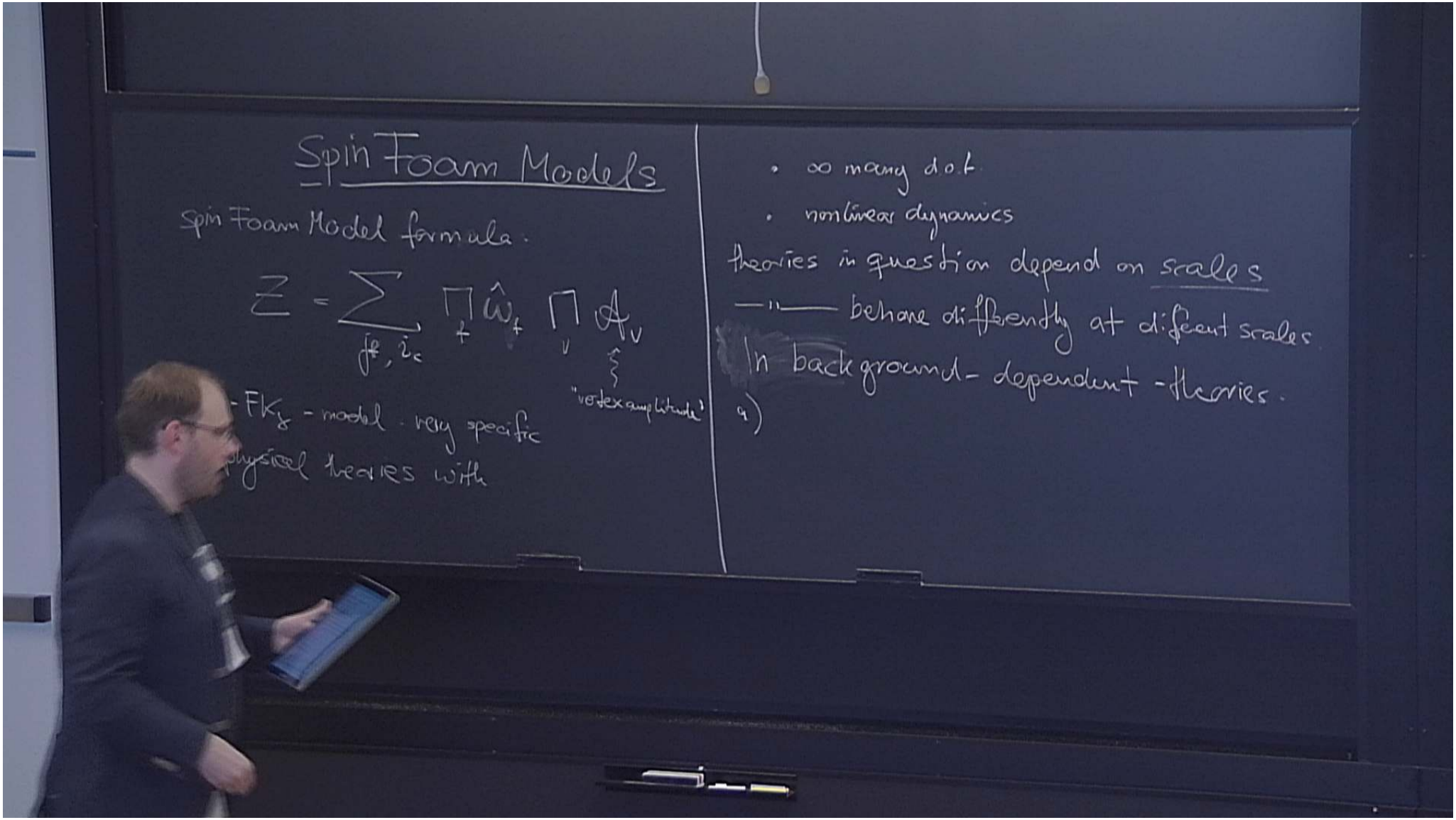
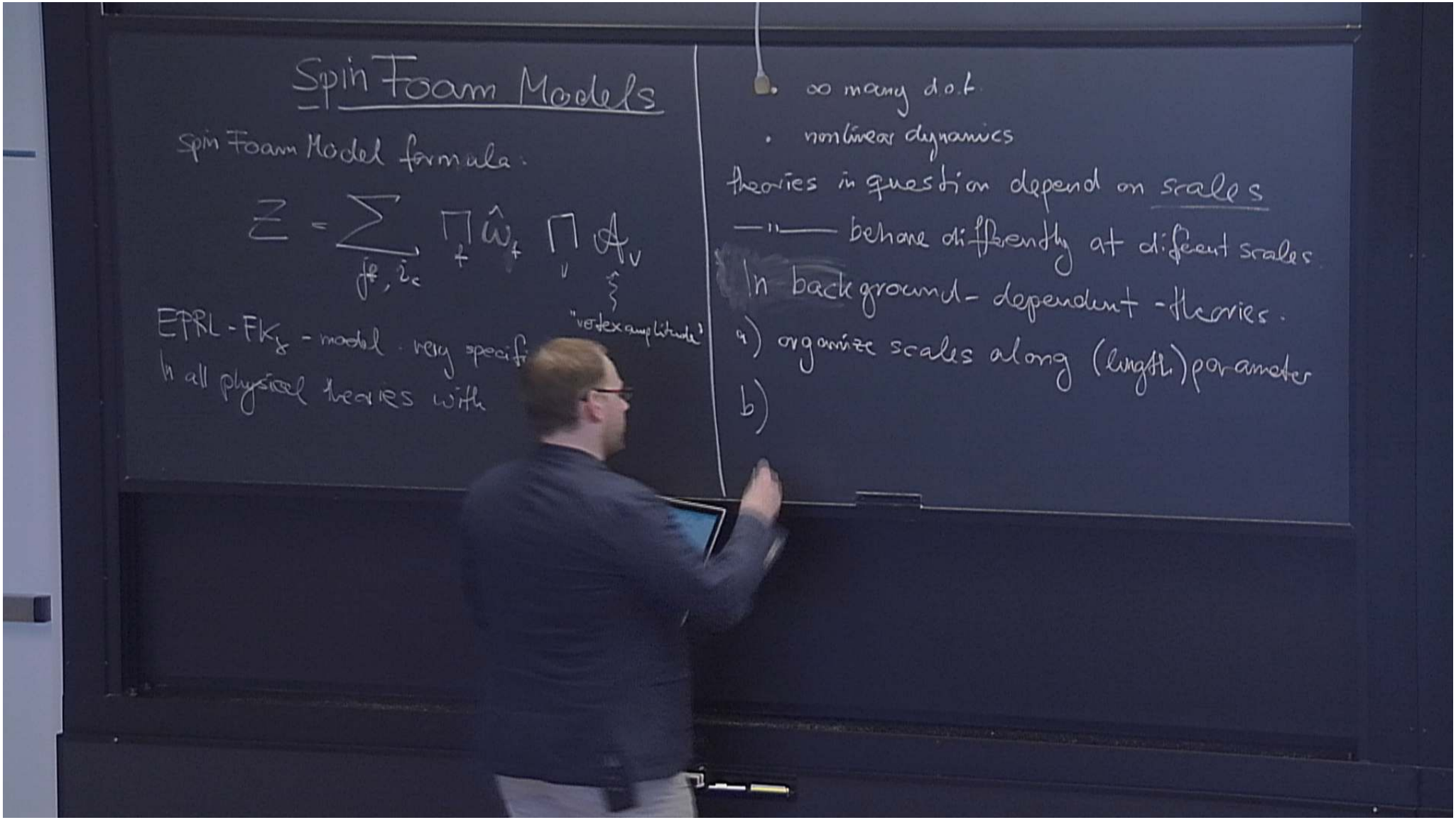
Title: TBA

Date: Sep 28, 2015 11:00 AM

URL: http://pirsa.org/15090077

Abstract:


Spin Foam Models
Spin Foam Model formula.
$Z = \sum_{j_f, i_e} \prod_f \hat{w}_f \prod_v A_v$
"vertex amplitude"
- FK_γ - model - very specific
physical theories with
• ∞ many d.o.f.
• nonlinear dynamics
theories in question depend on scales
—"— behave differently at different scales
In background-dependent-theories.
4)


Spin Foam Models
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"vertex amplitude"
EPRL - FK$_\gamma$ - model very specifi
In all physical theories with
• ∞ many d.o.f.
• nonlinear dynamics
theories in question depend on scales
—"— behave differently at different scales
In background-dependent-theories:
a) organize scales along (length) parameter
b)

Example. fine structure constant α.

$$\alpha(\text{Room temperature}) \approx \frac{1}{137}$$

$$\alpha(m_Z = 91\,\text{GeV}) \approx \frac{1}{128}$$

Similarly for spin foam models.

Spin Foam Models

Spin Foam Model formula:

$$Z = \sum_{j_f, i_e} \prod_f \hat{\omega}_f \prod_v A_v$$

"vertex amplitude"

EPRL - FK_γ - model. very specific

In all physical theories with

- ∞ many d.o.f.
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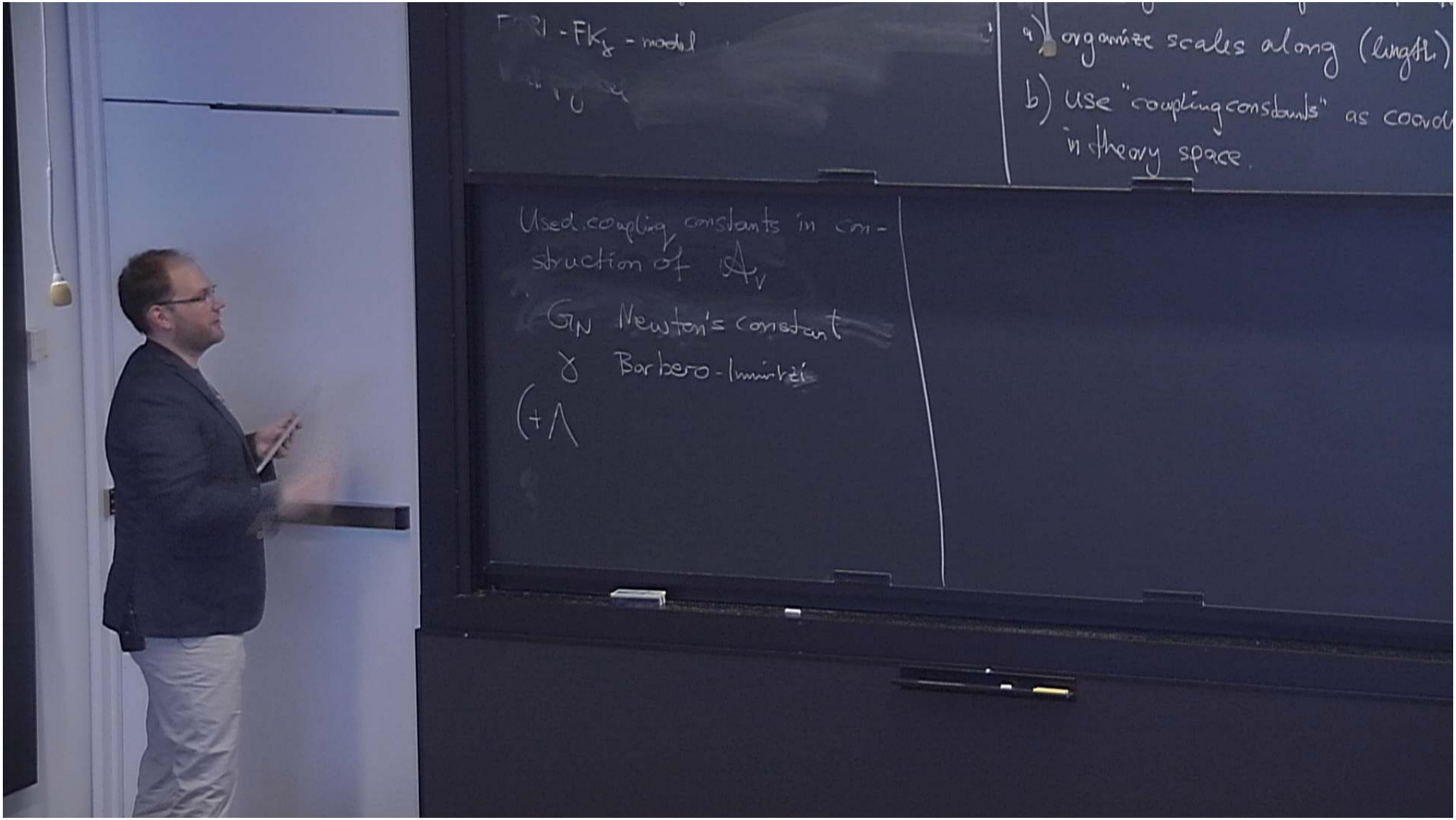
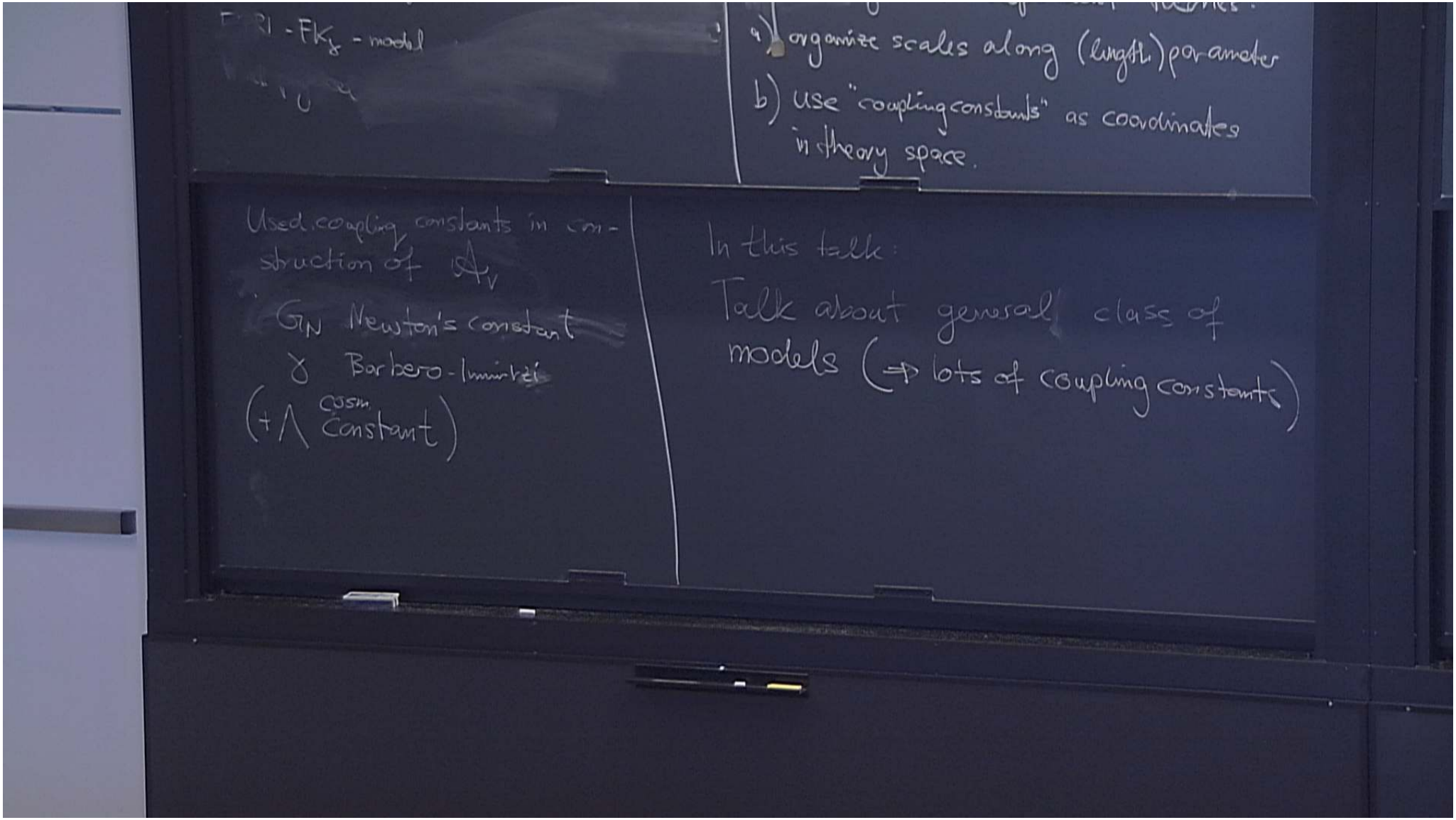
—"— behave differently at different scales.

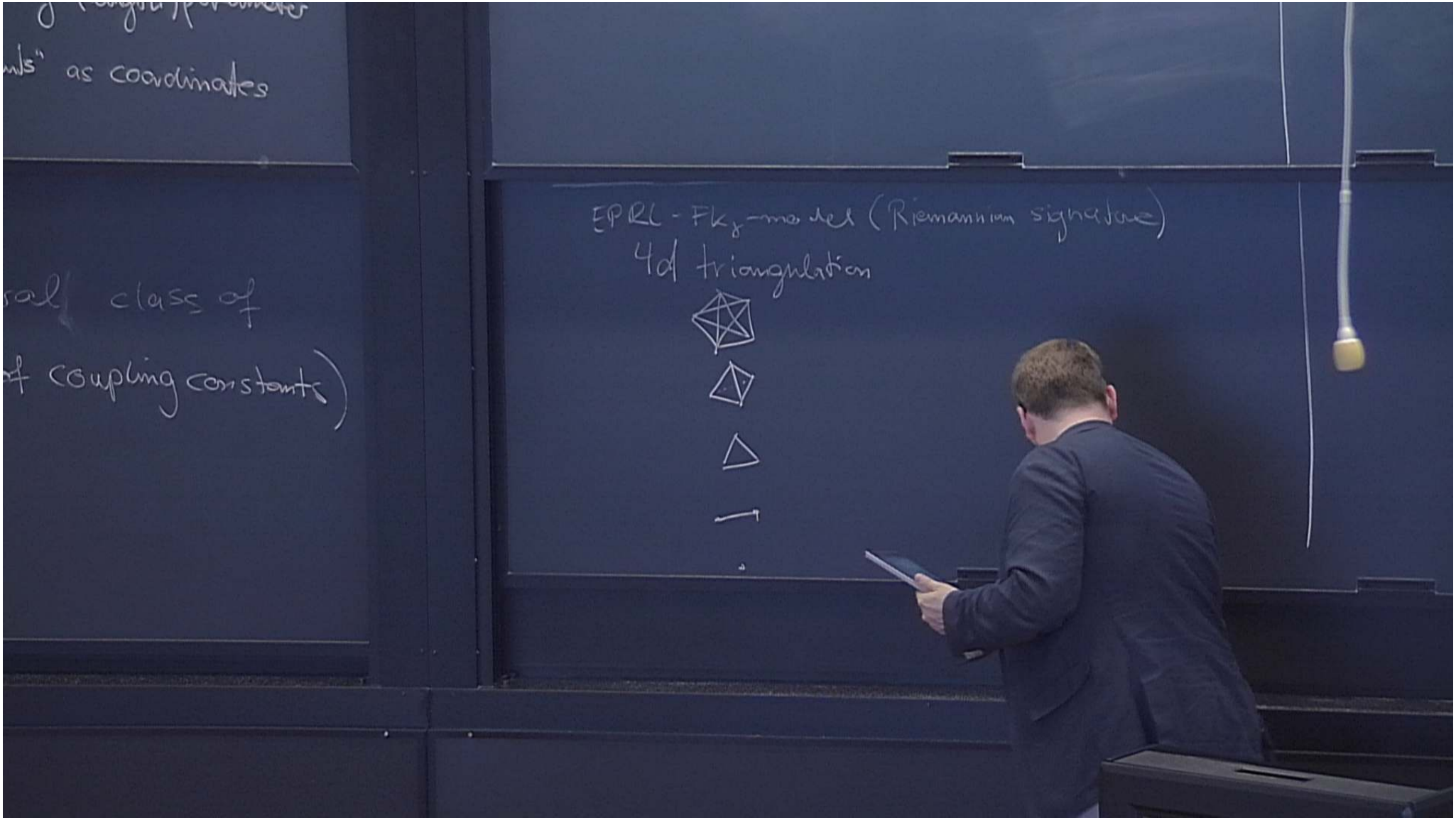
In background - dependent - theories.

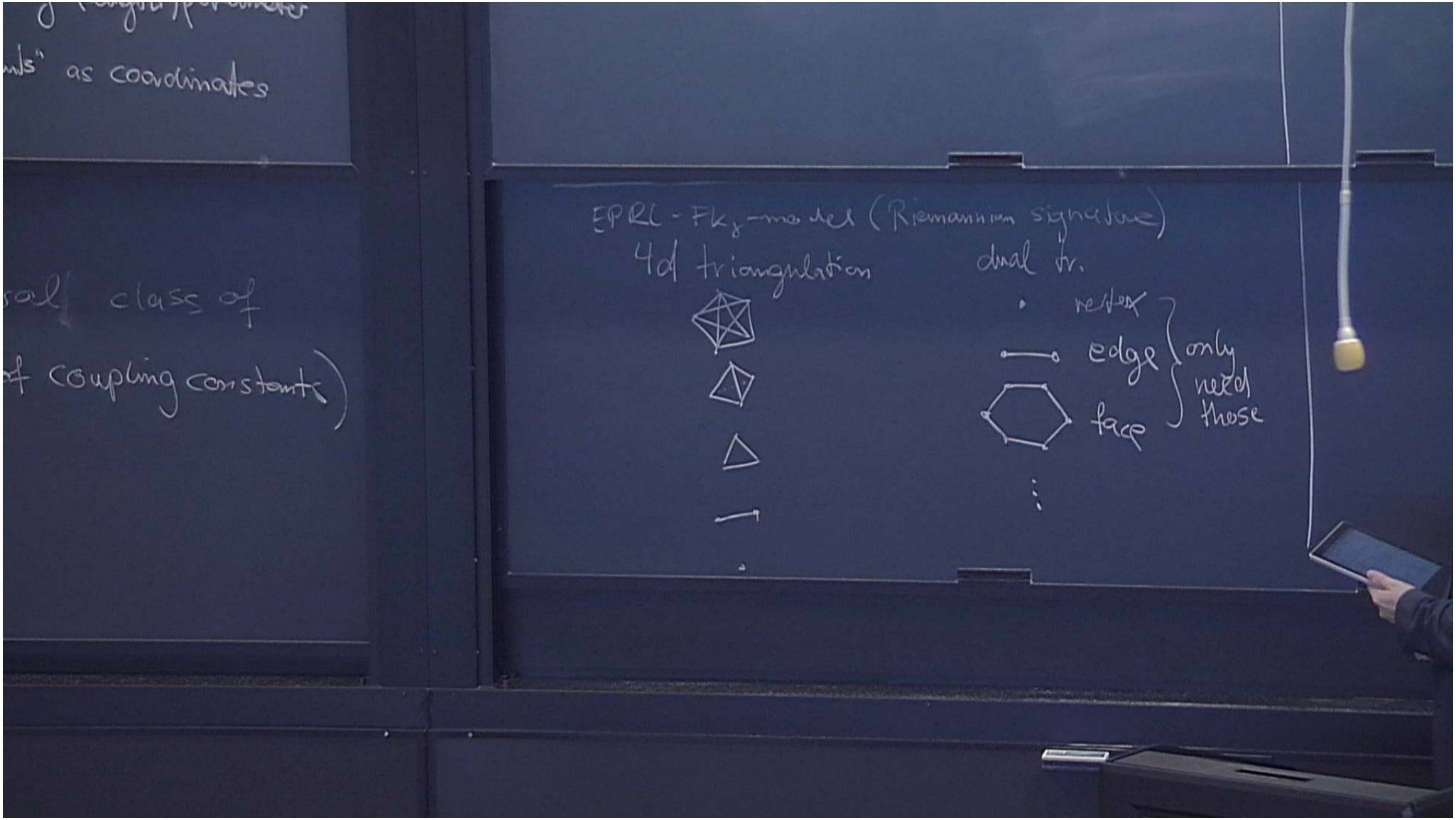
a) organize scales along (length) parameter

b) use "coupling constants" as coordinates in theory space.


- FK$_g$ - model
a) organize scales along (length)
b) use "coupling constants" as coordi
in theory space.
Used coupling constants in con-
struction of A_V
G_N Newton's constant
γ Barbero-Immirzi
(+ Λ


- FK$_\gamma$ - model
a) organize scales along (length) parameter
b) use "coupling constants" as coordinates in theory space.
Used coupling constants in construction of A_V
G_N Newton's constant
γ Barbero-Immirzi
($+\Lambda$ cosm. constant)
In this talk:
Talk about general class of models ($\Rightarrow$ lots of coupling constants)


as coordinates
class of
coupling constants)
EPRL - FK$_\gamma$-model (Riemannian signature)
4d triangulation


nts' as coordinates
sal class of
f coupling constants)
EPRL - Fk - model (Riemannian signature)
4d triangulation
dual tr.
vertex
edge
face
only need those

SF Models (Operator formalism)

EPRL - FK_γ - model (Riemannian signature)

4d triangulation

dual tr.

Generalization: 2-complex

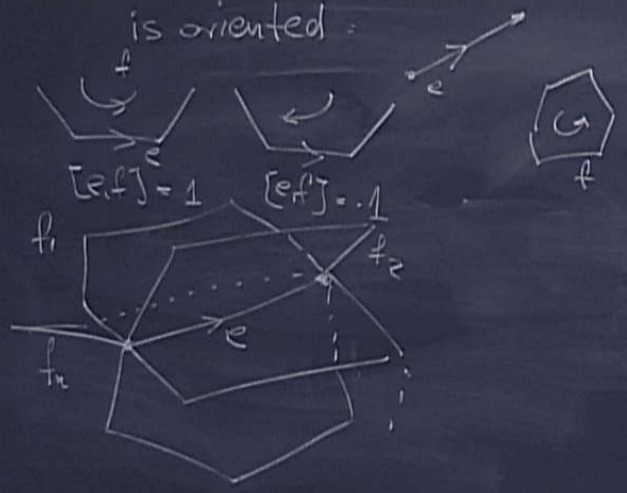
G_N Newton's constant
γ Barbero-Immirzi
talk about general class of
models (⇒ lots of coupling constants)
is oriented
[e,f] = 1
[e,f] = -1
G
f
Gauge group G (compact Lie)
⇒ Irrep g_f
Edge-Hilbert space
$\mathcal{H}_e$ = ⊗
[e,f] = 1

G_N Newton's constant

γ Barbero-Immirzi

talk about general class of models ($\Rightarrow$ lots of coupling constants)

is oriented:



Gauge group G (G compact Lie)

$\Rightarrow$ Irrep ρ_f

Edge-Hilbert space

$\mathcal{H}_e = \bigotimes_{[e,f]=1} V_{\rho_f} \otimes \bigotimes_{[e,f]=-1} V_{\rho_f}^{+}$

(f-dim. irred. unitary repn)

EPRL - FK - model (Riemannian signature)

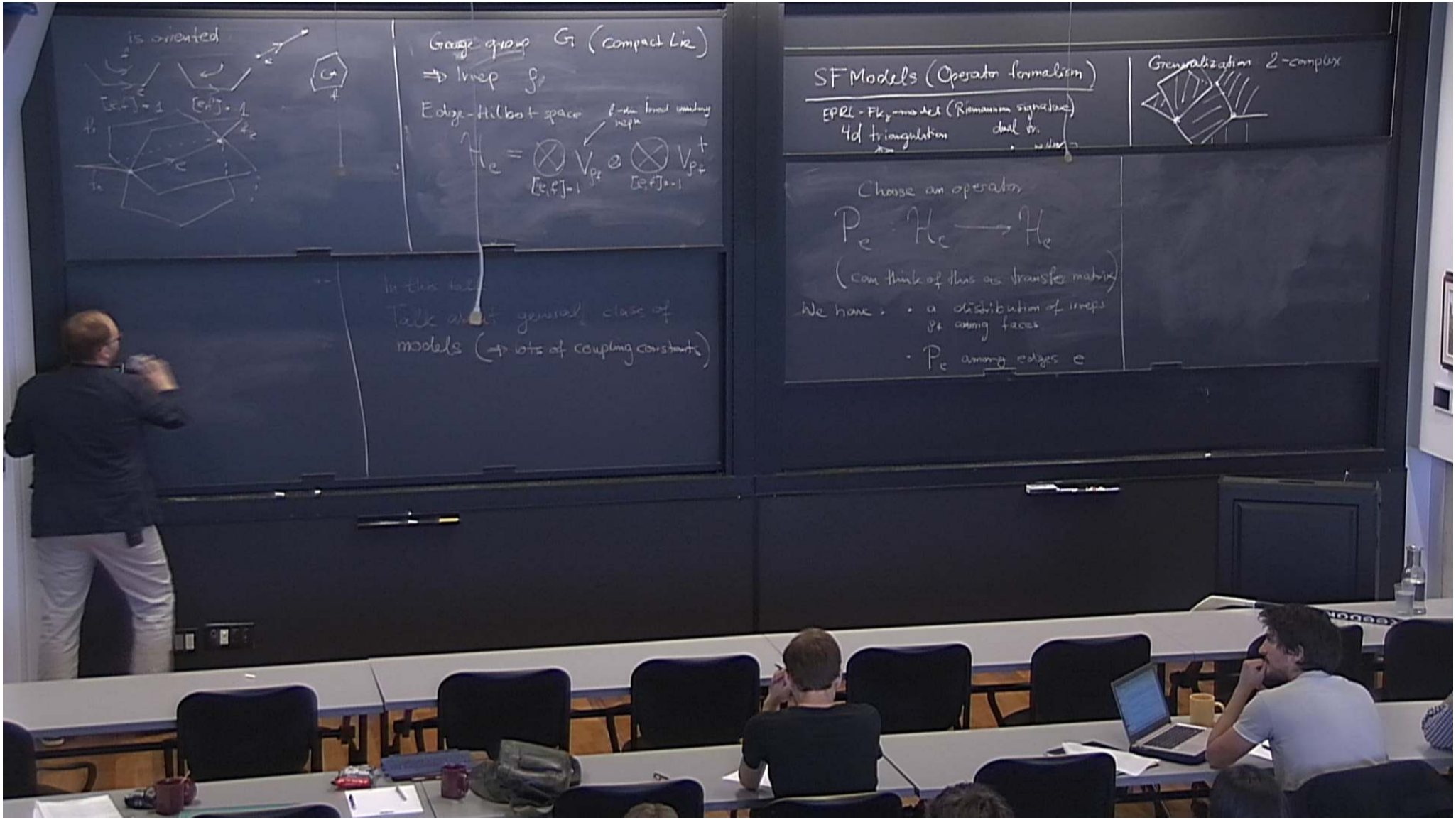
Choose an operator:

$$P_e : \mathcal{H}_e \longrightarrow \mathcal{H}_e$$

(can think of this as transfer matrix)

We have:

- a distribution of irreps ρ_f among faces.
- P_e among edges e


is oriented
Gauge group G (compact Lie)
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SF Models (Operator formalism)
EPRL - FK - model (Riemannian signature)
4d triangulation
dual tr.
Generalization 2-complex
Choose an operator
(can think of this as transfer matrix)
We have:
a distribution of irreps
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In this talk
Talk about general class of
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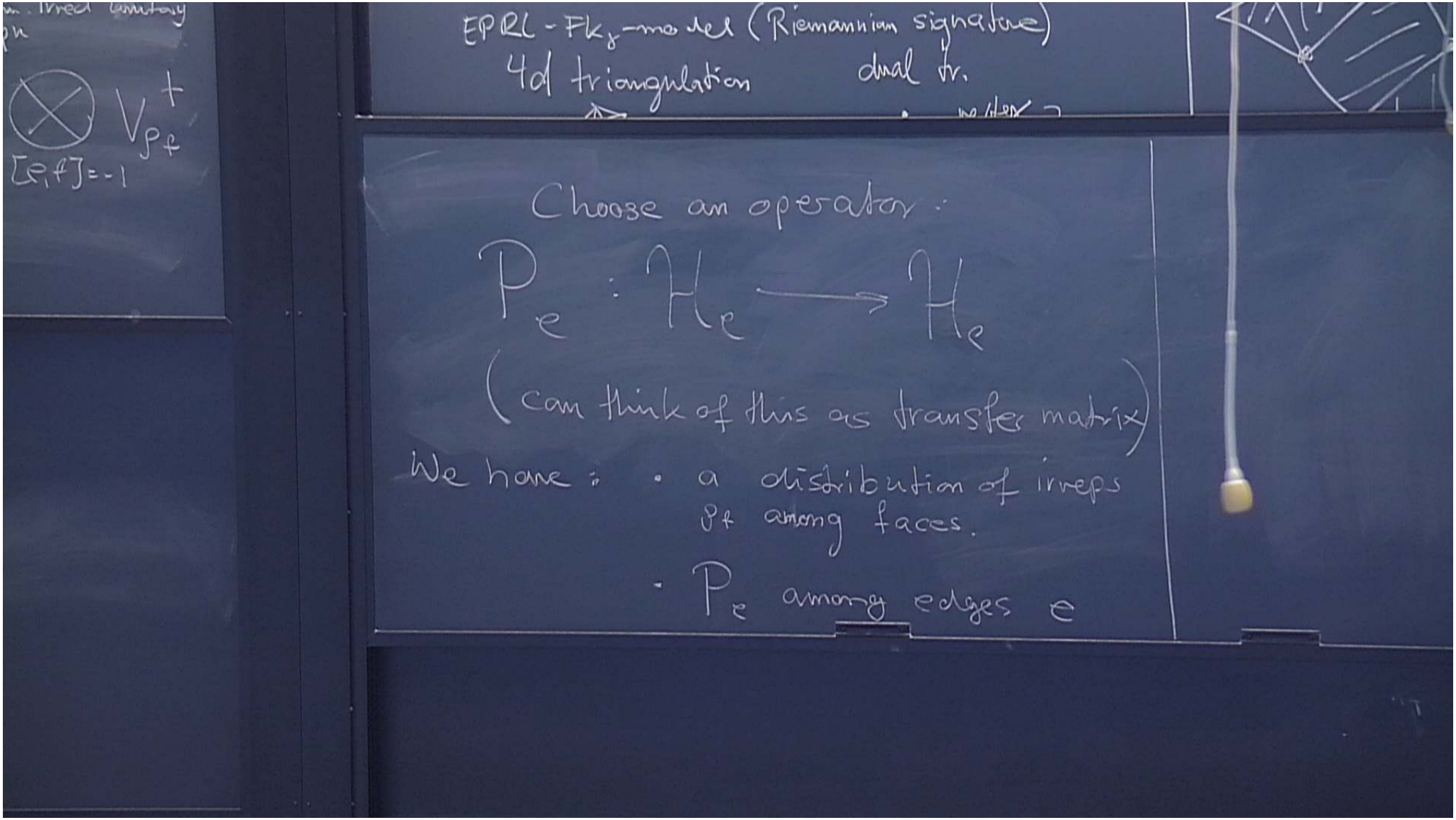

EPRL - Fk_γ - model (Riemannian signature)
4d triangulation
dual tr.
Choose an operator:
$P_e : \mathcal{H}_e \longrightarrow \mathcal{H}_e$
(can think of this as transfer matrix)
We have:
• a distribution of irreps ρ_f among faces.
• P_e among edges e
$V_{\rho_f}^{\iota}$
$[e, f] = -1$

4d triangulation ⟷ dual tr.

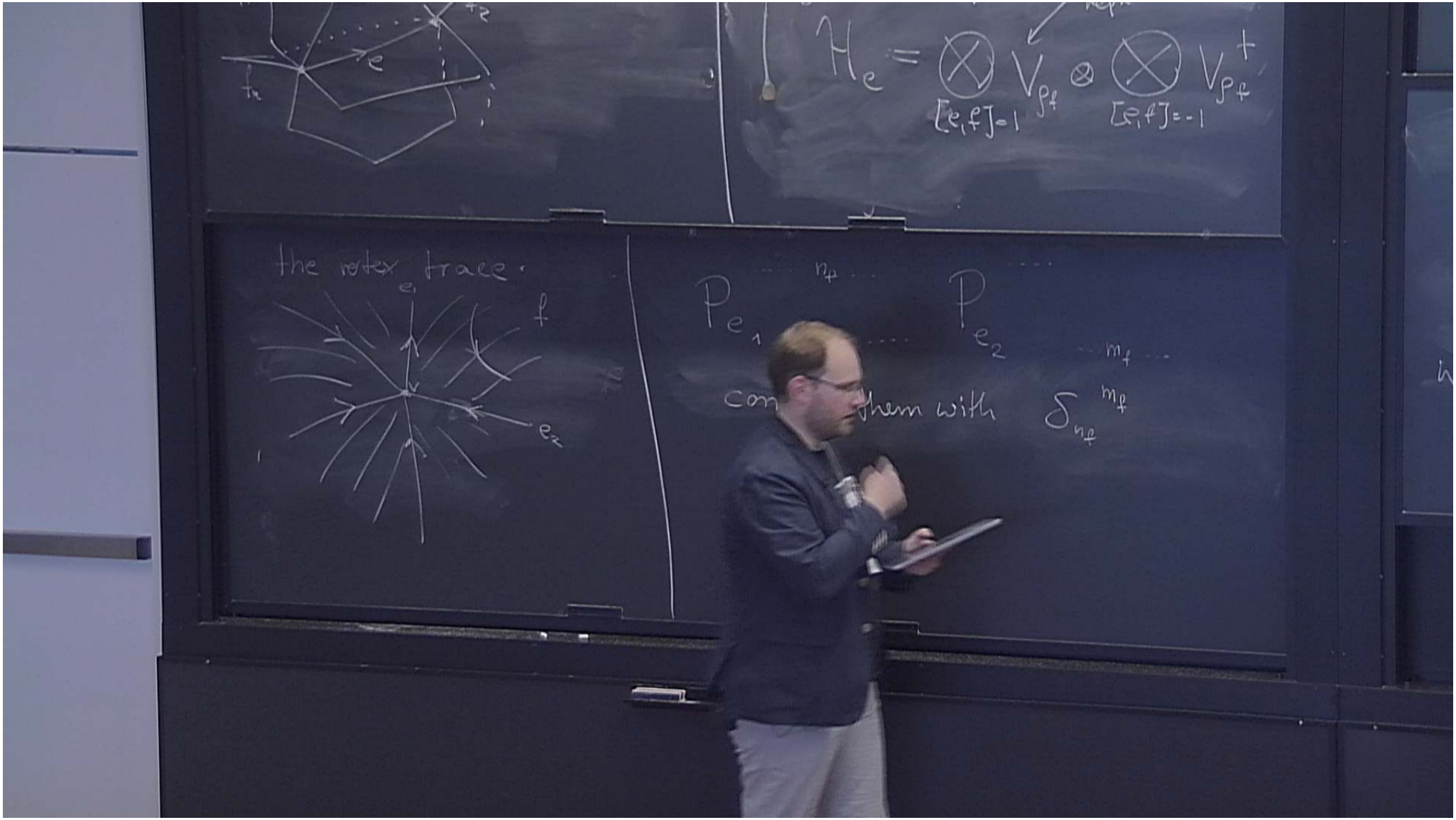
Choose an operator:

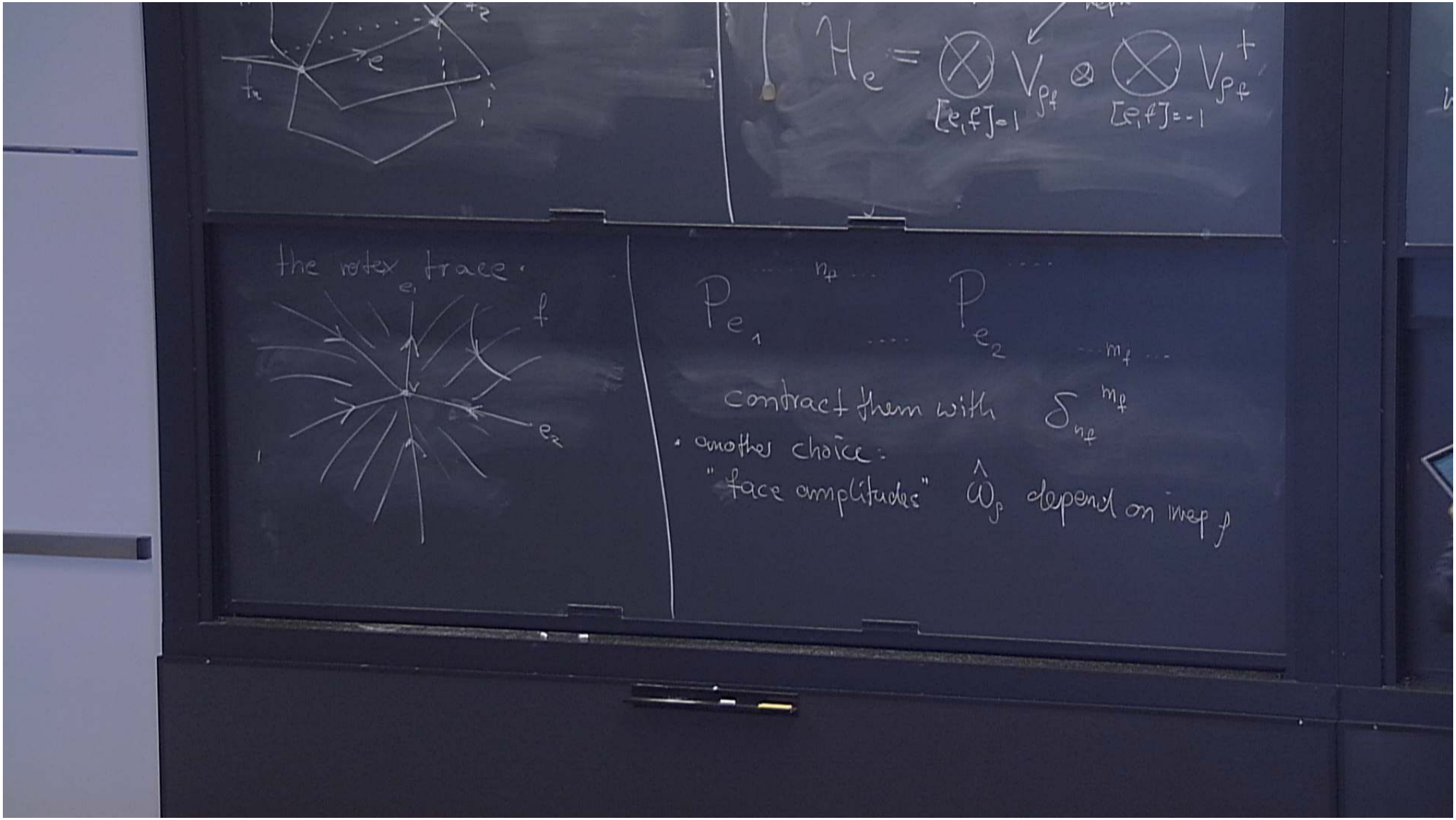
$$P_e : \mathcal{H}_e \longrightarrow \mathcal{H}_e$$

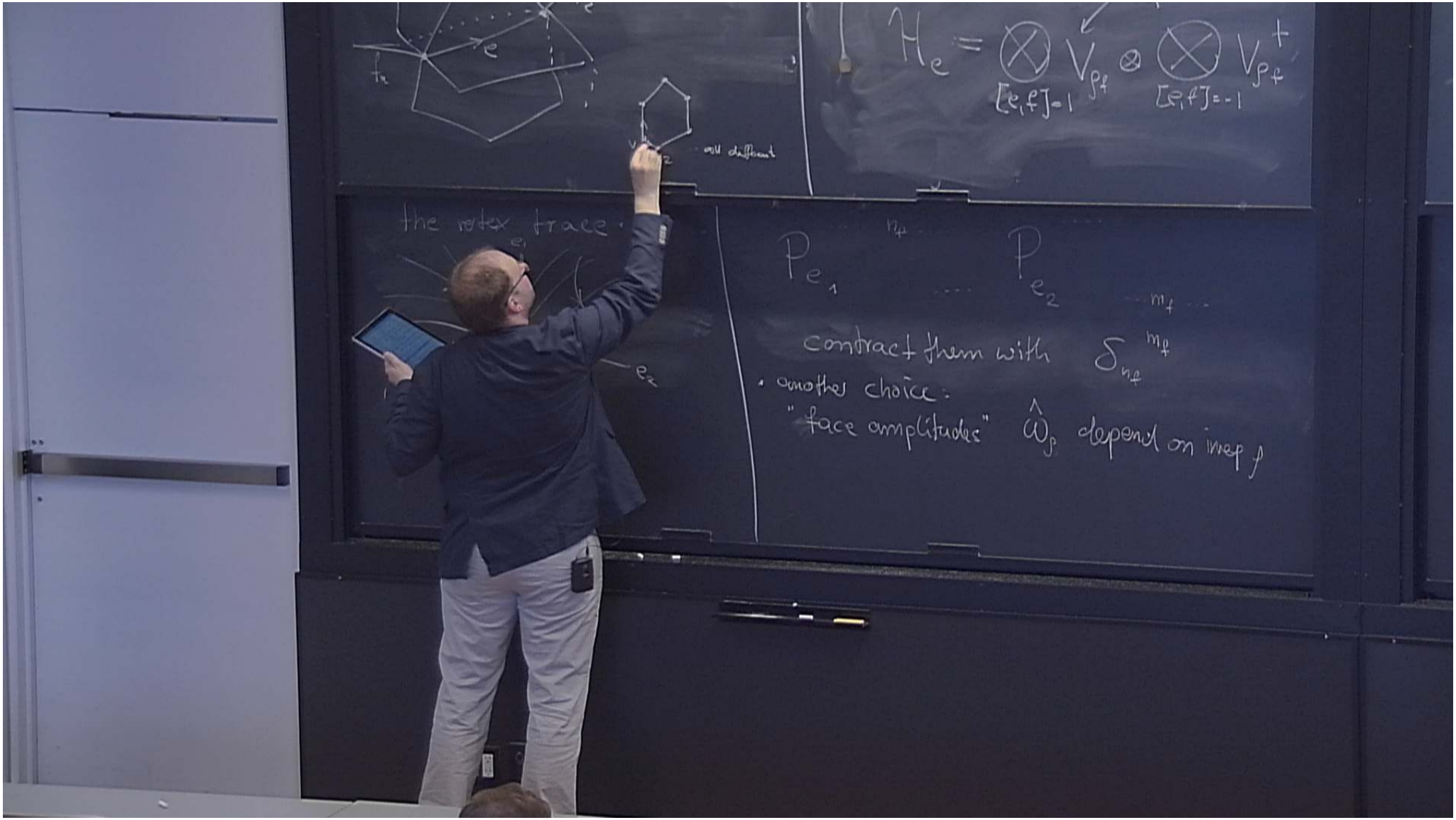
($\mathcal{H}_e$: starting vertex of e; $\mathcal{H}_e$: the ending vertex of e)

(can think of this as transfer matrix)

We have:
- a distribution of irreps ρ_f among faces.
- P_e among edges e


the vertex trace.
$\mathcal{H}_e = \bigotimes_{[e,f]=1} V_{\rho_f} \otimes \bigotimes_{[e,f]=-1} V_{\rho_f}^{\dagger}$
P_{e_1}
P_{e_2}
n_f
m_f
$\delta_{n_f}^{m_f}$


$\mathcal{H}_e = \bigotimes_{[e,f]=1} V_{\rho_f} \otimes \bigotimes_{[e,f]=-1} V_{\rho_f}^+$
the vertex trace:
P_{e_1} ... n_f ... P_{e_2} ... m_f ...
contract them with $\delta_{n_f}^{m_f}$
• another choice:
"face amplitudes" $\hat{\omega}_\rho$ depend on irrep ρ


the vertex trace
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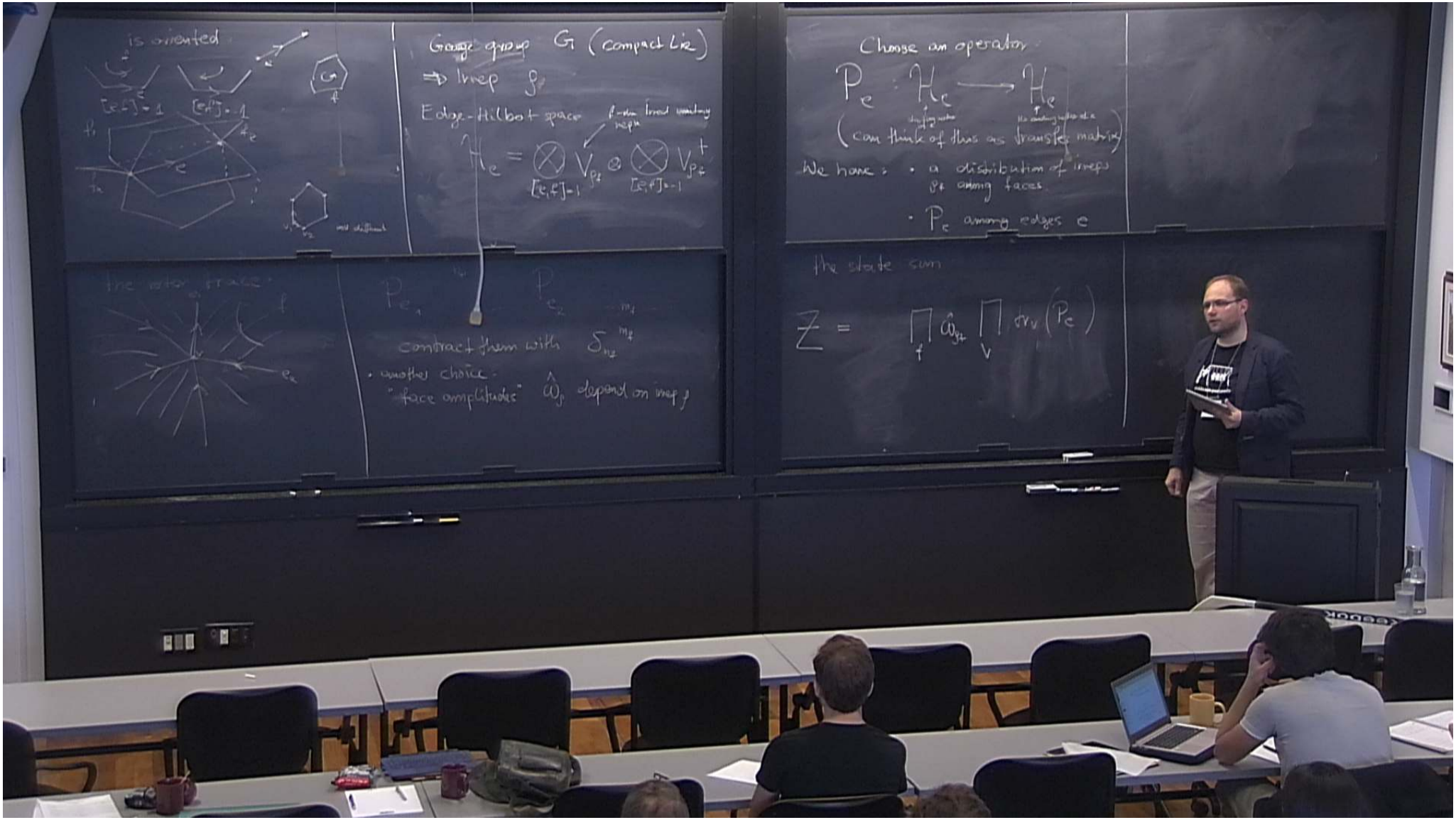
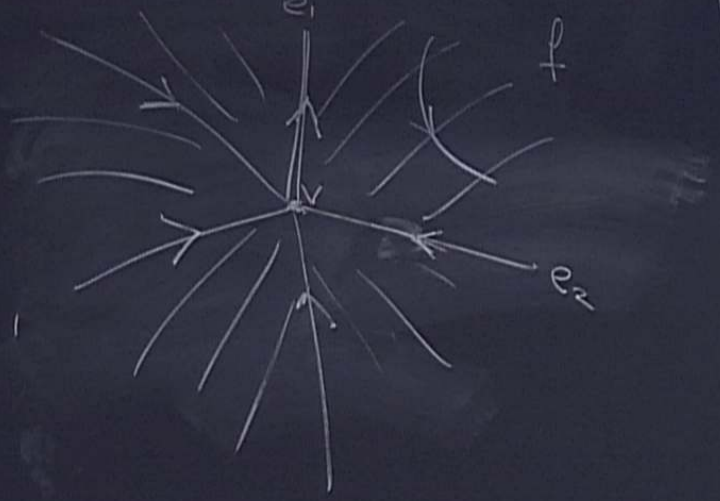

is oriented
Gauge group G (compact Lie)
⇒ Irrep
Edge-Hilbert space
Choose an operator
We have:
the state sum
contract them with
another choice:
"face amplitudes"
depend on irrep

v_1 v_2 ... all different

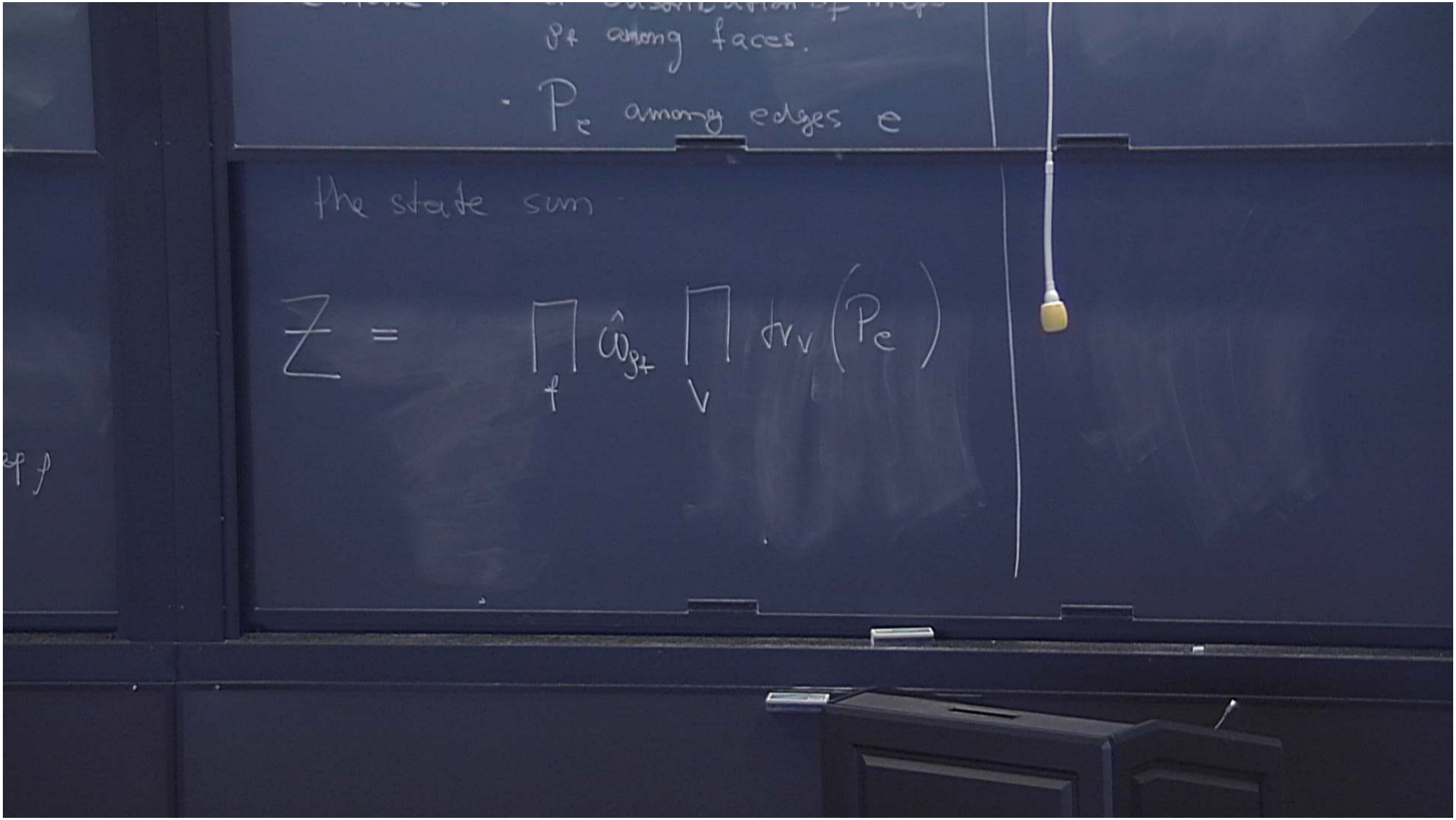
the vertex trace:

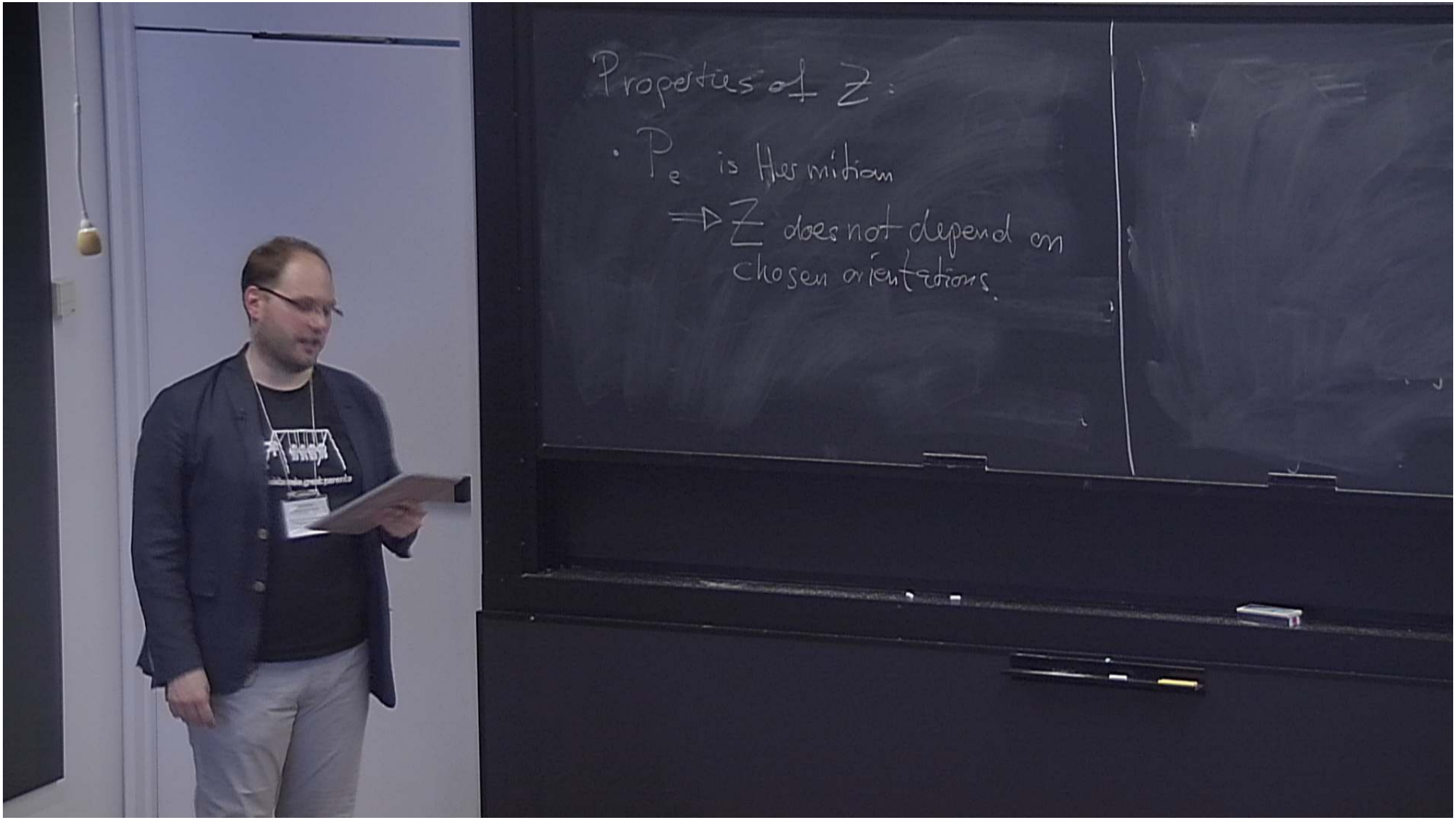


$P_{e_1} \cdots n_f \cdots P_{e_2} \cdots m_f \cdots$

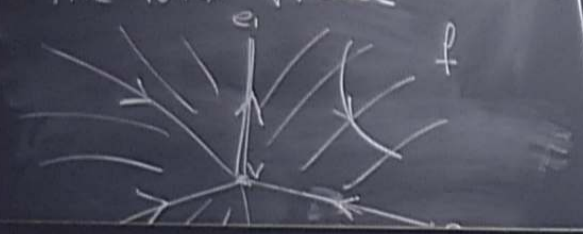
contract them with $\delta_{n_f}^{m_f}$

- another choice:
  "face amplitudes" $\hat{\omega}_f$ depend on irrep f


p_f among faces.
P_e among edges e
the state sum
$Z = \prod_f \hat{\omega}_{g_f} \prod_V dr_V(P_e)$


Properties of Z:
· P_e is Hermitian
⇒ Z does not depend on
chosen orientations.

the vertex trace:



$P_{e_1}^{\dots n_f \dots} \quad P_{e_2}^{\dots m_f \dots}$

contract them with $\delta_{n_f}^{m_f}$

Properties of Z:

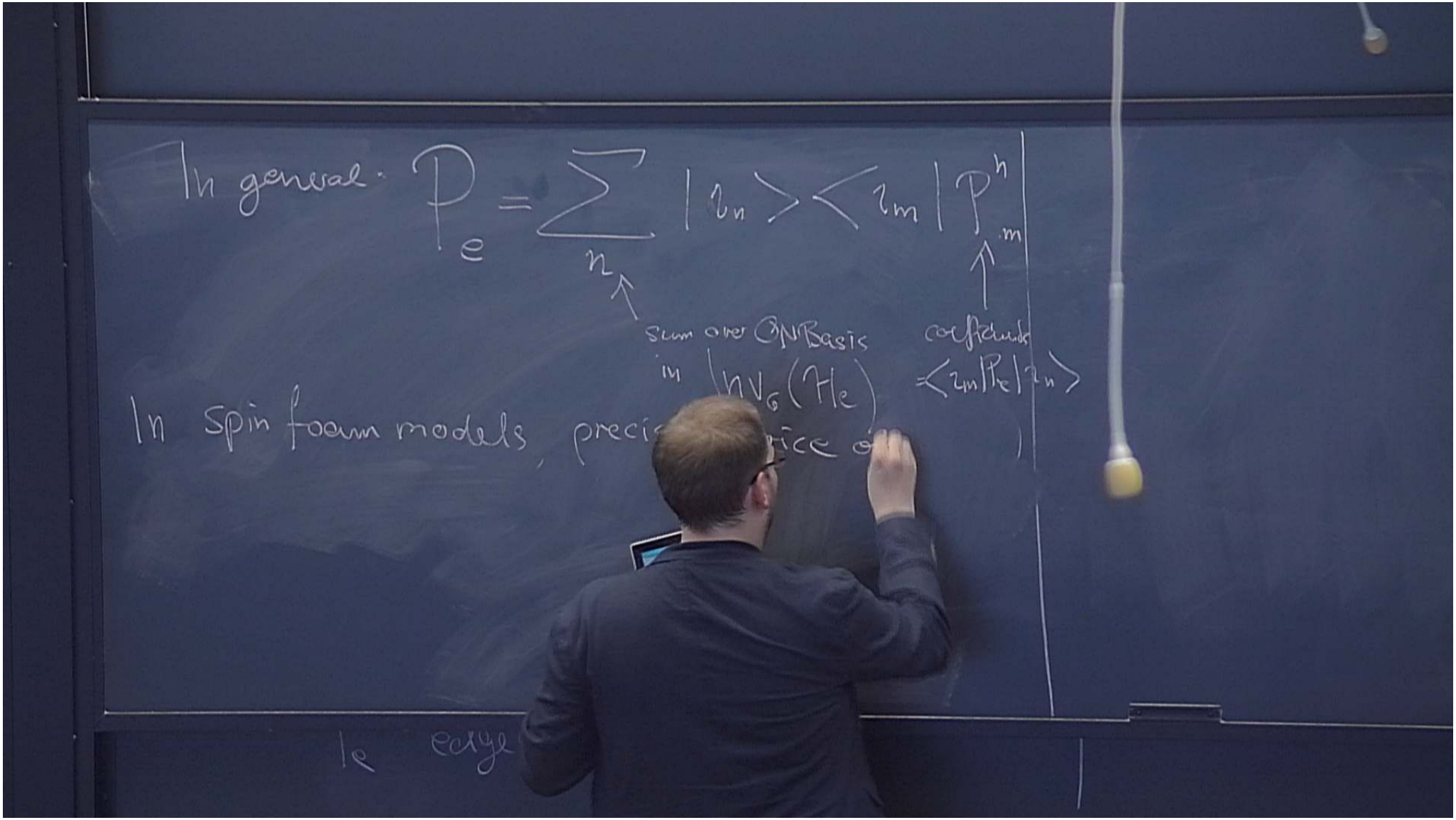
- P_e is Hermitian
  $\Rightarrow Z$ does not depend on chosen orientations.
- $P_e^2 = P_e \Rightarrow Z$ is invariant [under] trivial subdivi[sion] edges.

$g P_e = P_e g \quad g \in G$

$\Rightarrow Z$ is gauge-invariant

In general $\mathcal{H}_e = V_1 \otimes V_2 \otimes \dots \otimes V_n \otimes V_{n+1}^{+} \otimes \dots \otimes V_k^{+}$

$\mathrm{Inv}_G(\mathcal{H}_e)$ = set of g-invariant elements


In general: $P_e = \sum_n |i_n\rangle\langle i_m| P^n_m$
sum over ON Basis
in $\mathrm{Inv}_G(\mathcal{H}_e)$
coefficients
$= \langle i_m | P_e | i_n \rangle$
In spin foam models, preci
ice o
le edge

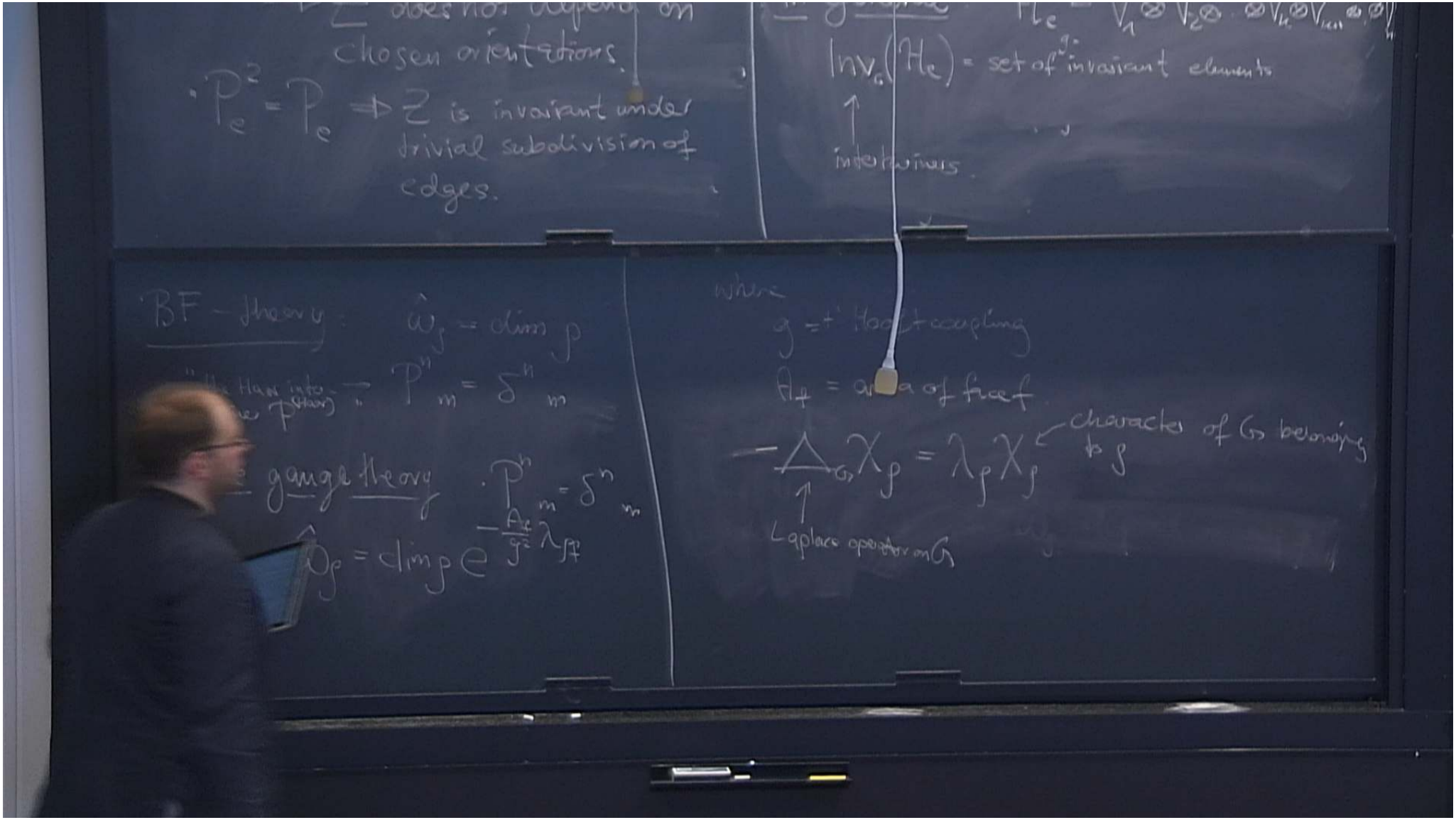
In general: $P_e = \sum_n |\gamma_n\rangle\langle\gamma_m| P^n_m$

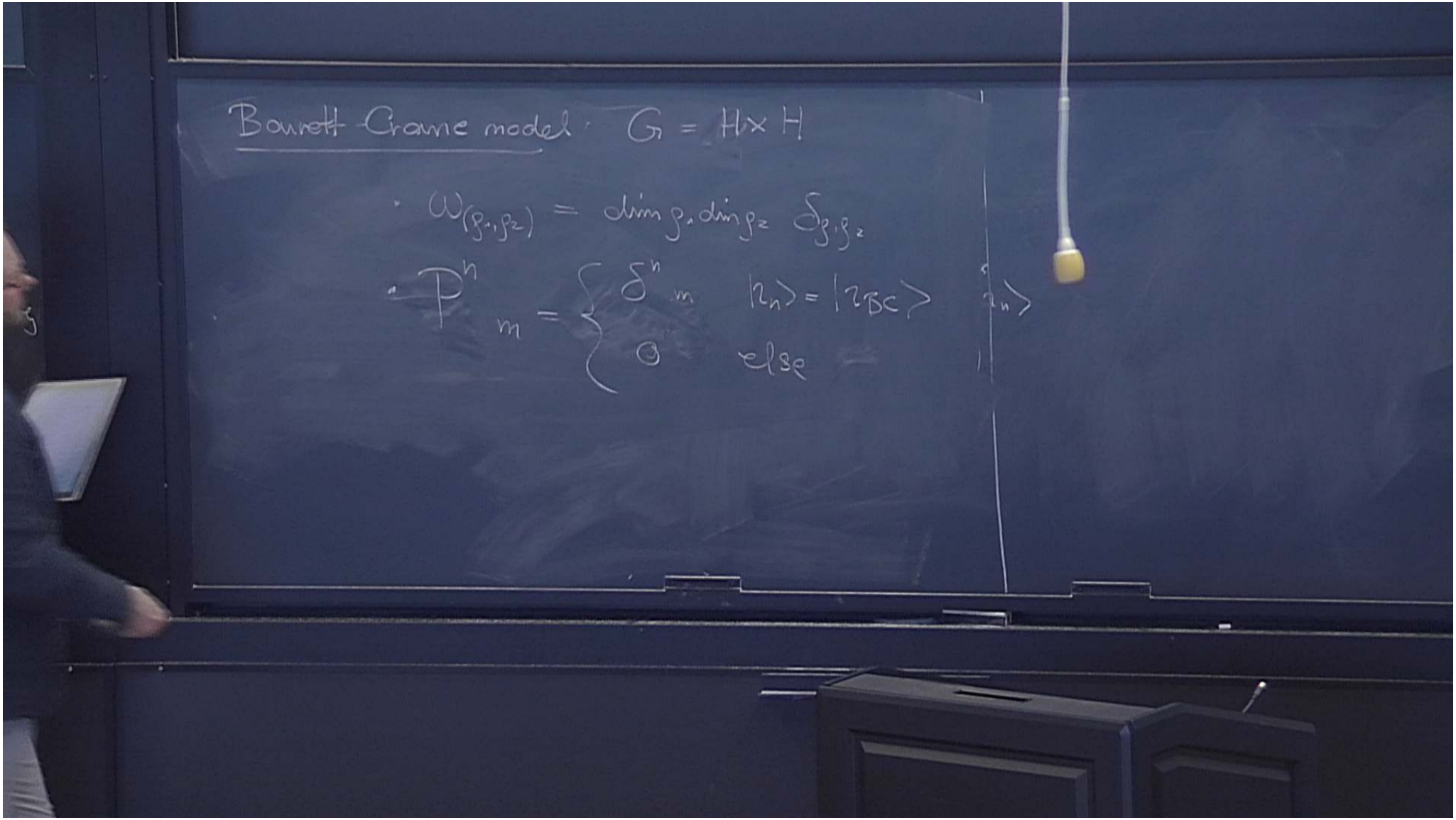
n: sum over ONBasis in $\mathcal{H}_{V_\Gamma}(\mathcal{H}_e)$

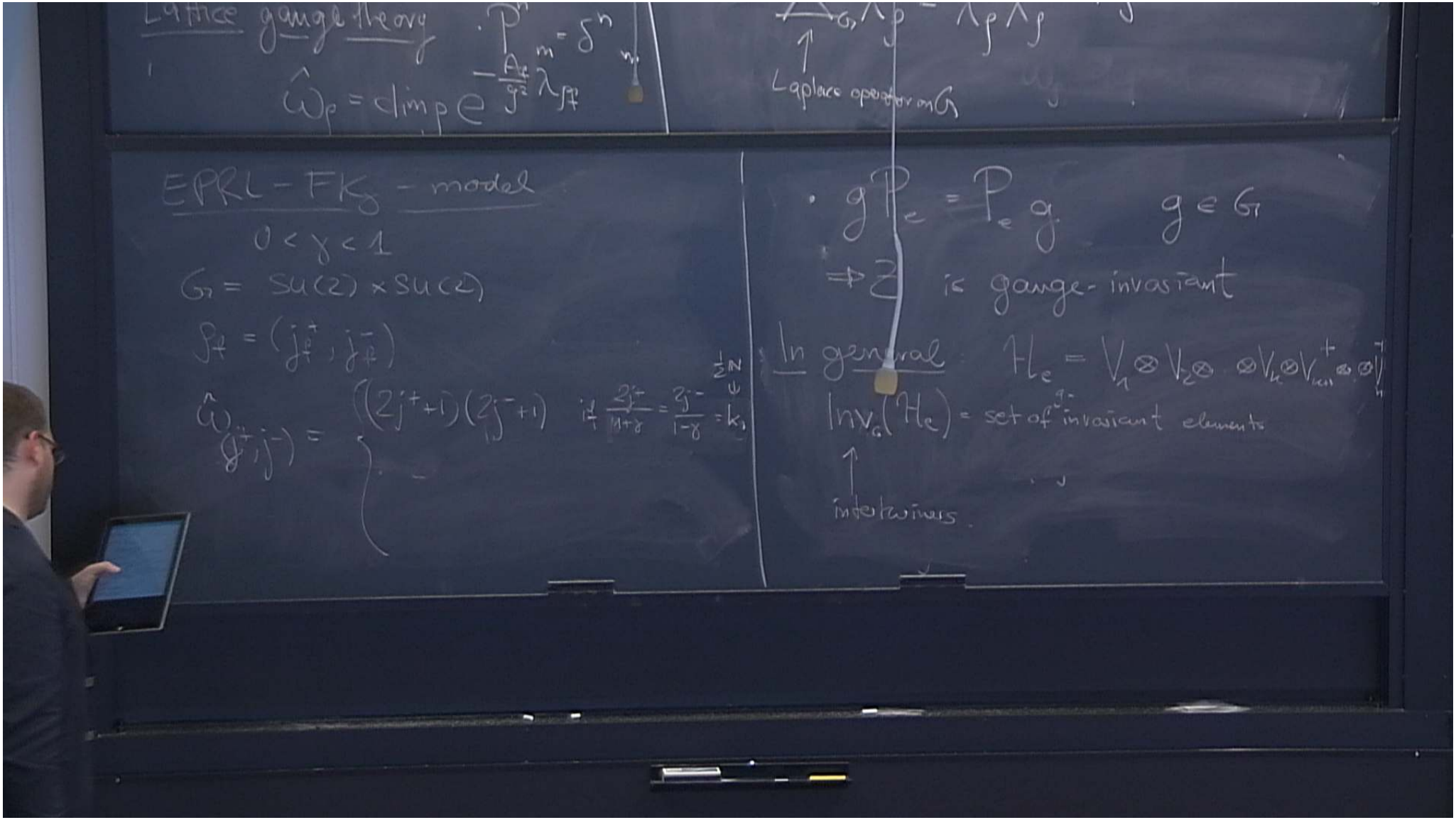
P^n_m: coefficients $= \langle\gamma_m|P_e|\gamma_n\rangle$

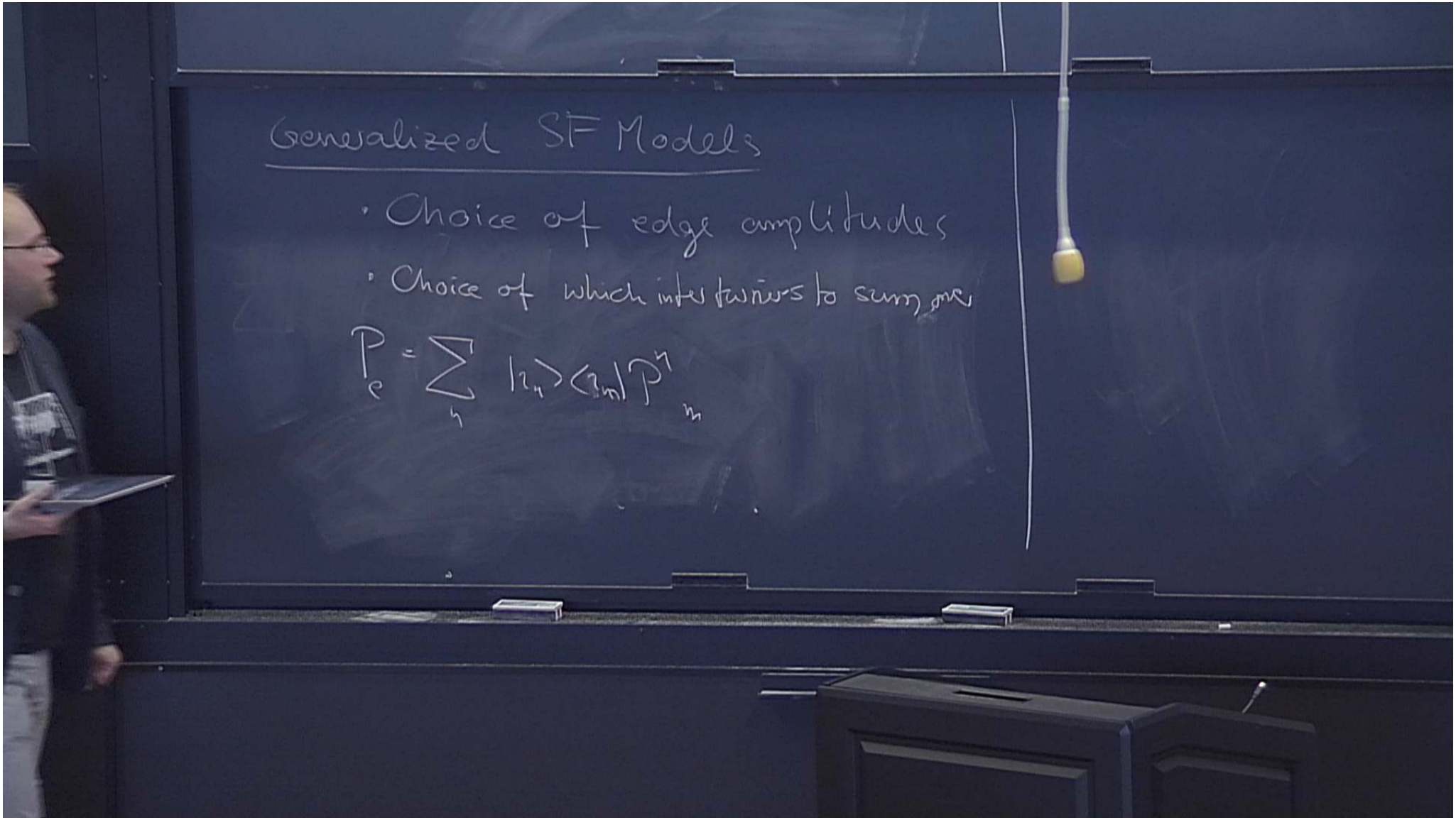
In spin foam models, precise choice of P^n_m $\hat{=}$ choice of implementation of simplicity constr.

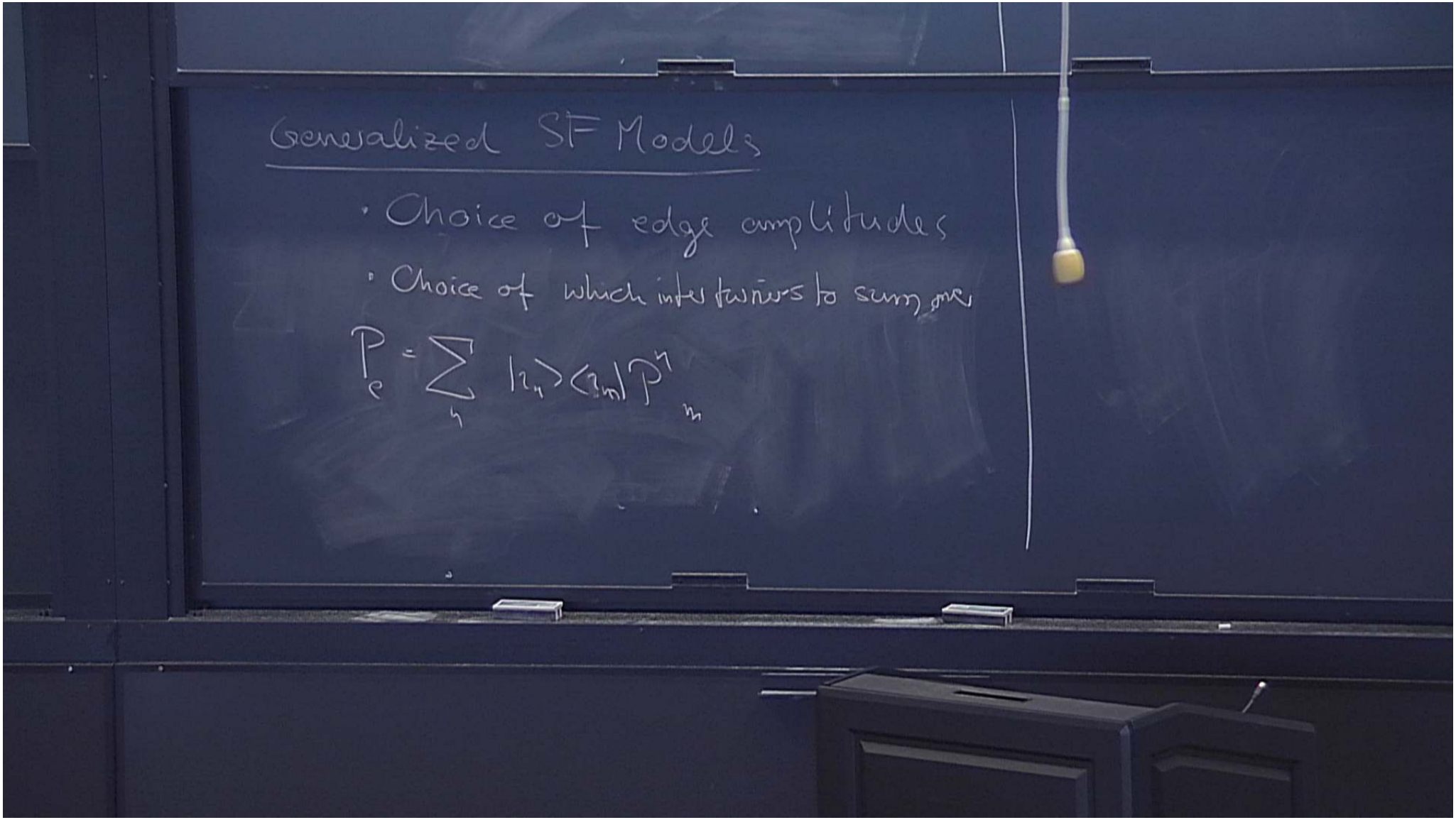
le edge operators


chosen orientations.
$P_e^2 = P_e \Rightarrow Z$ is invariant under trivial subdivision of edges.
$\mathrm{Inv}_G(H_e)$ = set of g-invariant elements
intertwiners
BF-theory
$\hat{\omega}_\rho = \dim \rho$
$P^n_m = \delta^n_m$
gauge theory
$P^n_m = \delta^n_m$
$\hat{\omega}_\rho = \dim \rho\, e^{-\frac{A_f}{g^2}\lambda_{\rho_f}}$
where
g = ... coupling
A_f = area of face f
$-\Delta_G \chi_\rho = \lambda_\rho \chi_\rho$
← characters of G belonging to ρ
↑ Laplace operator on G


Bartlett-Crane model: $G = H \times H$
$\omega_{(g_1,g_2)} = \dim g_1 \dim g_2 \, \delta_{g_1,g_2}$
$P^h{}_m = \begin{cases} \delta^n{}_m & |z_n\rangle = |z_{BC}\rangle \\ 0 & \text{else} \end{cases}$
$|z_n\rangle$


Lattice gauge theory
$\hat{\omega}_\rho = \dim \rho \, e^{-\frac{A_f}{g^2}\lambda_{\rho f}}$
Laplace operator on G
EPRL - FK$_\gamma$ - model
$0 < \gamma < 1$
$G = SU(2) \times SU(2)$
$\rho_f = (j_f^+, j_f^-)$
$\hat{\omega}_{(j^+, j^-)} = (2j^+ + 1)(2j^- + 1)$ if $\frac{2j^+}{1+\gamma} = \frac{2j^-}{1-\gamma} = k_1$
$\frac{1}{2}\mathbb{N}$
$g P_e = P_e g$ $g \in G$
$\Rightarrow Z$ is gauge-invariant
In general $\mathcal{H}_e = V_1 \otimes V_2 \otimes \cdots \otimes V_k \otimes V_{k+1}^+ \otimes \cdots \otimes V_n^+$
$\mathrm{Inv}_G(\mathcal{H}_e)$ = set of g-invariant elements
intertwiners


Generalized SF Models
• Choice of edge amplitudes
• Choice of which interwiners to sum over
$P_e = \sum_{\eta} |z_\eta\rangle\langle z_\eta| P^\eta_{nn}$


Generalized SF Models
• Choice of edge amplitudes
• Choice of which interwiners to sum over
$P_e = \sum_{\eta} |z_\eta\rangle\langle z_m| P^\eta_m$