Title: TBA

Date: Sep 28, 2015 09:05 AM

URL: http://pirsa.org/15090076

Abstract:

covariant methods in LQG

renormalization in background independent theories: foundations and techniques

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28 september 2015
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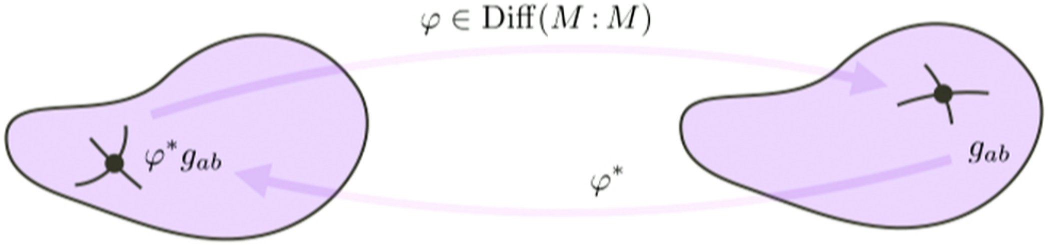
Renormalization in Background Invariant Theories

Background Invariance



background-invariant theory: $S[X] = S[\varphi^* X]$

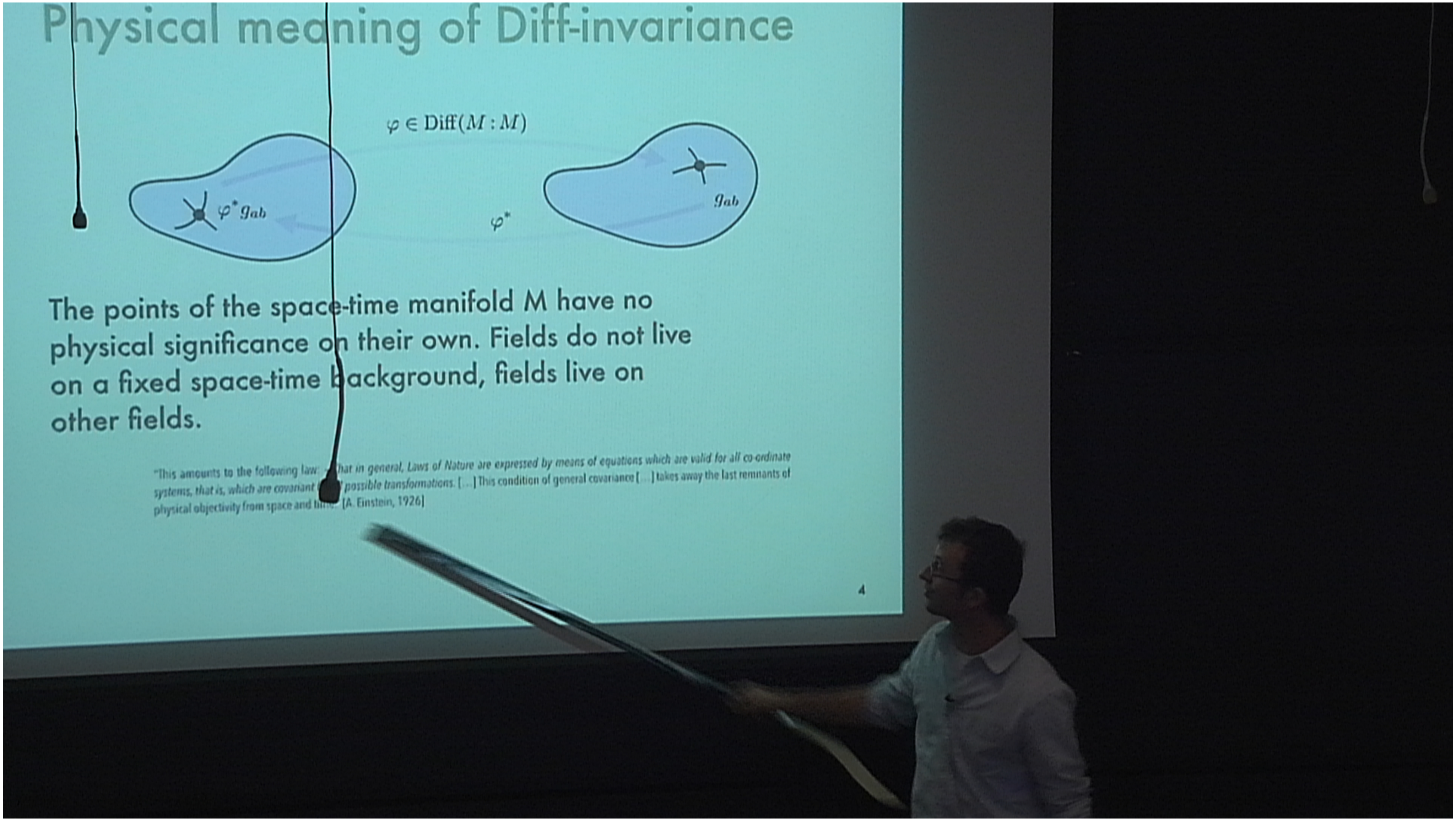
topological theories:

$$S_{BF}[B,A] = \int_M \mathrm{Tr}\big(B \wedge F[A]\big)$$

$$S_{\mathrm{CS}}[A] = \int_M \mathrm{Tr}\big(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\big)$$

general relativity:
(+matter)

$$\boxed{S_{\mathrm{EH}}[g] = \frac{1}{16\pi G}\int_M d^4 v_g \big(R[g] - 2\Lambda\big) - \frac{1}{8\pi G}\int_{\partial M} d^3 v_h K}$$



Physical meaning of Diff-invariance

The points of the space-time manifold M have no physical significance on their own. Fields do not live on a fixed space-time background, fields live on other fields.

"This amounts to the following law: — That *in general, Laws of Nature are expressed by means of equations which are valid for all co-ordinate systems, that is, which are covariant* [illegible] *possible transformations.* [...] This condition of general covariance [...] takes away the last remnants of physical objectivity from space and time." [A. Einstein, 1926]

Renormalization (e.g. lattice QCD)

- **lattice action:** $\frac{1}{4g^2}\int d^4x\,\mathrm{Tr}\big(F_{\alpha\beta}F^{\alpha\beta}\big) \longrightarrow \frac{1}{2g^2}\sum_{\text{plaqs}}\mathrm{Tr}\Big(\mathrm{P}e^{-\oint_{\partial\text{plaq}}A}\Big) = -S_{\text{lattice}}$
- **path integral:** $\int \mathcal{D}[A]\cdots = \prod_{\text{links}}\int_{SU(N)} dU_{\text{link}}\cdots$
- **Wilson loop observables:** $\int \mathcal{D}[A]\,\mathrm{e}^{-S}\,\mathrm{Tr}\Big(\mathrm{P}e^{-\oint_{\text{loop}}A}\Big) \propto \mathrm{e}^{-\tau(g,a)\mathrm{Area}}$
- **beta function:** $a\frac{\partial g}{\partial a} = -\beta(g)$
- **continuum limit (2-nd order phase transition):**

  $a \to 0,\ g \to 0,\ \tau(a,g) = \text{const.}$

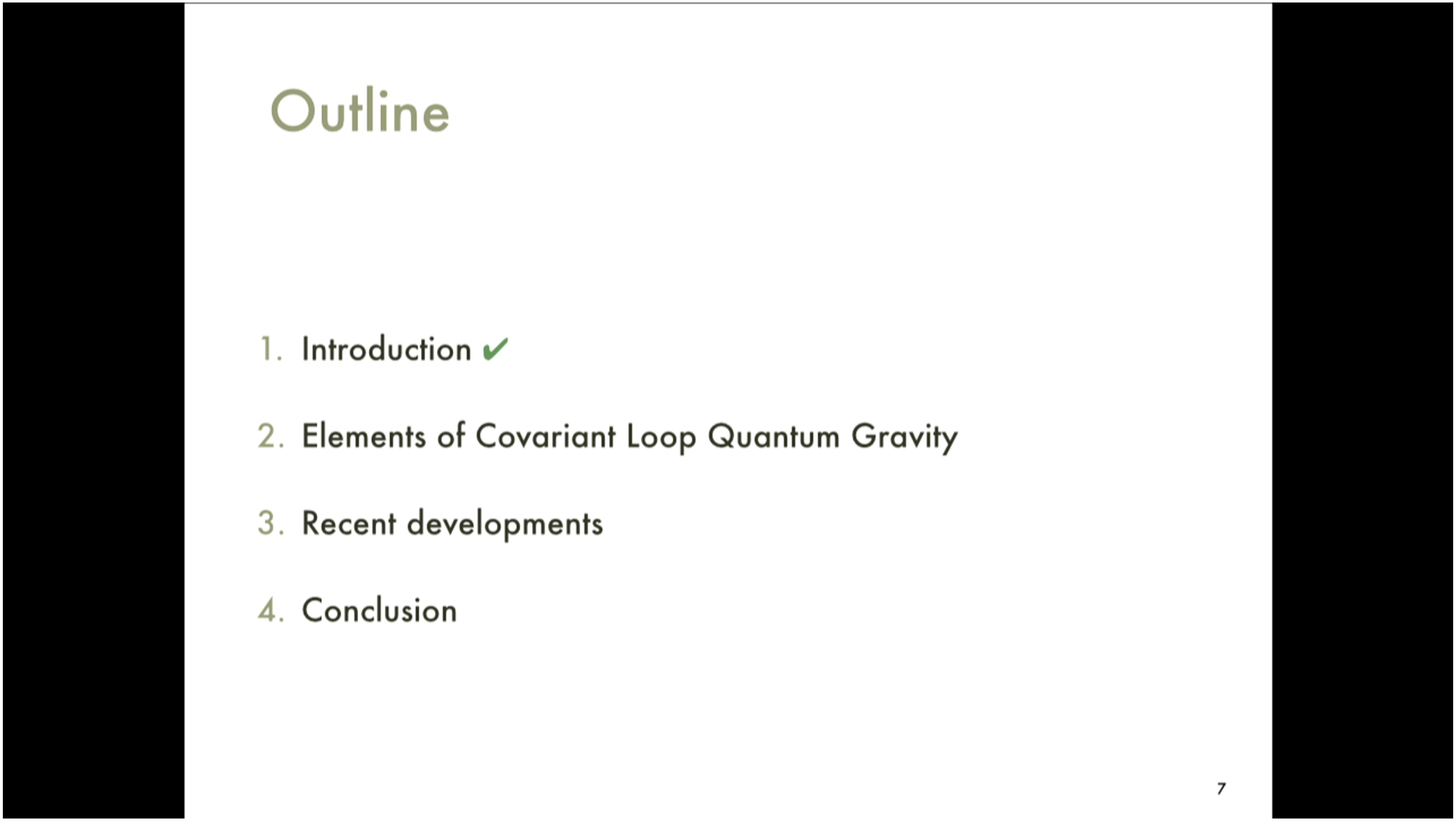
  all correlation lengths diverge in units of a

Perfect action and Ditt-invariance

- Gravity is different: The lattice spacing is itself a function of the dynamical field: $a = \int_{\text{link}} \mathrm{d}t \sqrt{|g_{\alpha\beta}(X)\dot{X}^{\alpha}\dot{X}^{\beta}|}$
- In LQG area is quantized: $A_j = 8\pi\gamma\,\hbar G\,\sqrt{j(j+1)}$
- No limit to a critical value $a \rightarrow 0$, $g \rightarrow 0$, only $N \rightarrow \infty$
- Discretized theory approaches Ditt-invariance

$$S_N(q_{n-1}, q_n) + S_N(q_n, q_{n+1}) \xrightarrow{N\to\infty} S_N(q_{n-1}, q_{n+1})$$

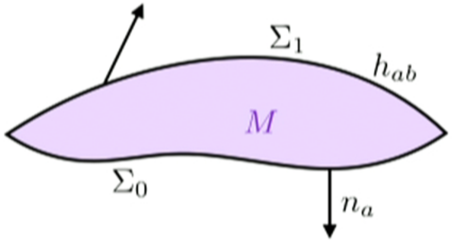
[dittrich, bahr, rovelli, steinhaus]



Outline

1. Introduction ✓
2. Elements of Covariant Loop Quantum Gravity
3. Recent developments
4. Conclusion

General structure



quantum gravity in finite regions:

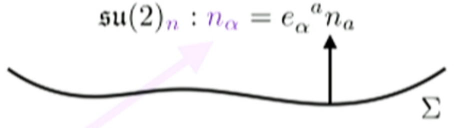
boundary Hilbert space	$\Psi[h_{ab}] \in \mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \bar{\mathcal{H}}_{\Sigma_0}$
amplitude map	$\mathcal{A}_M[\Psi] = \int_{g_{ab} \text{ on } M} \mathcal{D}[g_{ab}]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S_{\mathrm{EH}}[g_{ab}]}\, \Psi[h_{ab}]$
probabilities	$P(\Psi_i \vert \Phi) = \dfrac{\left\vert \mathcal{A}_M[\Psi_i \otimes \bar{\Phi}]\right\vert^2}{\sum_k \left\vert \mathcal{A}_M[\Psi_k \otimes \bar{\Phi}]\right\vert^2}$

path integral formally solves the WdW equation:

$$\forall \Psi : \mathcal{A}_M\left[\hat{H}[N]\Psi\right] = 0$$

[atiyah–segal, witten,..., oeckl, rovelli,...]

Hilbert space

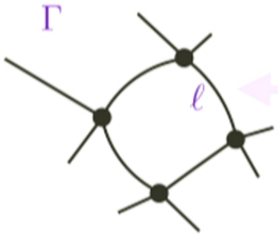

$\mathfrak{su}(2)_n : n_\alpha = e_\alpha{}^a n_a$

canonical variables

Ashtekar connection	$A^i{}_a = \Gamma^i{}_a[e] + \gamma K^i{}_a$
electric field	$E_i{}^a = \frac{1}{2}\tilde{\eta}^{abc}\epsilon_{ilm}e^l{}_b e^m{}_c$

Hilbert space

spin network functions	$\Psi_f[A] = f\big(h_\ell[A], h_{\ell'}[A], \dots\big) \in \mathrm{Cyl}_\Gamma$
Hilbert space	$\mathcal{H} = \overline{\bigcup_\Gamma \mathrm{Cyl}_\Gamma/\mathrm{Diff}}, \quad \mathcal{H}_\Gamma = \mathrm{Cyl}_\Gamma/\mathrm{Diff}$



$$h_\ell[A] = \mathrm{Pexp}\Big(-\int_\ell A\Big)$$

$$f \in L^2\big(SU(2)^{\#\mathrm{links}}/_\Gamma\, SU(2)^{\#\mathrm{nodes}}\big)$$

[ashtekar, rovelli, smolin, isham, thiemann, lewandowski, fleischhack, okolow, sahlmann, barbero, immirzi,...]

Hilbert space

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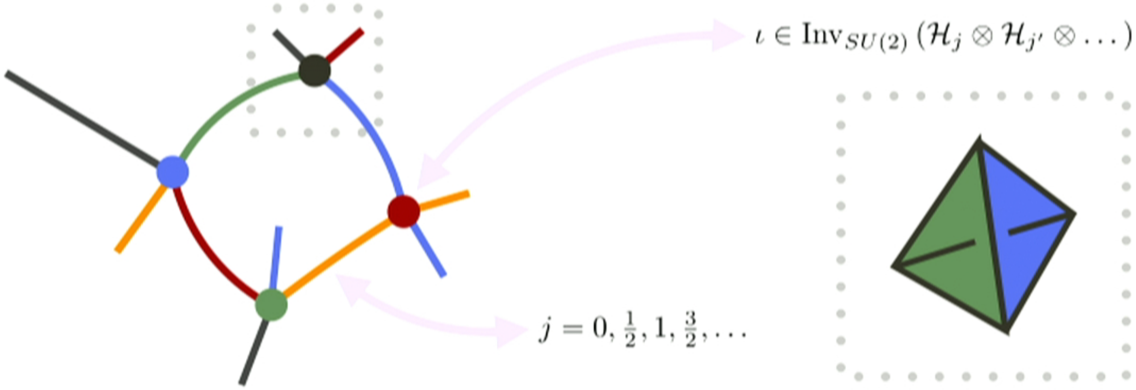
[ashtekar, rovelli, smolin, isham, thiemann, lewandowski, fleischhack, okolow, sahlmann, barbero, immirzi,...]

Quantum geometry

$[L_i^{\triangle}, L_j^{\triangle'}] = \mathrm{i}\delta_{\triangle\triangle'}\epsilon_{ij}{}^{k} L_k^{\triangle}$

spins, intertwiners and fuzzy polyhedra

simplicial fluxes	$\widehat{E}_i^{\triangle} = \int_{\triangle} d^2 s_a \big(h_{\triangle}[A]\big)_i{}^l \widehat{E}_l{}^a = 8\pi\gamma\,\hbar G\, L_i^{\triangle}$
area operator	$\widehat{\mathrm{Ar}}_{\triangle} = \sqrt{\delta^{ij}\,\widehat{E}_i^{\triangle}\widehat{E}_j^{\triangle}}$
area spectrum	$\mathrm{Ar}_{\vec{j}} = 8\pi\gamma\,\hbar G \sum_i \sqrt{j_i(j_i+1)}$



$\iota \in \mathrm{Inv}_{SU(2)}\,(\mathcal{H}_j \otimes \mathcal{H}_{j'} \otimes \dots)$

$j = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

[ashtekar, rovelli, smolin, lewandowski, thiemann,..., freidel, speziale,..., bianchi, doná, haggard]

Semi-classical limit

$$\mathcal{A}_\Delta\left[\Psi_{\{\ell\}}\right] \overset{j\to\infty}{\overset{\gamma j=\text{const.}}{\sim}} \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S_{\text{Regge}}[\{\ell\}]}, \quad S_{\text{Regge}}\left[\{\ell\}\right] = \frac{1}{8\pi G}\sum_{\triangle} \Xi_{\triangle}^{\{\ell\}} \mathrm{Ar}_{\triangle}^{\{\ell\}}$$

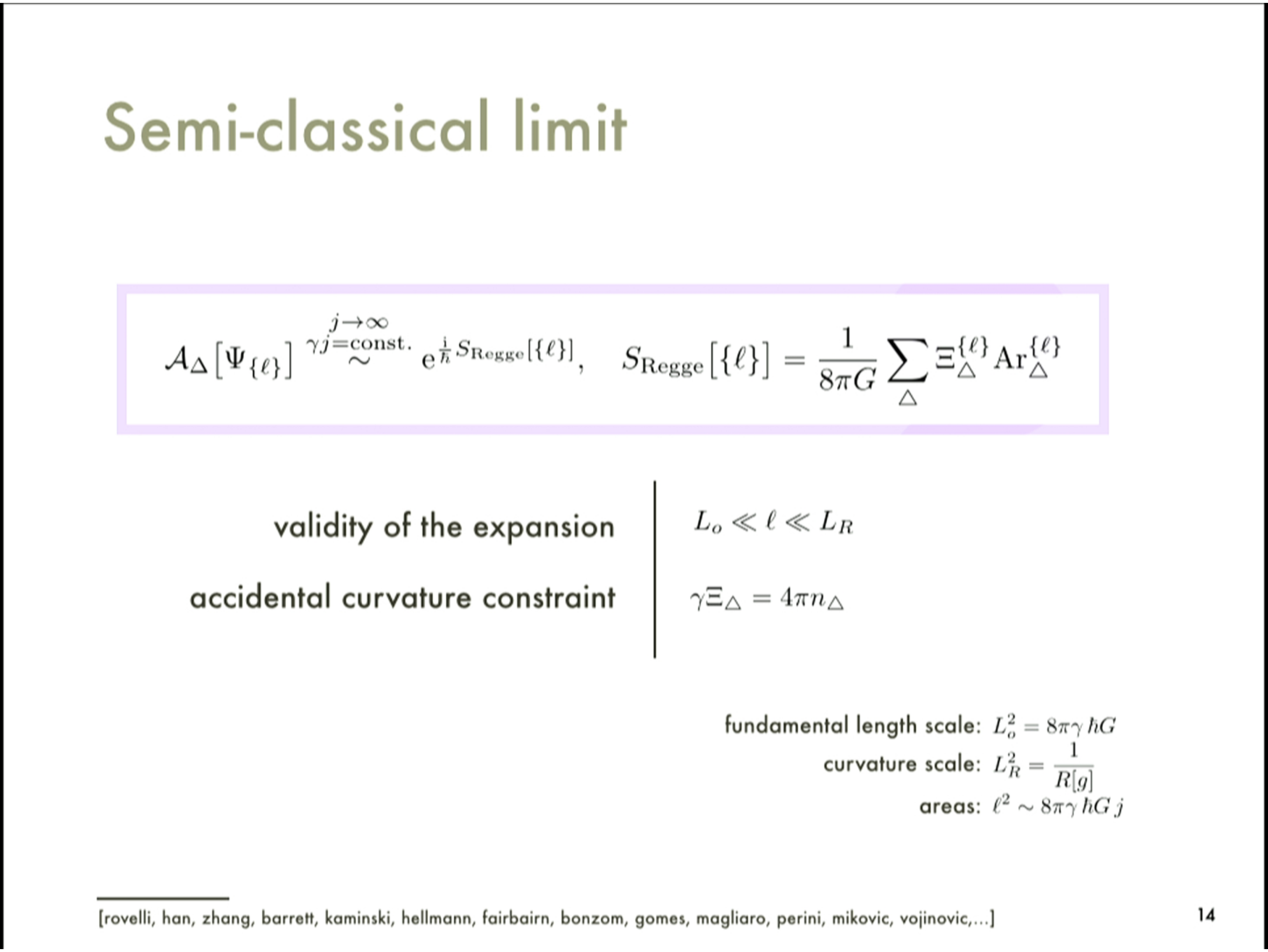
validity of the expansion	$L_o \ll \ell \ll L_R$
accidental curvature constraint	$\gamma\Xi_\triangle = 4\pi n_\triangle$

fundamental length scale: $L_o^2 = 8\pi\gamma\,\hbar G$

curvature scale: $L_R^2 = \dfrac{1}{R[g]}$

areas: $\ell^2 \sim 8\pi\gamma\,\hbar G\, j$

[rovelli, han, zhang, barrett, kaminski, hellmann, fairbairn, bonzom, gomes, magliaro, perini, mikovic, vojinovic,...]



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Simplicity constraints

$n_\alpha = e_\alpha{}^a n_a$

Holst action $S[A,e] = \frac{1}{16\pi G}\int_M (* - \gamma^{-1})\,(e_\alpha \wedge e_\beta) \wedge F^{\alpha\beta}[A]$

simplicial fluxes $\Pi^{\triangle}_{\alpha\beta} = \frac{1}{16\pi G}\int_\triangle (h_\triangle[A])_{\alpha\beta}{}^{\mu\nu}\,(* - \gamma^{-1})\,(e_\mu \wedge e_\nu)$

simplicity constraints

$$\boxed{\begin{array}{l} \underleftarrow{(e_\alpha \wedge e_\beta)}\, n^\beta = 0 \Rightarrow K_\alpha + \gamma L_\alpha = 0 \\ \qquad\qquad (\hat{K}_\alpha + \gamma \hat{L}_\alpha)\big|(\gamma j, j), jm\big\rangle_n \approx 0 \end{array}}$$

$L_\alpha = 2 * \Pi_{\alpha\beta} n^\beta, \quad K_\alpha = 2\Pi_{\alpha\beta} n^\beta$

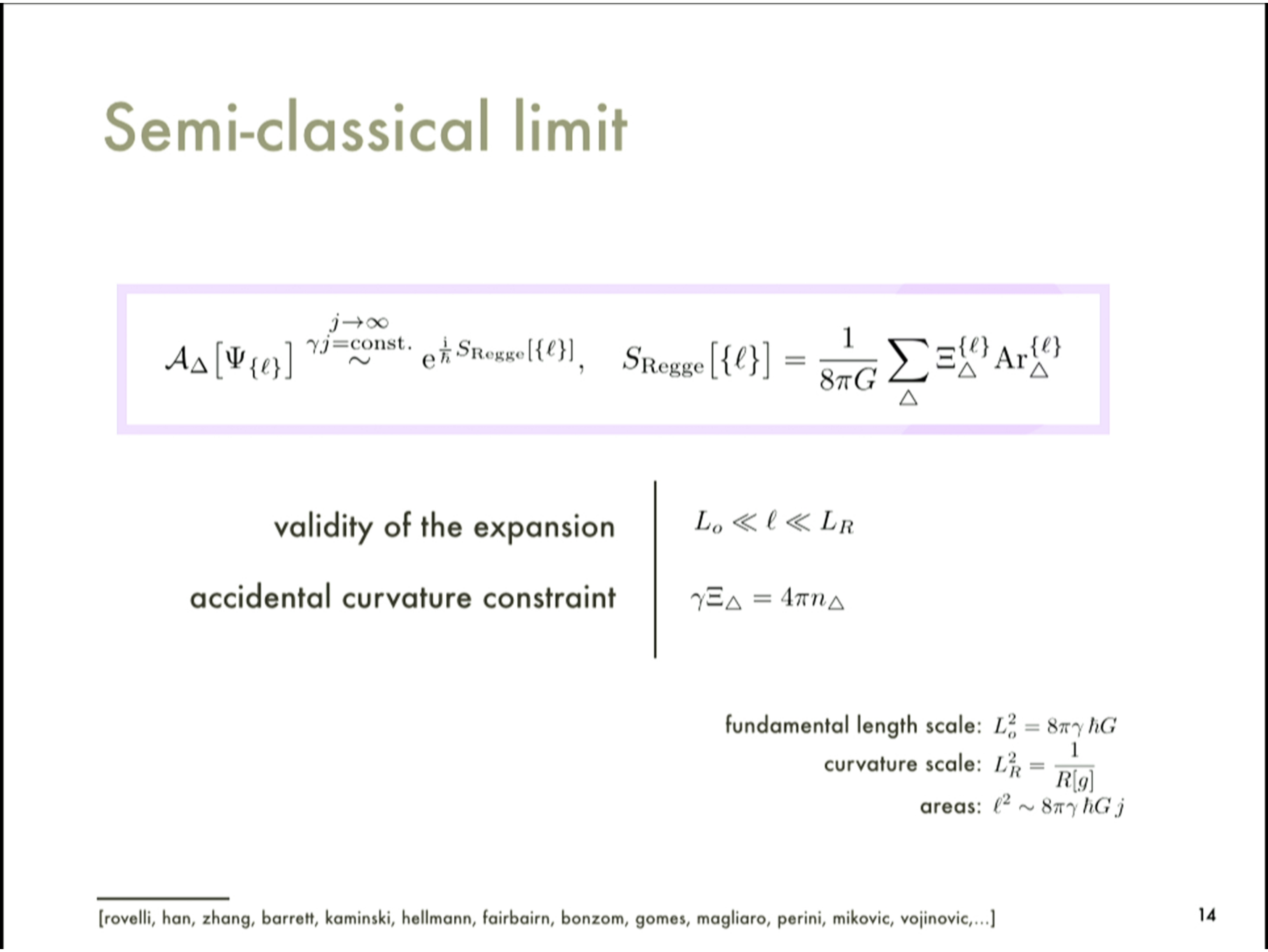
$Y^n_\gamma : \mathcal{H}_j \to \mathcal{H}_{\gamma j, j},\ |j,m\rangle \mapsto \big|(\gamma j, j), jm\big\rangle_n$

vertex amplitude

$$\mathcal{A}[\Psi_f] = \int_{\substack{SL(2,\mathbb{C}) \\ \text{connections } A}} \mathcal{D}[A]\,\delta(F[A])\,\Psi_{Y_\gamma^{n_1,\dots,n_5} f}[A] = \big(P_{SL(2,\mathbb{C})} \circ Y_\gamma^{n_1,\dots,n_5} f\big)\,(\mathbb{1},\dots)$$

local Lorentz invariance

[livine, alexandrov, dupuis, ww]



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Summing vs. refining

$$\Gamma \prec \Gamma' \prec \Gamma'' : \iota_{\Gamma'\Gamma''} \circ \iota_{\Gamma\Gamma'} = \iota_{\Gamma\Gamma''}$$

$$\Psi_\Gamma \sim \Psi_{\Gamma'} \Leftrightarrow \exists \Gamma'' : \iota_{\Gamma''\Gamma}(\Psi_\Gamma) = \iota_{\Gamma''\Gamma'}(\Psi_{\Gamma'})$$

inductive limit on the boundary	$\iota_{\Gamma\Gamma'} : \mathcal{H}_\Gamma \to \mathcal{H}_{\Gamma'}, \quad \mathcal{H} = \lim_{\Gamma\to\infty} \mathcal{H}_\Gamma = \overline{\bigcup_\Gamma \mathcal{H}_\Gamma / \sim}$
refining	$\mathcal{A}(h_\ell, h_{\ell'}, \dots) = \lim_{\Delta\to\infty} N_\Delta \mathcal{A}_\Delta(h_\ell, h_{\ell'}, \dots)\big\rvert_{(\partial\Delta)^*_1=\Gamma}$
summing	$\mathcal{A}(h_\ell, h_{\ell'}, \dots) = \sum_\Delta \frac{\lambda^{\#_{\mathrm{vertices}}}}{\mathrm{sym}_\Delta} \mathcal{A}_\Delta(h_\ell, h_{\ell'}, \dots)\big\rvert_{(\partial\Delta)^*_1=\Gamma}$

- embedding maps from AL vacuum? from new BF vacuum instead?
- summing = refining?

[ashtekar, thiemann, lewandowski, okolow, sahlmann, fleischhack,..., rovelli, smerlak,..., dittrich, geiller, bahr, steinhaus, martin-benito]

Summing: GFT

$$S_{\mathrm{GFT}}[\varphi] = \int_{SU(2)^4} d\vec{h} \int_{SU(2)^4} d\vec{h}' \, \bar{\varphi}(\vec{h})\, \mathcal{K}(\vec{h}, \vec{h}')\, \varphi(\vec{h}') + \lambda \times (\text{interaction term})$$

$$\boxed{\int \mathcal{D}[\varphi] \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S_{\mathrm{GFT}}[\varphi]} = \sum_{\Delta} \frac{\lambda^{\#\text{vertices}}}{\text{sym}_{\Delta}} \mathcal{A}_{\Delta}}$$

wavefunction of a single tetrahedron

field operator	$\varphi(h_1, \dots h_4) \longrightarrow \hat{\varphi}(\vec{h}) : [\hat{\varphi}(\vec{h}), \hat{\varphi}^{\dagger}(\vec{h}')] = \prod_{i=1}^{4} \delta(h_i, h_i')$
Fock space	$\mathcal{H} = \mathcal{F}\left(L^2(SU(2)^4/_{SU(2)})\right) + \text{glueing constraints}$
GFT condensates	$\lvert f \rangle = \frac{1}{N_f} \exp\left(\int_{SU(2)^4} d\vec{h} f(\vec{h}) \varphi^{\dagger}(\vec{h}) \right) \lvert 0_{\mathrm{AL}} \rangle$

further applications: Gross–Pitaevskii equation for condensate wavefunction f can give LQC

[rovelli, reissenberger, oriti, freidel, krajewski, ryan, ben goulon, bonzom, rivasseau, carrozza, sindoni, thürigen,...]

Refining: finite amplitudes

$$\mathcal{A} = \lim_{\Delta\to\infty} \mathcal{A}_\Delta : \forall\epsilon > 0\ \exists\Delta_\varepsilon\ \forall\Delta \succ \Delta_\varepsilon : |\mathcal{A} - \mathcal{A}_\Delta| < \varepsilon$$

$$\boxed{\mathcal{A} = \lim_{\Delta\to\infty} N_\Delta \mathcal{A}_\Delta, \quad N_\Delta = \left(\frac{w}{j_{\max}}\right)^{\#\text{vertices}}}$$

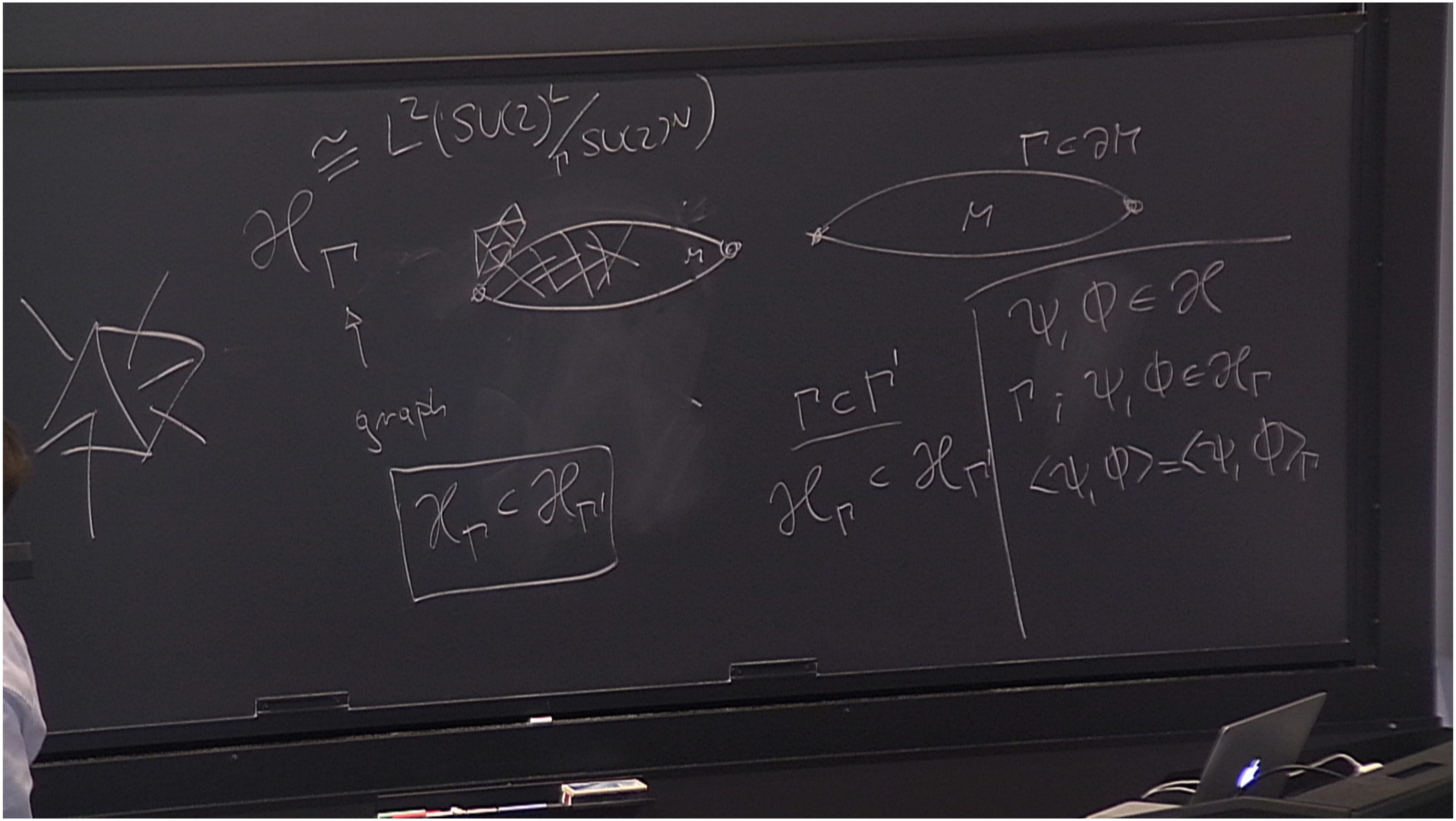
- idea: cosmological constant provides a physical IR cutoff, this is realized in 3d gravity (Turaev–Viro amplitudes)

$$SU(2) \to SU(2)_q, \quad q = \exp\left(\frac{\mathrm{i}\pi}{k+2}\right), \quad k = \frac{1}{\hbar G\sqrt{\Lambda}}, \quad j_{\max} = \frac{1}{2\hbar G\sqrt{\Lambda}}$$

- conjecture: the same happens in 4d with $SL(2,\mathbb{C})$ turning into $SL(2,\mathbb{C})_q$

$$SL(2,\mathbb{C}) \xrightarrow{?} SL(2,\mathbb{C})_q, \quad q \overset{?}{=} \exp\left(\frac{2\pi}{n}\frac{1}{\gamma+\mathrm{i}}\right), \quad n = \frac{3}{2}\frac{1}{\gamma\hbar G\Lambda}, \quad j_{\max} \overset{?}{\sim} \frac{1}{\hbar G\Lambda}$$

[rovelli, vidotto, han, smerlak,..., dittrich, martin-benito, steinhaus,...]



$$\mathcal{H}_\Gamma \cong L^2\left(SU(2)^L / _\Gamma SU(2)^N\right)$$

$$\boxed{\mathcal{H}_\Gamma \subset \mathcal{H}_{\Gamma'}}$$

$\Gamma \subset \partial M$

$$\frac{\Gamma \subset \Gamma'}{\mathcal{H}_\Gamma \subset \mathcal{H}_{\Gamma'}}$$

$\Psi, \Phi \in \mathcal{H}$

$\Gamma ; \Psi, \Phi \in \mathcal{H}_\Gamma$

$\langle \Psi, \Phi \rangle = \langle \Psi, \Phi \rangle_\Gamma$

How to make this concrete

geometry locally de sitter

simplicity constraints

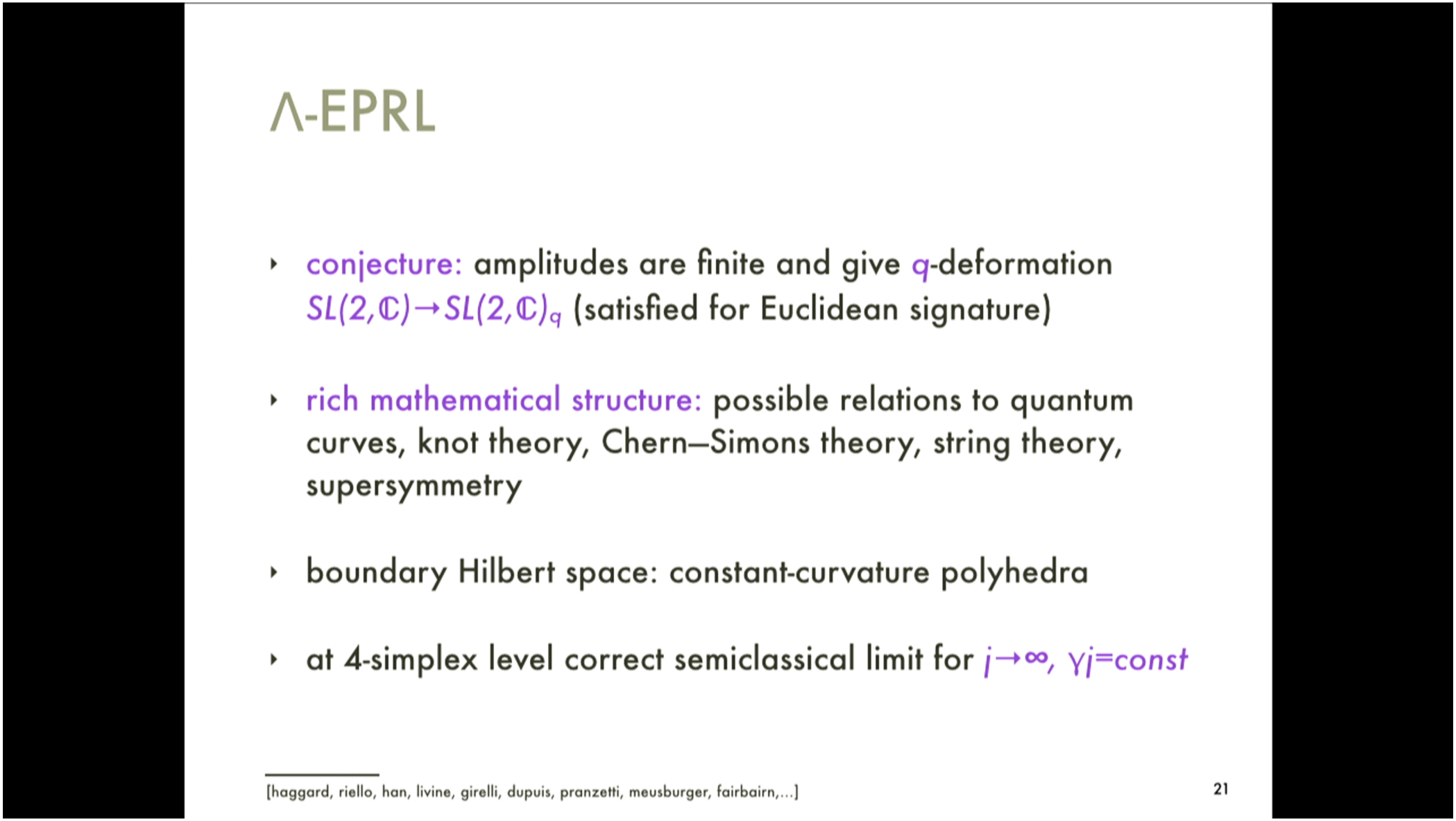
$$\mathcal{A}^{\Lambda}[\Psi] = \int \mathcal{D}[A] \mathrm{e}^{\frac{3}{2}\frac{1}{\ell^2\Lambda}\frac{\gamma+\mathrm{i}}{\gamma} S_{\mathrm{CS}}[A] - \mathrm{cc.}} \big(Y_\gamma \Psi\big)[A]$$

Λ-EPRL vertex amplitudes as *SL(2,ℂ)*
Chern–Simons evaluation of γ-simple
4-simplex Wilson graph operators

$$S_{\mathrm{CS}}[A] = \int_{\partial M} \mathrm{Tr}\Big(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\Big)$$

$$A^i{}_a = \Gamma^i{}_a[e] + \mathrm{i}K^i{}_a$$

[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,...]



Λ-EPRL

- conjecture: amplitudes are finite and give q-deformation $SL(2,\mathbb{C})\rightarrow SL(2,\mathbb{C})_q$ (satisfied for Euclidean signature)
- rich mathematical structure: possible relations to quantum curves, knot theory, Chern–Simons theory, string theory, supersymmetry
- boundary Hilbert space: constant-curvature polyhedra
- at 4-simplex level correct semiclassical limit for $j\rightarrow\infty$, $\gamma j=const$

[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,…]

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[haggard, riello, han, livine, girelli, dupuis, pranzetti, meusburger, fairbairn,...]

4. Conclusion

Gravity is different

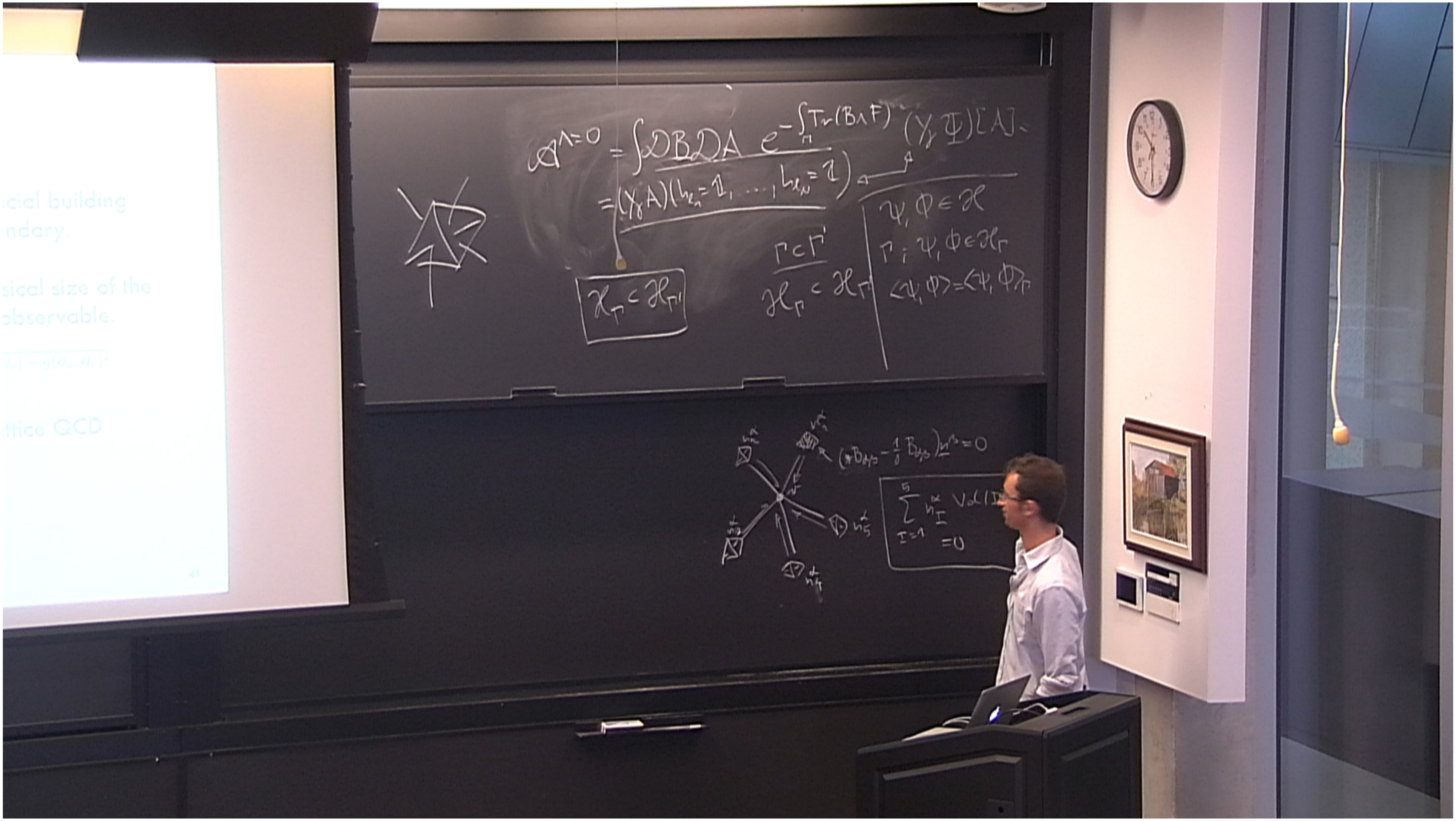
- Continuum limit: Limit of infinitely many simplicial building blocks $N \to \infty$, in both the bulk and at the boundary.

- The theory has no lattice constant a, the physical size of the simplicial building blocks is itself a quantum observable.

$$\mathrm{Area}(f) = \int_f du\, dv \sqrt{g(\partial_u, \partial_u) g(\partial_v, \partial_v) - g(\partial_u, \partial_v)^2}$$

- No limit to critical values $a \to 0$, $g \to 0$ as in lattice QCD

- New methods needed!


icial building
ndary.
sical size of the
observable.
ttice QCD
$\mathcal{A}^{\Lambda=0} = \int \mathcal{D}B\mathcal{D}A\, e^{-\int_\Pi \mathrm{Tr}(B\wedge F)} (\psi_\gamma \Psi)[A] =$
$= (\psi_\gamma A)(h_{e_1}=\mathbb{1}, \dots, h_{e_N}=\mathbb{1})$
$\mathcal{H}_\Gamma \subset \mathcal{H}_{\Gamma'}$
$\Gamma \subset \Gamma'$
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$\Psi, \Phi \in \mathcal{H}$
$\Gamma ;\ \Psi, \Phi \in \mathcal{H}_\Gamma$
$\langle\Psi, \Phi\rangle = \langle\Psi, \Phi\rangle_\Gamma$
n_1^α
n_2^α
n_3^α
n_4^α
n_5^α
$(*B_{\alpha\beta} - \frac{1}{\gamma} B_{\alpha\beta}) n^\beta = 0$
$\sum_{I=1}^{5} n_I^\alpha \mathrm{Vol}(I$
$= 0$