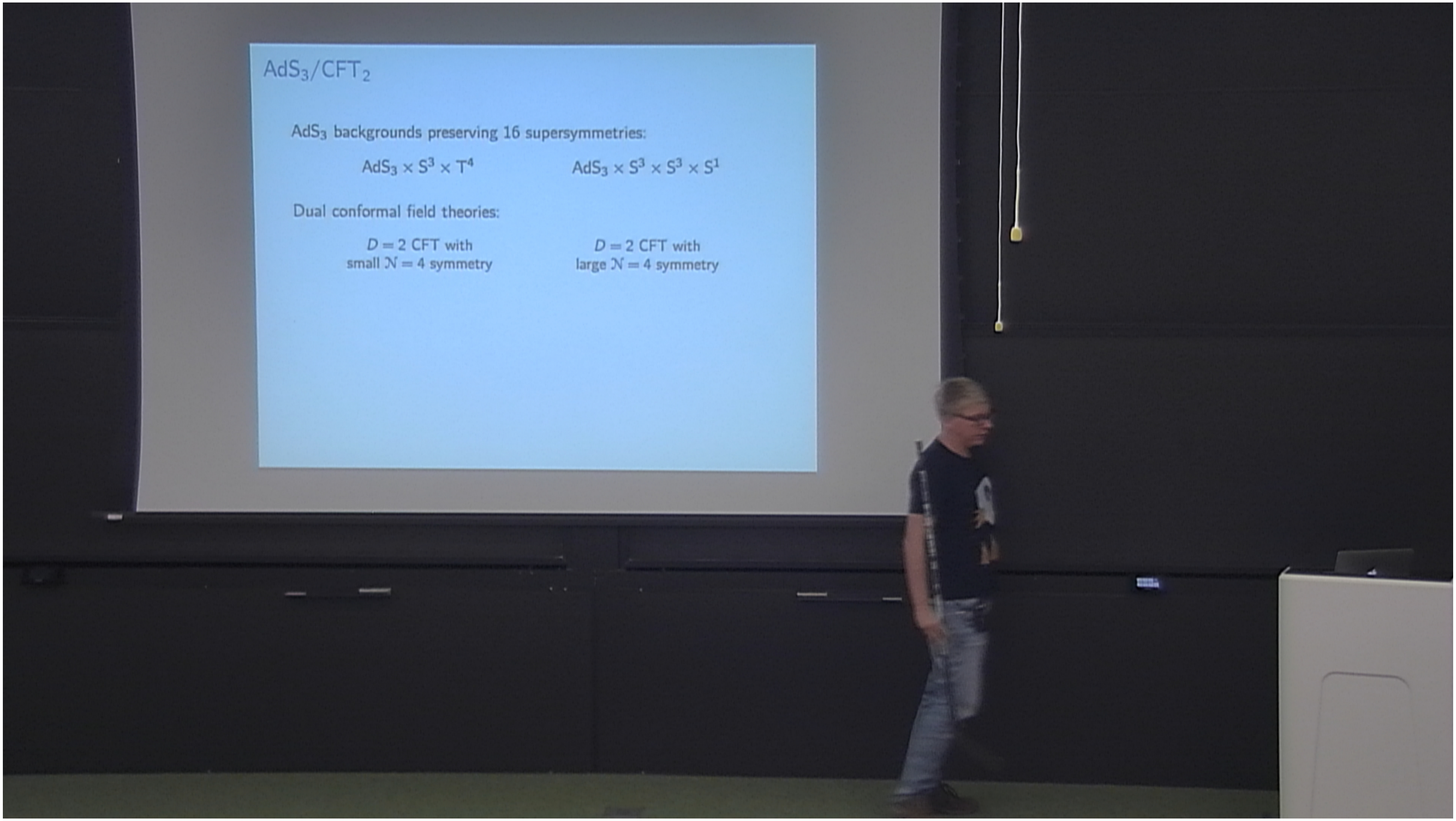
$AdS_3 \times S^3 \times T^4$

$AdS_3 \times S^3 \times S^3 \times S^1$

Dual conformal field theories:

$D = 2$ CFT with small $\mathcal{N} = 4$ symmetry

$D = 2$ CFT with large $\mathcal{N} = 4$ symmetry


AdS3/CFT2
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AdS3 × S3 × T4
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Sigma model on $\mathrm{Sym}_N(T^4)$
with small $\mathcal{N} = 4$ symmetry

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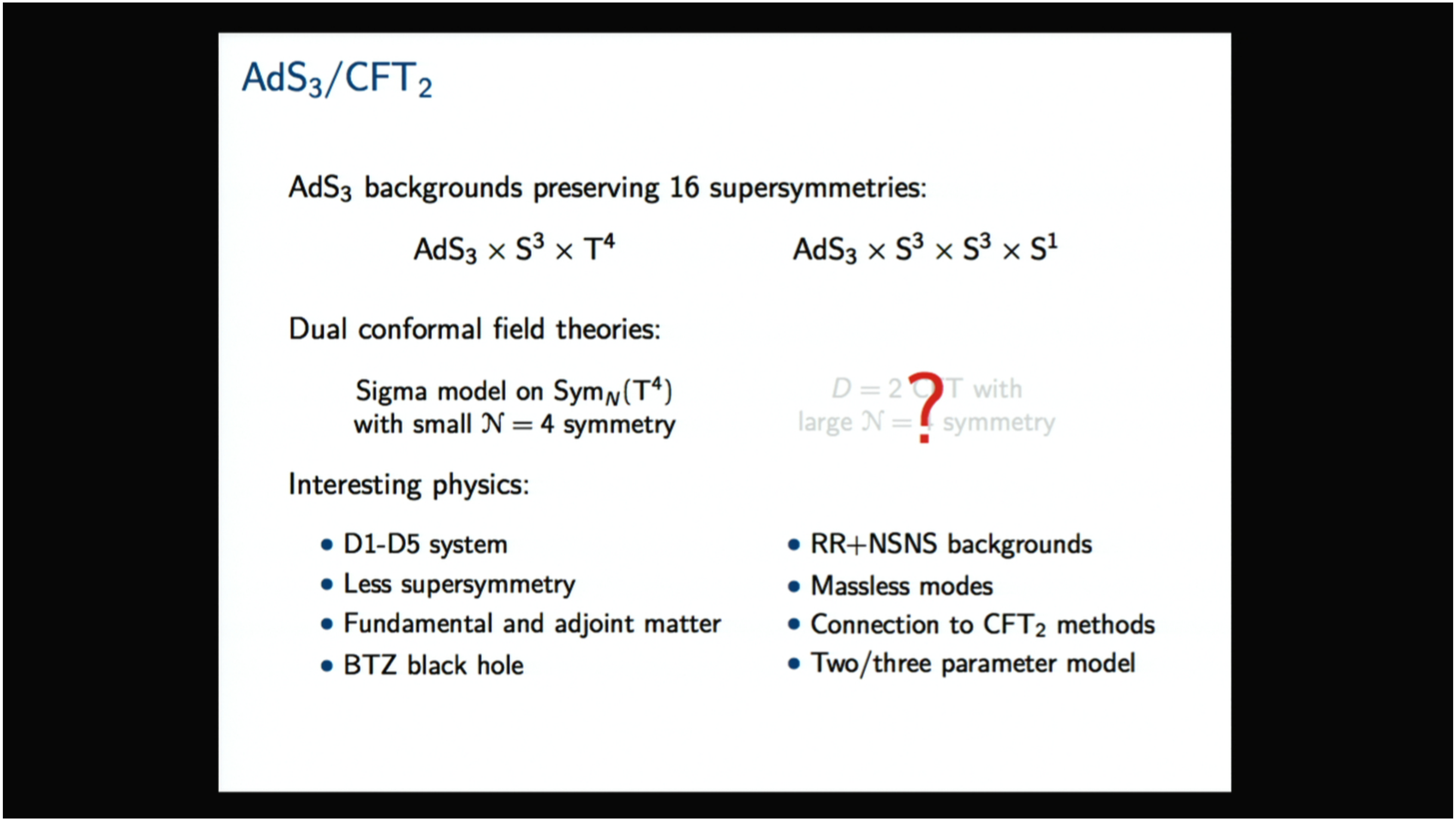
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Interesting physics:
• D1-D5 system
• Less supersymmetry
• Fundamental and adjoint matter
• BTZ black hole
• RR+NSNS backgrounds
• Massless modes
• Connection to CFT$_2$ methods
• Two/three parameter model

AdS_3/CFT_2

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Super-coset sigma models:

$$\frac{PSU(1,1|2) \times PSU(1,1|2)}{SL(2) \times SU(2)} \times U(1)^4 \qquad\qquad \frac{D(2,1;\alpha) \times D(2,1;\alpha)}{SL(2) \times SU(2) \times SU(2)} \times U(1)$$

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Super-cosets with $\mathbb{Z}_4$ automorphism

→ Classical integrability

AdS_3/CFT_2 and integrability

Use integrability to solve spectral problem:

- Energy spectrum of strings in $AdS_3 \times S^3 \times M^4$
- Free strings: $g_s \to 0$ / $N \to \infty$
- Light-cone gauge:

$$E - J = H_{\text{w.s.}}(\lambda)$$

- All-loop integrable worldsheet S matrix and dispersion relations from symmetries

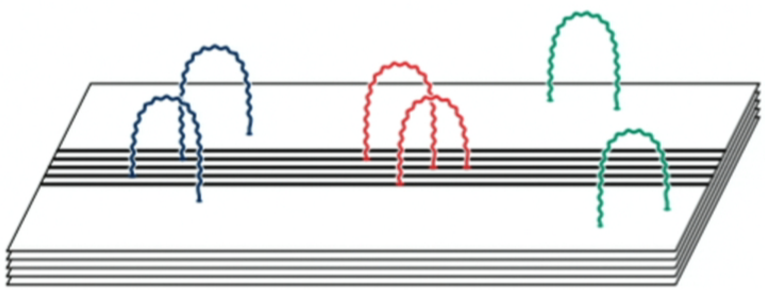
[Borsato, Lloyd, OOS, Sfondrini, Stefański, Torrielli '12–'15]

Today:

- CFT_2 dual of IIB string theory on $AdS_3 \times S^3 \times T^4$ supported by R-R three-form flux
- Integrability for the spectral problem in the dual CFT

The D1-D5 system

	$\mathbb{R}^{1,5}$						T^4			
	0	1	2	3	4	5	6	7	8	9
N_c D1-branes:	•	•								
N_f D5-branes:	•	•					•	•	•	•



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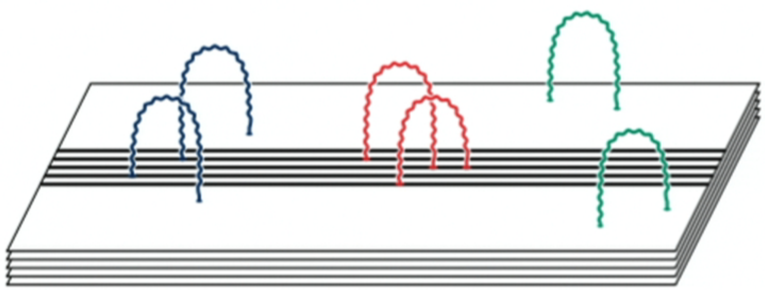

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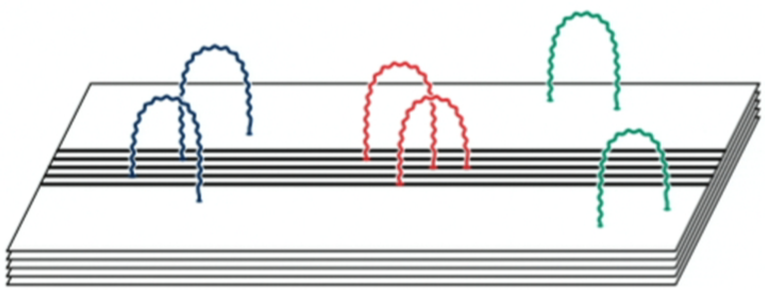

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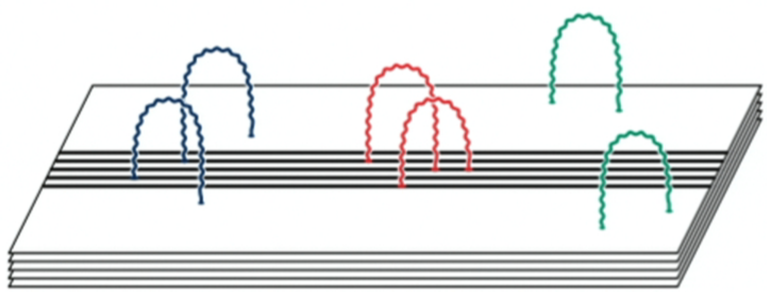
World-volume: 2D $\mathcal{N} = (4,4)$ $U(N_c)$ gauge theory with adjoint and fundamental matter



Field content

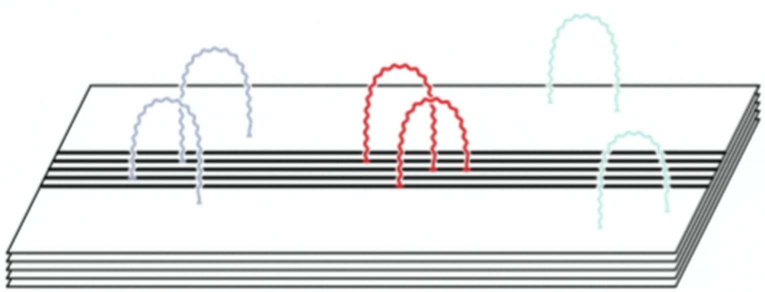
- D1-D1 strings: $U(N_c)$ adjoint

  $(4,4)$ vector multiplet: ϕ, ψ_l, ψ_r, A_μ, D

  $(4,4)$ hypermultiplets: $T^\pm$, $\chi_l^\pm$, $\chi_r^\pm$

Non-dynamic 2D gauge field

Three auxiliary scalars



Two-dimensional $\mathcal{N} = (4,4)$ UV gauge theory

- Action determined by supersymmetry (Reduction of 4D $\mathcal{N} = 2$ SYM)

$$\mathcal{L}_{\mathrm{UV}}(\phi, T, H) = \mathcal{L}_{\phi}(\phi) + \mathcal{L}_{T}(T, \phi) + \mathcal{L}_{H}(H, \phi)$$

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- Adjoint hyper: $\mathcal{L}_{T} = \text{tr}(|\nabla T|^2 + [T, \phi]^2 + T[D, T])$
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  Dimensionful coupling

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- Not conformal – flows to conformal fixed point in the IR
- Kinetic term for the vector multiplet is irrelevant

Two-dimensional $\mathcal{N} = (4, 4)$ UV gauge theory

- Two separate branches of the moduli space:
  - Coulomb branch: $\langle \phi \rangle \neq 0$ D1 and D5 branes separated
  - Higgs branch: $\langle H \rangle \neq 0$ D1 and D5 branes coincide

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  ↑ Different $SO(4)$ R-symmetry
- Two different CFTs
  - $\mathrm{CFT}_C \oplus \mathrm{CFT}_H$

[Witten '95, '97]

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  - Moduli space metric is hyper-Kähler

CFT_2 on the Higgs branch: $\mathcal{L}_{\text{IR}}$

IR action $\mathcal{L}_{\text{IR}}$ on the Higgs branch: [Witten '96]

1. Drop the kinetic term for the vector multiplet (irrelevant)

$$\mathcal{L}_{\text{IR}}(\phi, T, H) = \mathcal{L}_{\phi}(\phi) + \mathcal{L}_T(T, \phi) + \mathcal{L}_H(H, \phi)$$

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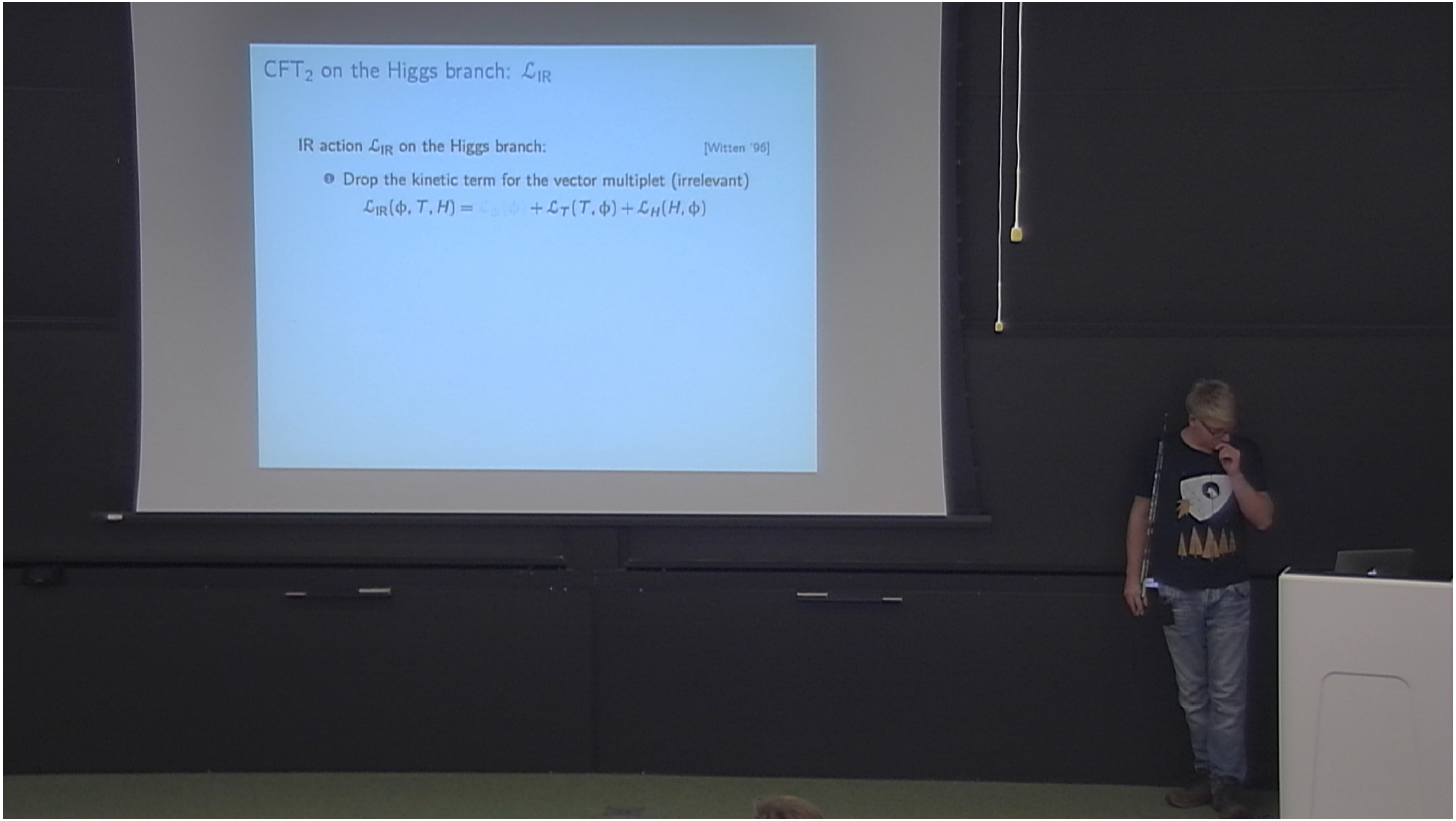

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$$\dim(A) = \dim(\phi) = 1 \quad \dim(\psi) = 3/2 \quad \dim(D) = 2$$

Hypermultiplets carry canonical dimensions

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3. Vector multiplet enters quadratically as auxiliary fields

CFT$_2$ on the Higgs branch: $\mathcal{L}_{\text{ADHM}}$

- Vector multiplet auxiliary: [Witten '97; Aharony, Berkooz '99]
  Integrate out ϕ using equations of motion

$$\mathcal{L}_{\text{IR}}(\phi, T, H) \quad \to \quad \mathcal{L}_{\text{ADHM}}(T, H)$$

- Obtain $\mathcal{N} = (4,4)$ sigma model with target space

$$\mathcal{M}_{N_c, N_f}$$

  Moduli space of N_c instantons in 4D $su(N_f)$ gauge theory

- Conventional picture of Higgs branch:
  D- and F-flatness condition equivalent to ADHM construction

- $\mathcal{L}_{\text{ADHM}}$ has small instanton singularity:
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CFT_2 on the Higgs branch: states near the origin

- States localised near the origin of the Higgs branch not captured by $\mathcal{L}_{ADHM}$

Near small instanton singularity

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- For such states integrate out the fundamental hypermultiplets

[Witten '97, Aharony, Berkooz '99]

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[Witten '97, Aharony, Berkooz '99]

$$\mathcal{L}_{\text{IR}}(\phi, T, H) \quad \rightarrow \quad N_f \mathcal{L}_{\text{eff}}(\phi) + \mathcal{L}_T(T, \phi)$$

N_f hypermultiplets integrated out

Effective coupling constant $1/N_f$

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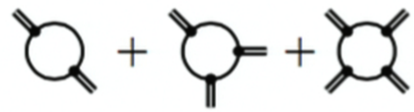
$N_f \mathcal{L}_{eff}(\phi) =$  $+ \cdots$

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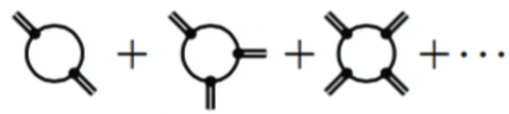

CFT_2 on the Higgs branch: $\mathcal{L}_{\text{eff}}$

- Vector multiplet becomes dynamical
- Two-point functions determined by conformal symmetry

$$\langle \phi(x)\phi(0)\rangle = \frac{1}{|x|^2} \quad \langle A_\mu(x)A_\nu(0)\rangle = \frac{\eta_{\mu\nu}}{|x|^2} \quad \langle D(x)D(0)\rangle = \frac{1}{|x|^4}$$

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Gauge field
1 d.o.f

“Auxiliary”
3 d.o.f

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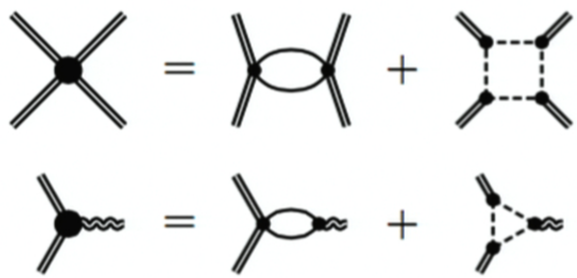

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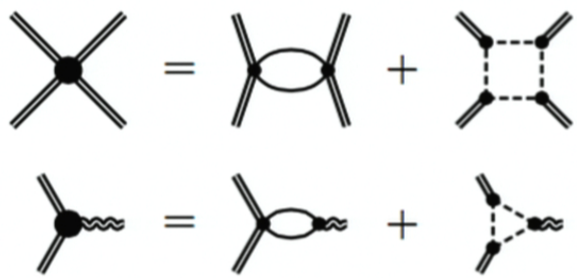

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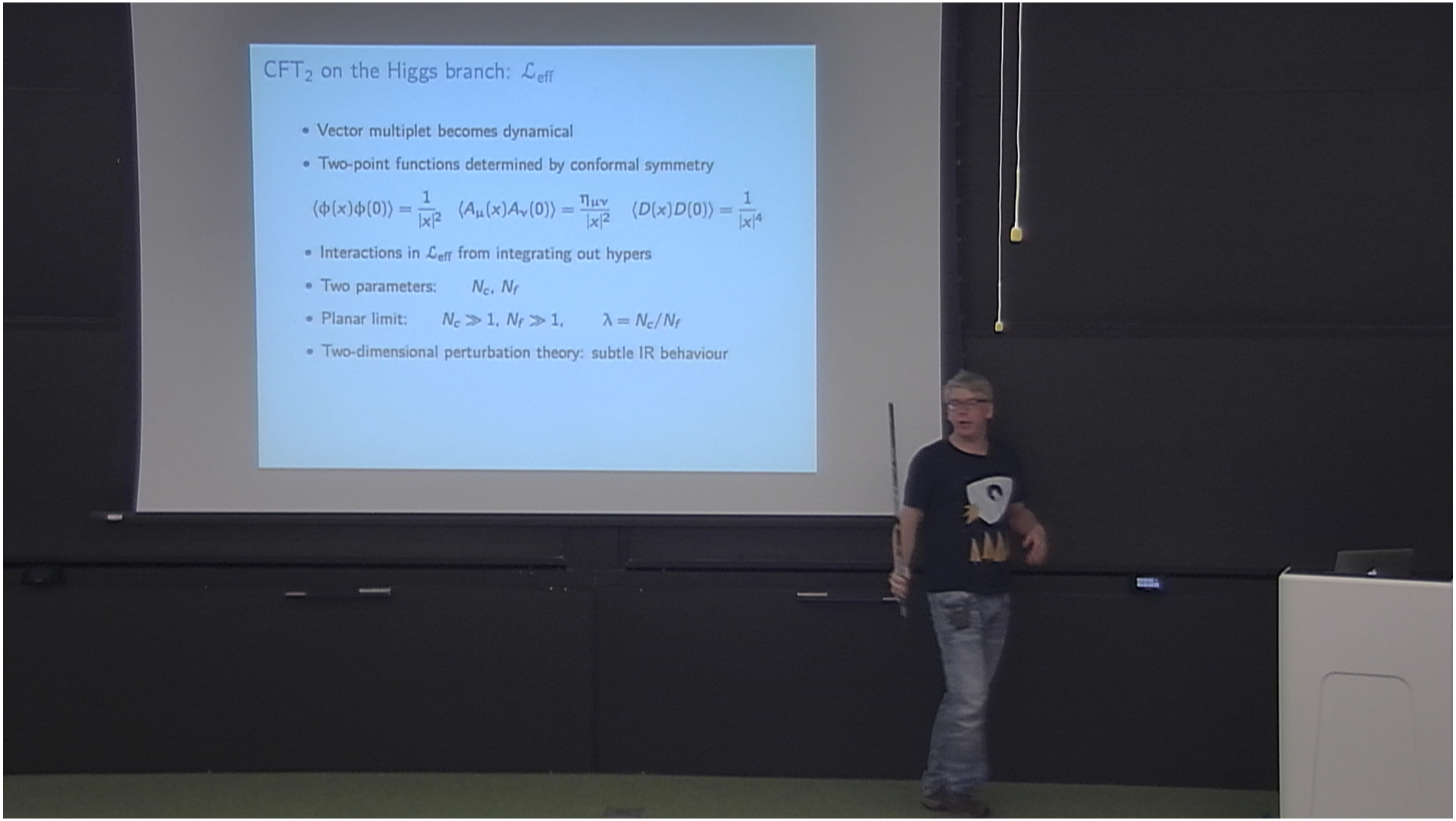
- Two parameters: $N_c,\ N_f$
- Planar limit: $N_c \gg 1,\ N_f \gg 1, \qquad \lambda = N_c/N_f$
- Two-dimensional perturbation theory: subtle IR behaviour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- Planar limit: $N_c \gg 1,\ N_f \gg 1, \qquad \lambda = N_c/N_f$
- Two-dimensional perturbation theory: subtle IR behaviour

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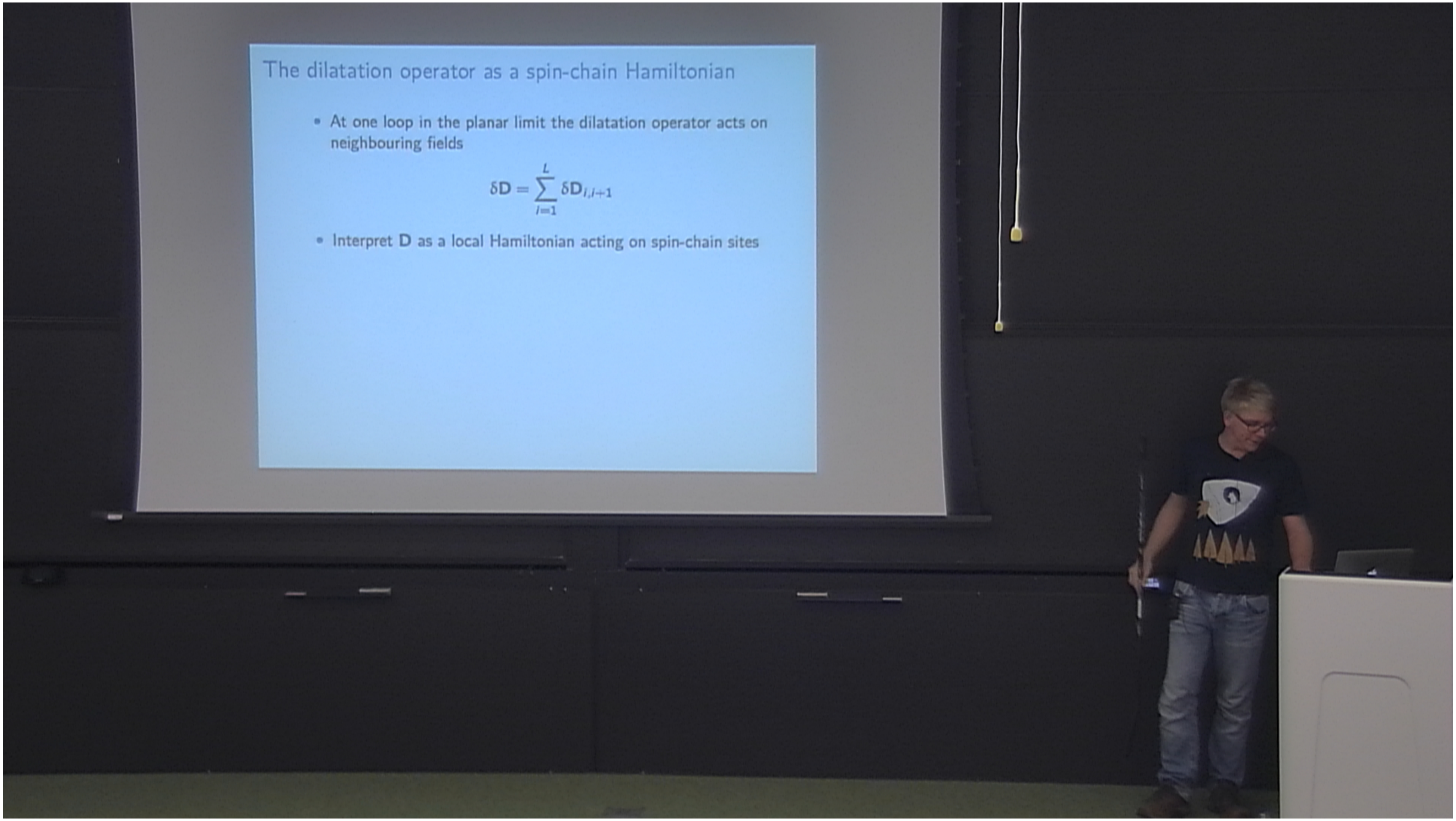
The dilatation operator as a spin-chain Hamiltonian
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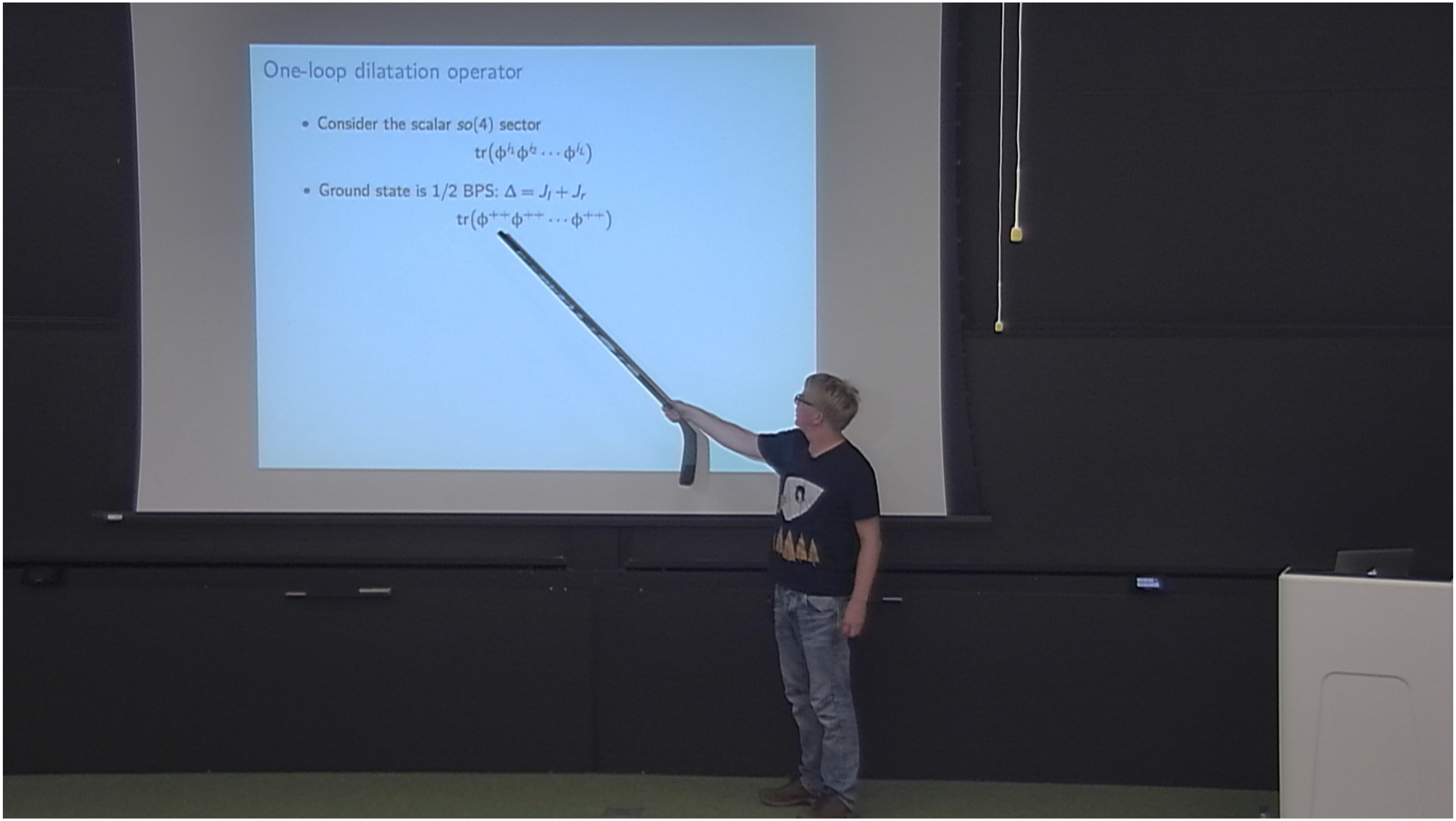
One-loop dilatation operator
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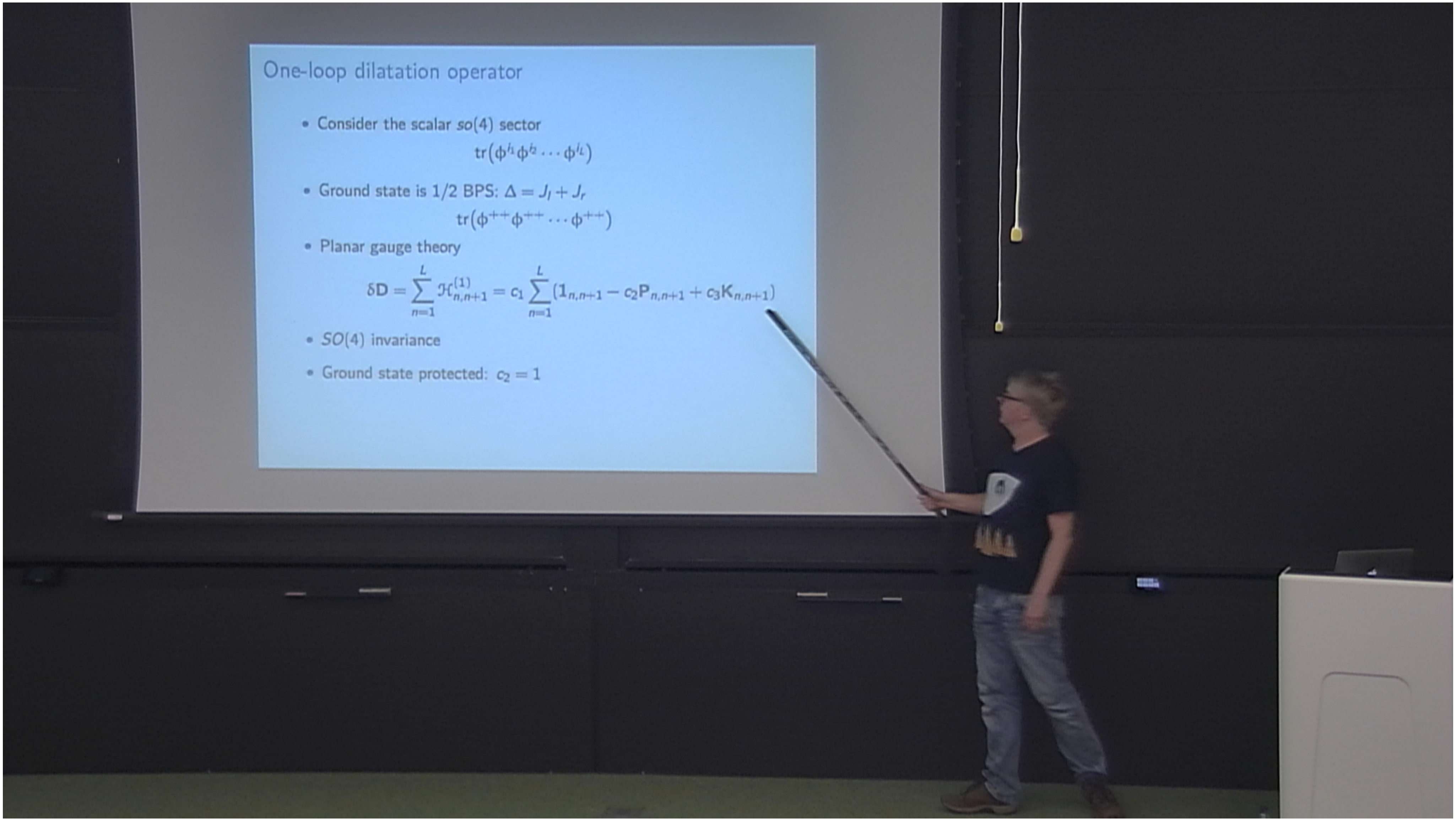
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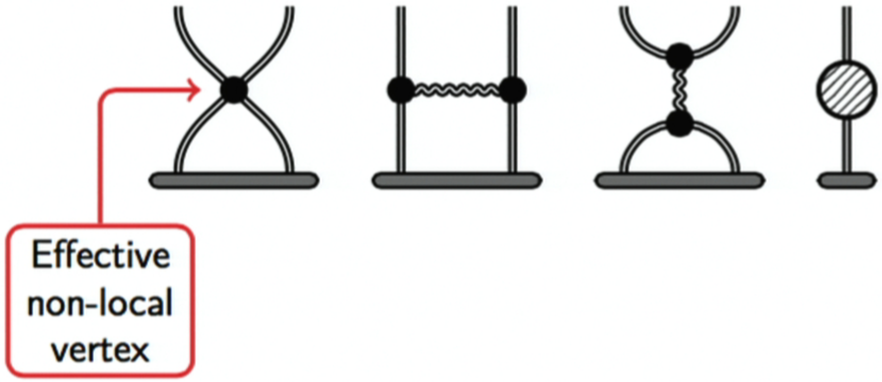

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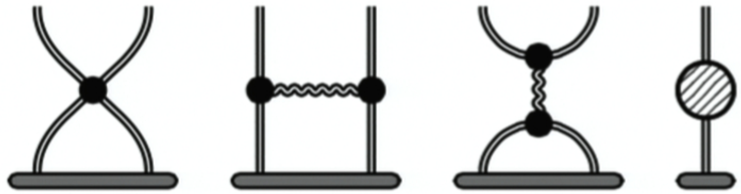

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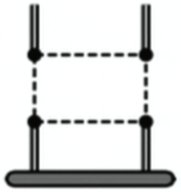

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- Integrability for $SO(N)$ if $c_3 = \frac{2}{N-2}$

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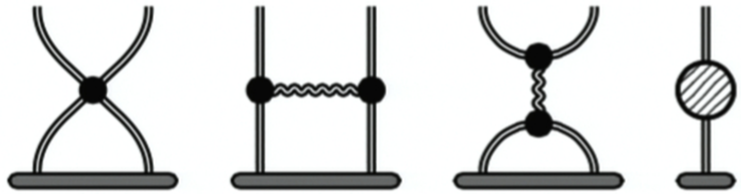

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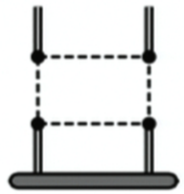

- Dilatation operator in the $so(4)$ sector

$$\delta \mathbf{D} \propto \frac{N_c}{N_f} \sum_{n=1}^{L} \left(\mathbf{1}_{n,n+1} - \mathbf{P}_{n,n+1} + \mathbf{K}_{n,n+1} \right)$$

Integrable $so(4)$ spin-chain Hamiltonian!

Summary

- AdS_3/CFT_2 duality from D1-D5 system
- Integrability in string theory and conformal field theory
- D1-D5 system leads to $\mathcal{N} = (4,4)$ gauge theory
- Flows to superconformal field theory in the IR
- Effective action at the origin of the Higgs branch
- Spin-chain constructed from the field strength multiplet and adjoint hypermultiplets
- Perturbative calculation of Hamiltonian in the $SO(4)$ sector:

One-loop dilatation operator in $SO(4)$ sector ⟷ Hamiltonian of integrable $SO(4)$ spin-chain

Open questions

- A local description of the effective action? (WZW?)
- Full spin-chain Hamiltonian at one-loop?
- Massless modes from the adjoint hypermultiplet?
- Three-point functions?
- How do we take the winding modes on T^4 into account?
- Interesting generalisations
  - Mixed NSNS- and RR-flux [Cagnazzo, Zarembo '12]
  - $AdS_3 \times S^3 \times S^3 \times S^1$
    Recent proposal for UV $\mathcal{N} = (0, 4)$ gauge theory [Tong '14]
- Connections to other points of the CFT_2 moduli space
  - WZW?
  - $\text{Sym}_N(T^4)$
  - Higher spins