

Title: PSI 2015/2016 Quantum Theory - Lecture 12

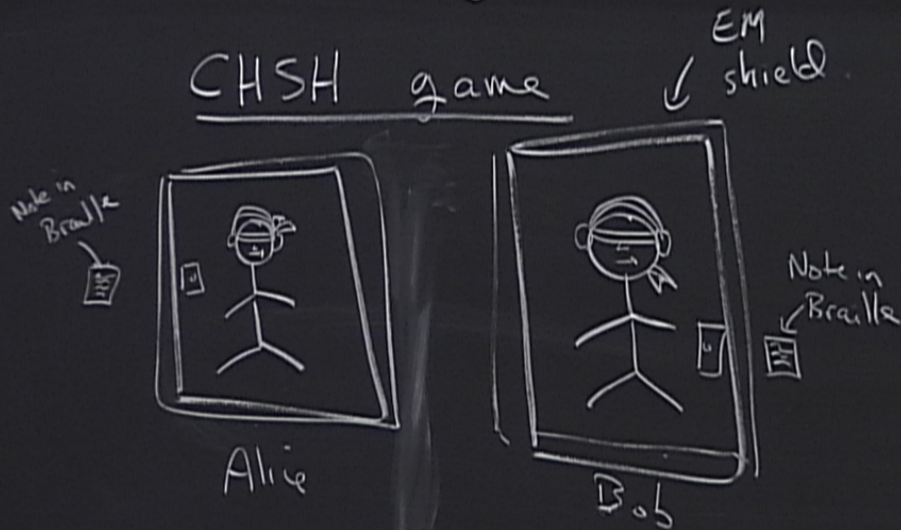
Date: Sep 23, 2015 10:45 AM

URL: <http://pirsa.org/15090053>

Abstract:

Non-locality

CHSH game



Rules

1. Prior to time t_0 (start of game) Alice & Bob can concert any plan they want.
 2. After t_0 , they can not communicate.
- They are advised in advance of the full rules.

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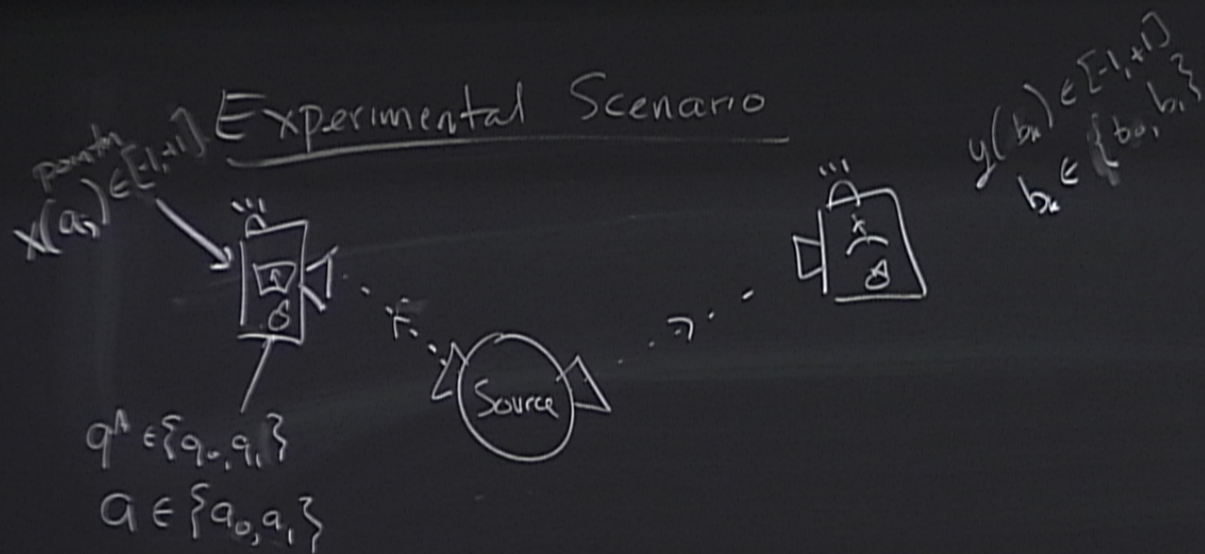
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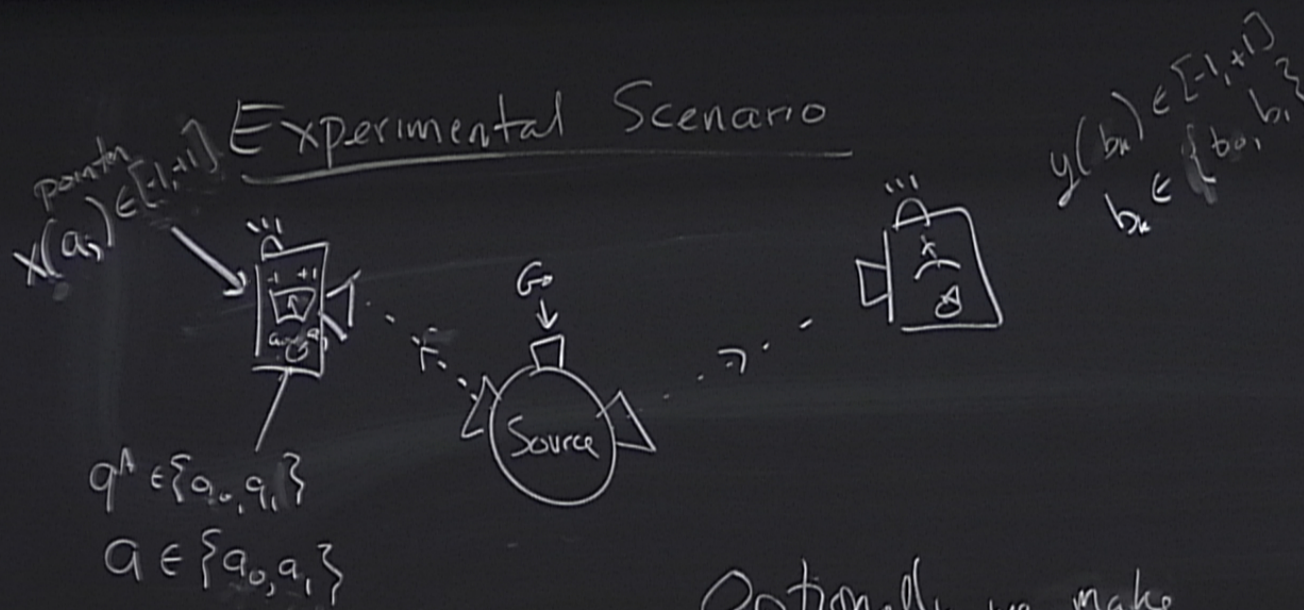
5. Constraint

Alice	Bob	Win a given round if Answers
q_0	q_0	$x_a = y_b$
q_0	q_1	$x_a = y_b$
q_1	q_0	$x_a = y_b$
q_1	q_1	$x_a \neq y_b$

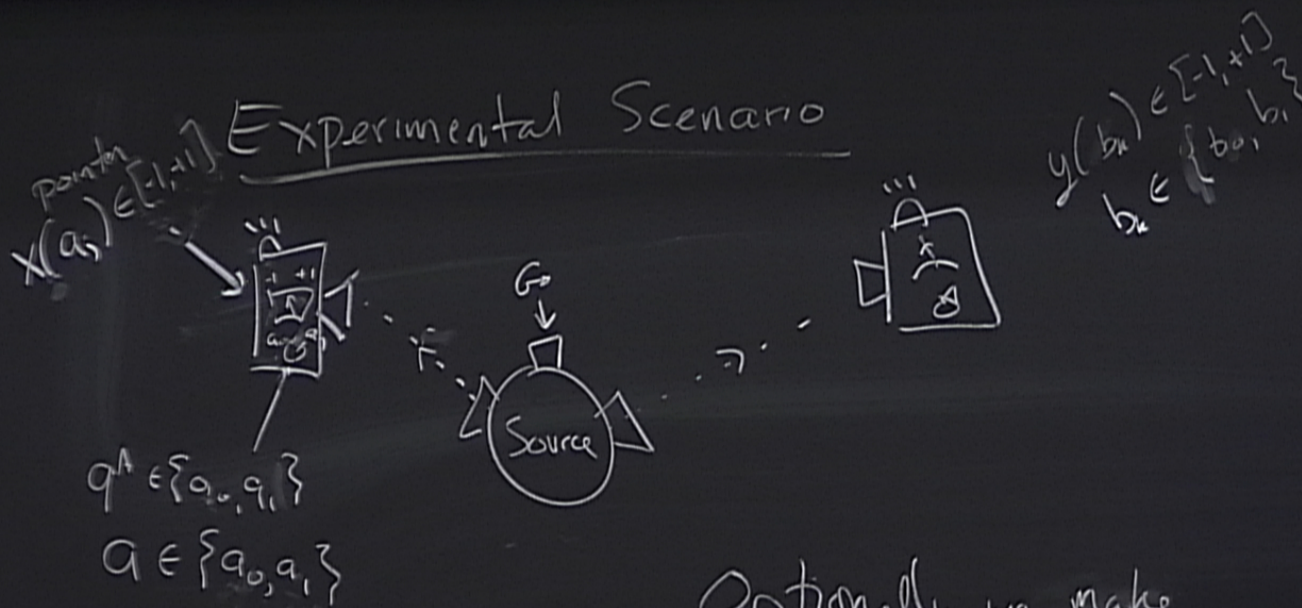
6. What is maximum fraction
of rounds they can win?

$$(x_a, y_b) \in \{(0,0), (0,1), (1,0), (1,1)\}$$





Optionally, we make
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5. Constraint

Alice	Bob
q_0	q_0
q_0	q_1
q_1	q_0
q_1	q_1

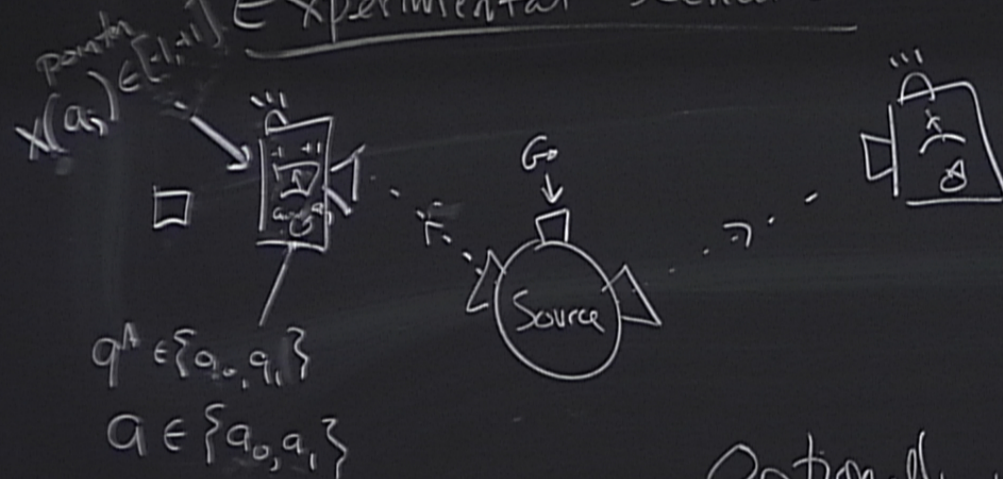
6. What is maximum
of rounds they

$$S := \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle$$

$\langle \cdot \rangle$ is an average over runs

$$\langle a_k b_n \rangle := \lim_{\substack{N \rightarrow \infty \\ \uparrow \\ \# \text{ of runs}}} \frac{1}{N} \sum_{n=1}^N X_n(a_k) Y_n(b_n)$$

Experimental Scenario



Optionally, we make the detectors space-like separated to guarantee no communication.

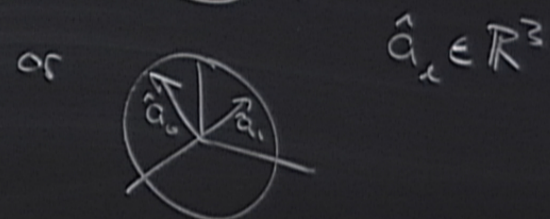
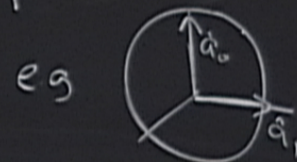
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$\langle \cdot \rangle$ is an average over runs

$$\langle a_k b_k \rangle := \lim_{\substack{N \rightarrow \infty \\ \uparrow \\ \# \text{ of runs}}} \frac{1}{N} \sum_{n=1}^N X_n(a_k) Y_n(b_k) =: \langle X(a_k) Y(b_k) \rangle$$

Review

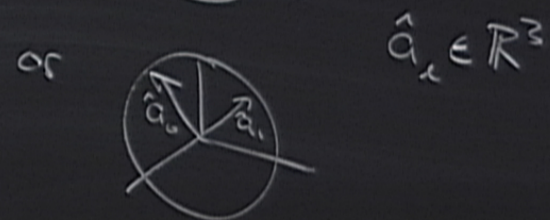
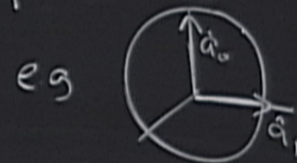
$$\langle a_e b_n \rangle = \text{Tr}(\hat{\rho} \vec{\sigma} \cdot \hat{a}_e \otimes \vec{\sigma} \cdot \hat{b}_n)$$



$\hat{a}_e \in \mathbb{R}^3$

Review

$$\langle a_e b_n \rangle = \text{Tr}(\hat{\rho} \vec{\sigma} \cdot \hat{a}_e \otimes \vec{\sigma} \cdot \hat{b}_n)$$



$$\hat{a}_e \in \mathbb{R}^3$$

Bell's Theorem in form
of CHSH Inequality

$$-2 \leq S \leq 2$$

$$\text{If } S = \int d\lambda \rho(\lambda) S(\lambda)$$

$$\text{where } S(\lambda) = \sum_{i,j \in \{0,1\}} \langle a_i b_j \rangle (-1)^{ij}$$

- λ may be determined probabilistically at source
- λ represents any common cause properties that play a role in determining outcomes x & y .

$$\sum_i (-1)^{i_j}$$

• λ can vary from run to run.

Model the source $\rho(\lambda)$

Realism: $\langle a_x b_y \rangle = \int d\lambda \rho(\lambda) \sum_{xy} x \cdot y \Pr(xy | a_x, b_y, \lambda)$

Local Causality: $\Pr(xy | a_x, b_y, \lambda) = \Pr(x | a_x, \lambda) \Pr(y | b_y, \lambda)$
(Bell locality)

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(Bell locality)
(No communication rule)

Local Causality is implied by

(i) Parameter Independence: $P_r(x)$ does not depend on remote "choice" b_k .

$$P_r(x|a, b, \lambda, y) = P_r(x|a, \lambda, y)$$

(ii) Outcome Independence: $P_r(x)$ does not depend on outcome "y"

$$P_r(x|a, \lambda, y) = P_r(x|a, \lambda)$$

• λ can vary from run to run

Model the source $p(\lambda)$

Realism: $\langle a_x b_y \rangle = \int d\lambda p(\lambda) \sum_{x,y} x \cdot y \Pr(x, y | a_x, b_y, \lambda)$

Local Causality: $\Pr(x, y | a_x, b_y, \lambda) = \Pr(x | \lambda, a_x) \Pr(y | \lambda, b_y)$
(Bell locality)
(No communication rule)

"Factorisation"

Local

We're not denying correlations

Uncorrelated

$$Pr(x, y | a, b) = Pr(x | a) Pr(y | b)$$

Correlated

$$Pr(x, y | a, b, \lambda) = Pr(x | a, \lambda) Pr(y | b, \lambda)$$

Common cause
(shared backward
light cone).
Can produce correlations

- λ can vary from run to run

Model the source $\rho(\underline{\lambda})$

Realism:

$$\langle a_x b_y \rangle = \int d\lambda \rho(\lambda) \sum_{xy} x \cdot y P(x, y | a_x, b_y, \lambda)$$

Local Causality:

(Bell locality)

(No communication rule)

$$P(x, y | a_x, b_y, \lambda) = P(x | \lambda, a_x) P(y | \lambda, b_y)$$

'Factorization'

We have

$$\langle a_i b_j \rangle_\lambda = \sum_{x,y} x \cdot y \, P_r(x|a_i, \lambda) P_r(y|b_j, \lambda)$$

$$= \underbrace{\left[\sum_x x P_r(x|a_i, \lambda) \right]}_{\langle a_i \rangle_\lambda} \cdot \underbrace{\left[\sum_y y P_r(y|b_j, \lambda) \right]}_{\langle b_j \rangle_\lambda}$$

$$S(\lambda) = \langle a_0 \rangle \langle b_0 \rangle + \langle a_1 \rangle \langle b_0 \rangle + \langle a_0 \rangle \langle b_1 \rangle - \langle a_1 \rangle \langle b_1 \rangle$$

$$\langle a_i \rangle \in [-1, +1]$$

$$\langle b_i \rangle \in [-1, +1]$$

Rearrange
terms

$$S(\lambda) = \langle a_i \rangle (\langle b_0 \rangle + \langle b_i \rangle) + \langle a_i \rangle (\langle b_0 \rangle - \langle b_i \rangle)$$

$$\leq |\langle b_0 \rangle + \langle b_i \rangle| + |\langle b_0 \rangle - \langle b_i \rangle|$$

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$$\leq |\langle b_0 \rangle + \langle b_i \rangle| + |\langle b_0 \rangle - \langle b_i \rangle|$$

w.l.o.g. $\langle b_0 \rangle \geq \langle b_i \rangle$

$$S(\lambda) \leq 2|\langle b_0 \rangle| \leq 2$$

$$S = \int d\lambda p(\lambda) S(\lambda) \leq 2$$

By symmetrical argument

$$-2 \leq S \leq 2$$

Punchline is with q.m.

$$S = 2\sqrt{2}$$

for certain choices of input

state & directions of measurement