Title: Tensor network renormalization

Date: Sep 29, 2015 09:00 AM

URL: http://pirsa.org/15090079

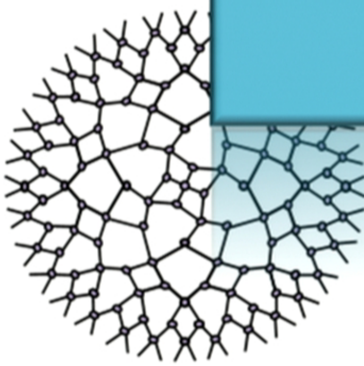
Abstract: I will present a pedagogical introduction on the application of tensor networks to the renormalization group. This program has resulted in a non-perturbative, real-space RG approach for lattice systems and the multi-scale entanglement renormalization ansatz (MERA). The MERA is currently of interest in a wide range of research areas, from statistical mechanics to condensed matter, from quantum field theory to holography in quantum gravity.

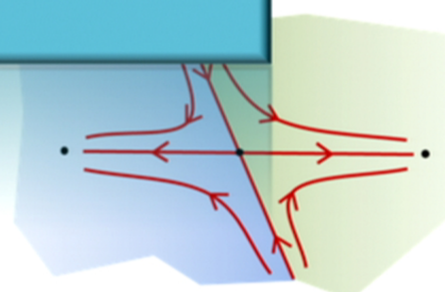
Perimeter Institute

RENORMALIZATION IN BACKGROUND INDEPENDENT THEORIES:
FOUNDATIONS AND TECHNIQUES

September 28th – October 2nd 2015

Tensor Network Renormalization (an introduction)





Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

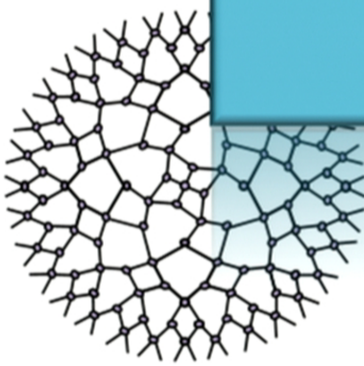
JOHN TEMPLETON FOUNDATION

Perimeter Institute

RENORMALIZATION IN BACKGROUND INDEPENDENT THEORIES:
FOUNDATIONS AND TECHNIQUES

September 28th – October 2nd 2015

Tensor Network Renormalization
(an introduction)



work with

GLEN EVENBLY
(UC Irvine)

Guifre Vidal

PERIMETER PI INSTITUTE FOR TH

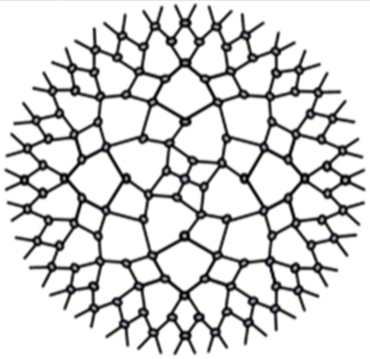
SIMONS FOUNDATION

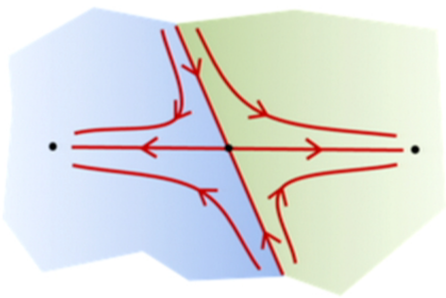
MPLETON
DATION

disclaimer:

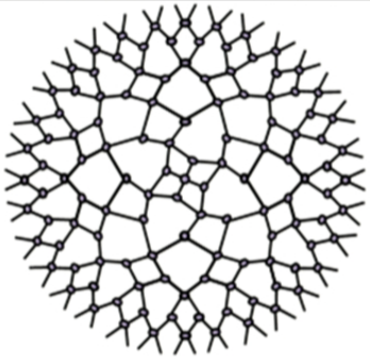
this workshop

RENORMALIZATION
IN BACKGROUND INDEPENDENT THEORIES:
FOUNDATIONS AND TECHNIQUES





disclaimer:

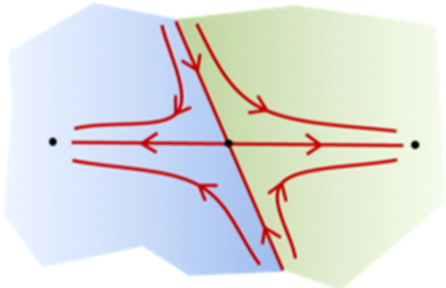


this workshop

RENORMALIZATION
IN BACKGROUND INDEPENDENT THEORIES:
FOUNDATIONS AND TECHNIQUES

this talk

RENORMALIZATION
IN BACKGROUND INDEPENDENT THEORIES:
FOUNDATIONS AND TECHNIQUES

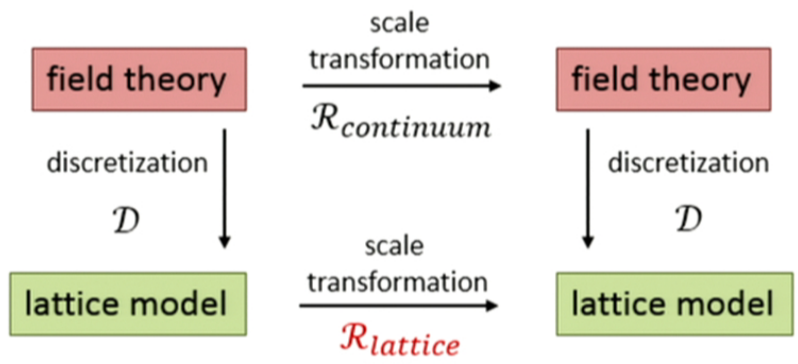


goal: Define a proper* RG transformation on the lattice

goal: Define a proper* RG transformation on the lattice

(*) Requirement:

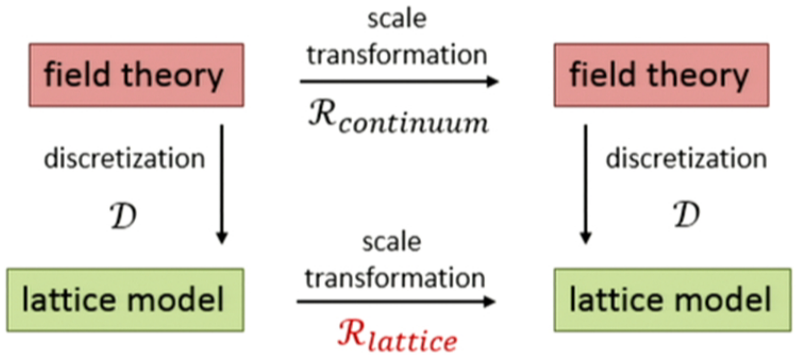
- consistency with the *continuum*



goal: Define a proper* RG transformation on the lattice

(*) Requirement:

- consistency with the *continuum*

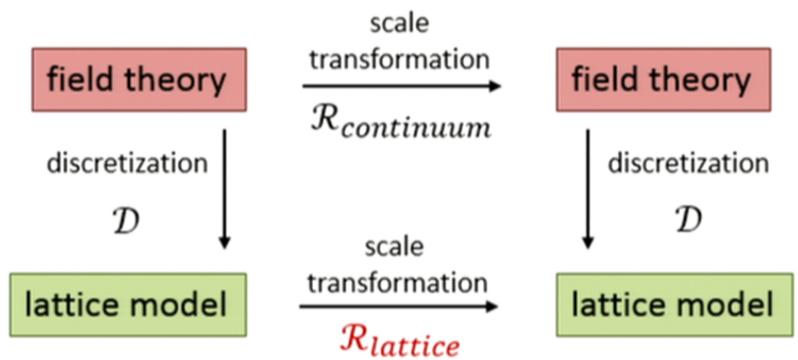


$$\mathcal{D} \circ \mathcal{R}_{continuum} = \mathcal{R}_{lattice} \circ \mathcal{D}$$

goal: Define a proper* RG transformation on the lattice

(*) Requirement:

- consistency with the *continuum*



$$\mathcal{D} \circ \mathcal{R}_{continuum} = \mathcal{R}_{lattice} \circ \mathcal{D}$$

- including *explicit scale invariance* at expected RG fixed-points

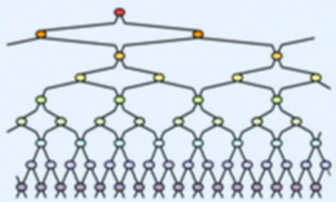
Our strategy:

- proper removal of short-range entanglement/correlations

outline:

part I

1+1D quantum systems:
ground state wave-function



multi-scale entanglement renormalization ansatz

(MERA)

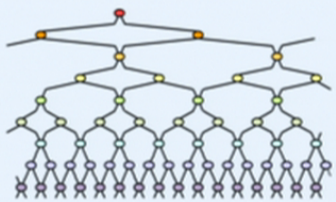
part II

part III

outline:

part I

1+1D quantum systems:
ground state wave-function

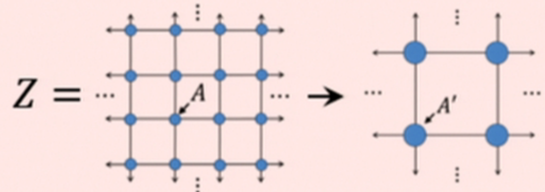


multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)



tensor network renormalization

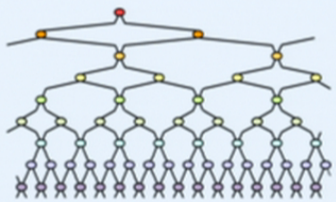
(TNR)

part III

outline:

part I

1+1D quantum systems:
ground state wave-function

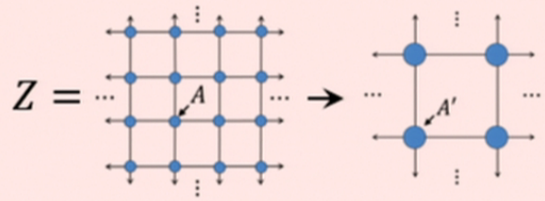


multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)



tensor network renormalization

(TNR)

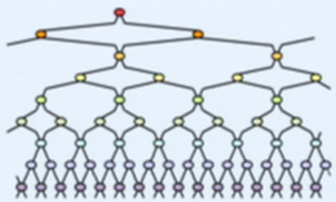
part III

TNR $\Rightarrow$ MERA

outline:

part I

1+1D quantum systems:
ground state wave-function

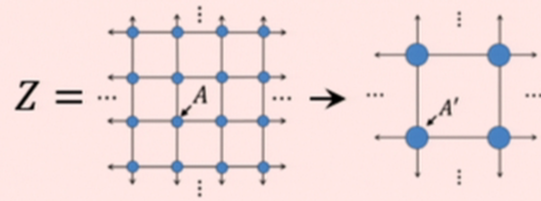


multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)



tensor network renormalization

(TNR)

part III

TNR ⇒ MERA

local scale
transformations

appetizer: What can MERA / TNR do for you?

appetizer: What can MERA / TNR do for you?

input

1D quantum / 2D classical Hamiltonian

- **on the lattice**
- **at a critical point**

appetizer: What can MERA / TNR do for you?

input

1D quantum / 2D classical Hamiltonian

- **on the lattice**
- **at a critical point**

→

output

Numerical extraction of conformal data of underlying CFT:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

appetizer: What can MERA / TNR do for you?

input

1D quantum / 2D classical Hamiltonian

- **on the lattice**
- **at a critical point**

→

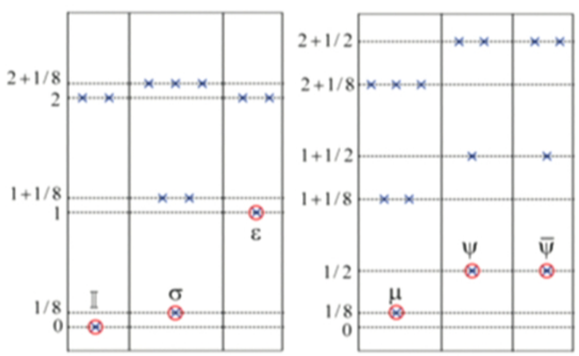
output

Numerical extraction of conformal data of underlying CFT:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



appetizer: What can MERA / TNR do for you?

input

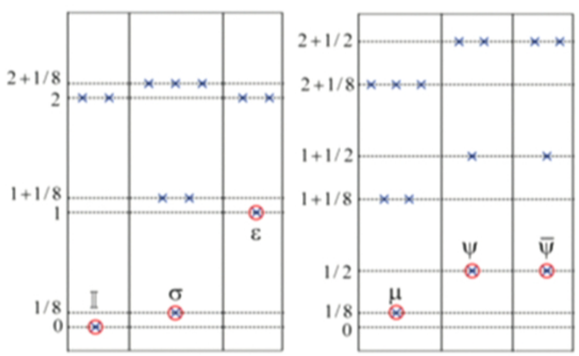
1D quantum / 2D classical Hamiltonian
- **on the lattice**
- **at a critical point**

output

Numerical extraction of conformal data of underlying CFT:
- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
  and conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

appetizer: What can MERA / TNR do for you?

input

1D quantum / 2D classical Hamiltonian

- **on the lattice**
- **at a critical point**

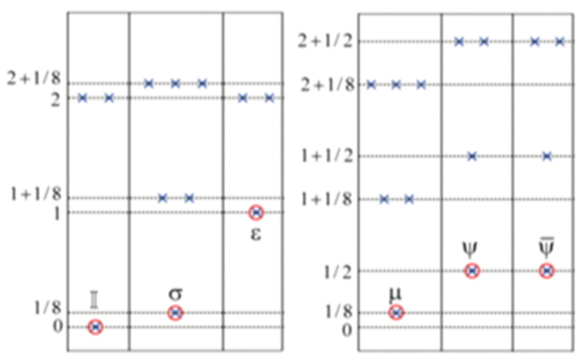
output

Numerical extraction of conformal data of underlying CFT:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
  and conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $\quad C_{\epsilon\mu\mu} = -\frac{1}{2}$

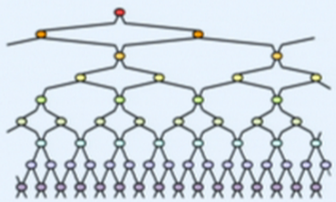
$C_{\epsilon\psi\bar{\psi}} = i$ $\quad C_{\epsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $\quad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$

part I

1+1D quantum systems:
ground state wave-function



multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)

tensor network renormalization

(TNR)

part III

TNR $\Rightarrow$ MERA

local scale
transformations

Many-body wave-function of N spins

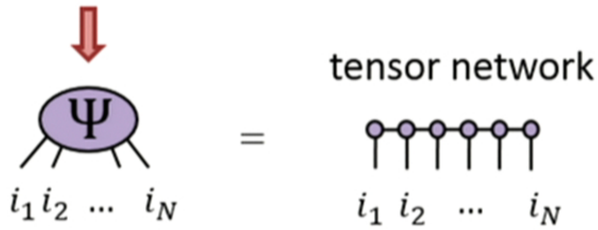
$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N
parameters

Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

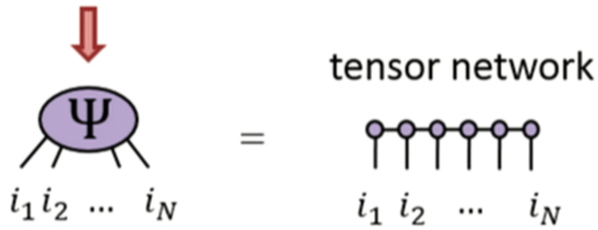
2^N parameters



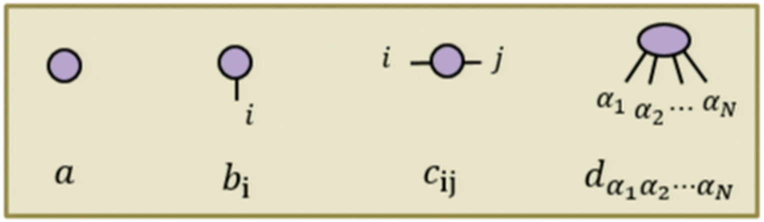
Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



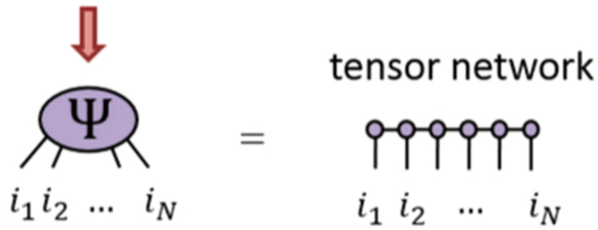
graphical notation



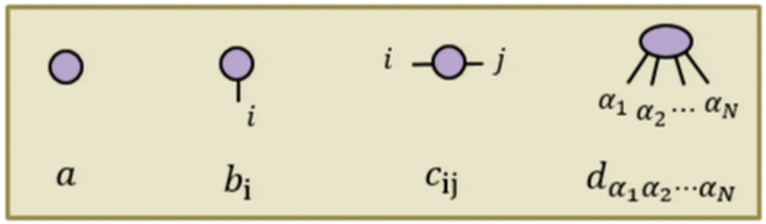
Many-body wave-function of N spins

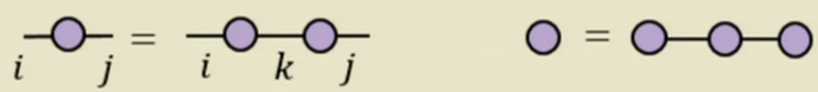
$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



graphical notation





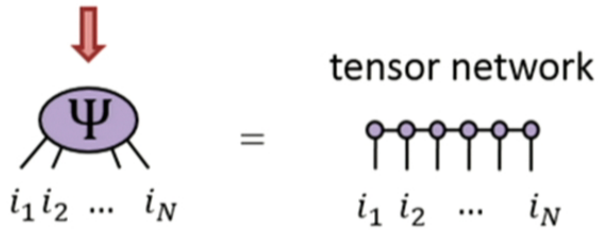
$$T_{ij} = \sum_k R_{ik} S_{kj}$$

$$a = \vec{y}^{\dagger} \cdot M \cdot \vec{x}$$

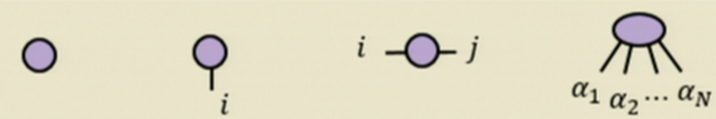
Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

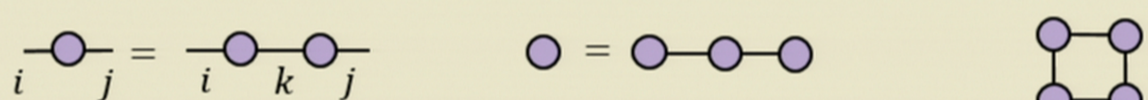
2^N parameters



graphical notation



a $\quad b_{\mathrm{i}}$ $\quad c_{\mathrm{ij}}$ $\quad d_{\alpha_1 \alpha_2 \cdots \alpha_N}$

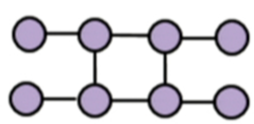


$$T_{ij} = \sum_k R_{ik} S_{kj}$$

$$a = \vec{y}^{\dagger} \cdot M \cdot \vec{x}$$

$$tr(ABCD)$$

why bother?

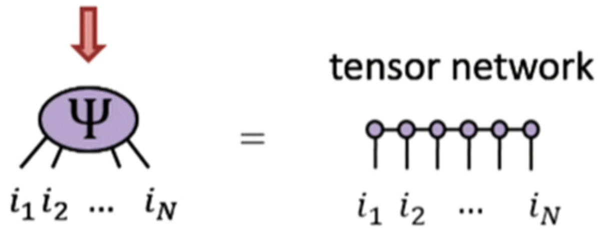


$$\sum_{ijklmnop} A_{ijk} B_{jlm} C_{nko} D_{kmr} x_i y_l z_n v_r$$

Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



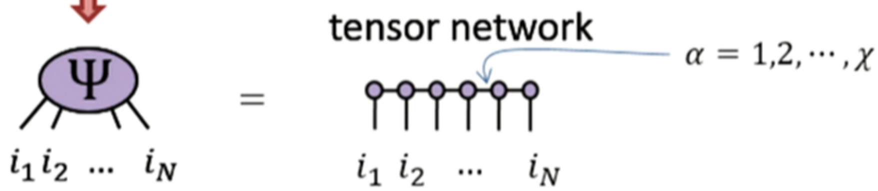
2^N parameters

inefficient

Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



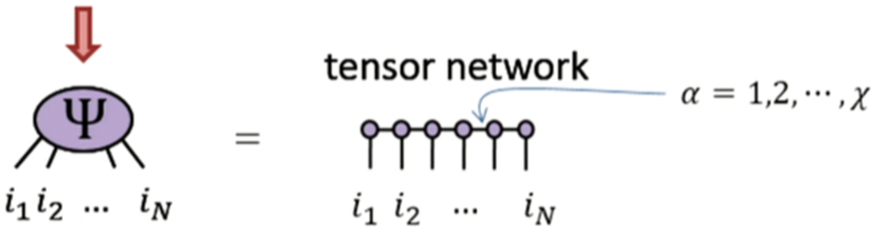
2^N parameters → $O(N)$ parameters

inefficient → **efficient**

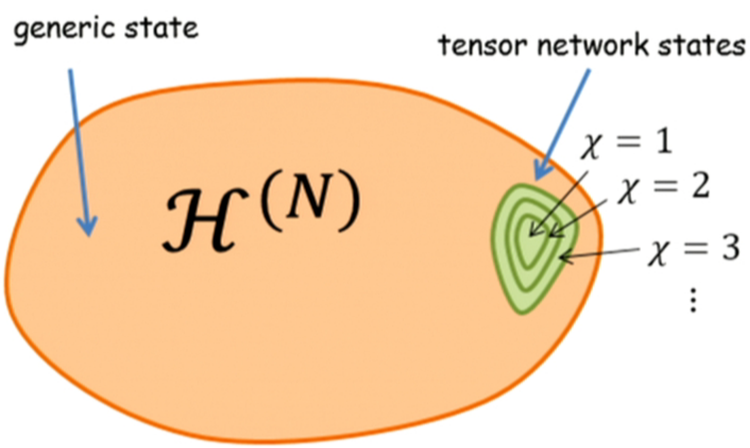
Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



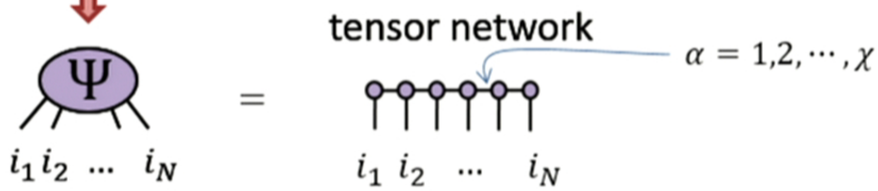
2^N parameters	→	$O(N)$ parameters
inefficient		**efficient**



Many-body wave-function of N spins

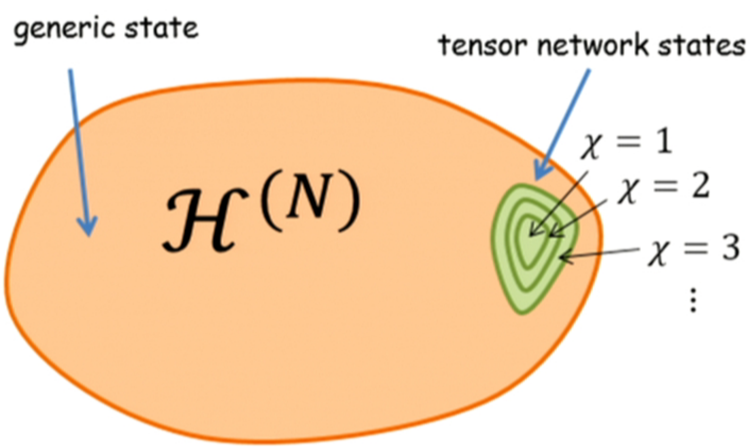
$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

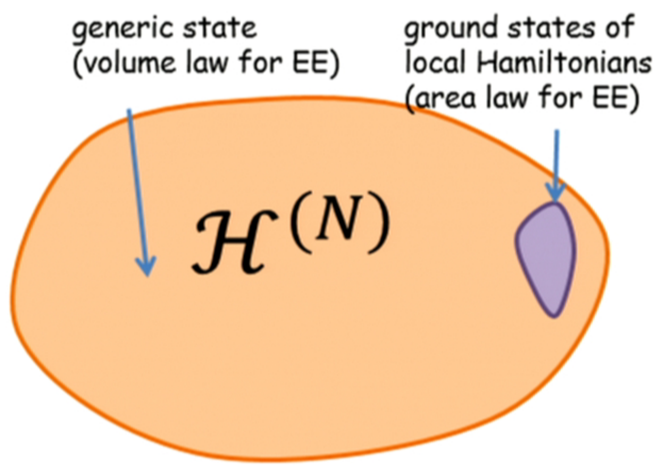
2^N parameters



2^N parameters → $O(N)$ parameters

inefficient → **efficient**





TENSOR NETWORKS

TENSOR NETWORKS

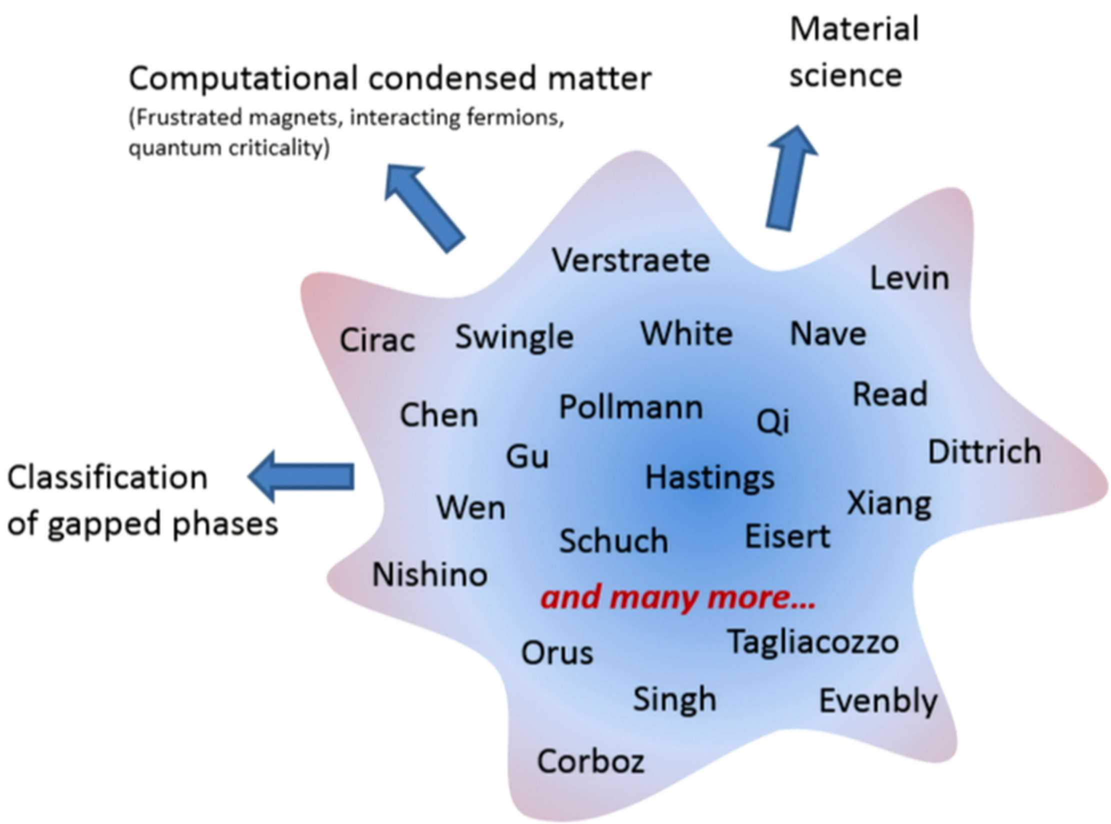
Computational condensed matter
(Frustrated magnets, interacting fermions, quantum criticality)

TENSOR NETWORKS

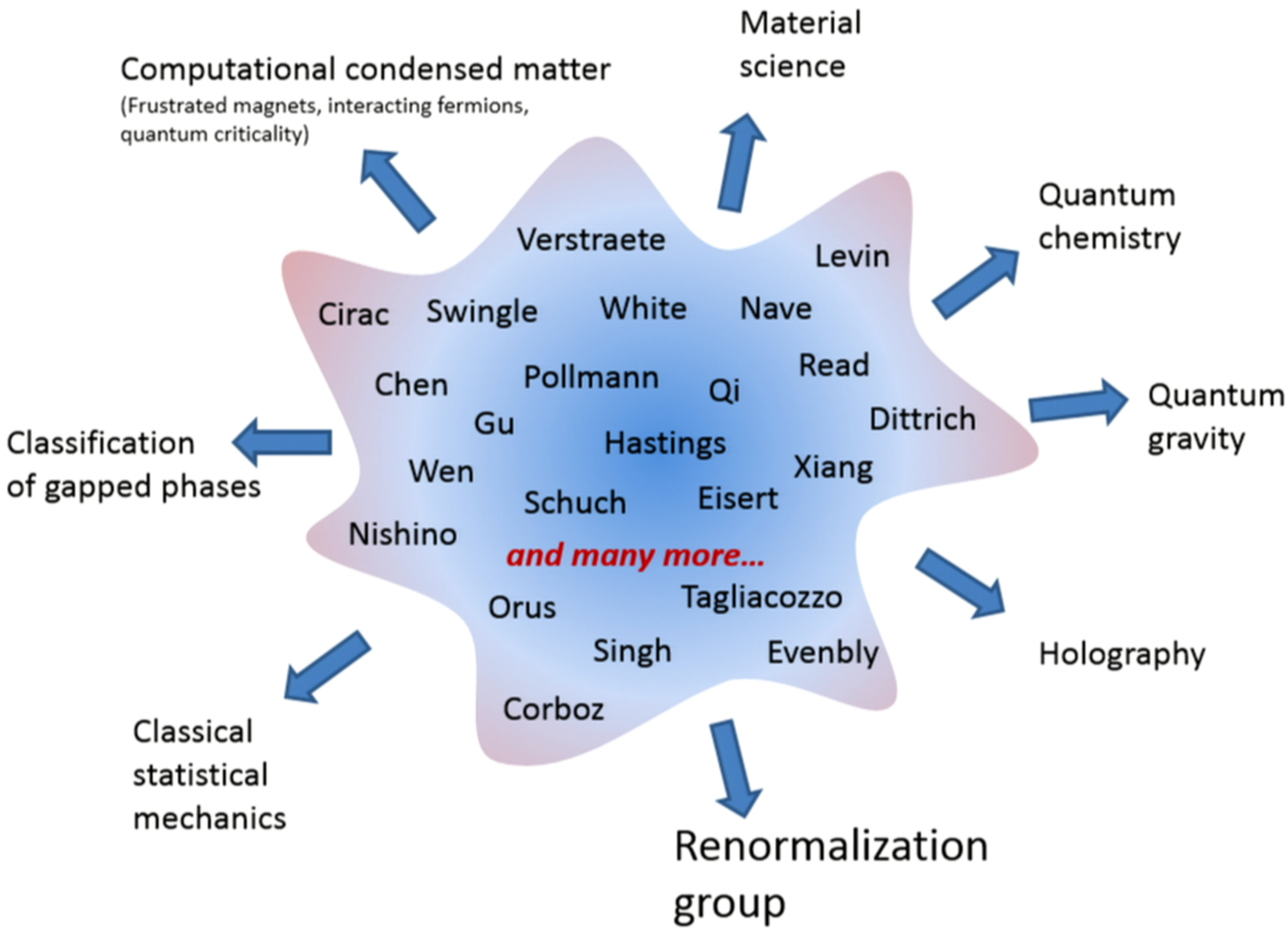
Computational condensed matter
(Frustrated magnets, interacting fermions, quantum criticality)

Classification of gapped phases

TENSOR NETWORKS

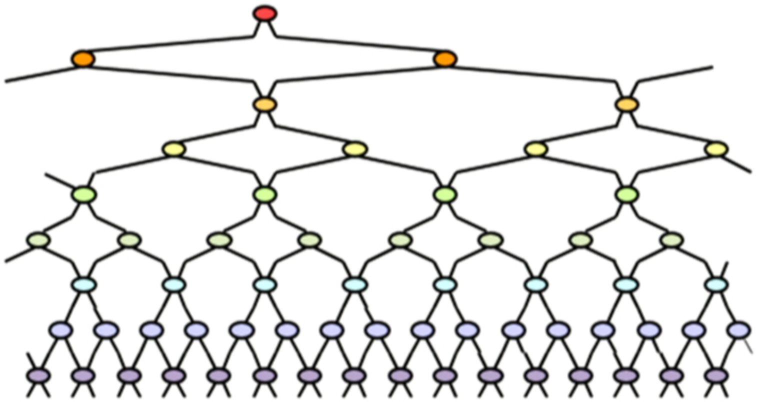


TENSOR NETWORKS



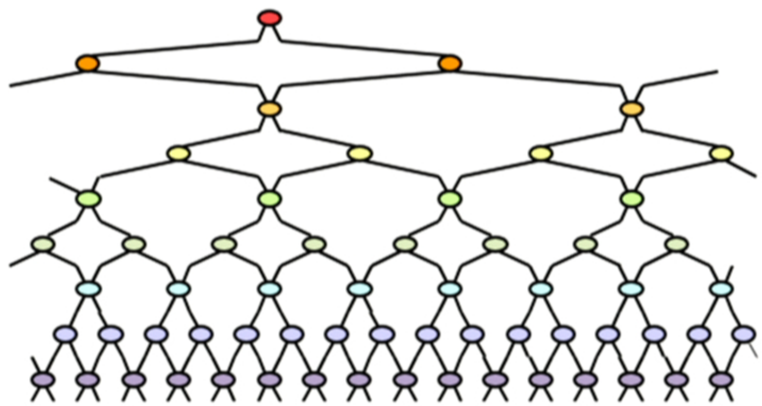
Multi-scale entanglement renormalization ansatz

(MERA)



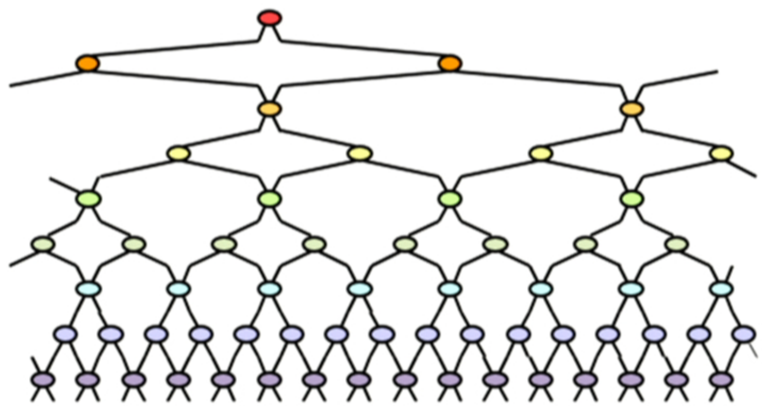
Multi-scale entanglement renormalization ansatz

(MERA)



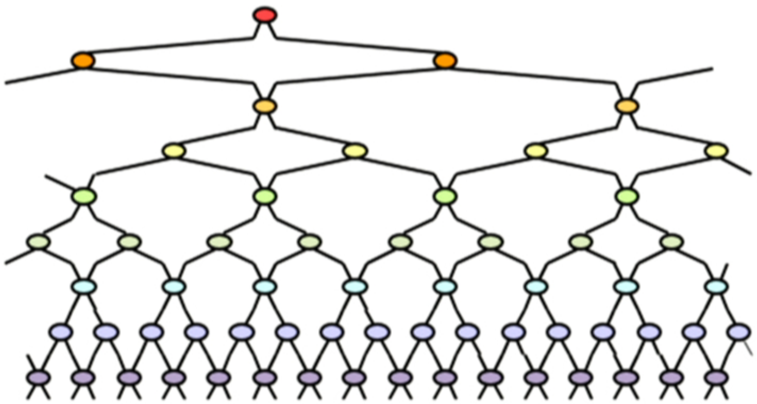
- Variational class of states for 1d systems, which extends in space and scale

Multi-scale entanglement renormalization ansatz (MERA)



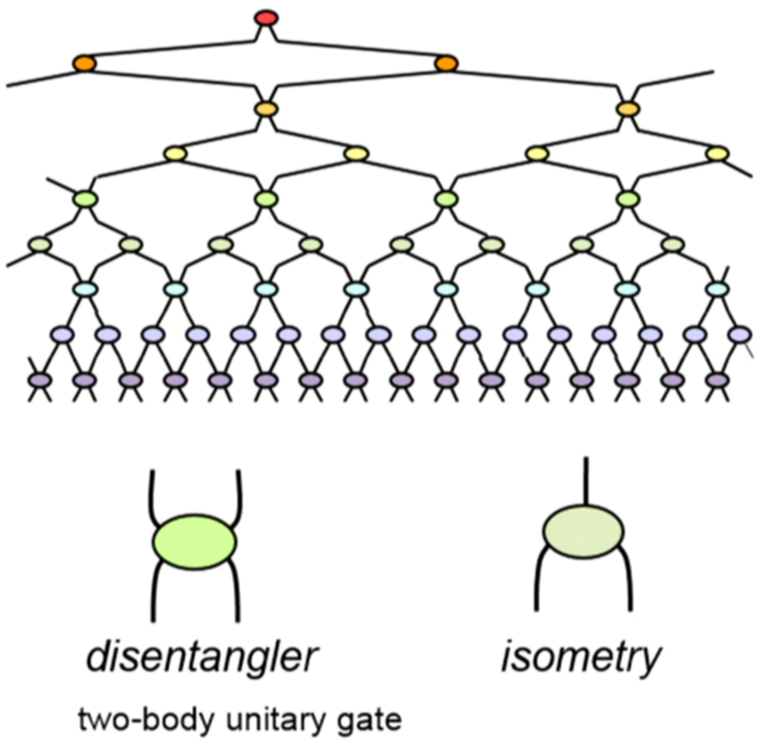
- Variational class of states for 1d systems, which extends in space and scale
- Variational parameters for different length scales stored in different tensors

Multi-scale entanglement renormalization ansatz (MERA)

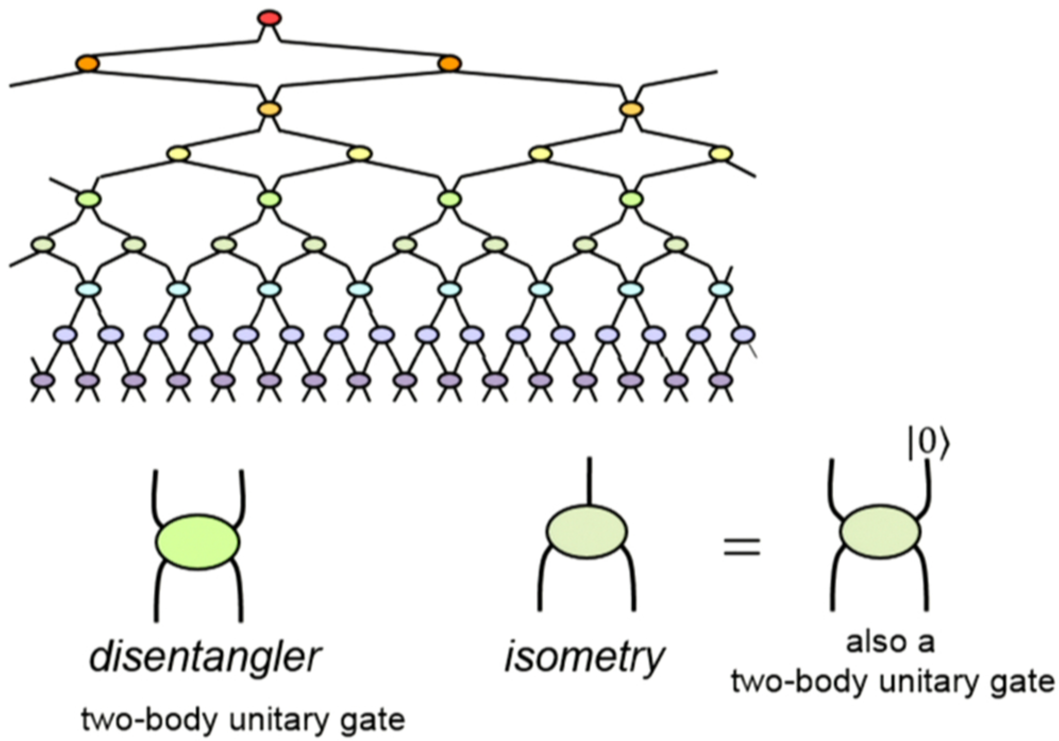


- Variational class of states for 1d systems, which extends in space and scale
- Variational parameters for different length scales stored in different tensors
- It is secretly a **quantum circuit** and an **RG transformation**

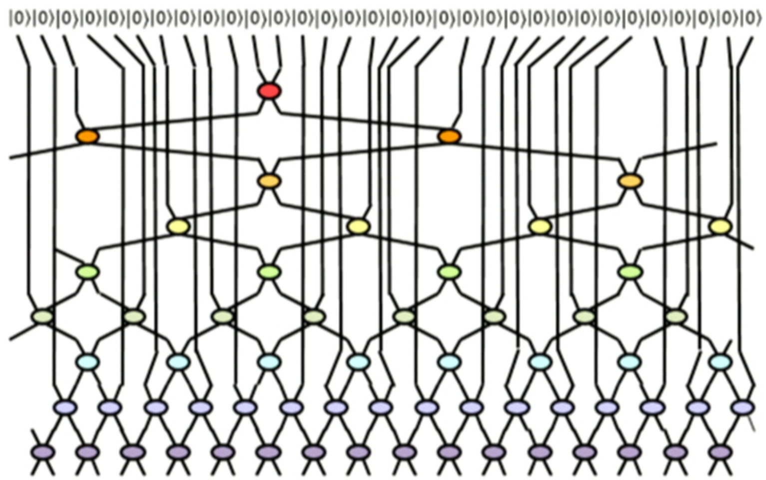
MERA as a quantum circuit

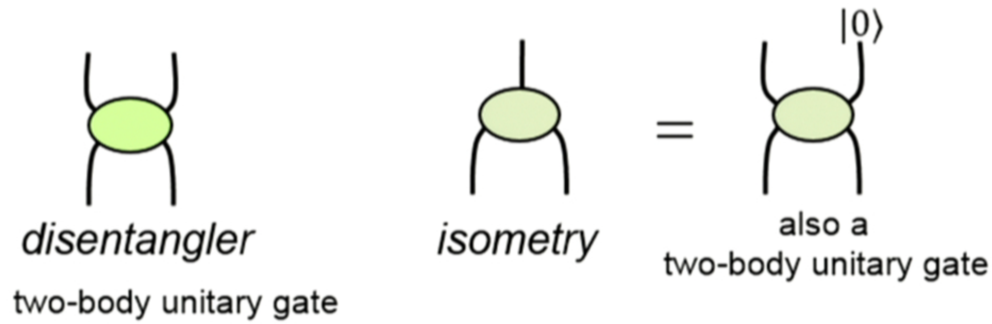


MERA as a quantum circuit

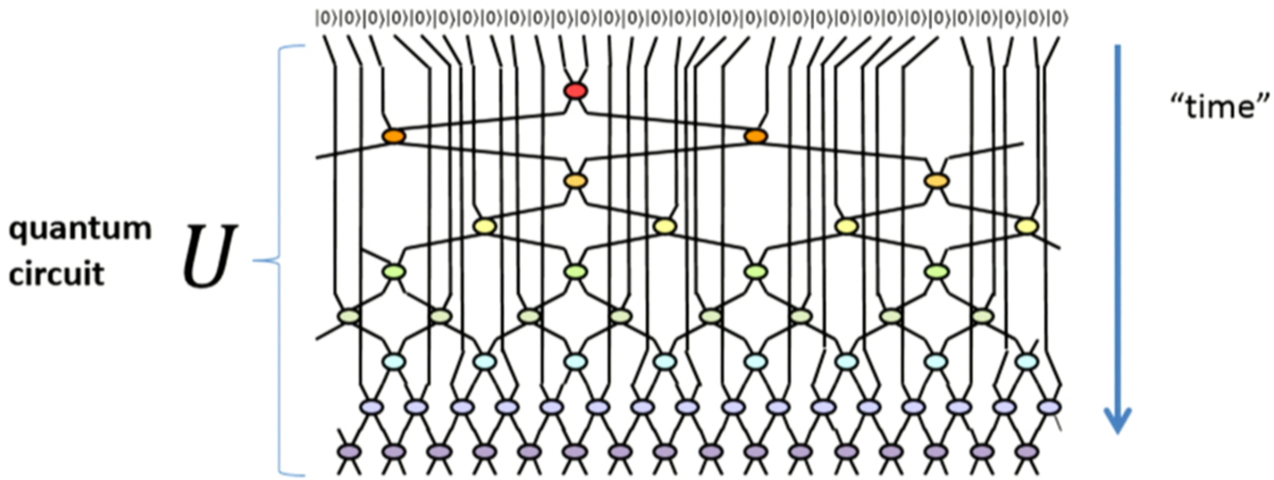


MERA as a quantum circuit

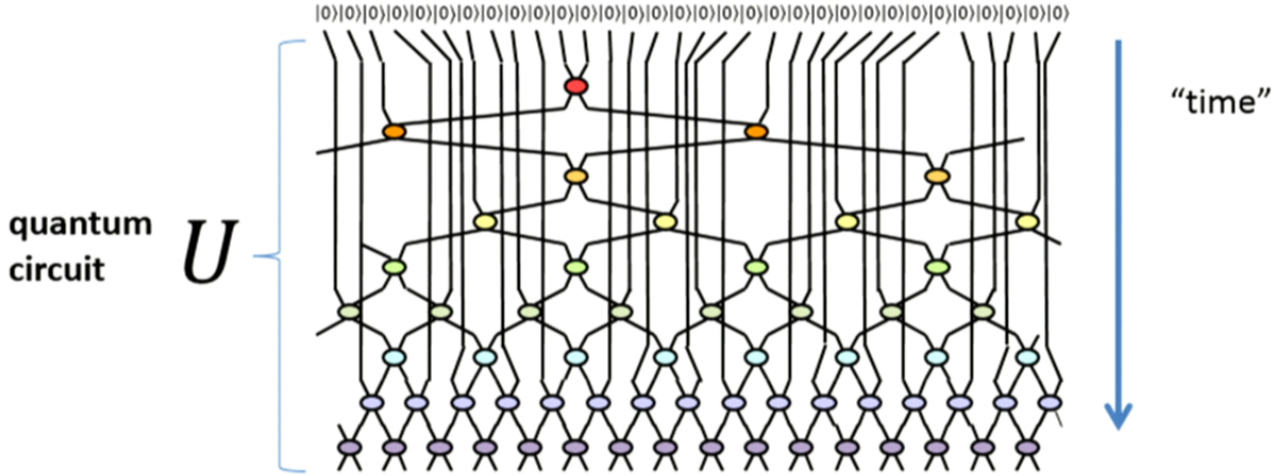




MERA as a quantum circuit

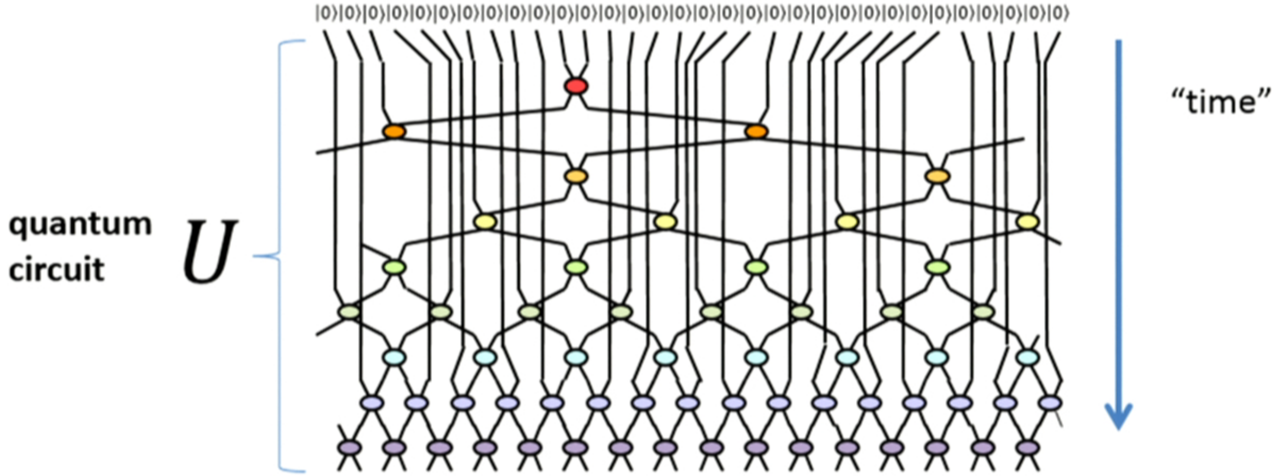


MERA as a quantum circuit



ground state ansatz $|\Psi\rangle = U\,|0\rangle^{\otimes N}$

MERA as a quantum circuit



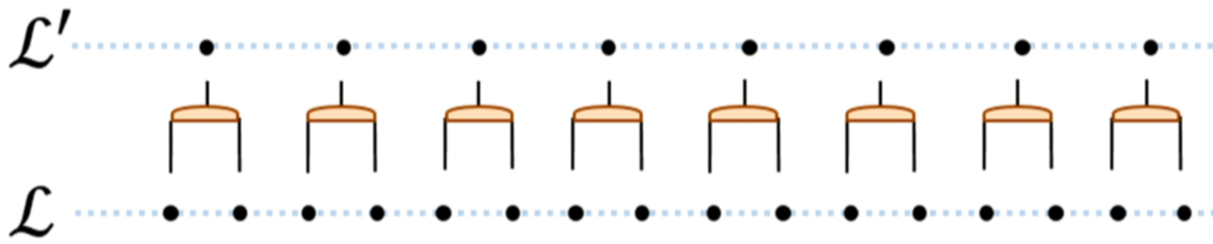
ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

Entanglement introduced by gates at different "times" (= length scales)

MERA as RG Transformation

[Vidal 05]

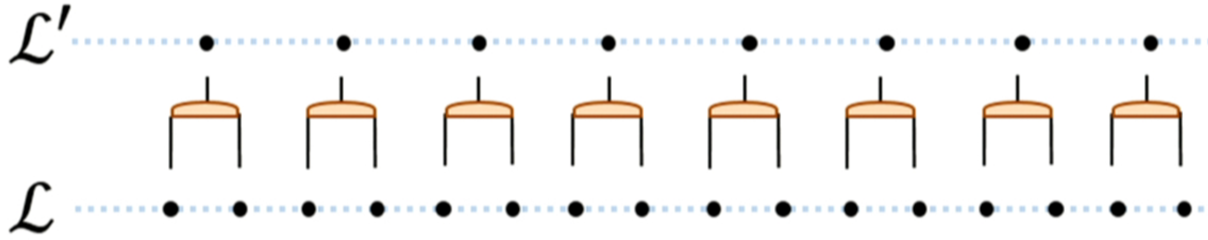
Kadanoff (1966) + White (1992)
blocking variational optimization



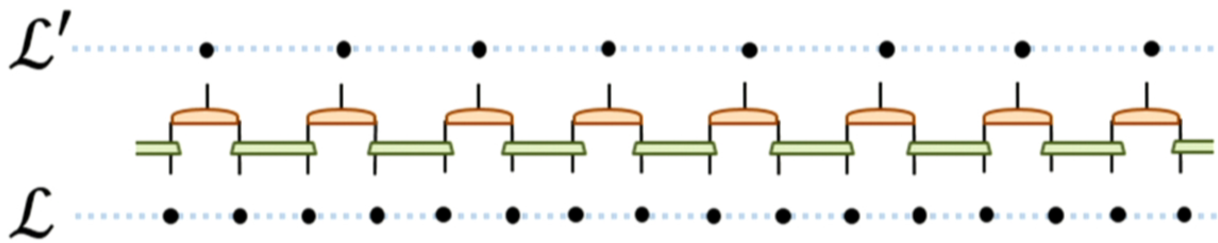
MERA as RG Transformation

[Vidal 05]

Kadanoff (1966) + White (1992)
blocking variational optimization



Entanglement renormalization (2005)

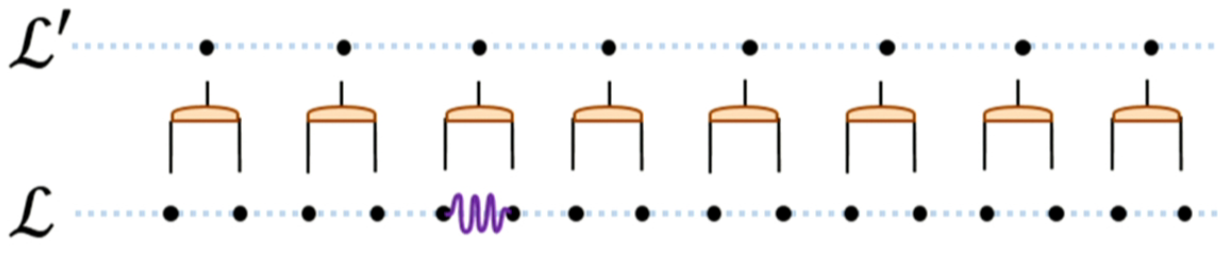


MERA as RG Transformation

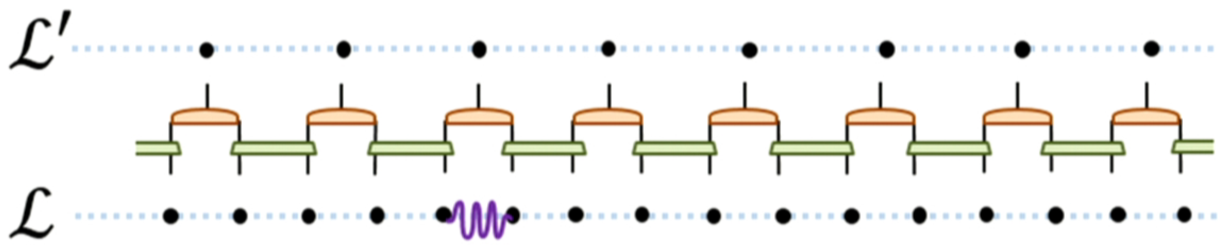
[Vidal 05]

Kadanoff (1966) + White (1992)

blocking variational optimization



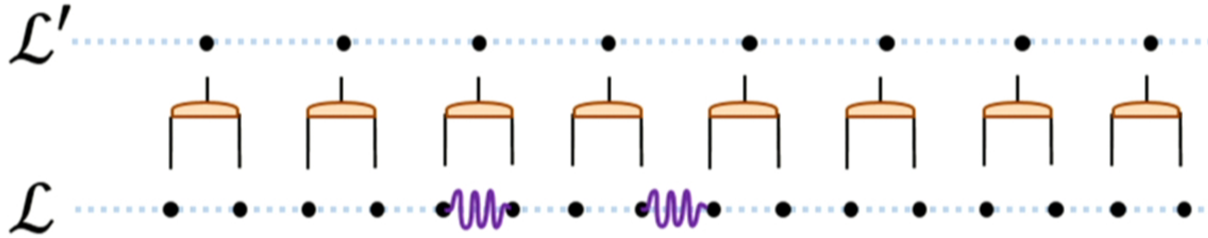
Entanglement renormalization (2005)



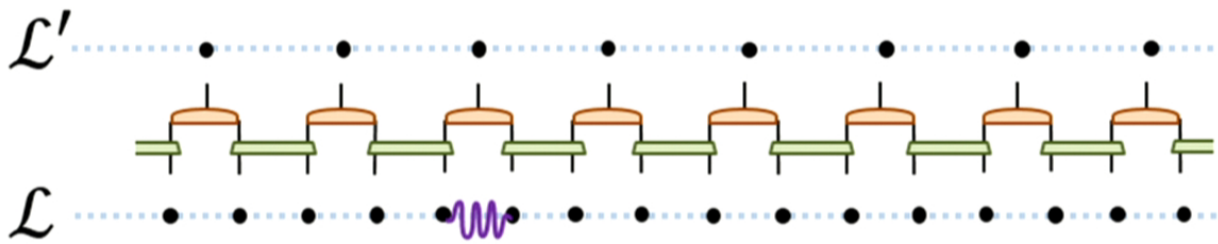
MERA as RG Transformation

[Vidal 05]

Kadanoff (1966) + White (1992)
blocking variational optimization



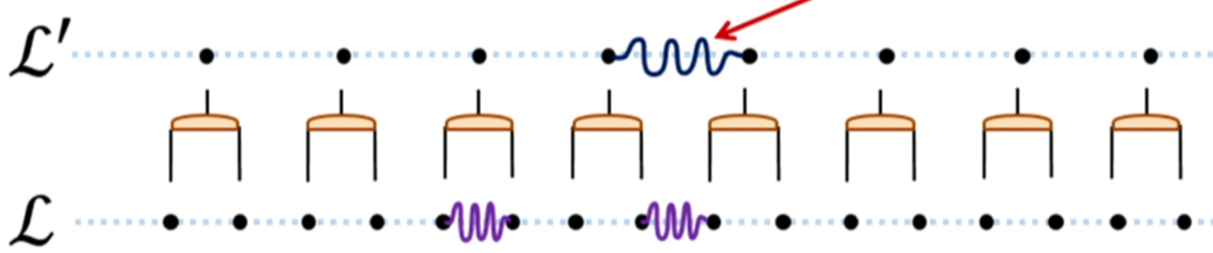
Entanglement renormalization (2005)



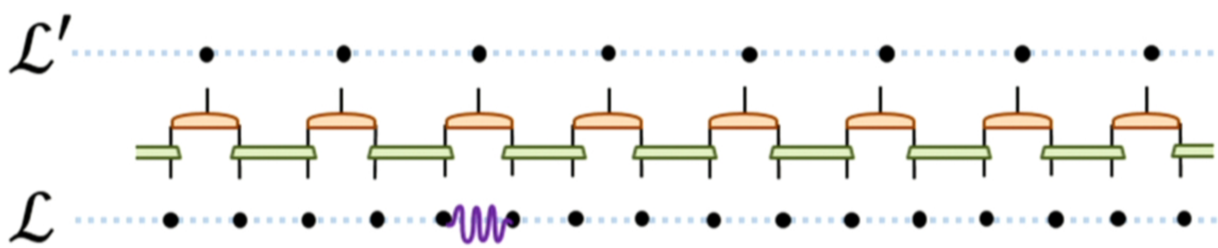
MERA as RG Transformation

[Vidal 05]

Kadanoff (1966) + White (1992)
blocking variational optimization



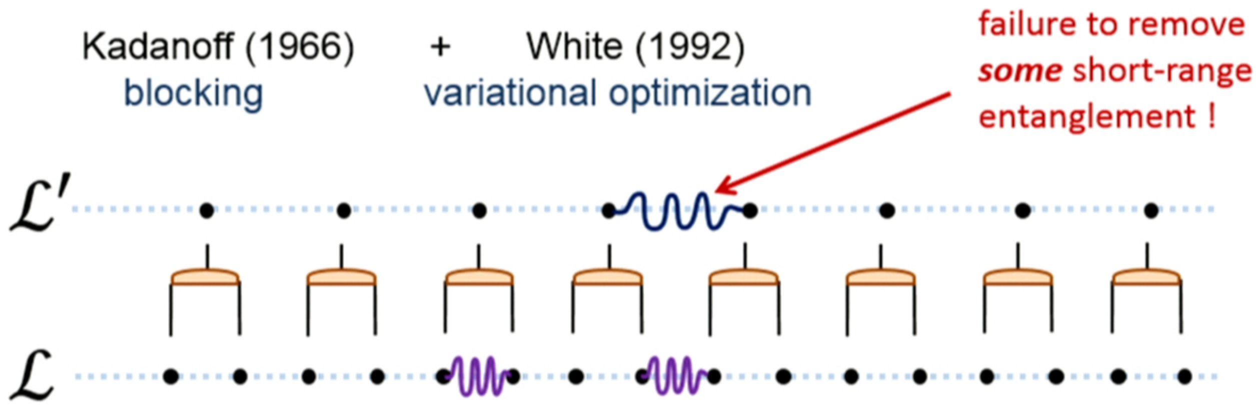
Entanglement renormalization (2005)



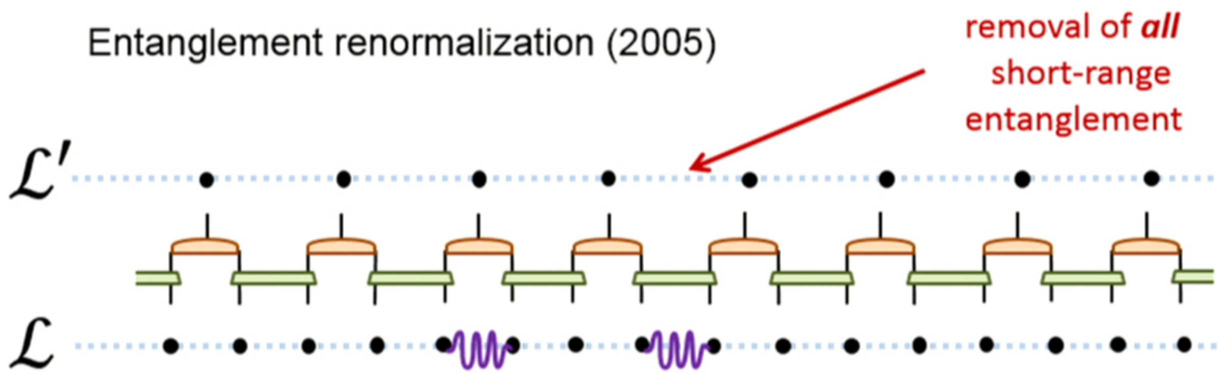
MERA as RG Transformation

[Vidal 05]

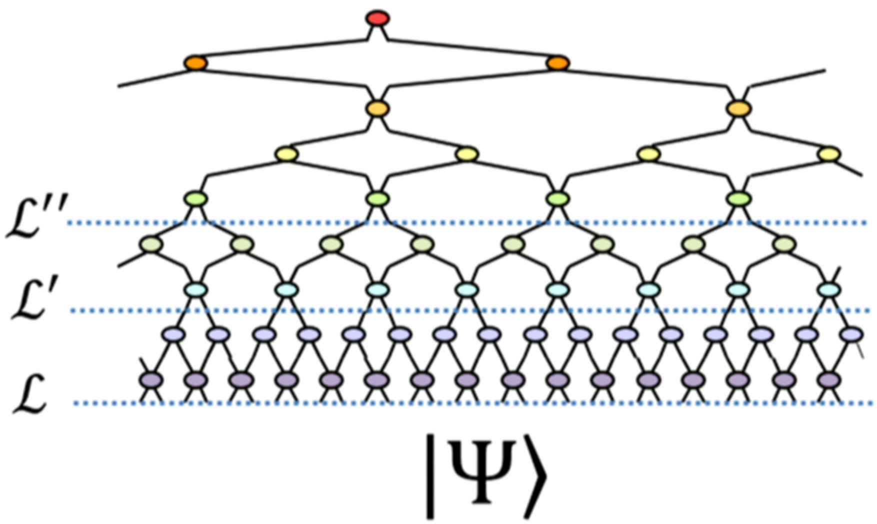
Kadanoff (1966) + White (1992)
blocking variational optimization



Entanglement renormalization (2005)

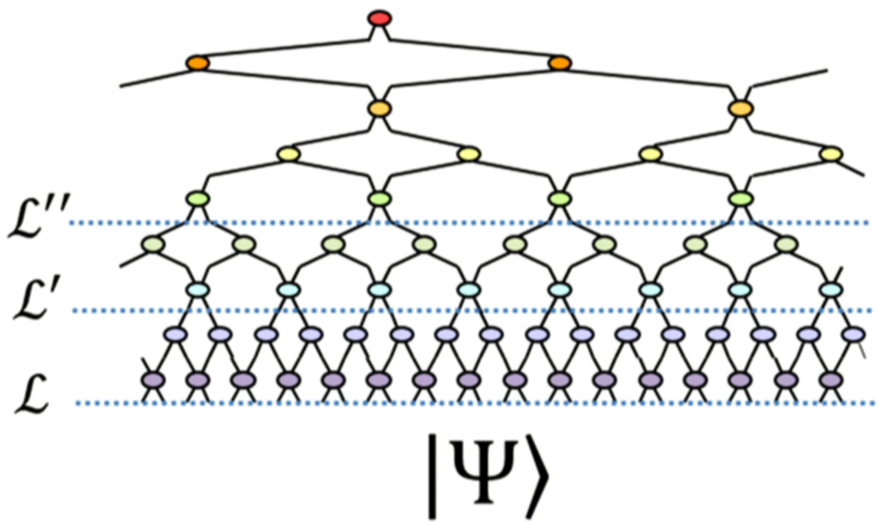


MERA as a sequence of ground state wave-functions



$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

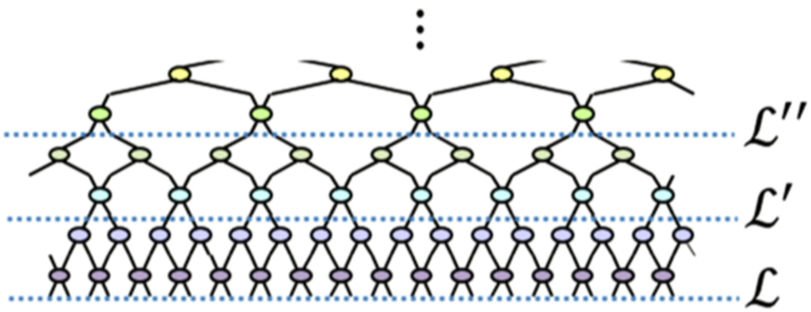
MERA as a sequence of ground state wave-functions

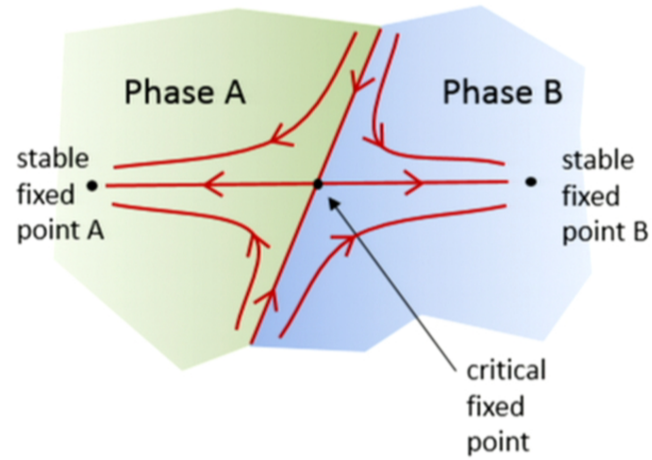


$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

MERA defines an RG flow in the space of wave-functions

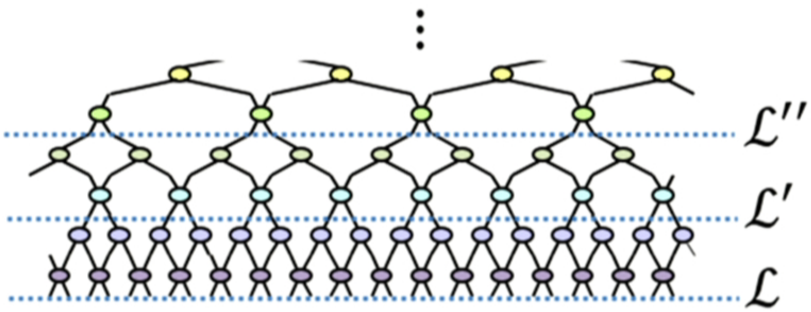
$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

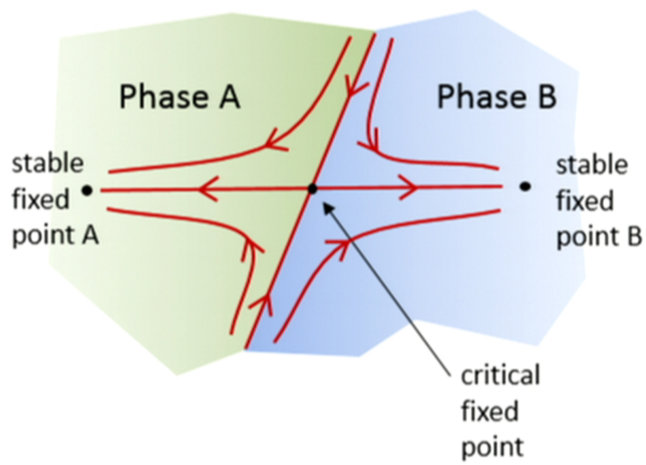




MERA defines an RG flow
in the space of wave-functions

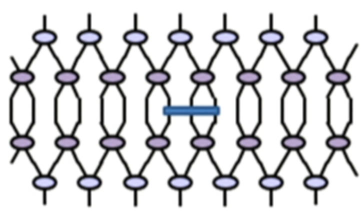
$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$





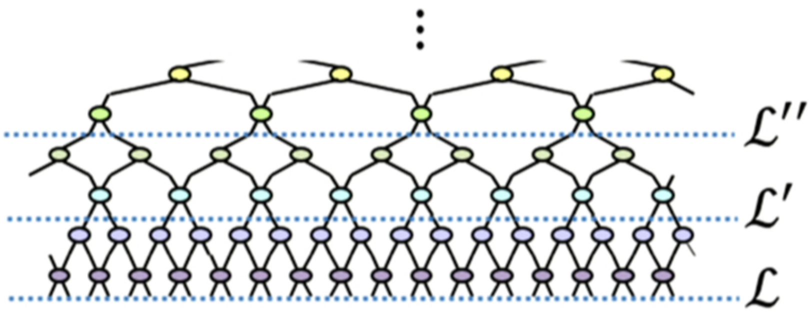
... and in the space of Hamiltonians

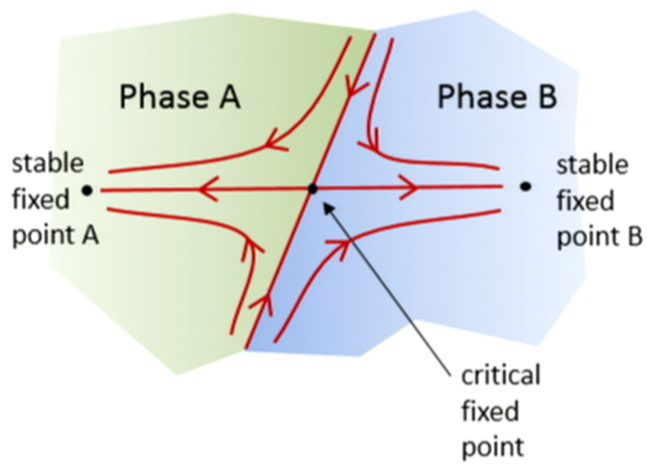
$$H \to H' \to H'' \to \cdots$$



MERA defines an RG flow
in the space of wave-functions

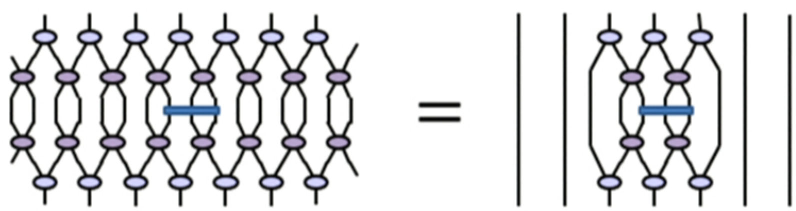
$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$





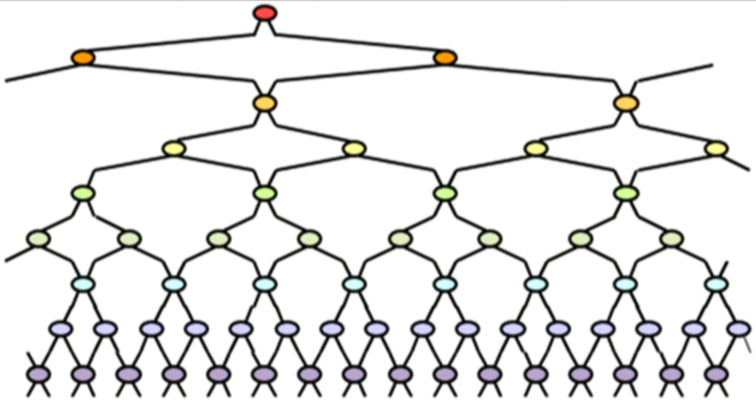
... and in the space of Hamiltonians

$H \rightarrow H' \rightarrow H'' \rightarrow \cdots$



Summary so far

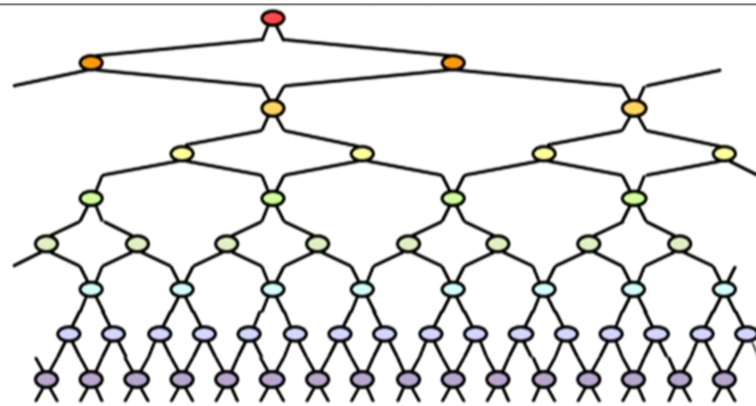
MERA



- Variational parameters for different length scales

Summary so far

MERA



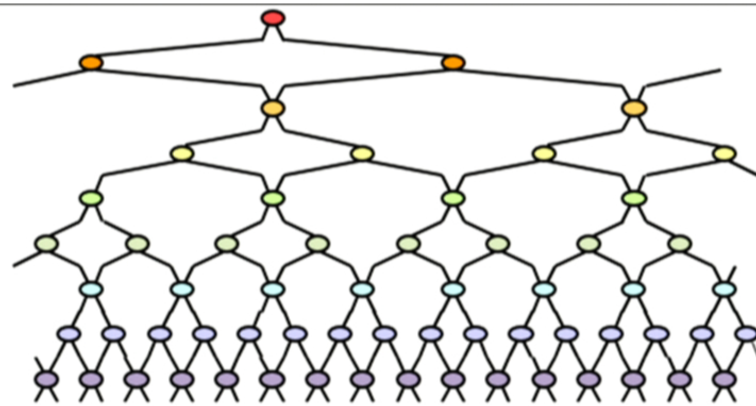
- Variational parameters for different length scales
- It is secretly a **quantum circuit** → *"entanglement at different length scales"*

and an **RG transformation**

"removes short-range entanglement" $|\Psi\rangle \rightarrow |\Psi'\rangle$ $H \rightarrow H'$ *"preservation of locality"*

Summary so far

MERA



- Variational parameters for different length scales
- It is secretly a **quantum circuit** → *"entanglement at different length scales"*

and an **RG transformation**

"removes short-range entanglement" $|\Psi\rangle \rightarrow |\Psi'\rangle$ $H \rightarrow H'$ *"preservation of locality"*

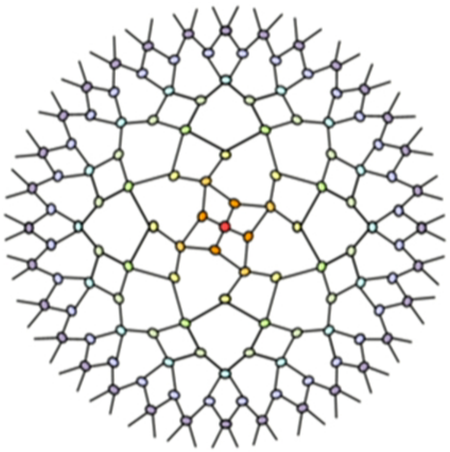
- blah, blah, blah…

Does it work?

input

1D quantum Hamiltonian

- **on the lattice**
- **at a critical point**



[optimization by energy minimization]

input

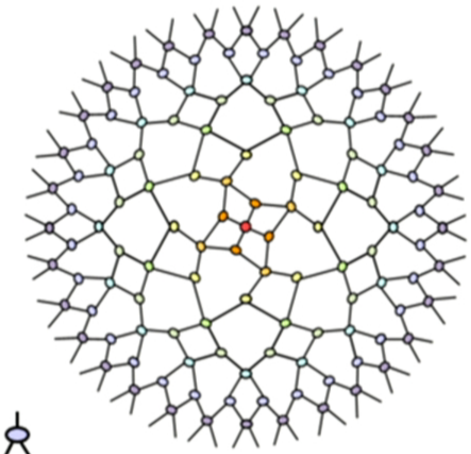
1D quantum Hamiltonian

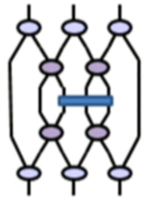
- **on the lattice**
- **at a critical point**

output

Numerical determination of conformal data:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$





[optimization by energy minimization]

input

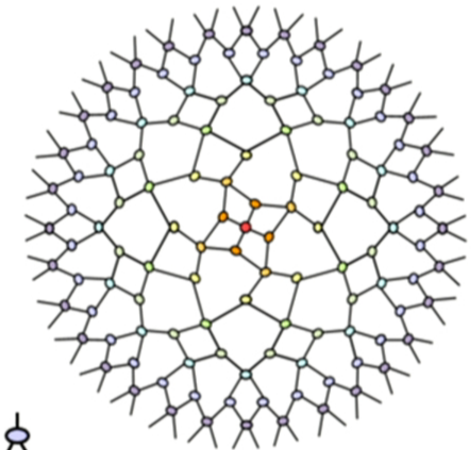
1D quantum Hamiltonian

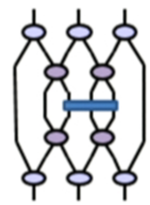
- **on the lattice**
- **at a critical point**

output

Numerical determination of conformal data:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

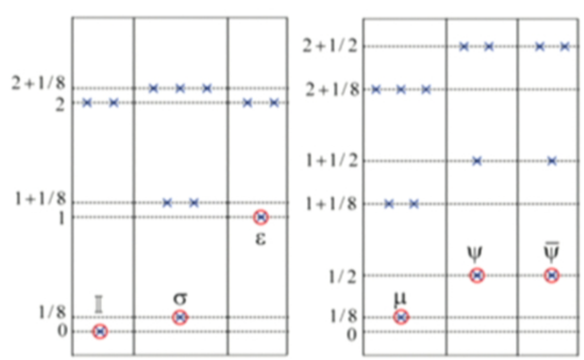




[optimization by energy minimization]

Pfeifer, Evenbly, Vidal 08

e.g. critical Ising model (approx. an hour on your laptop)



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

input

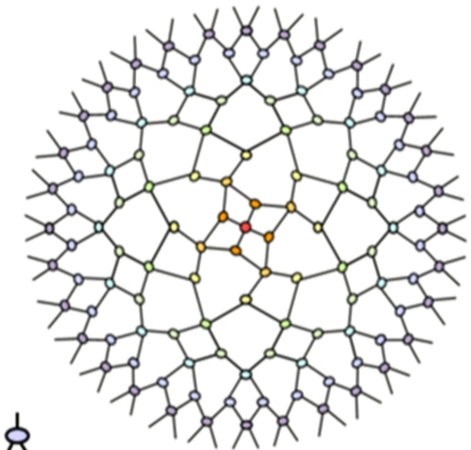
1D quantum Hamiltonian

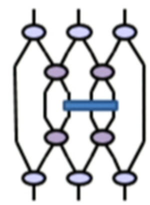
- **on the lattice**
- **at a critical point**

output

Numerical determination of conformal data:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

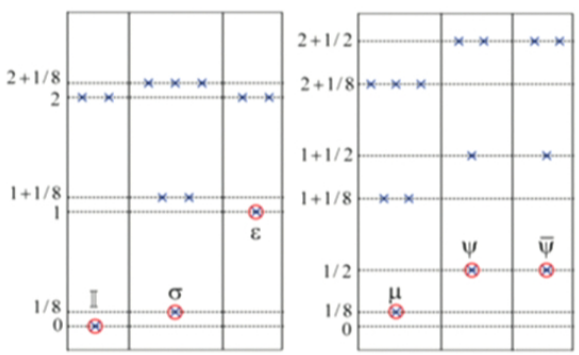




[optimization by energy minimization]

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $\quad C_{\epsilon\mu\mu} = -\frac{1}{2}$

$C_{\epsilon\psi\bar{\psi}} = i$ $\quad C_{\epsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $\quad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$

input

1D quantum Hamiltonian

- **on the lattice**
- **at a critical point**

output

Numerical determination of confo

- **central charge** c
- **scaling dimensions** Δ_α **and conformal spins** s_α
- **OPE coefficients** $C_{\alpha\beta\gamma}$

Also:

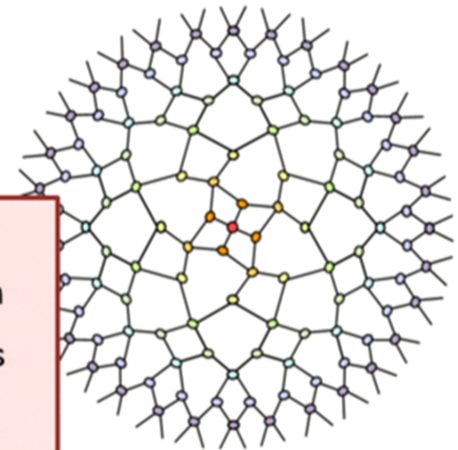
1+1 critical systems with

- ➢ impurities/defects
- ➢ boundaries
- ➢ interfaces

2+1 gapped phases with

- ➢ topological order
- ➢ frustrated spins
- ➢ interacting fermions

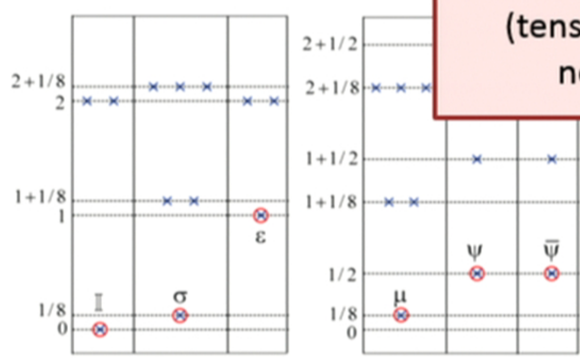
(tensor networks have no sign problem)



[optimization by energy minimization]

Pfeifer, Evenbly, Vidal 08

e.g. critical Ising model (appro



$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $\quad C_{\epsilon\mu\mu} = -\frac{1}{2}$

$C_{\epsilon\psi\bar{\psi}} = i$ $\quad C_{\epsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $\quad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$

part I

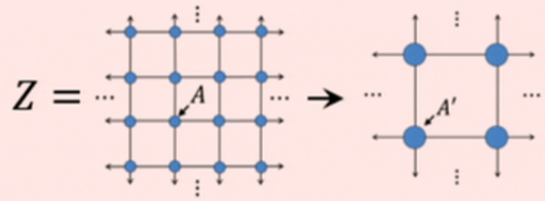
1+1D quantum systems:
ground state wave-function

multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)



tensor network renormalization

(TNR)

part III

TNR ⇒ MERA

local scale transformations

Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

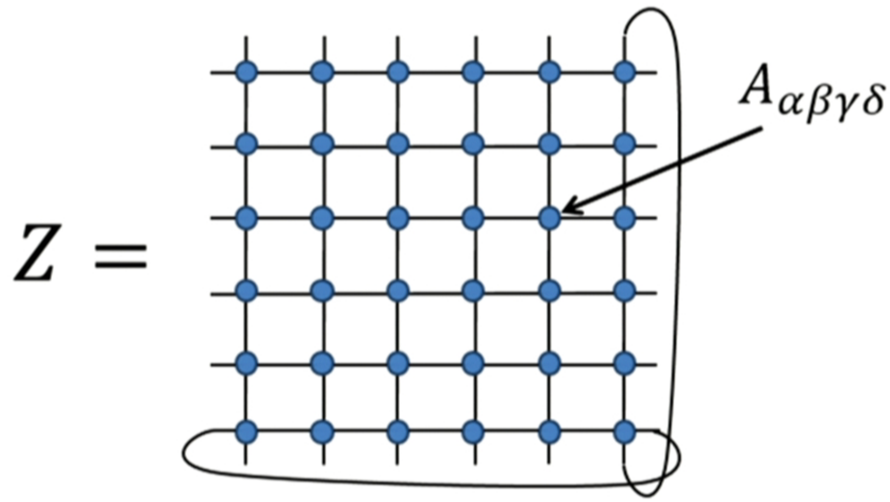
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

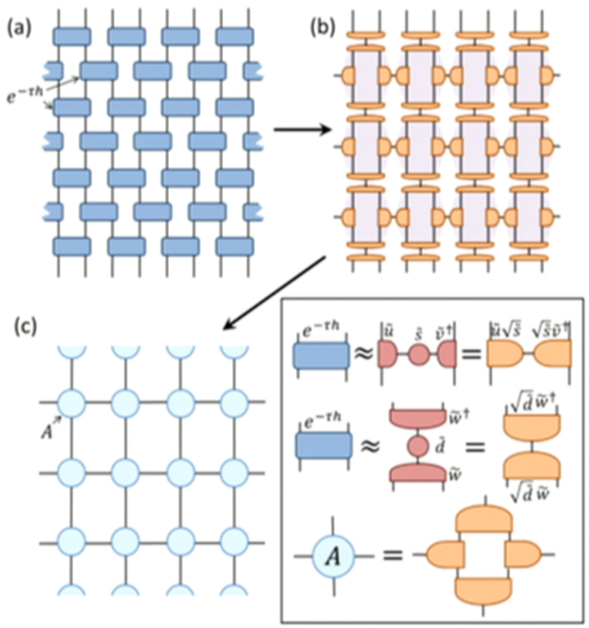
as a tensor network



Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$

$$H_q^{1d} = \sum_i (\sigma_z^i + \sigma_x^i \sigma_x^{i+1})$$



Euclidean path integral

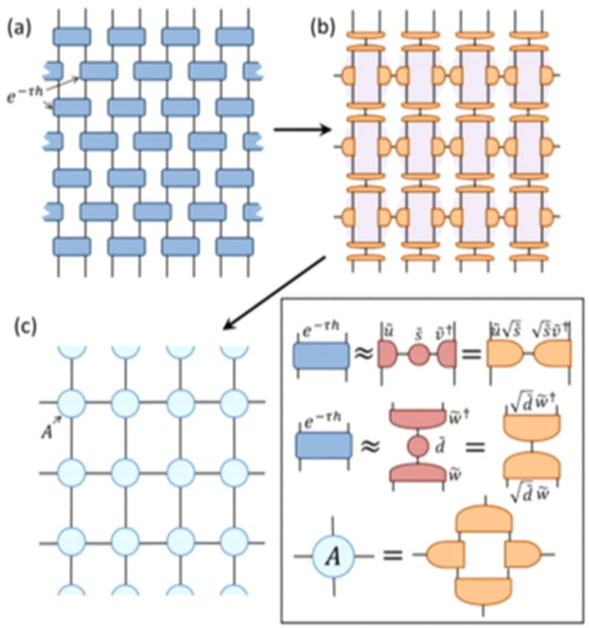
$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

$$H_q^{1d} = \sum_i (\sigma_z^i + \sigma_x^i \sigma_x^{i+1})$$

Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

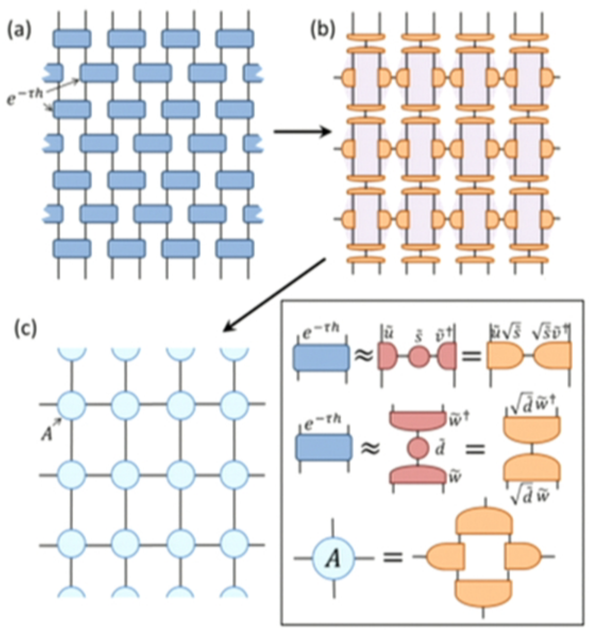
$$H_{cl}^{2d} = \sum_{<i,j>} \sigma^i \sigma^j$$



Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

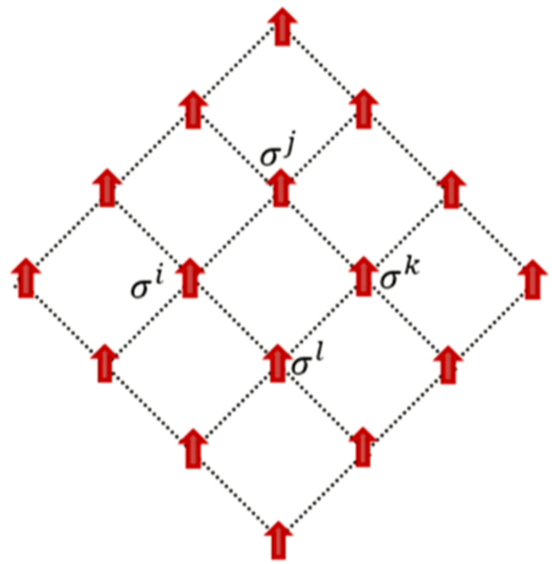
$$H_q^{1d} = \sum_i (\sigma_z^i + \sigma_x^i \sigma_x^{i+1})$$



Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

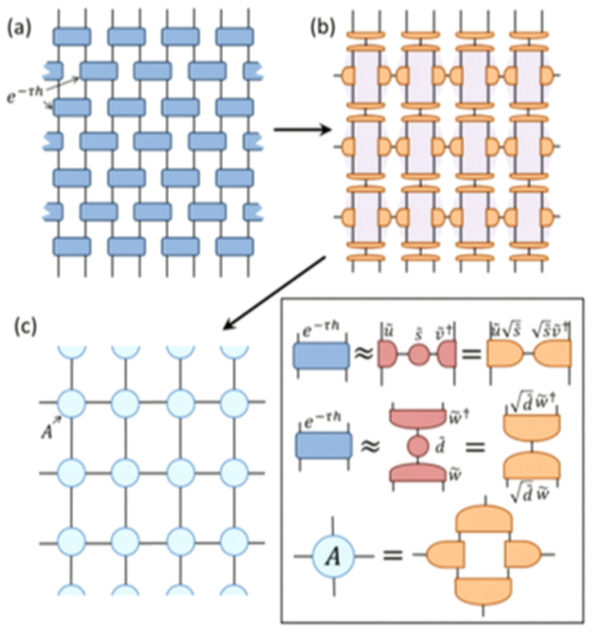
$$H_{cl}^{2d} = \sum_{<i,j>} \sigma^i \sigma^j$$



Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

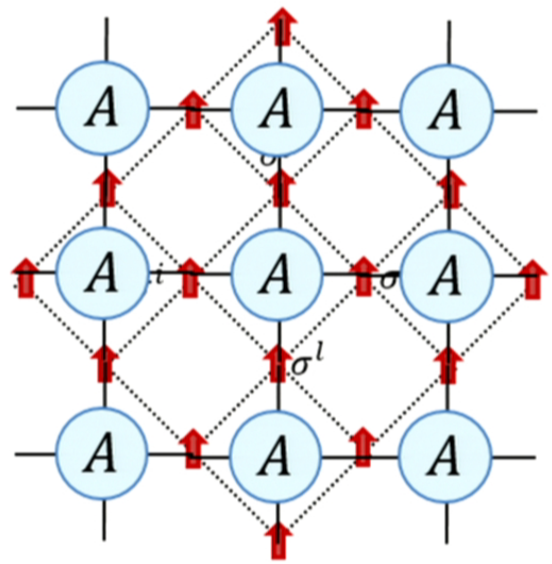
$$H_q^{1d} = \sum_i (\sigma_z^i + \sigma_x^i \sigma_x^{i+1})$$



Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

$$H_{cl}^{2d} = \sum_{<i,j>} \sigma^i \sigma^j$$

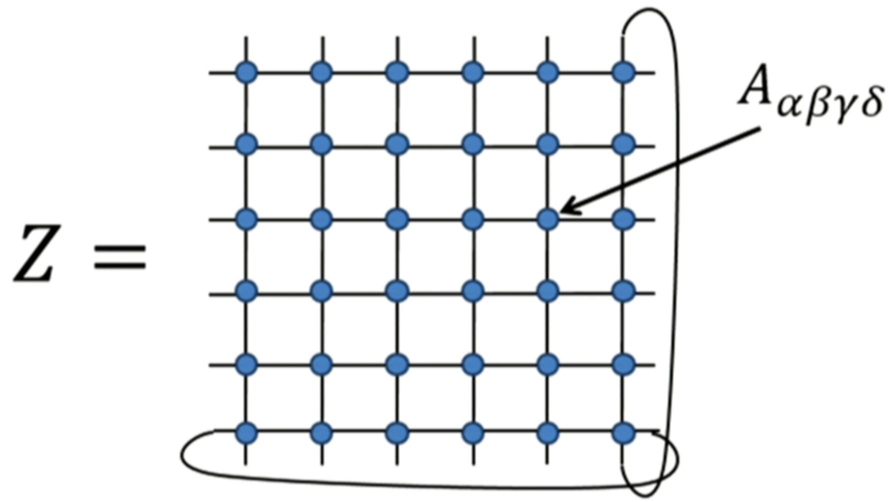


$$A_{ijkl} = e^{-(\sigma_i\sigma_j + \sigma_j\sigma_k + \sigma_k\sigma_l + \sigma_l\sigma_i)/T}$$

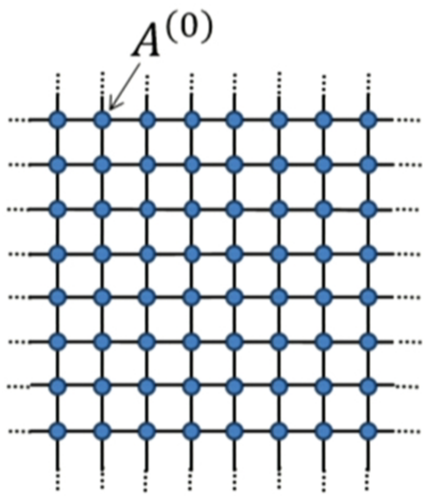
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

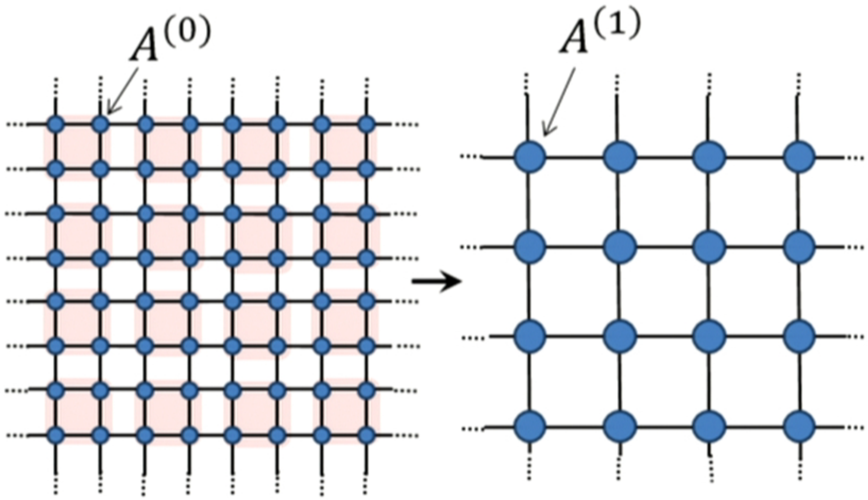
as a tensor network



goal: Define a proper* RG flow in the space of tensors

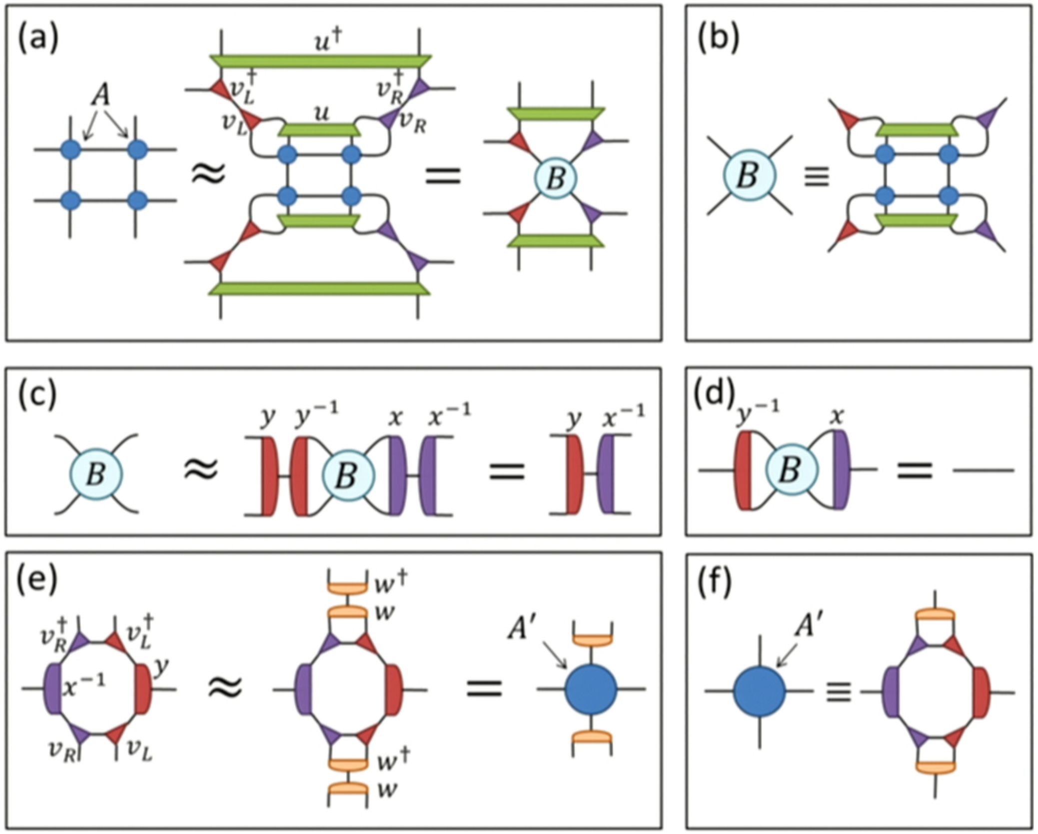


goal: Define a proper* RG flow in the space of tensors



Tensor Network Renormalization (TNR)

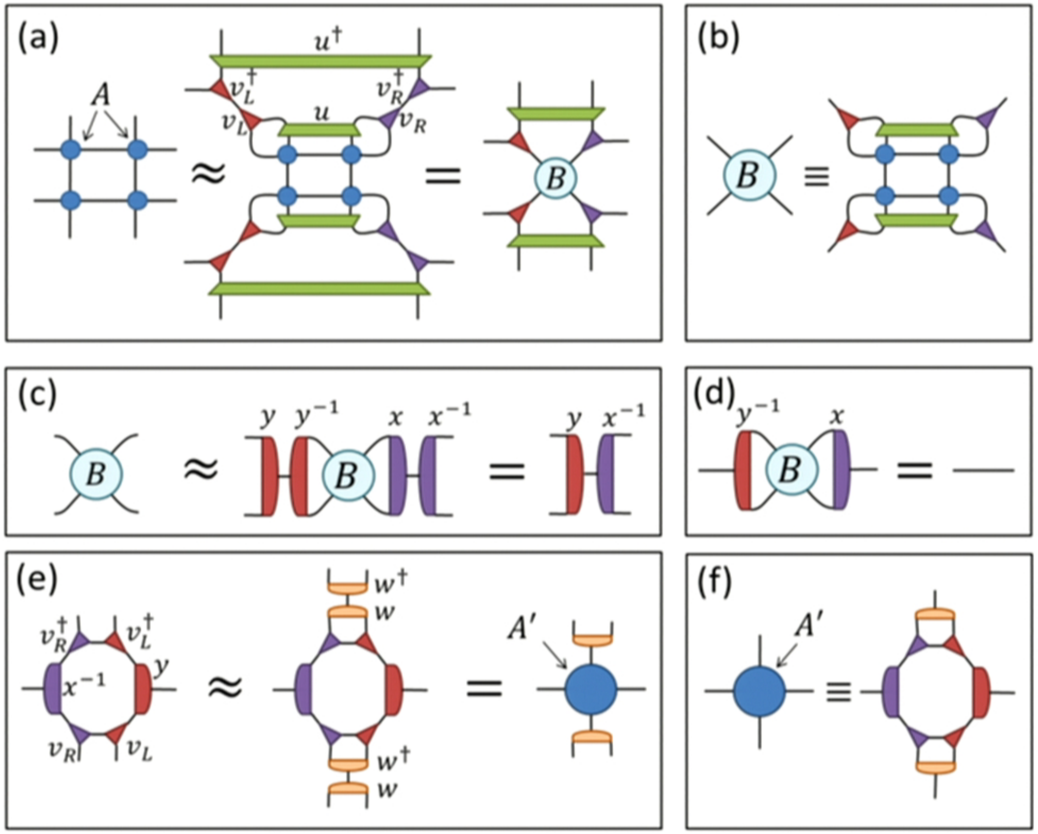
Evenbly, Vidal PRL 2015



For a detailed description see:
Glen Evenbly, "Algorithms for tensor network renormalization"
arXiv:1509.07484

Tensor Network Renormalization (TNR)

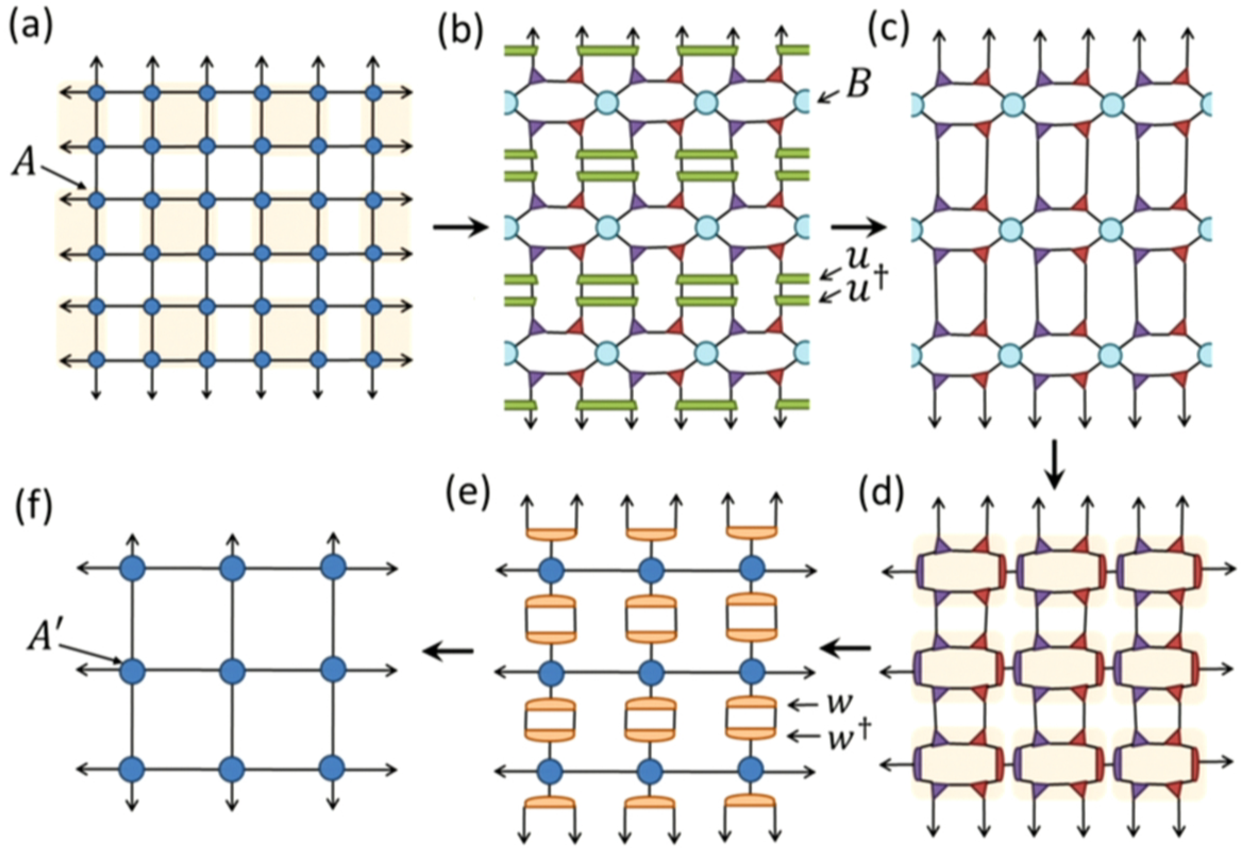
Evenbly, Vidal PRL 2015



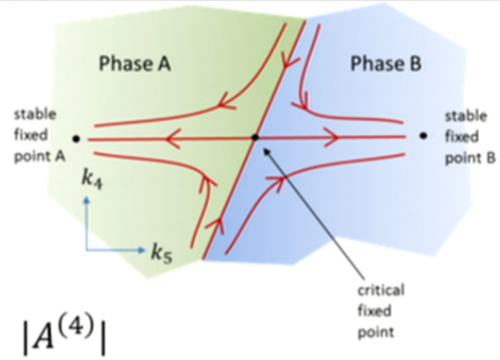
For a detailed description see:
Glen Evenbly, "Algorithms for tensor network renormalization"
arXiv:1509.07484

Tensor Network Renormalization (TNR)

Evenbly, Vidal PRL 2015



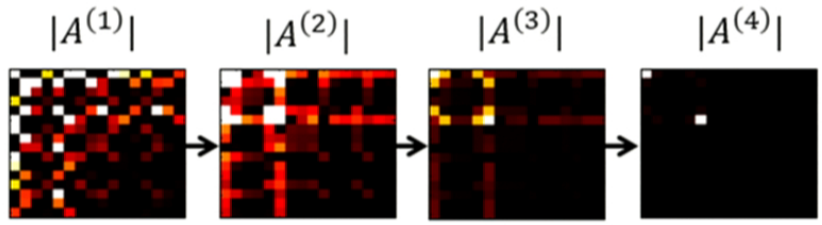
TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$

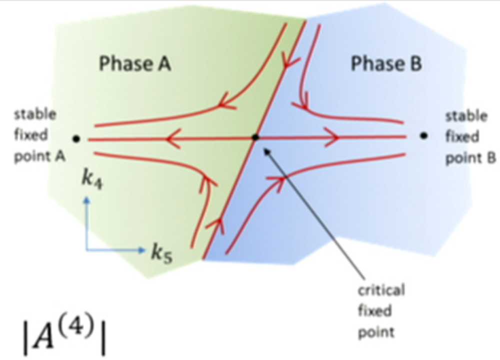
below critical

$T = 0.9\, T_c$

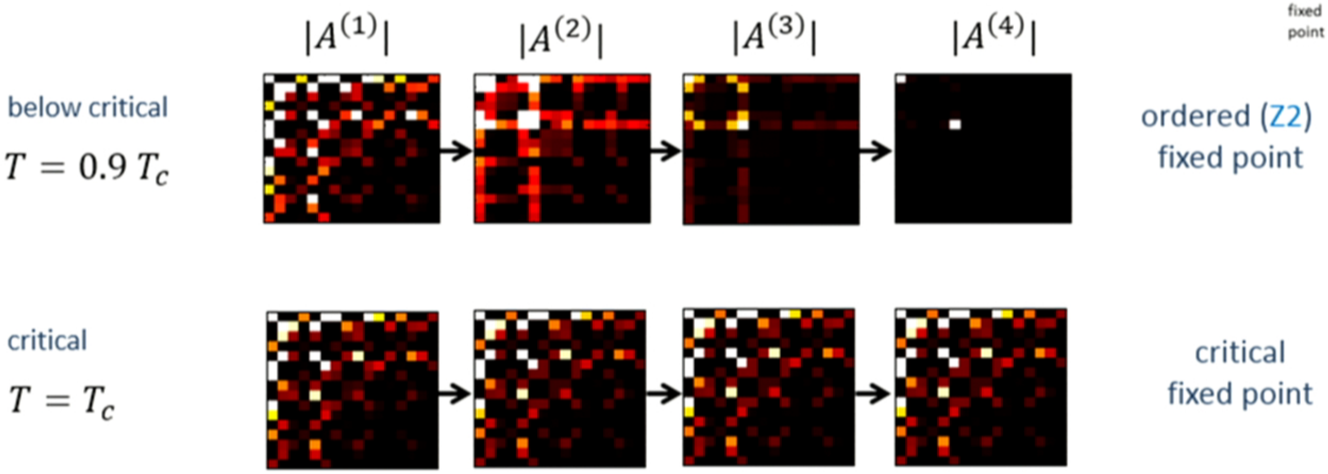


ordered (Z2)
fixed point

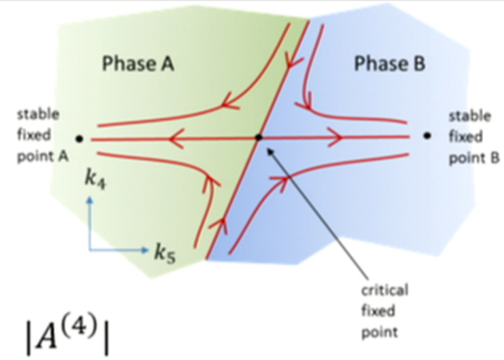
TNR -> proper RG flow
Example: 2D classical Ising



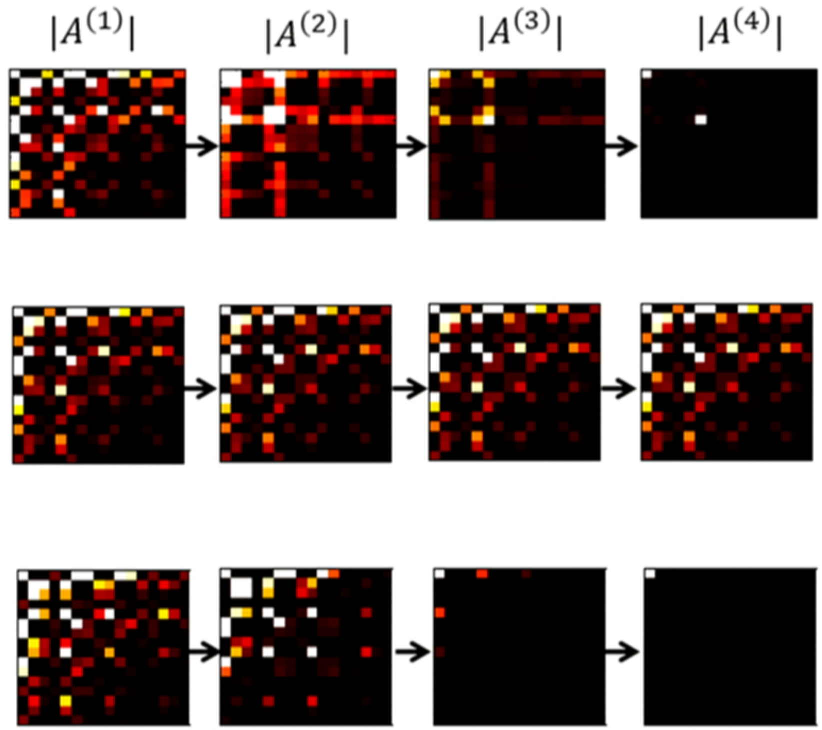
$$A \to A' \to A'' \to \cdots \to A^{fp}$$



TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$



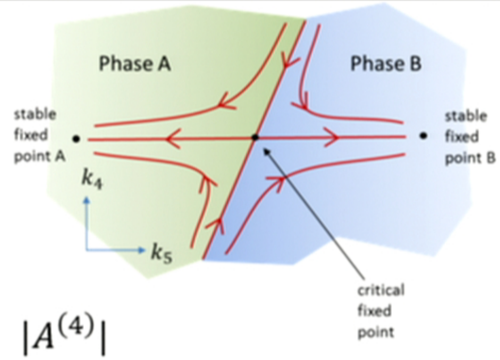
below critical
$T = 0.9\ T_c$
→ ordered (Z2) fixed point

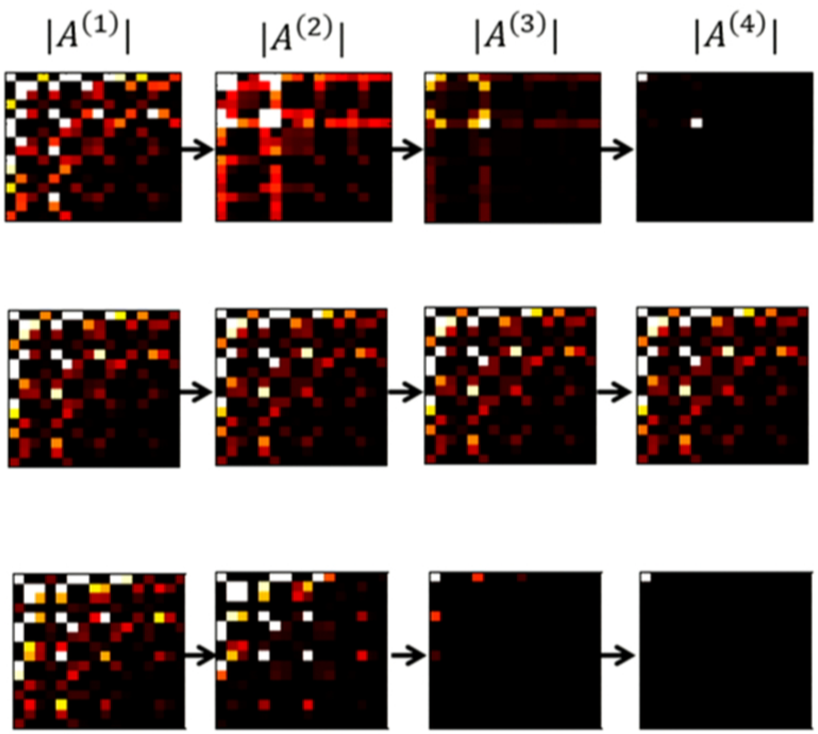
critical
$T = T_c$
→ critical fixed point

above critical
$T = 1.1\ T_c$
→ disordered (trivial) fixed point

TNR -> proper RG flow
Example: 2D classical Ising

$$A \to A' \to A'' \to \cdots \to A^{fp}$$





below critical $T = 0.9\, T_c$ — ordered (Z2) fixed point

also TEFR of Gu, Wen (2009)

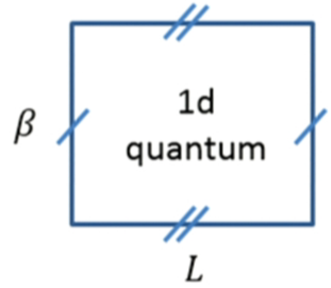
critical $T = T_c$ — critical fixed point

above critical $T = 1.1\, T_c$ — disordered (trivial) fixed point

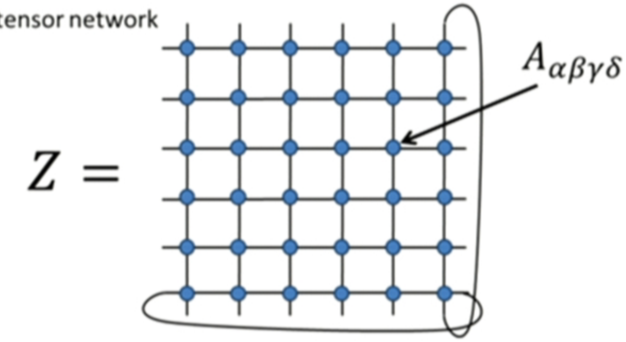
also TEFR of Gu, Wen (2009)

Euclidean path integral

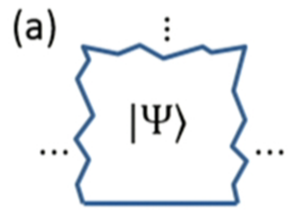
$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$



as a tensor network



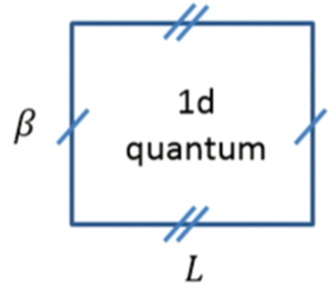
Euclidean time evolution on different geometries



ground state

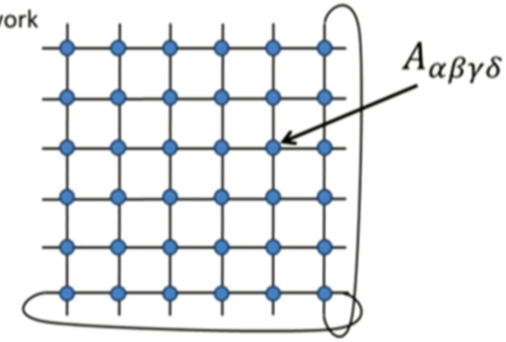
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

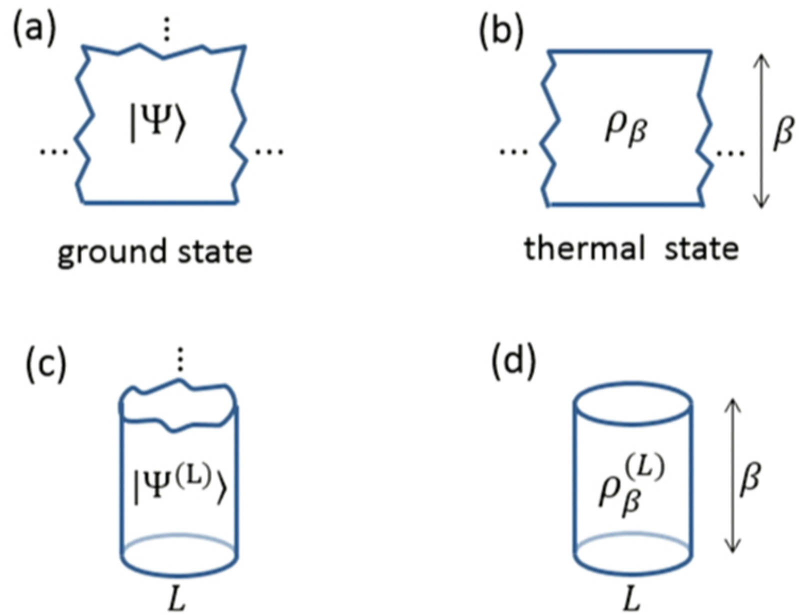


as a tensor network

$$Z =$$



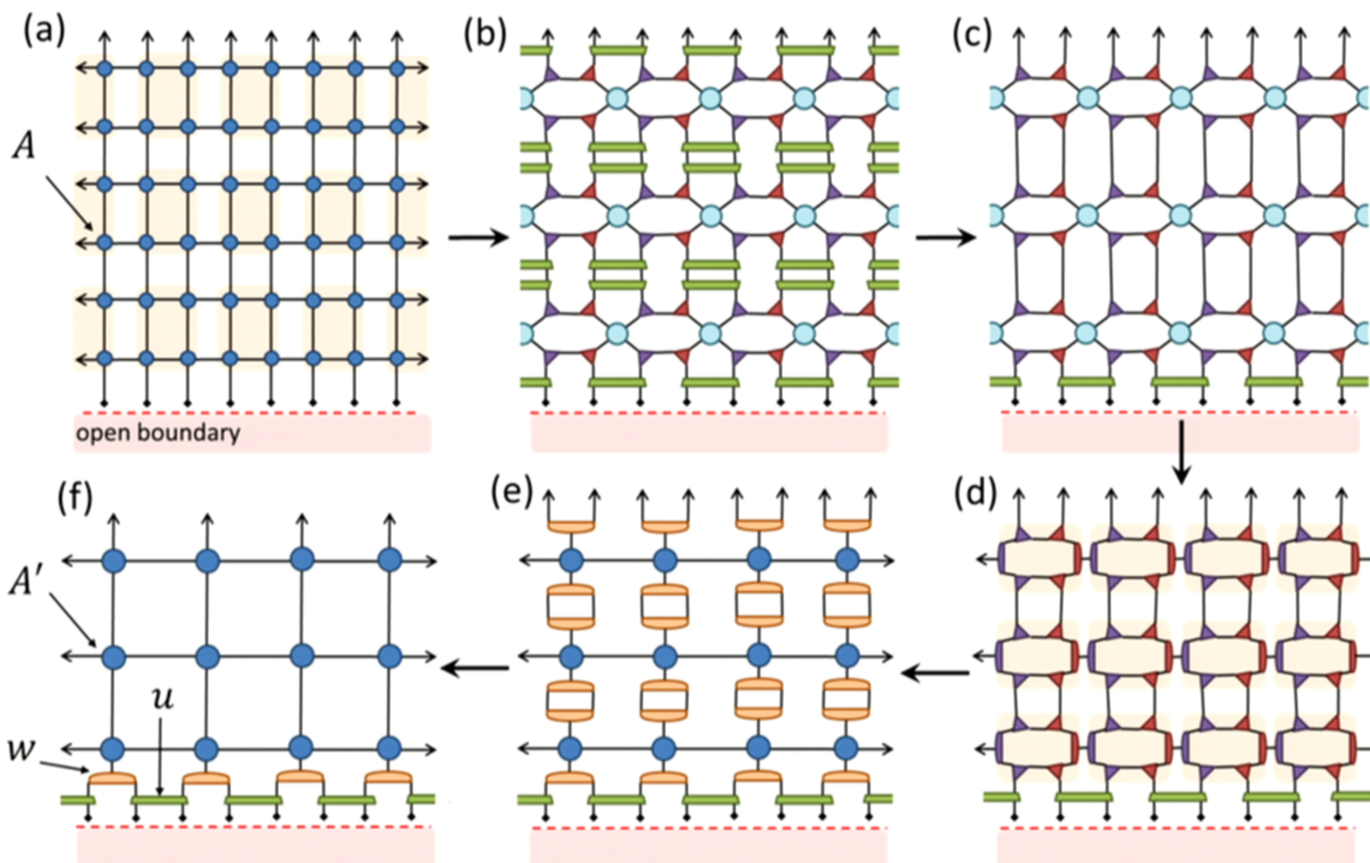
Euclidean time evolution on different geometries



Upper half plane

[Evenbly, Vidal, 15]

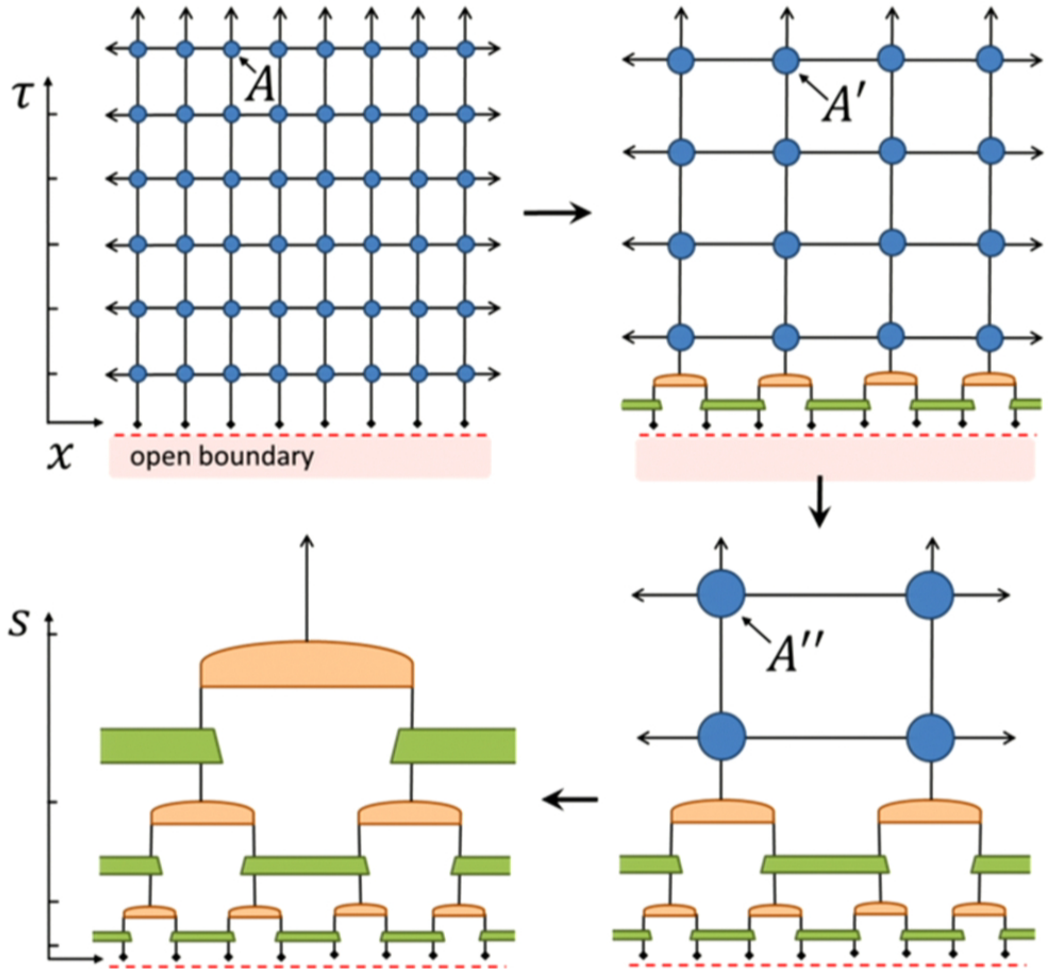
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$



Upper half plane

[Evenbly, Vidal, 15]

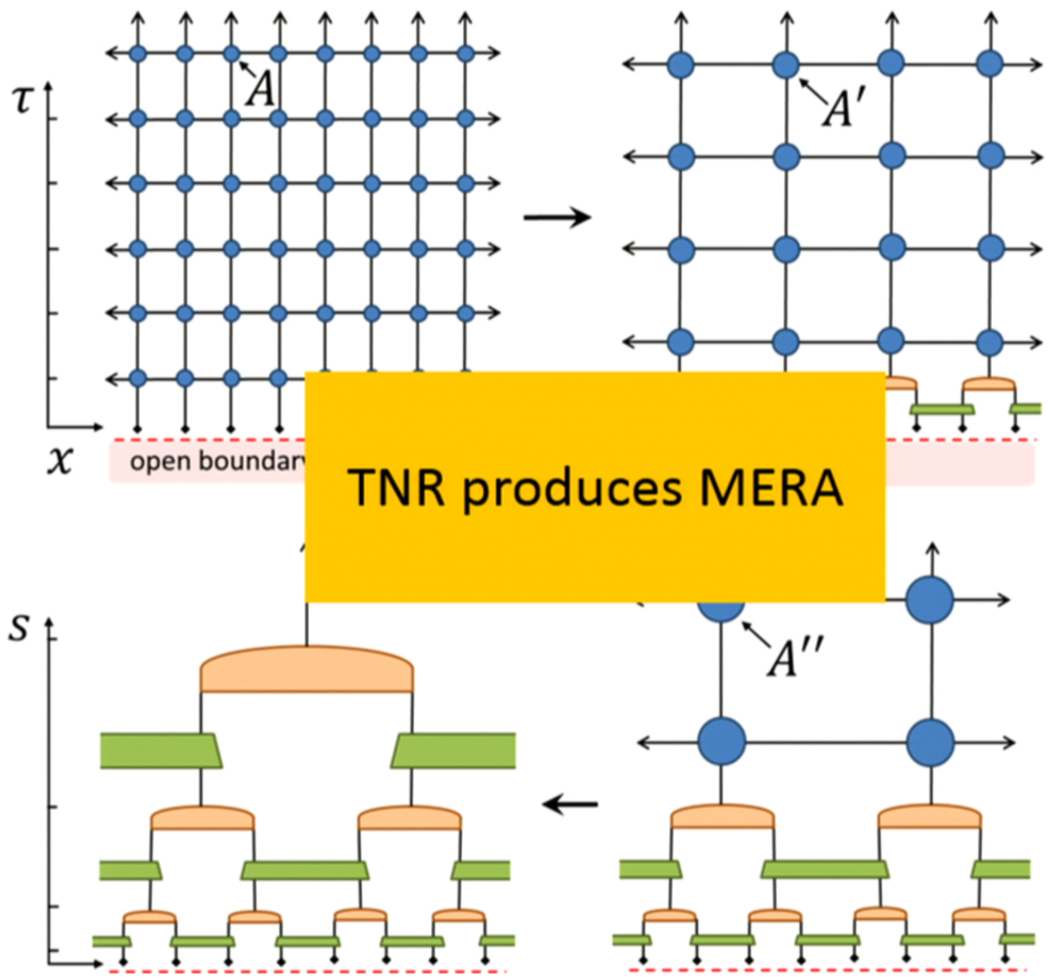
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$



[Evenbly, Vidal, 15]

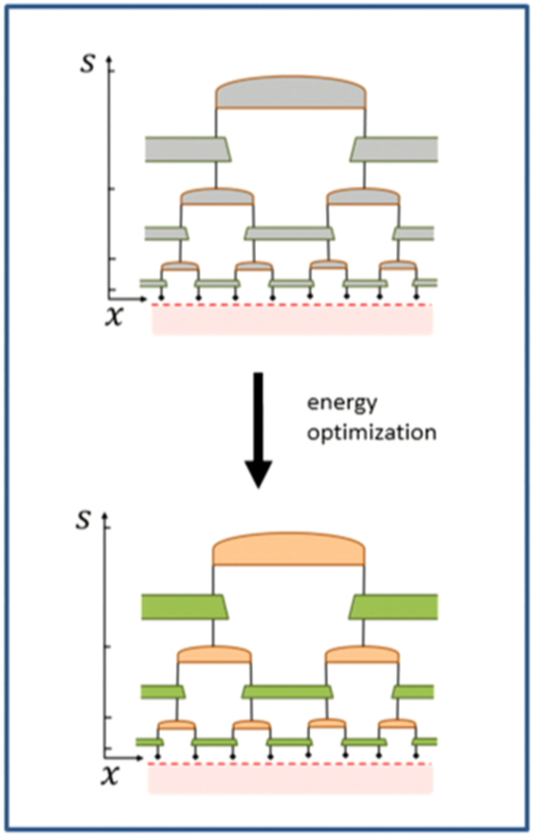
Upper half plane

$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$

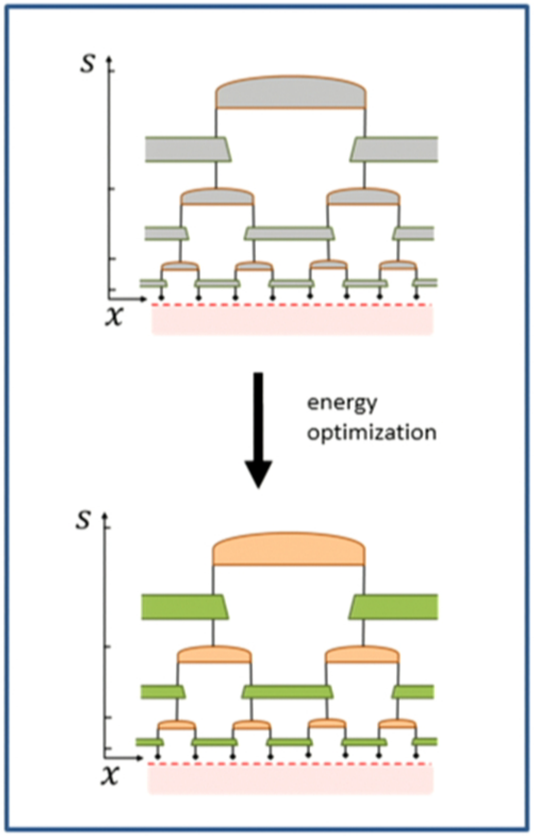


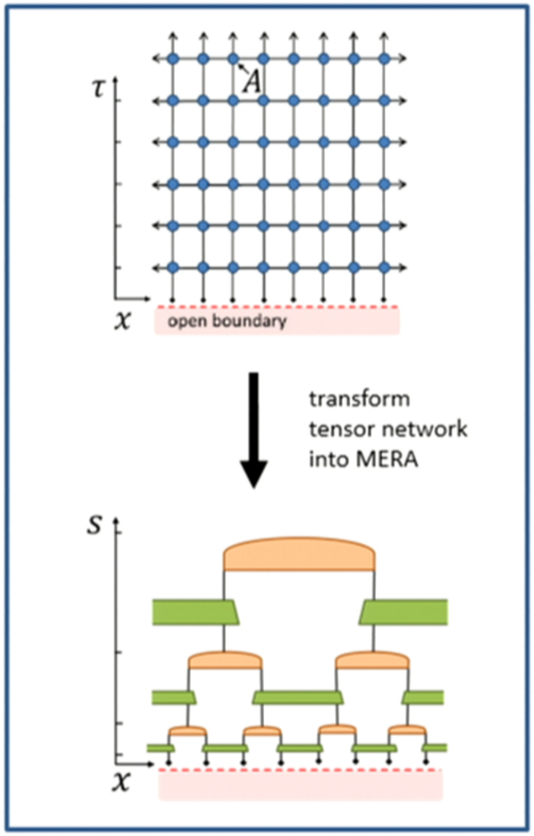
TNR produces MERA

MERA = variational ansatz

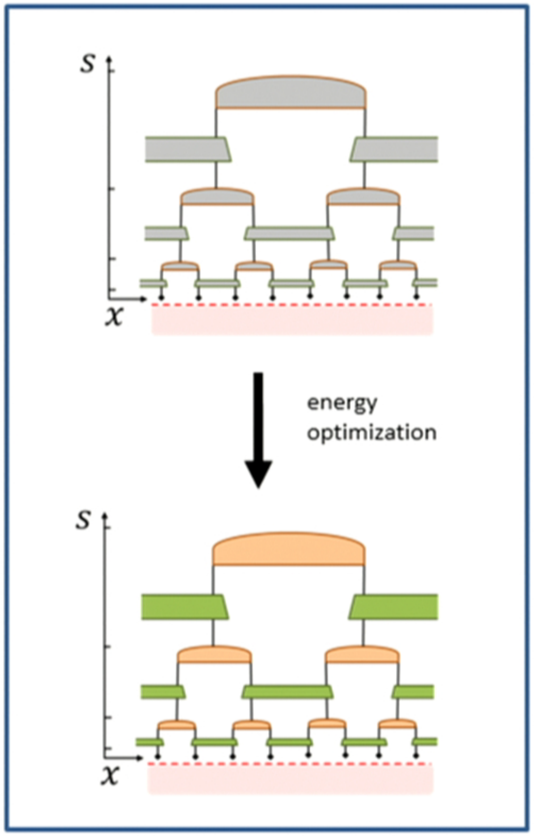


MERA = variational ansatz → MERA = by-product of TNR



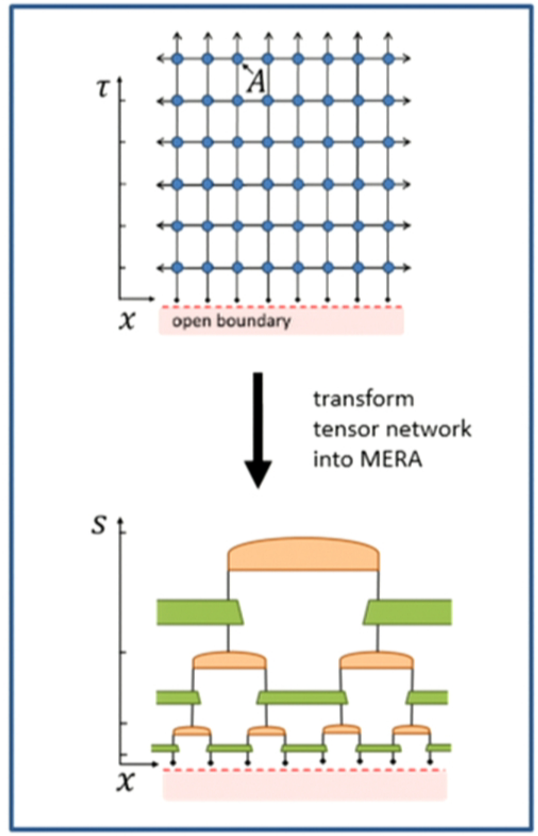


MERA = variational ansatz → MERA = by-product of TNR



energy minimization

- 1000s of iterations over scale
- local minima
- correct ground ?



TNR -> MERA

- single iteration over scale
- rewrite tensor network for ground state
- certificate of accuracy

part I

1+1D quantum systems:
ground state wave-function

multi-scale entanglement renormalization ansatz

(MERA)

part II

1+1D quantum systems:
Euclidean path integral
(or 2D statistical partition function)

tensor network renormalization

(TNR)

part III

TNR ⇒ MERA

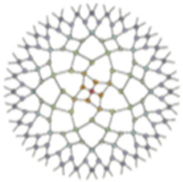
local scale transformations

summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)

summary / conclusions:

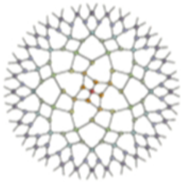
- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)

- Efficient representation of ground states (MERA) 

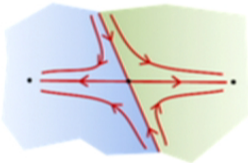
summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)

- Efficient representation of ground states (MERA)



- Proper* RG transformation (x3 !)



tensors

$A \to A' \to A'' \to \cdots$

wave-functions

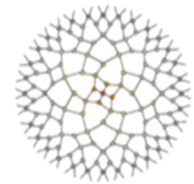
$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$

Hamiltonians

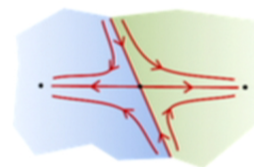
$H \to H' \to H'' \to \cdots$

summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts
  (quantum circuits, entanglement)
- Efficient representation of ground states (MERA)



- Proper* RG transformation (x3 !)



tensors

$$A \to A' \to A'' \to \cdots$$

wave-functions

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

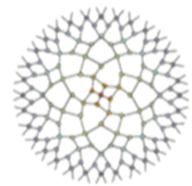
Hamiltonians

$$H \to H' \to H'' \to \cdots$$

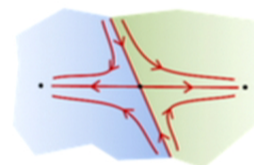
- Key ingredient: **removal of short-range entanglement**

summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)
- Efficient representation of ground states (MERA)



- Proper* RG transformation (x3 !)



tensors

$$A \to A' \to A'' \to \cdots$$

wave-functions

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

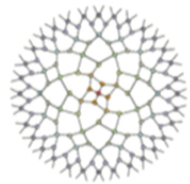
Hamiltonians

$$H \to H' \to H'' \to \cdots$$

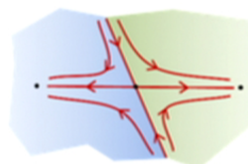
- Key ingredient: **removal of short-range entanglement**
- Very accurate in 1+1 dimensions (Ising model, etc)

summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)
- Efficient representation of ground states (MERA)



- Proper* RG transformation (x3 !)



tensors

$$A \to A' \to A'' \to \cdots$$

wave-functions

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

Hamiltonians

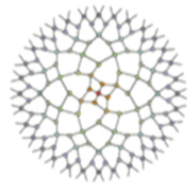
$$H \to H' \to H'' \to \cdots$$

- Key ingredient: **removal of short-range entanglement**
- Very accurate in 1+1 dimensions (Ising model, etc)

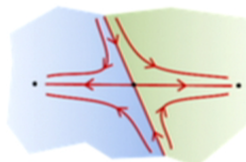
What about 2+1, 3+1 dimensions?

summary / conclusions:

- Reformulation of Wilson's RG using quantum information tools/concepts (quantum circuits, entanglement)
- Efficient representation of ground states (MERA)



- Proper* RG transformation (x3 !)



tensors

$$A \to A' \to A'' \to \cdots$$

wave-functions

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

Hamiltonians

$$H \to H' \to H'' \to \cdots$$

- Key ingredient: **removal of short-range entanglement**
- Very accurate in 1+1 dimensions (Ising model, etc)

What about 2+1, 3+1 dimensions?

- Continuum limit? (we managed to extract CFT data from the lattice!)