Title: Relative locality and Non commutative geometry

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Abstract:

# Geometry of Relative Locality

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PI 15

based on 1307.7080, 1405.3949, and 1502.08005…
with R.G Leigh (Univ. Illinois) and D. Minic (Virginia tech)

# Non Locality

•We expect that any theory of quantum gravity will involve some non-locality.
The question is what type? and how do deal with it? without opening Pandora's Box

•Built in Locality in FT and GR:

locality of asympt. states,
locality of interactions
locality of RG =sep.of scales.

# Non Locality

- Both QM and GR exhibits non locality:

- QM: Heisenberg non locality $\Delta x \Delta p \geq \hbar/2$

  Entanglement non locality :"Spooky action"

  Aharanov-Bohm non locality = non locality of interferences

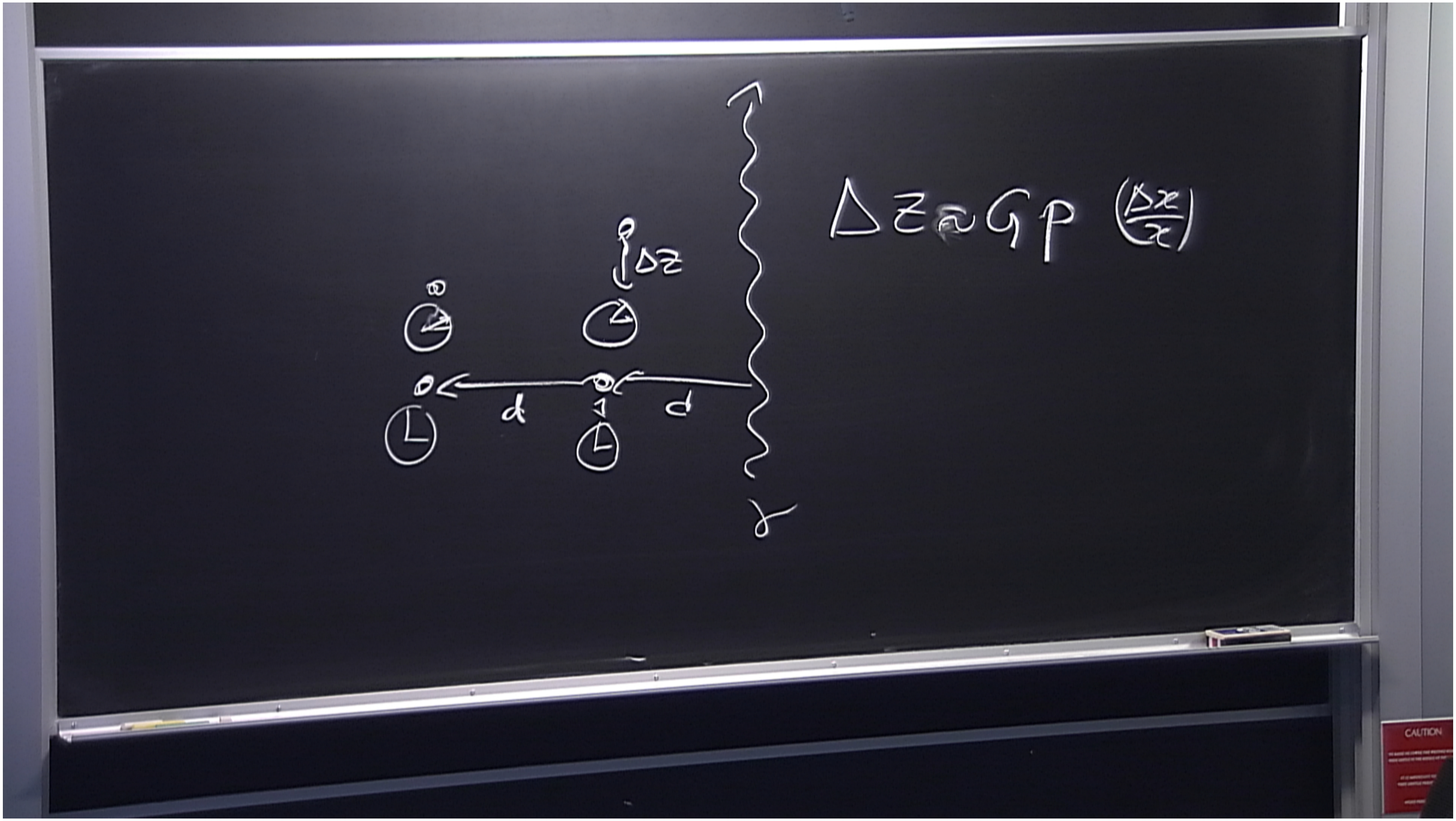
  measured by modular operators, with no classical analog

- GR: Diffeomorphism invariance $\longrightarrow$ gravity observables are non local (Generalised Gauss Law)= Holography.

  Due to causality there is no-screening, the gravity charges = E are all positive.

  Non local memory effect: $\Delta z \sim Gp\frac{\delta x}{x}$

- Non locality respecting local causality.


ΔZ ≈ Gρ (Δx/x)
ΔZ
d
d
γ

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# What kind of non locality?

A new take on quantum gravity: It should emerge from a theory that both has a fundamental delocalisation scale and satisfy the relativity principle

Here I will take the point of view, that attempts to define quantum geometry via the interaction of its probes

- Relative locality

Relative Locality is taken as the organizational feature allowing us to tame non locality.

In relative locality processes among probes defines via localization its own notion of space-time: Locality is relative

- Born Duality

ability to change polarization

# What is Relative locality?

Absolute locality is the hypothesis that the concept of spacetime is independent of the nature of probed used. It is a universal notion.

Relative locality is on the contrary exploring the idea that spacetime is a notion which depends on the quantum nature of probe used i-e energy and quantum numbers.

The usual spacetime notion is adapted to probes which are Point-like and classical.

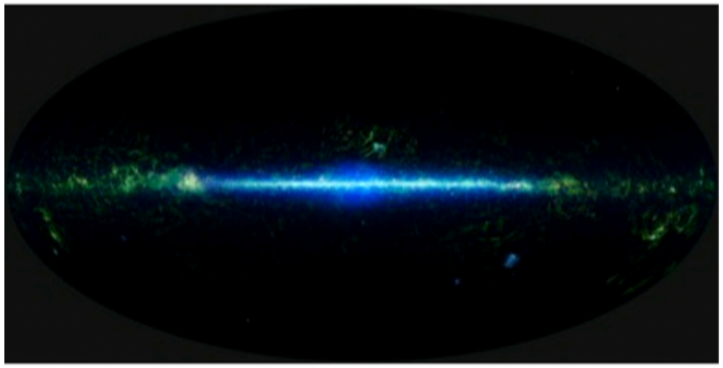
What is the proper notion of quantum spacetime adapted to quantum and non-local probes ?

Spacetime is relative to energy-momentum in phase space.

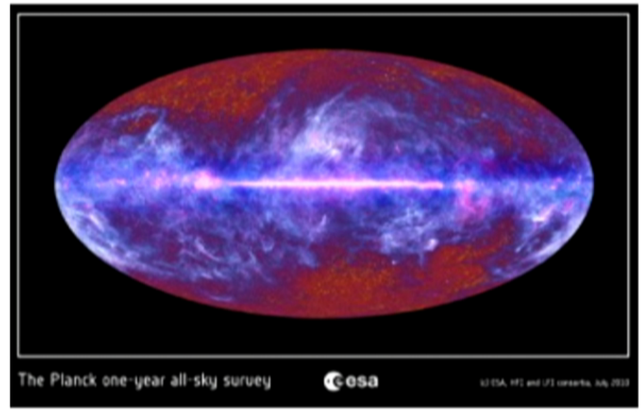
Why? How to implement it? what are the consequences for spacetime and effective field theory?

# Relative Locality: Illustration

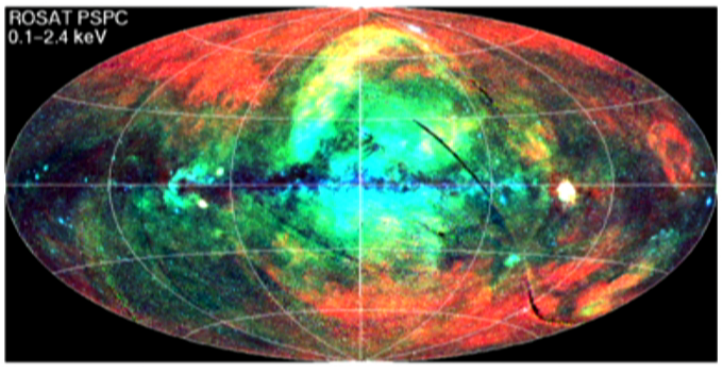
full sky survey:



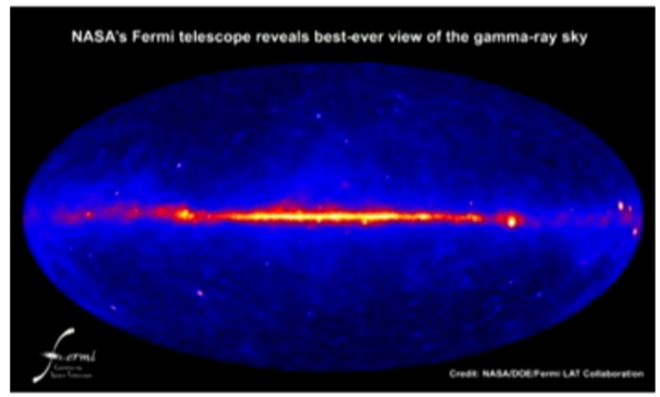
Wise infrared




Planck microwave




Rosat X-ray

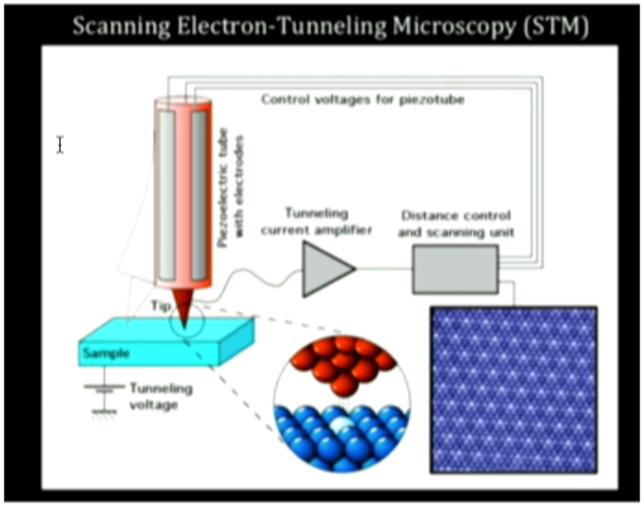



Fermi Gamma ray

# Relative Locality: An Illustration

What do we see when we
visualise the electron quantum wave function?

How do quantum electron localise inside a crystal ?




Tunelling current $I(\boldsymbol{r}, z, E)$

sample position

e Voltage

Tunelling conductivity

$$\frac{\mathrm{d}I}{\mathrm{d}V} \propto n(\boldsymbol{r}, E)$$

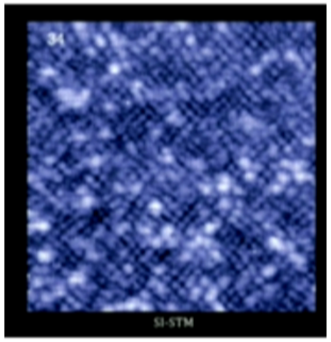
Local density of state

$$n(\boldsymbol{r}, E) = \frac{1}{N} \sum_i |\Psi_i(\boldsymbol{r})|^2 \delta(E - E_i).$$

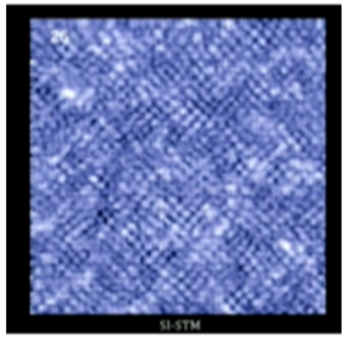
energy dependent measurement of the electron wave function

# Quantum Visualization

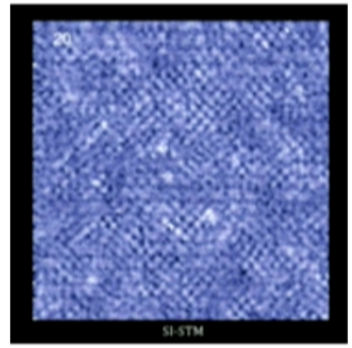
Thanks to S Davis



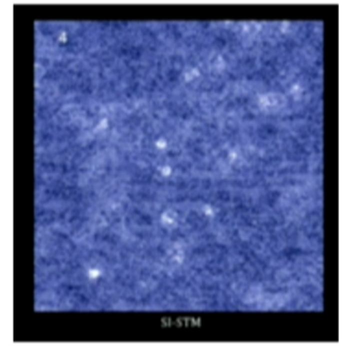

High E
disordered




Quasi-Particle interferences:
Friedel oscillations




ordered




Low E
translation invariant

The classical question: is it ordered or disordered? is ill-defined in QM

In the same region of space we can have different eigenstates of different energy, it is disordered at a given energy and ordered at another.

The energy here is not the energy put in the sample but the energy used to look at the wave function. beholder's eye

Analogy: Quantum crystal = spacetime : electrons= probes.

# Geometry of Relative locality

This analogy suggests that the proper geometrical setting appropriate to discuss the quantum geometry is Phase space

We cannot talk about localization property or fixed spacetime in a quantum mechanical setting without giving information about the energy scale involved

What does relative locality has to do with Quantum Gravity or Asymptotic Safety or String theory?

Discreteness of space-time ⟷ non trivial momentum geometry

RL geo ⟷ effective description of the interacting Fixed point

ST is a non local theory ⟷ Relative Locality provides a geometrical framework for understanding T-Duality geometry

# Geometry of Relative locality

This analogy suggests that the proper geometrical setting appropriate to discuss the quantum geometry is Phase space (of a single probe)

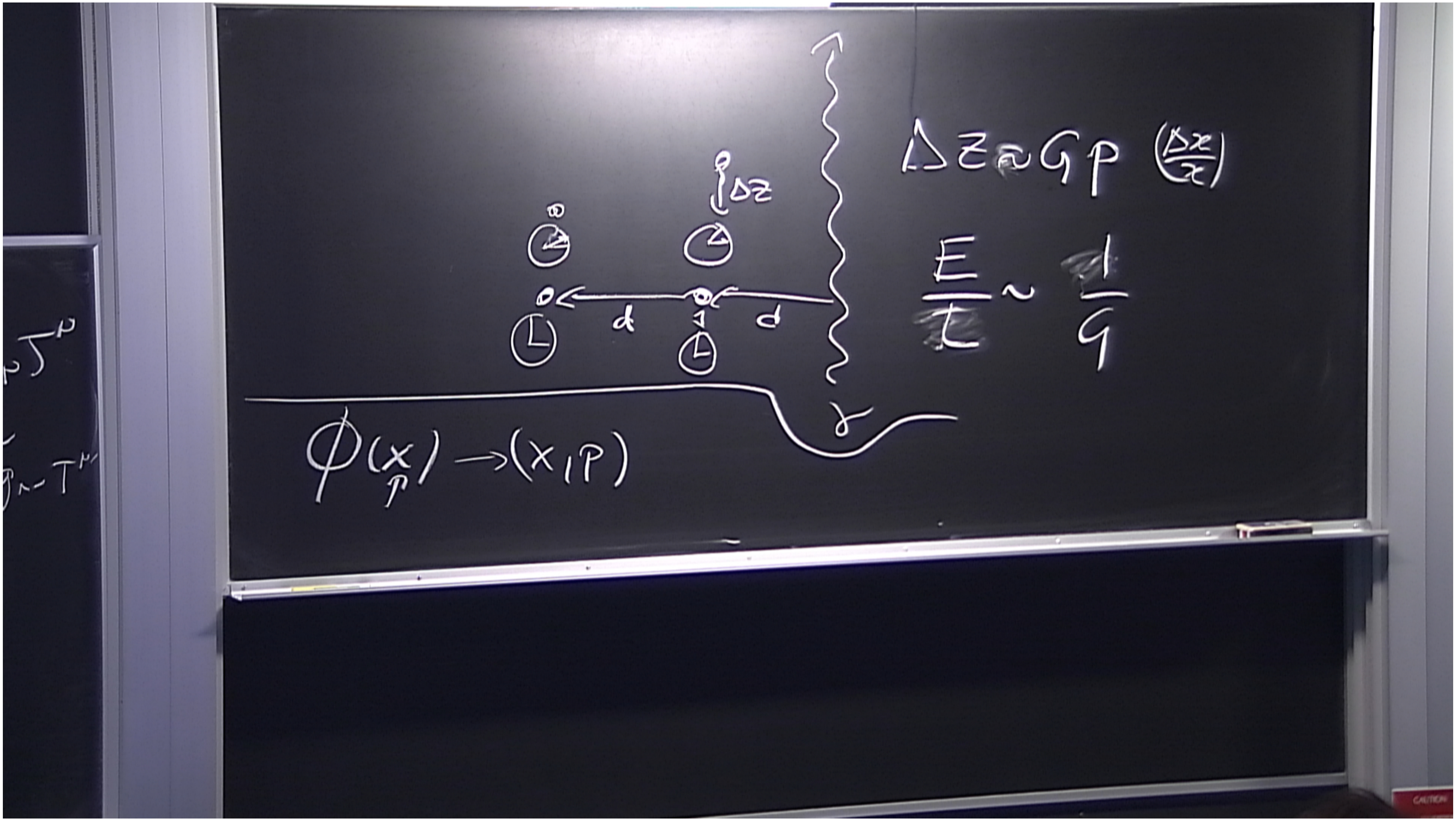
The geometry of spacetime is encoded in a Lorentzian metric.

What is the geometry of the relativistic phase space?

To unify space-time with energy and momenta we need, a fundamental length scale and energy scale

$$\mathbb{X}^A = \begin{pmatrix} x^\mu \\ \tilde{x}_\mu \end{pmatrix} \in \boldsymbol{P} \quad \text{with} \quad \{x, \tilde{x}\} = \frac{1}{2\pi}$$

$$x = \frac{q}{\sqrt{\hbar G}} \qquad \tilde{x} = \frac{p}{2\pi}\sqrt{\frac{G}{\hbar}}$$


ΔZ ≈ Gp (Δx/x)
E/t ~ 1/G
ΔZ
d
d
γ
φ(x, p) → (x, p)

# Non Commutative geometry

The main theme here is that non-commutative geometry should not be understood as a generalization of Riemmanian geometry but as the generalisation of Born geometry

Born Geometry = Phase space geometry.

# Geometry of Phase space I

Phase space $\boldsymbol{P}$ naturally possess 3 natural structures:
A symplectic structure $\omega$ and 2 metrics.
The Quantum metric $H$ and the locality metric $\eta$

$$(\boldsymbol{P}, \omega, H, \eta)$$

Symplectic structure $\omega^T = -\omega$

In Darboux coordinates $\omega_{AB} \mathrm{d}\mathbb{X}^A \mathrm{d}\mathbb{X}^B = \frac{1}{\hbar} \mathrm{d}p_a \wedge \mathrm{d}q^a$

At the quantum level $[x, \tilde{x}] = \frac{i}{2\pi}$

# Geometry of Phase space II $(\boldsymbol{P}, \omega, H, \eta)$

The quantum metric H, needs a conversion factor

Equivalence principle = universality of G

In Darboux coordinates

$$\mathrm{d}s_H^2 = H_{AB}\mathrm{d}\mathbb{X}^A\mathrm{d}\mathbb{X}^B = \frac{1}{\hbar}\left(\frac{\mathrm{d}q^2}{G} + G\mathrm{d}p^2\right)$$

signature $(2, 2(d-1))$

For weakly gravitating objects $G\Delta E << \Delta L$

This metric reduces to the usual spacetime metric.

$$\mathrm{d}s_H^2 \propto \mathrm{d}q^2$$

gravitational tension is huge $\frac{c^2}{G} \sim 10^{17}\,\frac{\mathrm{kg}}{\mathring{A}}$

In relative locality the spacetime metric is the leftover of the quantum metric

# Geometry of Phase space III $(\boldsymbol{P}, \omega, H, \eta)$

The locality metric $\eta$

In Darboux coordinates

$$ds^2_\eta = \eta_{AB} \mathrm{d}\mathbb{X}^A \mathrm{d}\mathbb{X}^B = \frac{2}{\hbar} \mathrm{d}p \mathrm{d}q$$

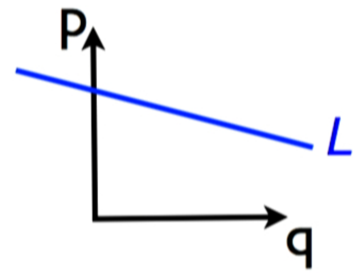
signature $(d, d)$

Vector tangents to spacetime are null with respect to $\eta$
Vector tangents to momentum space are also null wrt $\eta$

It determines a Bilagrangian structure

# Geometry of Phase space

In relative locality space-time is not an absolute notion it is a Lagrangian manifold.

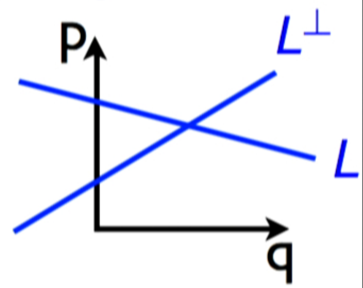



A subset of max dim of P such that $\omega|_L = 0$

Which one?

In order to do physics we need to define what is space-time for probes and what is their momentum space.

Energy-momentum space is a transverse Lagrangian manifold $L^{\perp}$




This defines a Bilagrangian.

$\eta$ uniquely determines a Bilagrangian structure: spacetime and momentum space are null with respect to it

# Geometry of Phase space

Spacetime = null subspace for $\eta$

Absolute locality: P= TM = Flatness of the locality metric

=The splitting between space and momentum space is non dynamical

Relative locality: allowing $\eta$ to be curved.

allowing symmetries to mix p and q

The spacetime metric is the pull back of H on the Lagrangian

$$g_{\mu\nu}(x,p) \equiv H|_L$$

It is a rainbow metric in general

# Geometry of Phase space

Phase space naturally possess 2 metrics.
The Quantum metric $H$ and the polarisation metric $\eta$

$\eta$ defines the splitting space/momentum

$H$ projected on L defines the space-time metric

In QM: $\begin{cases} \eta & \text{is arbitrary} \\ H & \text{is flat} \end{cases}$ ← Born duality

In GR: $\begin{cases} \eta & \text{is fixed and flat} \\ H|_L & \text{is curved} \end{cases}$ ← absolute locality

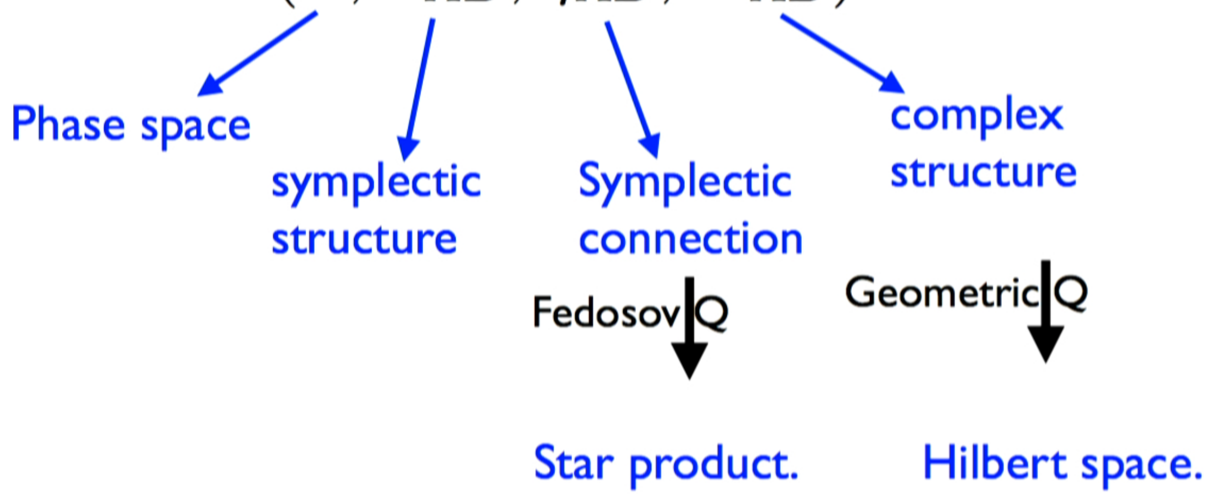
In QG: we expect both to be curved

2 strategies: Quantise GR or Gravitise QM.

# Geometry of Quantization

Remarkably, in the non relativistic case the same structure appears in the geometry of quantization!
Phase space geometry= geometry of quantization

$$(P, \omega_{AB}, \eta_{AB}, H_{AB})$$



# Bilagrangian and Fedosov

In 1992 Fedosov proved a foundational result about quantization. He showed that given a torsionless symplectic connection there exists a non-commutative star product.

$$\nabla\omega = 0$$

$$\nabla \to f * g \qquad f * g - g * f = \frac{\hbar}{i}\{f, g\} + \cdots$$

A choice of torsionless symplectic connection

is uniquely characterized by a Polarisation metric

$$\mathrm{d}s^2_\eta = \mathrm{d}p\mathrm{d}q$$

A polarisation metric determines a choice of operator ordering

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# Meta-string

- String theory provides a natural example of relativistic quantum and non-local probes $\longrightarrow$ a test case for relative locality

Bosonic string theory backgrounds are originally defined as a set of consistent 2-d CFT with c= 26.

- Looking at the space of CFT around the flat one we found that it is bigger than the set of Polyakov sigma models
- In order to explore all CFT and incorporate T-duality we have to introduce phase space as a target of strings and the geometry of relative locality! $(\boldsymbol{P}, \omega, H, \eta)$
- quantum strings propagate in non commutative phase space and the effective fields live in a modular spacetime

# Modular space-time

- Fields are functions on phase space $\Phi(q,p)$
But they are not any function! That would violate causality

Fields on Modular space-time form a commutative sub algebra of the Heisenberg algebra Non-commutative perspective

$$[q,p] = i\hbar$$

Modular variables are quantum observables without classical analog Aharonov

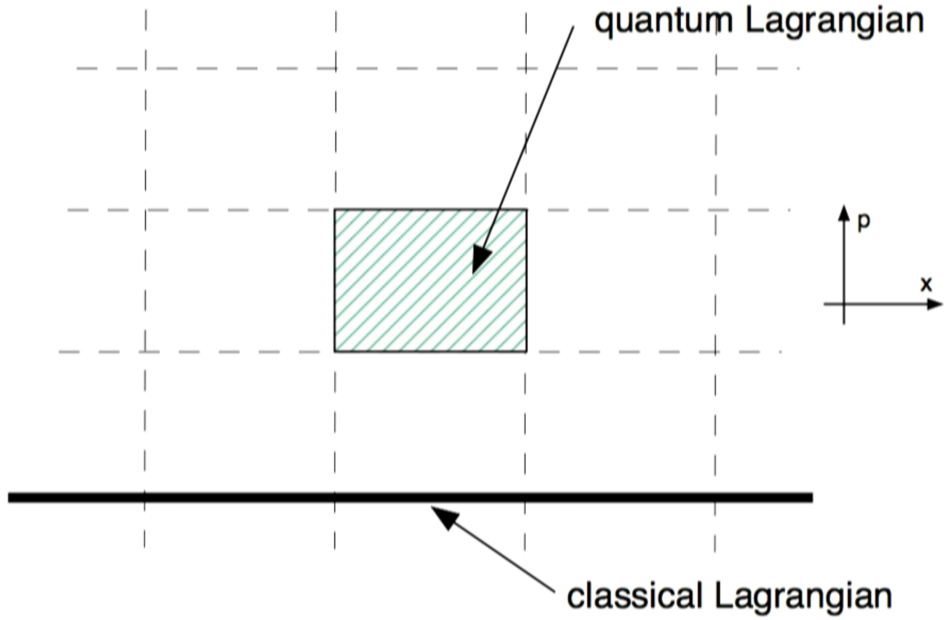
$$[p]_{\frac{h}{R}} = p \bmod \left(\frac{h}{R}\right) \qquad [q]_R = q \bmod (R)$$

They commute quantum mechanically but not classically !

$$[[q],[p]] = 0$$

# Modular space-time

•The effective flat spacetime of the dyonic excitation defined as a commutative algebra of a non commutative one. It captures both geometrical and purely QM elements




Modular spacetime is quantum in essence!

# Fields on modular spacetime

- A modular field satisfy two equations associated with the two metrics H, $\eta$ For massless field they look like

$$\Box_H \Phi = 0 \qquad \Box_\eta \Phi = 0$$

+ interactions local in P.

$\Box_\eta \Phi = \eta^{AB}\partial_A \partial_B = \partial_{p_a}\partial_{q^a}$ One recover usual field when $\partial_p \Phi = ($

Fundamental excitations are not particles: they are dyons whose length is proportional to their momenta


$$P = \eta^{-1} H(\Delta \mathbb{X})$$

Lorentz symmetry is extended $(q,p) \to (\Lambda q, \Lambda^{-1} p)$

$\beta^T = -\beta$ $\qquad (q,p) \to (\alpha q + \beta p, \alpha p + \beta q)$

Bogolioubov like mixing of space and momenta

# Main prediction

$$\Box_H \Phi = 0 \qquad \Box_\eta \Phi = 0$$

Dyons whose length is proportional to their momenta


Extended Lorentz symmetry mixing space and momenta

One of the main prediction of this new approach is the fact that separation of scale as we know it is no longer tenable

Their is a fundamental UV-IR mixing in fundamental physics

Effective field theory does not survive, a new locality notion, modular locality is needed.

# Epilogue

•"We all agree your theory is crazy. The question which divides us is whether it is crazy enough to have a chance of being correct."

Neils Bohr to Pauli