Title: TBA

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URL: http://pirsa.org/15090062

Abstract:

What happens, operationally, as one approaches the Planck Scale?

Resolve a distance more and more precisely.

=> **increasing momentum uncertainty,**

=> **increasing curvature uncertainty,**

=> **increasing distance uncertainty.**

➔ **Cannot resolve distances below** 10^(-35)m.

What is the structure of spacetime ?

Paradox:

General relativity:

- Fields live on a differentiable spacetime manifold.

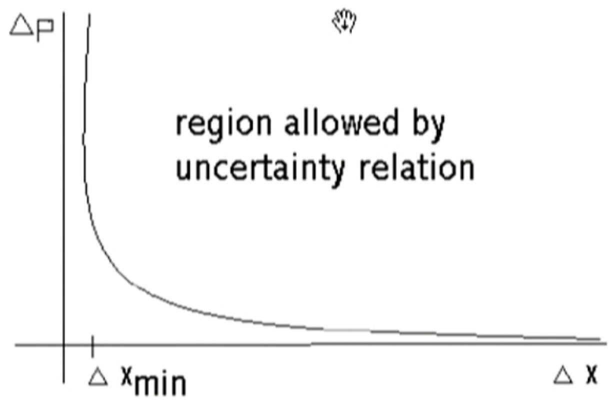
Quantum field theory:

- QFT generally only well defined if spacetime is discrete.

Possible resolution

Studies in quantum gravity and string theory

=>

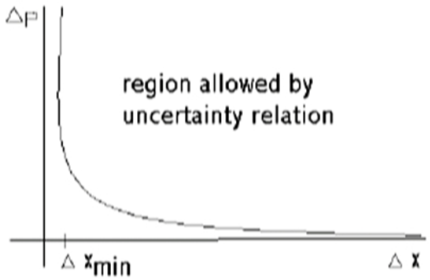


Arises also from Quantum Groups / NC Geometry

A. Kempf, J. Math. Phys. **35**, 4483 (1994)

- Quantum group $SU_q(n)$-symmetric CCRs:

$$\begin{aligned} a_i a_j - q a_j a_i &= 0 \quad \text{for} \quad i < j \\ a_i^\dagger a_j^\dagger - q a_j^\dagger a_i^\dagger &= 0 \quad \text{for} \quad i > j \\ a_i a_j^\dagger - q a_j^\dagger a_i &= 0 \quad \text{for} \quad i \neq j \\ a_i a_i^\dagger - q^2 a_i^\dagger a_i &= 1 + (q^2 - 1) \sum_{j<i} a_j^\dagger a_j \end{aligned}$$

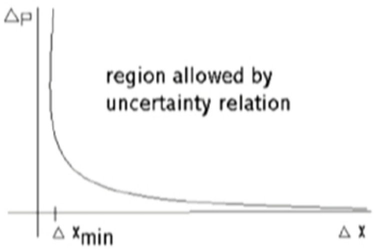


$$x_r := L_r(a_r^\dagger + a_r) \qquad \text{and} \qquad p_r := iK_r(a_r^\dagger - a_r)$$

- For example, in 1 dimension this reduces to:

$$[x, p] = i\hbar + i\hbar(q^2 - 1)\left(\frac{x^2}{4L^2} + \frac{p^2}{4K^2}\right)$$

- Notice: Compatible with quantum group and other, including conventional symmetries.
- **So what is really the math underlying this limited localizability?** →→→



➔ ***Fields must possess a finite bandwidth !***

(the other direction is clear)

Bandlimitation is a central concept in information theory!

So … ?

➔ **Spacetime can be both discrete *and* continuous,**

in the same mathematical way that information is.

D. Aasen, T. Bhamre, A. Kempf, Phys. Rev. Lett. **110**, 121301 (2013)
A. Kempf, Phys. Rev. Lett. **103**, 231301 (2009)
A. Kempf, R. Martin, Phys. Rev. Lett. **100**, 021304 (2008)
A. Kempf, Phys. Rev. Lett. **92**, 221301 (2004)
A. Kempf, Phys. Rev. Lett. **85**, 2873 (2000)

Rôle of bandlimitation in information theory ?

Information can be:

- continuous (e.g., music):

- discrete (letters, digits, etc): R 7 2 S B

Unified in 1949 by Shannon, through: <u>Sampling theory</u>

Applications ubiquitous:

- communication engineering & signal processing
- scientific data taking, e.g., in astronomy.

Shannon's sampling theorem

- Assume f is bandlimited, i.e:

$$f(x) = \int_{-\omega_{\max}}^{\omega_{\max}} \tilde{f}(\omega)\, e^{-2\pi i \omega x}\, d\omega$$

- Take samples of f(x) at Nyquist rate:

$$x_{n+1} - x_n = (2\omega_{\max})^{-1}$$

- **Then, exact reconstruction is possible:**

samples

$$f(x) = \sum_n f(x_n)\, \frac{\sin[\,2\pi\,(x - x_n)\,\omega_{\max}\,]}{\pi\,(x - x_n)\,\omega_{\max}}$$

Properties of bandlimited functions

- **Differential operators are also finite difference operators.**
- **Differential equations are also finite difference equations.**
- **Integrals are also series:**

$$\int_{-\infty}^{\infty} f(x)^* g(x)\, dx = \frac{1}{2\omega_{\max}} \sum_{n=-\infty}^{\infty} f(x_n)^* g(x_n)$$

Remark:

Useful also as a summation tool for series

(traditionally used, e.g., in analytic number theory)

Covariant "bandlimitation" ?

Cut off of the spectrum of

the Laplacian or d'Alembertian.

$$Z[J] = \int_F e^{iS[\phi]+i\int J\phi\ d^nx} D[\phi]$$

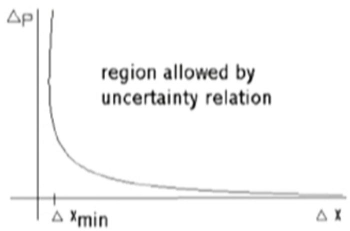
➔The space of fields, F, in the QFT path integral

is spanned by the eigenfunctions w. eigenvalues:

$$\lambda_i < \Lambda$$

What if physical fields are "bandlimited"?

Fields possess equivalent representations



- **on a differentiable spacetime manifold**

  (which shows preservation of external symmetries)

- **on any lattice of sufficiently dense spacing**

  (which shows UV finiteness of QFTs).

Density of degrees of freedom ?

For euclidean QFT, we can use [Gilkey 1975]:

Consider any compact 4-dim Riemannian manifold. Then:

$$N = \frac{1}{16\pi^2}\int d^4x\sqrt{|g|}\ \left\{\frac{\Lambda^2}{2} + \frac{\Lambda}{6}R + O(R^2, \Lambda^{-1})\right\}$$

Can now read off:

- Cosmological constant is density of DOF: $\frac{N}{V} = \frac{\Lambda^2}{32\pi}$
- Curvature is local perturbation of density of DOF.

Re-expressing the Einstein action

=> Einstein action takes the simple form:

$$S = \frac{6\pi}{\Lambda}\frac{1}{16\pi^2}\int d^4x\sqrt{|g|}\ \left\{\frac{\Lambda^2}{2} + \frac{\Lambda}{6}R + O(R^2,\Lambda^{-1})\right\}$$

$$= \frac{6\pi}{\Lambda}N$$

$$= \frac{6\pi}{\Lambda}\,Tr(1)$$

Notice: The Einstein action is the integral over the density of degrees of freedom, where the cosmological constant sets the baseline, modulated by curvature.

Matter actions are traces as well

E.g.:

$$S_{matter} = \int d^n x \sqrt{|g|} \; \tfrac{1}{2}\phi(x)(\Delta + m^2)\phi(x)$$

$$= \sum_{i=1}^{N} \tfrac{1}{2}\phi_i(\lambda_i + m^2)\phi_i$$

$$= Tr[\; \tfrac{1}{2}(\Delta + m^2)\,|\,\phi)(\phi\,|\;]$$

Actions are traces, and gravity could be a leading constant.

But what is bandlimitation for spacetime itself ?

Is there a Shannon-like reconstruction of space from discrete sets of samples?

With bandwidth / min uncertainty cutoff,

what could supercede rulers and clocks ?

Idea:
Noise correlator as proxy for distance

- Quantum field correlators indicate distance.

- Does the entanglement structure of the vacuum encode spacetime's curvature ?

Idea: correlators as proxy for distances

At N points x_i of a finite piece of the manifold,

sample the propagator's matrix elements:

$$< x_a \,|\, 1/\Delta \,|\, x_b >$$

- Work w. Aslanbeigi and Saravani: One can reconstruct the metric.
- Basis independent information ➔ eigenvalues of Δ .

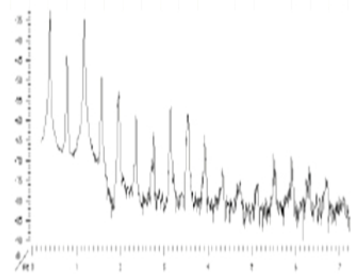
•Does the spectrum tell the shape?

Spectral Geometry:

- " How far is shape determined by sound ? "

$$-d^2\phi/dt^2 = \Delta_g \phi$$

⇔

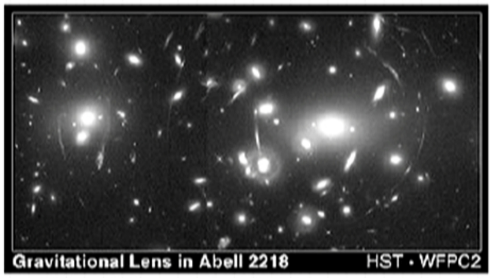


(M,g) spec(Δ)

There are some positive results, e.g., on shapes of revolution!

Prospect:

Can one hear a spacetime's curvature in its quantum noise?

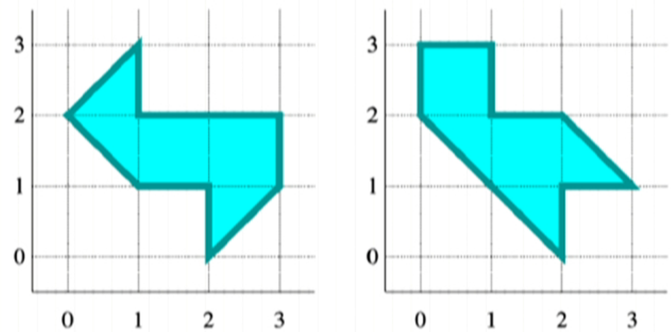


⇔

Deep link between gravity and quantum theory?

Problem !

- **Spectral geometry has counter examples !**
- Work by Milnor, Sunada, Gordon …



Give up now?

No, we seem so close !

Let's try to fix spectral geometry !

A fresh (and more modest) start for spectral geometry

Notice: *Spectral geometry is hard due to nonlinearity*

Idea: *Try infinitesimal spectral geometry*

Perturbations of the shape

Describe small shape changes, e.g., through a function f :

$$g_{\mu\nu}(x) \to (1 + f(x))\, g_{\mu\nu}(x)$$

Then expand f in the eigenbasis $\{b_n(x)\}$ of the Laplacian Δ:

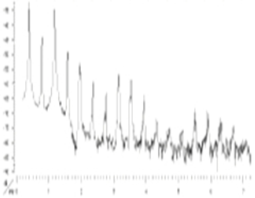
$$f = \Sigma_n\, f_n\, b_n(x)$$

–> Shape changes described by: $\{f_n\}$

Perturbations of the spectrum

Corresponding small change of spectrum

$$\lambda_n \rightarrow \lambda_n + \mu_n$$



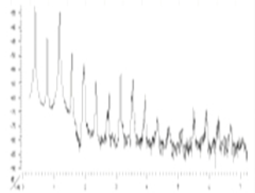
is described by coefficients $\{\mu_n\}$.

➔ There exists a matrix S: $\mu_n = \sum_m S_{nm} f_m$

Calculate S in: $\mu_n = \Sigma_m S_{nm} f_m$

We have $\Delta b_n = \lambda_n b_n$

Now when changing the shape:



$g_{\mu\nu}(x) \rightarrow (1+f(x))\, g_{\mu\nu}(x)$

$\Delta \rightarrow \Delta + \Sigma f_n \Delta'_n$

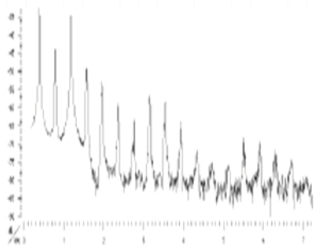
$\lambda_n \rightarrow \lambda_n + \mu_n$ with $\mu_n = (b_n, \Sigma f_m \Delta'_m b_n)$

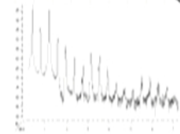
Thus: $S_{nm} = (b_n, \Delta'_m b_n)$ Is S invertible?

Finite vision and finite hearing

Consider geometry and acoustics only up to a cutoff scale:

Cut off spectrum of Δ at some N.





Counting dimensions

\# of eigenvalues = # of eigenfunctions

- $\{f_n\}$ and $\{\mu_n\}$ both now have N coefficients.
- $\{S_{nm}\}$ is a generic square NxN matrix,

  which should generically be invertible.

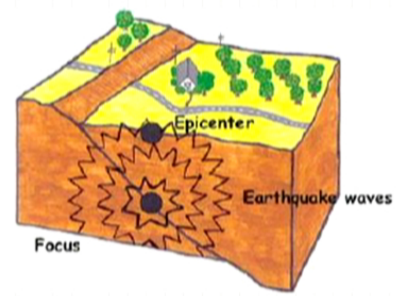
➔ This should work !
But weren't there counter examples ?

We missed tensor perturbations !

In dimensions d>2, not every perturbation of a Riemannian manifold can be described by a scalar function *f*.

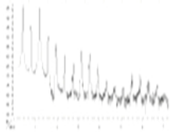
$$g_{\mu\nu}(x) \rightarrow (1+f(x))\, g_{\mu\nu}(x)$$

Need to use scalar, vector and tensor perturbations:



$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \delta s_{\mu\nu}(x) + \delta v_{\mu\nu}(x) + \delta h_{\mu\nu}(x)$$

(Seismic waves of different types carry independent information too)



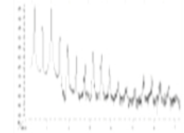
Infinitesimal spectral geometry

Consider the change of spectra of scalar, vector and tensor Laplacians: (Δ on symmetric tensors not unique)

➔ # of shape & sound dimensions matches again!

➔ Yes, one may be able to hear small shape changes – when listening to the spectra of *all* types of waves.

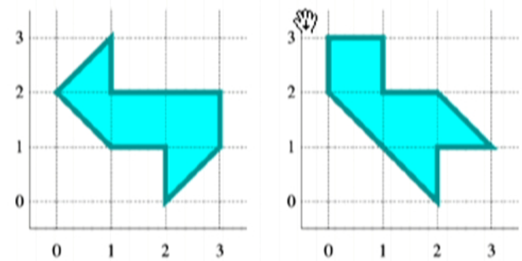
Key challenges:



- From 2 to 4 -dimensional compact Riemannian manifolds
  - Use the spectra of Laplacians on 0,1, 2 tensors
  - Use the Dirac operator for fermions (and relate to Connes')
- From infinitesimal to finite spectral geometry
- Lorentzian signature

Work with Mikhail Panine (UW)

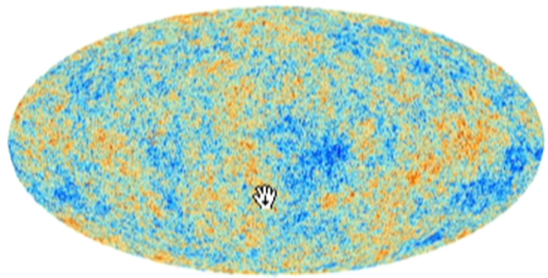
- We showed that infinitesimal spectral geometry can be applied even to the spectral geometry of planar domains !



- See the next talk

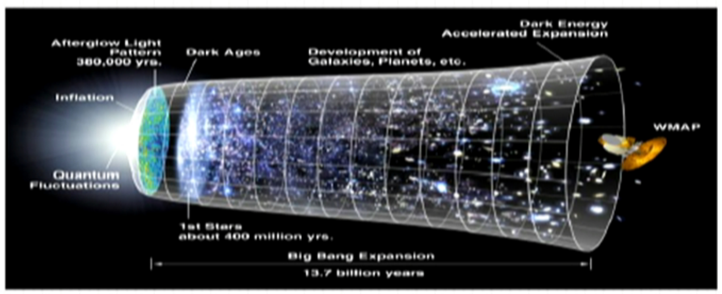
Experimental predictions ?

CMB is closest to Planck scale

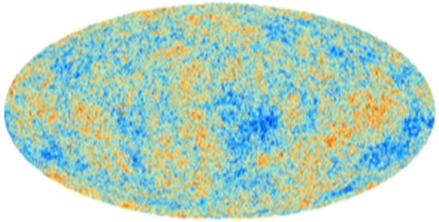


Why?

Hubble scale in inflation only about 5 orders from Planck scale.



Applied to cosmology



Multiple groups have non-covariant predictions for CMB.

- Characteristic, O(10^-5) or O(10^-10) modulations
- Characteristic deviation from scalar/tensor consistency relation in B-polarization data.

Big challenge:

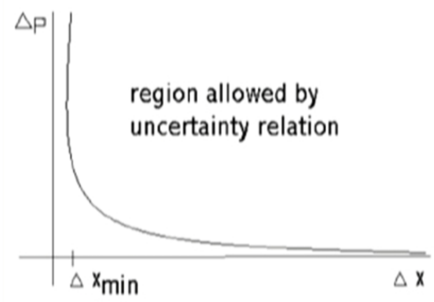
Predictions with local Lorentz covariant bandlimit cutoff!

Upcoming work with:

Aidan Chatwin-Davies (CalTech) and Robert Martin (U. Cape Town)

Summary

- Philosophy: Only information theoretic concepts may survive at Planck scale



- Found: Spacetime may be bandlimited

- Thus discrete = continuous

  for spacetime, as is the case for information.

- Notion of distance replaced by info-theoretic notion of correlation.

- Key challenge: make it all Lorentz-covariant