Title: TBA

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Abstract:

What is a theory of quantum gravity?

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Perimeter Institute

Renormalization in background independent theories
Perimeter Institute,
PI, Sep 2015

How to solve
Quantum Gravity?

It has already been claimed to have been solved.

But what is
a theory of quantum
gravity?

How to construct
a theory of quantum
gravity?

(Can we come up with a practical
procedure.)

Principles

- Background independence:
  Do not prefer a (metric) background. Ensure independence of auxiliary structures.

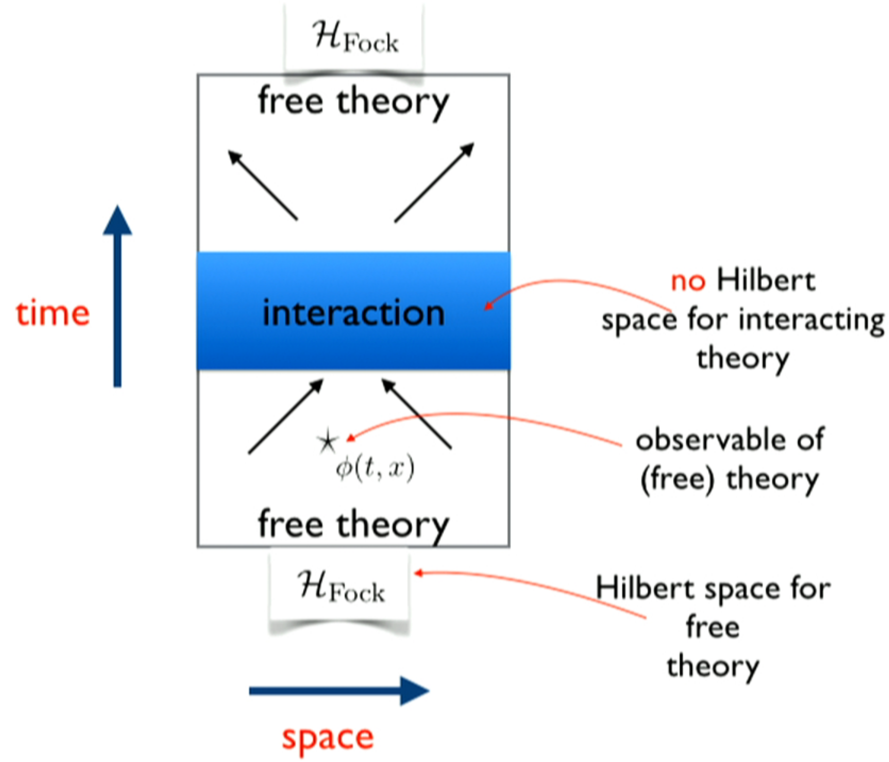
  But: What is the vacuum state? Peaked on some solution?

- Diffeomorphism symmetry and relational formulation.
  No additional degrees of freedom (which serve as a kind of aether).

  Path integral: Hamilton-Jacobi equations and projector property.
  Canonical formalism: constraint algebra.
  So far this has not been realized. Why is this the case? Are there better criteria?

Challenges

- Know QFT best if formulated on a fixed background geometry.
- Structure of QFT: vacuum plus excitations even non-perturbatively (GNS construction)
  What is vacuum?
- Very few non-perturbative methods: Discretization



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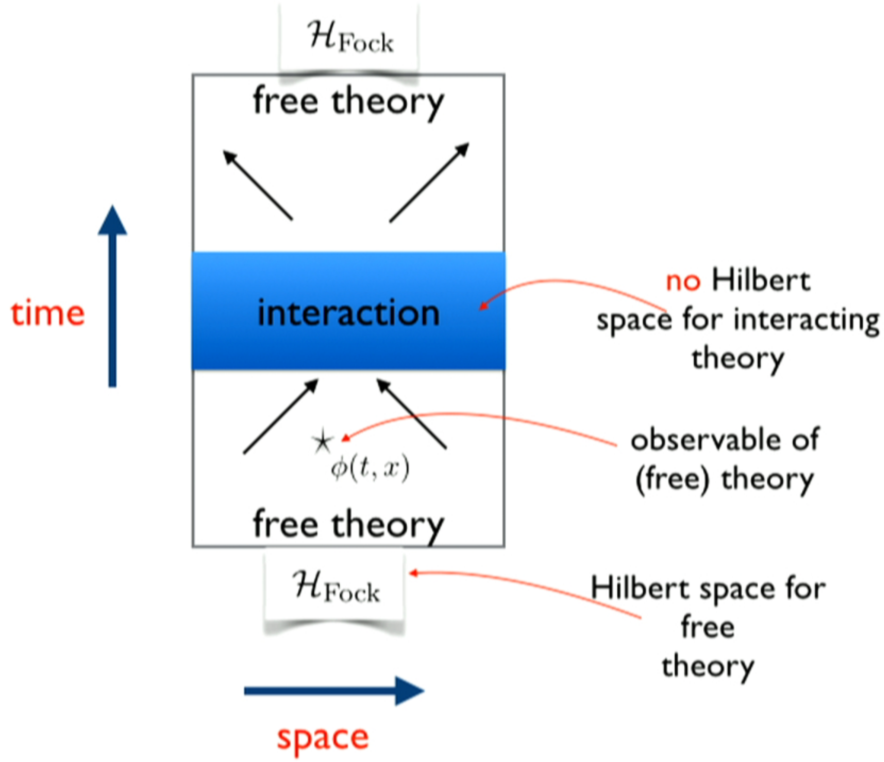


Discretization

first step of ‘renormalization’

What does it mean?
In a background independent context without lattice scale?

Discretizations

- What does it mean? In a background independent context without lattice scale?

(a) Kinematical choice (which should be determined by the dynamics however)

What observables are described with discrete theory.
Relation of these discrete observables to continuum?

Examples:
- scalar fields at (which?) set of points
- connection variables vs Wilson loop

Remark: More involved in canonical theory (mirror symplectic structure of continuum)

Discretizations

(b) Dynamics

One way to derive action for discrete observables:

-embed/ understand discrete configurations as continuum configuration
-find Hamilton's principal function (Hamilton-Jacobi) for this kind of boundary data

Usefulness of resulting discrete action depends on choice of "embedding".

Examples: Scalar field

[BD 12]

Embedding a la LQG: -set scalar field to zero at all points not probed
-dynamically highly unstable

Piecewise linear embedding: -set scalar field to be piecewise linear
-not exactly preserved, but can be used in iterative procedure
[BD 12]

"Perfect" embedding: (on regular slicing, free theory)
-set (spatial) Fourier modes above a cut-off to zero
-leads to perfect discretization
-non-local action

[similar (but different) to Kempf]

Examples: Massless scalar field on null lines

[BD-Steinhaus, Kyrankyi-Asante PSI essay and wip]

Another strategy:

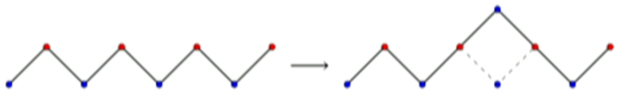
1. What kind of solutions do we want to probe?
2. Encode the finite subset of solutions in parameters / initial data.
3. Find (exact) dynamics for these data.

Massless scalar field, pick solutions with f,g piecewise linear. $\phi(u,v) = f(u) + g(v)$

Discontinuities move along null lines.

Canonical data: Scalar fields at points along zig-zag null line.

These data can be "perfectly" evolved.



But action is non-local.

“Discretizing solution space”

Disadvantage: need to know the solution space.

Gravity: consider configurations with bound on spectra of Dirac / Laplace operator?
[Kempf]

But: How to extract dynamics? How accessible is local physics from this?

What is a good discretization?

Perfect discretization:
Observables in discrete theory coincide with observables in continuum theory.
(Assuming we know how observables match to each other.)

But a perfect discretization requires basically to solve the theory.
Equivalent to fully effective action:
Evaluation of coarser observables does not change if we go to refined description.

What is a good discretization?

Good discretization, if we have a lattice scale/constant:
Observables in discrete theory approximate with observables in continuum theory up to orders in lattice constant.
(For observables describing larger length scales than lattice scale.)

If we do not have a lattice constant?
Can make sense of it if we have a (semi-classical) boundary state with a length scale (eg. curvature radius, variation of curvature).
We can then consider the regime: length scale(state) >> induced length scale discretization.
We demand that the discretization reproduces continuum observables (of larger length scales) in this regime.

Without a boundary state?

How do we treat this issue in quantum gravity?

- choose discrete 'boundary states' (spin networks)
- construct amplitudes from gluing amplitudes for basic building blocks

$$Z(j_{bdry}) = \sum_{j_{bulk}} \prod_{f} \omega_f(j) \prod_{v} \mathcal{A}_v(j)$$

What is the status of this theory?

(a) Fundamental? What do the discrete structures mean?

(b) Auxiliary? Have to take the continuum limit? What does it mean?

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Canonical formulation

- LQG-Kinematics: Inductive Hilbert space construction

[Ashtekar, Isham, Lewandowski, …
Bahr, BD, Geiller]

Hilbert spaces of probe states are labelled with discrete structures.
Can embed coarser into finer Hilbert spaces.
Require consistency of observables/dynamics with respect to this embedding.

- Problem: Consistent formulation of canonical dynamics, see next slide.

- View point: It is easier to derive a consistent canonical formulation from a covariant one, after the covariant one has been constructed.

- Some problems:
-How to deal with inconsistent constraints? (consistent constraints equivalent to perfect discr.)
-How to formulate continuum limit in canonical framework?

[BD, Thiemann, Gambini-Pullin: master constraints to uniform discretizations,
Lanery-Thiemann]

Are spin foams (in this formulation) fundamental?

- Diffeo-symmetry broken in Regge calculus (hoped for semi-classical limit for spin foams) [Bahr, BD]
- Leading to additional degrees of freedom

- In canonical theory: pseudo constraints [Gambini,Pullin, Bahr, BD, Hoehn]

- Not triangulation invariant: even under moves under which Regge calculus is invariant (one-loop "anomaly") [BD, Kaminski, Steinhaus]

- Meaning of large discretizations as boundary states?

- Why should we believe in what the (discretized) theory tells us at small scales?

  Leads to: How can we make sure small scale dynamics is consistent with larger scales?

  NB: Construction of amplitudes rather from IR to UV then from UV to IR.

Are spin foams (in this formulation) fundamental?

- Can we take just the continuum limit in the bulk?

- How to use this in iterative solution to the problem of taking the continuum limit?

- Require gluing principle but only with continuum limit on the glued boundary components?

- Does universality hold with respect to all details of the amplitude?
- If not what do these choices mean? (Are these fundamental?)

- Why should this particular choice of boundary discretization / type of boundary state be preferred? Rather let dynamics choose the type of boundary states, which then would also be connected to dynamical vacuum. These would then gives states which we can actually prepare with finite effort.

Spin foams: an approximation / first guess?

- What would be a full theory of quantum gravity?
- How to construct it?

- What properties should the final theory satisfy?

It should be consistent.

- Keeping in mind that we still want to have some way to extract predictions.

Can we use discretization as an approximation tool?
Constructive framework to extract predictions?

In the following: consider quantum theory.

- What properties should the final theory satisfy? It should be consistent.

Have conditions on amplitudes that can be checked (at least approximately) without taking the continuum limit.

With the perspective to use these conditions to construct quantum gravity amplitudes.

And to investigate whether such a theory can actually exist.

Boundary and Bulk formulation

[Zapata et al]

Consistent Boundary formulation

[BD 12, Bahr, Steinhaus 13, BD14]

- define dynamics via (boundary) amplitudes
- require consistency of amplitudes wrt inductive structure of boundary Hilbert space
- allow for general embeddings (encode vacuum)
- do not require 'gluing principle', at least not on discrete level

Cylindrical consistent measure formulation

[Bahr 11,14]

- define dynamics via bulk amplitudes (or measure)
- work on projective limit of bulk configuration space
- require consistency of measure with respect to projective structure
- do not require 'gluing principle', at least not on discrete level
- non-local amplitudes

NB: Inductive structures might be more general than those obtained from projective structures.
Allow for "dynamical embedding maps".

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Inductive limit Hilbert spaces

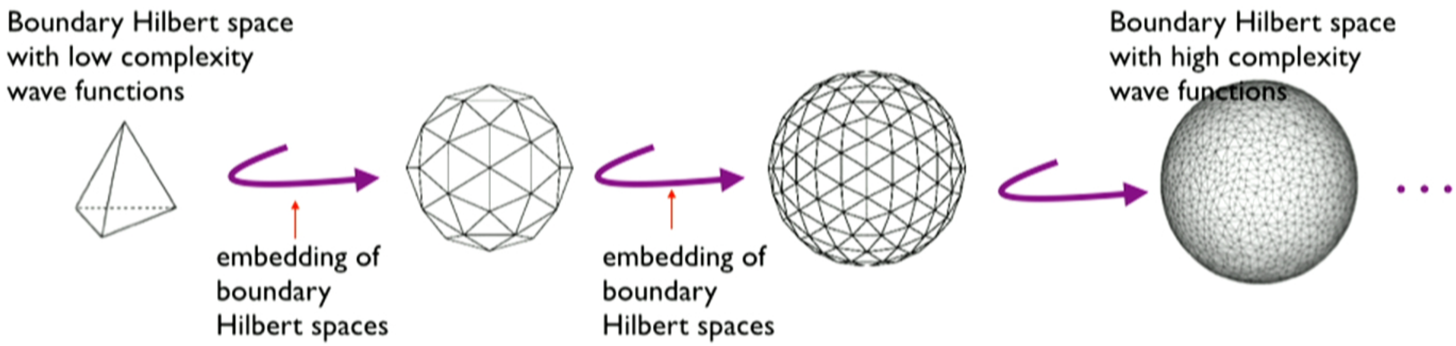
A (partially) ordered family of Hilbert spaces that carry probe states.

(Partial) ordering into coarser and finer states/ Hilbert spaces.

(Consistent) Embedding maps: How to understand coarser states as finer states.
This is put in as equivalence relation.

(Consistent) Inner product / Observables: Well defined prescription on equivalence classes of states.

(Consistent) Physical inner product = (Consistent) amplitudes.



Inductive limit Hilbert spaces

(1) A directed partially ordered set of labels $\{\alpha\}$, representing discrete structures, e.g. graphs or triangulations. The ordering induces a notion of coarser and finer.

(2) Hilbert spaces $\mathcal{H}_\alpha$ associated to these labels.

(3) Embedding maps $\iota_{\alpha\alpha'} : \mathcal{H}_\alpha \to \mathcal{H}_{\alpha'}$ for each pair of labels with $\alpha \prec \alpha'$, i.e. α' is finer than α. These embedding maps have to satisfy the consistency condition $\iota_{\alpha'\alpha''} \circ \iota_{\alpha\alpha'} = \iota_{\alpha\alpha''}$ for any triple $\alpha \prec \alpha' \prec \alpha''$.

The inductive limit of Hilbert spaces is given by the

$$\mathcal{H} \quad := \quad \overline{\cup_\alpha \mathcal{H}_\alpha / \sim}$$

where the equivalence relation is defined as follows: two elements $\psi_\alpha \in \mathcal{H}_\alpha$ and $\psi'_{\alpha'} \in \mathcal{H}_{\alpha'}$ are equivalent $\psi_\alpha \sim \psi'_{\alpha'}$ iff there exist a refinement α'' of α and α' such that $\iota_{\alpha\alpha''}(\psi_\alpha) = \iota_{\alpha'\alpha''}(\psi'_{\alpha'})$.

Two elements are equivalent if they become equal under refinement eventually.

Cylindrically consistent (kinematical) observable

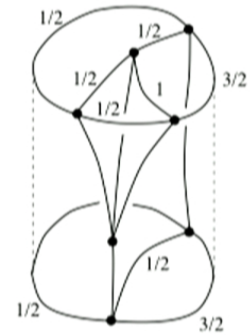
Observables $\mathcal{O} = \{\mathcal{O}_\alpha\}_\alpha$ are cylindrically consistent iff

$$\iota_{\alpha\alpha'}(\mathcal{O}_\alpha \psi_\alpha) \quad = \quad \mathcal{O}_{a'} \iota_{\alpha\alpha'}(\psi_\alpha) \quad .$$

Any calculation done on α gives the same result as on any refined α' – if all ingredients are cylindrically consistent.

Cylindrically consistent transition amplitude

$\alpha := (\alpha_{\text{initial}}, \alpha_{\text{final}})$



$\mathcal{A}_\alpha : \mathcal{H}_{\alpha_{\text{initial}}} \otimes \mathcal{H}^\star_{\alpha_{\text{final}}} \to \mathbb{C}$

Amplitudes $\mathcal{A} = \{\mathcal{A}_\alpha\}_\alpha$ are cylindrically consistent iff

$$\mathcal{A}_\alpha(\psi_\alpha) \quad = \quad \mathcal{A}_{a'}(\iota_{\alpha\alpha'}(\psi_\alpha)) \quad .$$

Amplitudes can be evaluated on 'coarse' states - result does not change under boundary refinements.

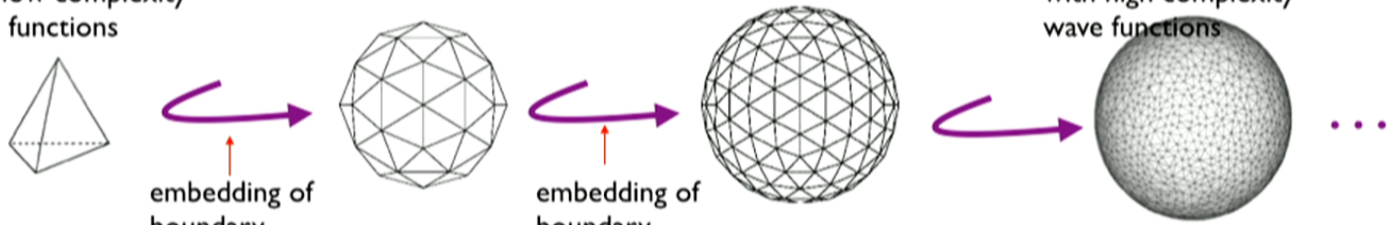
Very strong condition!

How to express the continuum dynamics

[BD NJP 12, BD 14]

Boundary Hilbert space
with low complexity
wave functions

Boundary Hilbert space
with high complexity
wave functions



embedding of
boundary
Hilbert spaces

embedding of
boundary
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restricts to

$\mathcal{A}_{vac}^{\text{low com}}(\psi_{\text{low com}}) \longleftarrow \mathcal{A}_{vac}^{\text{med com}}(\psi_{\text{med com}}) \longleftarrow \mathcal{A}_{vac}^{\text{high com}}(\psi_{\text{high com}}) \cdots$

(cylindrical) consistency condition

A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

27

[BD NJP 12, BD 14]

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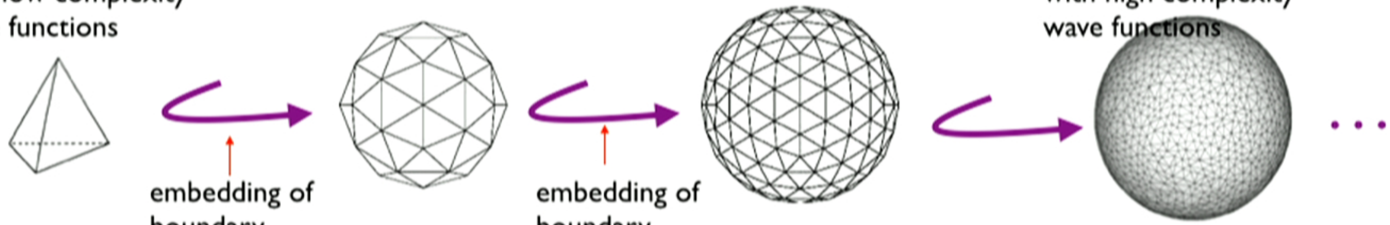

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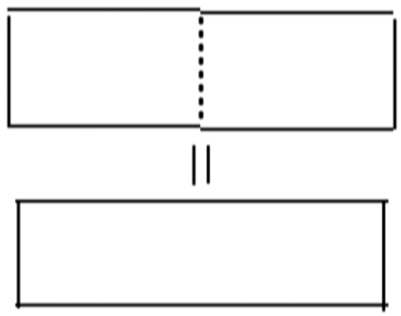
(Counter-) Arguments for its existence?

From this requirement one should be able to derive e.g.

restriction on matter coupling.

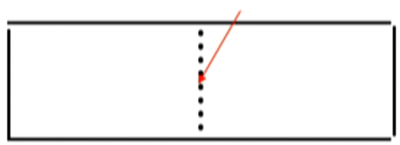
Problems with gluing principle



'Gluing axiom':
used to get more complicated from simpler amplitudes

Amplitude for more complicated boundary state
=
glued from amplitudes for less complicated boundary state

Integrating over data associated to discretization misses out on most of the continuum data. Even on the necessary discrete data (depending on boundary discretization).



However this gluing axiom does not hold in the discrete:
Restricting to discretization we never obtain a full resolution of identity for the continuum Hilbert space.
(This leads eventually to non-local amplitudes if one wants to represent continuum physics.) [e.g. Bahr, BD, He NJP 11]

Cylindrically consistency replaces gluing principle.

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Bulk formulation and Wilson renormalization flow

[Bahr 11,14]

X configuration variables,
$\mathcal{C}_\alpha$ configuration space based on (bulk and boundary) discretization α
$\pi_{\alpha',\alpha} : \mathcal{C}_{\alpha'} \to \mathcal{C}_{\alpha'}$ from finer to coarser

Projections = Blocking functions:

$$\int_{\pi_{\alpha',\alpha}(X')=X} \mathcal{A}_{\alpha'}(X')\, \mathcal{D}X' \quad = \quad \mathcal{A}_\alpha(X)$$

Wilsonian Renormalization flow

Amplitudes satisfying Renormalization flow equation give cylindrically consistent measure:

$$\int \mathcal{O}_\alpha(\pi_{\alpha',\alpha}(X'))\mathcal{A}_{\alpha'}(X')\, \mathcal{D}X' \quad = \quad \int \mathcal{O}_\alpha(X)\mathcal{A}_\alpha(X)\, \mathcal{D}X$$

cylindrically consistent measure condition

To the Consistent Boundary Formulation

Choose discrete structure representing bulk and boundary.
Choose as observable a boundary wave function:

$$\mathcal{O}_\alpha = \psi_\alpha$$

Condition for cylindrical consistent measure translates to:

$$\int \psi_\alpha(\pi_{\alpha',\alpha}(X'_{bdry}))\mathcal{A}_{\alpha'}(X'_{bulk}, X'_{bdry})\,\mathcal{D}X'_{bulk}\mathcal{D}X'_{bdry} \quad = \quad \int \psi_\alpha(X_{bdry})\mathcal{A}_\alpha(X_{bulk}, X_{bdry})\,\mathcal{D}X_{bulk}\mathcal{D}X_{bdry}$$

$$\Rightarrow \qquad \int \psi_\alpha(\pi_{\alpha',\alpha}(X'_{bdry}))\mathcal{A}_{\alpha'}(X'_{bdry})\,\mathcal{D}X'_{bdry} \quad = \quad \int \psi_\alpha(X_{bdry})\mathcal{A}_\alpha(X_{bdry})\,\mathcal{D}X_{bdry}$$

Gives consistent amplitude conditions if we choose: $\iota_{\alpha,\alpha'}(\psi_\alpha) = \psi_\alpha \circ \pi_{\alpha',\alpha}$

Can all possible embeddings be derived from projections?
Or can one define projections from ("dynamical") embedding maps.
(Dont even know how to do it for the BF embedding. Requires "Flux representation".)

[BD, Geiller 14,
Baratin, Dittrich, Oriti, Tambornino]

Using embeddings seems more more general.
Non-localities of amplitudes are shifted into more boundary data.

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cylindrically consistent measure condition

Cylindrically consistency encodes renormalization flow

Cylindrical consistent amplitudes encode renormalization flow.
Thus should encode the running of coupling constant.

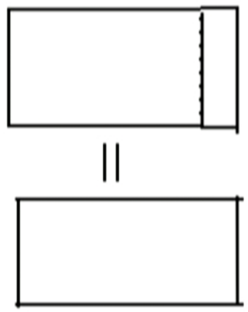
In a naive discretization: S = Grav + coup Matter

Should extract running of coupling from (local approximation to) consistent amplitudes

S = Grav + coup(metric, discretiz.) Matter

[Bahr 14: Example 2D Gauge Theory]

Derivation of Canonical Dynamics



derivation of Hamiltonian constraints from (consistent) amplitudes

Wheeler-deWitt equation

$$\mathcal{A}(\psi_b) \quad = \quad \int \mathcal{D}X_{b_i} \mathcal{A}(\psi_{b_1,b_i}) \mathcal{A}(\psi_{b_2,\overline{b_i}})$$

Need (a priori) continuum limit for gluing. Might not need continuum limit if we have 'perfect/ sufficiently good' dynamical embedding maps.

[generalizing Barrett-Crane derivation of 3D Hamiltonian]

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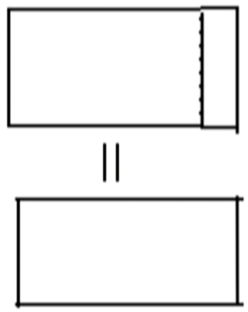

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[generalizing Barrett-Crane derivation of 3D Hamiltonian]

Constructing amplitudes means constructing beta-functions

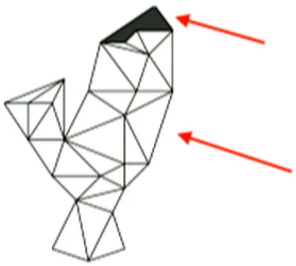
"Solving quantum gravity" includes solving beta functions.

In particular consistent Hamiltonian constraints need to include the coupling constant flow:

H = Grav + coup(metric, discretiz.) Matter

Constructing consistent amplitudes

Evaluating path integral via coarse graining: higher dimensions?



Coarse graining of non-topological theories leads to non-local couplings: hard to control.

[Bahr, BD, He 11]
[BD 12]

Even weak notion of diffeo-symmetry / triangulation independence needs non-local amplitudes.

[BD, Kaminski, Steinhaus 14]

Iterative process: coarse graining

$$\mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) \quad = \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha'})^\dagger | \mathbf{K}_{\alpha\alpha'}\psi_\alpha \rangle \quad = \quad \mathcal{A}^{imp}_{\alpha}(\psi_\alpha)$$

(dual) vacuum amplitude

coarse graining (by time evolution) of this amplitude

Fixed point of iteration process satisfies consistency by construction:

$$\begin{aligned} \mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) \quad &= \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha'})^\dagger | \mathbf{K}_{\alpha\alpha'}\psi_\alpha \rangle \\ &\overset{\mathrm{Conv}}{=} \quad \langle \psi_\emptyset | (\mathbf{K}_{\emptyset\alpha})^\dagger | \psi_\alpha \rangle \quad = \mathcal{A}_\alpha(\psi_\alpha) \quad . \end{aligned}$$

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(Can we come up with a practical
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Renormalization built into very definition of quantum gravity.

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Practical implementation: Guifre's talk

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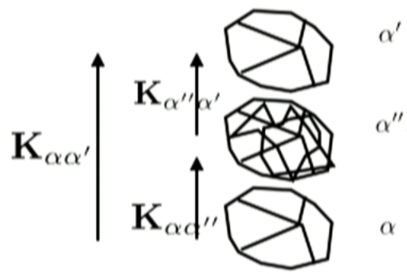
Constructing amplitudes

- Construct amplitude for simplest boundary (i.e. simplex) as a first approximation to final answer.
- Use this amplitude in the usual gluing scheme to build amplitudes for more complicated 'transitions'.



$$\mathbf{K}_{\alpha''\alpha'} \circ \mathbf{K}_{\alpha\alpha''} = \mathbf{K}_{\alpha\alpha'}$$

convolution property

$$\mathcal{A}_{\alpha'}(\iota_{\alpha\alpha'}\psi_\alpha) = \langle\psi_\emptyset|(\mathbf{K}_{\emptyset\alpha'})^\dagger|\mathbf{K}_{\alpha\alpha'}\psi_\alpha\rangle = \mathcal{A}^{imp}_\alpha(\psi_\alpha)$$

(dual) vacuum amplitude

coarse graining (by time evolution) of this amplitude