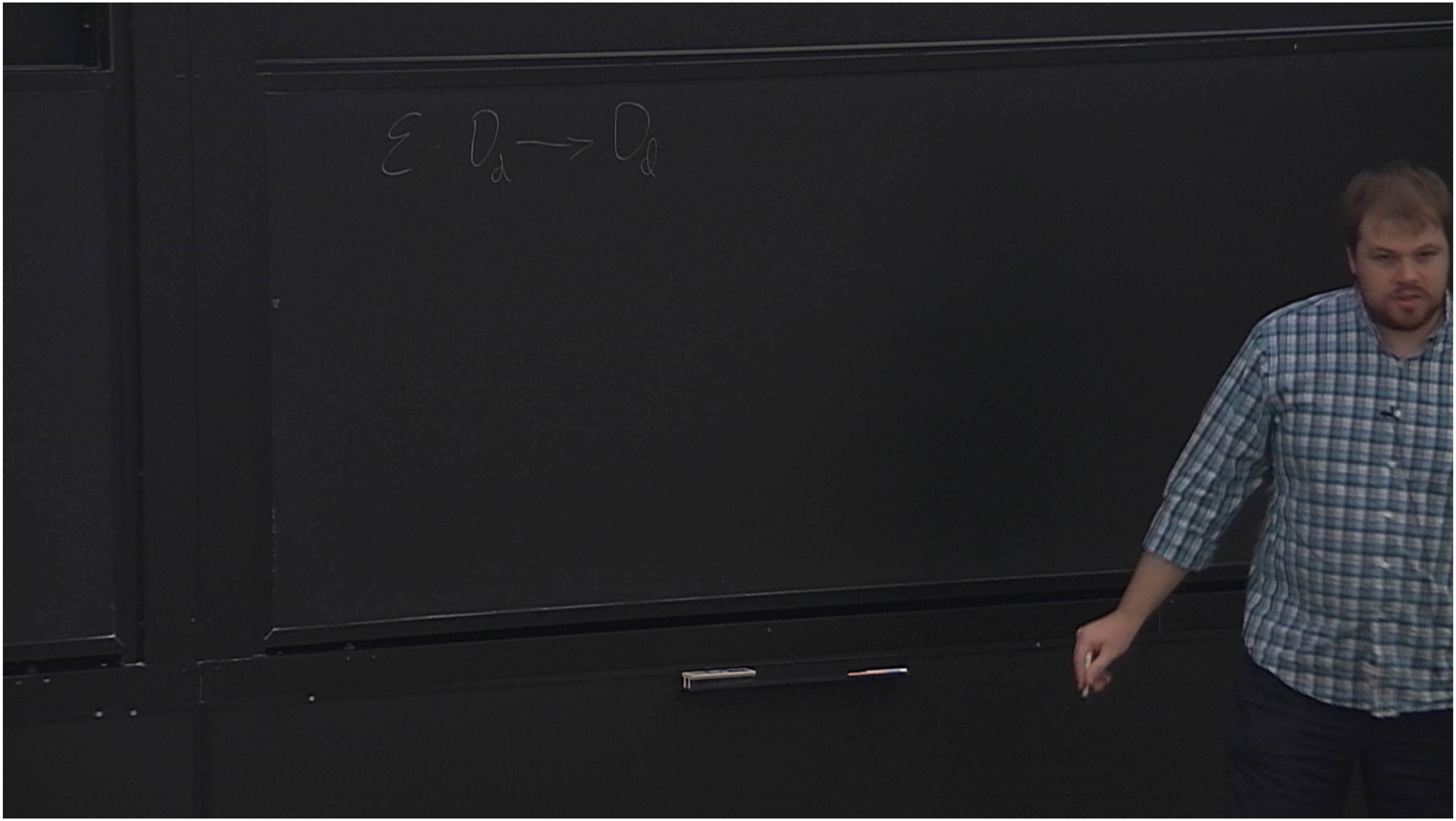
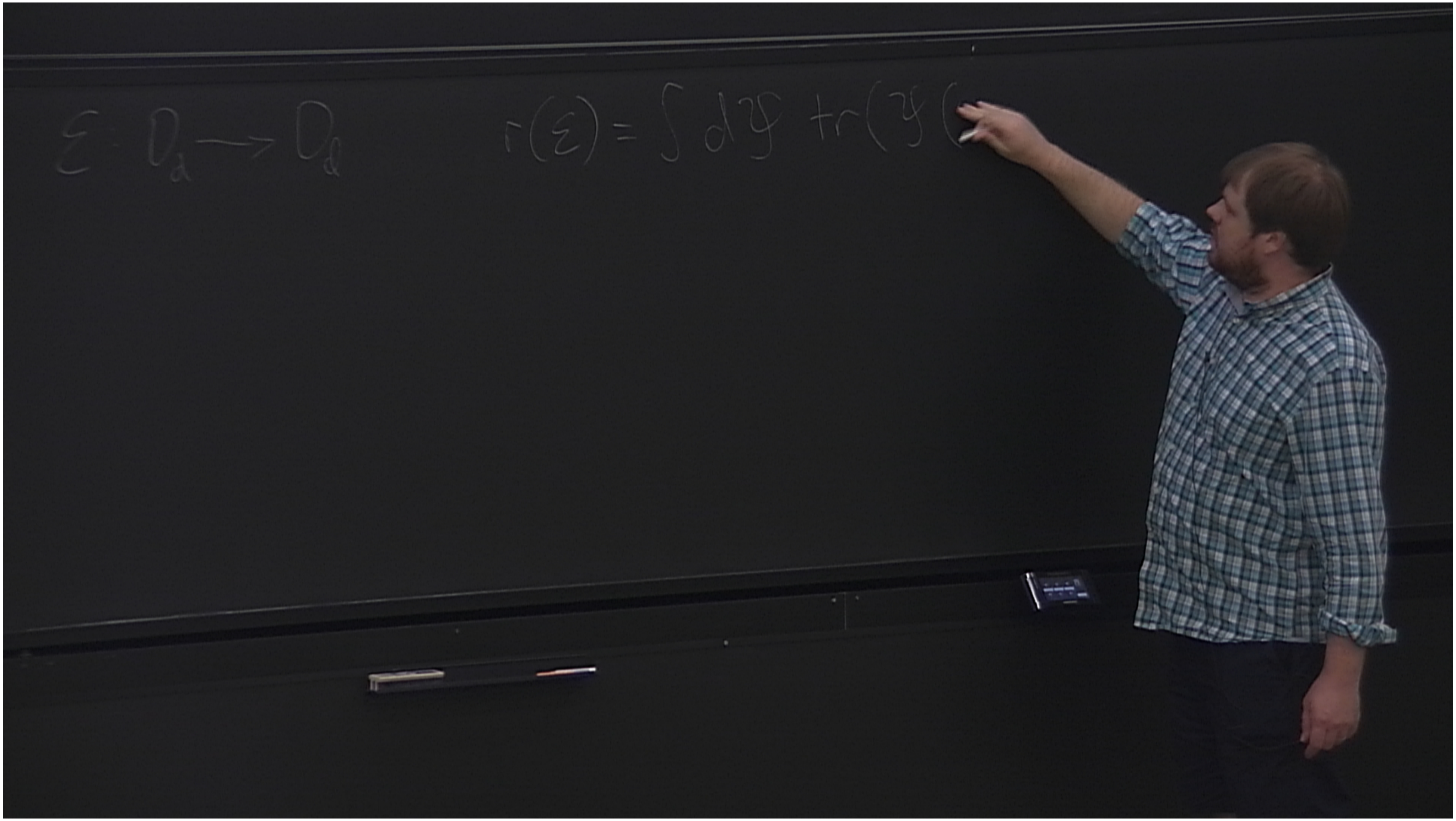
Title: Characterizing the coherence of errors

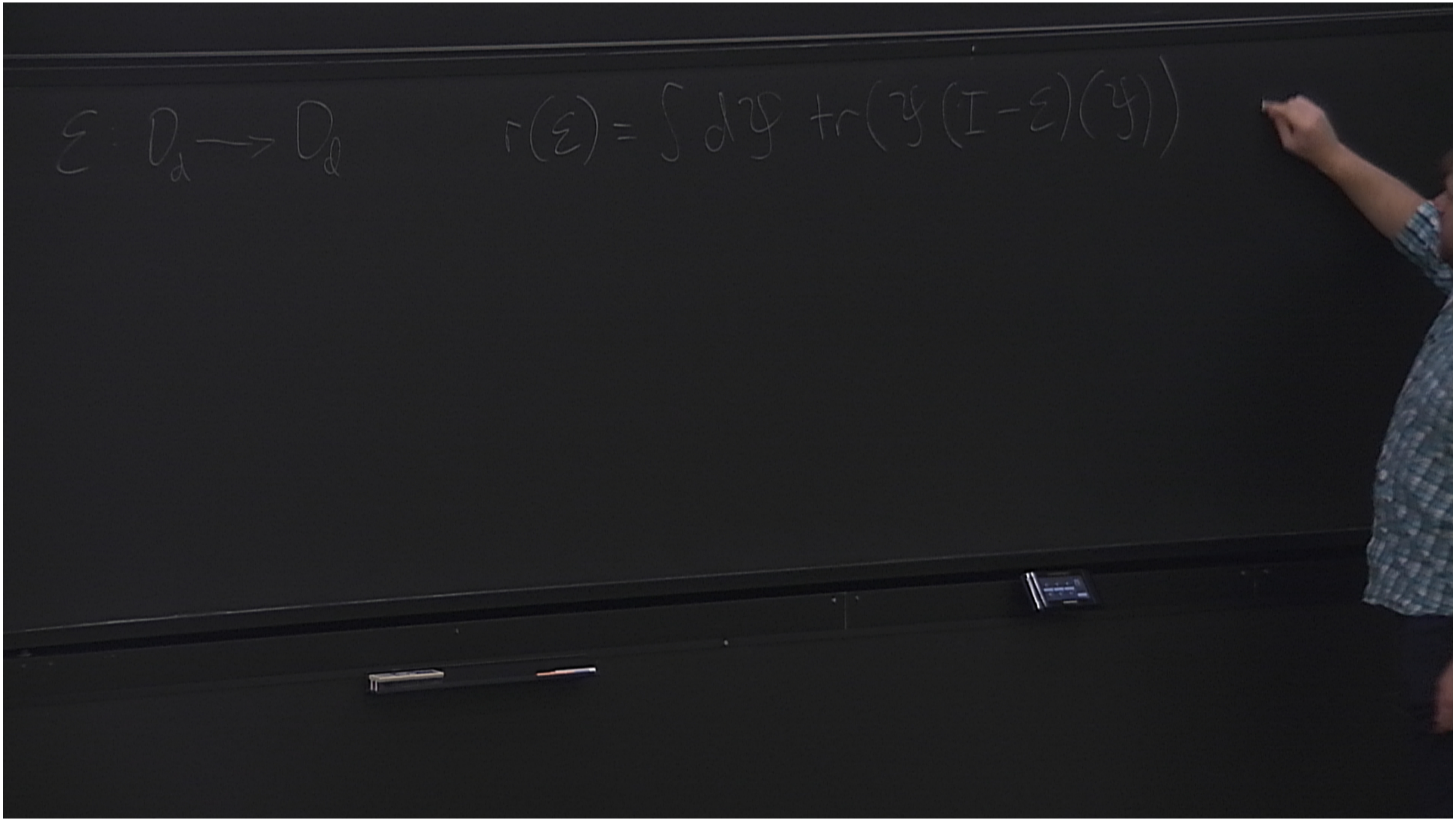
Date: Sep 30, 2015 04:00 PM

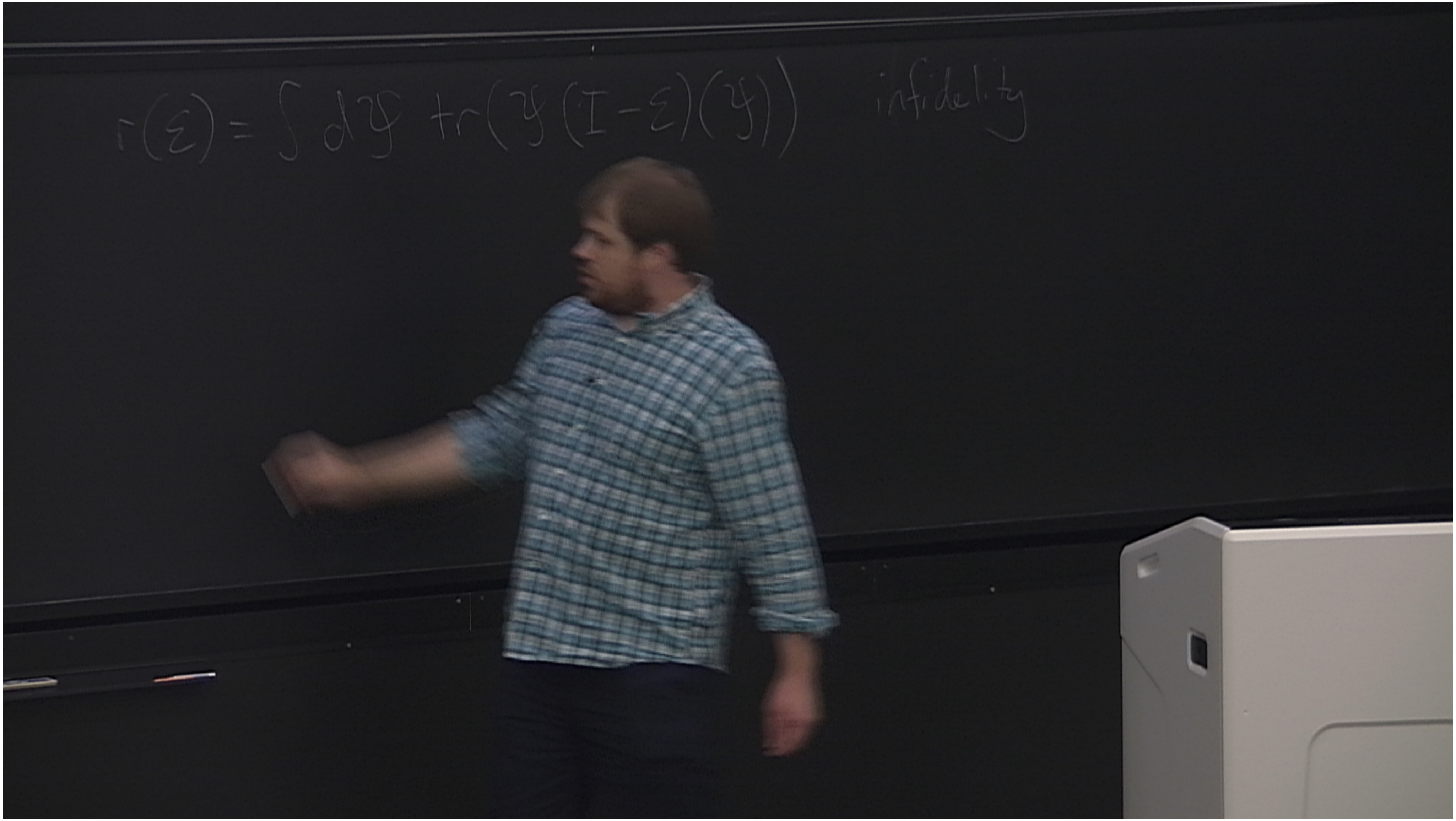
URL: http://pirsa.org/15090088

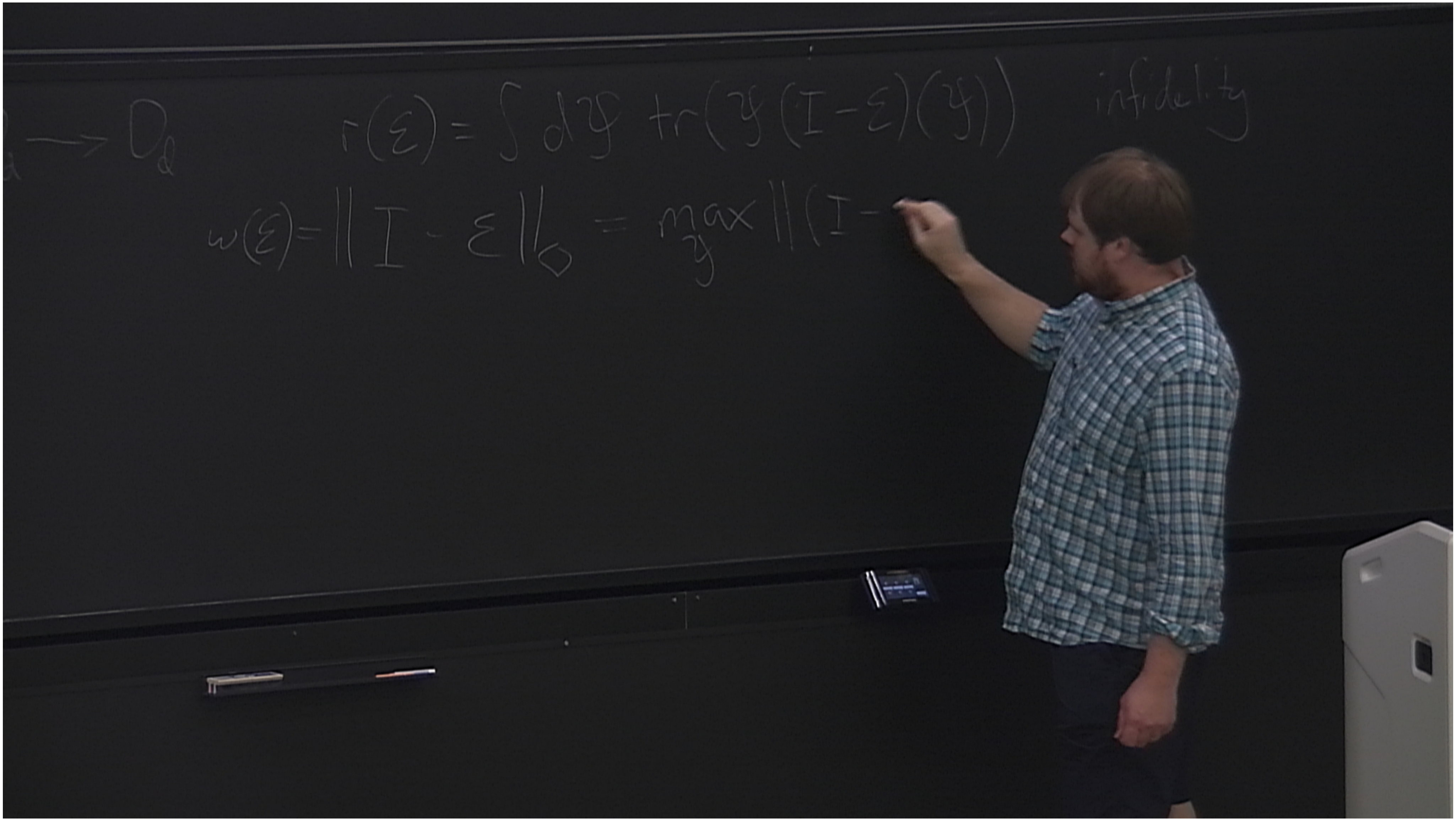
Abstract: <p>I will introduce the unitarity, a parameter quantifying the coherence of a channel and show that it is useful for two reasons. First, it can be efficiently estimated via a variant of randomized benchmarking. Second, it captures useful information about the channel, such as the optimal fidelity achievable with unitary corrections and an improved bound on the diamond distance.</p>


$\mathcal{E}: D_d \longrightarrow D_d$

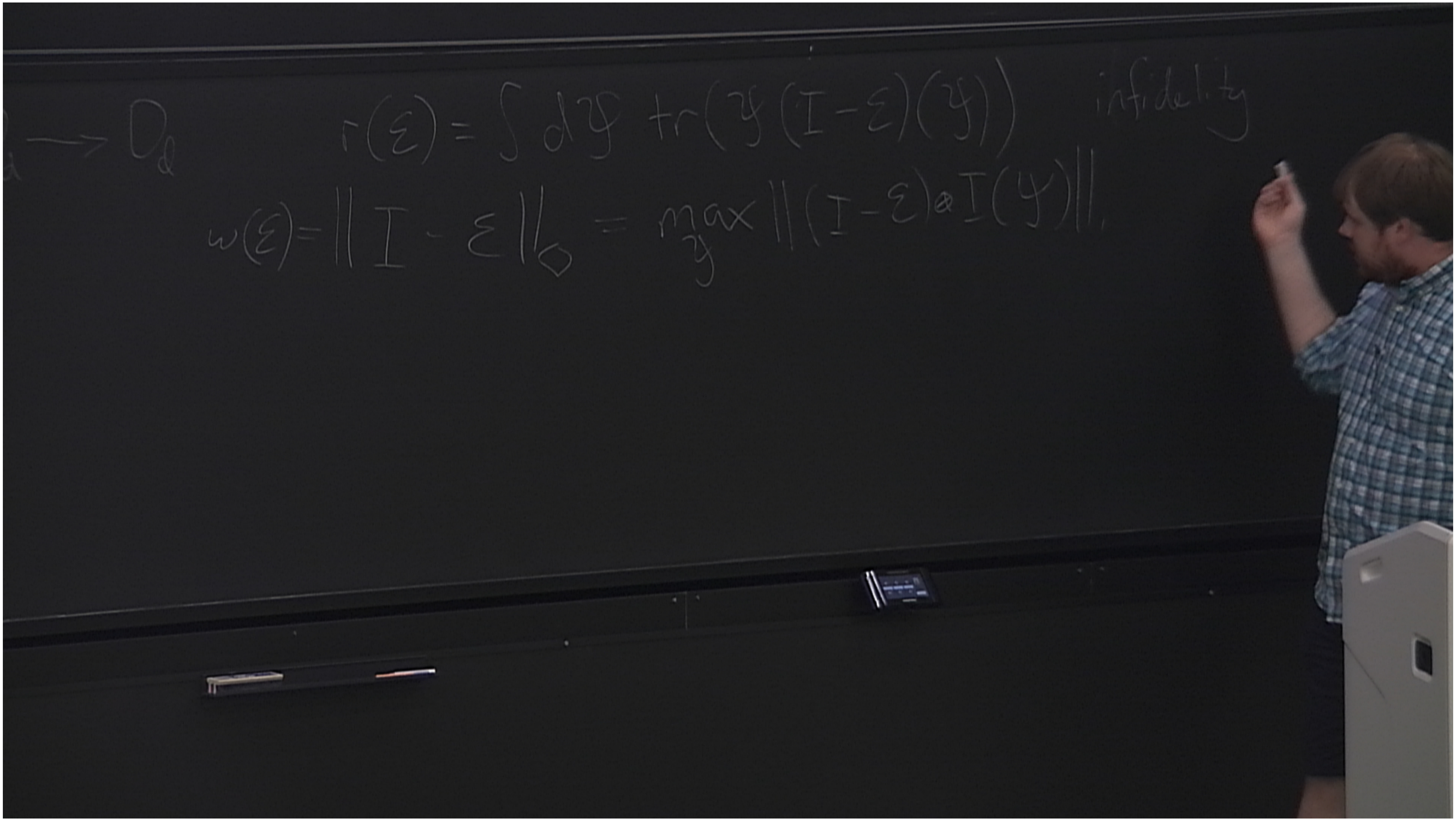

$\mathcal{E}: D_d \longrightarrow D_d$
$r(\mathcal{E}) = \int d\psi \; tr(\psi($

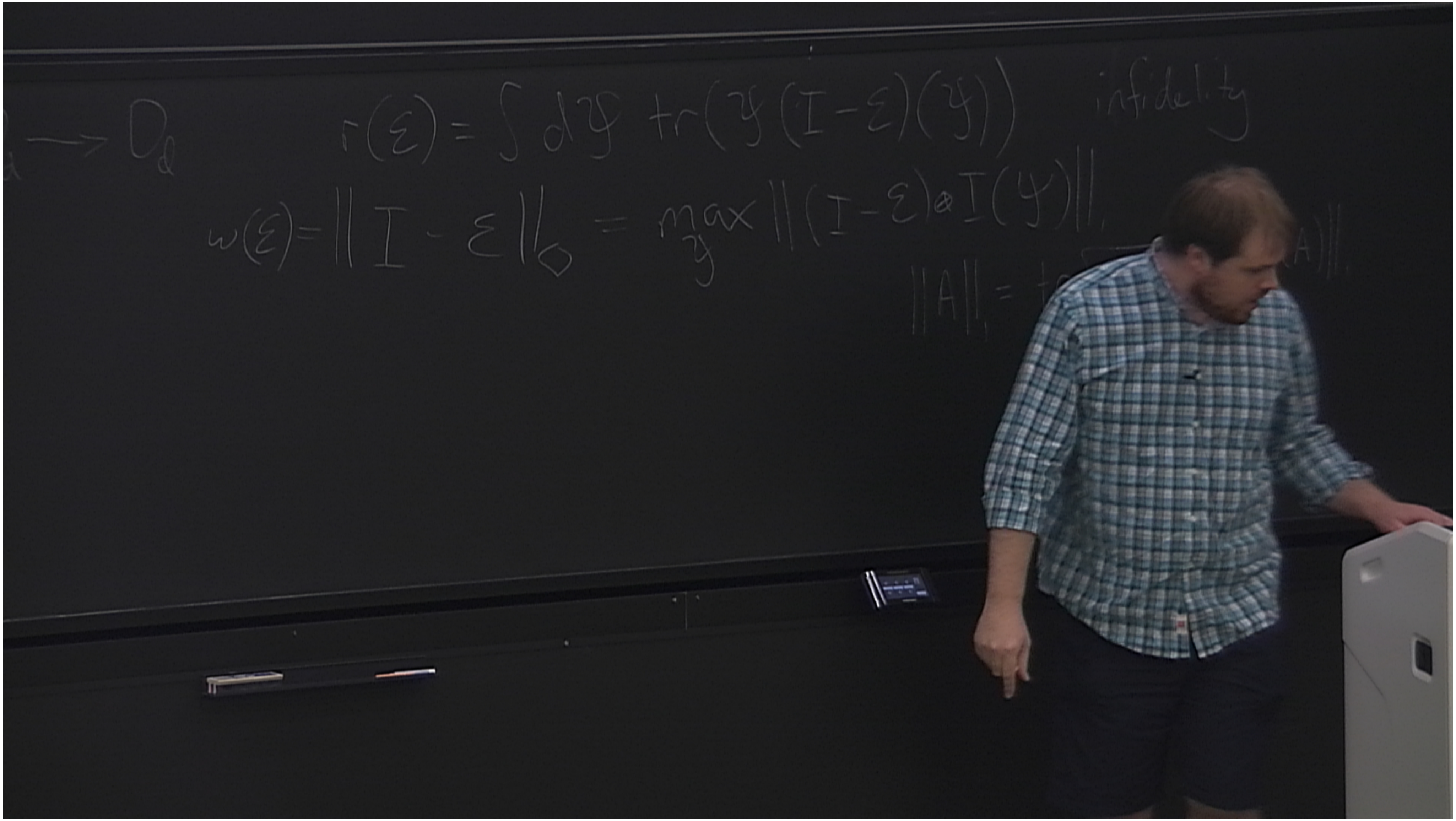

$\mathcal{E} : D_d \longrightarrow D_d$
$r(\mathcal{E}) = \int d\psi \; tr(\psi (I - \mathcal{E})(\psi))$

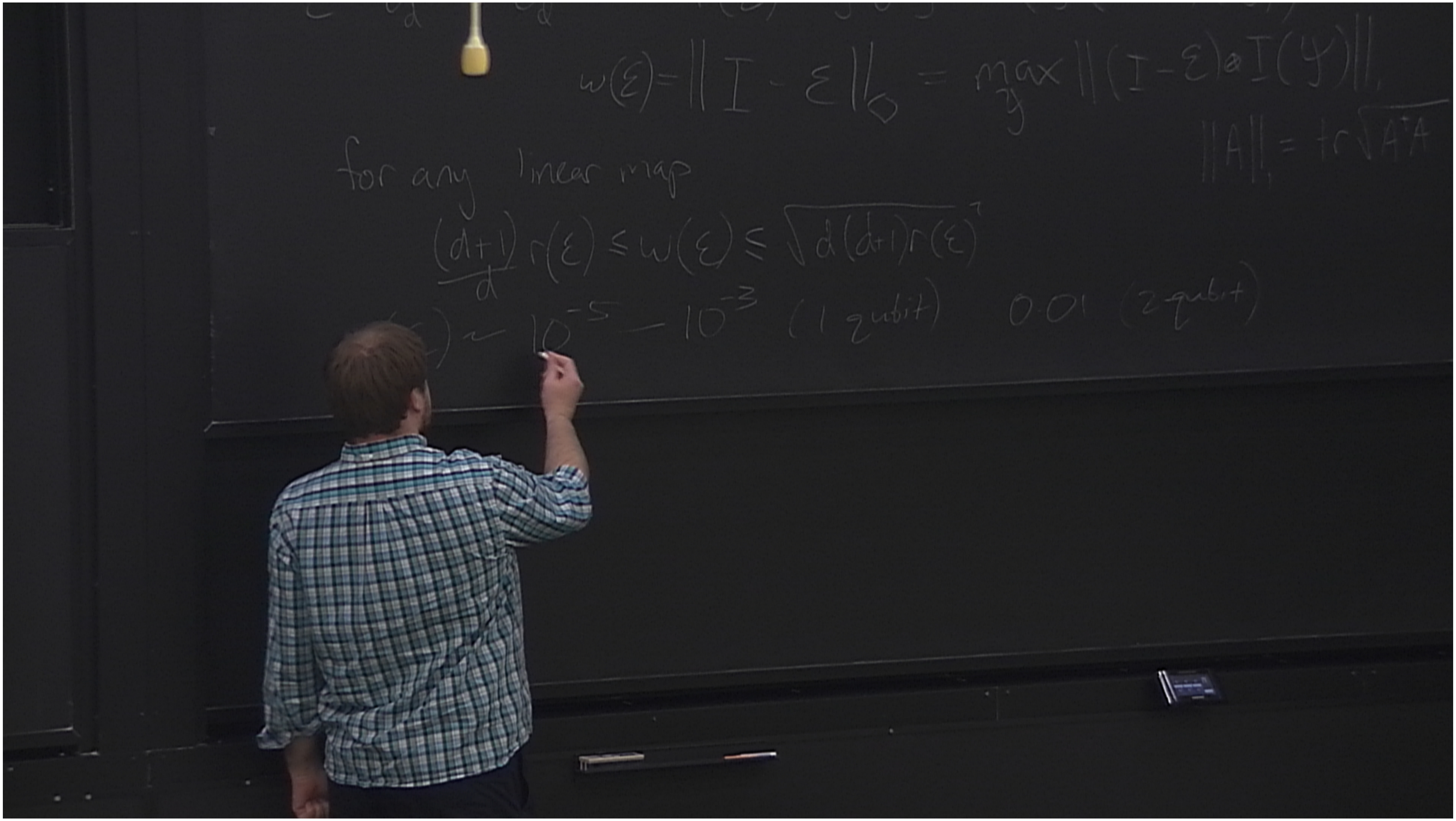

r(ε) = ∫ dψ tr(ψ(I − ε)(ψ))
infidelity

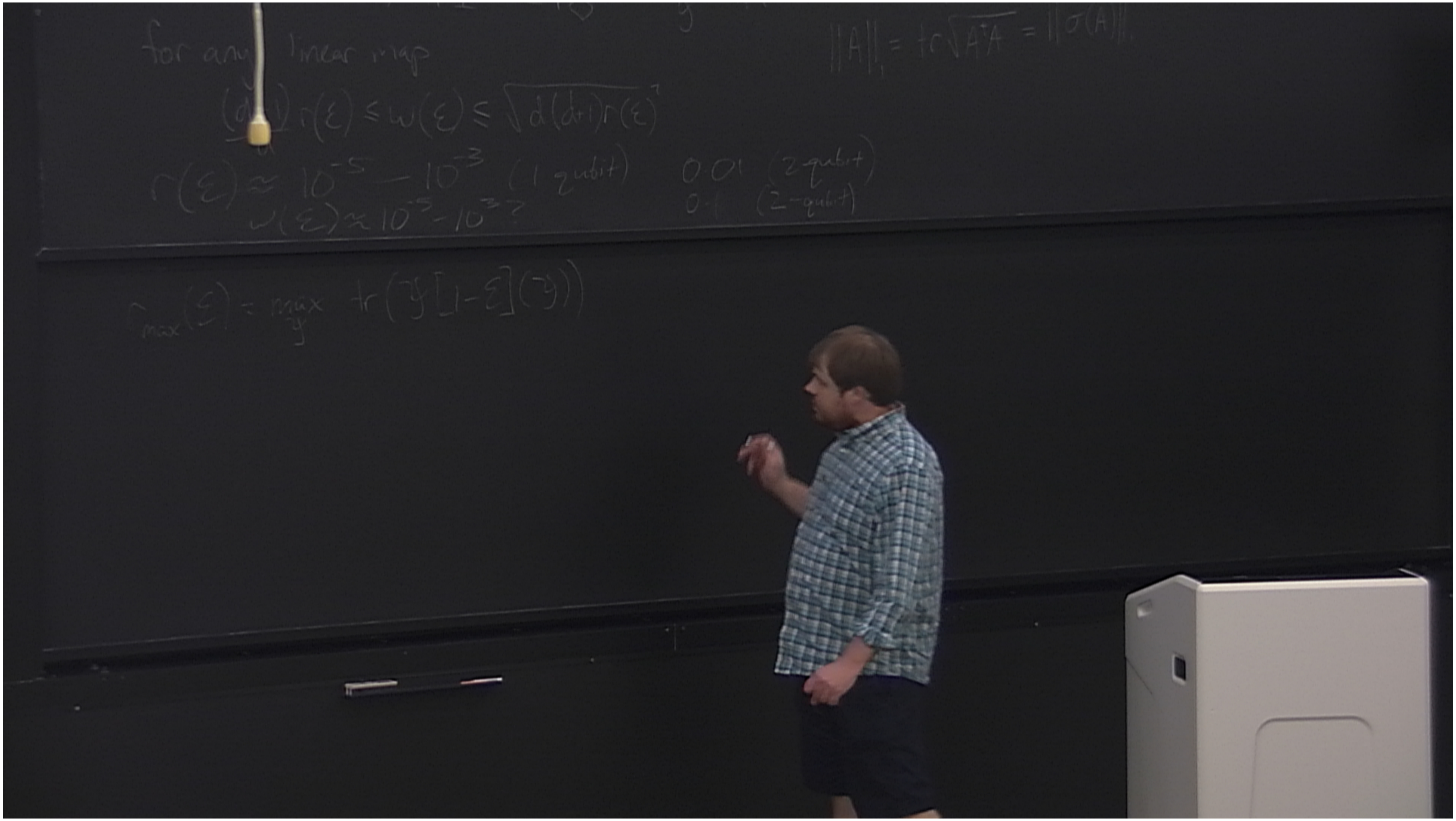


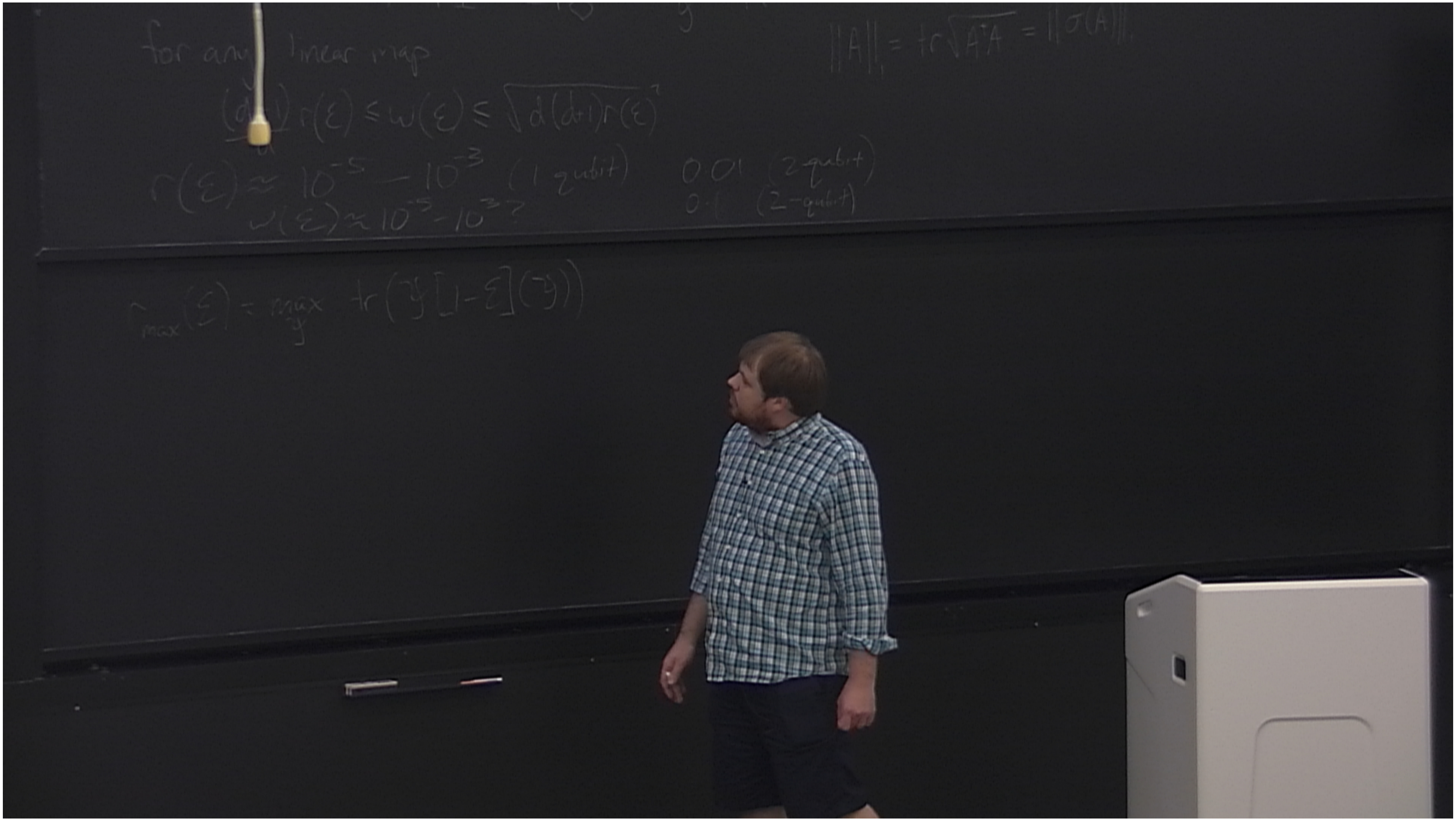
$r(\mathcal{E}) = \int d\psi \, \mathrm{tr}(\psi (I - \mathcal{E})(\psi))$
infidelity
$w(\mathcal{E}) = \| I - \mathcal{E} \|_\diamond = \max_\psi \| (I -$

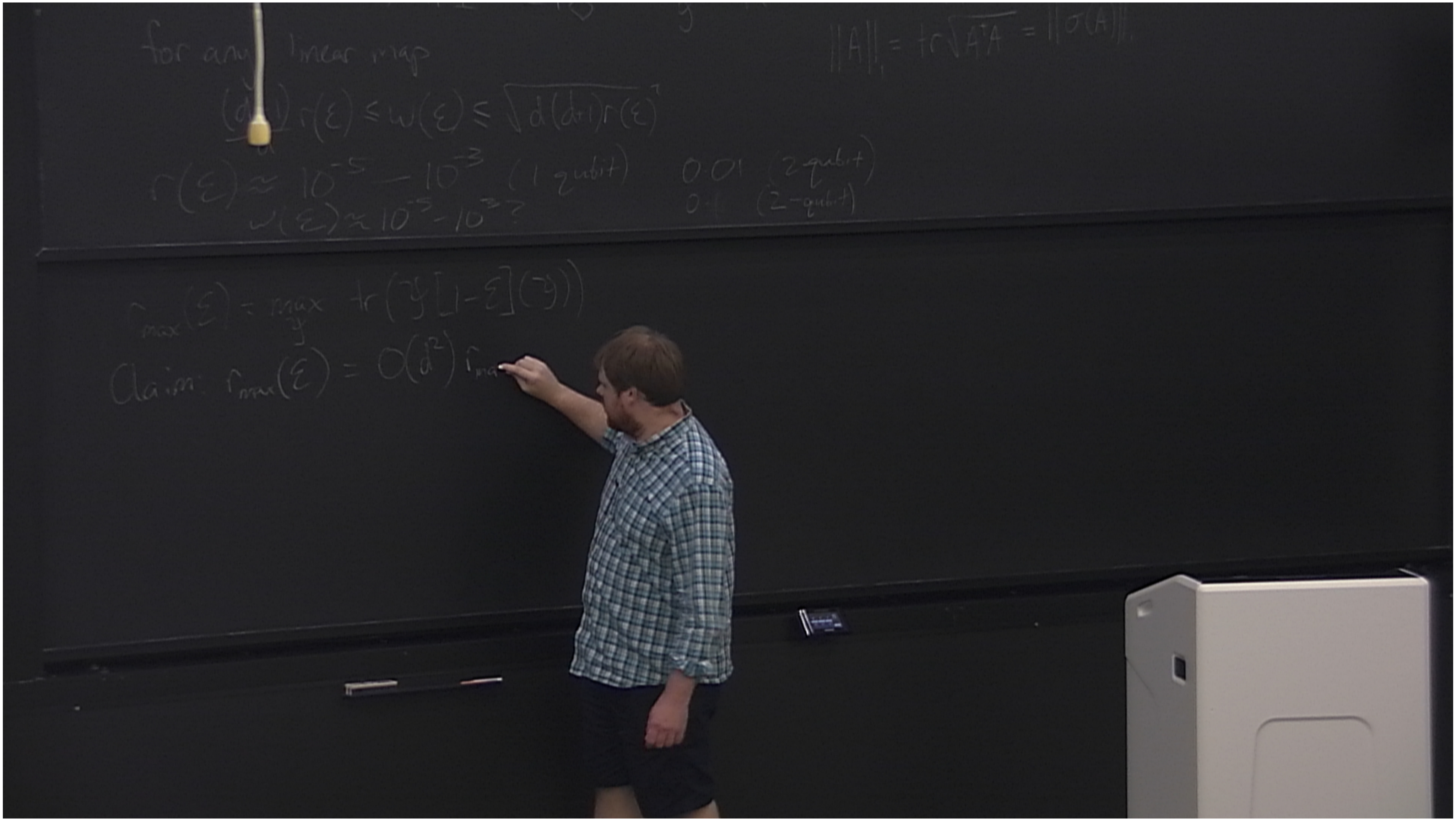

$\longrightarrow D_d$
$r(\mathcal{E}) = \int d\psi \, tr(\psi (I - \mathcal{E})(\psi))$ infidelity
$w(\mathcal{E}) = \| I - \mathcal{E} \|_\diamond = \max_\psi \| (I - \mathcal{E}) \otimes I(\psi) \|_1$

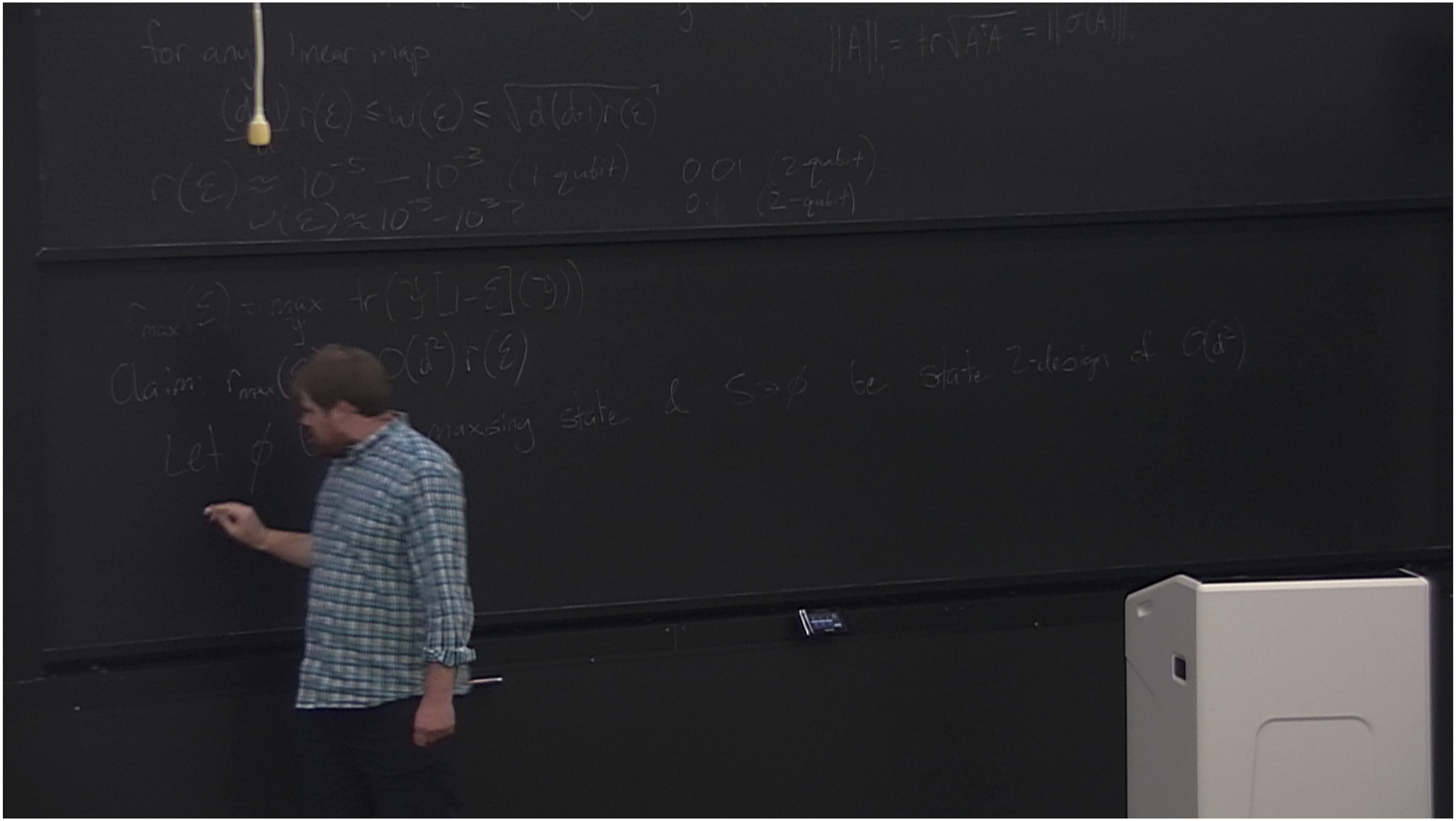

r(E) = ∫ dψ tr(ψ (I − E)(ψ))
infidelity
w(E) = || I − E ||◇ = max ||(I − E)⊗I(ψ)||1
||A||1 = tr

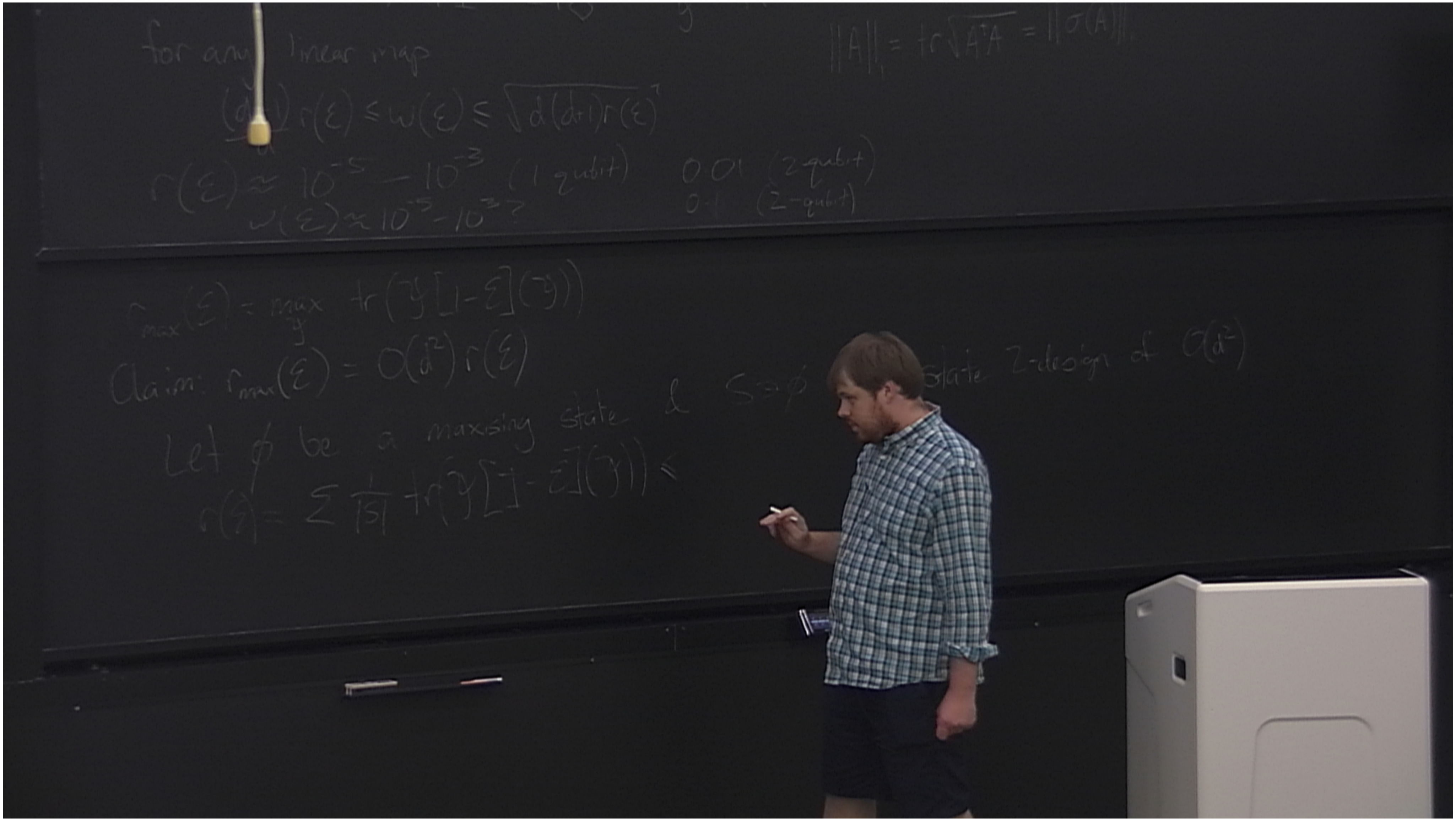

w(E) = ||I - E||◇ = max_ψ ||(I - E)⊗I(ψ)||_1
||A||_1 = tr√(A†A)
for any linear map
((d+1)/d) r(E) ≤ w(E) ≤ √(d(d+1)r(E))
r(E) ~ 10^-5 – 10^-3 (1 qubit) 0.01 (2-qubit)

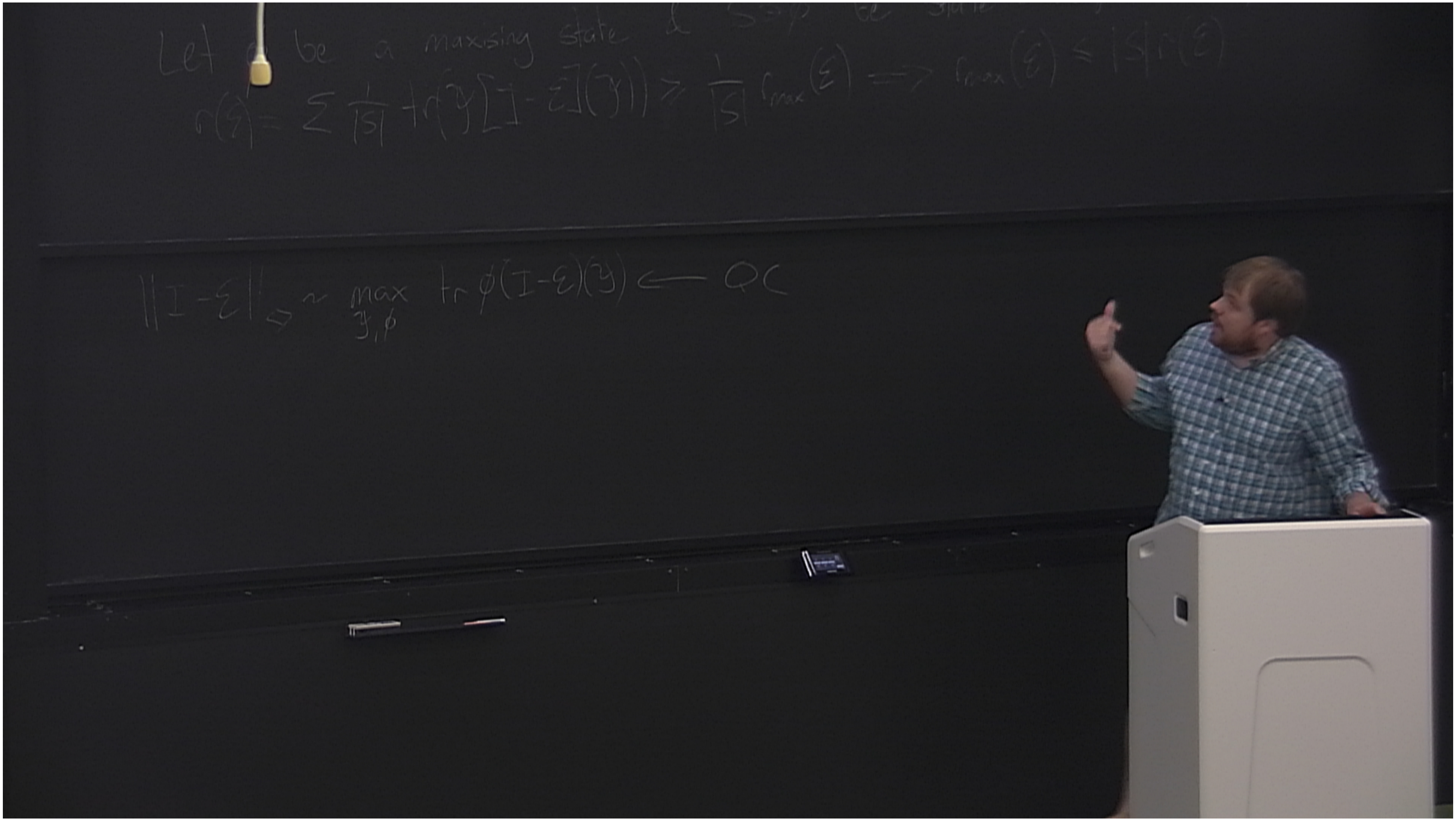

for any linear map
||A||1 = tr√(A†A) = ||σ(A)||1
(d+1) r(ε) ≤ w(ε) ≤ √(d(d+1) r(ε))
r(ε) ≈ 10^-5 — 10^-3 (1 qubit) 0.01 (2-qubit)
w(ε) ≈ 10^-5 - 10^-3 ? 0.1 (2-qubit)
r max(ε) = max ψ tr(ψ[1-ε](ψ))

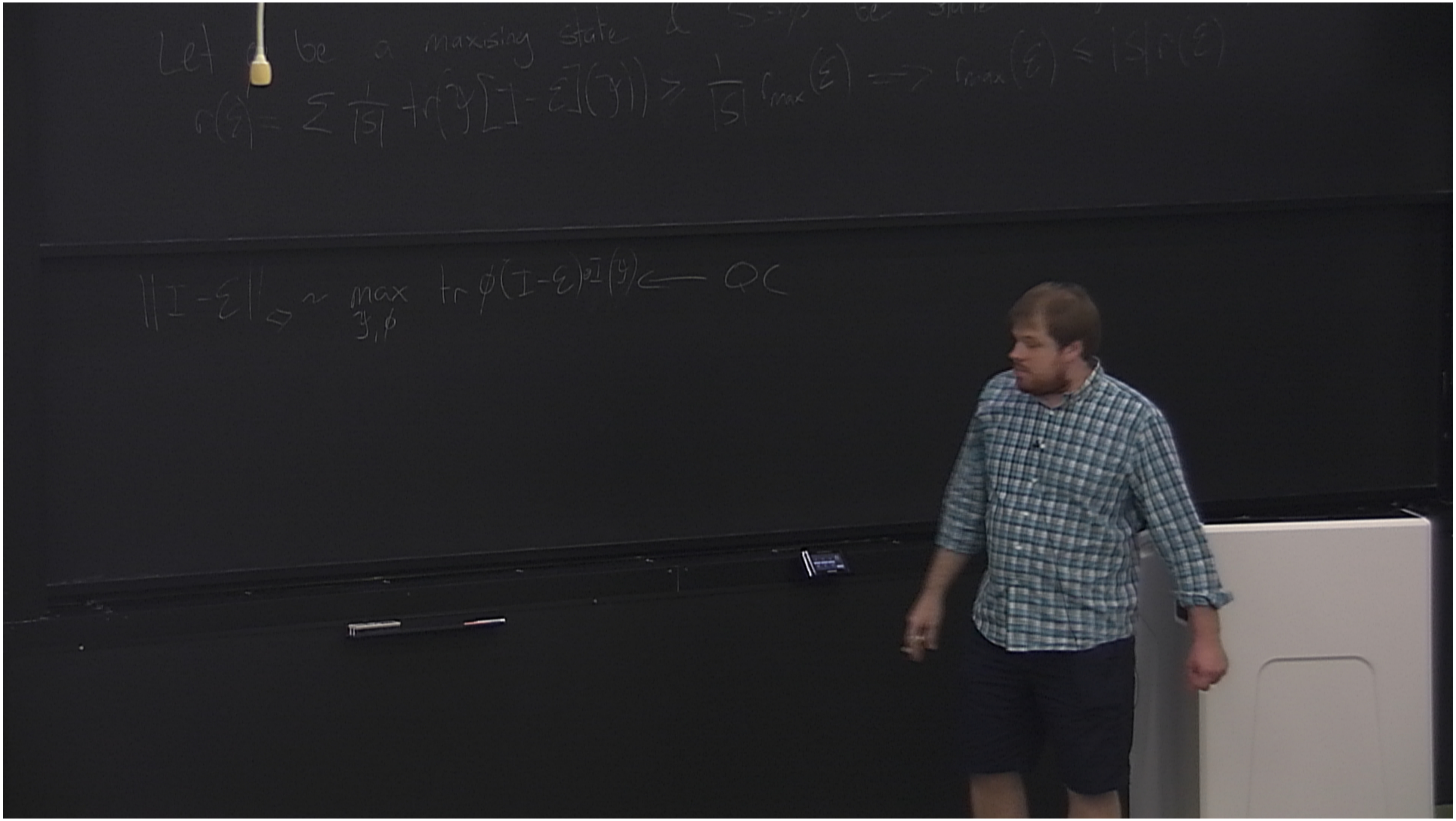

for any linear map
$\|A\|_1 = tr\sqrt{A^\dagger A} = \|\sigma(A)\|_1$
$r(\mathcal{E}) \approx 10^{-5} - 10^{-3}$ (1 qubit)
0.01 (2-qubit)
0.1 (2-qubit)
$w(\mathcal{E}) \approx 10^{-5} - 10^{-3}$?

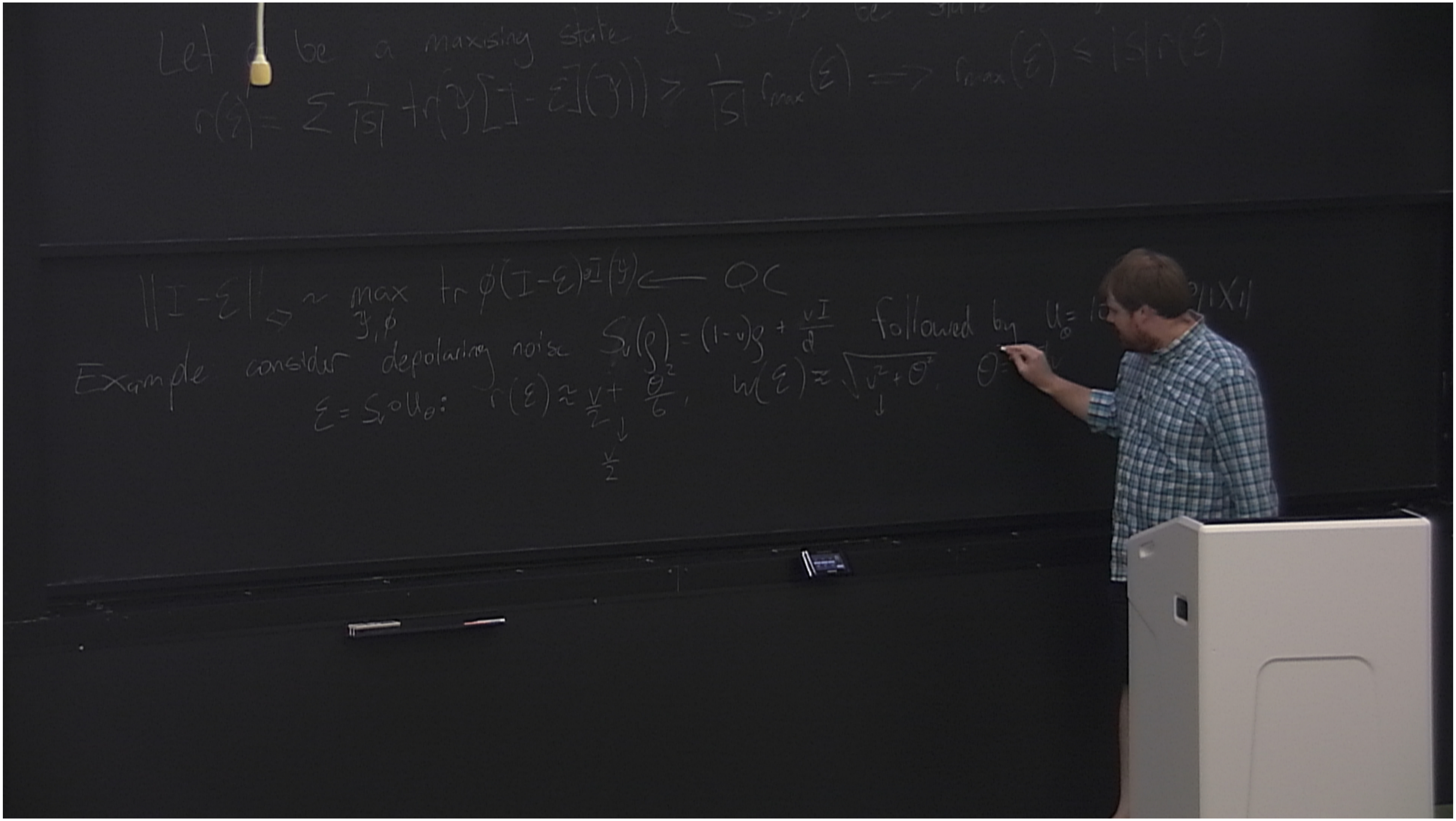

for any linear map
r(E) ≈ 10^-5 – 10^-3 (1 qubit)
0.01 (2-qubit)
(2-qubit)
Claim: r_max(E) = O(d^2)

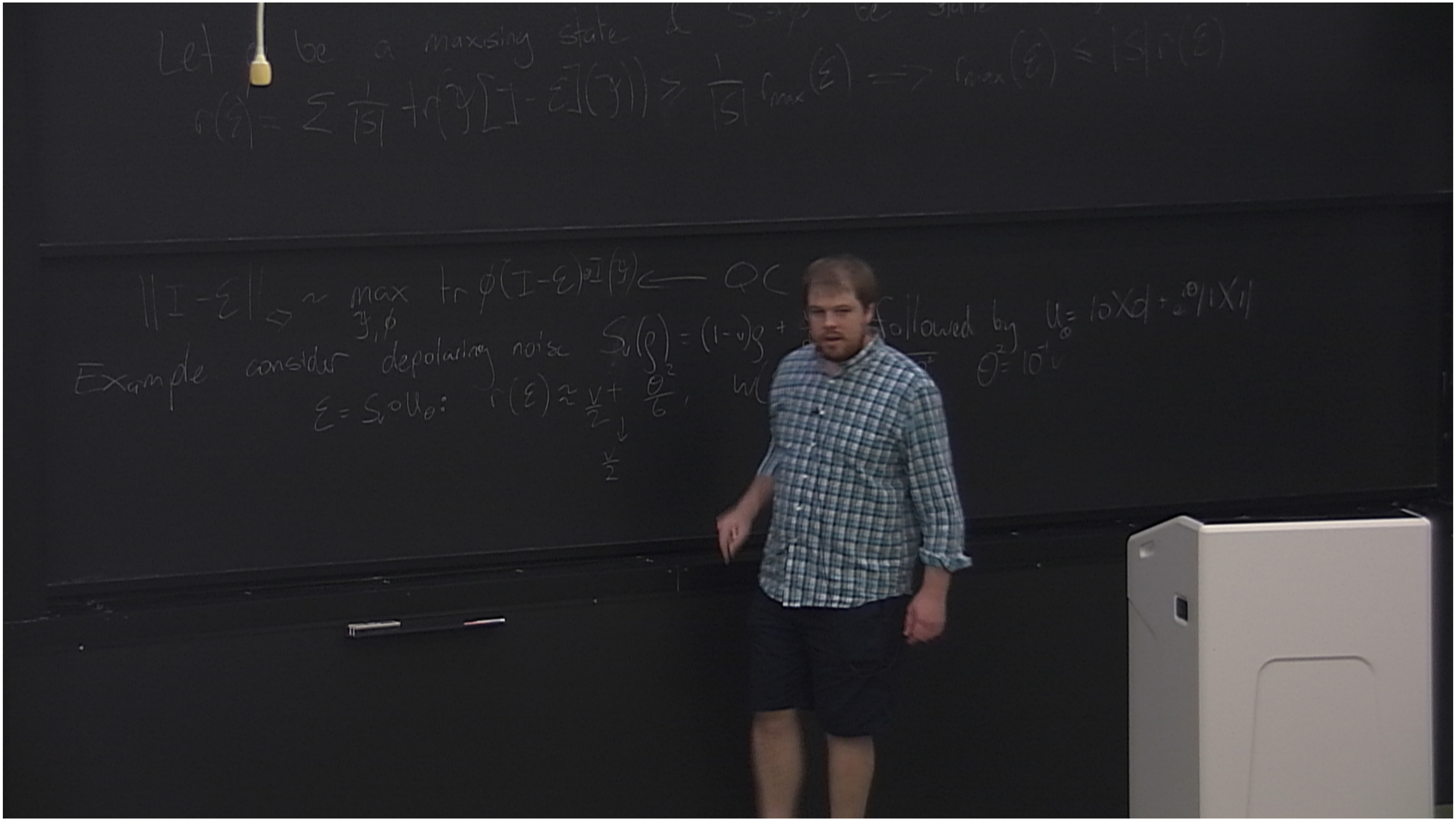

for any linear map
||A||_1 = tr √(A†A) = ||σ(A)||_1
r(ε) ≤ w(ε) ≤ √(d(d+1)r(ε))
r(ε) ≈ 10^-5 — 10^-3 (1 qubit)
0.01 (2-qubit)
w(ε) ≈ 10^-5 - 10^-3 ?
0.1 (2-qubit)
r_max(ε) = max tr(ψ[1-ε](ψ))
Claim: r_max(
O(d^2) r(ε)
Let φ
maxising state & S∋φ be state 2-design of O(d^2)

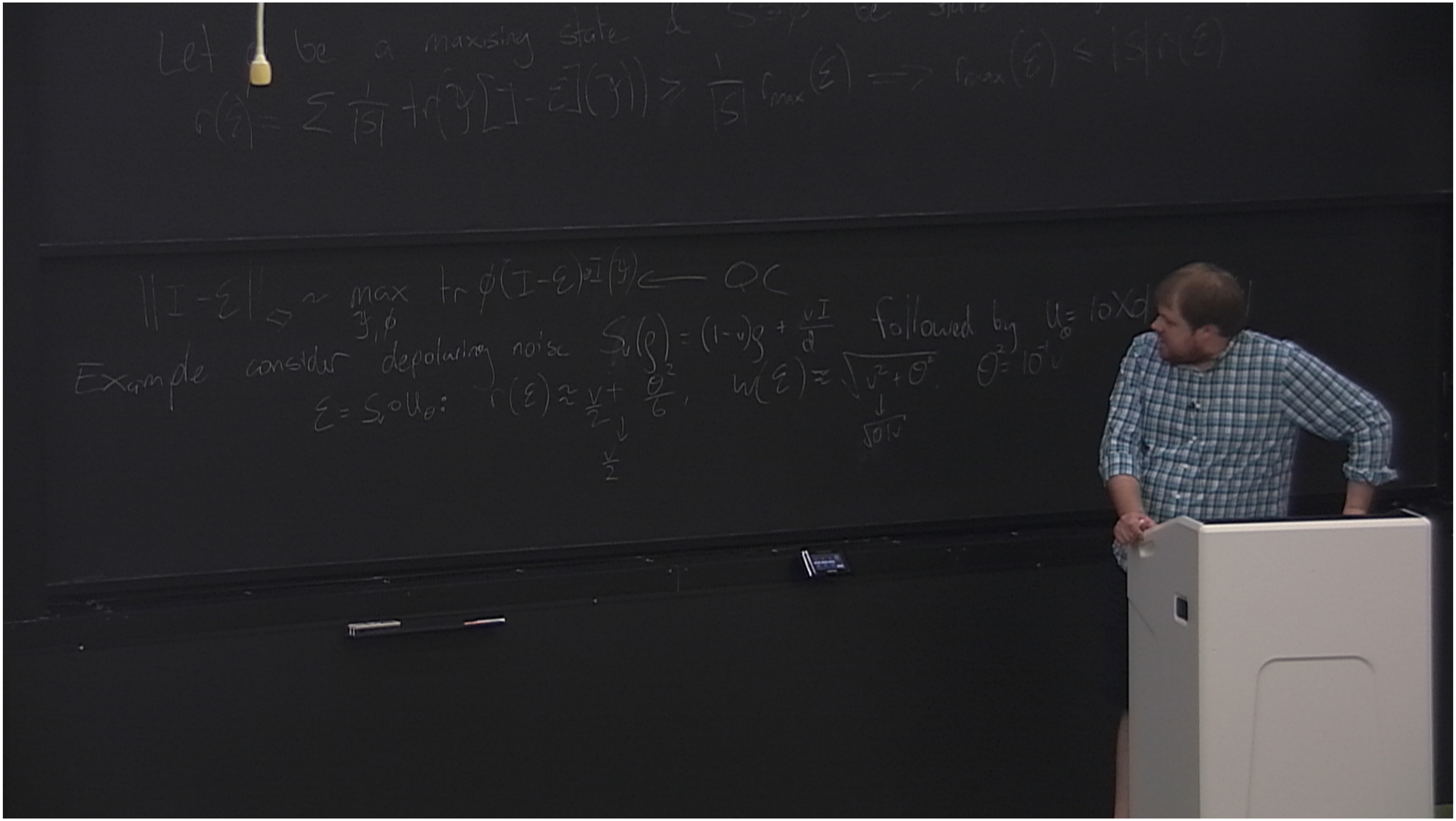

for any linear map
Claim:
Let φ be a maximising state &
state 2-design of
(1 qubit)
(2-qubit)


Let ρ be a maxising state
$\|I-\mathcal{E}\|_\diamond \sim \max_{\xi,\phi} \mathrm{tr}\,\phi(I-\mathcal{E})(\xi) \leftarrow QC$

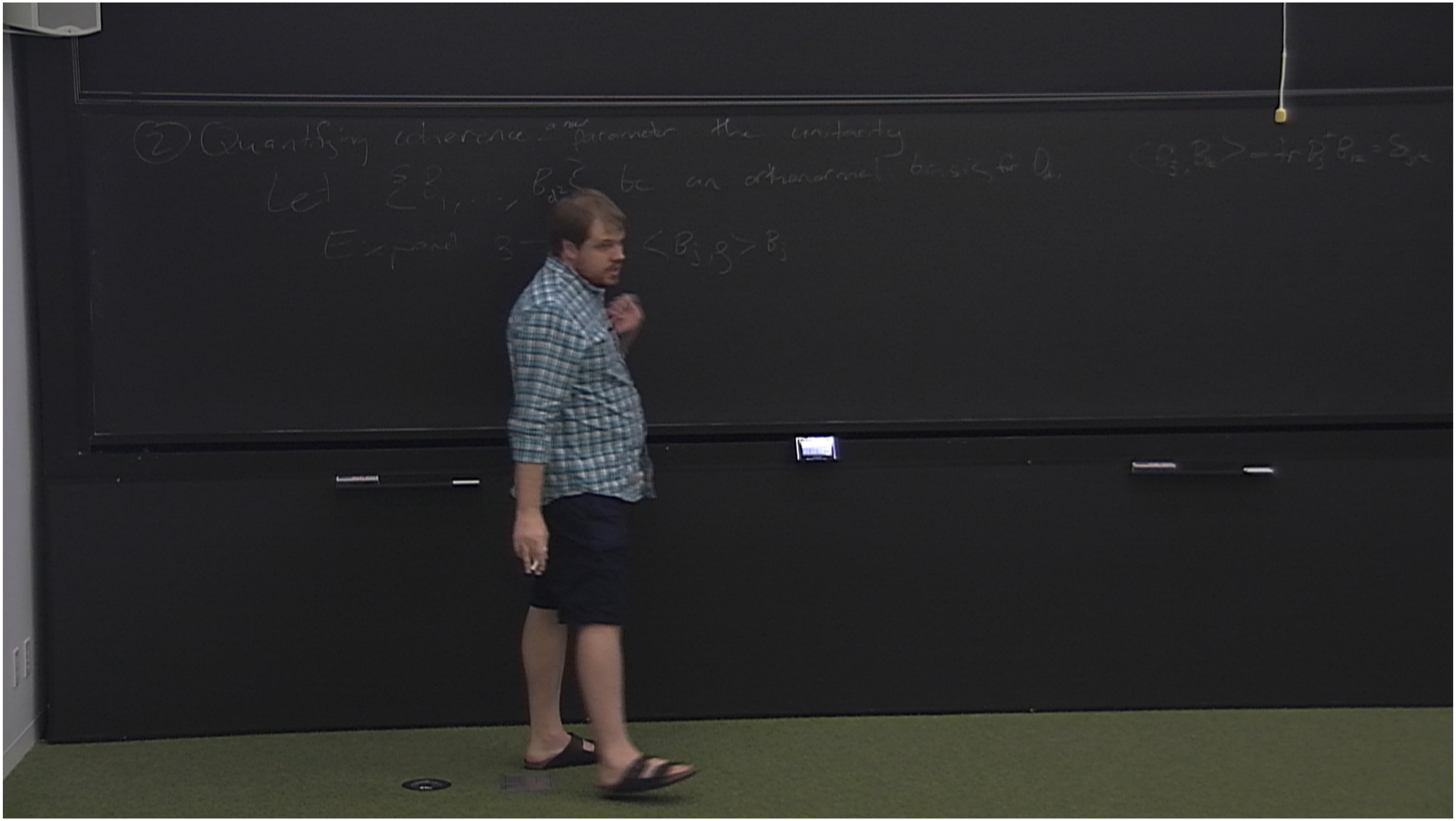


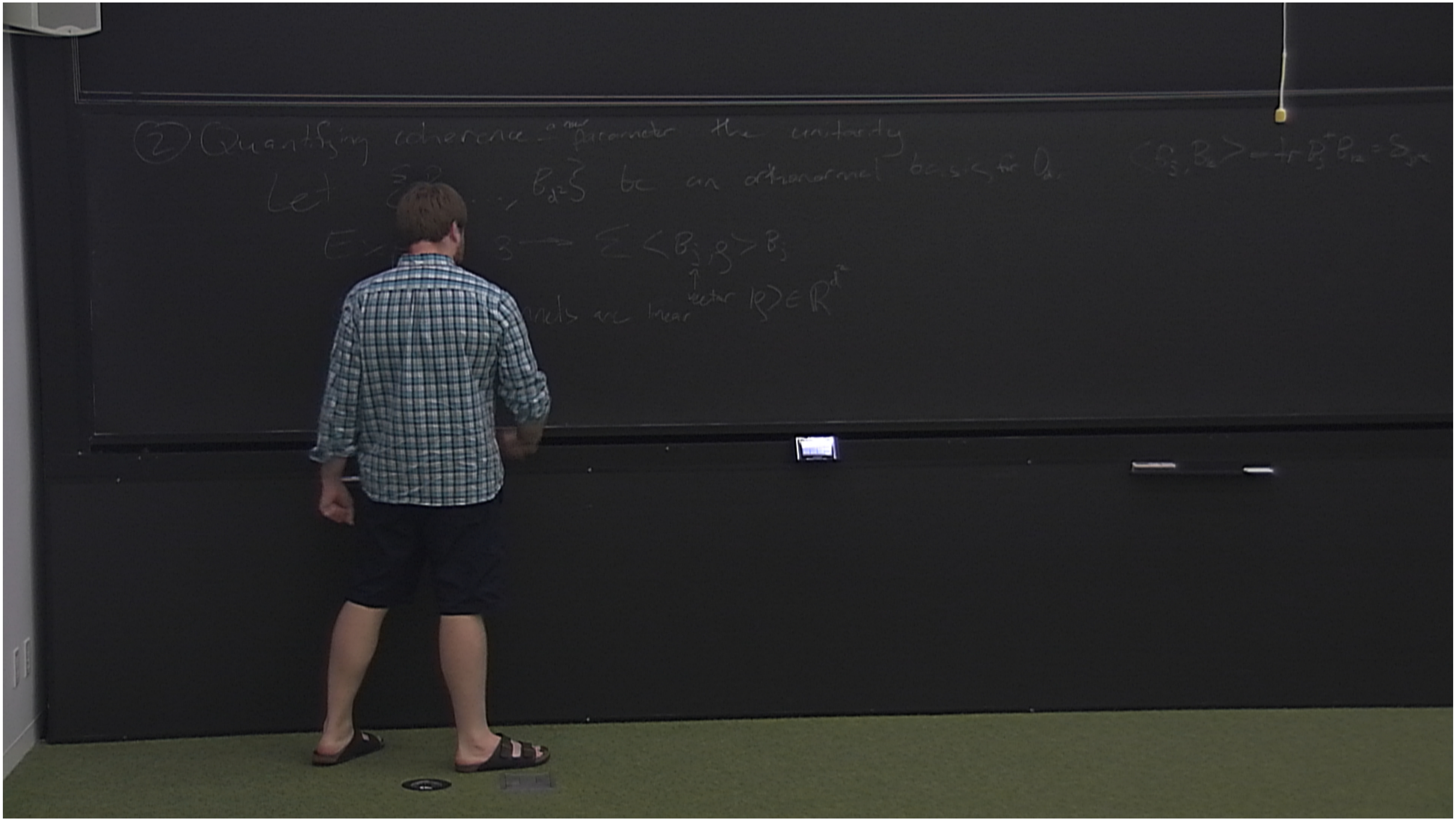



Let ρ be a maxising state
Example consider depolarizing noise
followed by

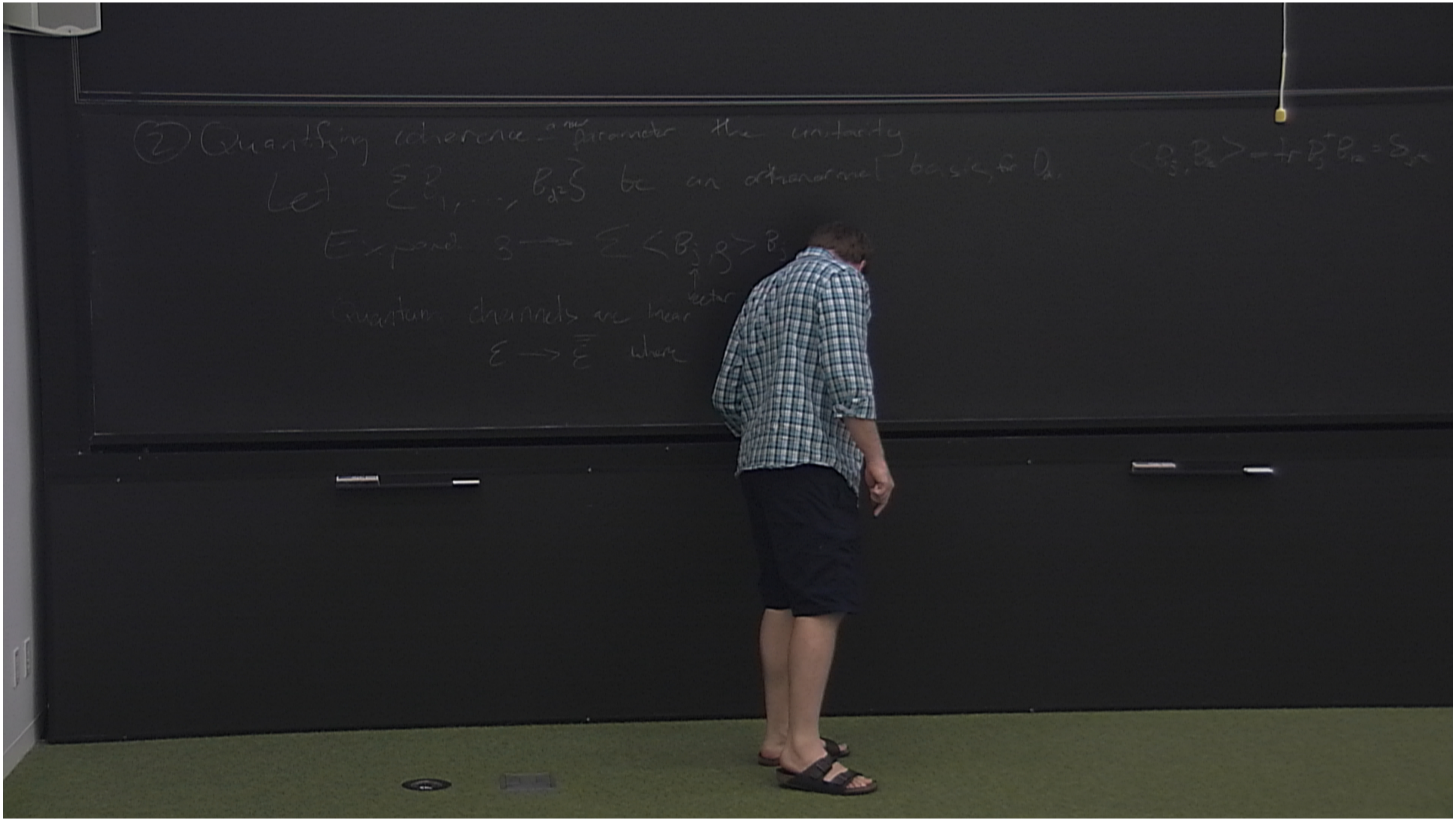


max
QC
Example consider depolarizing noise
followed by

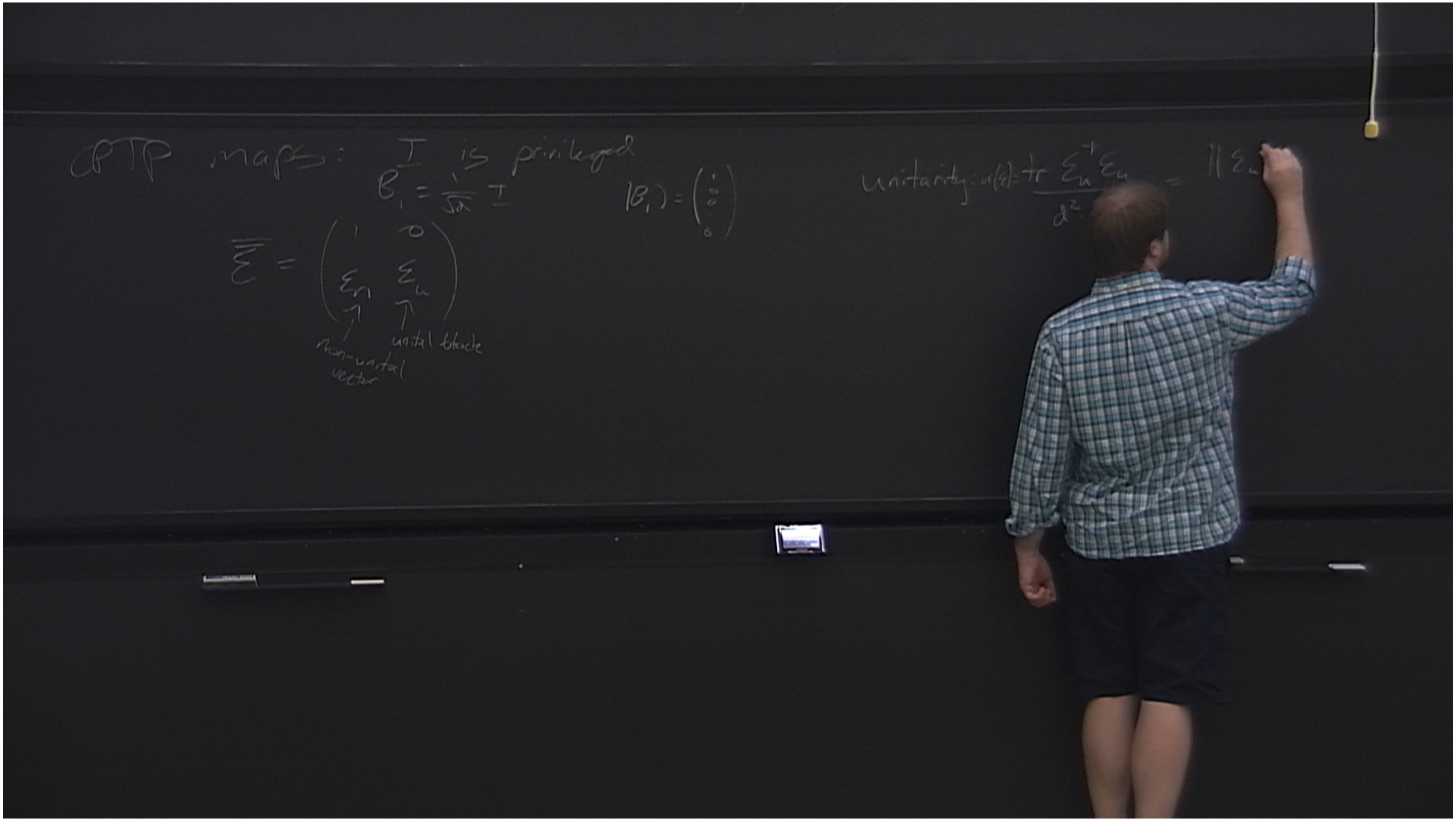

(2) Quantifying coherence: a new parameter the unitaerty
Let {B_1, ..., B_{d^2}} be an orthonormal basis for O_d.
Expand

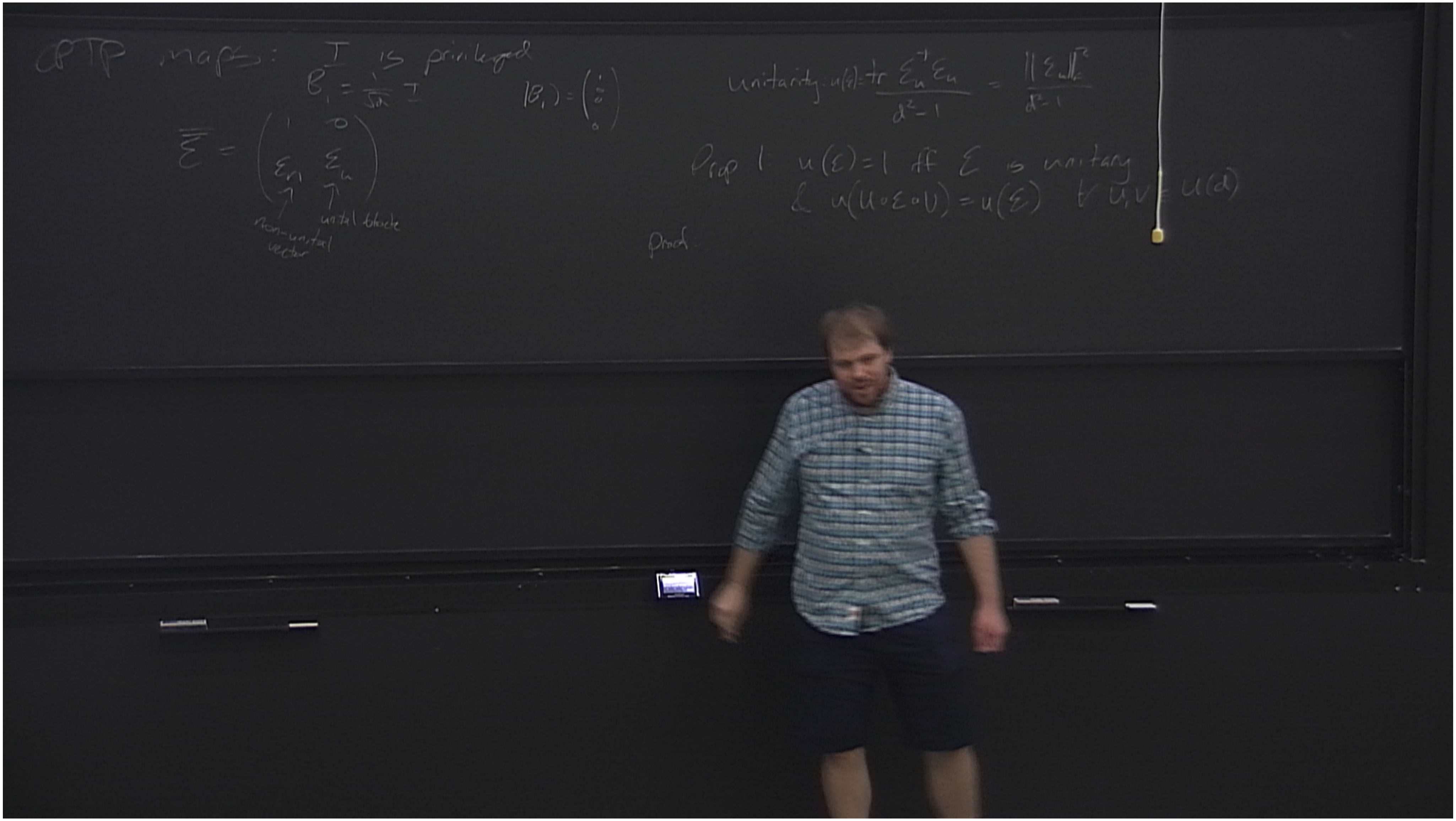


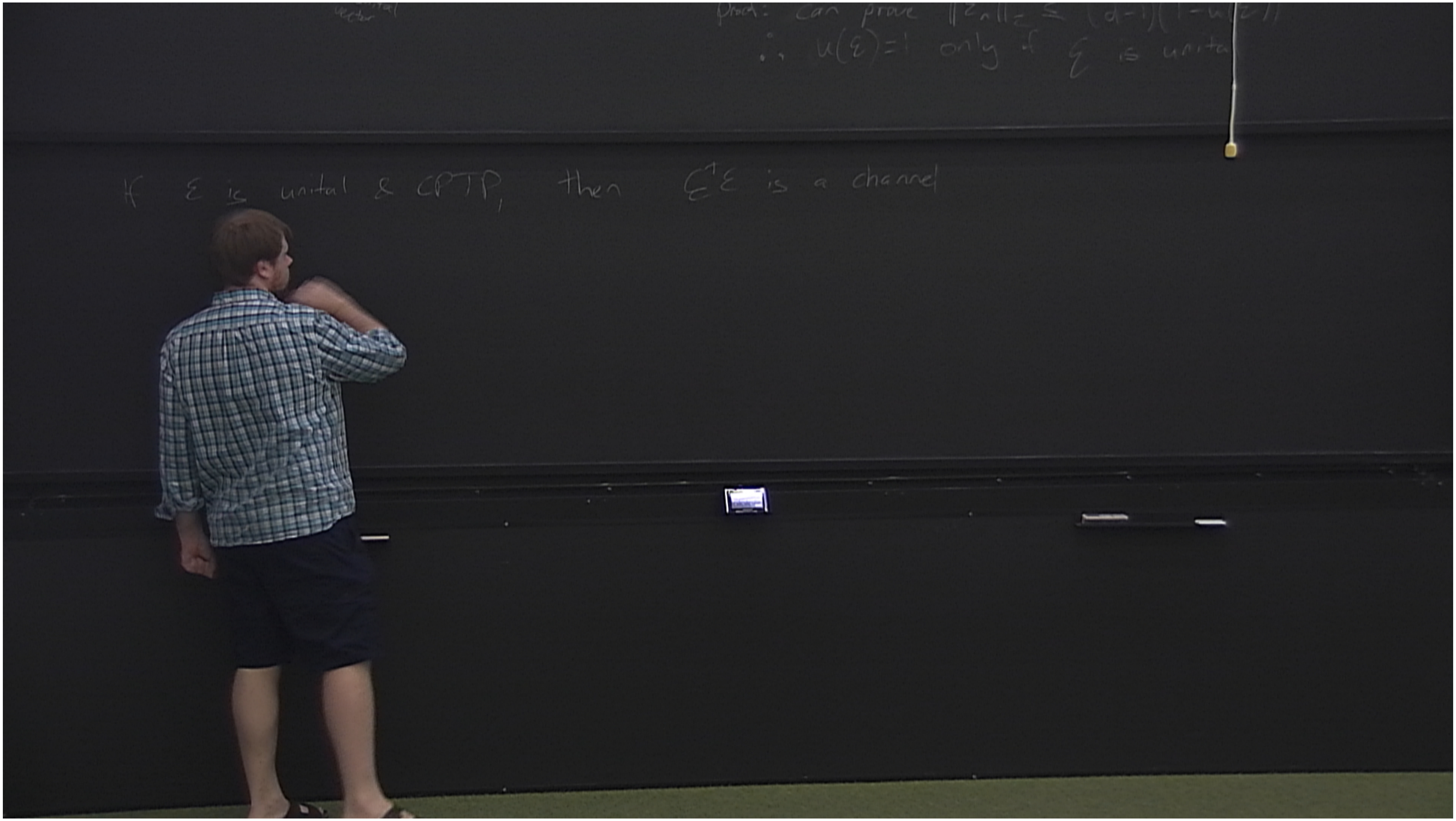
② Quantifying coherence
Let
be an orthonormal basis for

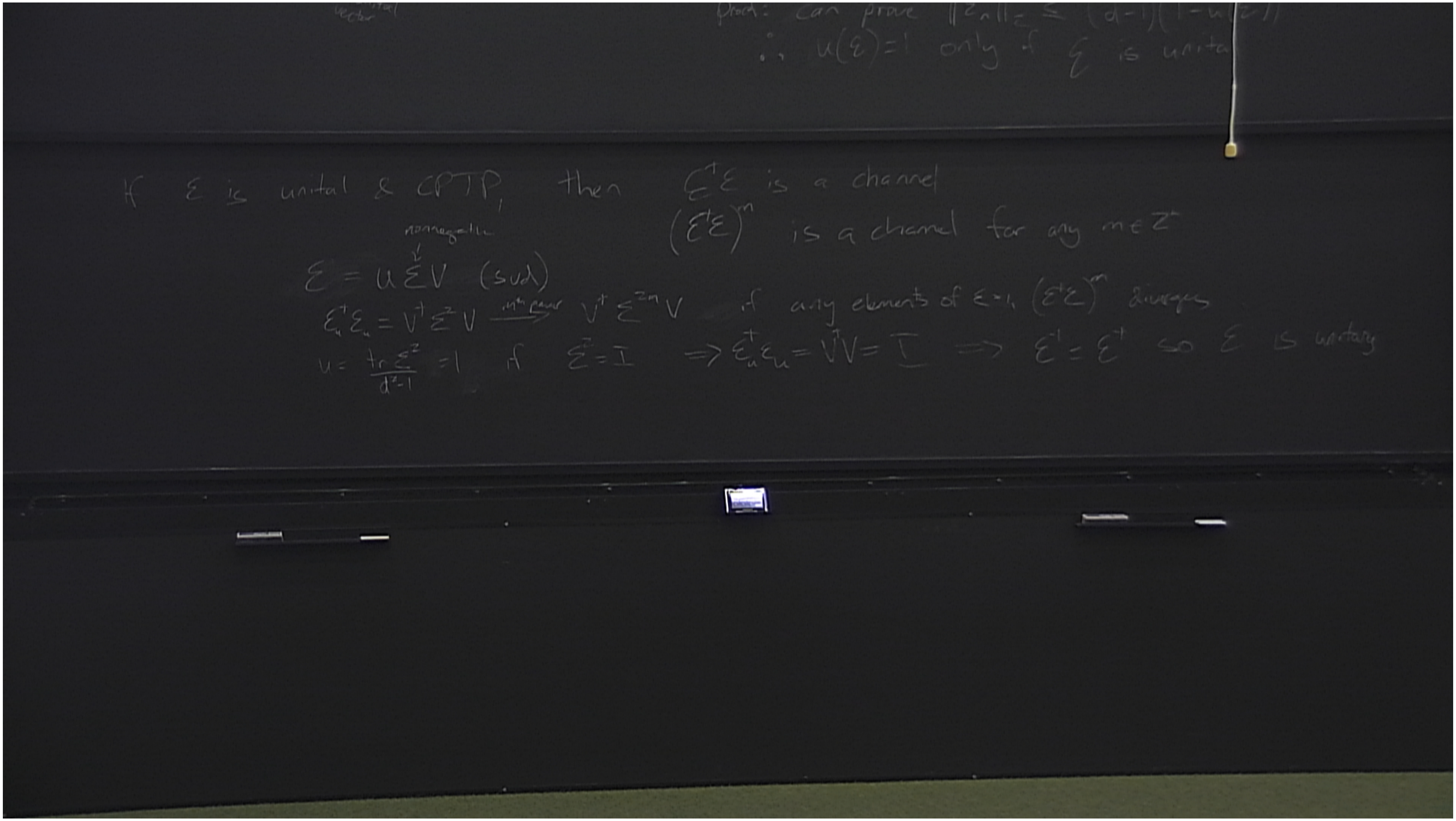


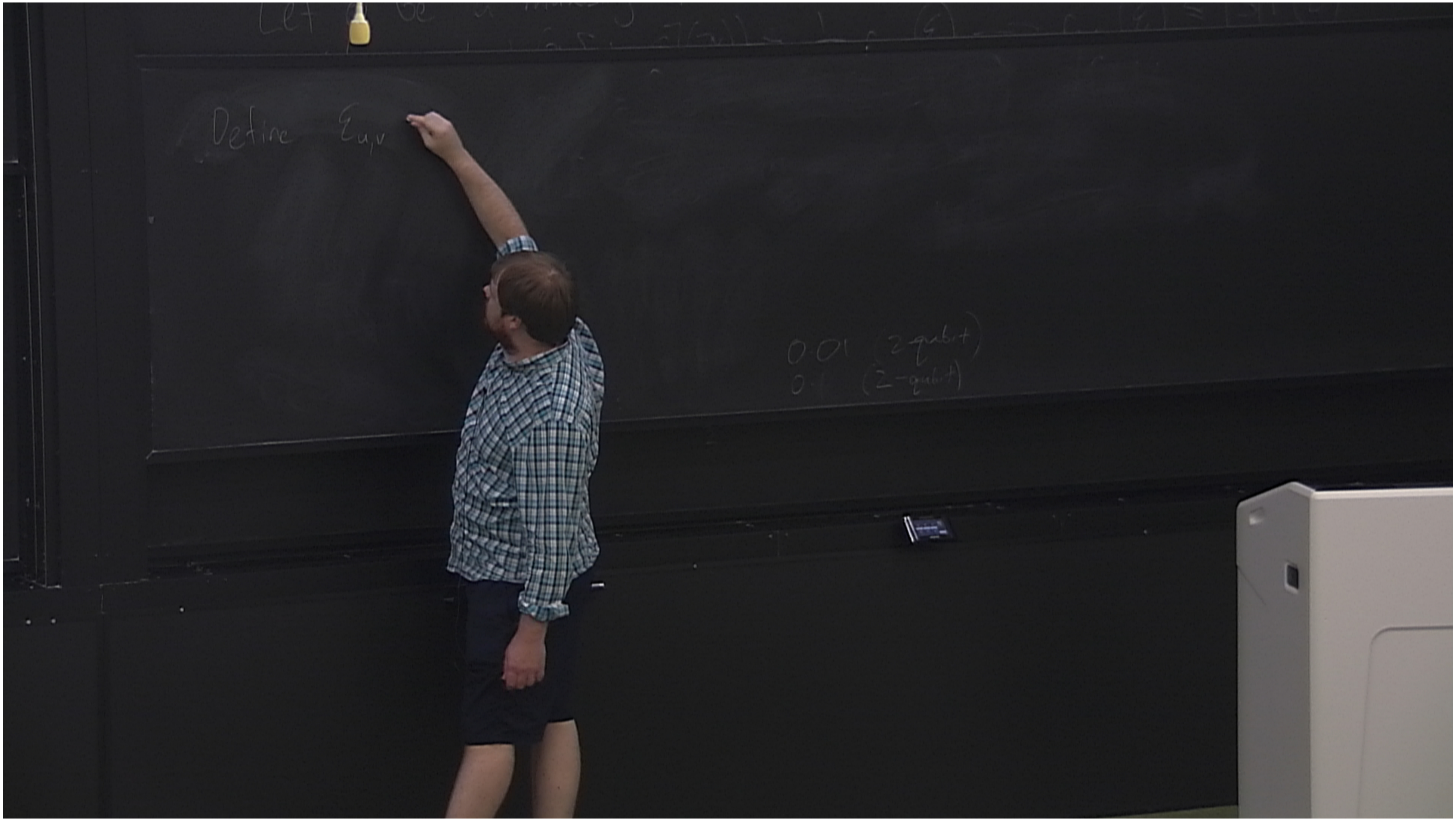
② Quantifying coherence
the unitarity
Let {B_1, …, B_{d^2}} be an orthonormal basis
Expand
vector
Quantum channels are linear
where

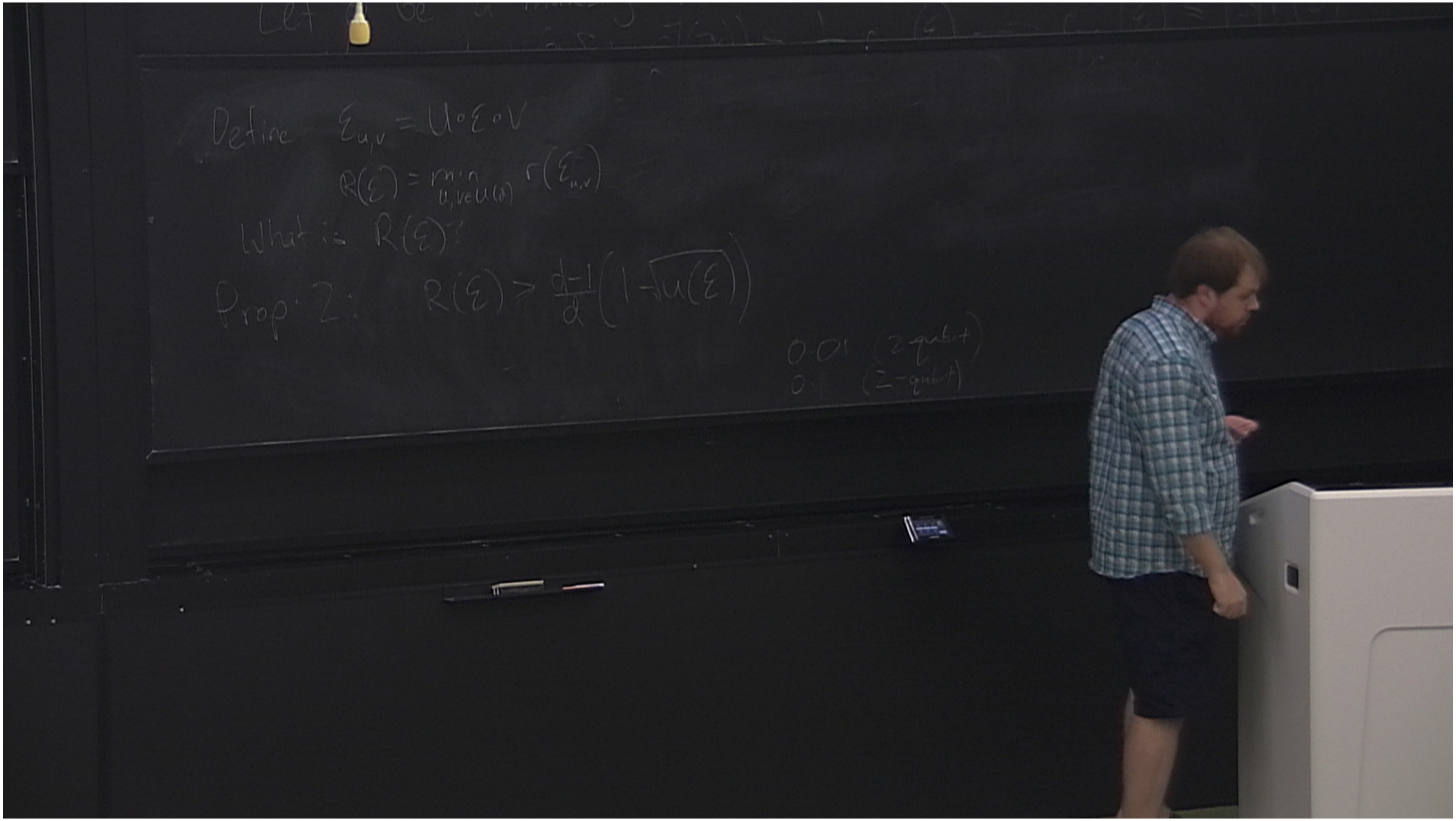

CPTP maps: I is privileged
unital block
non-unital vector
unitarity

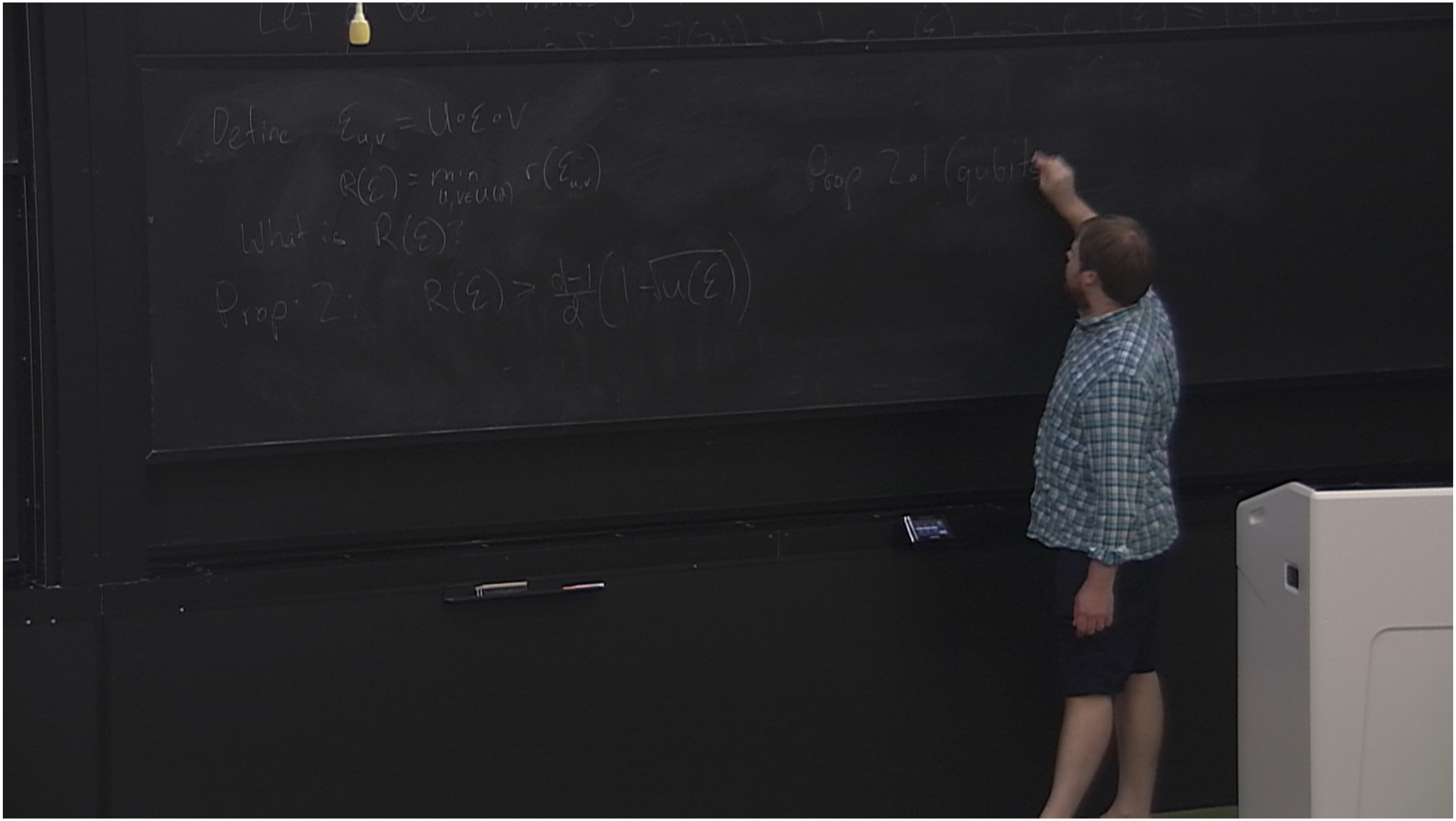

CPTP maps: I is privileged
unitarity
non-unital vector
unital block
Prop 1
iff
is unitary
Proof

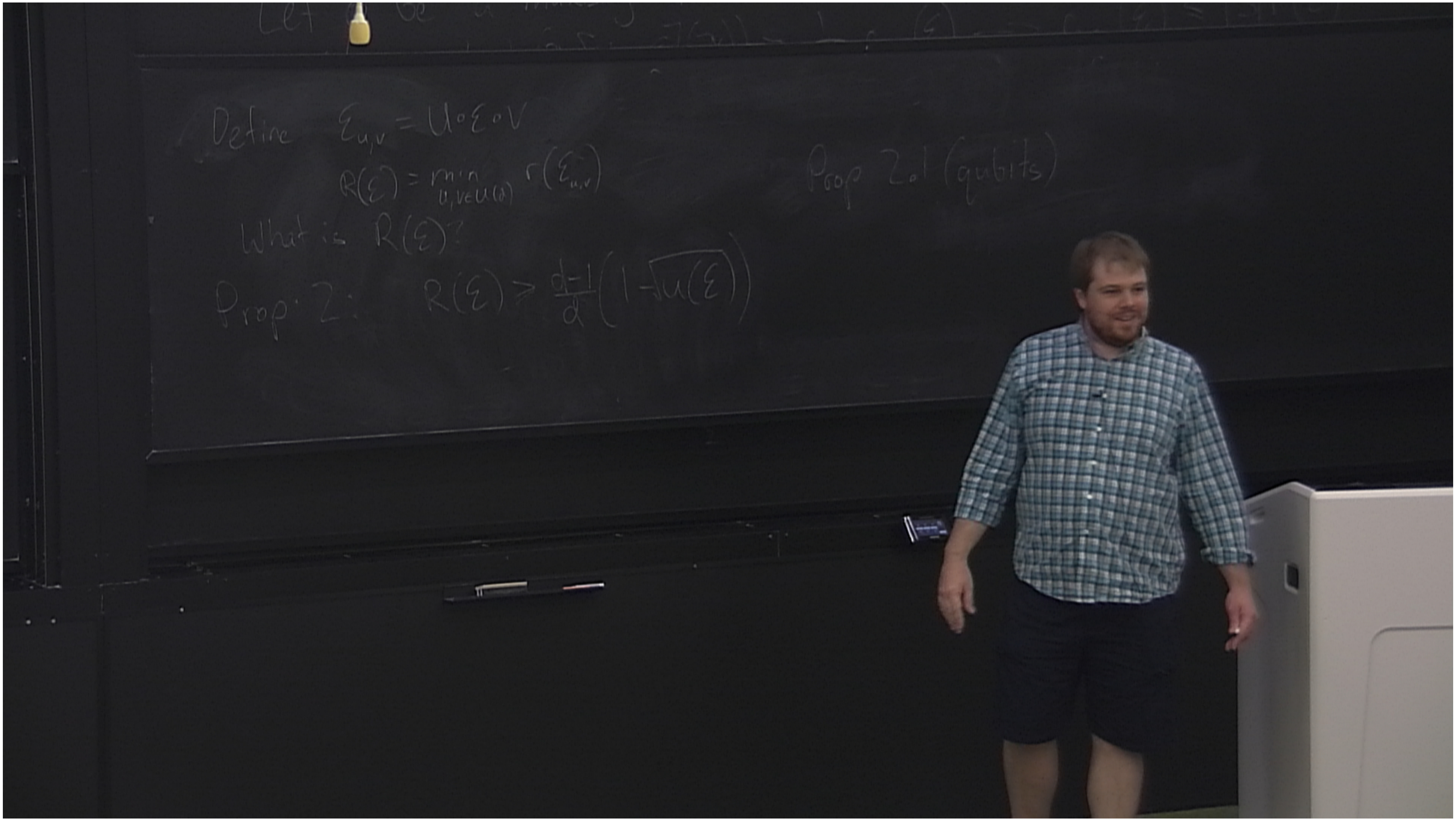

If ℰ is unital & CPTP, then ℰ†ℰ is a channel

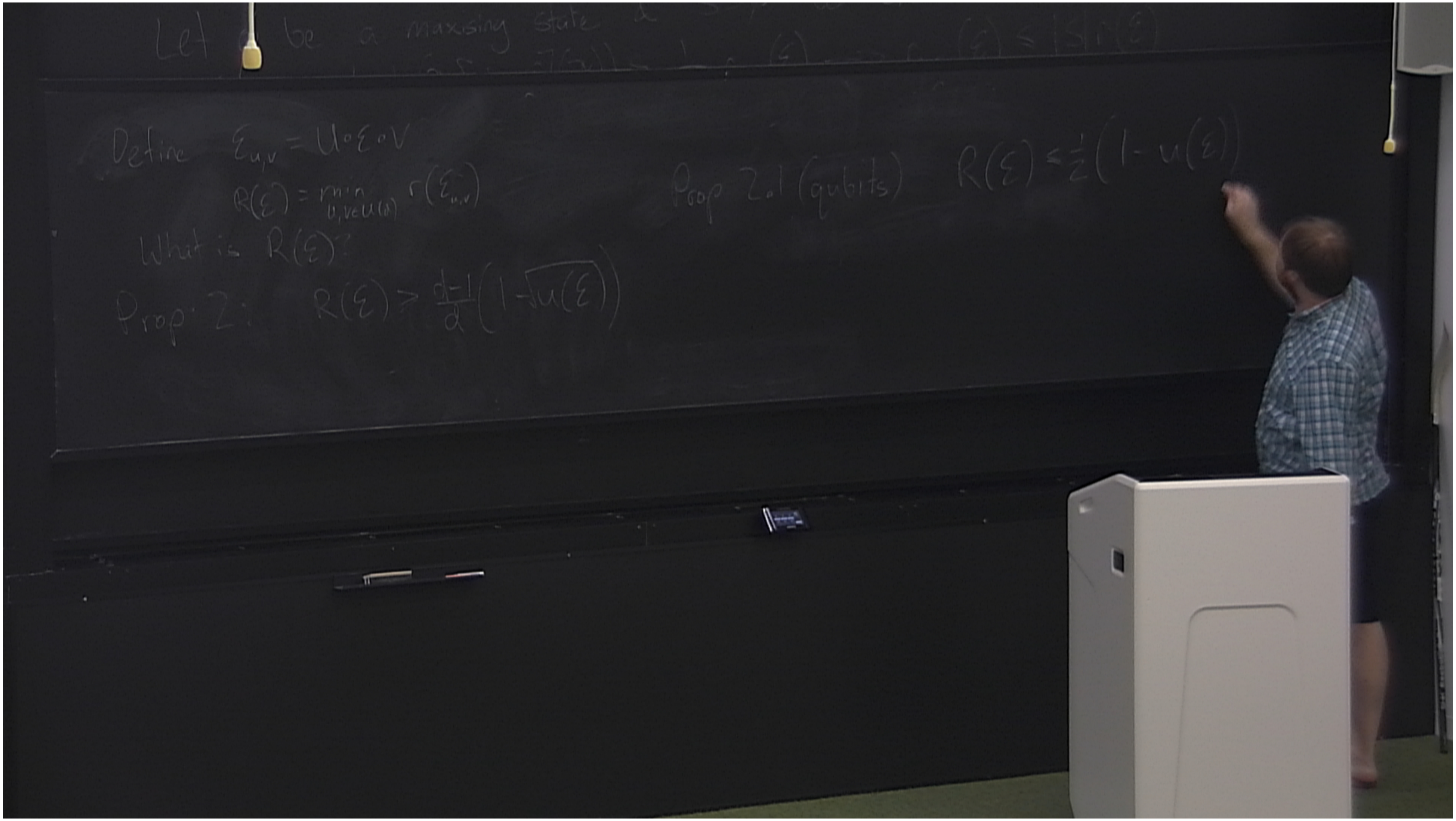

Proof: can prove $\|\mathcal{E}_n\|_2 \le (d-1)(1-u(\mathcal{E}))$
∴ $u(\mathcal{E})=1$ only if $\mathcal{E}$ is unital
If $\mathcal{E}$ is unital & CPTP, then $\mathcal{E}^\dagger\mathcal{E}$ is a channel
$(\mathcal{E}^\dagger\mathcal{E})^m$ is a channel for any $m \in \mathbb{Z}^+$
nonnegative
$\mathcal{E} = U\Sigma V$ (svd)
$\mathcal{E}_u^\dagger\mathcal{E}_u = V^\dagger\Sigma^2 V \xrightarrow{m^{th}\text{ power}} V^\dagger\Sigma^{2m}V$ if any elements of $\Sigma > 1$, $(\mathcal{E}^\dagger\mathcal{E})^m$ diverges
$u = \frac{\mathrm{tr}\,\Sigma^2}{d^2-1} = 1$ if $\Sigma^2 = I$ $\Rightarrow \mathcal{E}_u^\dagger\mathcal{E}_u = V^\dagger V = I \Rightarrow \mathcal{E}^{-1} = \mathcal{E}^\dagger$ so $\mathcal{E}$ is unitary

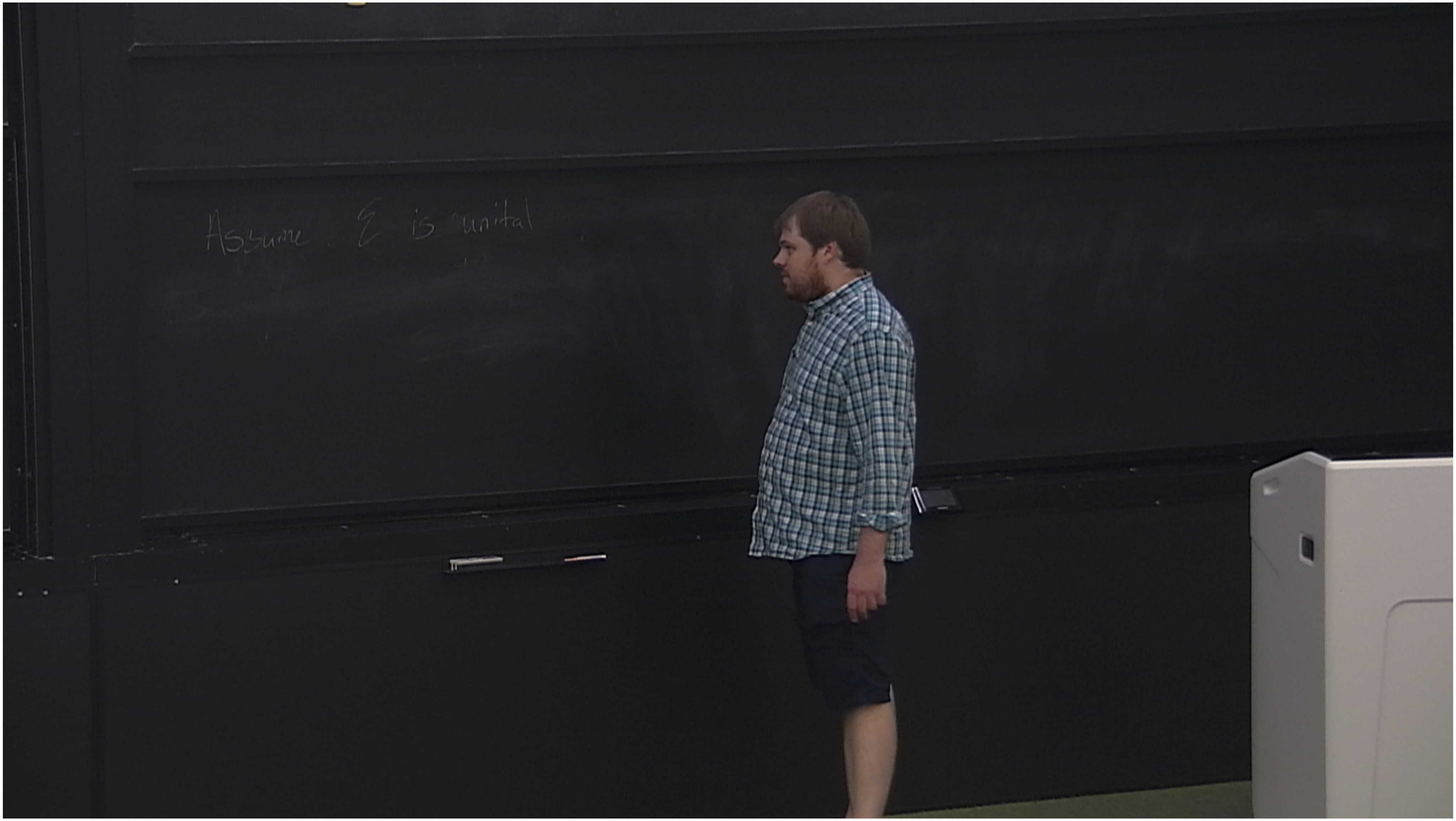

Define E u,v
0.01 (2-qubit)
(2-qubit)

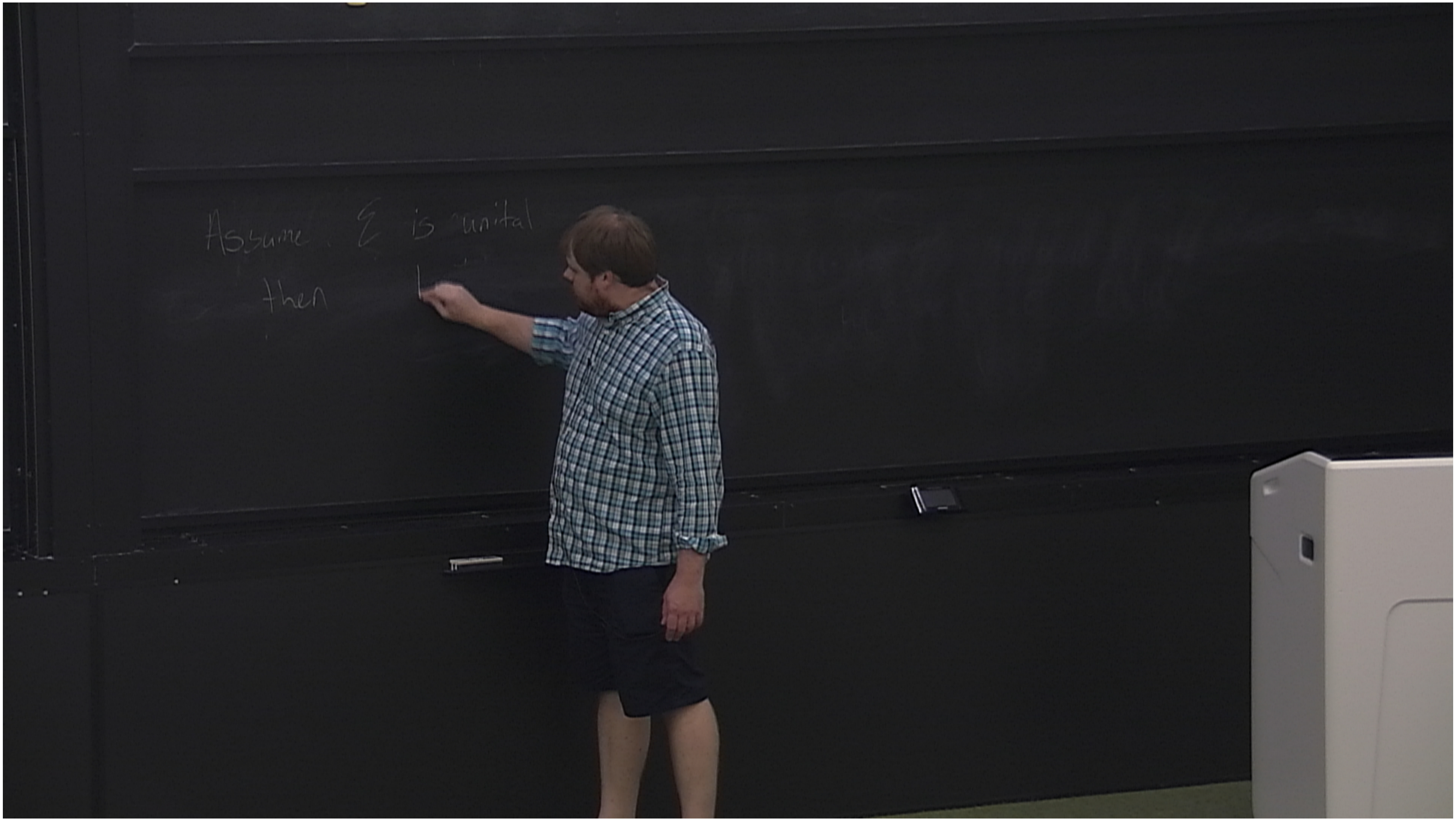

Define ε_{u,v} = U∘ε∘V
R(ε) = min_{U,V∈U(d)} r(ε_{u,v})
What is R(ε)?
Prop. 2: R(ε) ≥ (d−1)/d (1 − √u(ε))
0.01 (2-qubit)
0.1 (2-qubit)

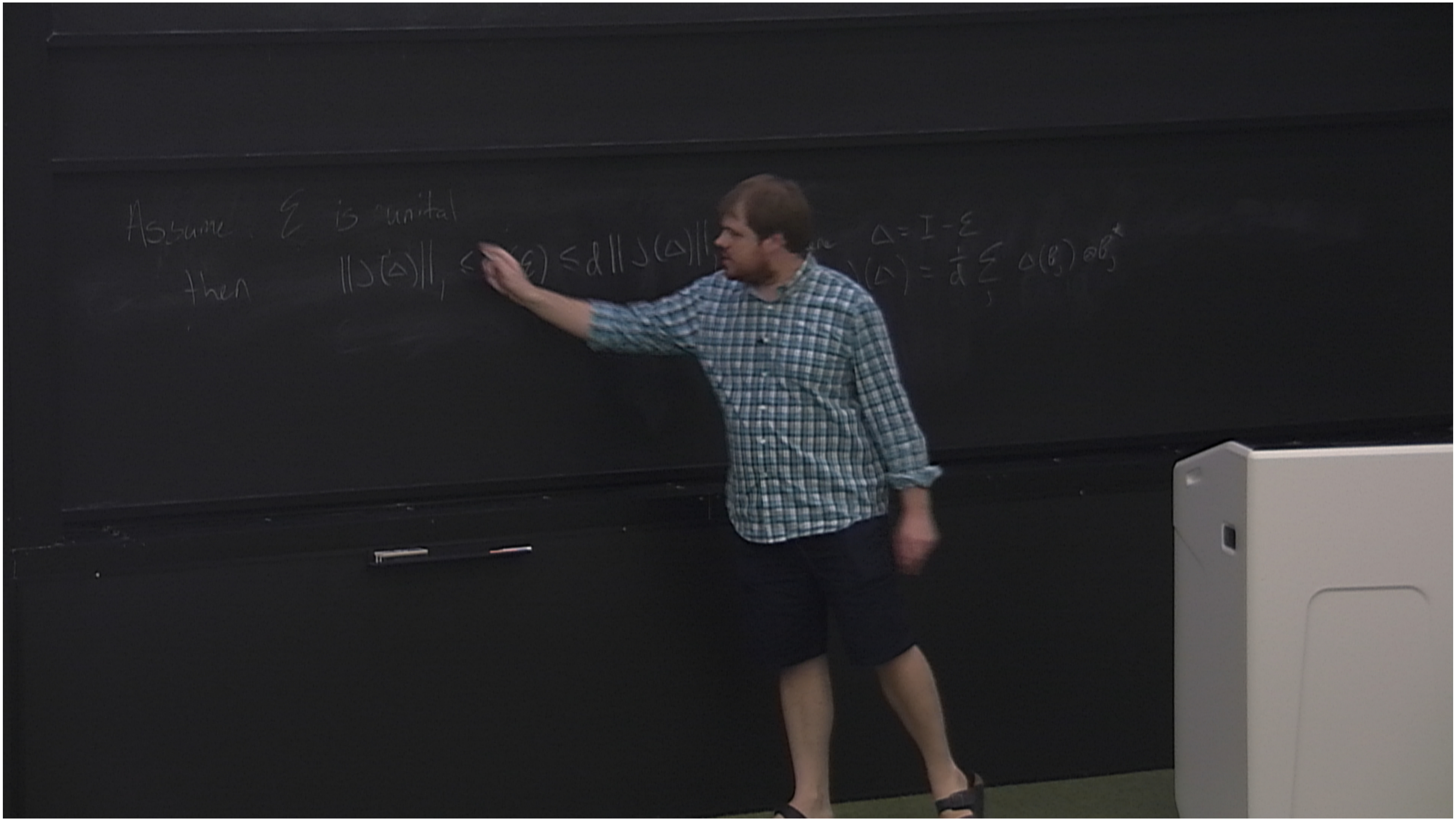

Define E_{u,v} = U∘E∘V
R(E) = min_{U,V∈U(d)} r(E_{u,v})
What is R(E)?
Prop 2: R(E) ≥ (d−1)/d (1−√u(E))
Prop 2.1 (qubits


Define Σ_{u,v} = U∘Σ∘V
R(Σ) = min_{U,V∈U(d)} r(Σ_{u,v})
What is R(Σ)?
Prop 2: R(Σ) ≥ (d−1)/d (1 − √u(Σ))
Prop 2.1 (qubits)

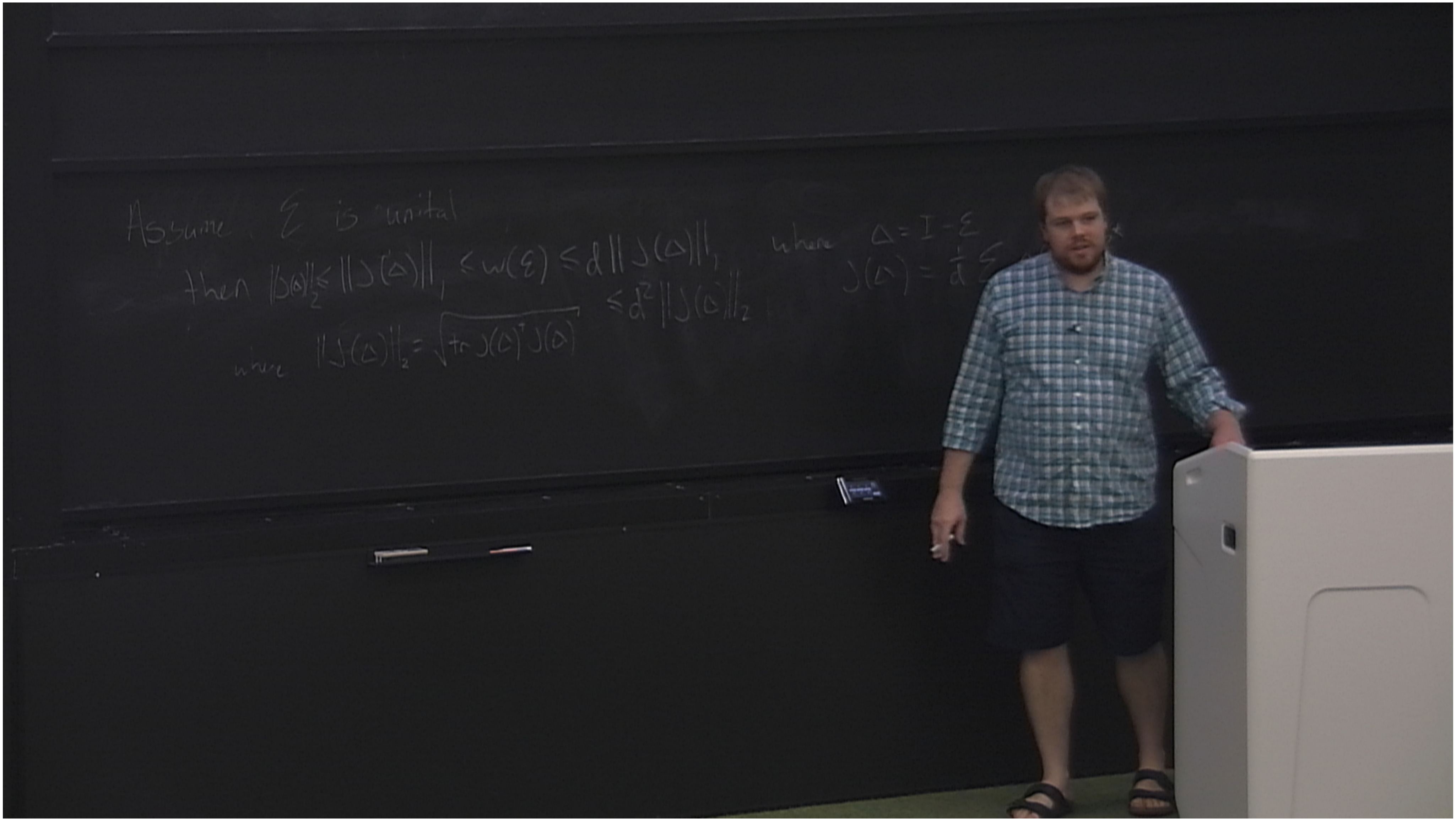

Define ℰ_{u,v} = U∘ℰ∘V
R(ℰ) = min_{U,V∈U(d)} r(ℰ_{u,v})
What is R(ℰ)?
Prop 2: R(ℰ) ≥ (d-1)/d (1-√u(ℰ))
Prop 2.1 (qubits) R(ℰ) ≤ 1/2 (1-u(ℰ))

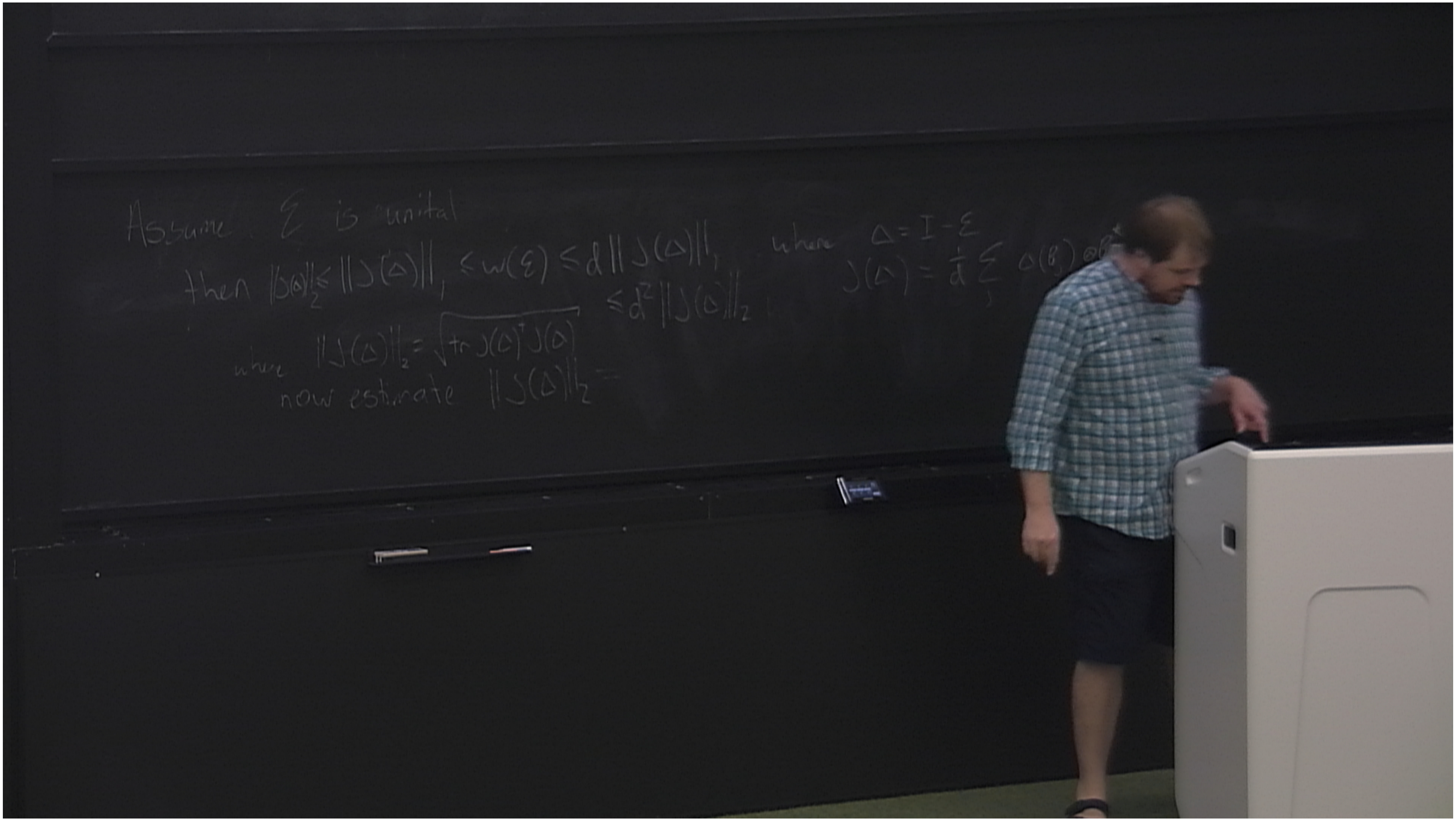

Assume Ɛ is unital

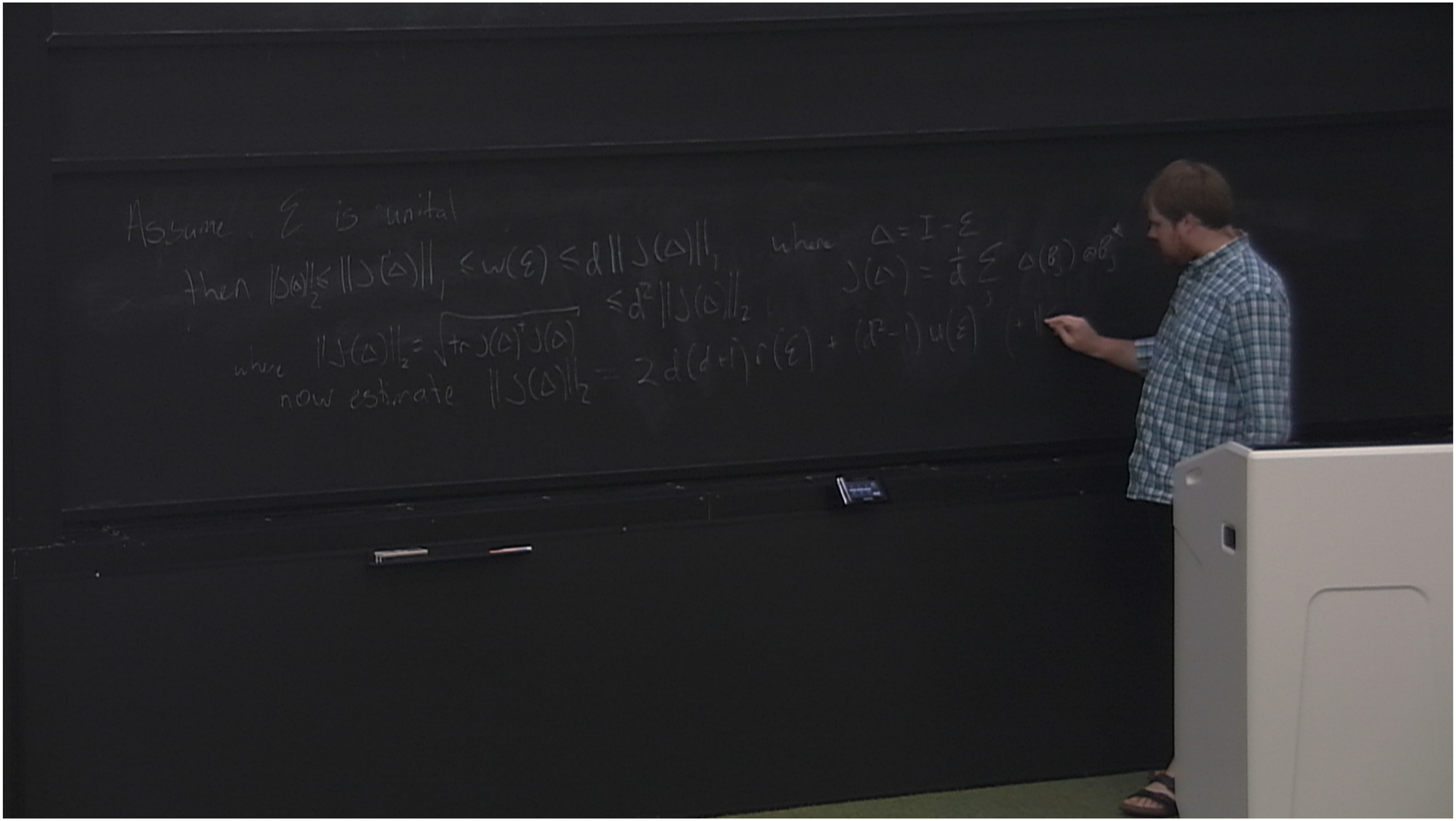

Assume ℰ is unital
then



Assume $\mathcal{E}$ is unital
then


Assume Ɛ is unital
then ||J(Δ)||₂ ≤ ||J(Δ)||₁ ≤ w(Ɛ) ≤ d ||J(Δ)||₁ where Δ = I − Ɛ
≤ d² ||J(Δ)||₂
J(Δ) = 1/d Σ
where ||J(Δ)||₂ = √(tr J(Δ)† J(Δ))


Assume ℰ is unital
then ||J(Δ)||₂ ≤ ||J(Δ)||₁ ≤ w(ℰ) ≤ d ||J(Δ)||₁ where Δ = I − ℰ
J(Δ) = 1/d Σ Δ(B) ⊗ B
≤ d² ||J(Δ)||₂
where ||J(Δ)||₂ = √(tr J(Δ)† J(Δ))
now estimate ||J(Δ)||₂ =


Assume Ɛ is unital
then ||J(Δ)||_2 ≤ ||J(Δ)||_1 ≤ w(Ɛ) ≤ d||J(Δ)||_1 where Δ = I − Ɛ
J(Δ) = 1/d Σ_j Δ(B_j) ⊗ B_j*
≤ d^2 ||J(Δ)||_2
where ||J(Δ)||_2 = √(tr J(Δ)† J(Δ))
now estimate ||J(Δ)||_2 = 2d(d+1) r(Ɛ) + (d^2 − 1) u(Ɛ) (+ ||