Title: PSI 2015/2016 Quantum Theory - Lecture 9

Date: Sep 18, 2015 10:45 AM

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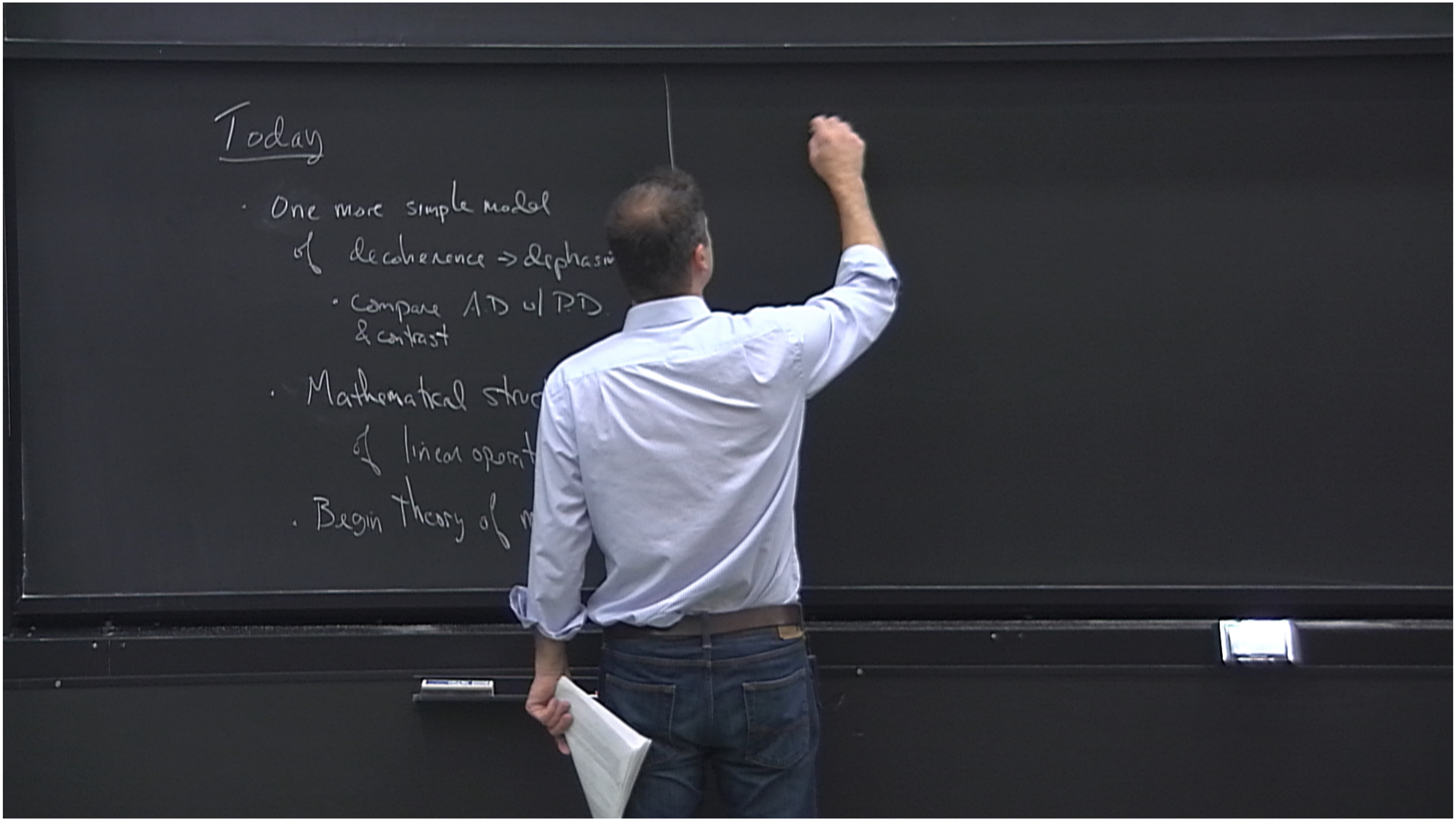
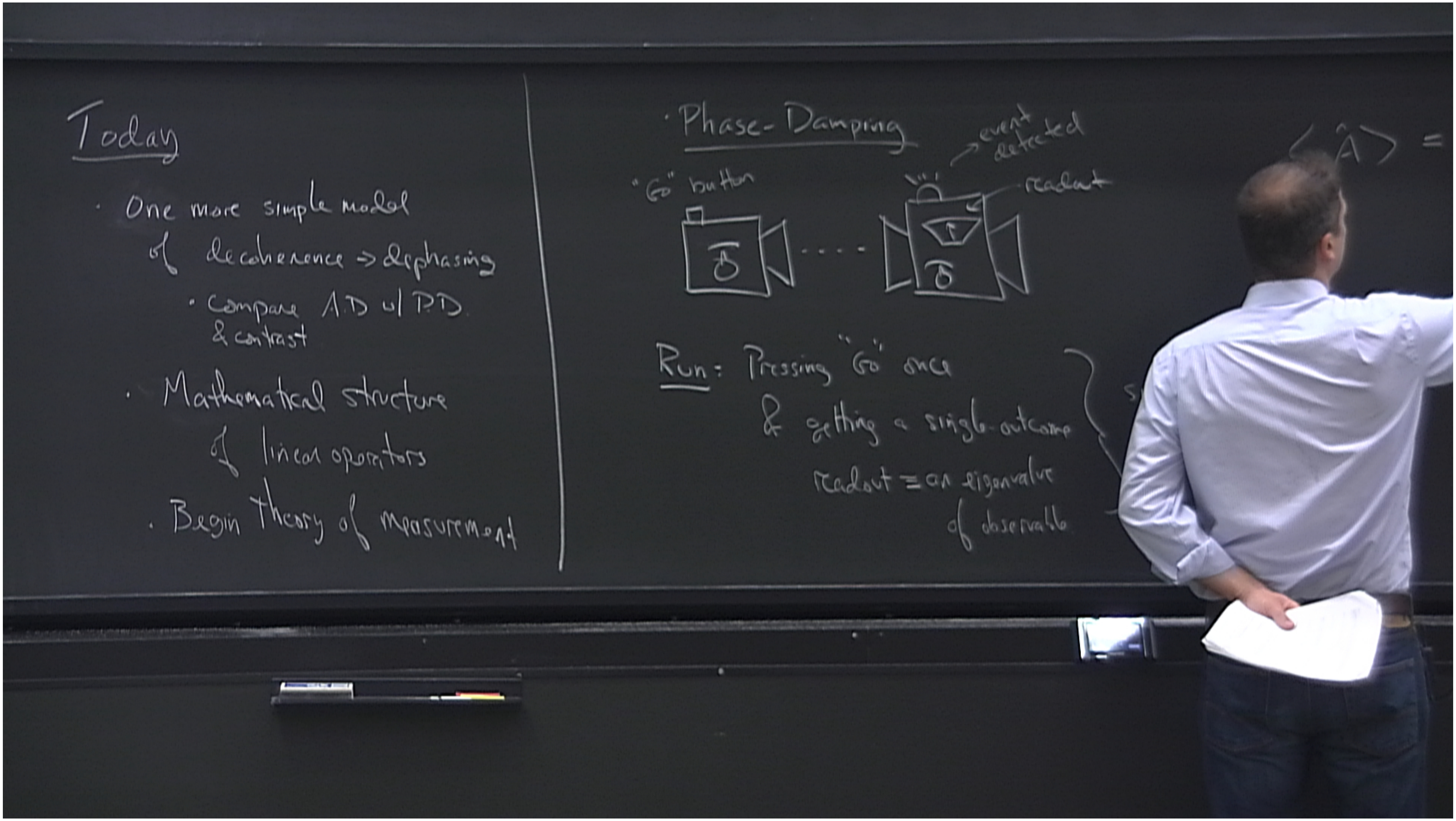
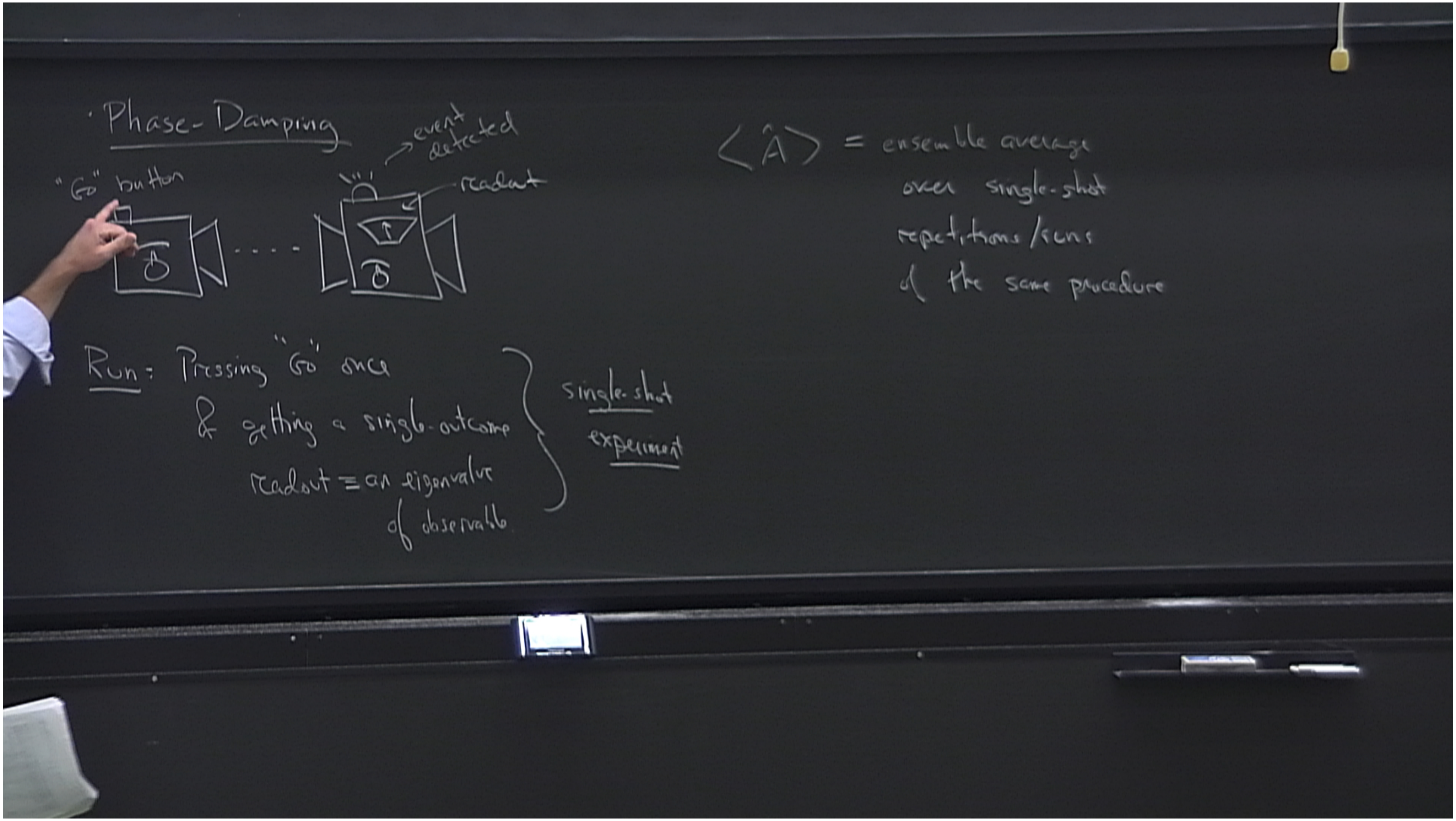
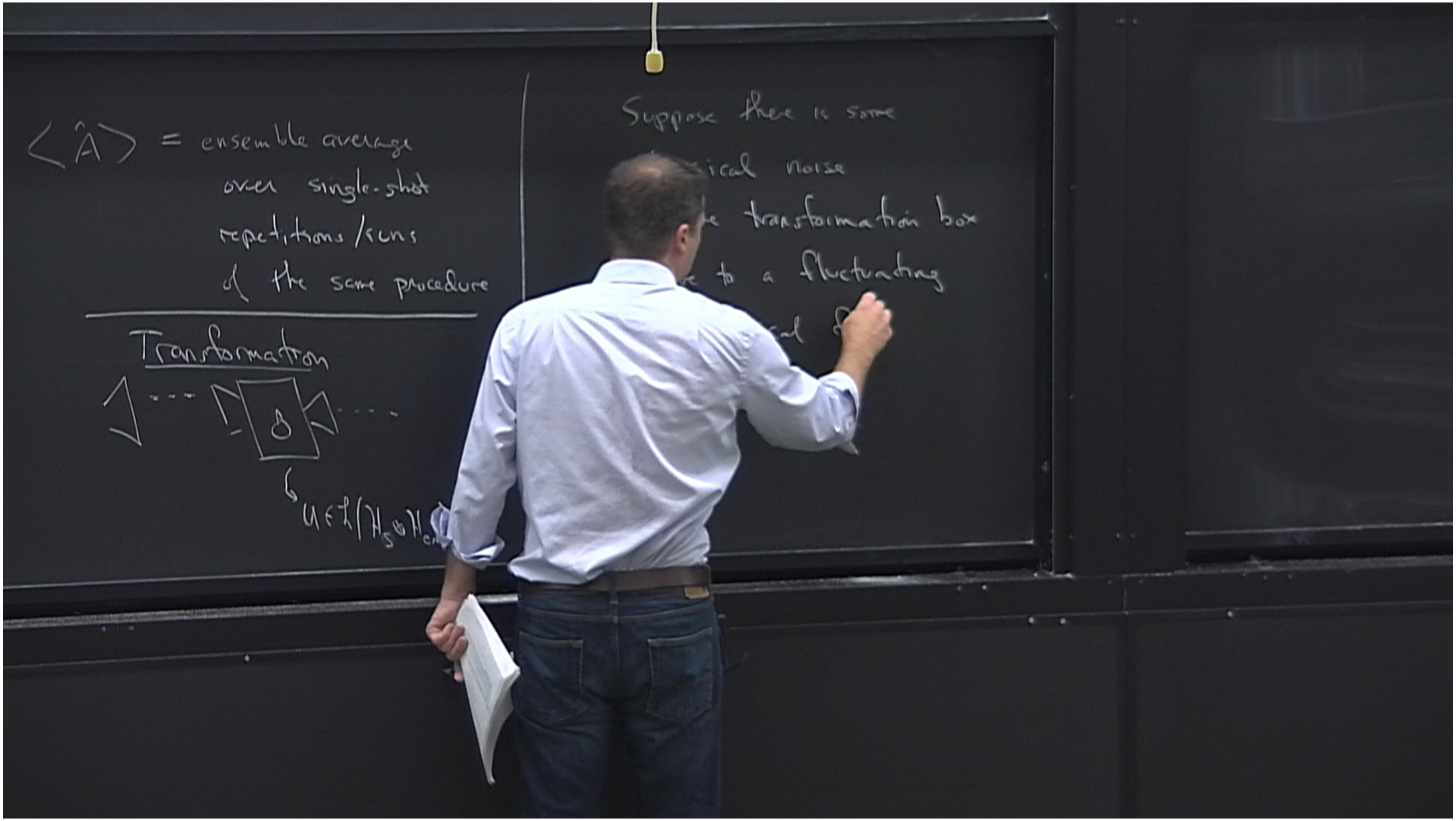
Abstract:


Today
· One more simple model
of decoherence → dephasi
· compare A.D w/ P.D
& contrast
· Mathematical struc
of linear operat
· Begin theory of


Today
· One more simple model
of decoherence ⇒ dephasing
· Compare A.D w/ P.D
& contrast
· Mathematical structure
of linear operators
· Begin theory of measurement
· Phase-Damping
event detected
"Go" button
readout
Run: Pressing "Go" once
& getting a single outcome
readout ≡ an eigenvalue
of observable
⟨Â⟩ =


· Phase-Damping
"Go" button
event detected
readout
Run: Pressing "Go" once
& getting a single-outcome
readout ≡ an eigenvalue
of observable
single-shot
experiment
$\langle \hat{A} \rangle$ = ensemble average
over single-shot
repetitions/runs
of the same procedure


⟨Â⟩ = ensemble average
over single-shot
repetitions/runs
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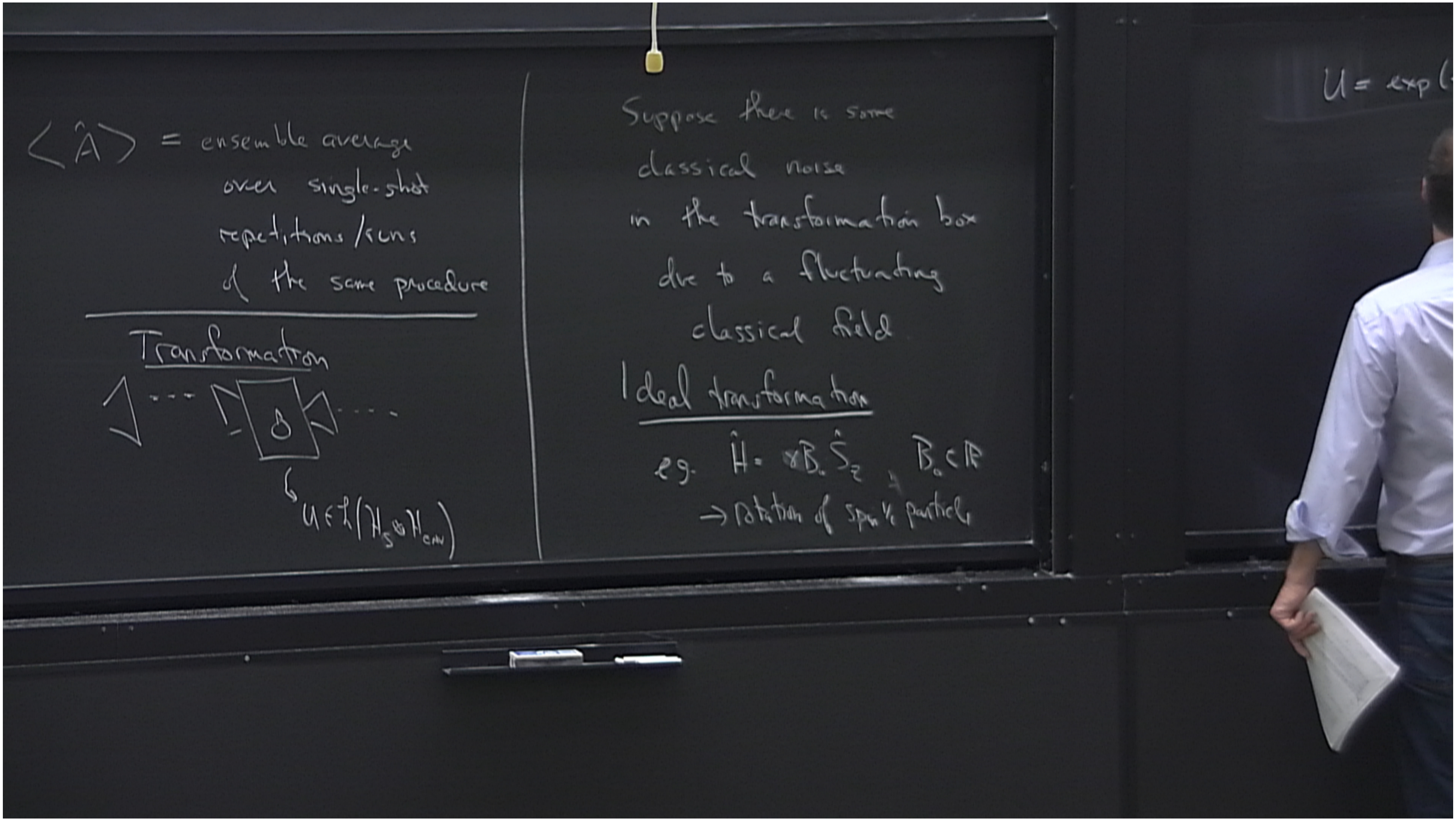
Transformation
Suppose there is some
noise
transformation box
to a fluctuating


⟨Â⟩ = ensemble average
over single-shot
repetitions / runs
of the same procedure
Transformation
U ∈ L(H_S ⊗ H_env)
Suppose there is some
classical noise
in the transformation box
due to a fluctuating
classical field
Ideal transformation
e.g. Ĥ = γB₀ Ŝ_z   B₀ ∈ ℝ
→ rotation of spin ½ particle
U = exp(


$U = \exp(-i \frac{Ht}{\hbar}) = \exp(-i\theta S_z)$
$\hbar = 1$
$= \frac{\omega t}{\hbar}$
$e^{-i\theta S_z}$
$\in O(3)$
space

$$U = \exp\left(-i\frac{Ht}{\hbar}\right) = \exp(-i\theta S_z)$$

$$\hbar = 1$$

$$\theta = \frac{\omega t}{\hbar}$$

$$e^{-i\theta S_z} = R_z(\theta) \in O(3)$$

$\Rightarrow$ rotation in 3D space

$$U = \exp\left(-i \frac{H t}{\hbar}\right) = \exp(-i\theta S_z)$$

$\hbar = 1$

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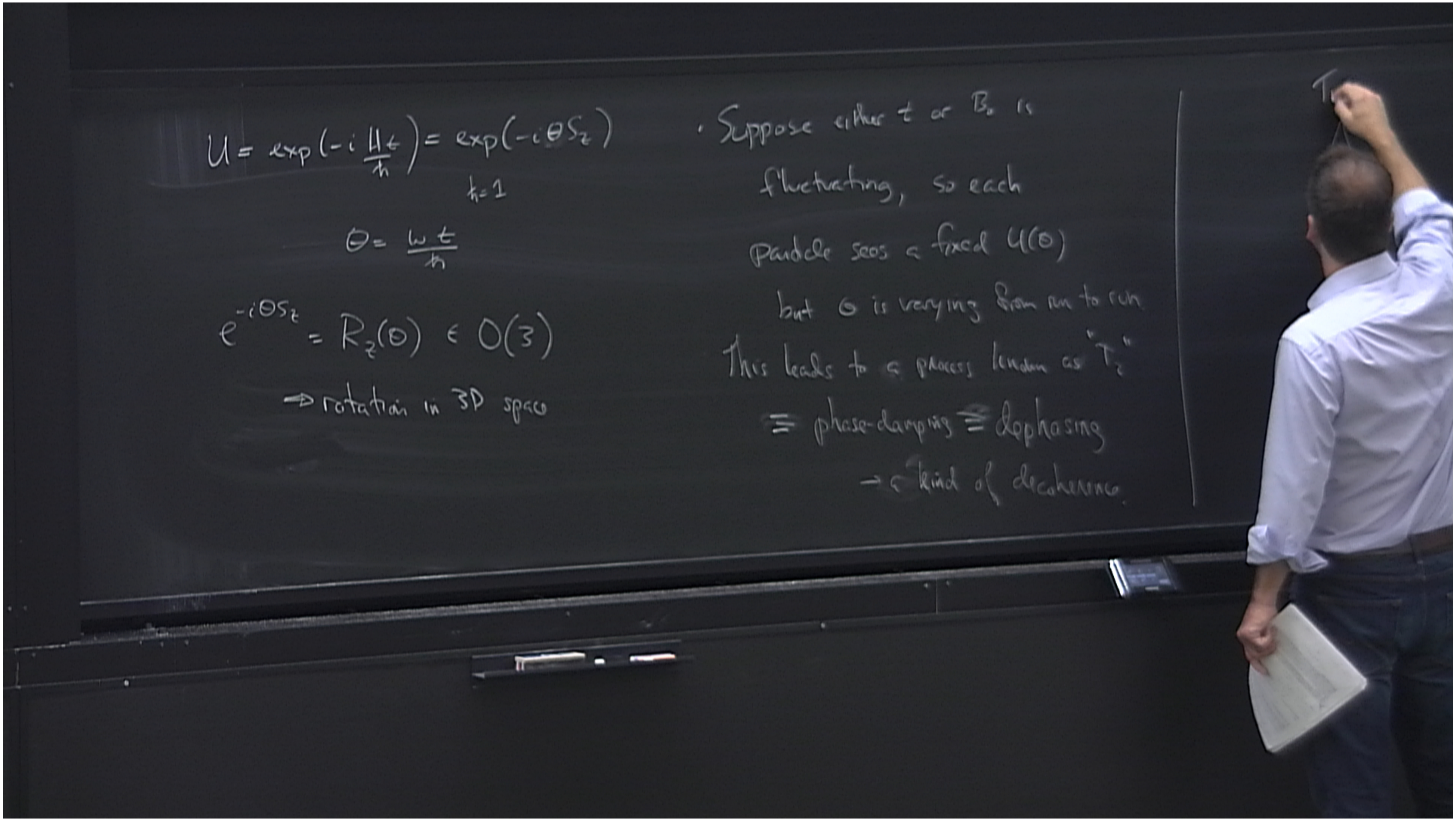
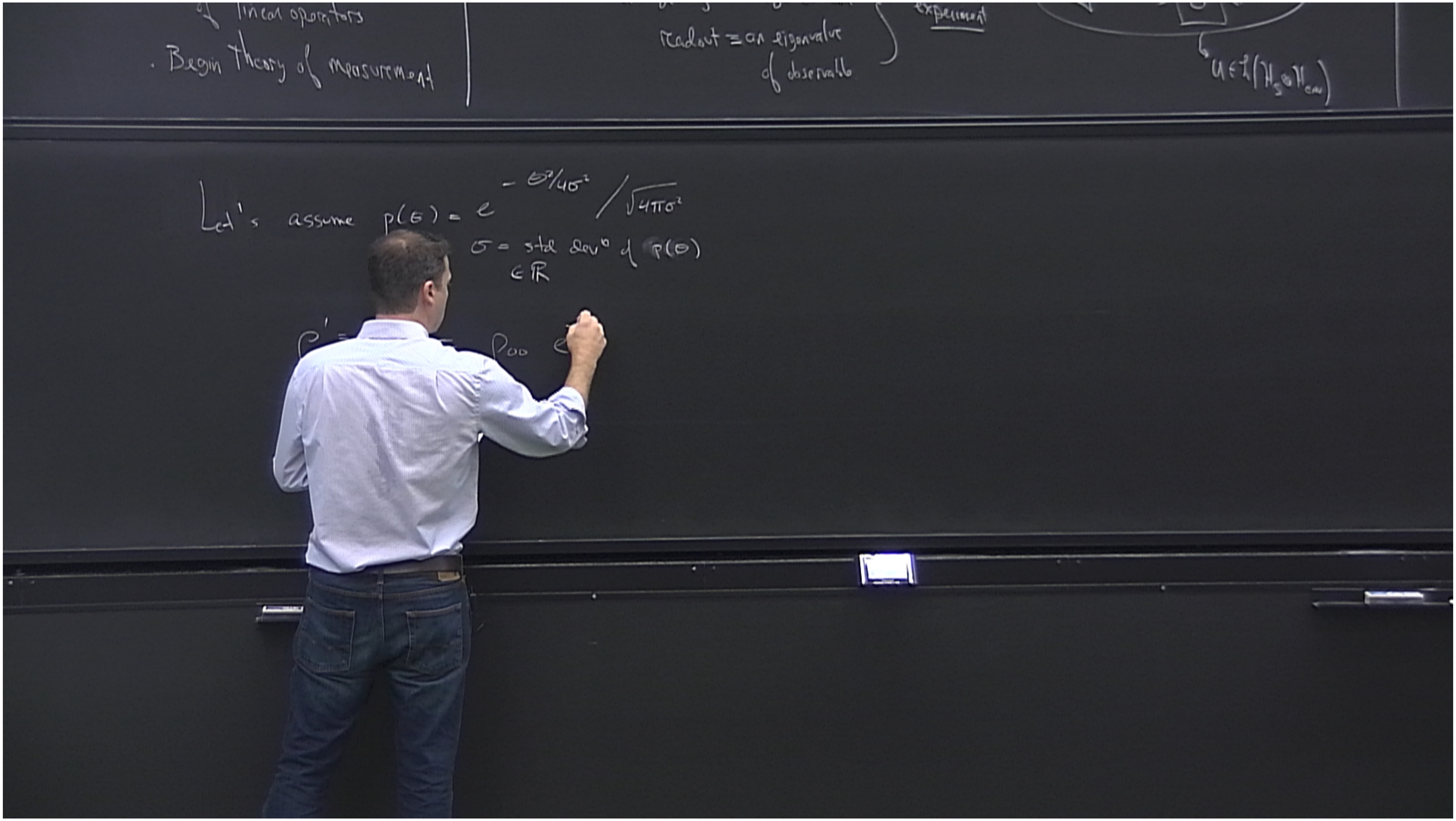
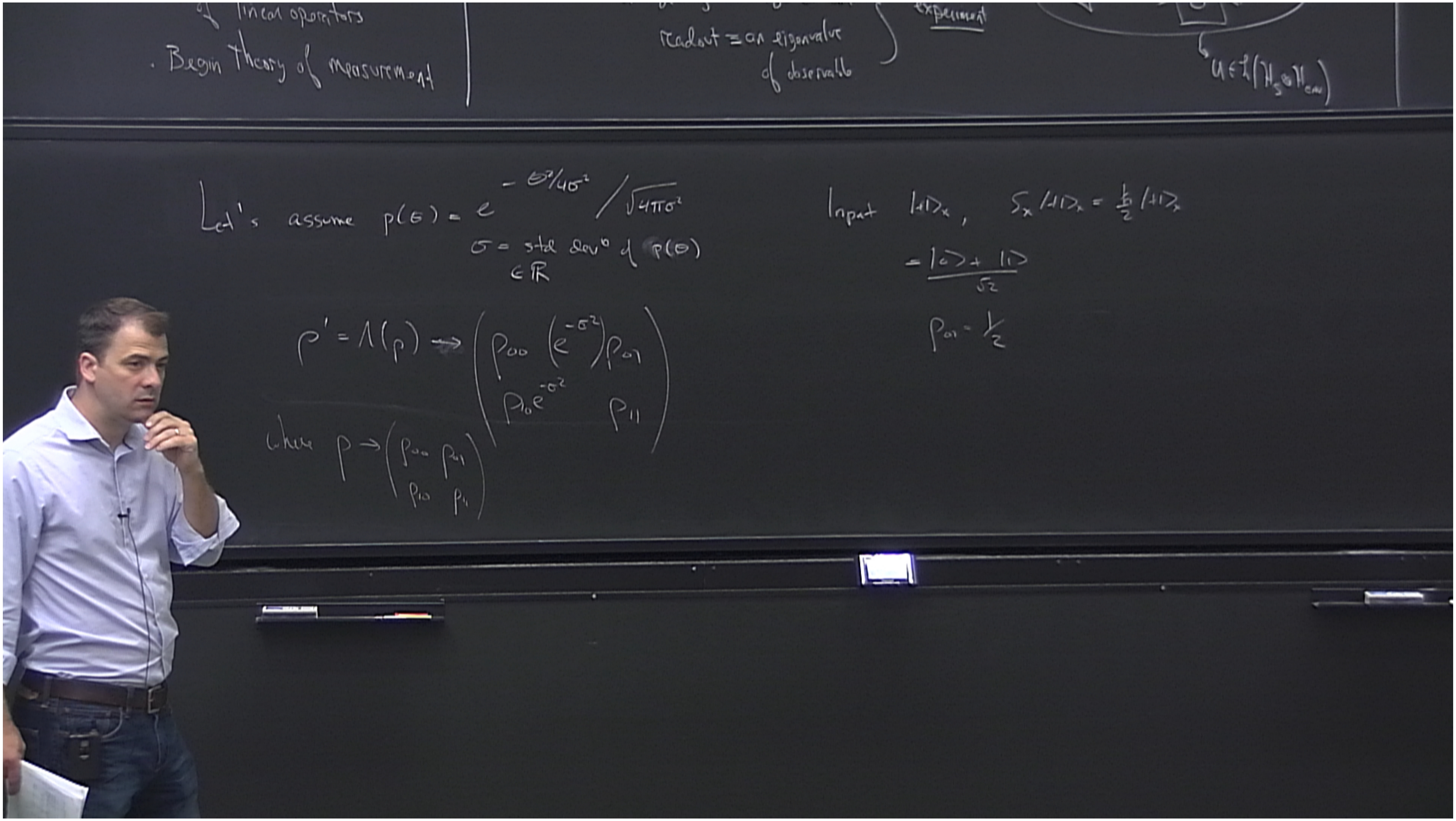
$\Rightarrow$ rotation in 3D space

Suppose either t or B_0 is fluctuating,


$U = \exp(-i \frac{H t}{\hbar}) = \exp(-i\theta S_z)$
$\hbar = 1$
$\theta = \frac{\omega t}{\hbar}$
$e^{-i\theta S_z} = R_z(\theta) \in O(3)$
$\Rightarrow$ rotation in 3D space
• Suppose either t or B_z is
fluctuating, so each
particle sees a fixed $U(\theta)$
but θ is varying from run to run
This leads to a process known as "T_2"
$\equiv$ phase-damping $\equiv$ dephasing
$\rightarrow$ a kind of decoherence


linear operators
. Begin Theory of measurement
readout ≡ an eigenvalue of observable
experiment
$U \in \mathcal{L}(\mathcal{H}_S \otimes \mathcal{H}_{env})$
Let's assume $p(\theta) = e^{-\theta^2/4\sigma^2} / \sqrt{4\pi\sigma^2}$
σ = std dev of $p(\theta)$
$\in \mathbb{R}$
$\rho' =$ ρ_{00} e


linear operators
. Begin theory of measurement
readout ≡ an eigenvalue
of observable
experiment
Let's assume p(θ) = e^(-θ²/4σ²) / √(4πσ²)
σ = std dev of p(θ)
∈ ℝ
ρ' = Λ(ρ) → (ρ00 (e^(-σ²))ρ01 ; ρ10 e^(-σ²) ρ11)
where ρ → (ρ00 ρ01 ; ρ10 ρ11)
Input |+1>x , Sx |+1>x = ħ/2 |+1>x
= (|0> + |1>)/√2
ρ01 = 1/2

of linear operators

. Begin theory of measurement

readout ≡ an eigenvalue of observable

experiment

$U \in \mathcal{L}(\mathcal{H}_S \otimes \mathcal{H}_{env})$

Let's assume $p(\theta) = e^{-\theta^2/4\sigma^2} / \sqrt{4\pi\sigma^2}$

σ = std dev^n of $p(\theta)$
$\in \mathbb{R}$

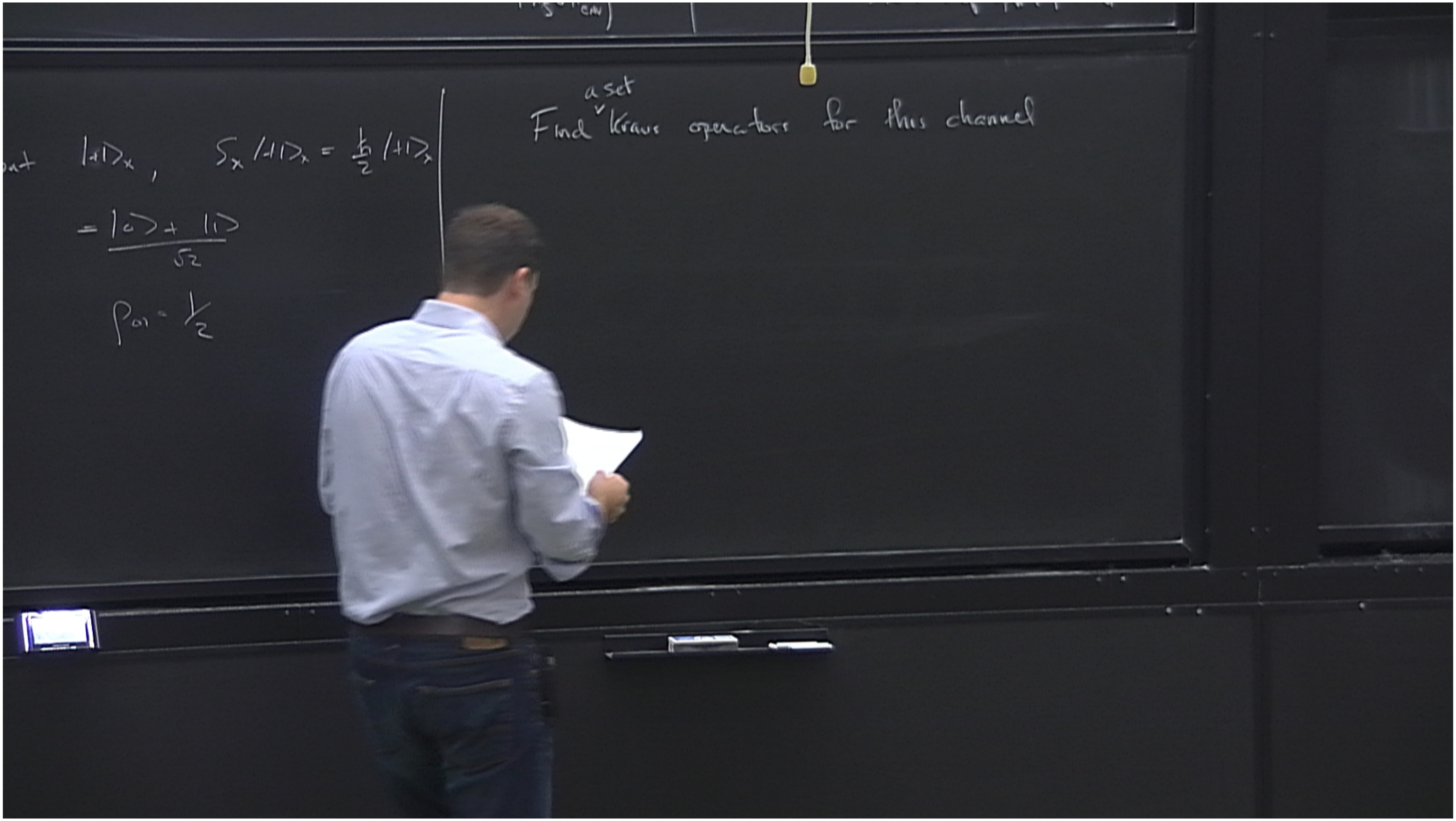
$$\rho' = \Lambda(\rho) \rightarrow \begin{pmatrix} \rho_{00} & (e^{-\sigma^2})\rho_{01} \\ \rho_{10} e^{-\sigma^2} & \rho_{11} \end{pmatrix}$$

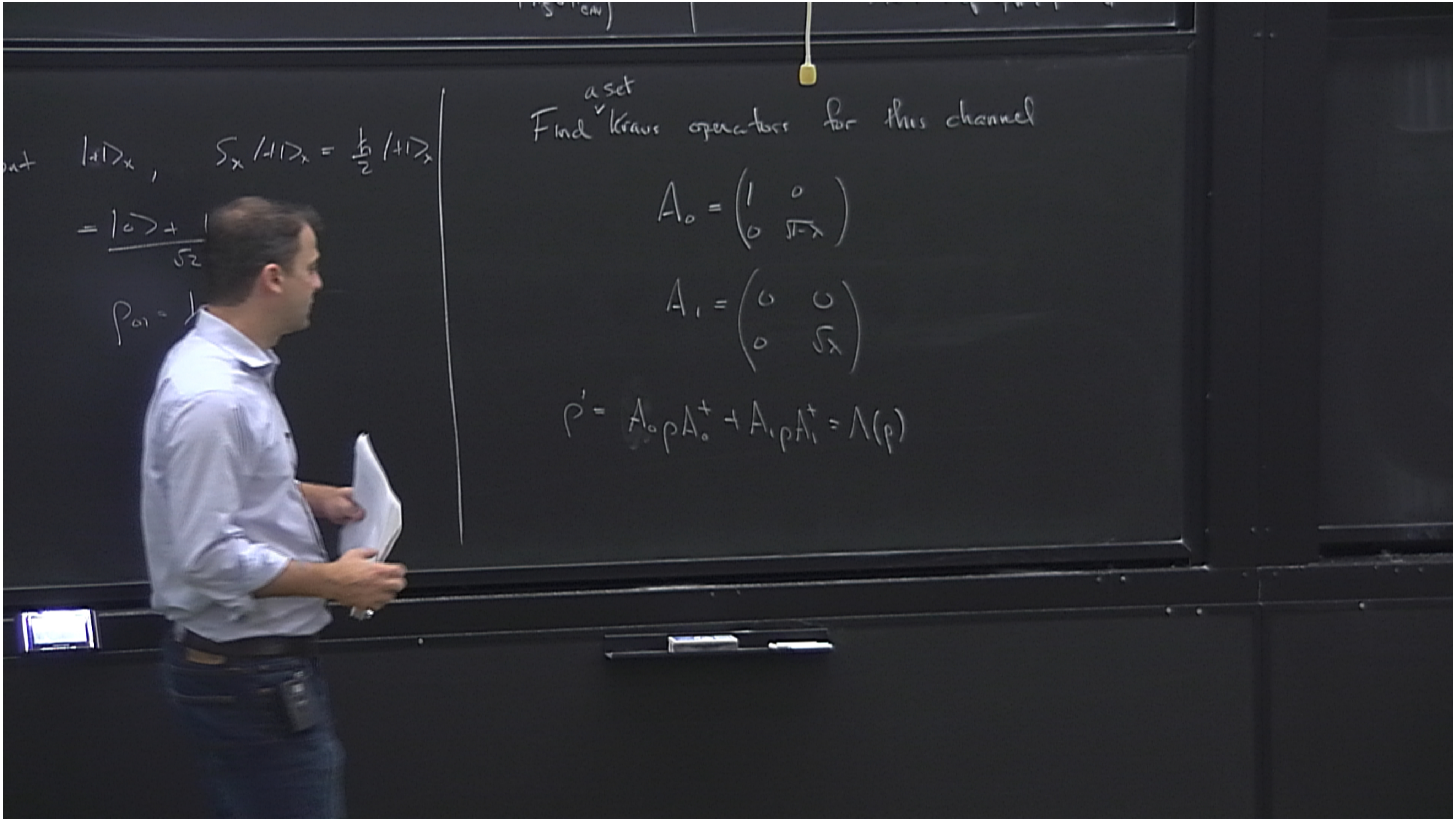
where $\rho \rightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

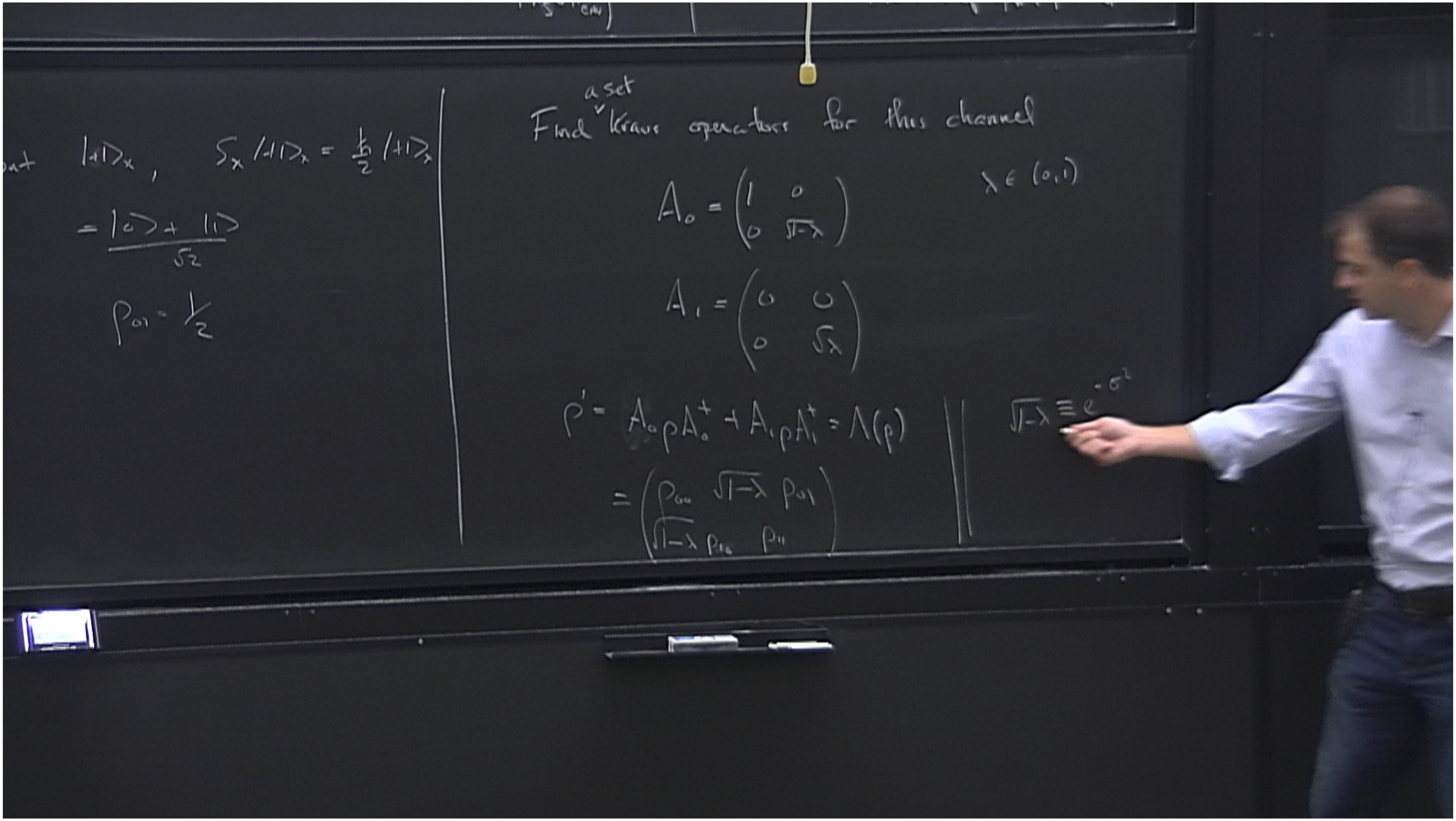
Input $|+1\rangle_x$, $S_x |+1\rangle_x = \frac{\hbar}{2} |+1\rangle_x$

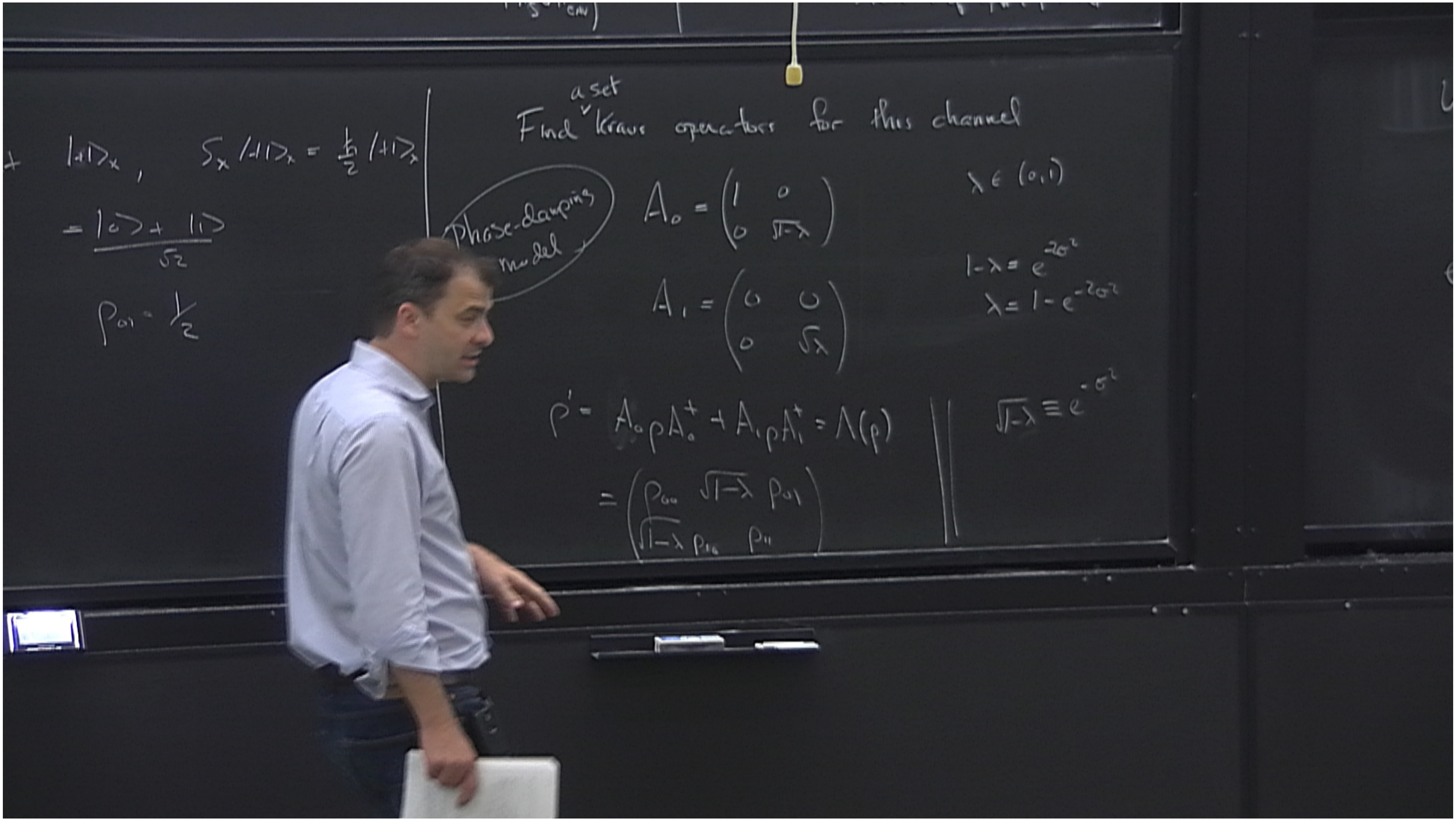
$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

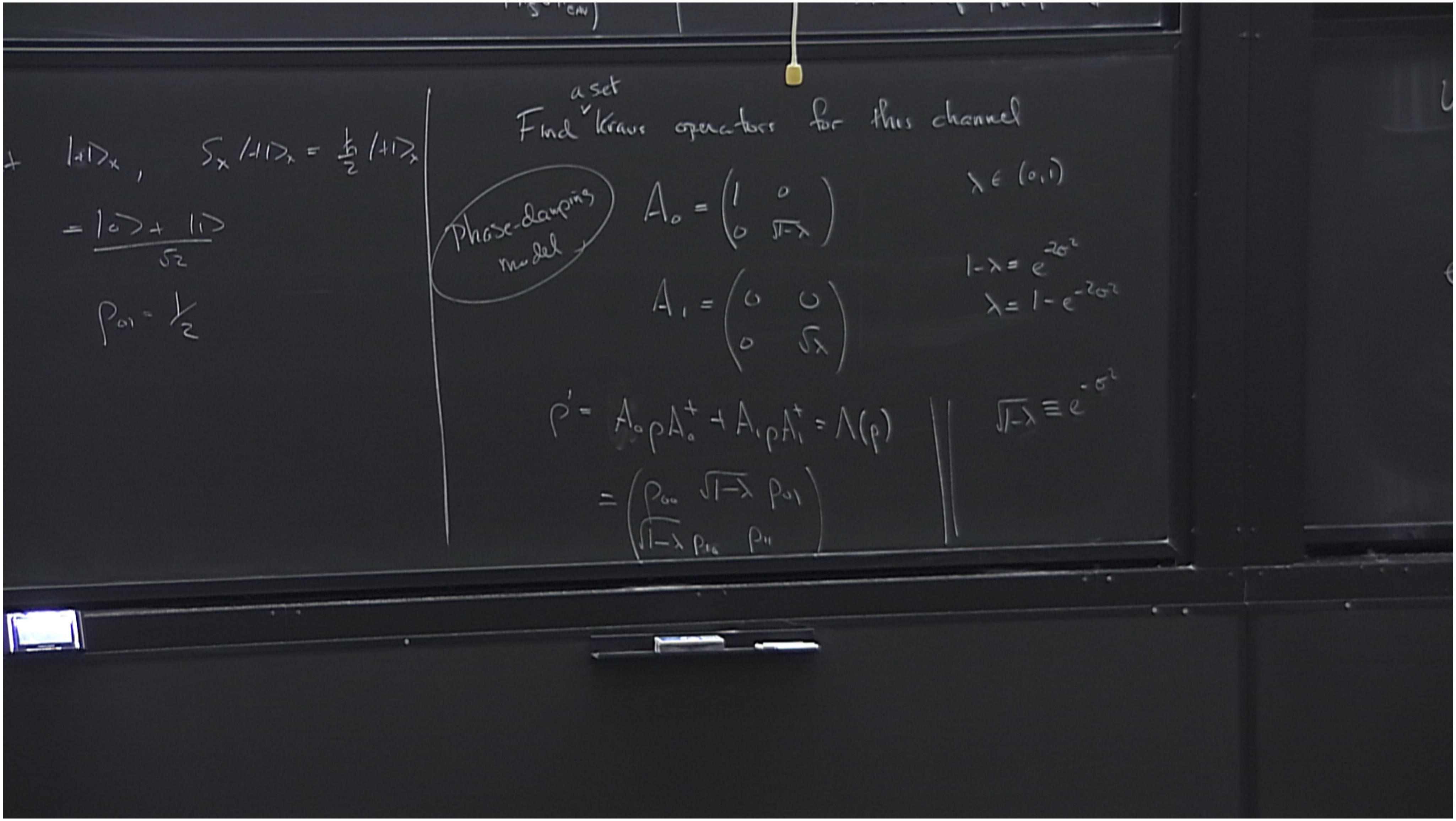
$\rho_{01} = 1/2$

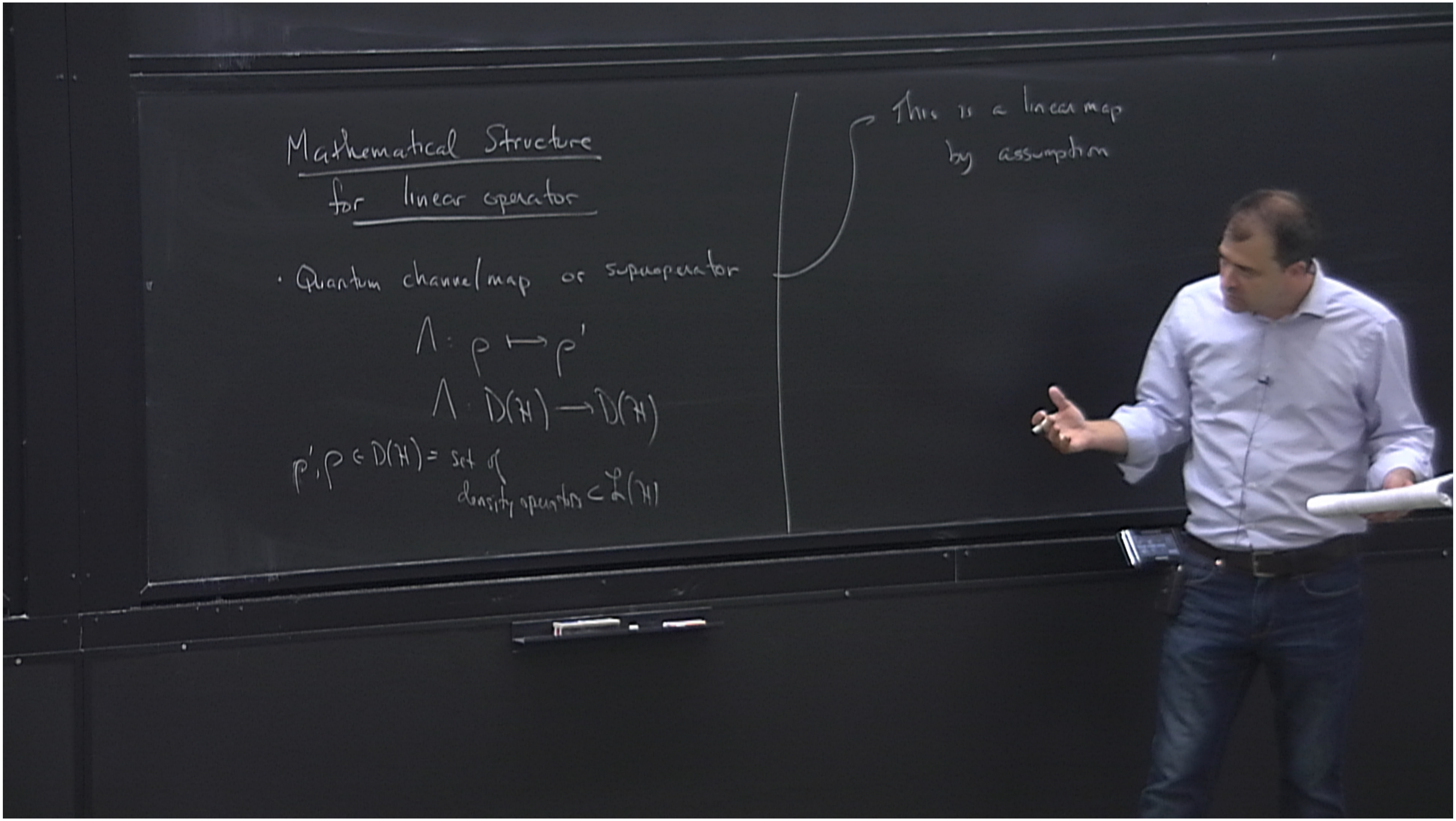

Find a set Kraus operators for this channel
|+1>x , Sx |+1>x = ħ/2 |+1>x
= (|0> + |1>)/√2
ρ01 = 1/2

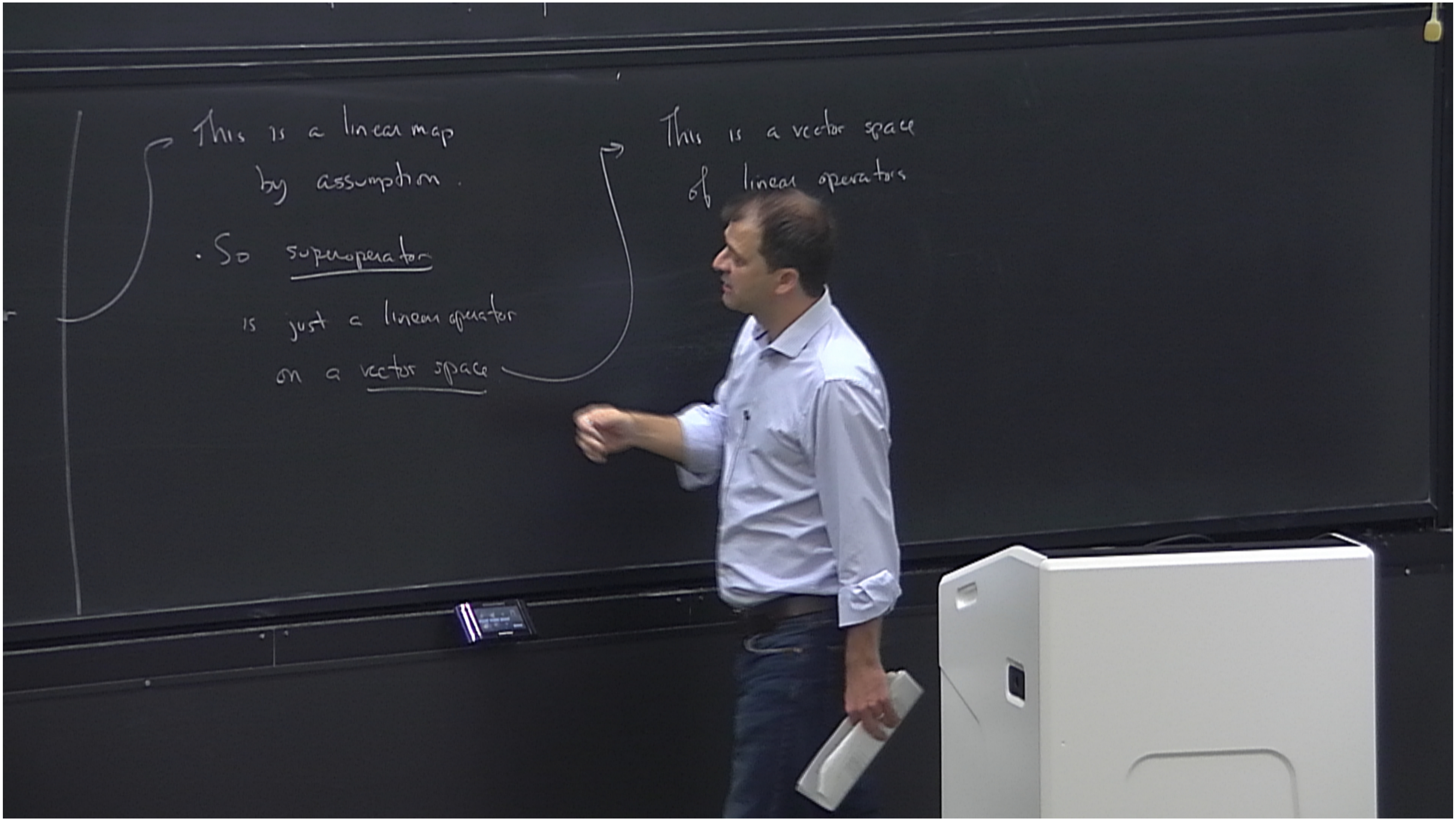

Find a set Kraus operators for this channel
$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-x} \end{pmatrix}$
$A_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{x} \end{pmatrix}$
$\rho' = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger = \Lambda(\rho)$
$|+1\rangle_x$, $S_x |+1\rangle_x = \frac{\hbar}{2} |+1\rangle_x$

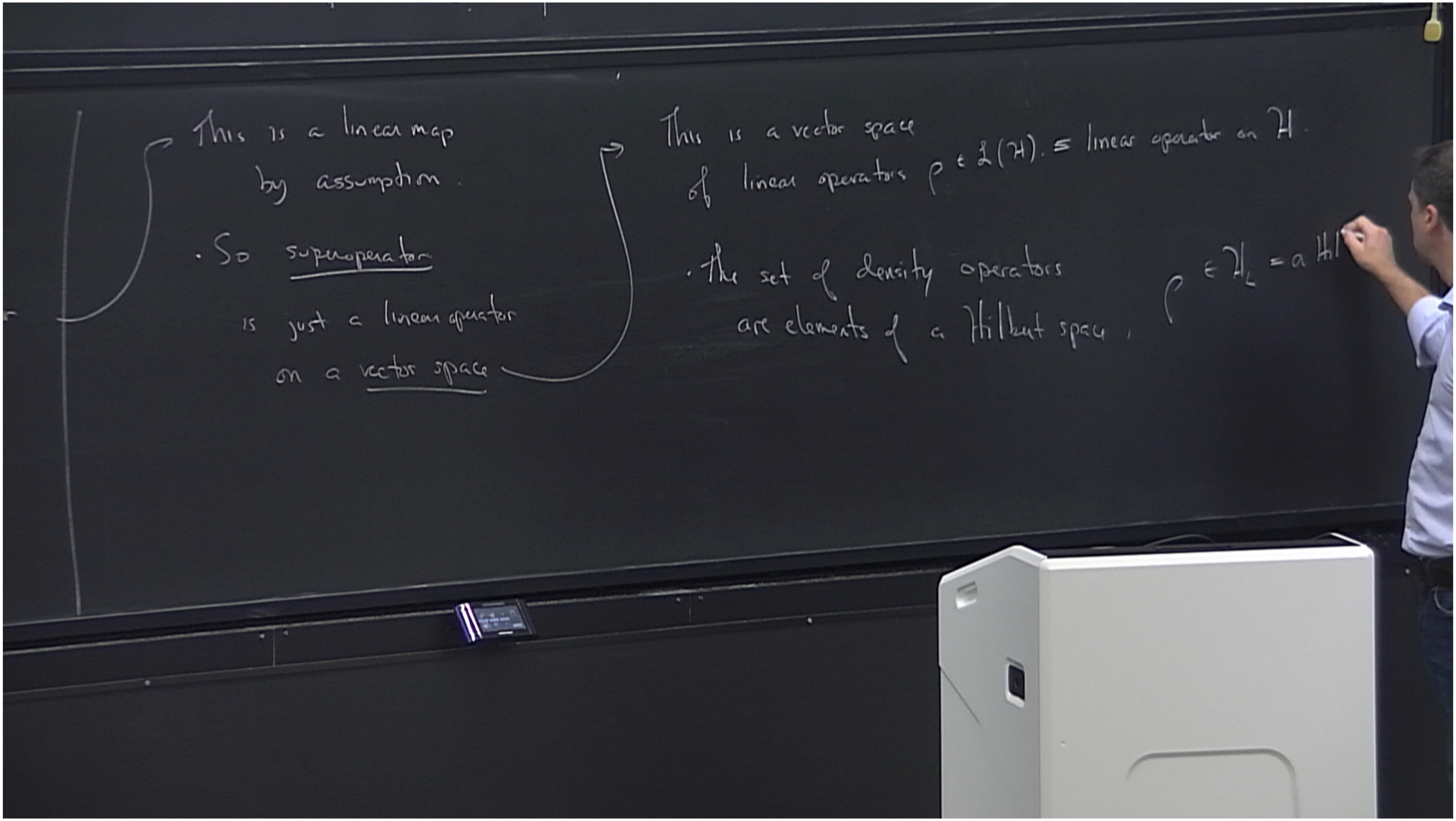

Find a set Kraus operators for this channel
$|{+1}\rangle_x$, $S_x|{+1}\rangle_x = \frac{\hbar}{2}|{+1}\rangle_x$
$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$\rho_{01} = 1/2$
$\lambda \in (0,1)$
$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}$
$A_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{pmatrix}$
$\rho' = A_0 \rho A_0^+ + A_1 \rho A_1^+ = \Lambda(\rho)$
$= \begin{pmatrix} \rho_{00} & \sqrt{1-\lambda}\,\rho_{01} \\ \sqrt{1-\lambda}\,\rho_{10} & \rho_{11} \end{pmatrix}$
$\sqrt{1-\lambda} \equiv e^{-\sigma^2}$

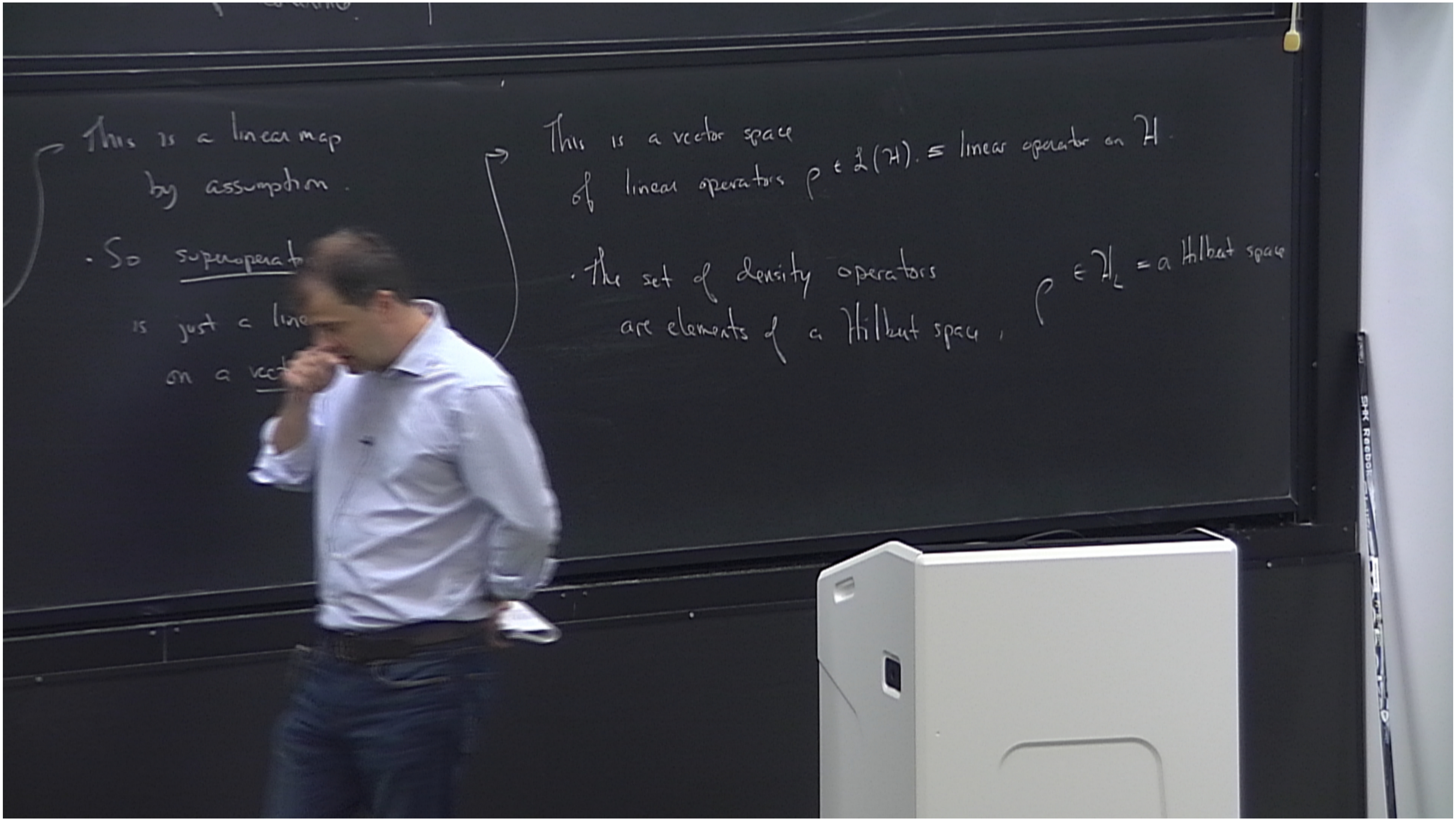

Find a set Kraus operators for this channel
Phase-damping model
A₀ = (1 0 / 0 √(1-λ))
A₁ = (0 0 / 0 √λ)
λ ∈ (0,1)
1-λ = e^(-2σ²)
λ = 1 - e^(-2σ²)
√(1-λ) ≡ e^(-σ²)
ρ' = A₀ρA₀⁺ + A₁ρA₁⁺ = Λ(ρ)
= (ρ₀₀ √(1-λ) ρ₀₁ / √(1-λ) ρ₁₀ ρ₁₁)
ρ₀₁ = 1/2


Find a set Kraus operators for this channel
Phase-damping model
$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}$
$A_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{pmatrix}$
$\lambda \in (0,1)$
$1-\lambda = e^{-2\sigma^2}$
$\lambda = 1 - e^{-2\sigma^2}$
$\sqrt{1-\lambda} \equiv e^{-\sigma^2}$
$\rho' = A_0 \rho A_0^+ + A_1 \rho A_1^+ = \Lambda(\rho)$
$= \begin{pmatrix} \rho_{00} & \sqrt{1-\lambda}\,\rho_{01} \\ \sqrt{1-\lambda}\,\rho_{10} & \rho_{11} \end{pmatrix}$
$|{+1}\rangle_x$, $S_x |{+1}\rangle_x = \frac{\hbar}{2} |{+1}\rangle_x$
$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$\rho_{01} = 1/2$


Mathematical Structure
for linear operator
· Quantum channel/map or superoperator
Λ : ρ ↦ ρ'
Λ : D(H) → D(H)
ρ', ρ ∈ D(H) = set of
density operators ⊂ L(H)
This is a linear map
by assumption


This is a linear map
by assumption.
• So superoperator
is just a linear operator
on a vector space
This is a vector space
of linear operators


This is a linear map
by assumption.
· So superoperator
is just a linear operator
on a vector space
This is a vector space
of linear operators $\rho \in \mathcal{L}(\mathcal{H})$. ≡ linear operator on $\mathcal{H}$.
· The set of density operators
are elements of a Hilbert space.
$\rho \in \mathcal{H}_L$ = a Hil


This is a linear map
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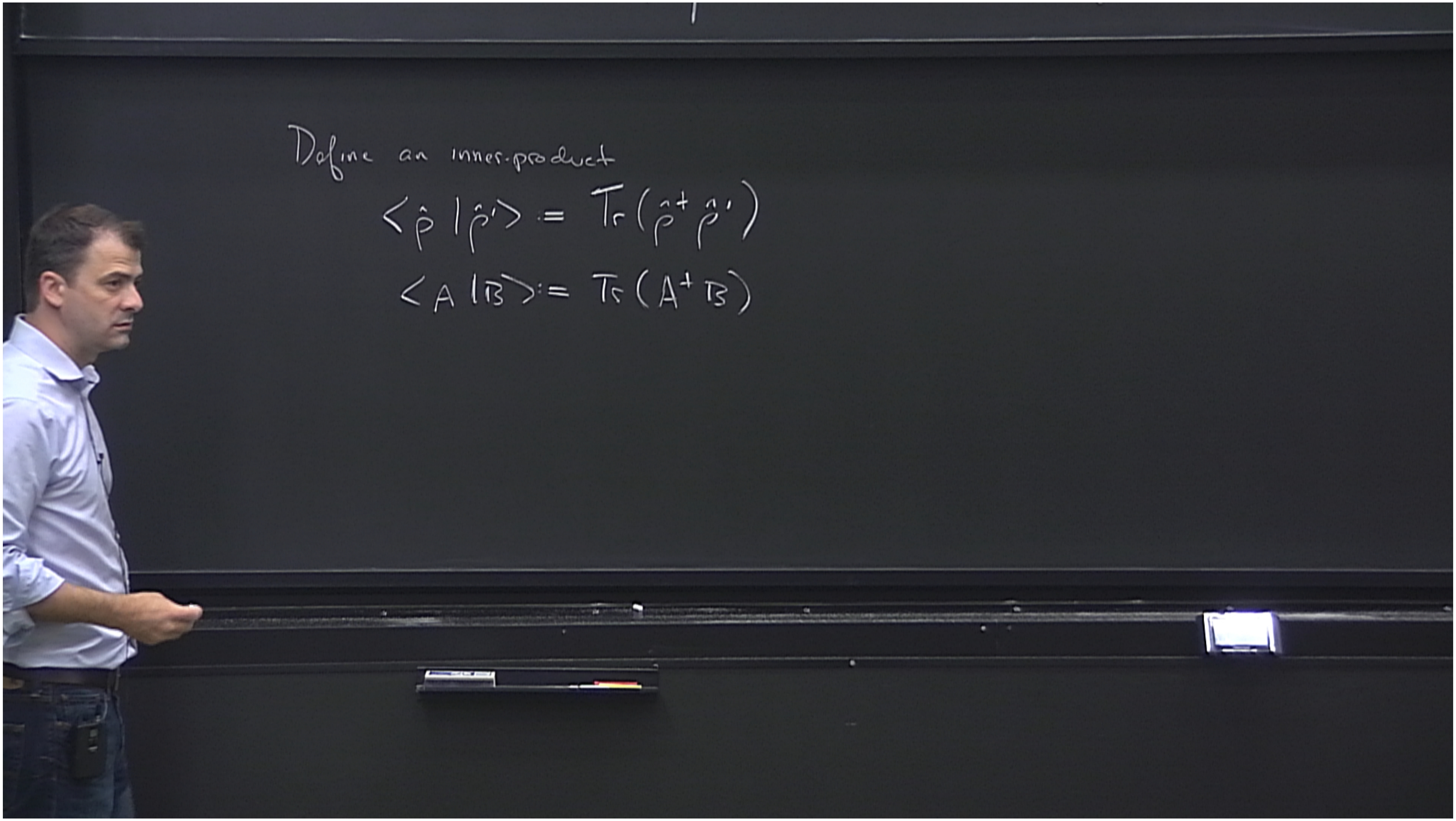
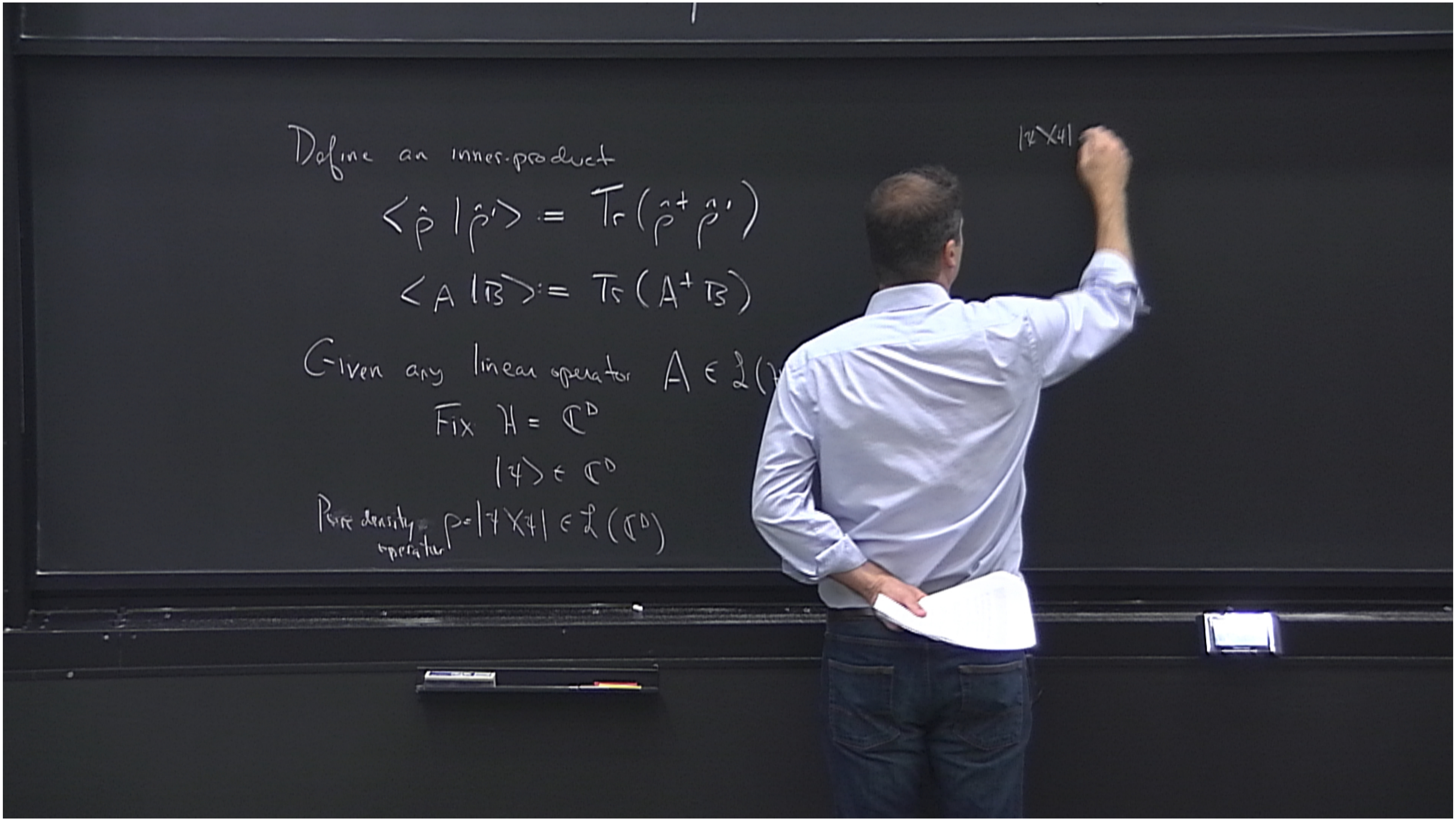
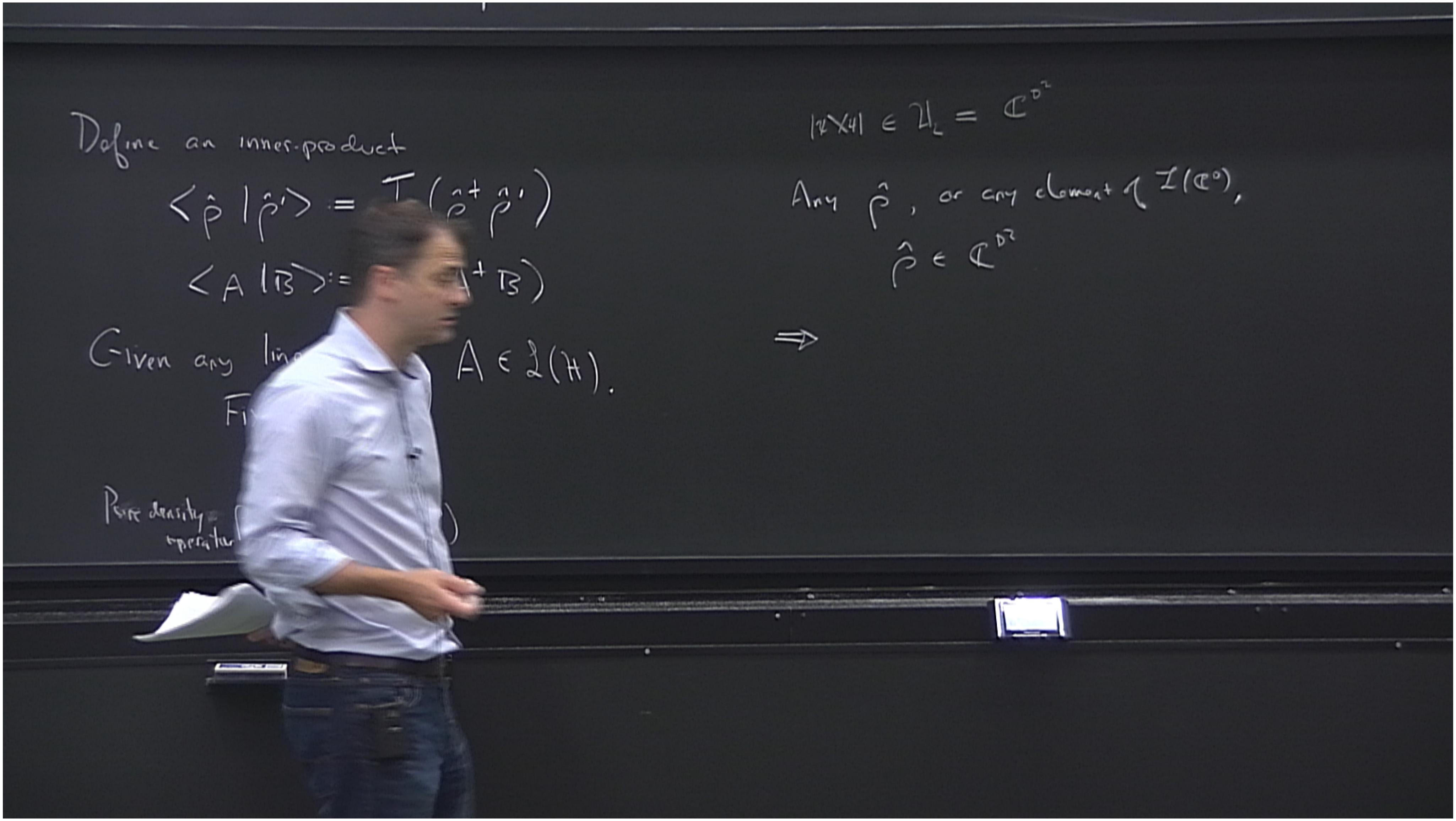

Define an inner-product
$\langle \hat{\rho} | \hat{\rho}' \rangle := \mathrm{Tr}(\hat{\rho}^{+} \hat{\rho}')$
$\langle A | B \rangle := \mathrm{Tr}(A^{+} B)$


Define an inner-product
$\langle \hat{\rho} | \hat{\rho}' \rangle := Tr(\hat{\rho}^\dagger \hat{\rho}')$
$\langle A | B \rangle := Tr(A^\dagger B)$
Given any linear operator $A \in \mathcal{L}($
Fix $H = \mathbb{C}^D$
$|\psi\rangle \in \mathbb{C}^D$
Pure density operator $\rho = |\psi\rangle\langle\psi| \in \mathcal{L}(\mathbb{C}^D)$
$|\psi\rangle\langle\psi|$



Define an inner-product
$\langle \hat{\rho} | \hat{\rho}' \rangle :=$
$\langle A | B \rangle :=$
Given any lin
$A \in \mathcal{L}(\mathcal{H})$.
Pure density operator
$|\psi\rangle\langle\psi| \in \mathcal{H}_L = \mathbb{C}^{D^2}$
Any $\hat{\rho}$, or any element of $\mathcal{L}(\mathbb{C}^D)$,
$\hat{\rho} \in \mathbb{C}^{D^2}$
$\Rightarrow$

Define an inner-product

$$\langle \hat{\rho} | \hat{\rho}' \rangle = \mathrm{Tr}(\hat{\rho}^\dagger \hat{\rho}')$$

$$\langle A | B \rangle := \mathrm{Tr}(A^\dagger B)$$

Given any linear operator $A \in \mathcal{L}(\mathcal{H})$.

Fix $\mathcal{H} = \mathbb{C}^D$

$|\psi\rangle \in \mathbb{C}^D$

Pure density operator $\rho = |\psi\rangle\langle\psi| \in \mathcal{L}(\mathbb{C}^D)$

$|\psi\rangle\langle\psi| \in \mathcal{H}_L = \mathbb{C}^{D^2}$

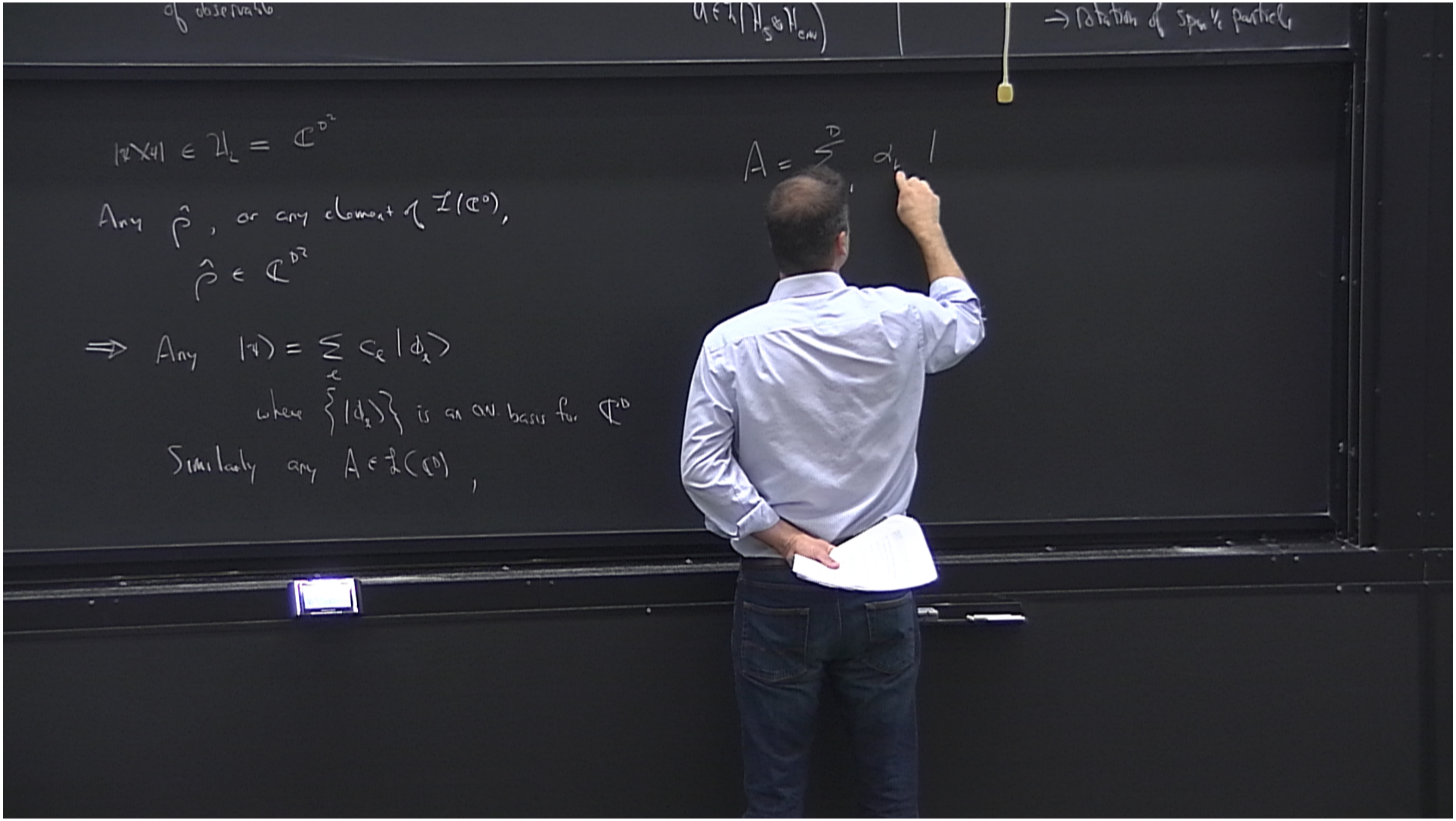
Any $\hat{\rho}$, or any element of $\mathcal{L}(\mathbb{C}^D)$,

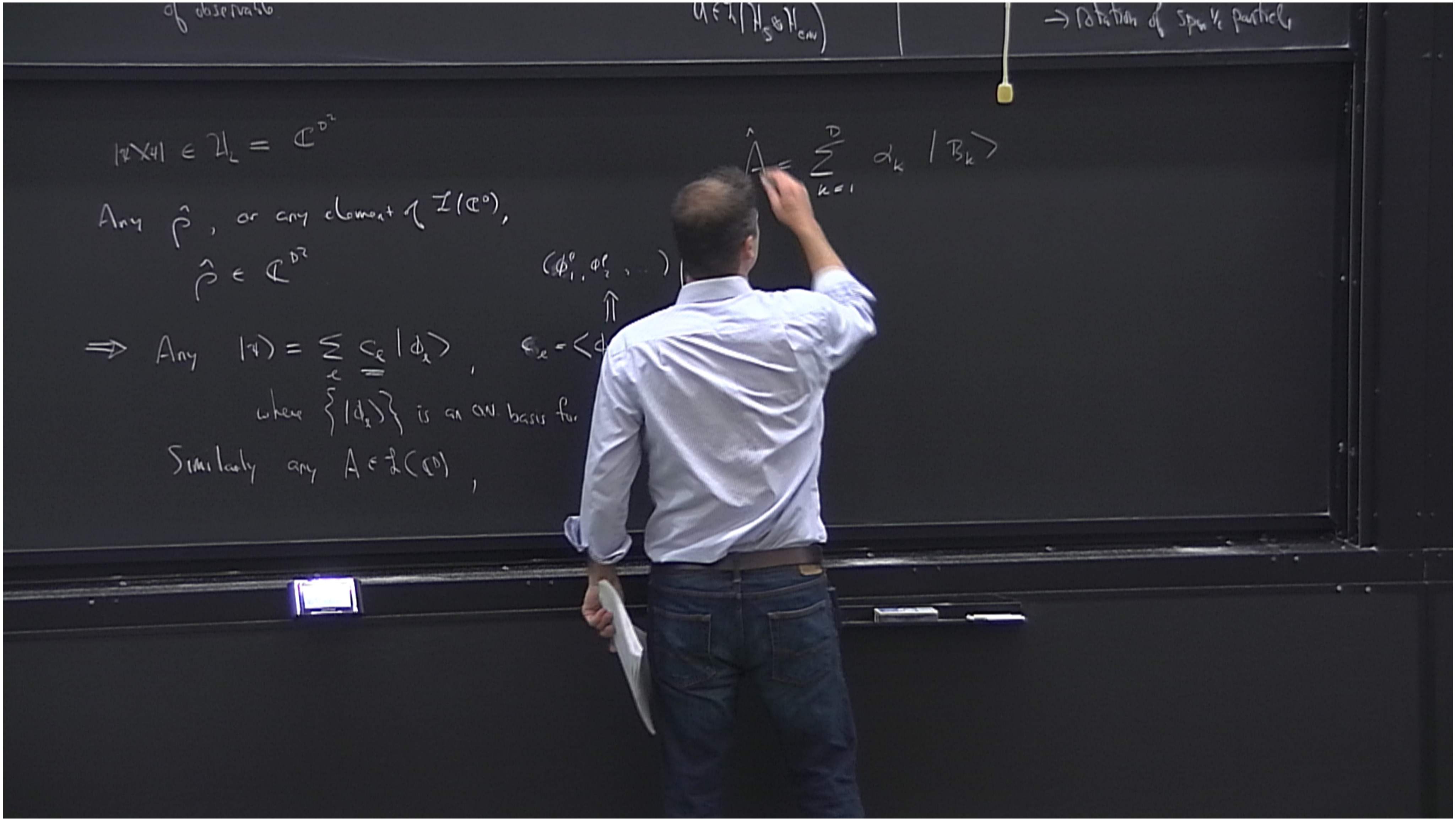
$\hat{\rho} \in \mathbb{C}^{D^2}$

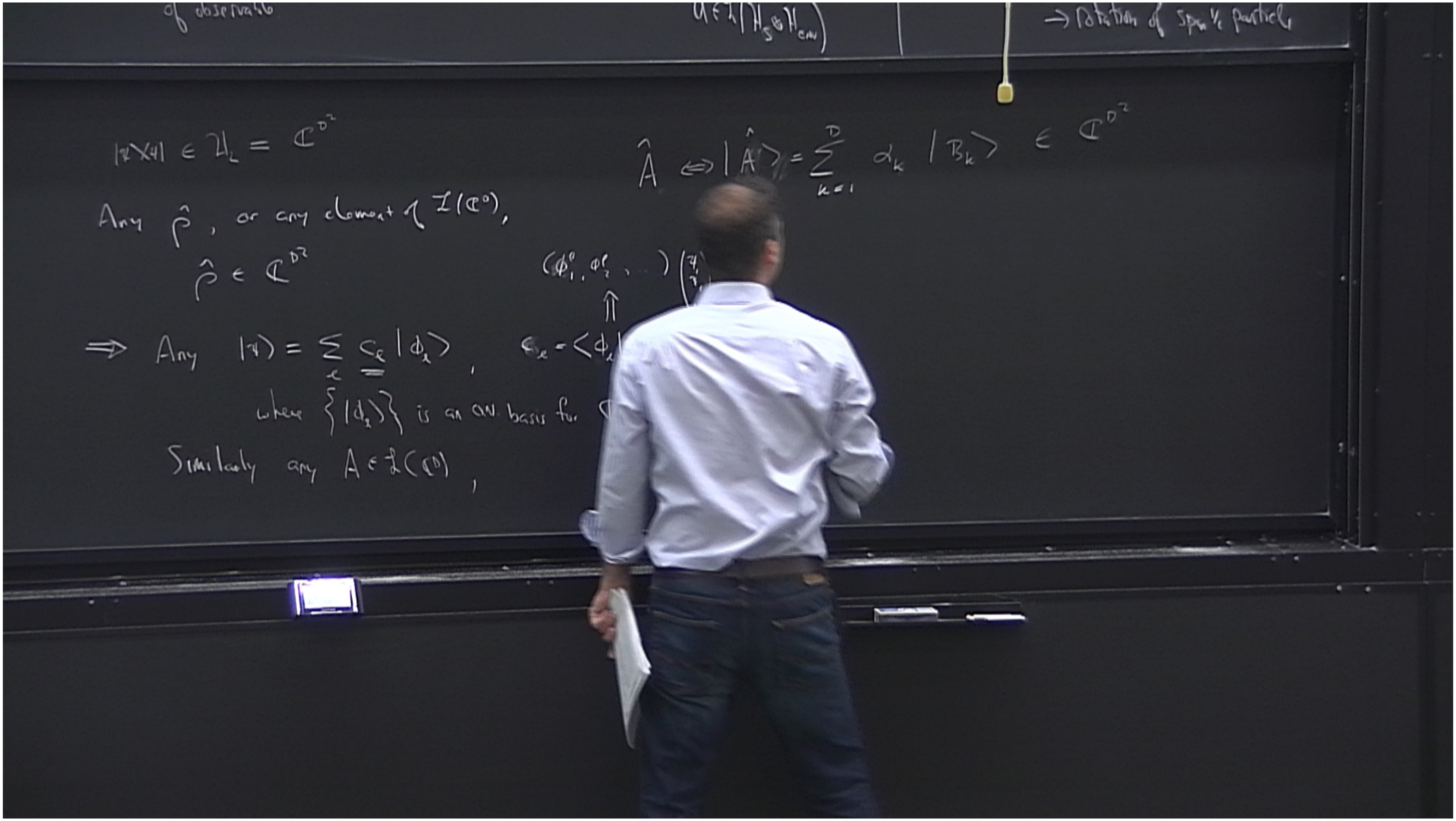
$\Rightarrow$ Any $|\psi\rangle = \sum_k c_k |\phi_k\rangle$

where $\{|\phi_k\rangle\}$ is an O.N. basis f

Similarly any $A \in \mathcal{L}(\mathbb{C}^D)$,


of observable
$U \in \mathcal{L}(H_S \otimes H_{env})$
→ rotation of spin ½ particle
$|\psi\rangle\langle\psi| \in \mathcal{H}_L = \mathbb{C}^{D^2}$
Any $\hat{\rho}$, or any element of $\mathcal{L}(\mathbb{C}^D)$,
$\hat{\rho} \in \mathbb{C}^{D^2}$
$\Rightarrow$ Any $|\psi\rangle = \sum_{\ell} c_\ell |\phi_\ell\rangle$
where $\{|\phi_\ell\rangle\}$ is an ON. basis for $\mathbb{C}^D$
Similarly any $A \in \mathcal{L}(\mathbb{C}^D)$,
$A = \sum^{D} \alpha_k |$


of observable
→ rotation of spin ½ particle
|ψ⟩⟨ψ| ∈ H_L = C^{D²}
Â = Σ_{k=1}^{D} α_k |B_k⟩
Any ρ̂, or any element of L(C^D),
ρ̂ ∈ C^{D²}
(φ_1^ℓ, φ_2^ℓ, ...)
⇒ Any |ψ⟩ = Σ_ℓ c_ℓ |φ_ℓ⟩, c_ℓ = ⟨φ
where {|φ_ℓ⟩} is an O.N. basis for
Similarly any A ∈ L(C^D),


of observable
→ rotation of spin ½ particle
$|\psi\rangle\langle\psi| \in \mathcal{H}_L = \mathbb{C}^{D^2}$
$\hat{A} \Leftrightarrow |\hat{A}\rangle = \sum_{k=1}^{D} \alpha_k |B_k\rangle \in \mathbb{C}^{D^2}$
Any $\hat{\rho}$, or any element of $\mathcal{L}(\mathbb{C}^D)$,
$\hat{\rho} \in \mathbb{C}^{D^2}$
$\Rightarrow$ Any $|\psi\rangle = \sum_\ell c_\ell |\phi_\ell\rangle$,
where $\{|\phi_\ell\rangle\}$ is an ON basis for
Similarly any $A \in \mathcal{L}(\mathbb{C}^D)$,

of observable

$U \in \mathcal{L}(H_S \otimes H_{env})$

$\rightarrow$ rotation of spin ½ particle

$|\psi\rangle\langle\psi| \in \mathcal{H}_L = \mathbb{C}^{D^2}$

Any $\hat{\rho}$, or any element of $\mathcal{L}(\mathbb{C}^D)$,

$\hat{\rho} \in \mathbb{C}^{D^2}$

$(\phi_1^{\ell}, \phi_2^{\ell}, \dots) \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}$

$\Uparrow$

$\Rightarrow$ Any $|\psi\rangle = \sum_{\ell} \underline{\underline{c_\ell}} |\phi_\ell\rangle$, $\quad c_\ell = \langle \phi_\ell | \psi \rangle \in \mathbb{C}$

where $\{|\phi_\ell\rangle\}$ is an ON basis for $\mathbb{C}^D$

Similarly any $A \in \mathcal{L}(\mathbb{C}^D)$,

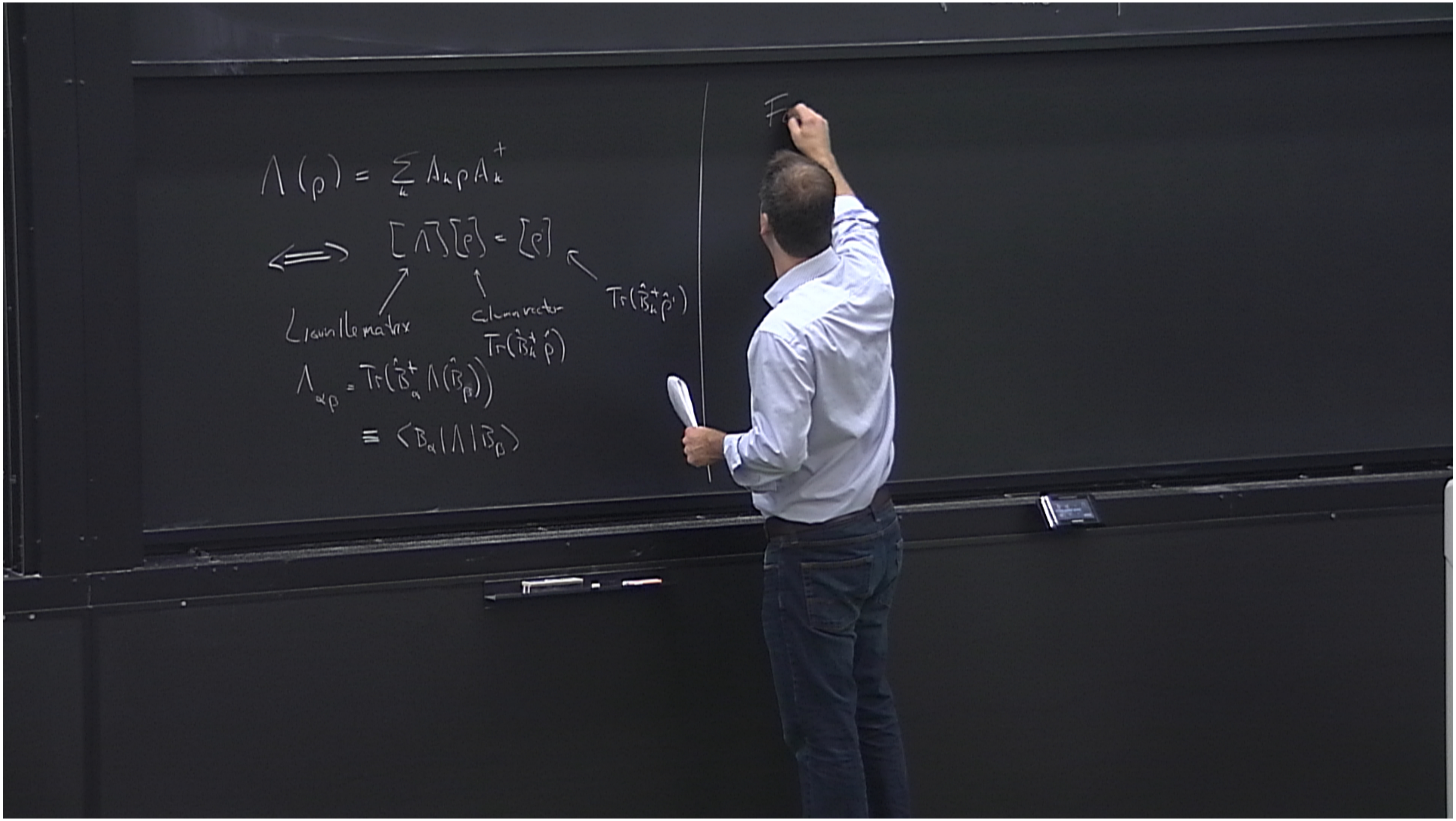
$$\hat{A} \Leftrightarrow |\hat{A}\rangle = \sum_{k=1}^{D^2} \alpha_k |\hat{B}_k\rangle \in \mathbb{C}^{D^2}$$

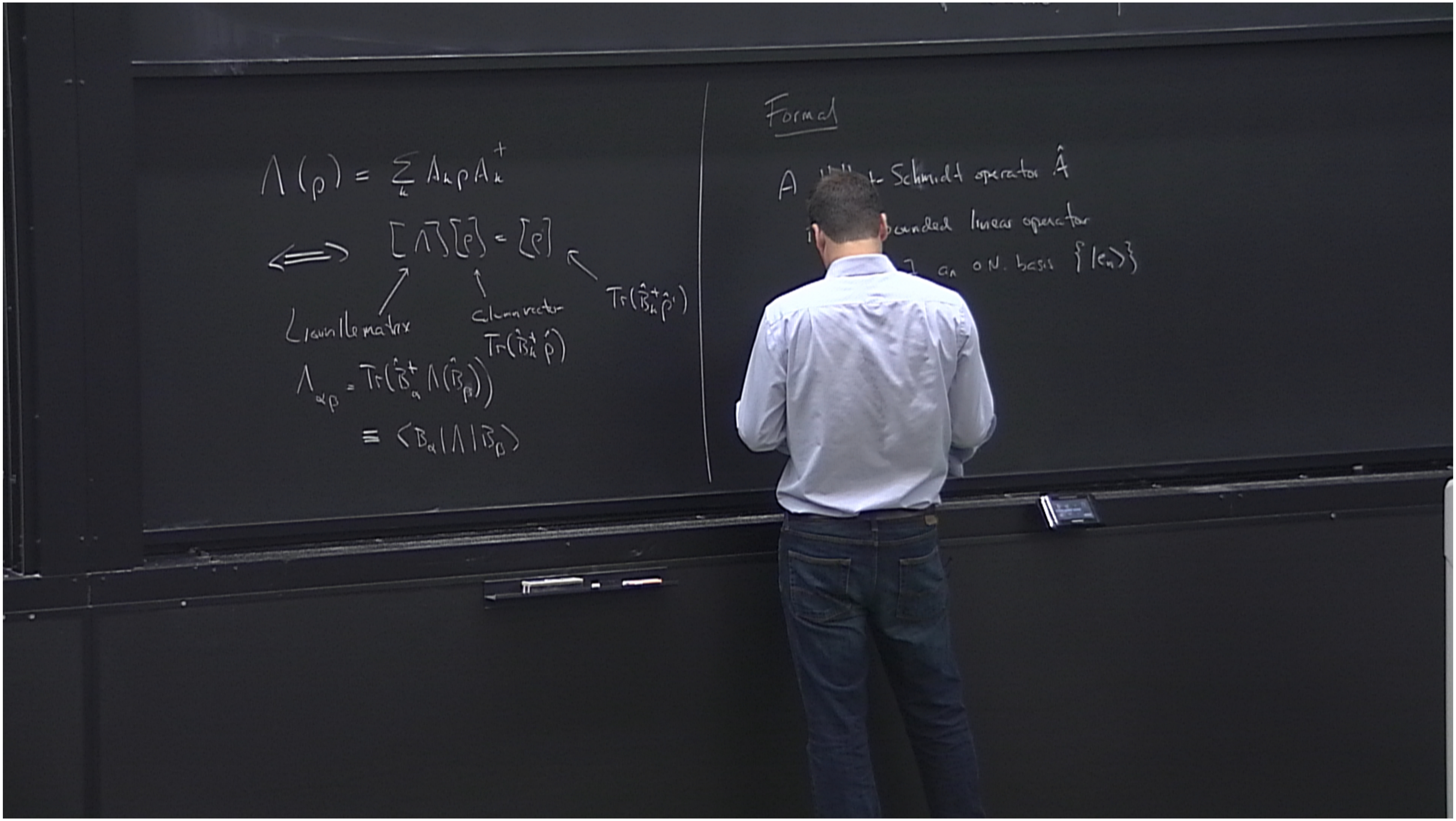
$$\alpha_k = \underline{\langle \hat{B}_k | \hat{A} \rangle} = \mathrm{Tr}(B_k^\dagger A) \in \mathbb{C}$$

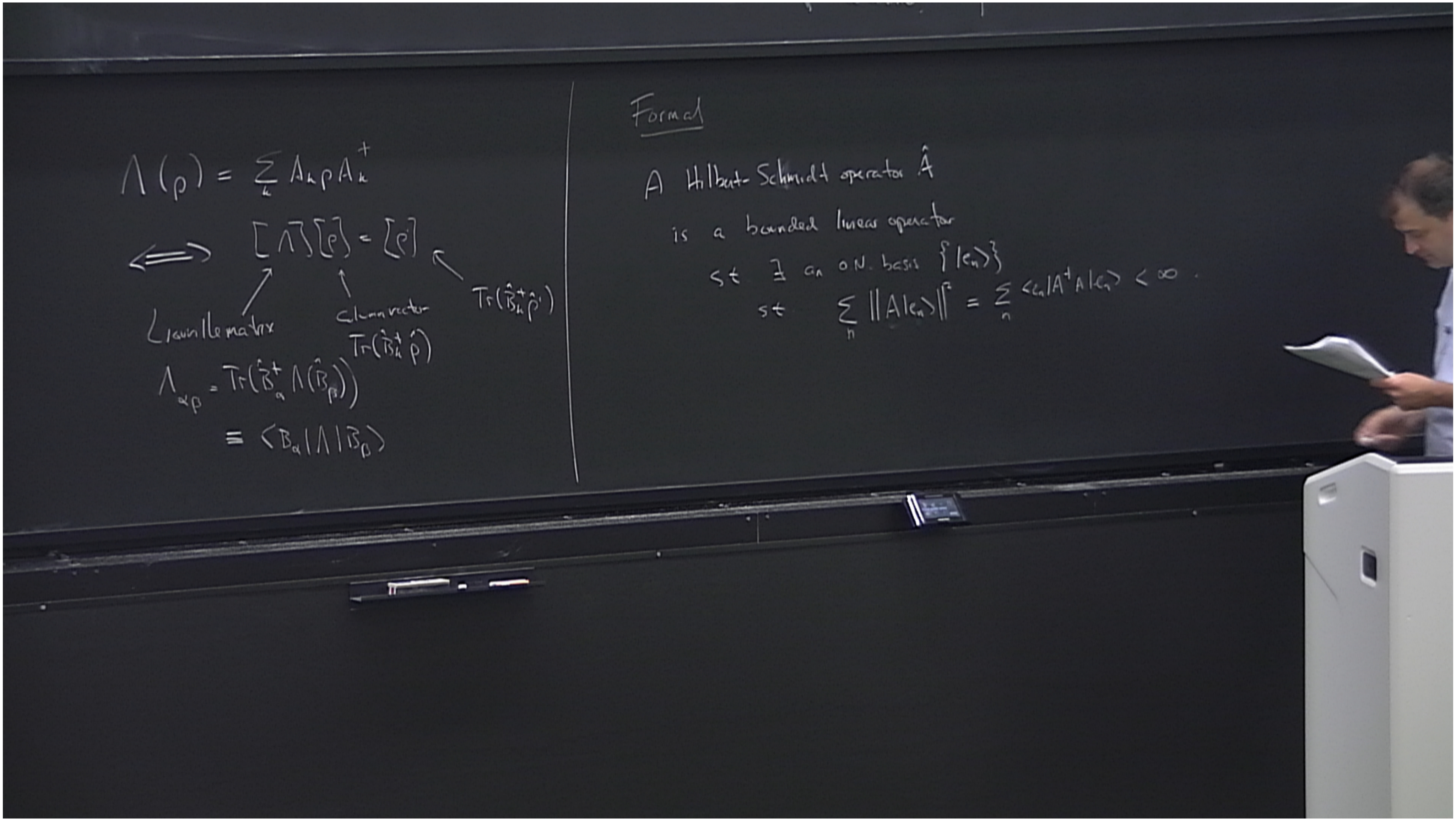
$\{|B_k\rangle\}$ is an ON basis

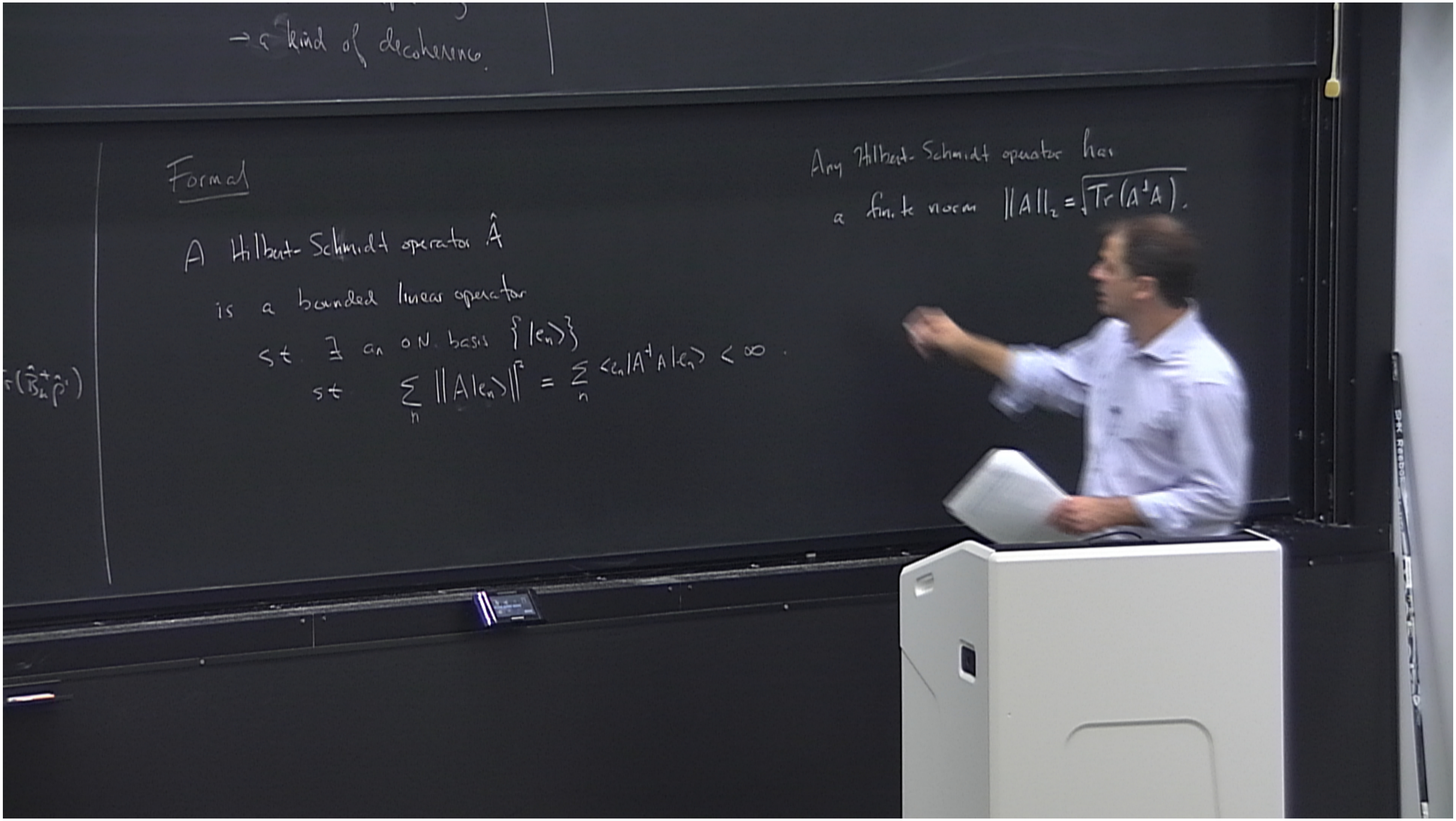
for $\mathbb{C}^{D^2} \cong \mathcal{L}(\mathbb{C}^D)$ = Liouville space.

$\mathrm{Tr}(B_k^\dagger B_\ell) = \delta_{k\ell}$


$\Lambda(\rho) = \sum_k A_k \rho A_k^\dagger$
$\Longleftrightarrow [\Lambda][\rho] = [\rho']$
Liouville matrix
Column vector
$Tr(\hat{B}_k^\dagger \hat{\rho})$
$Tr(\hat{B}_k^\dagger \hat{\rho}')$
$\Lambda_{\alpha\beta} = Tr(\hat{B}_\alpha^\dagger \Lambda(\hat{B}_\beta))$
$\equiv \langle B_\alpha | \Lambda | B_\beta \rangle$


Formal
Λ(ρ) = Σ_k A_k ρ A_k^†
⟺ [Λ][ρ] = [ρ']
Liouville matrix
Column vector
Tr(B̂_k^† ρ̂)
Tr(B̂_k^† ρ̂')
Λ_αβ = Tr(B̂_α^† Λ(B̂_β))
≡ ⟨B_α|Λ|B_β⟩
A t-Schmidt operator Â
ounded linear operator
an O.N. basis {|e_n⟩}


$\Lambda(\rho) = \sum_k A_k \rho A_k^\dagger$
$\Longleftrightarrow \quad [\Lambda][\rho] = [\rho']$
Liouville matrix
column vector
$Tr(\hat{B}_k^\dagger \hat{\rho})$
$Tr(\hat{B}_k^\dagger \hat{\rho}')$
$\Lambda_{\alpha\beta} = Tr(\hat{B}_\alpha^\dagger \Lambda(\hat{B}_\beta))$
$\equiv \langle B_\alpha | \Lambda | B_\beta \rangle$
Formal
A Hilbert-Schmidt operator $\hat{A}$
is a bounded linear operator
s.t. $\exists$ an O.N. basis $\{|e_n\rangle\}$
s.t. $\sum_n \| A|e_n\rangle \|^2 = \sum_n \langle e_n | A^\dagger A | e_n \rangle < \infty$.


→ a kind of decohering
Formal
A Hilbert-Schmidt operator $\hat{A}$
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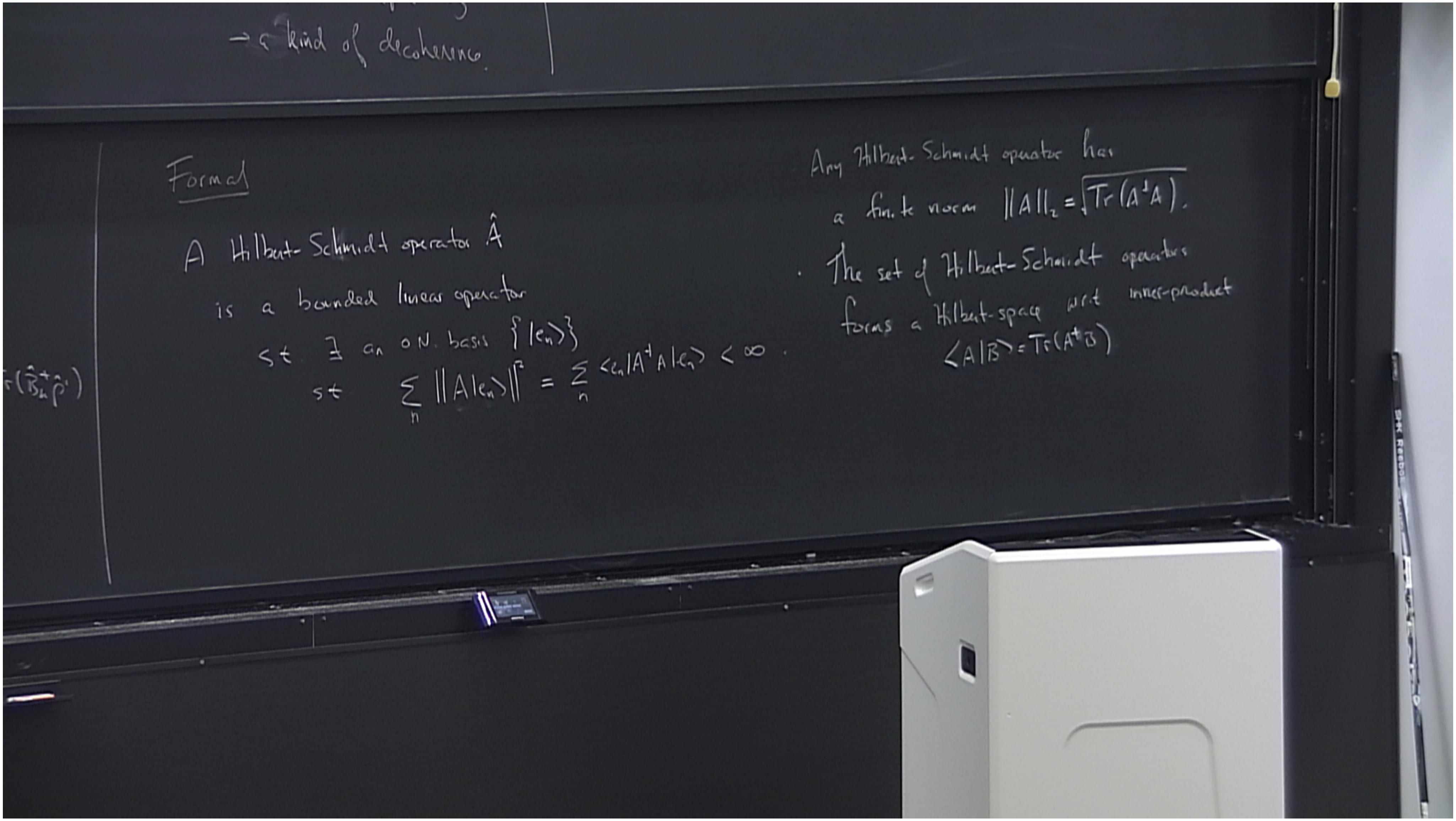
s.t. $\exists$ an O.N. basis $\{|e_n\rangle\}$
s.t. $\sum_n \| A|e_n\rangle \|^2 = \sum_n \langle e_n|A^\dagger A|e_n\rangle < \infty$.
Any Hilbert-Schmidt operator has
a finite norm $\|A\|_2 = \sqrt{\mathrm{Tr}(A^\dagger A)}$.


→ a kind of decoherence
Formal
A Hilbert-Schmidt operator $\hat{A}$
is a bounded linear operator
s.t $\exists$ an O.N. basis $\{|e_n\rangle\}$
s.t $\sum_n \| A|e_n\rangle \|^2 = \sum_n \langle e_n|A^\dagger A|e_n\rangle < \infty$.
Any Hilbert-Schmidt operator has
a finite norm $\|A\|_2 = \sqrt{\mathrm{Tr}(A^\dagger A)}$.
· The set of Hilbert-Schmidt operators
forms a Hilbert-space w.r.t inner-product
$\langle A|B\rangle = \mathrm{Tr}(A^\dagger B)$

$$\Lambda(\rho) = \sum_k A_k \rho A_k^+$$

$$\Longleftrightarrow \quad [\Lambda][\rho] = [\rho']$$

Liouville matrix

column vector $\mathrm{Tr}(\hat{B}_k^+ \hat{\rho})$

$\mathrm{Tr}(\hat{B}_k^+ \hat{\rho}')$

$$\Lambda_{\alpha\beta} = \mathrm{Tr}(\hat{B}_\alpha^+ \Lambda(\hat{B}_\beta))$$

$$\equiv \langle B_\alpha | \Lambda | B_\beta \rangle$$

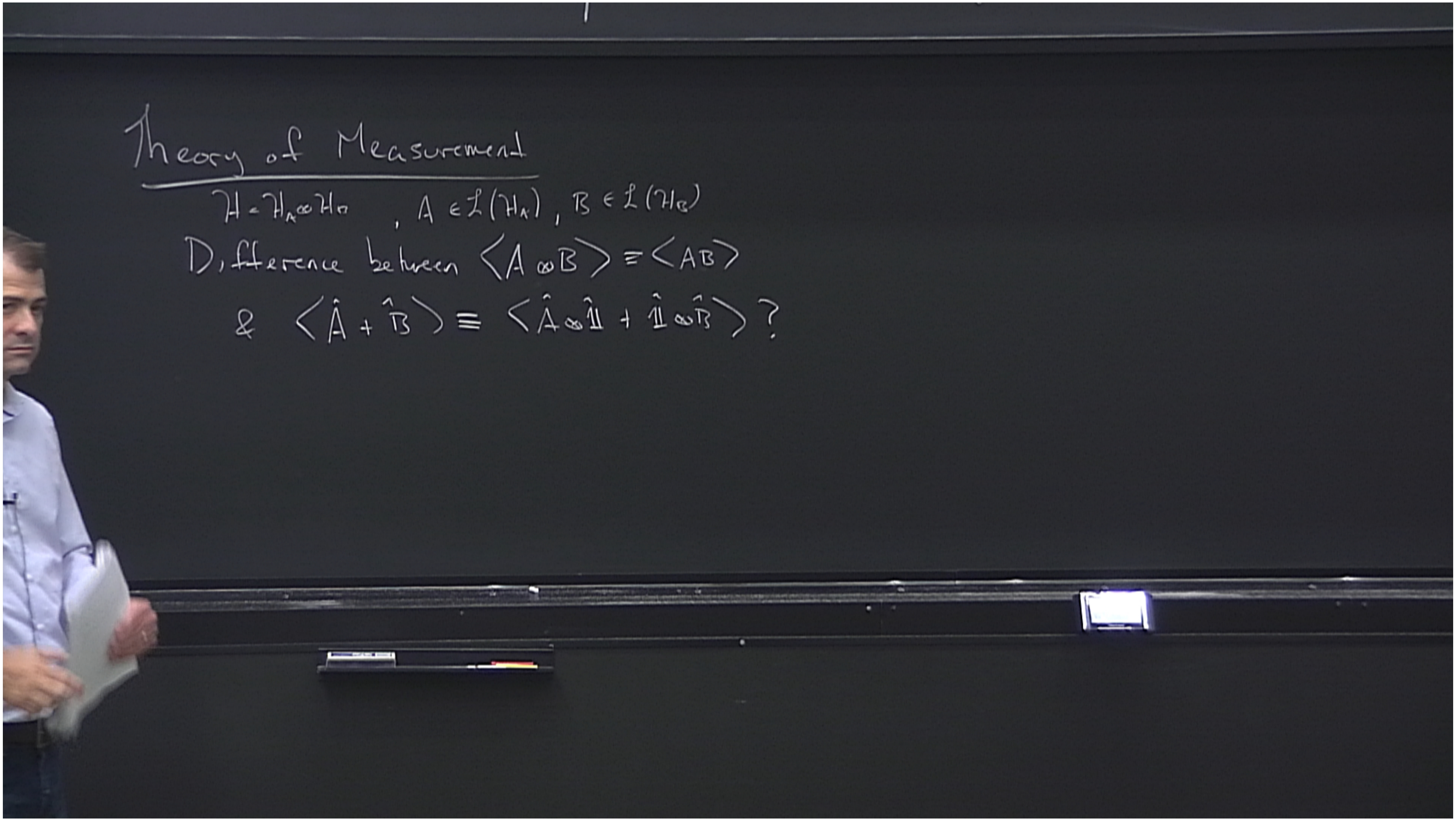
Formal

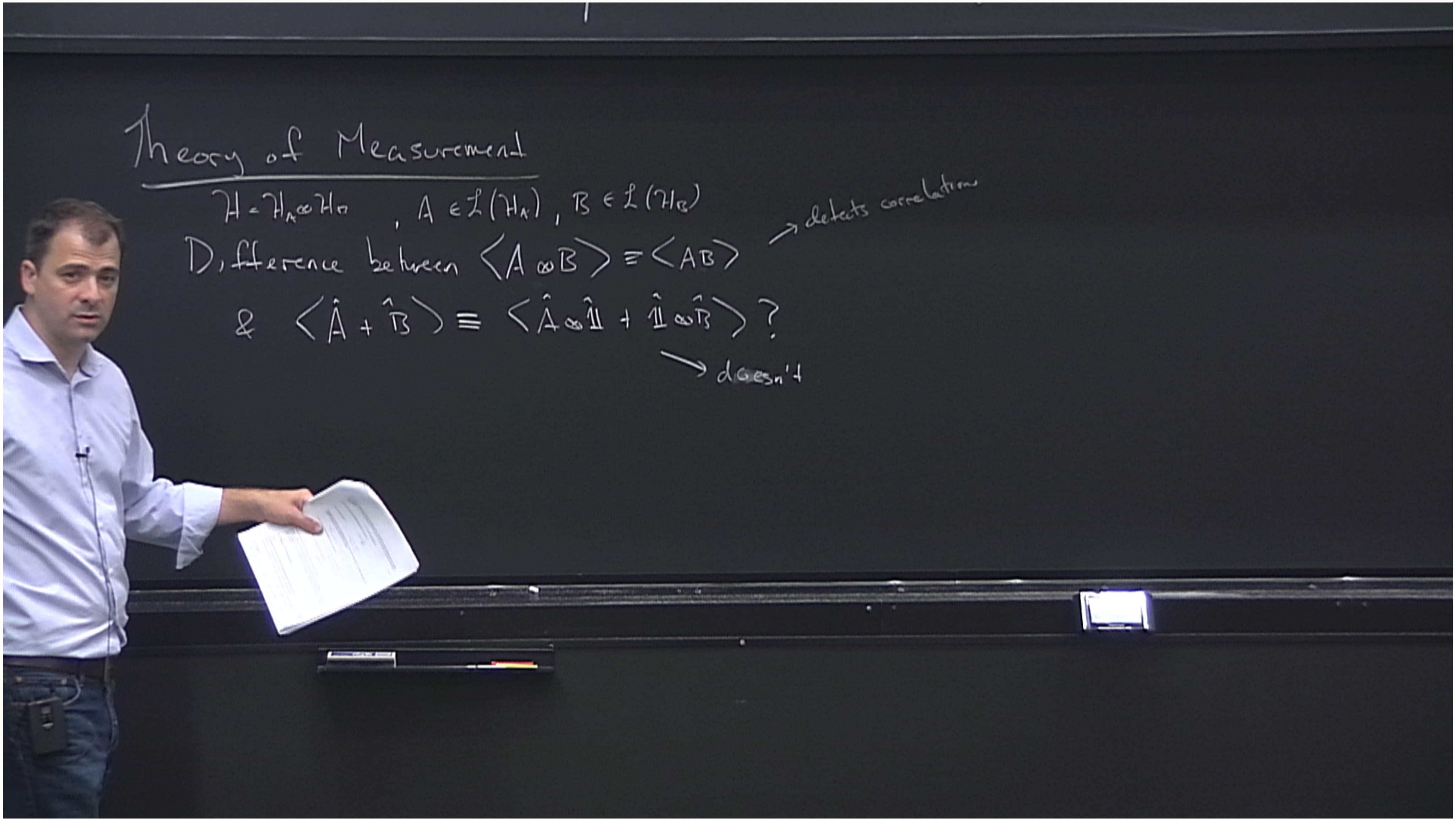
A Hilbert-Schmidt operator $\hat{A}$

is a bounded linear operator

s.t. $\exists$ an O.N. basis $\{|e_n\rangle\}$

s.t. $\sum_n \| A|e_n\rangle \|^2 = \sum_n \langle e_n | A^+ A$


Theory of Measurement
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, $A \in \mathcal{L}(\mathcal{H}_A)$, $B \in \mathcal{L}(\mathcal{H}_B)$
Difference between $\langle A \otimes B \rangle \equiv \langle AB \rangle$
& $\langle \hat{A} + \hat{B} \rangle \equiv \langle \hat{A} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{B} \rangle$?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→ detects correlations
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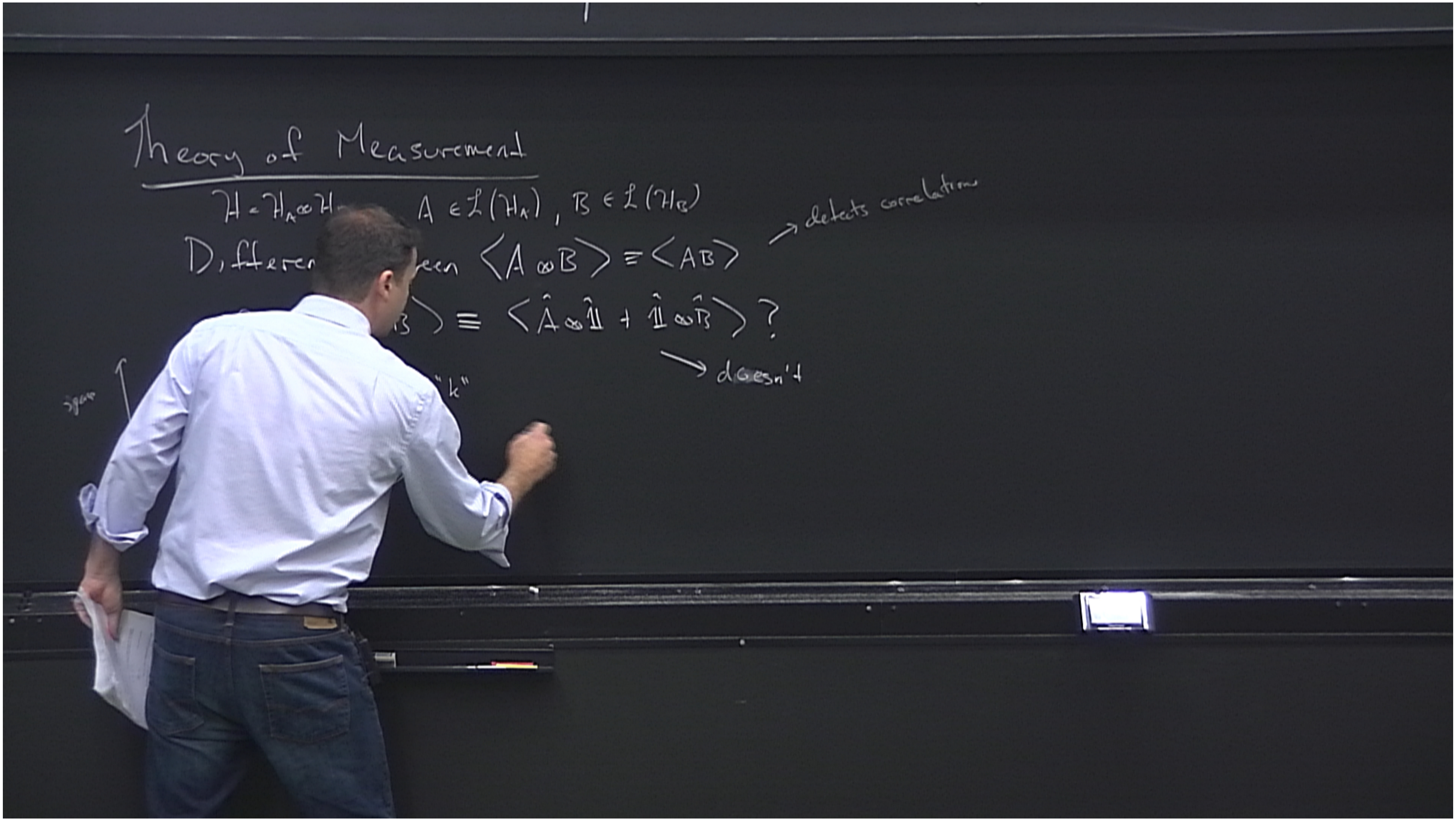
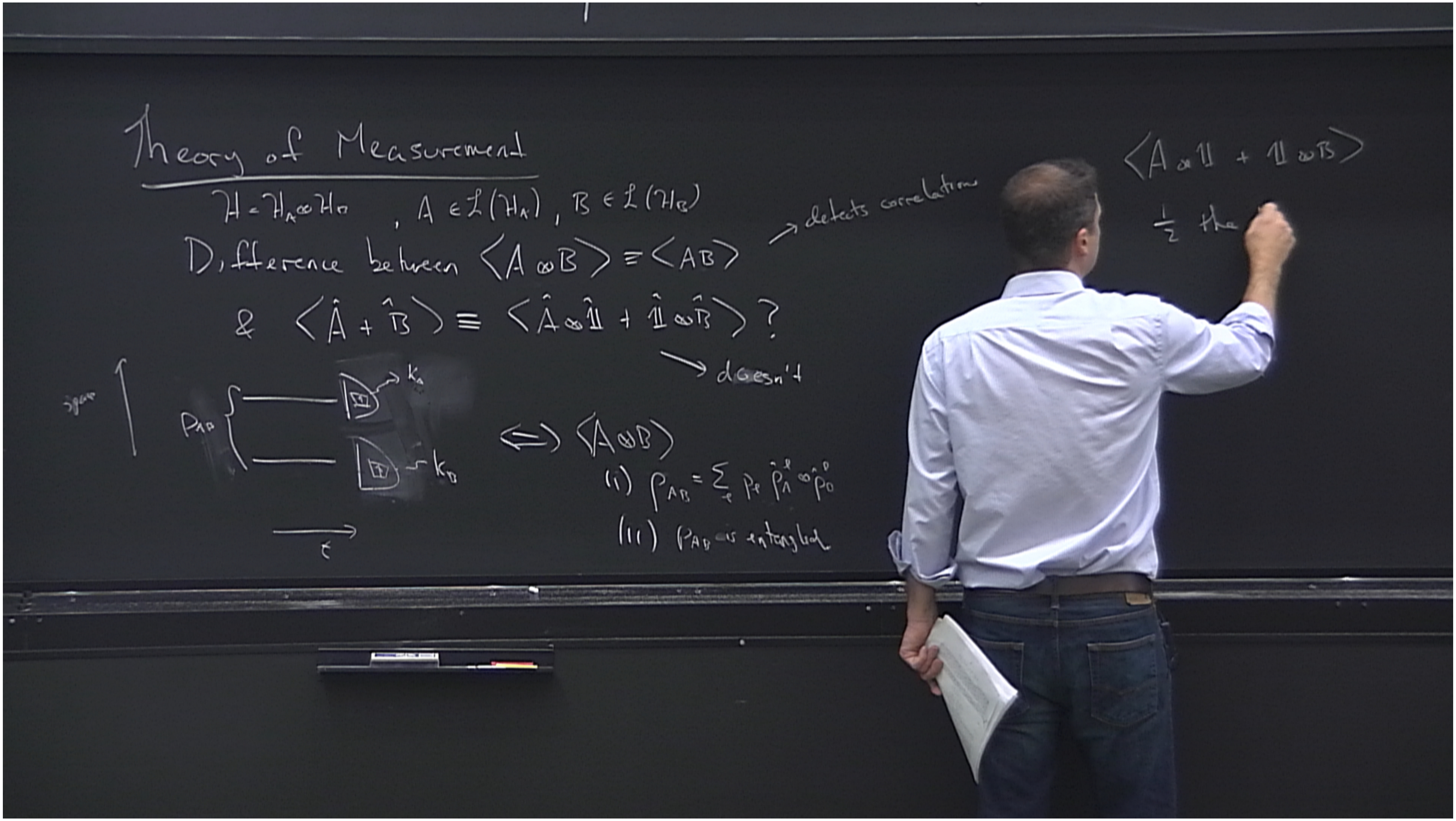
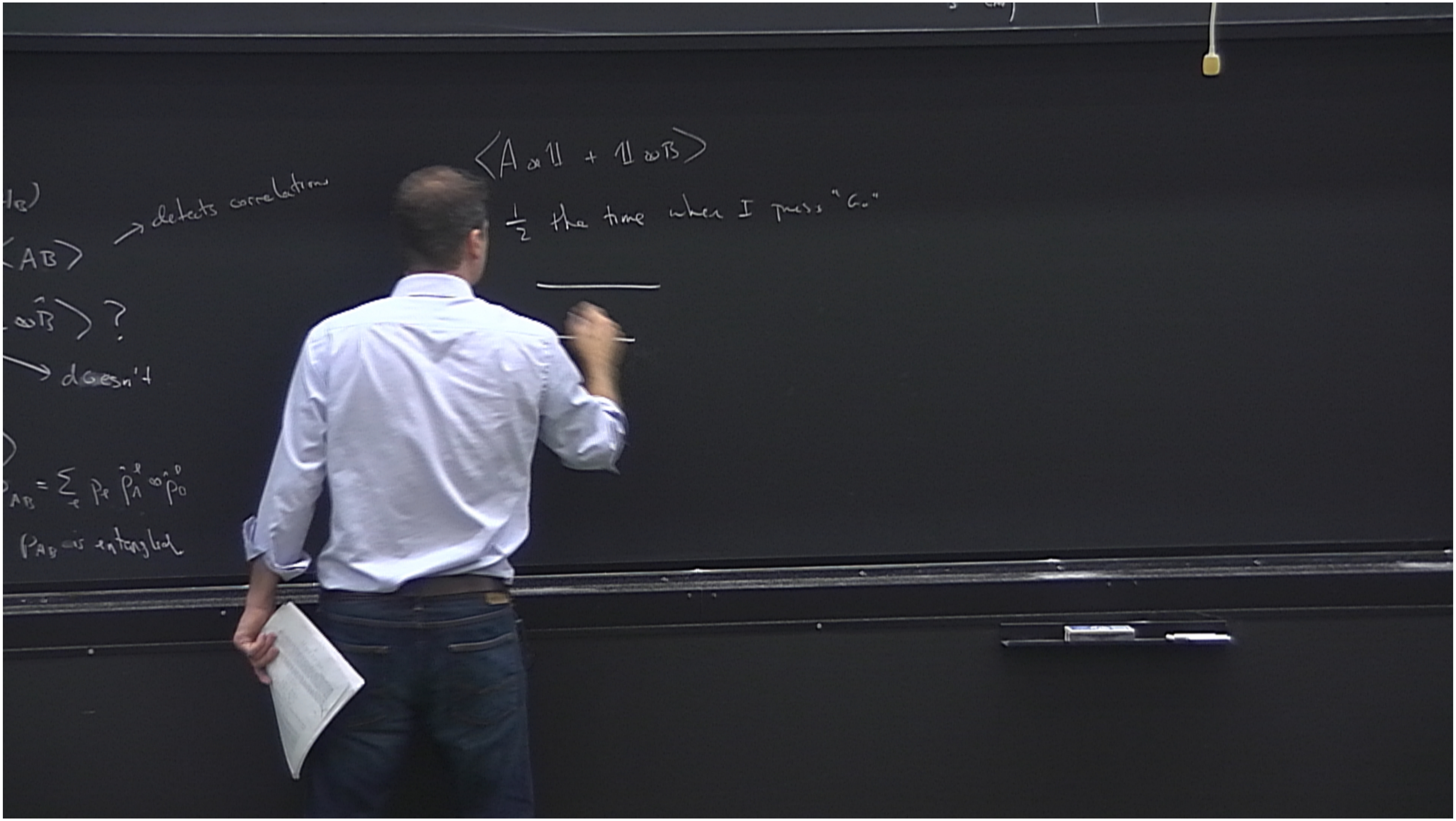
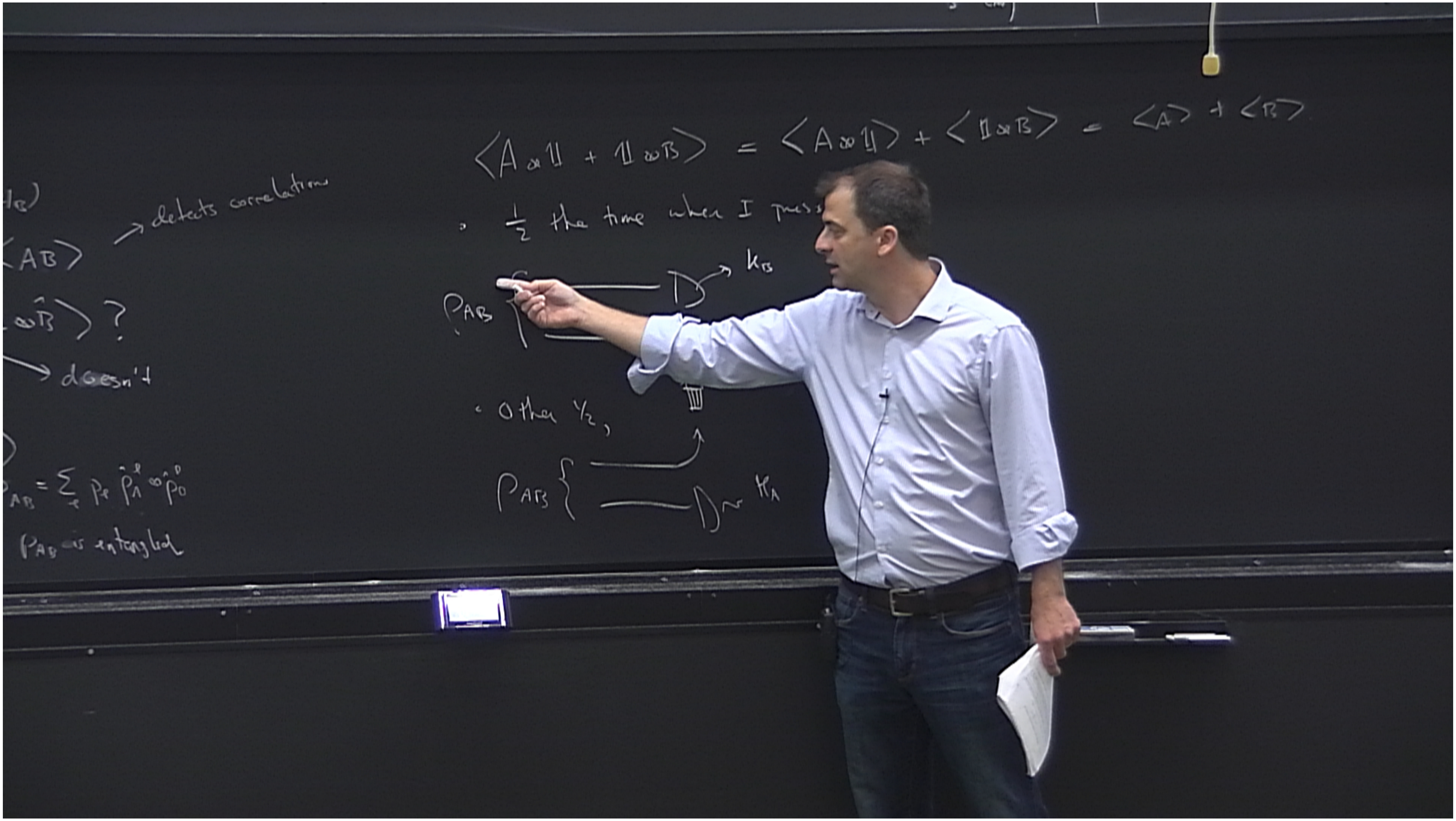
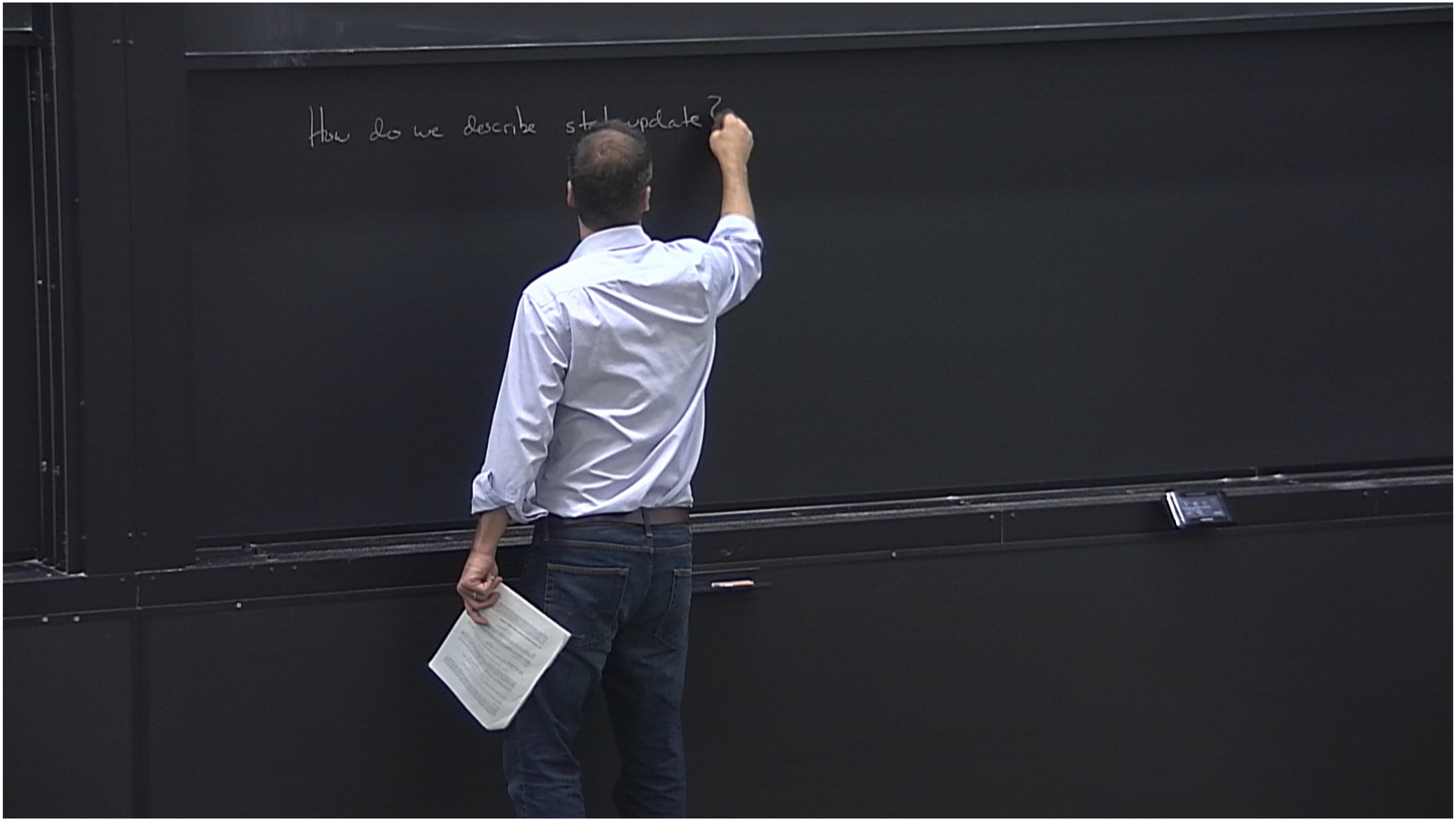
→ doesn't

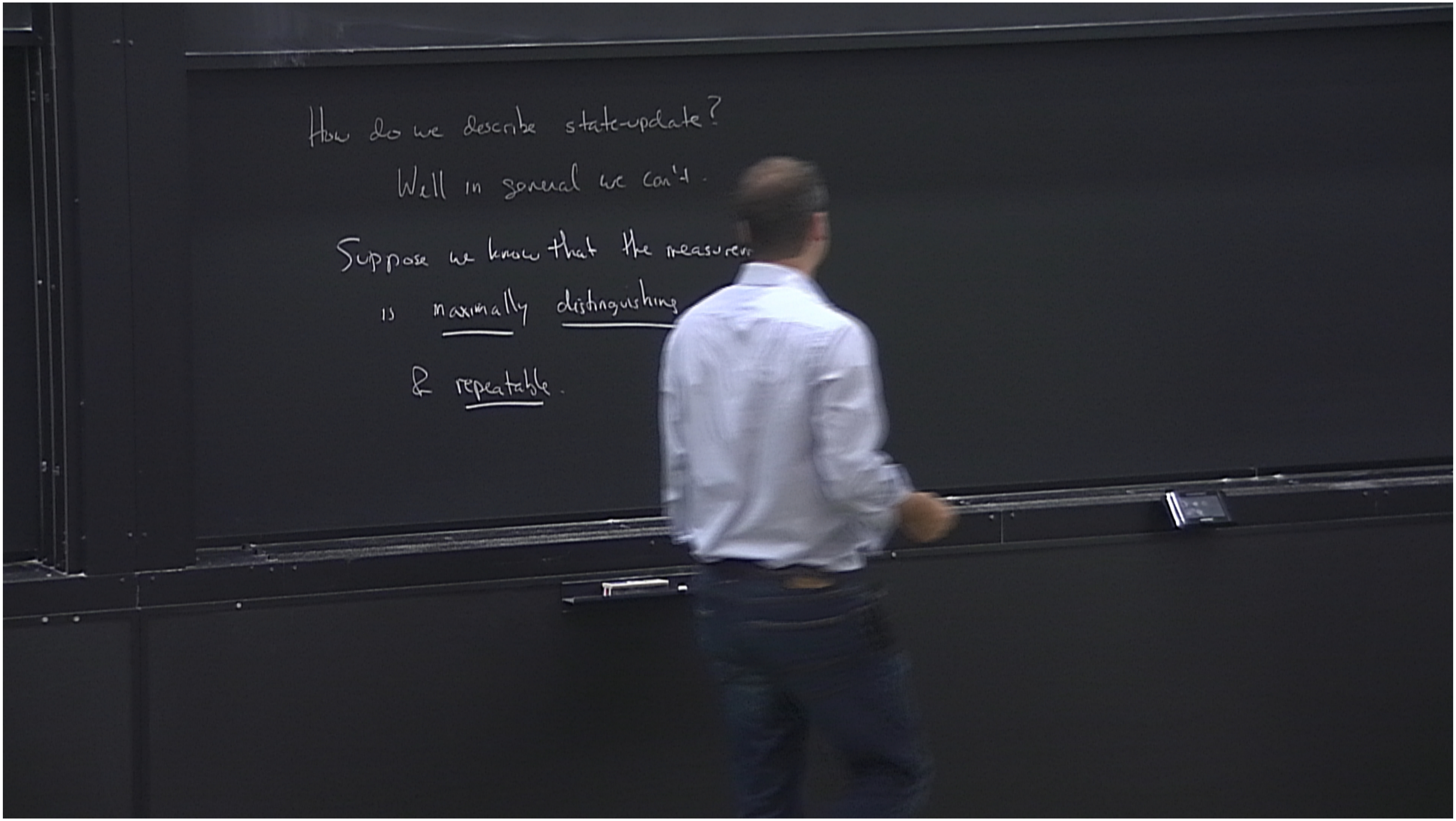

Theory of Measurement
A ∈ L(H_A), B ∈ L(H_B)
⟨A ⊗ B⟩ ≡ ⟨AB⟩
detects correlations
≡ ⟨Â ⊗ 𝟙 + 𝟙 ⊗ B̂⟩ ?
doesn't
"k"

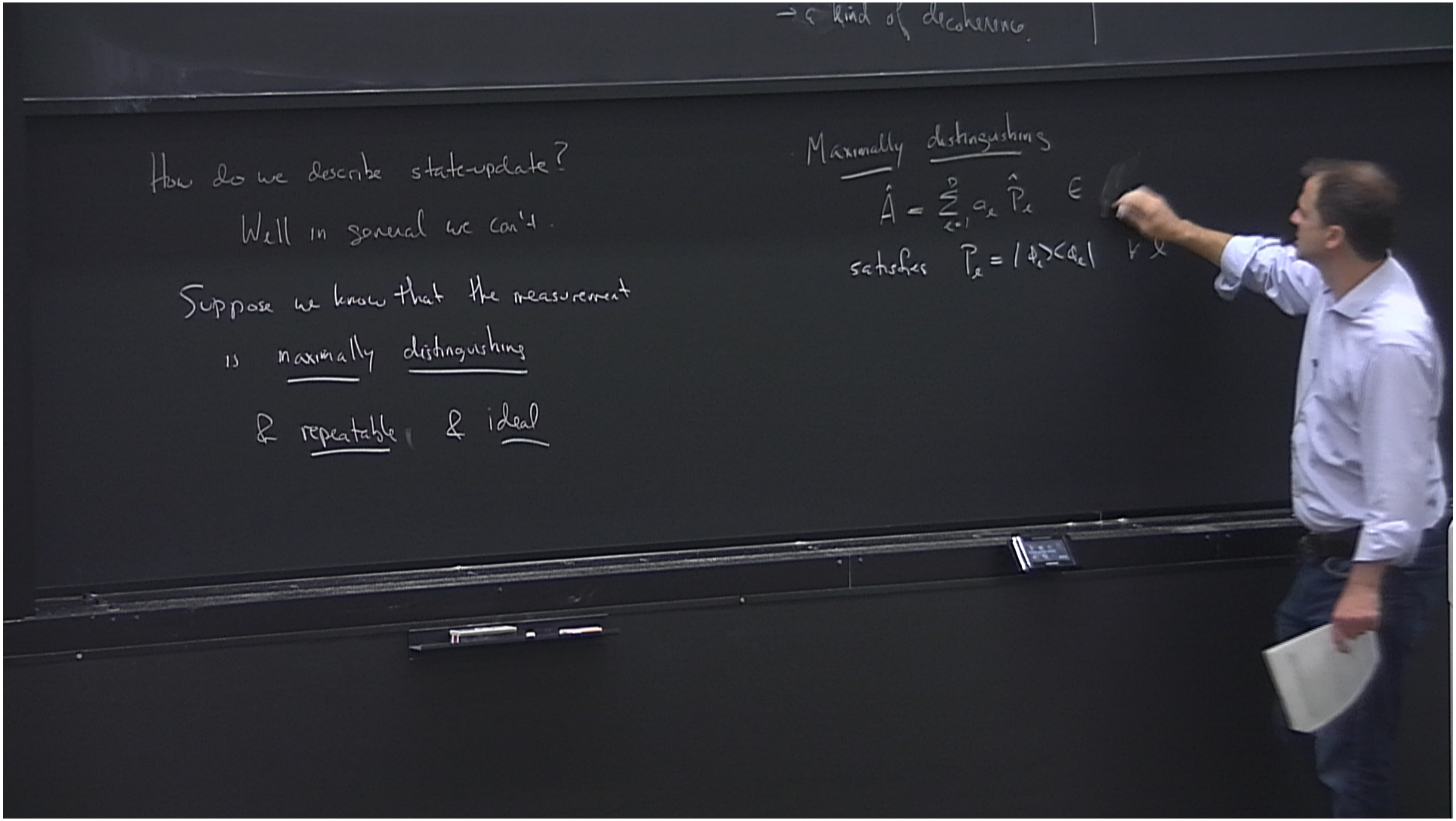

Theory of Measurement
H = H_A ⊗ H_B , A ∈ L(H_A) , B ∈ L(H_B)
Difference between ⟨A ⊗ B⟩ ≡ ⟨AB⟩
& ⟨Â + B̂⟩ ≡ ⟨Â ⊗ 1̂ + 1̂ ⊗ B̂⟩ ?
detects correlations
doesn't
ρ_AB
k_A
k_B
t
⟨A ⊗ B⟩
(i) ρ_AB = Σ_e p_e ρ̂_A^e ⊗ ρ̂_B^e
(ii) ρ_AB is entangled
⟨A ⊗ 1 + 1 ⊗ B⟩
1/2 the


⟨A ⊗ 𝟙 + 𝟙 ⊗ B⟩
½ the time when I press "Go"
→ detects correlation
⟨AB⟩
→ doesn't
ρAB = Σ pℓ ρA ⊗ ρB
ρAB is entangled


detects correlation
doesn't
½ the time when I press
Other ½
is entangled


How do we describe st update?


How do we describe state-update?
Well in general we can't.
Suppose we know that the measure
is maximally distinguishing
& repeatable.


→ a kind of decohering.
How do we describe state-update?
Well in general we can't.
Suppose we know that the measurement
is maximally distinguishing
& repeatable & ideal
Maximally distinguishing
$\hat{A} = \sum_{\ell=1}^{D} a_\ell \hat{P}_\ell \in$
satisfies $P_\ell = |\phi_\ell\rangle\langle\phi_\ell| \;\forall \ell$

→ a kind of decohering.

How do we describe state-update?

Well in general we can't.

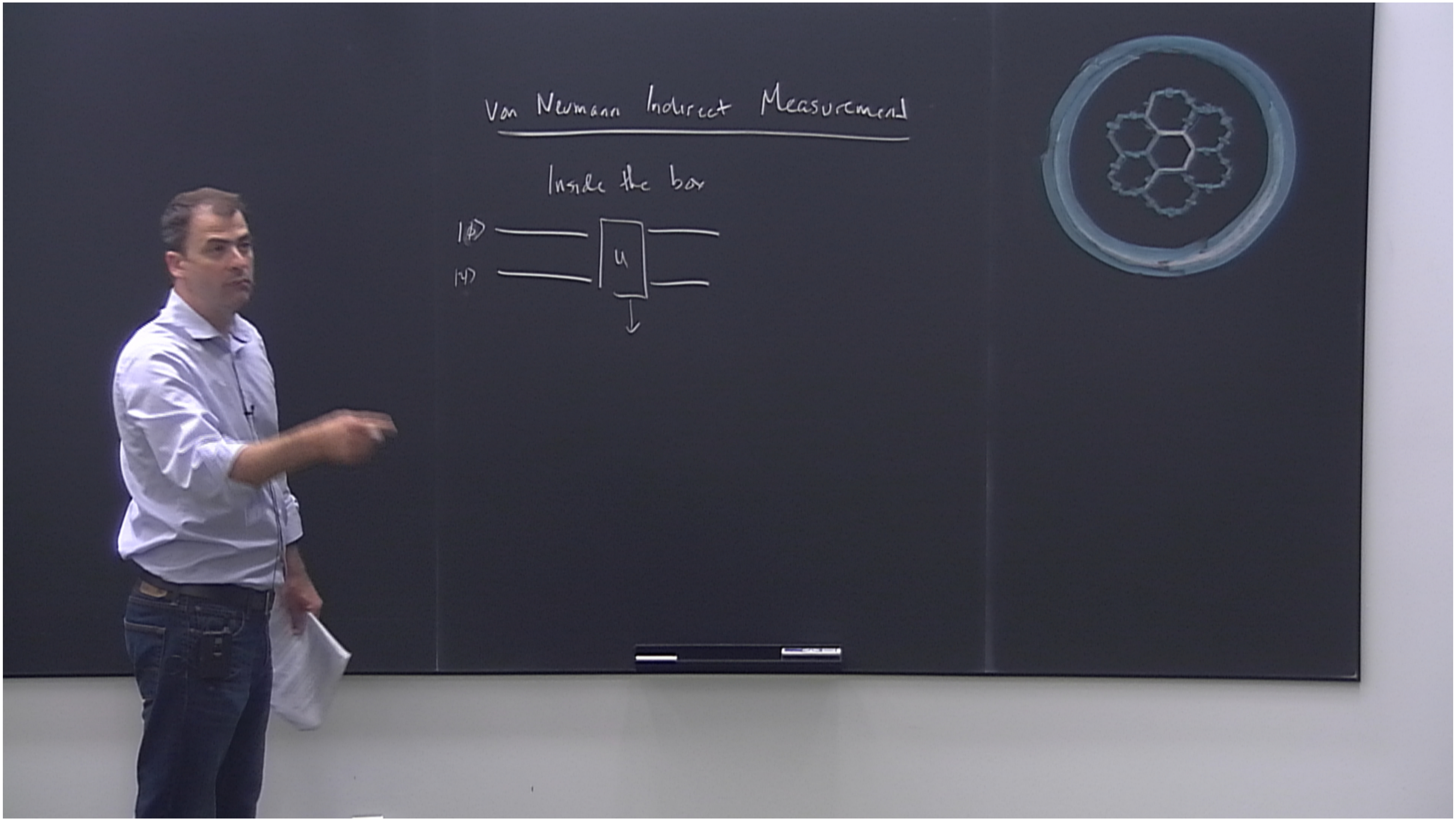
Suppose we know that the measurement is maximally distinguishing & repeatable & ideal

· Maximally distinguishing

$$\hat{A} = \sum_{\ell=1}^{D} a_\ell \hat{P}_\ell \in \mathcal{L}(\mathbb{C}^D) \simeq \mathbb{C}^{D^2}$$

satisfies $P_\ell = |\phi_\ell\rangle\langle\phi_\ell| \;\forall \ell$.

· Repeatable means that the same measurement procedure applied sequentially produces the same result (outcome)


Von Neumann Indirect Measurement
Inside the box
|φ⟩
|ψ⟩
U