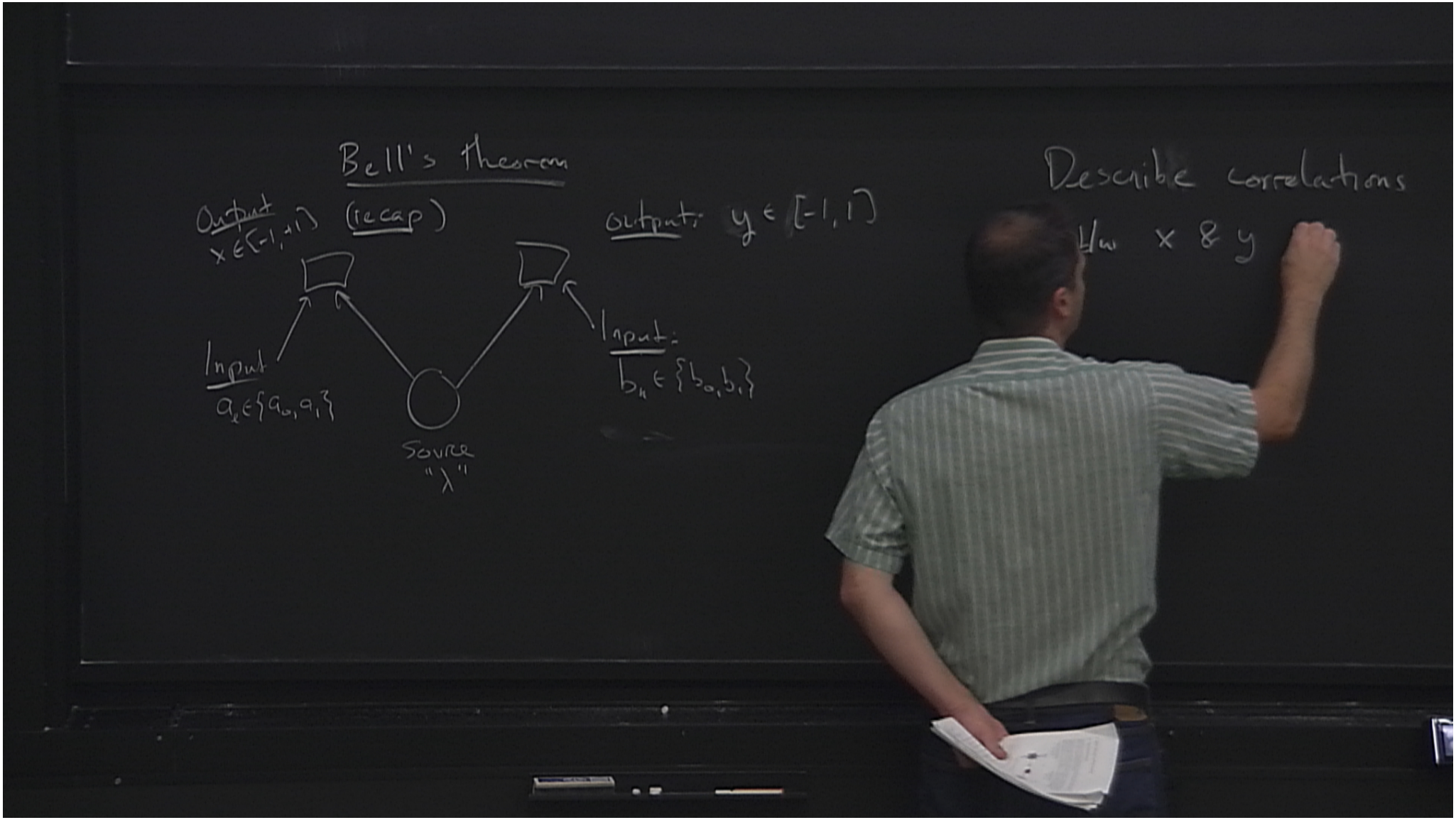
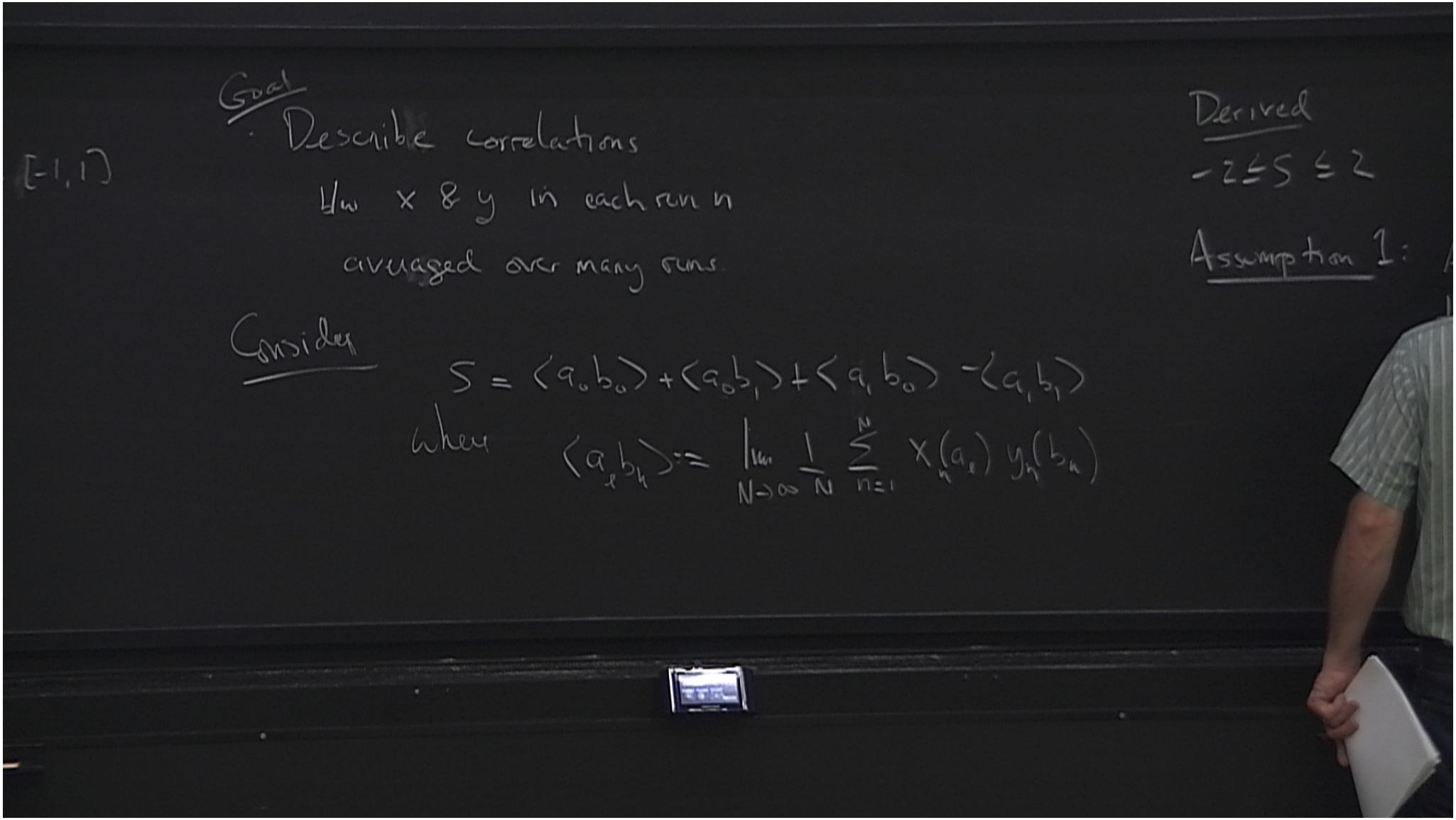
Title: PSI 2015/2016 Quantum Theory - Lecture 13

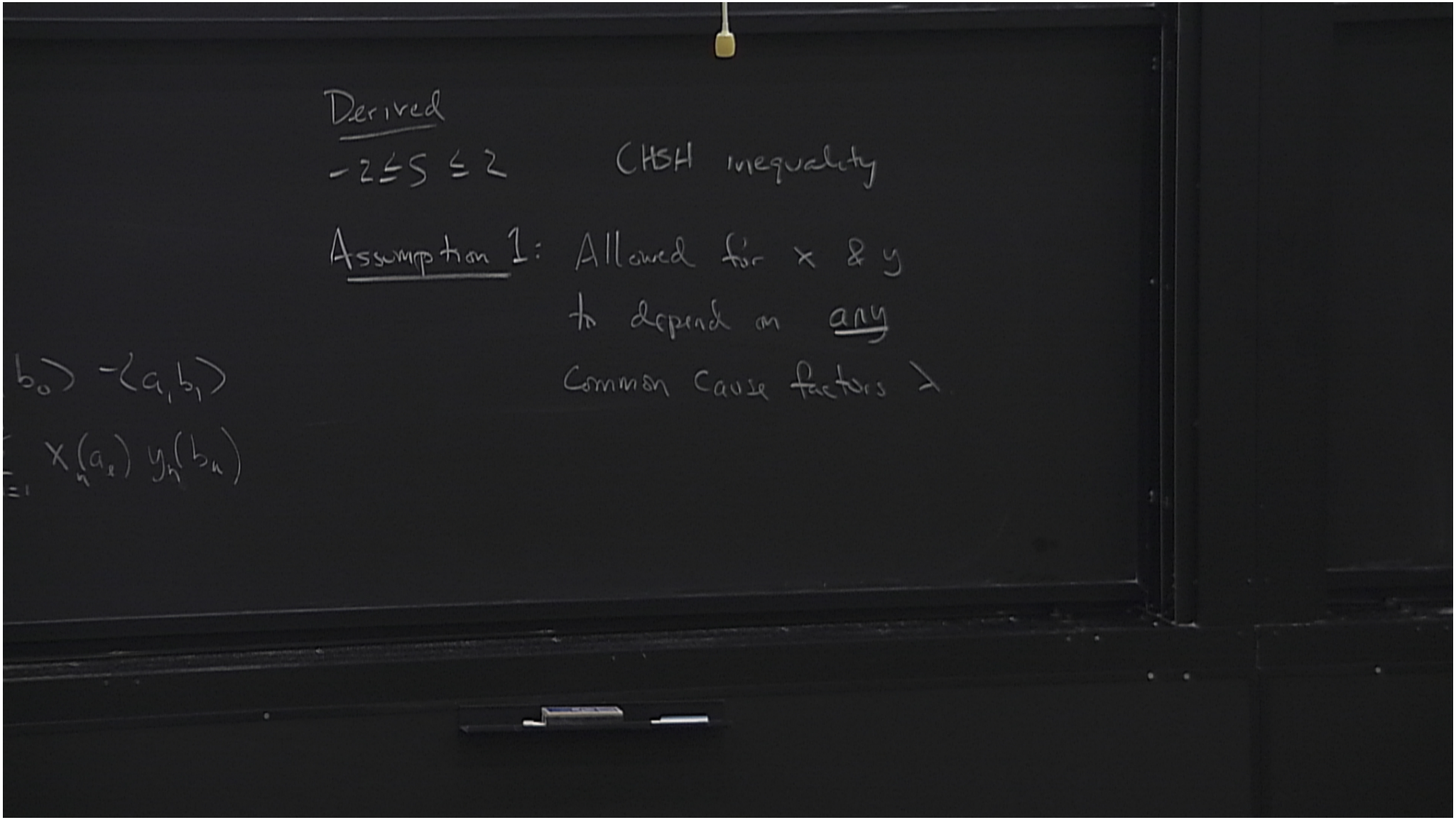
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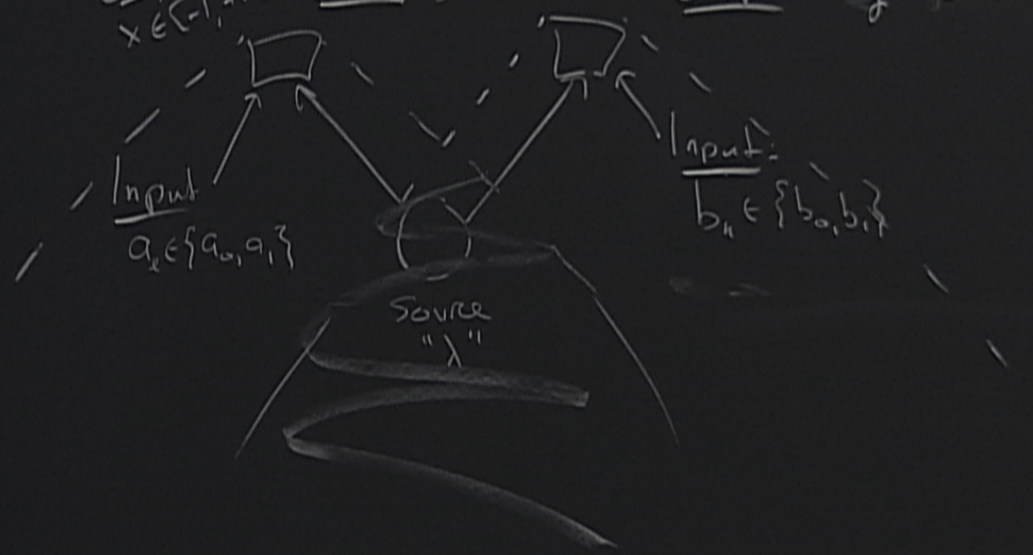
Abstract:


Bell's theorem
(recap)
Output
$x \in \{-1, +1\}$
Input
$a_x \in \{a_0, a_1\}$
Source
"λ"
output: $y \in \{-1, 1\}$
Input:
$b_x \in \{b_0, b_1\}$
Describe correlations
b/w x & y


Goal
Describe correlations
Hw x & y in each run n
averaged over many runs
[-1,1]
Derived
$-2 \leq S \leq 2$
Assumption 1:
Consider
$S = \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle$
where
$\langle a_s b_t \rangle := \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} x_n(a_s)\, y_n(b_t)$


Derived
$-2 \leq S \leq 2$ CHSH inequality
Assumption 1: Allowed for x & y
to depend on any
common cause factors λ
$b_0\rangle - \langle a_1 b_1\rangle$
$X_n(a_x)\, y_n(b_y)$

Bell's theorem



Goal

Describe correlations

b/w x & y in each ru

averaged over many ru

Consider

$S = \langle a_0 b_0 \rangle +$

when $\langle a_x b_y \rangle =$

s

h run n

y runs

$\rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle$

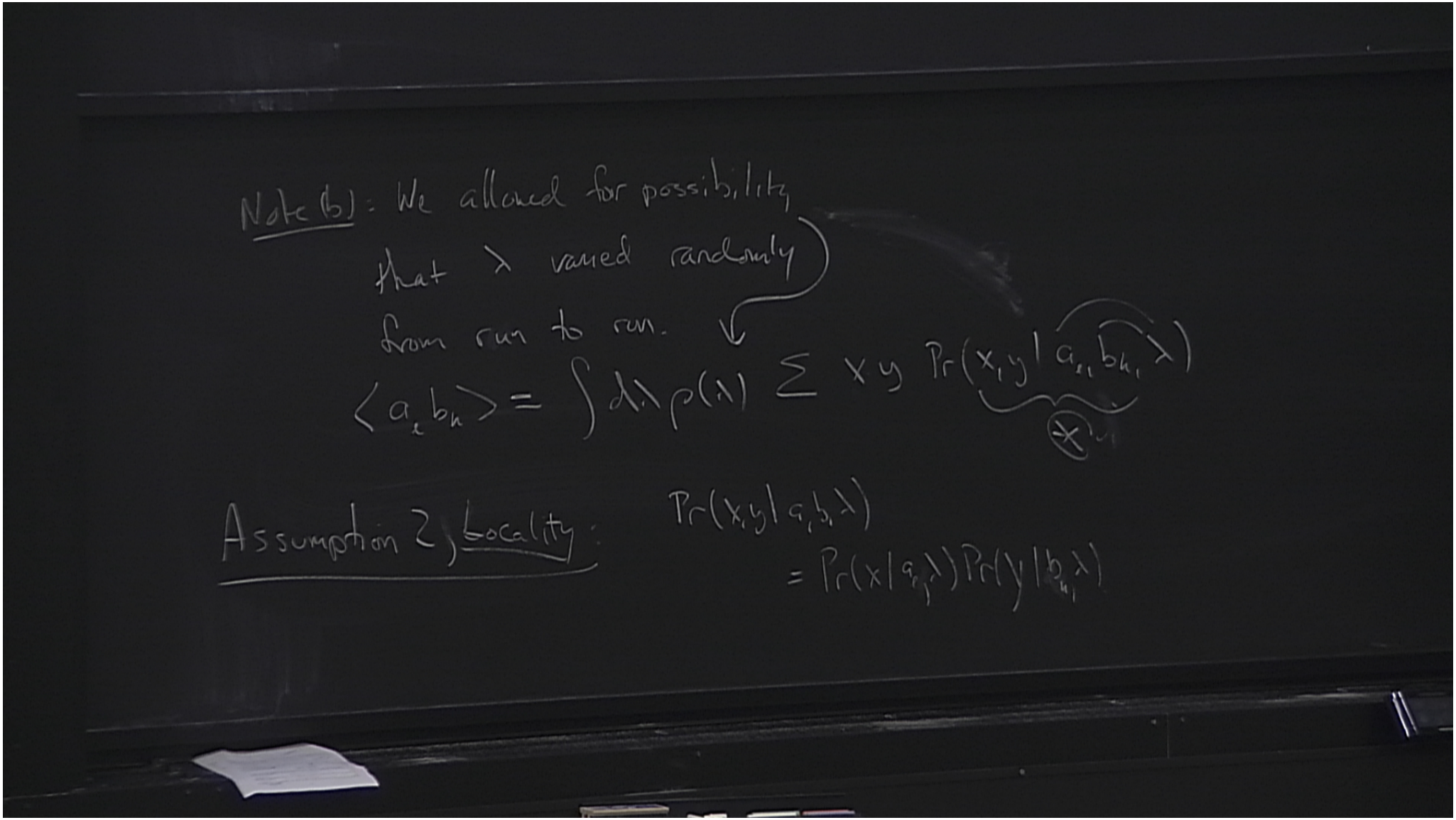
$b_n \rangle := \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} X(a_x, \lambda_n)\, Y(b_y, \lambda_n)$

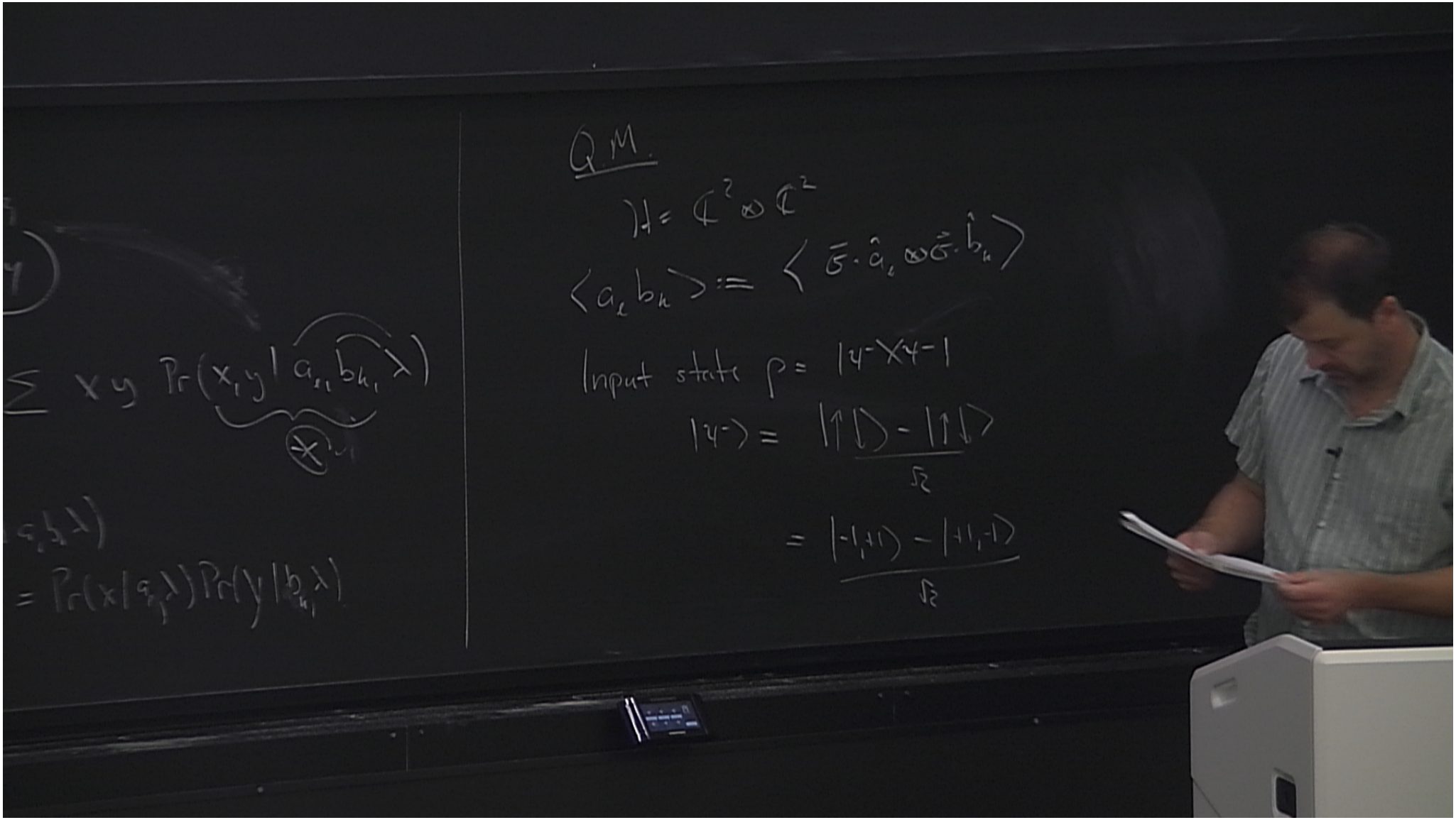
Derived

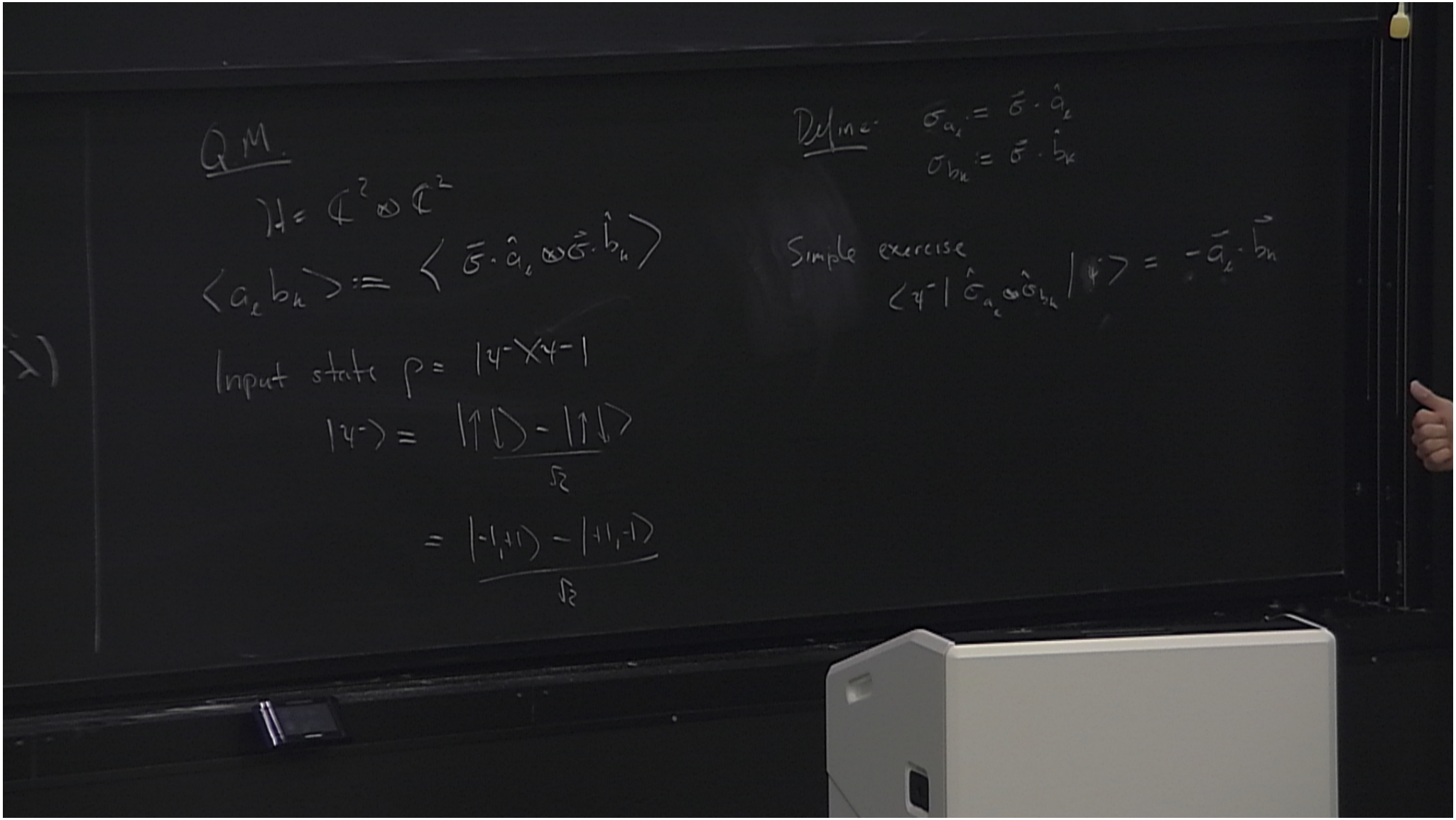
$-2 \leq S \leq 2$ CHSH inequality

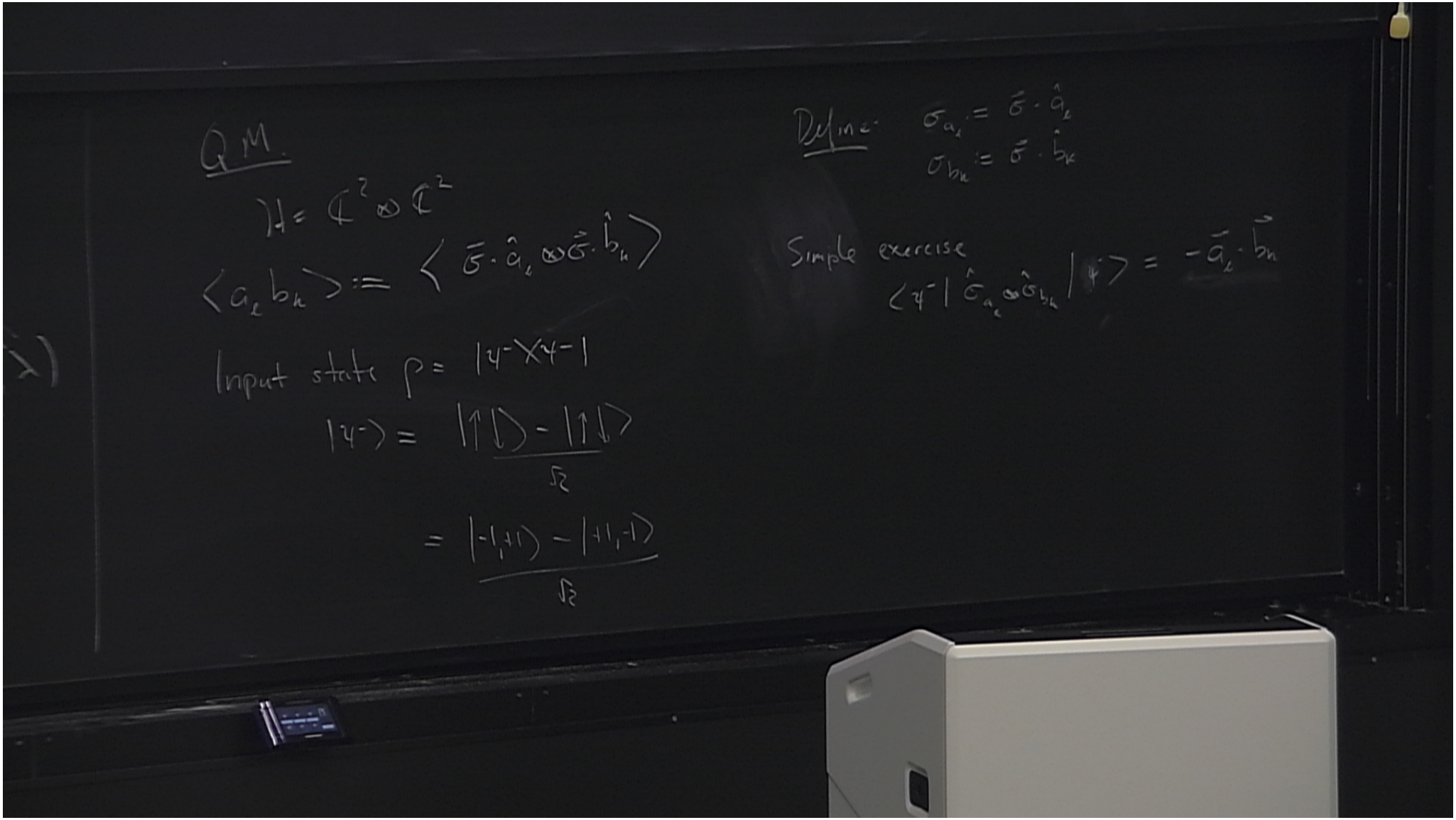
Assumption 1: "Realism" Allowed for x & y to depend on any common cause factors λ

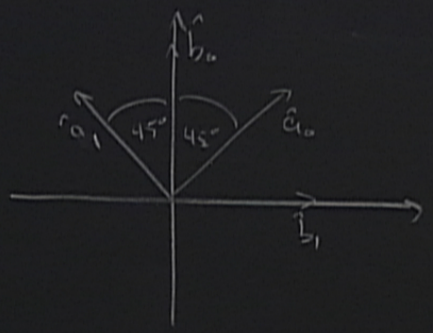
Note: we allowed for only a probabilistic dependence of x & y on λ


Note (b): We allowed for possibility that λ varied randomly from run to run.
$\langle a_x b_y \rangle = \int d\lambda\, \rho(\lambda) \sum xy \Pr(x,y \mid a_x, b_y, \lambda)$
Assumption 2) Locality:
$\Pr(x,y \mid a_x, b_y, \lambda)$
$= \Pr(x \mid a_x, \lambda) \Pr(y \mid b_y, \lambda)$


QM
$\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$
$\langle a_i b_k \rangle := \langle \vec{\sigma} \cdot \hat{a}_i \otimes \vec{\sigma} \cdot \hat{b}_k \rangle$
Input state $\rho = |\psi^-\rangle\langle\psi^-|$
$|\psi^-\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$
$= \frac{|-1,+1\rangle - |+1,-1\rangle}{\sqrt{2}}$
$\sum xy \Pr(x,y|a_i,b_k,\lambda)$
$= \Pr(x|a_i,\lambda)\Pr(y|b_k,\lambda)$

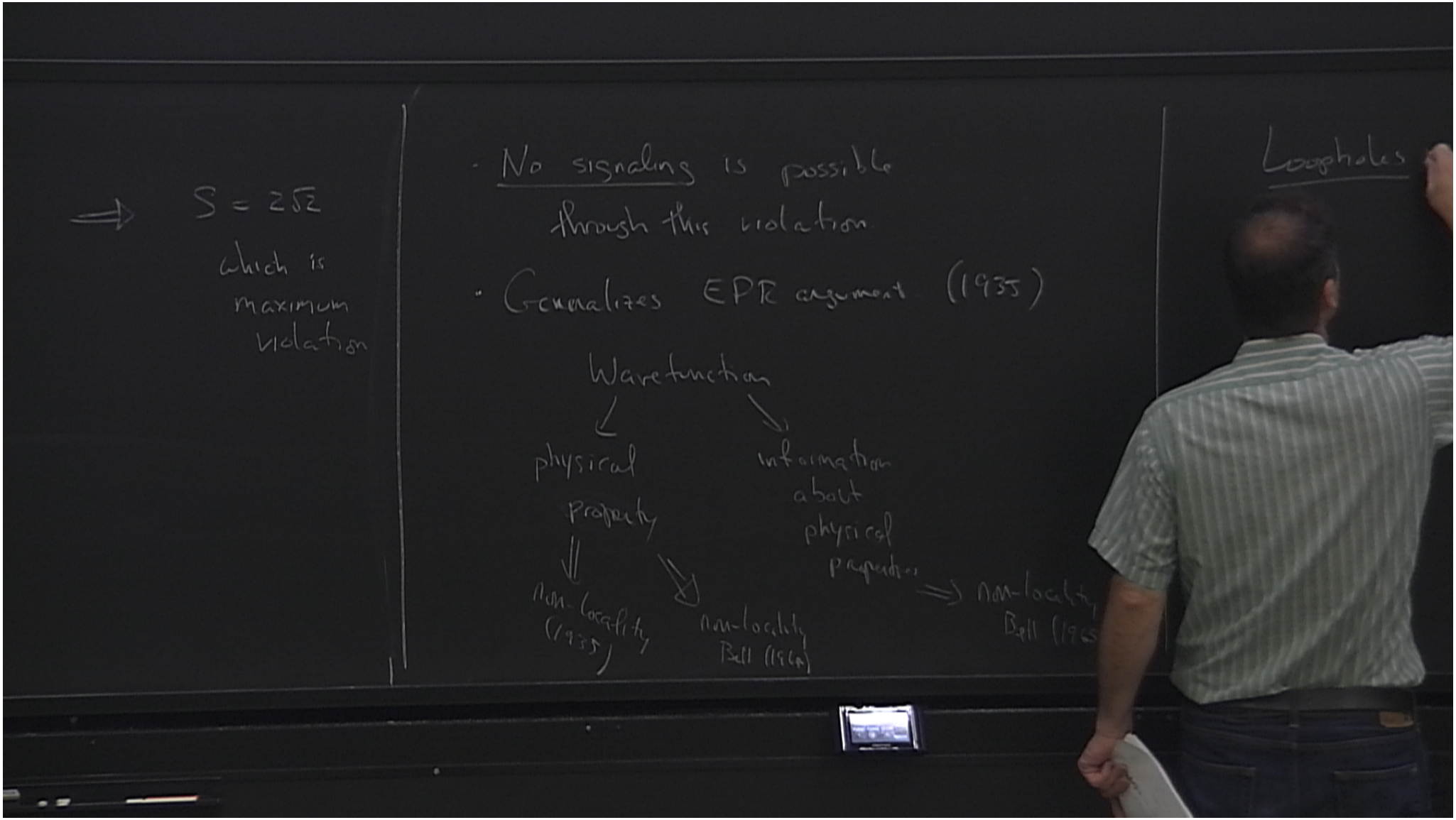

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$|\psi^-\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$
$= \frac{|-1,+1\rangle - |+1,-1\rangle}{\sqrt{2}}$
Define: $\sigma_{a_i} := \vec{\sigma} \cdot \hat{a}_i$
$\sigma_{b_k} := \vec{\sigma} \cdot \hat{b}_k$
Simple exercise
$\langle \psi^- | \hat{\sigma}_{a_i} \otimes \hat{\sigma}_{b_k} | \psi^- \rangle = -\vec{a}_i \cdot \vec{b}_k$

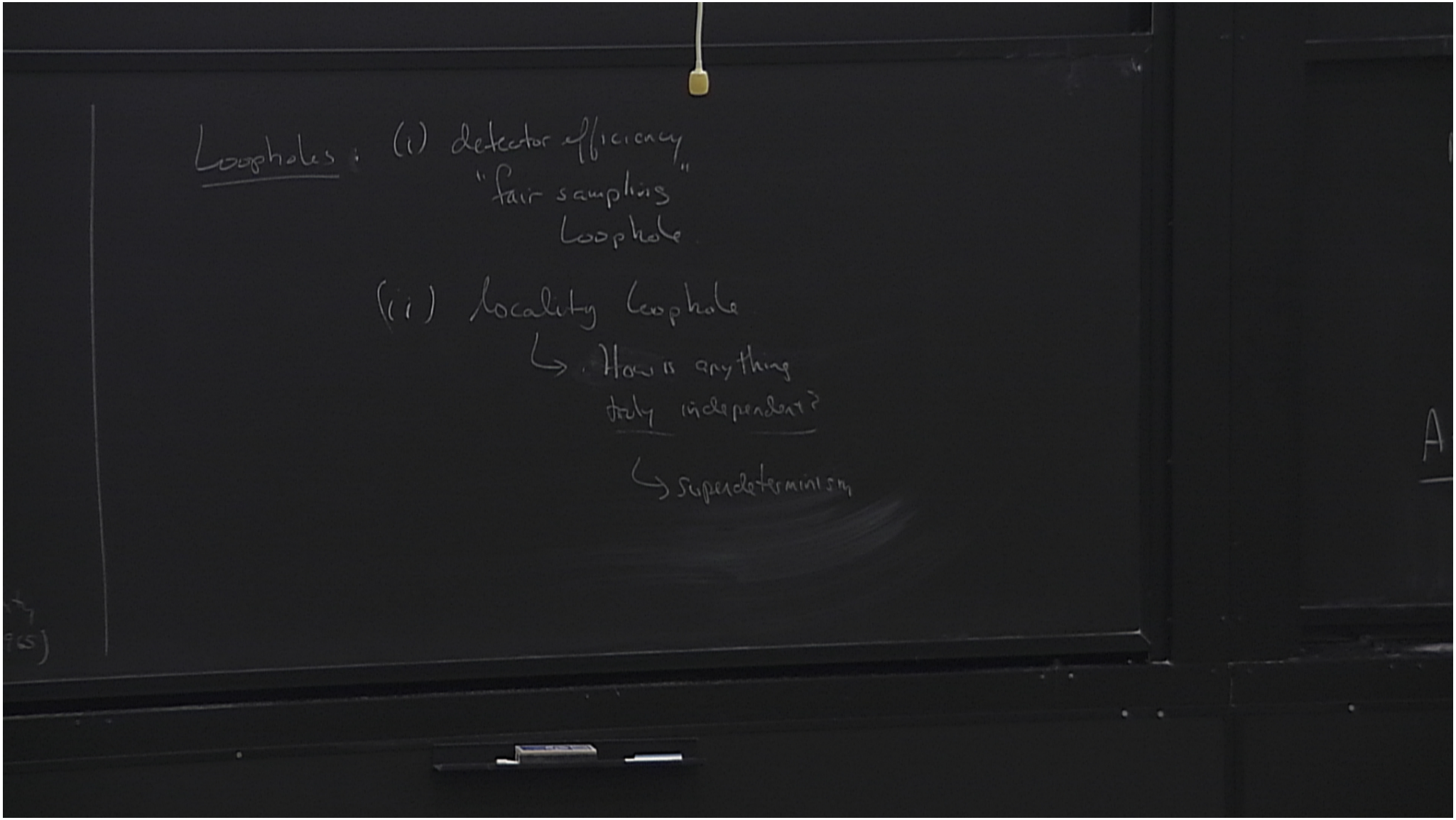

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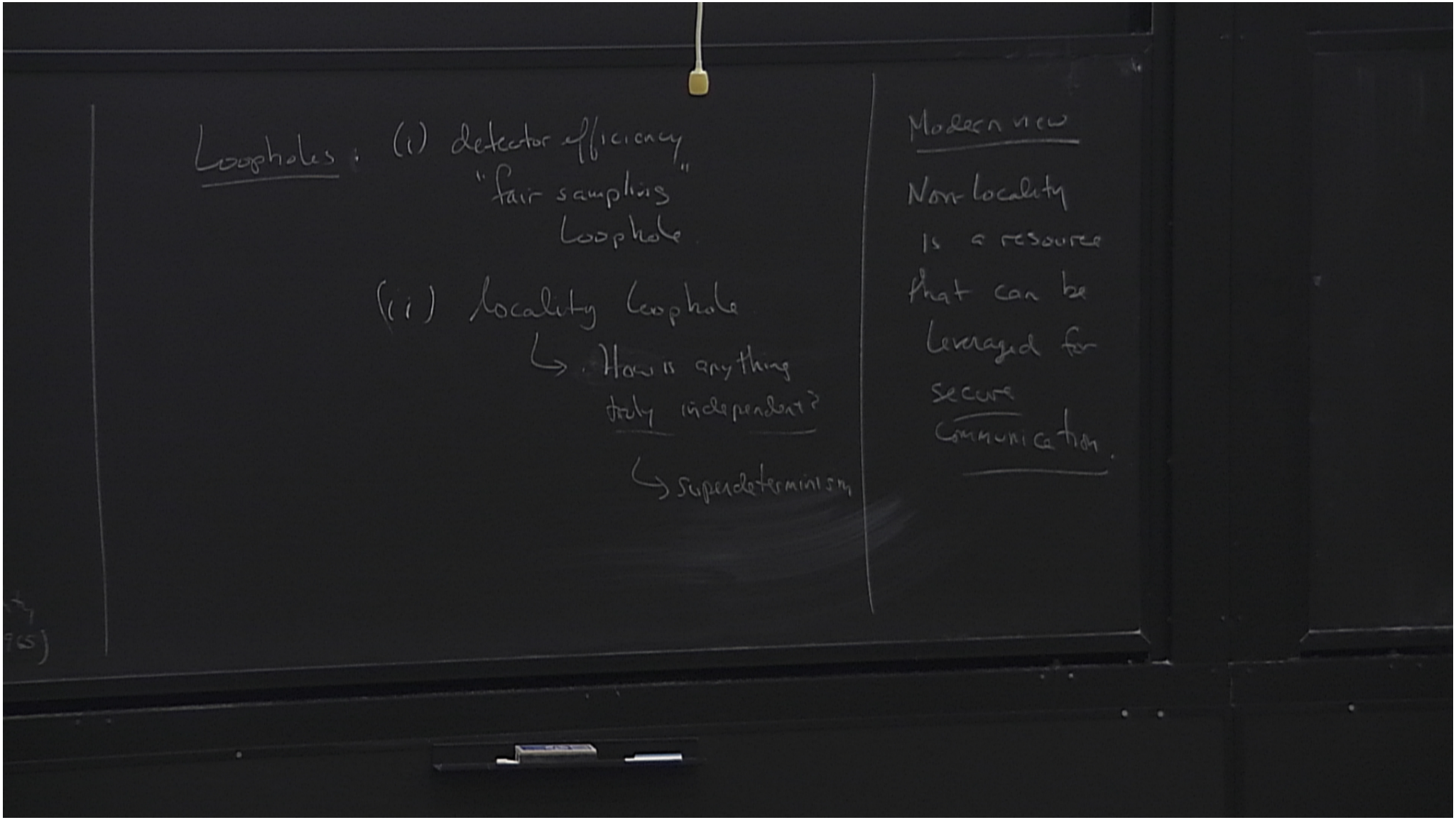


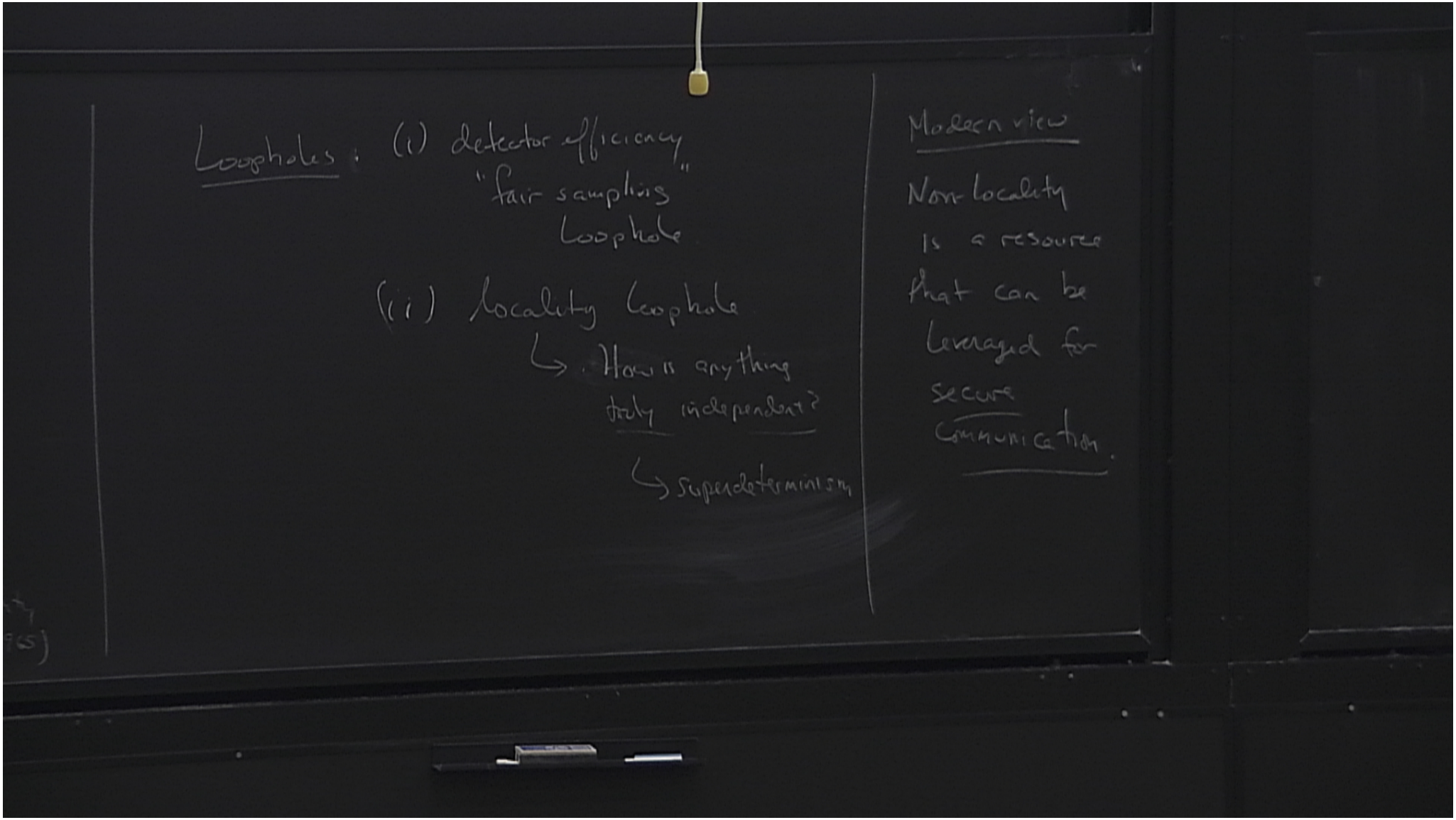
$\Rightarrow$ $S = 2\sqrt{2}$

which is maximum violation


$\Rightarrow S = 2\sqrt{2}$
which is maximum violation
· No signaling is possible through this violation
· Generalizes EPR argument (1935)
Wavefunction
physical property
information about physical properties
non-locality (1935)
non-locality Bell (1964)
non-locality Bell (19
Loopholes

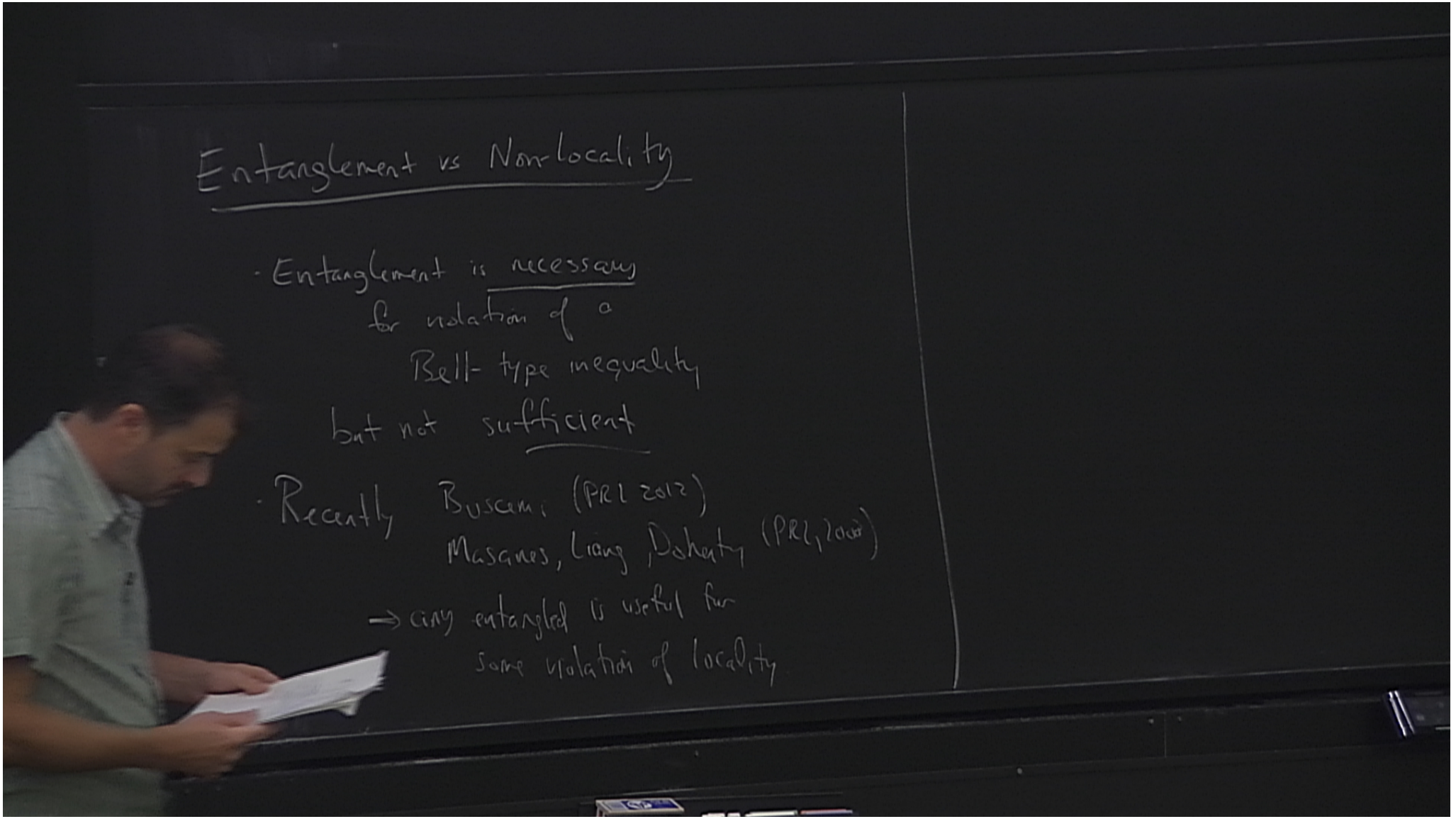

Loopholes: (i) detector efficiency
"fair sampling"
loophole
(ii) locality loophole
↳ How is anything
truly independent?
↳ superdeterminism

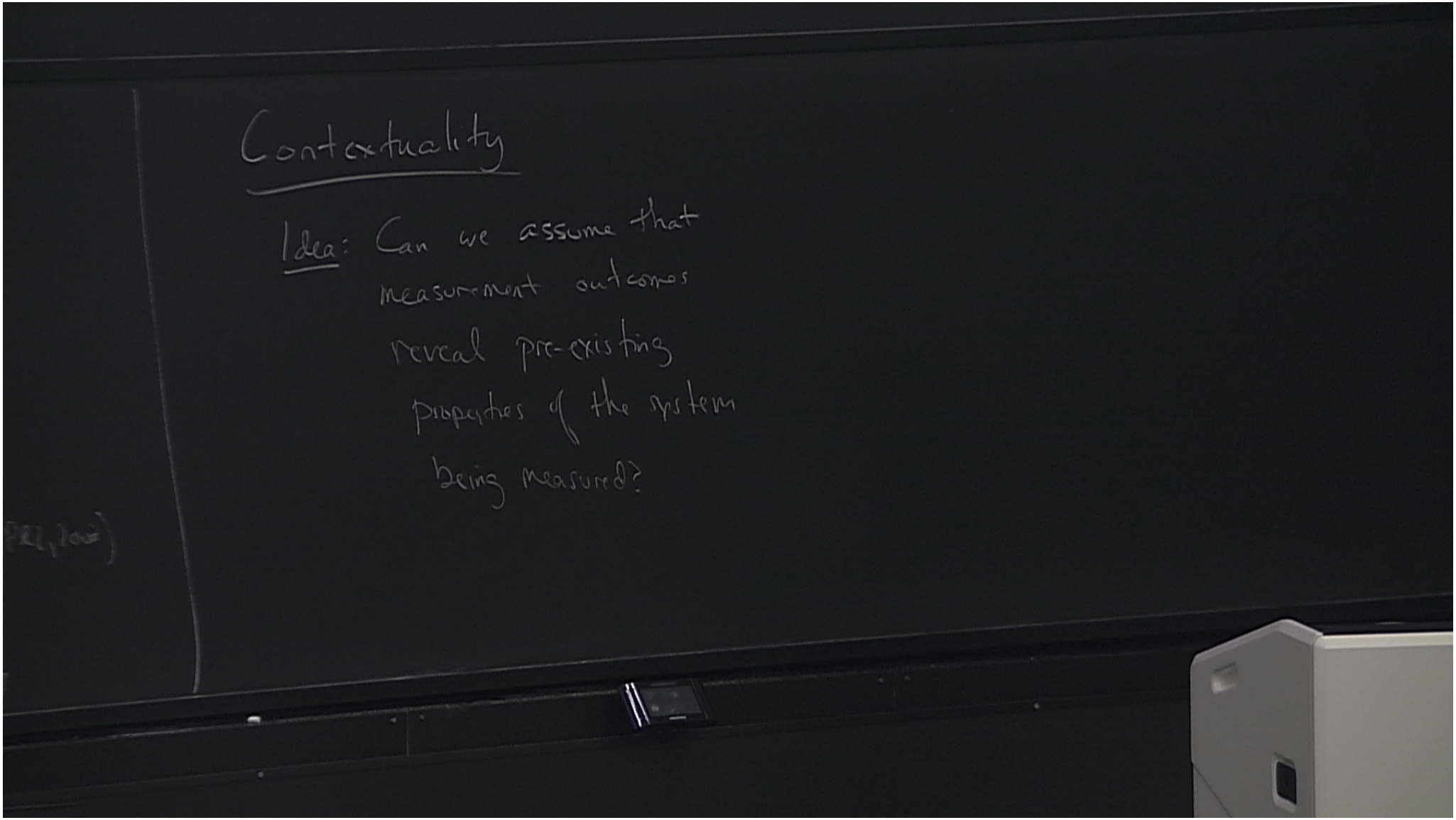

Loopholes: (i) detector efficiency
"fair sampling"
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↳ How is anything truly independent?
↳ superdeterminism
Modern view
Non-locality is a resource that can be leveraged for secure communication.


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Non-locality is a resource that can be leveraged for secure communication.

Entanglement vs Non-locality

- Entanglement is necessary for violation of a Bell-type inequality but not sufficient
- Recently Buscemi (PRL 2012)
  Masanes, Liang, Doherty (PRL, 2008)
  $\Rightarrow$ any entangled is useful for some violation of locality


Entanglement vs Non-locality
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· Recently Buscemi (PRL 2012)
Masanes, Liang, Doherty (PRL, 2008)
⇒ any entangled is useful for
some violation of locality


Contextuality
Idea: Can we assume that
measurement outcomes
reveal pre-existing
properties of the system
being measured?

Bell (1964)

Consider $\mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$, $\mathcal{H}^A \otimes \mathcal{H}^B$

$$\begin{array}{ccc} v(\hat{\sigma}_x^B) & v(\sigma_x^A) & v(\sigma_x^A \otimes \sigma_x^B) \\ v(\sigma_y^A) & v(\sigma_y^B) & v(\sigma_y^A \otimes \sigma_y^B) \\ v(\sigma_x^B \sigma_y^A) & v(\sigma_x^A \sigma_y^B) & v(\sigma_z^A \otimes \sigma_z^B) \end{array}$$

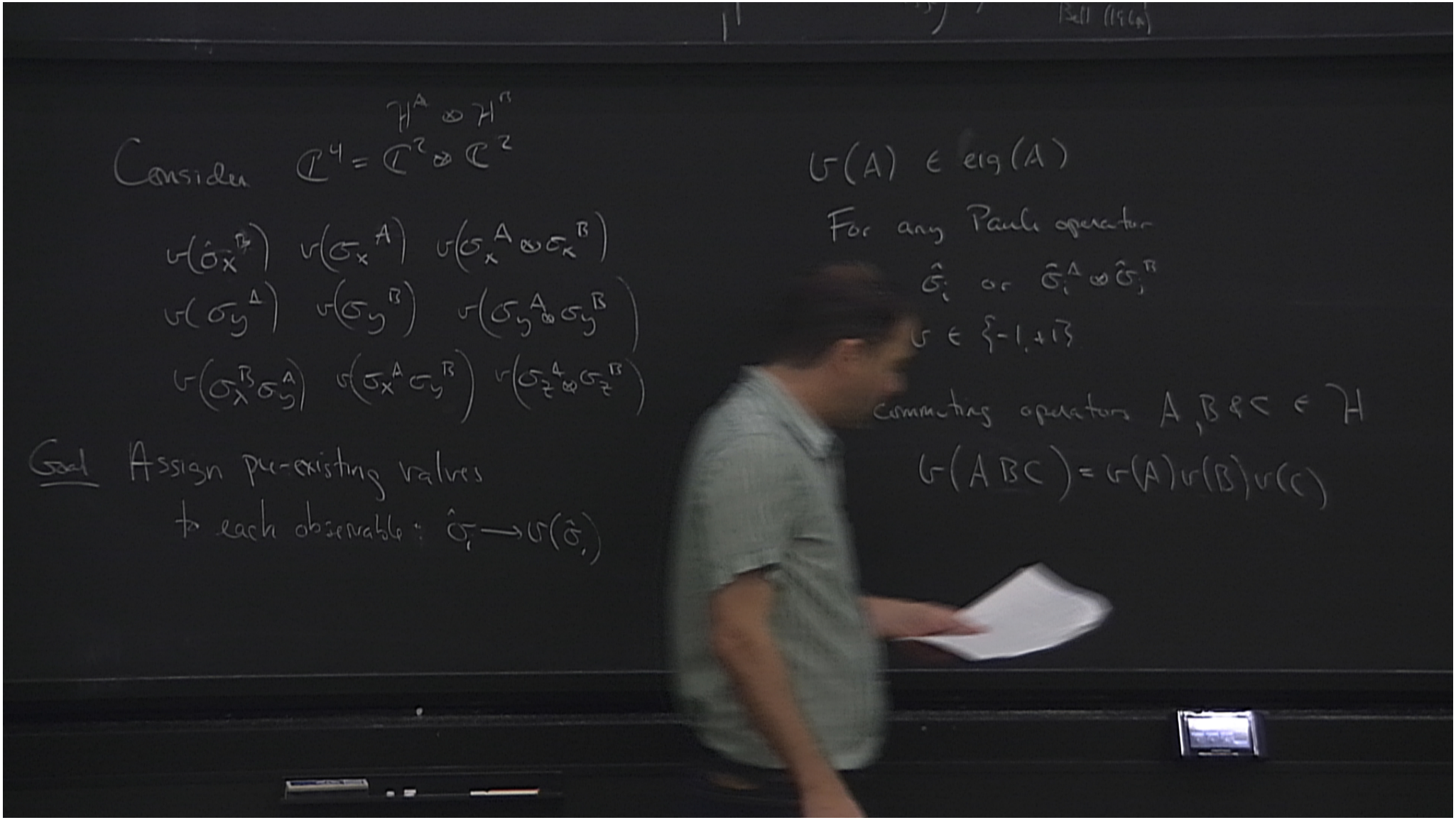
Goal: Assign pre-existing values to each observable: $\hat{\sigma}_i \rightarrow v(\hat{\sigma}_i)$

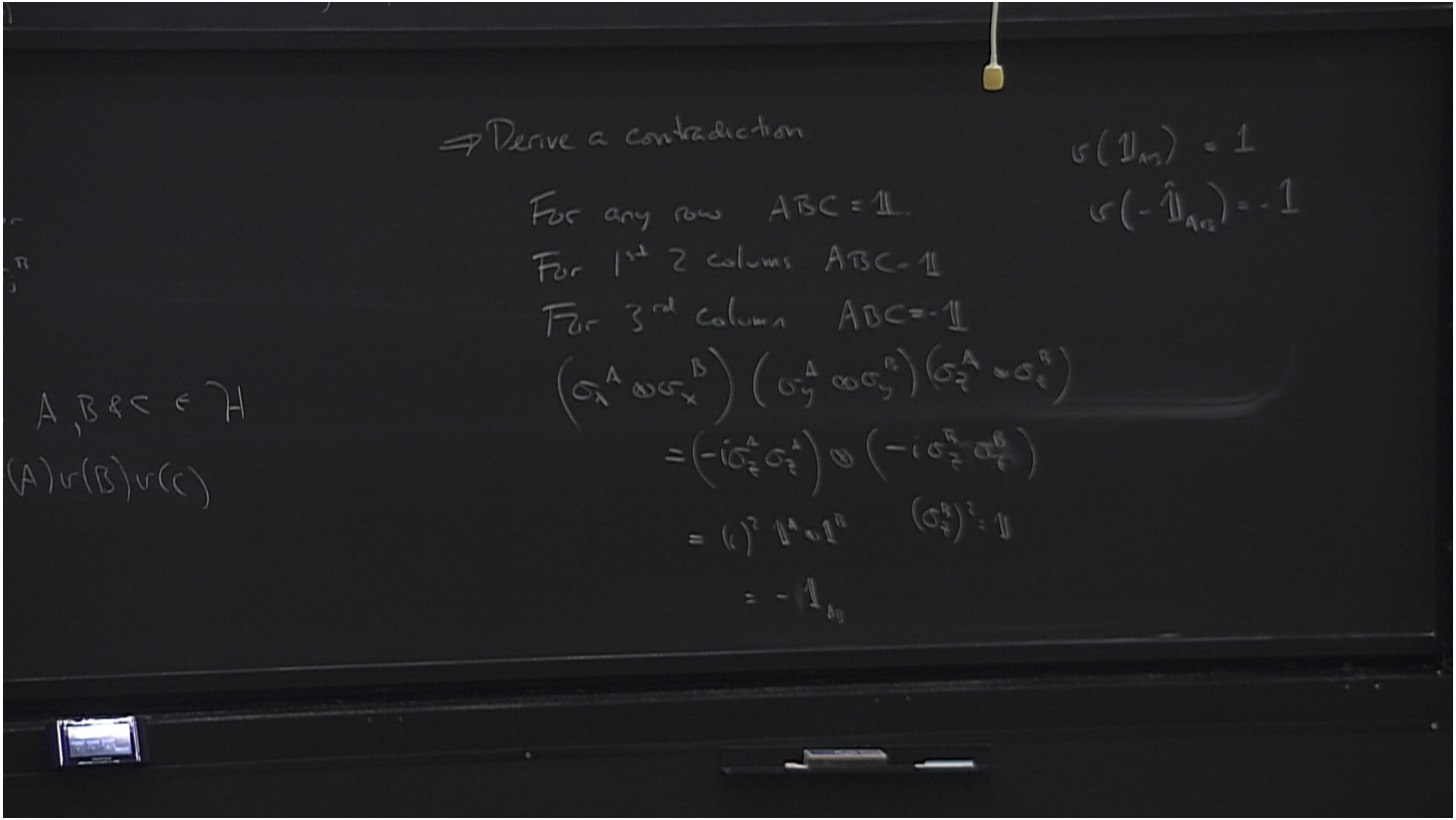
$v(A) \in \mathrm{eig}(A)$

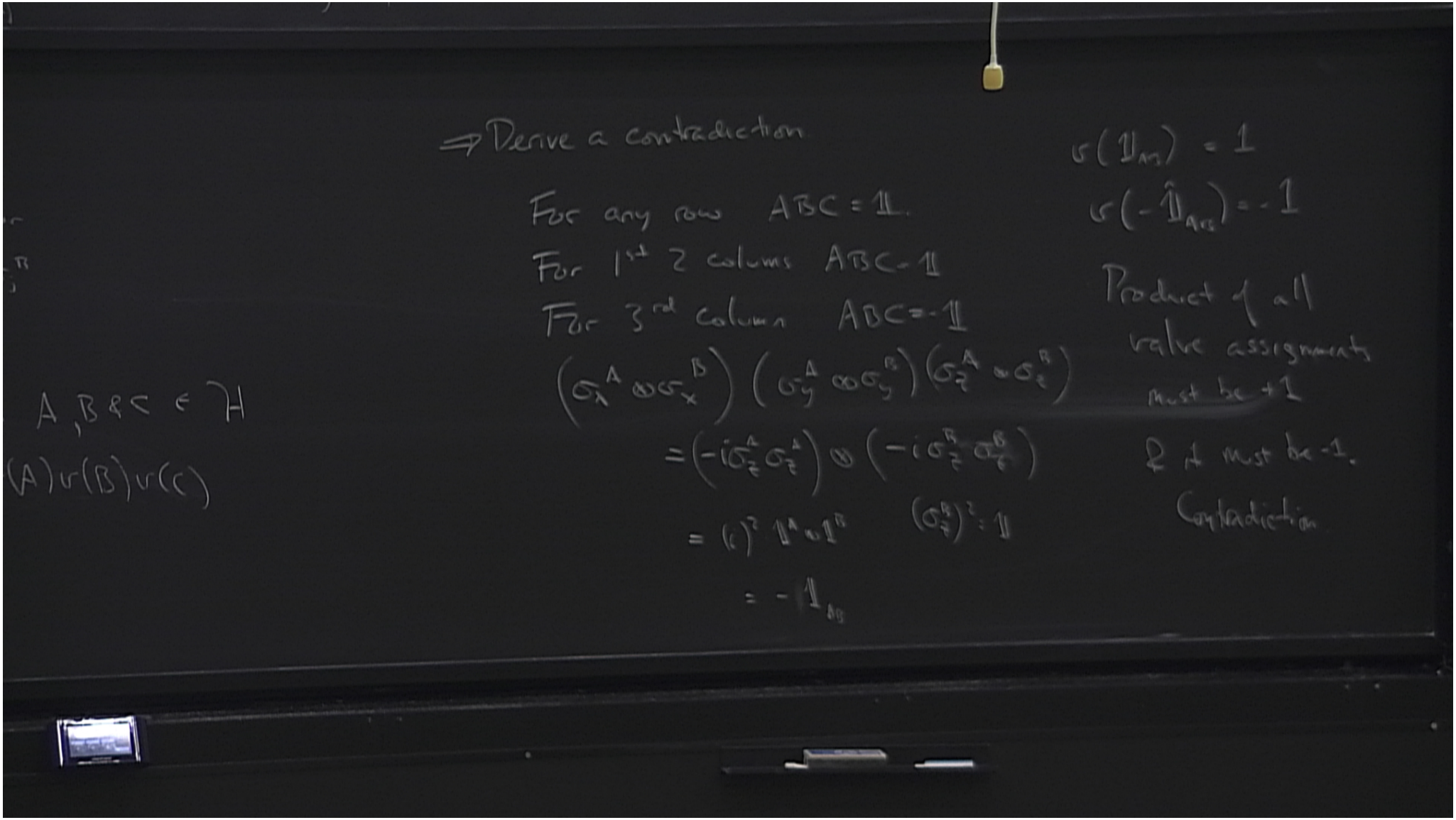
For any Pauli operator $\hat{\sigma}_i$ or $\hat{\sigma}_i^A \otimes \hat{\sigma}_j^B$

$v \in \{-1, +1\}$

For commuting operators


Bell (1964)
Consider $\mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$
$\mathcal{H}^A \otimes \mathcal{H}^B$
$v(\hat{\sigma}_x^B)$ $v(\sigma_x^A)$ $v(\sigma_x^A \otimes \sigma_x^B)$
$v(\sigma_y^A)$ $v(\sigma_y^B)$ $v(\sigma_y^A \otimes \sigma_y^B)$
$v(\sigma_x^B \sigma_y^A)$ $v(\sigma_x^A \sigma_y^B)$ $v(\sigma_z^A \otimes \sigma_z^B)$
Goal Assign pre-existing values
to each observable: $\hat{\sigma}_i \rightarrow v(\hat{\sigma}_i)$
$v(A) \in \mathrm{eig}(A)$
For any Pauli operator
$\hat{\sigma}_i$ or $\hat{\sigma}_i^A \otimes \hat{\sigma}_j^B$
$v \in \{-1, +1\}$
Commuting operators A, B & $C \in \mathcal{H}$
$v(ABC) = v(A)v(B)v(C)$


⇒ Derive a contradiction
For any row ABC = 𝟙
For 1st 2 colums ABC = 𝟙
For 3rd column ABC = -𝟙
A, B & C ∈ ℋ
v(A) v(B) v(C)
v(𝟙_A) = 1
v(-𝟙_A) = -1


⇒ Derive a contradiction
For any row ABC = 𝟙.
For 1st 2 colums ABC = -𝟙
For 3rd column ABC = -𝟙
$(\sigma_x^A \otimes \sigma_x^B)(\sigma_y^A \otimes \sigma_y^B)(\sigma_z^A \otimes \sigma_z^B)$
$= (-i\sigma_z^A\sigma_z^A) \otimes (-i\sigma_z^B\sigma_z^B)$
$= (i)^2\, \mathbb{1}^A \otimes \mathbb{1}^B \quad (\sigma_z^B)^2 = \mathbb{1}$
$= -\mathbb{1}_{AB}$
A, B & C ∈ ℋ
v(A) v(B) v(C)
v(𝟙_A) = 1
v(-𝟙_A) = -1
Product of all value assignments must be +1
& it must be -1.
Contradiction

Bell (1964)

$\mathcal{H}^A \otimes \mathcal{H}^B$

Consider $\mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$

$v(\hat{\sigma}_x^B)$ $v(\sigma_x^A)$ $v(\sigma_x^A \otimes \sigma_x^B)$

$v(\sigma_y^A)$ $v(\sigma_y^B)$ $v(\sigma_y^A \otimes \sigma_y^B)$

$v(\sigma_x^B \sigma_y^A)$ $v(\sigma_x^A \sigma_y^B)$ $v(\sigma_z^A \otimes \sigma_z^B)$

Assign pre-existing values to each observable: $\hat{\sigma}_i \longrightarrow v(\hat{\sigma}_i)$

$v(A) \in \mathrm{eig}(A)$

For any Pauli operator $\hat{\sigma}_i$ or $\hat{\sigma}_i^A \otimes \hat{\sigma}_j^B$

$v \in \{-1, +1\}$

For commuting operators A, B & $C \in \mathcal{H}$

$$v(ABC) = v(A)\,v(B)\,v(C)$$

non-locality (1935) | non-locality Bell (1964)

Consider $\mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$ $\quad \mathcal{H}^A \otimes \mathcal{H}^B$

Peres-Mermin square

$$\begin{cases} v(\hat{\sigma}_x^B) & v(\sigma_x^A) & v(\sigma_x^A \otimes \sigma_x^B) \\ v(\sigma_y^A) & v(\sigma_y^B) & v(\sigma_y^A \otimes \sigma_y^B) \\ v(\sigma_x^B \sigma_y^A) & v(\sigma_x^A \sigma_y^B) & v(\sigma_z^A \otimes \sigma_z^B) \end{cases}$$

Goal: Assign pre-existing values to each observable: $\hat{\sigma}_i \longrightarrow v(\hat{\sigma}_i)$

$v(A) \in \mathrm{eig}(A)$

For any Pauli operator $\hat{\sigma}_i$ or $\hat{\sigma}_i^A \otimes \hat{\sigma}_j^B$

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