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Abstract:

covariant methods in LQG

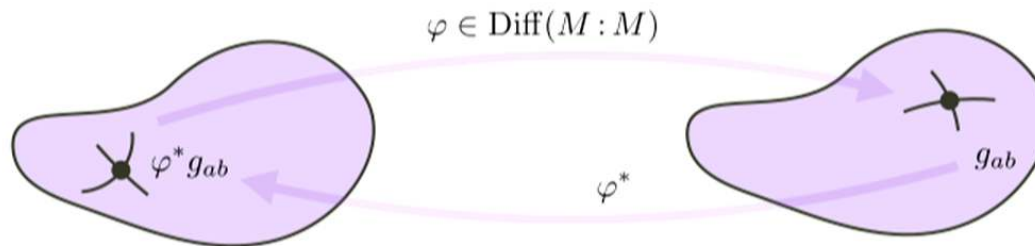
renormalization in background independent theories:
foundations and techniques

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Renormalization in Background Invariant Theories

Background Invariance



background-invariant theory: $S[X] = S[\varphi^* X]$

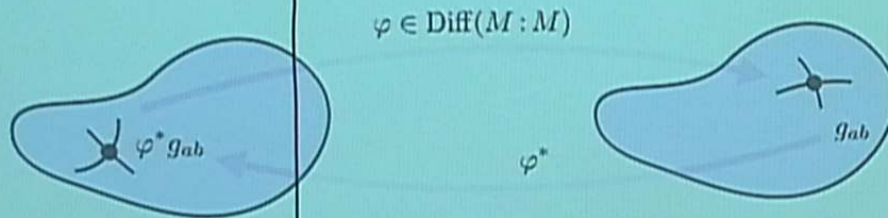
topological theories: $S_{BF}[B, A] = \int_M \text{Tr}(B \wedge F[A])$

$$S_{\text{CS}}[A] = \int_M \text{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$$

general relativity:
(+matter)

$$S_{\text{EH}}[g] = \frac{1}{16\pi G} \int_M d^4 v_g (R[g] - 2\Lambda) - \frac{1}{8\pi G} \int_{\partial M} d^3 v_h K$$

Physical meaning of Diff-invariance



The points of the space-time manifold M have no physical significance on their own. Fields do not live on a fixed space-time background, fields live on other fields.

"This amounts to the following law: that in general, Laws of Nature are expressed by means of equations which are valid for all co-ordinate systems, that is, which are covariant under all possible transformations: [...] This condition of general covariance [...] takes away the last remnants of physical objectivity from space and time. [A. Einstein, 1926]

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Renormalization (e.g. lattice QCD)

- ▶ **lattice action:** $\frac{1}{4g^2} \int d^4x \text{Tr}(F_{\alpha\beta} F^{\alpha\beta}) \rightarrow \frac{1}{2g^2} \sum_{\text{plaqs}} \text{Tr}(\text{Pe}^{-\oint_{\text{plaq}} A}) = -S_{\text{lattice}}$
- ▶ **path integral:** $\int \mathcal{D}[A] \dots = \prod_{\text{links}} \int_{SU(N)} dU_{\text{link}} \dots$
- ▶ **Wilson loop observables:** $\int \mathcal{D}[A] e^{-S} \text{Tr}(\text{Pe}^{-\oint_{\text{loop}} A}) \propto e^{-\tau(g,a)\text{Area}}$
- ▶ **beta function:** $a \frac{\partial g}{\partial a} = -\beta(g)$
- ▶ **continuum limit (2-nd order phase transition):**
 $a \rightarrow 0, g \rightarrow 0, \tau(a, g) = \text{const.}$
all correlation lengths diverge in units of a

Perfect action and Ditt-invariance

- ▶ Gravity is different: The lattice spacing is itself a function of the dynamical field: $a = \int_{\text{link}} dt \sqrt{|g_{\alpha\beta}(X) \dot{X}^\alpha \dot{X}^\beta|}$

- ▶ In LQG area is quantized: $A_j = 8\pi\gamma \hbar G \sqrt{j(j+1)}$

- ▶ No limit to a critical value $a \rightarrow 0$, $g \rightarrow 0$, only $N \rightarrow \infty$

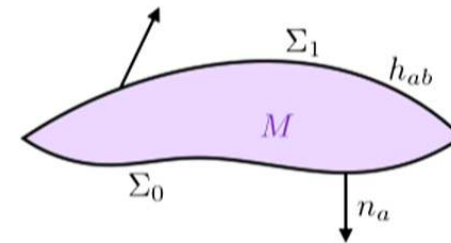
- ▶ Discretized theory approaches Ditt-invariance

$$S_N(q_{n-1}, q_n) + S_N(q_n, q_{n+1}) \xrightarrow{N \rightarrow \infty} S_N(q_{n-1}, q_{n+1})$$

Outline

1. Introduction ✓
2. Elements of Covariant Loop Quantum Gravity
3. Recent developments
4. Conclusion

General structure



quantum gravity in finite regions:

boundary Hilbert space

$$\Psi[h_{ab}] \in \mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \bar{\mathcal{H}}_{\Sigma_0}$$

amplitude map

$$\mathcal{A}_M[\Psi] = \int_{g_{ab} \text{ on } M} \mathcal{D}[g_{ab}] e^{\frac{i}{\hbar} S_{\text{EH}}[g_{ab}]} \Psi[h_{ab}]$$

probabilities

$$P(\Psi_i | \Phi) = \frac{|\mathcal{A}_M[\Psi_i \otimes \bar{\Phi}]|^2}{\sum_k |\mathcal{A}_M[\Psi_k \otimes \bar{\Phi}]|^2}$$

path integral formally solves the WdW equation:

$$\forall \Psi : \mathcal{A}_M[\hat{H}[N]\Psi] = 0$$

[atiyah-segal, witten,..., oeckl, roveli,...]

Hilbert space

canonical variables

Ashtekar connection
electric field

$$A^i_a = \Gamma^i_a[e] + \gamma K^i_a$$

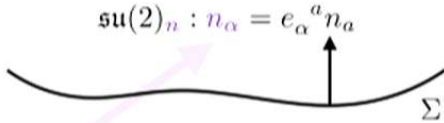
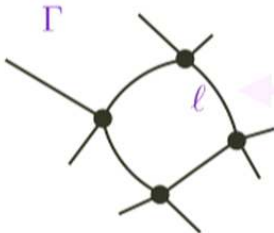
$$E_i^a = \frac{1}{2} \tilde{\eta}^{abc} \epsilon_{ilm} e^l_b e^m_c$$

Hilbert space

spin network functions
Hilbert space

$$\Psi_f[A] = f(h_\ell[A], h_e[A], \dots) \in \text{Cyl}_\Gamma$$

$$\mathcal{H} = \overline{\bigcup_\Gamma \text{Cyl}_\Gamma / \text{Diff}}, \quad \mathcal{H}_\Gamma = \text{Cyl}_\Gamma / \text{Diff}$$



$$h_\ell[A] = \text{Pexp}\left(-\int_\ell A\right)$$

$$f \in L^2(SU(2)^{\#\text{links}} /_\Gamma SU(2)^{\#\text{nodes}})$$

[ashtekar, rovelli, smolin, isham, thiemann, lewandowski, fleischhack, okolow, sahlmann, barbero, immerzi,...]

Quantum geometry

spins, intertwiners and fuzzy polyhedra

$$[L_i^\Delta, L_j^{\Delta'}] = i\delta_{\Delta\Delta'}\epsilon_{ij}^k L_k^\Delta$$

simplicial fluxes

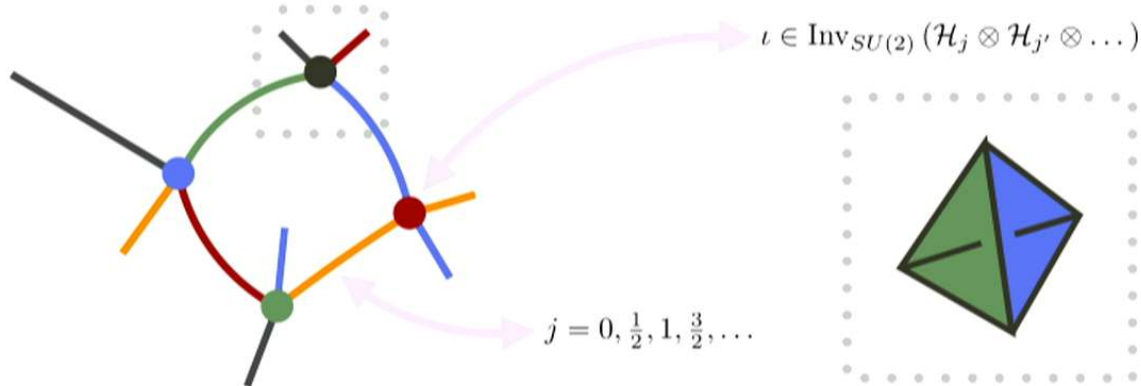
$$\widehat{E}_i^\Delta = \int_{\Delta} d^2 s_a (h_{\Delta}[A])_i^l \widehat{E}_l^a = 8\pi\gamma \hbar G L_i^\Delta$$

area operator

$$\widehat{Ar}_{\Delta} = \sqrt{\delta^{ij} \widehat{E}_i^\Delta \widehat{E}_j^\Delta}$$

area spectrum

$$Ar_j = 8\pi\gamma \hbar G \sum_i \sqrt{j_i(j_i + 1)}$$



[ashtekar, rovelli, smolin, lewandowski, thiemann,..., freidel, speziale,..., bianchi, doná, haggard]

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Semi-classical limit

$$\mathcal{A}_\Delta[\Psi_{\{\ell\}}] \stackrel{j \rightarrow \infty}{\sim} \gamma^{j=\text{const.}} e^{\frac{i}{\hbar} S_{\text{Regge}}[\{\ell\}]}, \quad S_{\text{Regge}}[\{\ell\}] = \frac{1}{8\pi G} \sum_{\Delta} \Xi_{\Delta}^{\{\ell\}} \text{Ar}_{\Delta}^{\{\ell\}}$$

validity of the expansion

$$L_o \ll \ell \ll L_R$$

accidental curvature constraint

$$\gamma \Xi_{\Delta} = 4\pi n_{\Delta}$$

fundamental length scale: $L_o^2 = 8\pi\gamma \hbar G$

curvature scale: $L_R^2 = \frac{1}{R[g]}$

areas: $\ell^2 \sim 8\pi\gamma \hbar G j$

[rovelli, han, zhang, barrett, kaminski, hellmann, fairbairn, bonzom, gomes, magliaro, perini, mikovic, vojnovic,...]

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Simplicity constraints



Holst action

$$S[A, e] = \frac{1}{16\pi G} \int_M (* - \gamma^{-1}) (e_\alpha \wedge e_\beta) \wedge F^{\alpha\beta}[A]$$

simplicial fluxes

$$\Pi_{\alpha\beta}^\Delta = \frac{1}{16\pi G} \int_\Delta (h_\Delta[A])_{\alpha\beta}{}^{\mu\nu} (* - \gamma^{-1}) (e_\mu \wedge e_\nu)$$

simplicity constraints

$$\begin{aligned} \overleftarrow{(e_\alpha \wedge e_\beta)} n^\beta = 0 &\Rightarrow K_\alpha + \gamma L_\alpha = 0 \\ (\hat{K}_\alpha + \gamma \hat{L}_\alpha) |(\gamma j, j), jm\rangle_n &\approx 0 \end{aligned}$$

$$L_\alpha = 2 * \Pi_{\alpha\beta} n^\beta, \quad K_\alpha = 2 \Pi_{\alpha\beta} n^\beta$$

$$Y_\gamma^n : \mathcal{H}_j \rightarrow \mathcal{H}_{\gamma j, j}, |j, m\rangle \mapsto |(\gamma j, j), jm\rangle_n$$

vertex amplitude

$$\mathcal{A}[\Psi_f] = \int_{\text{connections } A} \mathcal{D}[A] \delta(F[A]) \Psi_{Y_\gamma^{n_1, \dots, n_5} f}[A] = (P_{SL(2, \mathbb{C})} \circ Y_\gamma^{n_1, \dots, n_5} f)(\mathbb{1}, \dots)$$

local Lorentz invariance

[livine, alexandrov, dupuis, ww]

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Semi-classical limit

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[rovelli, han, zhang, barrett, kaminski, hellmann, fairbairn, bonzom, gomes, magliaro, perini, mikovic, vojnovic,...]

Summing vs. refining

$$\Gamma \prec \Gamma' \prec \Gamma'' : \iota_{\Gamma''} \circ \iota_{\Gamma'} = \iota_{\Gamma''}$$

$$\Psi_\Gamma \sim \Psi_{\Gamma'} \Leftrightarrow \exists \Gamma'' : \iota_{\Gamma''}(\Psi_\Gamma) = \iota_{\Gamma''}(\Psi_{\Gamma'})$$

inductive limit on
the boundary

$$\iota_{\Gamma'} : \mathcal{H}_\Gamma \rightarrow \mathcal{H}_{\Gamma'}, \quad \mathcal{H} = \lim_{\Gamma \rightarrow \infty} \mathcal{H}_\Gamma = \overline{\bigcup_{\Gamma} \mathcal{H}_\Gamma / \sim}$$

refining

$$\mathcal{A}(h_\ell, h_{\ell'}, \dots) = \lim_{\Delta \rightarrow \infty} N_\Delta \mathcal{A}_\Delta(h_\ell, h_{\ell'}, \dots) \Big|_{(\partial\Delta)_1^* = \Gamma}$$

summing

$$\mathcal{A}(h_\ell, h_{\ell'}, \dots) = \sum_{\Delta} \frac{\lambda^{\#\text{vertices}}}{\text{sym}_\Delta} \mathcal{A}_\Delta(h_\ell, h_{\ell'}, \dots) \Big|_{(\partial\Delta)_1^* = \Gamma}$$

- › embedding maps from AL vacuum? from new BF vacuum instead?
- › summing = refining?

Summing: GFT

$$S_{\text{GFT}}[\varphi] = \int_{SU(2)^4} d\vec{h} \int_{SU(2)^4} d\vec{h}' \bar{\varphi}(\vec{h}) \mathcal{K}(\vec{h}, \vec{h}') \varphi(\vec{h}') + \lambda \times (\text{interaction term})$$

$$\int \mathcal{D}[\varphi] e^{\frac{i}{\hbar} S_{\text{GFT}}[\varphi]} = \sum_{\Delta} \frac{\lambda^{\#\text{vertices}}}{\text{sym}_{\Delta}} \mathcal{A}_{\Delta}$$

wavefunction of a single tetrahedron

field operator	$\varphi(h_1, \dots, h_4) \longrightarrow \hat{\varphi}(\vec{h}) : [\hat{\varphi}(\vec{h}), \hat{\varphi}^\dagger(\vec{h}')] = \prod_{i=1}^4 \delta(h_i, h'_i)$
Fock space	$\mathcal{H} = \mathcal{F}(L^2(SU(2)^4/SU(2))) + \text{glueing constraints}$
GFT condensates	$ f\rangle = \frac{1}{N_f} \exp\left(\int_{SU(2)^4} d\vec{h} f(\vec{h}) \varphi^\dagger(\vec{h})\right) 0_{\text{AL}}\rangle$

further applications: Gross–Pitaevskii equation for condensate wavefunction f can give LQC

[rovelli, reissenberger, oriti, freidel, krajewski, ryan, ben goulon, bonzom, rivasseau, carrozza, sindoni, thürigen,...]

Refining: finite amplitudes

$$\mathcal{A} = \lim_{\Delta \rightarrow \infty} \mathcal{A}_\Delta : \forall \epsilon > 0 \exists \Delta_\epsilon \forall \Delta \succ \Delta_\epsilon : |\mathcal{A} - \mathcal{A}_\Delta| < \epsilon$$

$$\mathcal{A} = \lim_{\Delta \rightarrow \infty} N_\Delta \mathcal{A}_\Delta, \quad N_\Delta = \left(\frac{w}{j_{\max}} \right)^{\#\text{vertices}}$$

- idea: cosmological constant provides a physical IR cutoff, this is realized in 3d gravity (Turaev–Viro amplitudes)

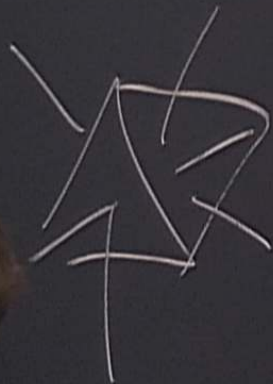
$$SU(2) \rightarrow SU(2)_q, \quad q = \exp\left(\frac{i\pi}{k+2}\right), \quad k = \frac{1}{\hbar G \sqrt{\Lambda}}, \quad j_{\max} = \frac{1}{2\hbar G \sqrt{\Lambda}}$$

- conjecture: the same happens in 4d with $SL(2, \mathbb{C})$ turning into $SL(2, \mathbb{C})_q$

$$SL(2, \mathbb{C}) \xrightarrow{?} SL(2, \mathbb{C})_q, \quad q \stackrel{?}{=} \exp\left(\frac{2\pi}{n} \frac{1}{\gamma + i}\right), \quad n = \frac{3}{2} \frac{1}{\gamma \hbar G \Lambda}, \quad j_{\max} \stackrel{?}{\sim} \frac{1}{\hbar G \Lambda}$$

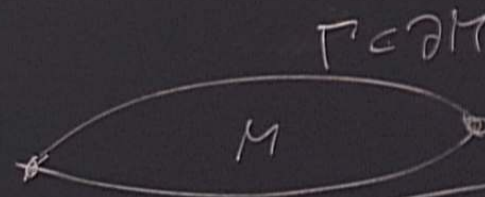
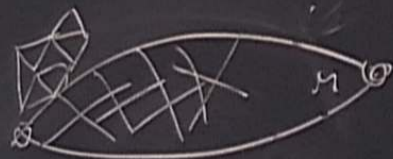
[rovelli, vidotto, han, smerlak, ..., dittrich, martin-benito, steinhaus, ...]

$$\mathcal{H} \cong L^2(\mathrm{SU}(2)^L / \pi \mathrm{SU}(2)^N)$$



graph

$$\mathcal{H}_\Gamma \subset \mathcal{H}_{\Gamma'}$$



$$\frac{\Gamma \subset \Gamma'}{\mathcal{H}_\Gamma \subset \mathcal{H}_{\Gamma'}}$$

$$\begin{aligned} &\psi, \phi \in \mathcal{H} \\ &\Gamma; \psi, \phi \in \mathcal{H}_\Gamma \\ &\langle \psi, \phi \rangle = \langle \psi, \phi \rangle_\Gamma \end{aligned}$$

How to make this concrete

geometry locally de sitter

simplicity constraints

$$\mathcal{A}^\Lambda[\Psi] = \int \mathcal{D}[A] e^{\frac{3}{2} \frac{1}{\ell^2 \Lambda} \frac{\gamma+1}{\gamma} S_{\text{CS}}[A] - \text{cc.}} (Y_\gamma \Psi)[A]$$

Λ -EPRL vertex amplitudes as $SL(2, \mathbb{C})$
 Chern–Simons evaluation of γ -simple
 4-simplex Wilson graph operators

$$\left\{ \begin{array}{l} S_{\text{CS}}[A] = \int_{\partial M} \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \\ A^i_a = \Gamma^i_a[e] + iK^i_a \end{array} \right.$$

[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,...]

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Λ -EPRL

- **conjecture:** amplitudes are finite and give q -deformation $SL(2, \mathbb{C}) \rightarrow SL(2, \mathbb{C})_q$ (satisfied for Euclidean signature)
- **rich mathematical structure:** possible relations to quantum curves, knot theory, Chern–Simons theory, string theory, supersymmetry
- boundary Hilbert space: constant-curvature polyhedra
- at 4-simplex level correct semiclassical limit for $j \rightarrow \infty$, $\gamma_j = \text{const}$

[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,...]

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[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,...]

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4. Conclusion

Gravity is different

- ▶ **Continuum limit:** Limit of infinitely many simplicial building blocks $N \rightarrow \infty$, in both the bulk and at the boundary.
- ▶ The theory has **no lattice constant α** , the physical size of the simplicial building blocks is itself a quantum observable.

$$\text{Area}(f) = \int_f du dv \sqrt{g(\partial_u, \partial_u)g(\partial_v, \partial_v) - g(\partial_u, \partial_v)^2}$$

- ▶ No limit to critical values **$\alpha \rightarrow 0, g \rightarrow 0$** as in lattice QCD
- ▶ **New methods needed!**

ical building
 ndary

 ical size of the
 observable.

 ical size of the
 observable.

 nice GCD

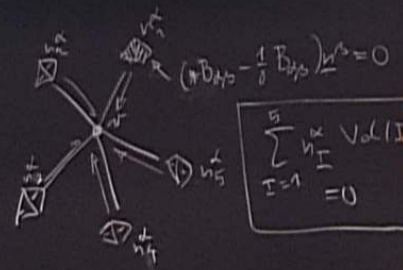
$$\mathcal{Z}^{\Lambda=0} = \int \mathcal{D}B \mathcal{D}A e^{-\int_{\Gamma} \text{Tr}(B \wedge F)} (\chi, \Psi) \mathcal{Z}_{\Gamma}$$

$$= (\chi, A) (h_{x_1} = 1, \dots, h_{x_n} = 1)$$

$\mathcal{Z}_{\Gamma} \subset \mathcal{Z}_{\Gamma'}$

$\Gamma \subset \Gamma'$
 $\mathcal{Z}_{\Gamma} \subset \mathcal{Z}_{\Gamma'}$

$\Psi, \Phi \in \mathcal{X}$
 $\Gamma_i \Psi, \Phi \in \mathcal{Z}_{\Gamma}$
 $\langle \Psi, \Phi \rangle = \langle \Psi, \Phi \rangle_{\Gamma}$



$$\sum_{I=1}^5 w_I^k \text{Vol}(I) = 0$$

