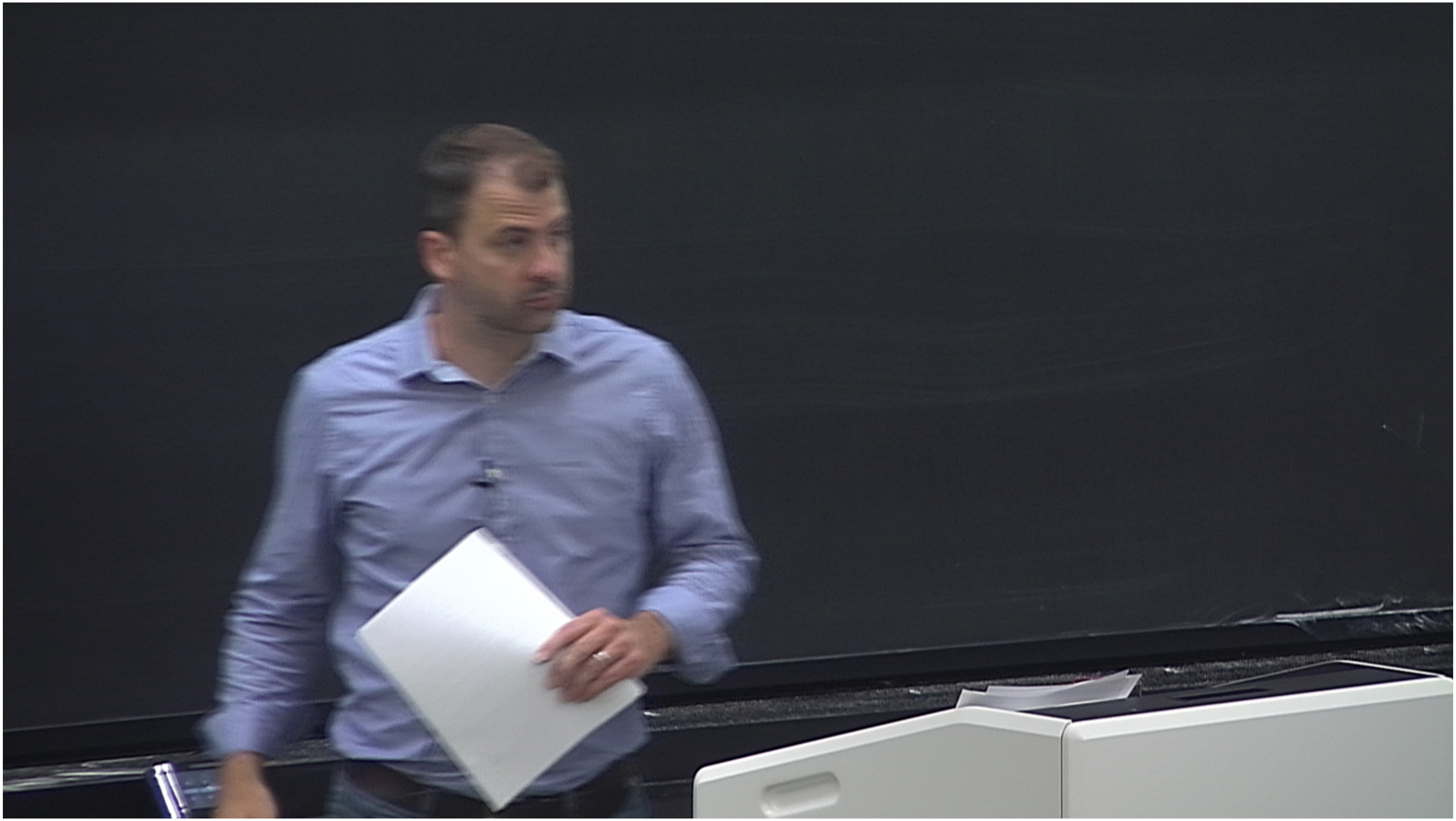
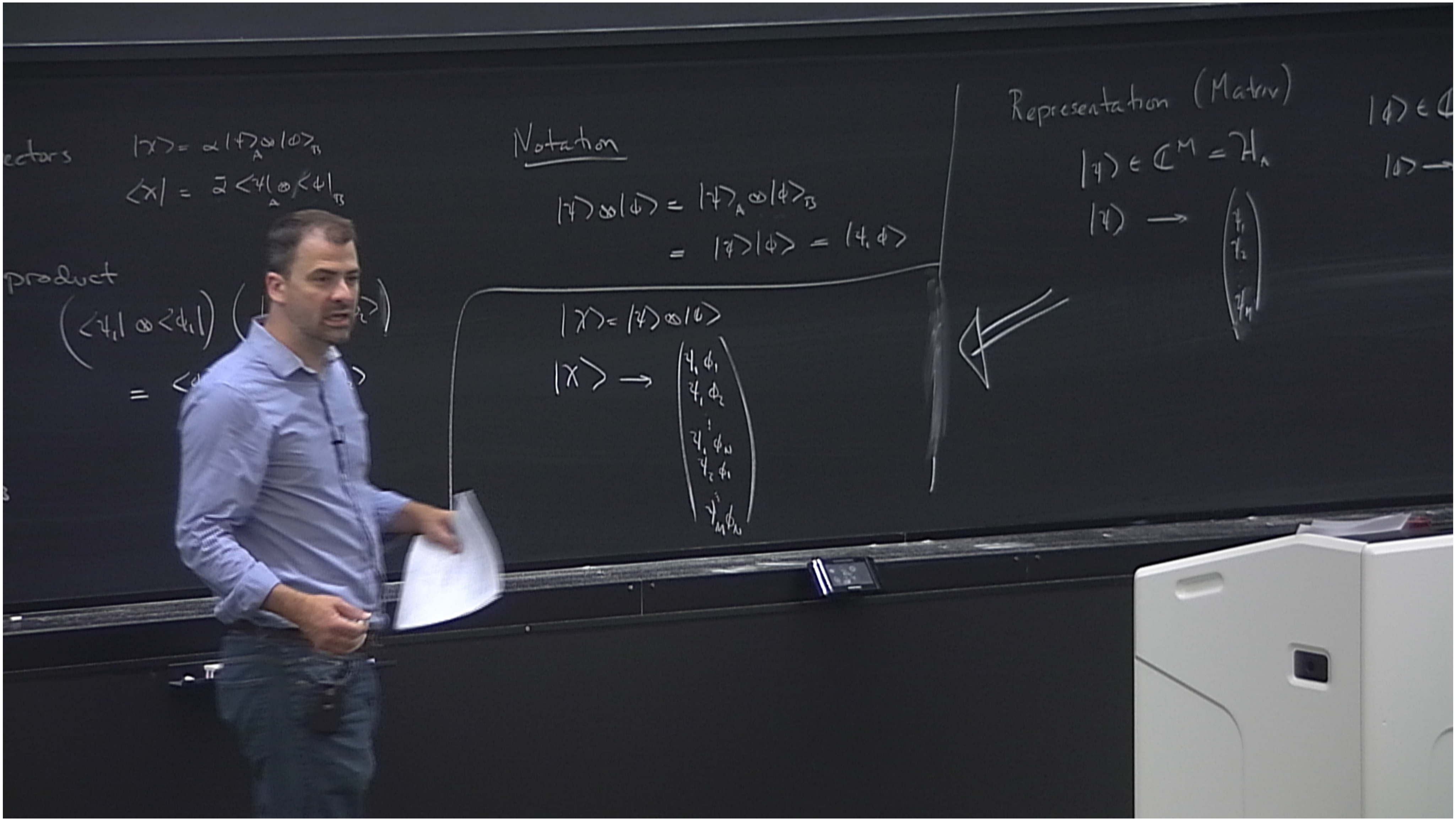
Title: PSI 2015/2016 Quantum Theory - Lecture 3

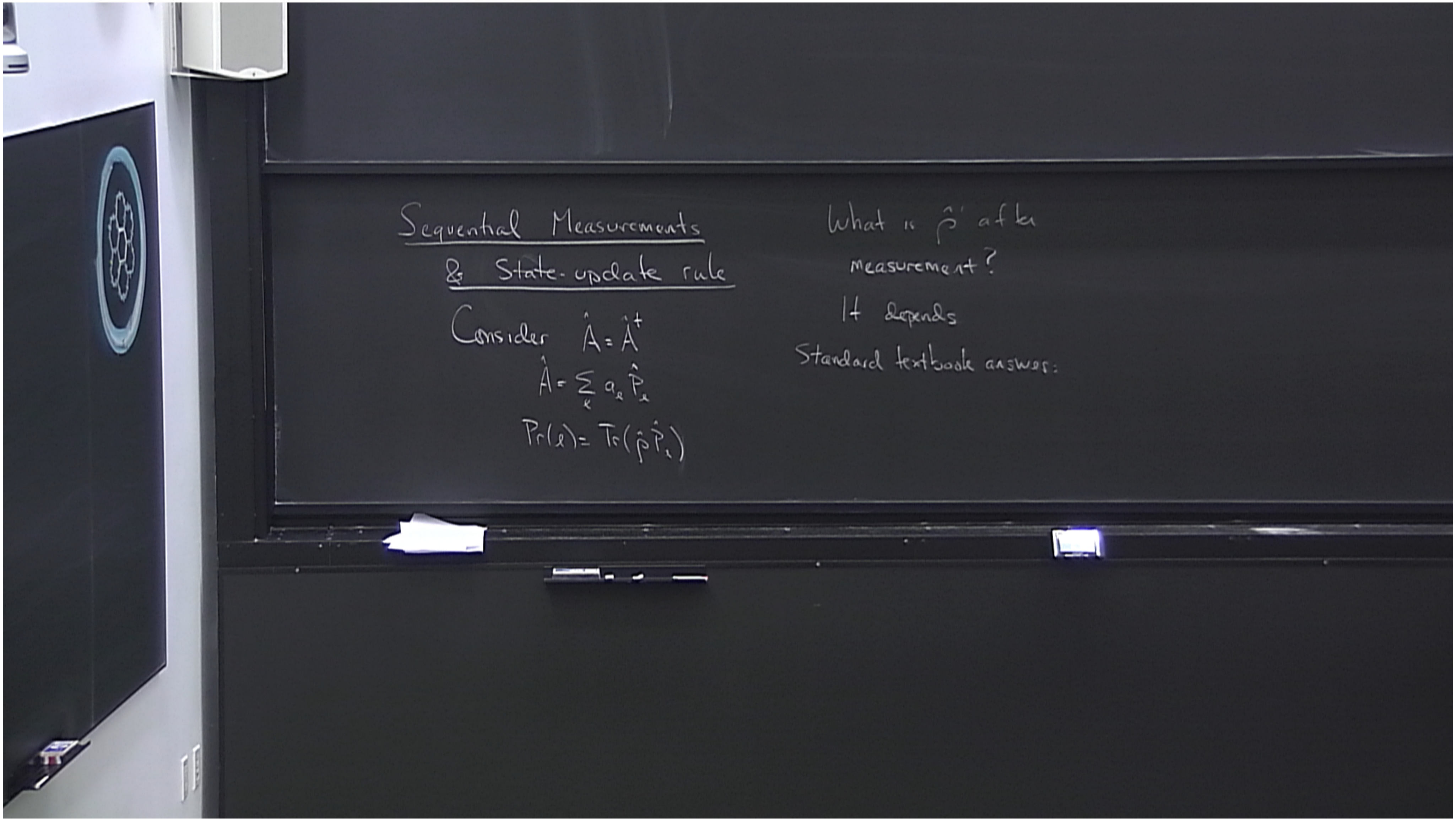
Date: Sep 10, 2015 10:45 AM

URL: http://pirsa.org/15090044

Abstract:




ectors
Notation
|ψ⟩⊗|φ⟩ = |ψ⟩_A⊗|φ⟩_B
= |ψ⟩|φ⟩ = |ψ,φ⟩
|χ⟩= |ψ⟩⊗|φ⟩
Representation (Matrix)
|ψ⟩ ∈ ℂ^M = ℋ_A
product


Sequential Measurements
& State-update rule
Consider $\hat{A} = \hat{A}^{\dagger}$
$\hat{A} = \sum_{\ell} a_{\ell} \hat{P}_{\ell}$
$Pr(\ell) = Tr(\hat{\rho} \hat{P}_{\ell})$
What is $\hat{\rho}'$ after
measurement?
It depends
Standard textbook answers:

Sequential Measurements & State-update rule

Consider $\hat{A} = \hat{A}^{\dagger}$

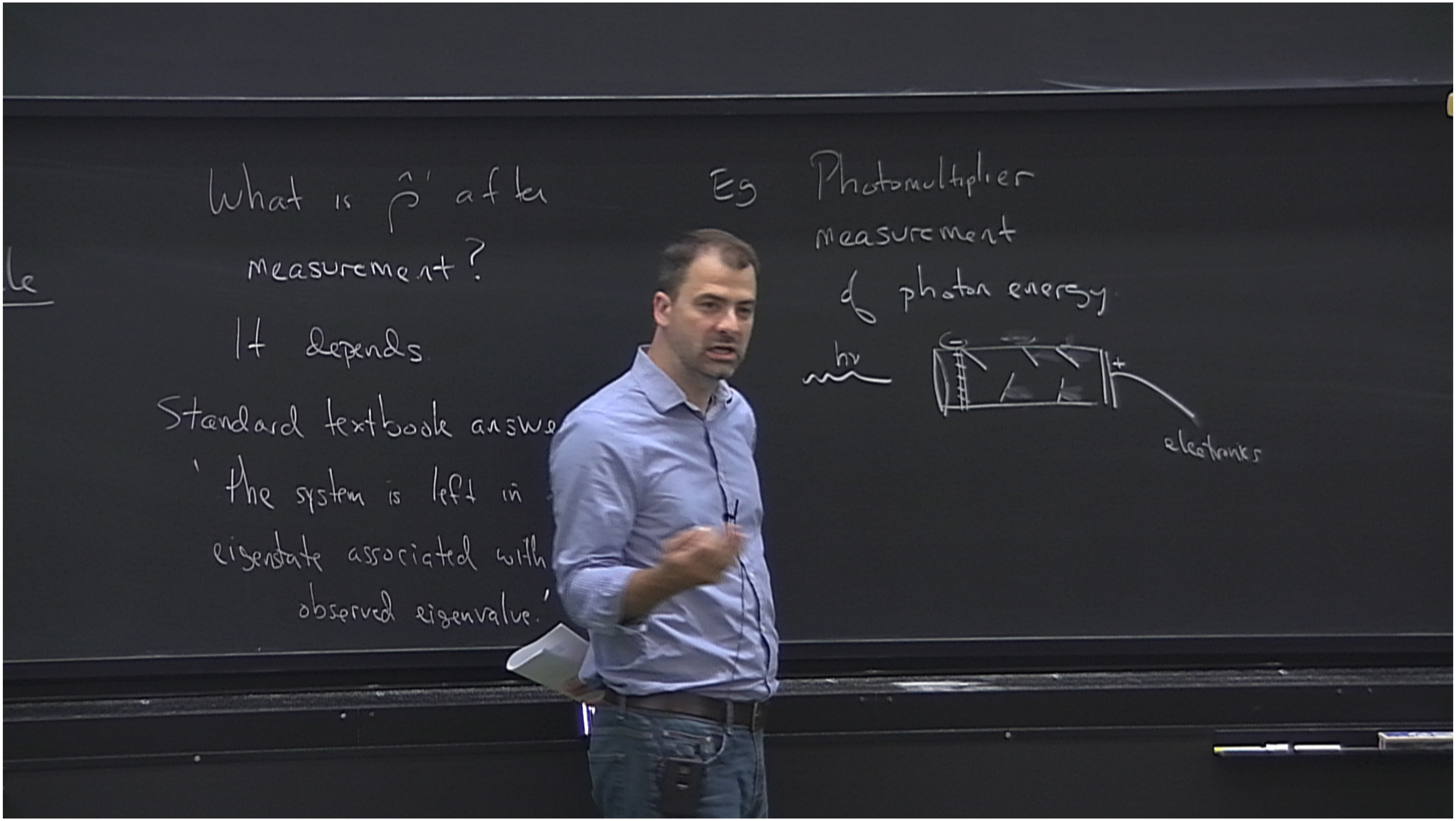
$$\hat{A} = \sum_{\ell} a_{\ell} \hat{P}_{\ell}$$

$$Pr(\ell) = Tr(\hat{\rho} \hat{P}_{\ell})$$

What is $\hat{\rho}'$ after measurement?

It depends.

Standard textbook answer:
'the system is left in the eigenstate associated with the observed eigenvalue.'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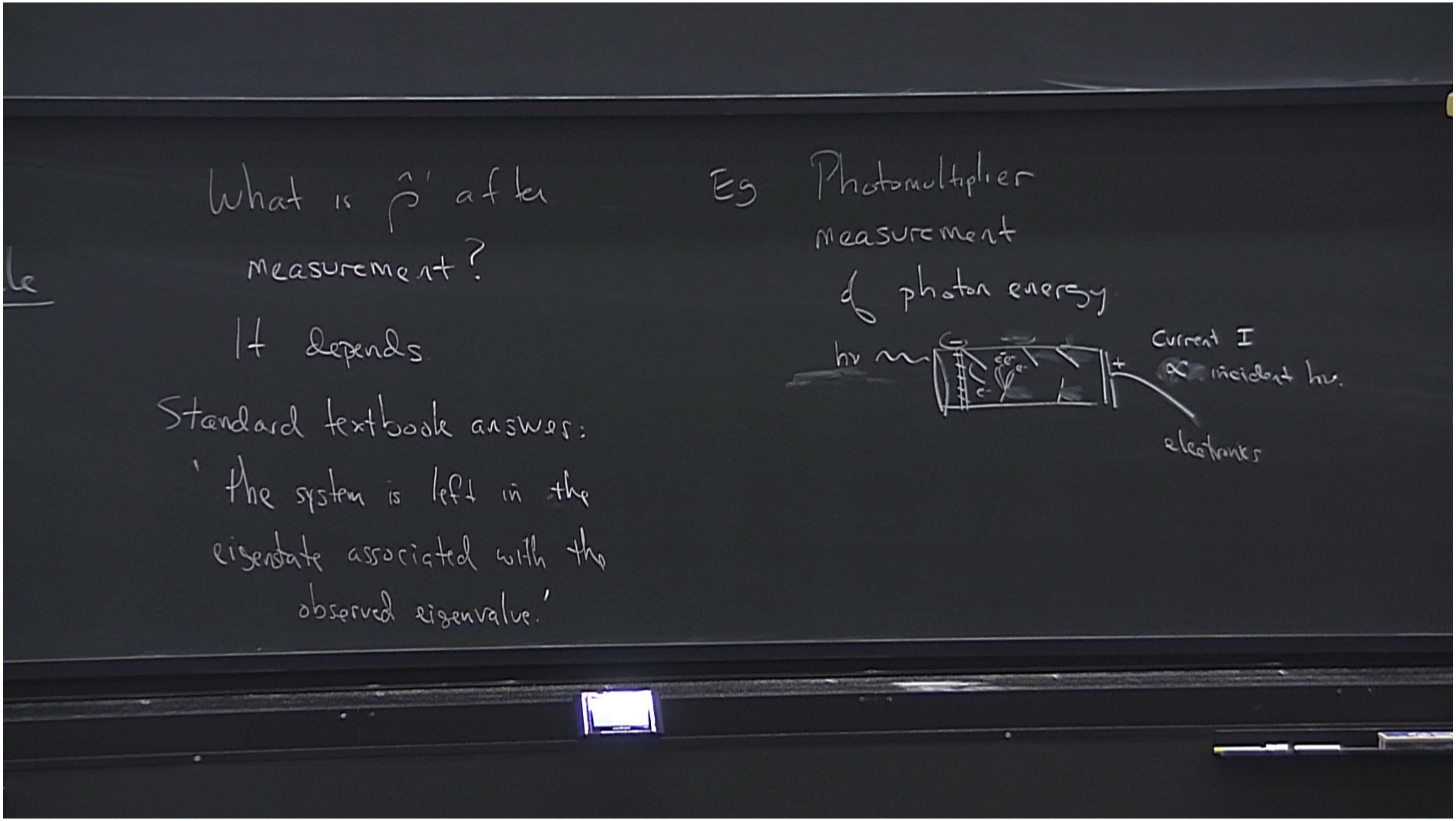
Eg Photomultiplier
measurement
of photon energy
hν
+
electronics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measurement of photon energy
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Current I ∝ incident hν.
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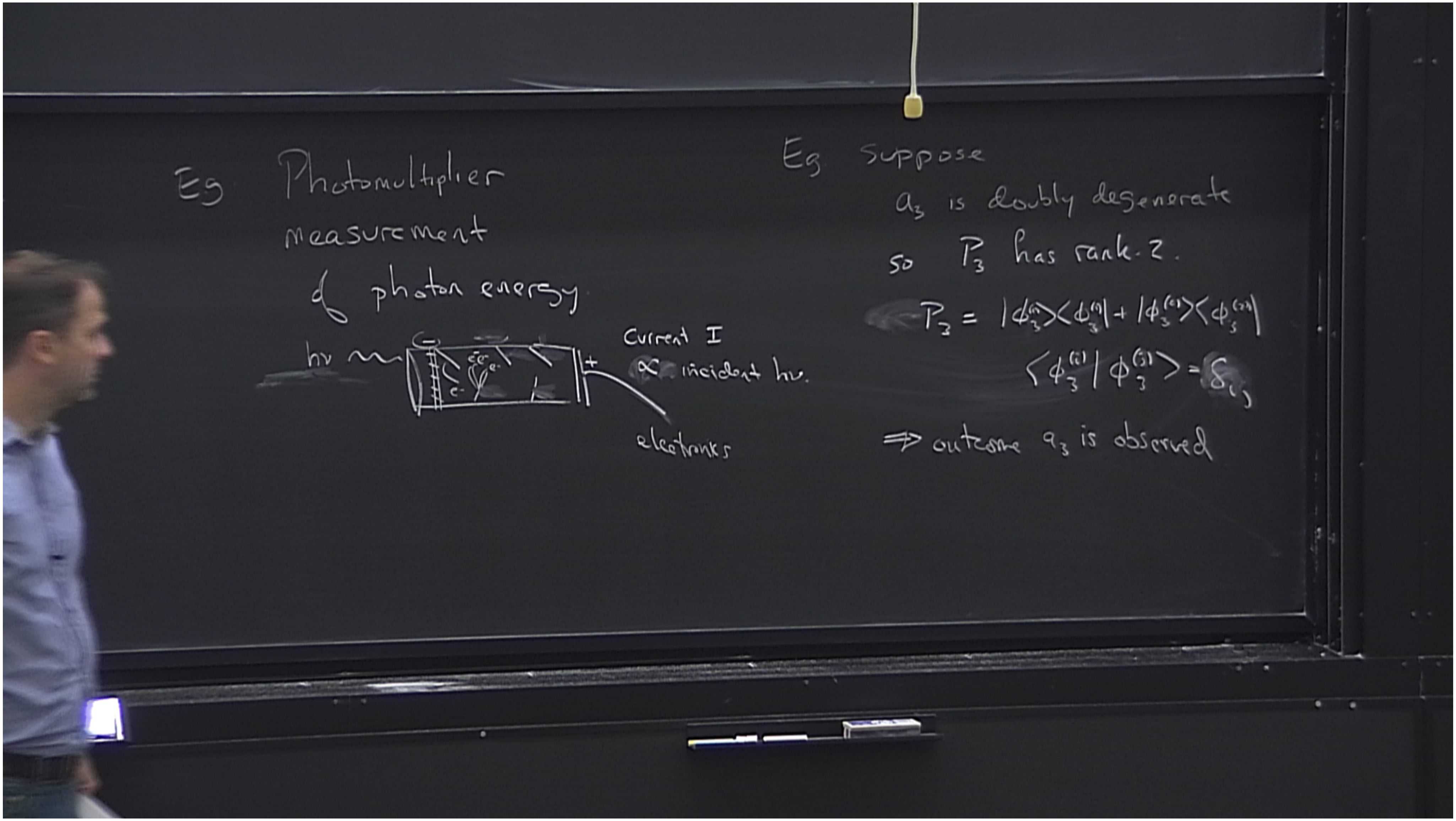

Eg Photomultiplier
measurement
of photon energy
$h\nu$
Current I
$\propto$ incident $h\nu$.
electrons
Eg suppose
a_3 is doubly degenerate
so P_3 has rank 2.
$P_3 = |\phi_3^{(1)}\rangle\langle\phi_3^{(1)}| + |\phi_3^{(2)}\rangle\langle\phi_3^{(2)}|$
$\langle\phi_3^{(i)}|\phi_3^{(j)}\rangle = \delta_{ij}$
$\Rightarrow$ outcome a_3 is observed

es:

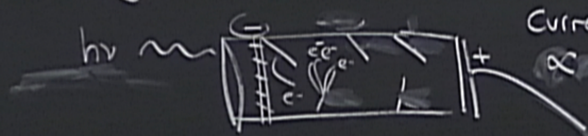
the

the

Eg Photomultiplier

measurement

of photon energy



Current I

$\propto$ incident $h\nu$

electrons

Eg suppose

a_3 is doubly degenerate

so P_3 has rank 2.

$$P_3 = |\phi_3^{(1)}\rangle\langle\phi_3^{(1)}| + |\phi_3^{(2)}\rangle\langle\phi_3^{(2)}|$$

$$\langle\phi_3^{(i)}|\phi_3^{(j)}\rangle = \delta_{ij}$$

$\Rightarrow$ outcome a_3 is observed

What is "the eigenstate" associated with a_3?

doubly degenerate

has rank-2.

$= |\phi_3^{(1)}\rangle\langle\phi_3^{(1)}| + |\phi_3^{(2)}\rangle\langle\phi_3^{(2)}|$

$\langle\phi_3^{(i)}|\phi_3^{(j)}\rangle = \delta_{ij}$

come q_3 is observed

is "the eigenstate"

ssociated with q_3?

Update rule for a certain class of ideal measurements is the Lüders' rule

$$\rho \to \rho' = \frac{\hat{P}_\ell \hat{\rho} \hat{P}_\ell}{Tr(\rho \hat{P}_\ell)}$$

when outcome ℓ is observed

Update rule for a certain class of ideal measurements is the Lüders' rule

$$\boxed{\rho \to \rho' = \frac{\hat{P}_\ell \hat{\rho} \hat{P}_\ell}{\mathrm{Tr}(\rho \hat{P}_\ell)}}$$

when outcome ℓ is observed

Example: $|\psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle + c_3|\phi_3\rangle$

$$\sum_i |c_i|^2 = 1$$

$$P_1 = |\phi_1\rangle\langle\phi_1|$$

$$P_\perp = |\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3|$$

$$\langle\phi_i|\phi_j\rangle = \delta_{ij} \quad \begin{cases} \langle\phi_1|\phi_2\rangle = \langle\phi_1|\phi_3\rangle = 0 \\ \langle\phi_2|\phi_3\rangle = 0 \end{cases}$$

Example: $|\psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle + c_3|\phi_3\rangle$

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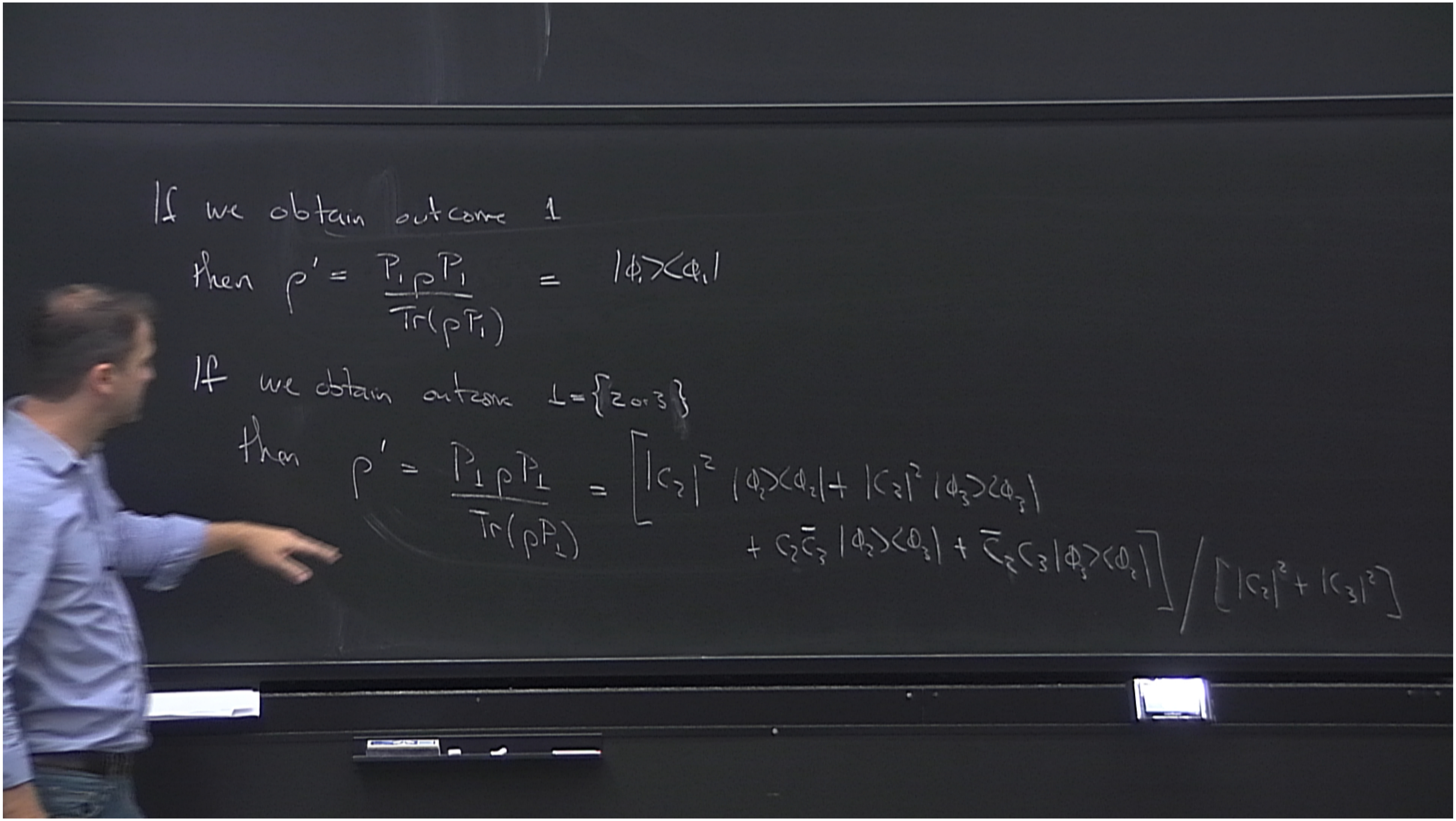
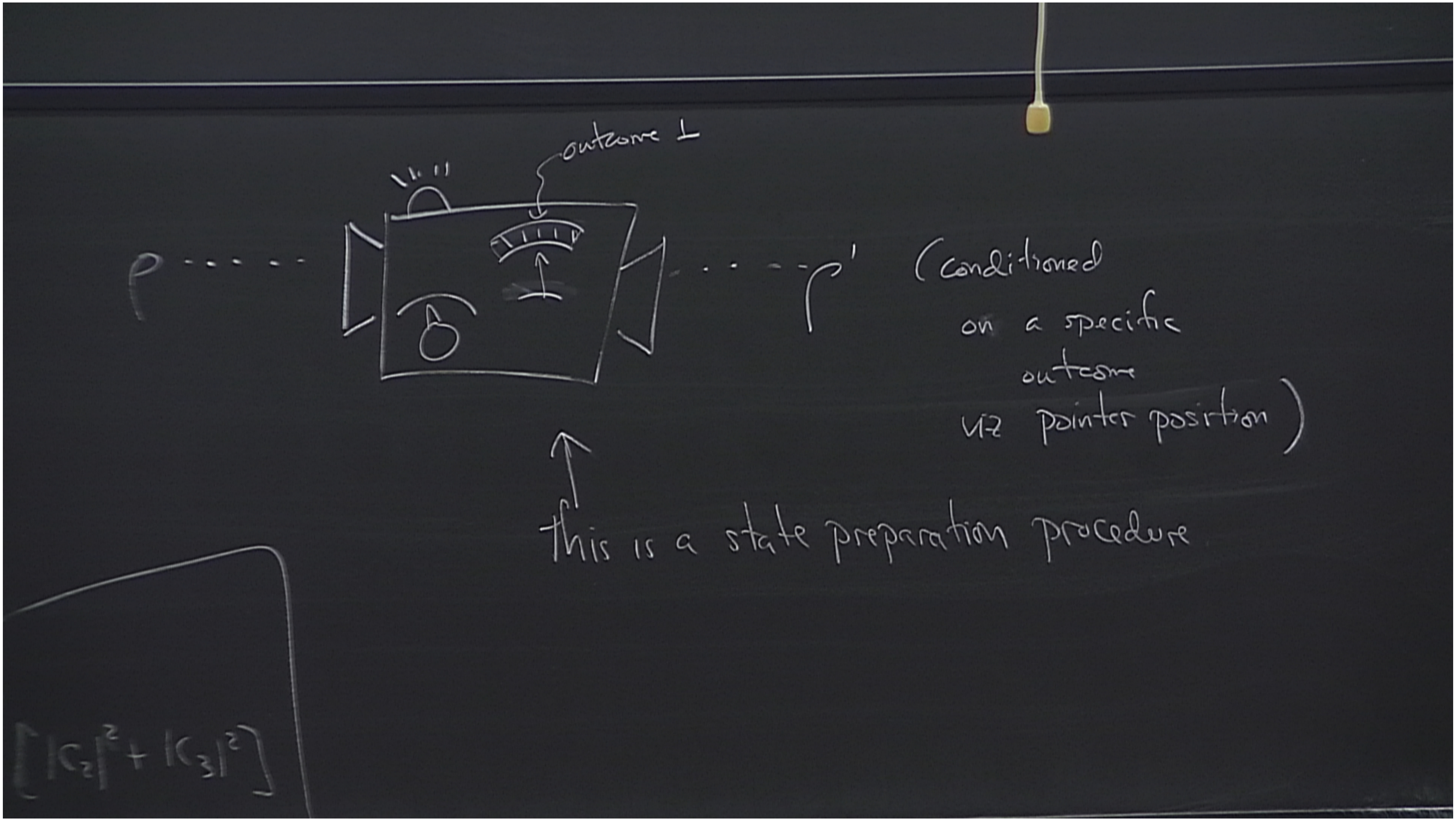
$$\Longrightarrow \quad \rho_{in} = |\psi\rangle\langle\psi|$$

$$\rho_{in} = |c_1|^2|\phi_1\rangle\langle\phi_1| + |c_2|^2|\phi_2\rangle\langle\phi_2| + |c_3|^2|\phi_3\rangle\langle\phi_3|$$
$$+ c_1\bar{c}_2|\phi_1\rangle\langle\phi_2| + c_1\bar{c}_3|\phi_1\rangle\langle\phi_3|$$
$$+ c_2\bar{c}_3|\phi_2\rangle\langle\phi_3| + c.c$$


If we obtain outcome 1
then $\rho' = \frac{P_1 \rho P_1}{Tr(\rho P_1)} = |\phi_1\rangle\langle\phi_1|$
If we obtain outcome $\perp = \{2 \text{ or } 3\}$
then $\rho' = \frac{P_\perp \rho P_\perp}{Tr(\rho P_\perp)} = \Big[|c_2|^2 |\phi_2\rangle\langle\phi_2| + |c_3|^2 |\phi_3\rangle\langle\phi_3|$
$+ c_2\bar{c}_3 |\phi_2\rangle\langle\phi_3| + \bar{c}_2 c_3 |\phi_3\rangle\langle\phi_2|\Big] \Big/ \Big[|c_2|^2 + |c_3|^2\Big]$


outcome 1
ρ
ρ'
(conditioned on a specific outcome viz pointer position)
this is a state preparation procedure
$[|c_2|^2 + |c_3|^2]$

Composite Systems & Entanglement

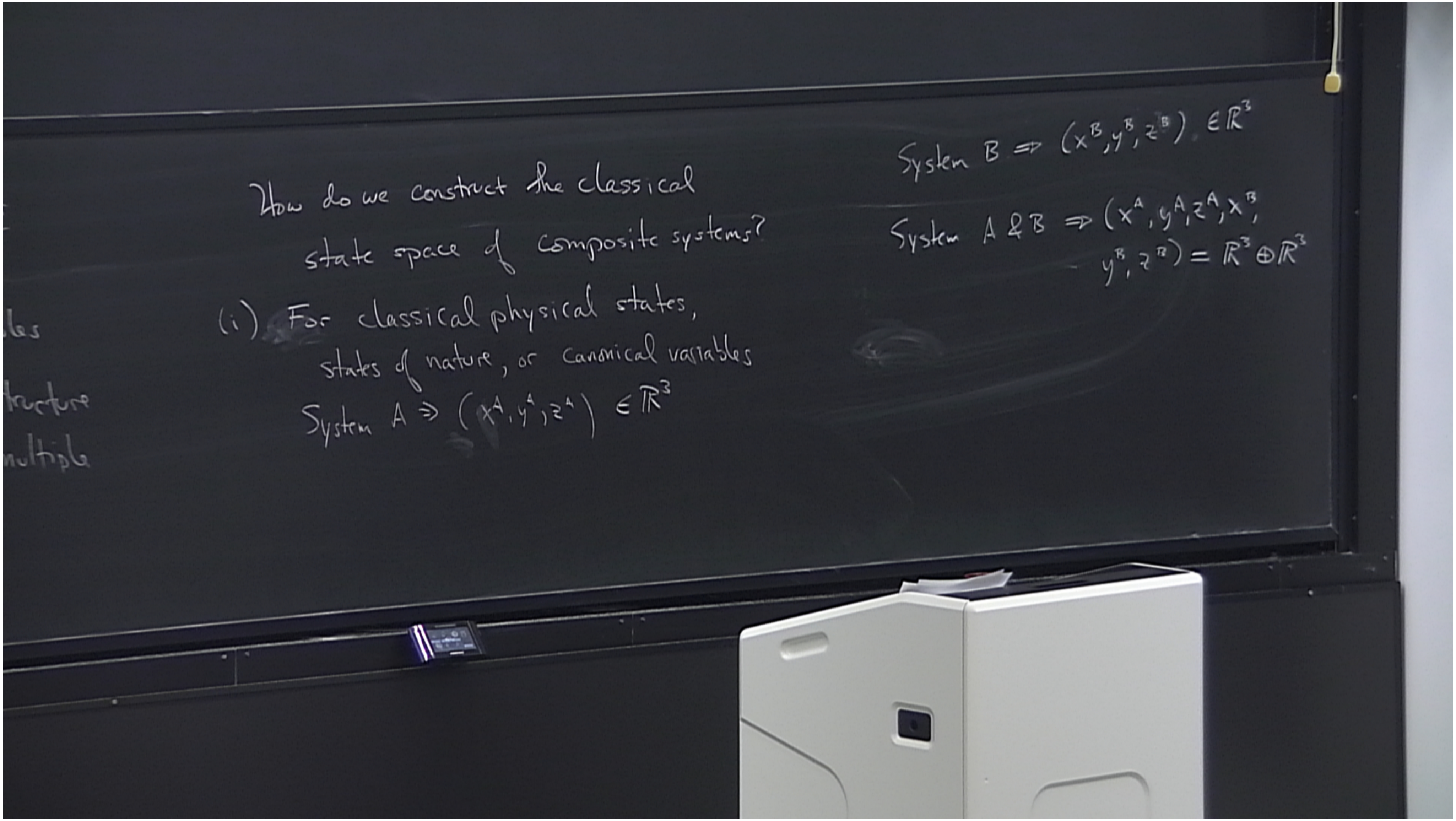
- Algebra of (accessible) observables determines the Hilbert space structure of a system composed of multiple (accessible) properties.

(i)

How do we construct the classical state space of composite systems?

(i) For classical physical states, states of nature, or canonical variables

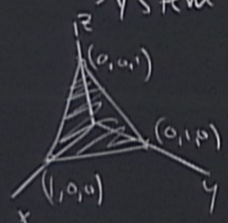
System A $\Rightarrow (x^A, y^A, z^A) \in \mathbb{R}^3$


How do we construct the classical state space of composite systems?
(i) For classical physical states, states of nature, or canonical variables
System A $\Rightarrow (x^A, y^A, z^A) \in \mathbb{R}^3$
System B $\Rightarrow (x^B, y^B, z^B) \in \mathbb{R}^3$
System A & B $\Rightarrow (x^A, y^A, z^A, x^B, y^B, z^B) = \mathbb{R}^3 \oplus \mathbb{R}^3$

$$+ c_2 c_3 |\phi_2\rangle\langle\phi_3| + \bar{c}_2 c_3 |\phi_3\rangle\langle\phi_2|\Big] \Big/ \Big[|c_2|^2 + |c_3|^2\Big]$$

(ii) Classical states of uncertainty,
probability states,
not physical properties,
but states of knowledge.

System A $\quad \vec{p}^A \in \Delta_2 \subset \mathbb{R}^3$



System B $\quad \vec{p}^B \in \Delta_2 \subset \mathbb{R}^3$

For systems A & B,

$$\vec{q}^{A\&B} \in \Delta_8 \subset \mathbb{R}^9 = \mathbb{R}^3 \otimes \mathbb{R}^3$$

- If I have n systems, each with $\vec{p}^i \in \mathbb{R}^3$
$i \in \{1, \dots, n\}$

then the "state space" for my state of knowledge combines $\mathbb{R}^{3^n}$

$\Rightarrow$ grows exponentially with n.

$$+ c_2 c_3 |\Phi_2\rangle\langle\Phi_3| + \bar{c}_2 c_3 |\Phi_3\rangle\langle\Phi_2| \Big] \Big/ \Big[|c_2|^2 + |c_3|^2 \Big]$$

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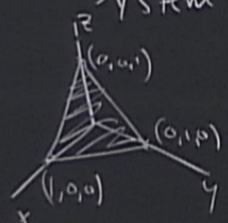

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then the "state space" for my state of knowledge combines $\mathbb{R}^{3^n}$

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n system A

.. B.

ssigned

$\otimes \mathcal{H}_B$

If $\mathcal{H}_A$ is spanned by $\{|a_k\rangle\}$, ← O.N. bases

& $\mathcal{H}_B$ is spanned by $\{|b_\ell\rangle\}$

Then $\mathcal{H}_A \otimes \mathcal{H}_B$ is spanned by $\{|a_k\rangle \otimes |b_\ell\rangle\}$

Hence any $|\chi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

$$|\chi\rangle = \sum_{\ell k} c_{\ell k}\, |a_\ell\rangle \otimes |b_k\rangle$$

Example: Let $\mathcal{H}_A = \mathbb{C}^M$

$\mathcal{H}_B = \mathbb{C}^N$

Then $\mathcal{H}_{AB} = \mathbb{C}^M \otimes \mathbb{C}^N \equiv \mathbb{C}^{M \cdot N}$

If $\mathcal{H}_A$ is spanned by $\{|a_k\rangle\}$, ← O.N. bases

& $\mathcal{H}_B$ is spanned by $\{|b_\ell\rangle\}$

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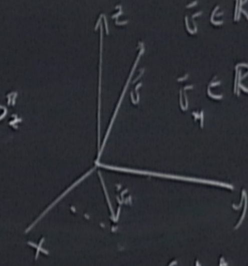
Hence any $|\chi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

$$|\chi\rangle = \sum_{\ell k} c_{\ell k} \, |a_\ell\rangle \otimes |b_k\rangle$$

$$q_{\substack{A=x \\ B=z}} = p_x^A \cdot p_z^B$$

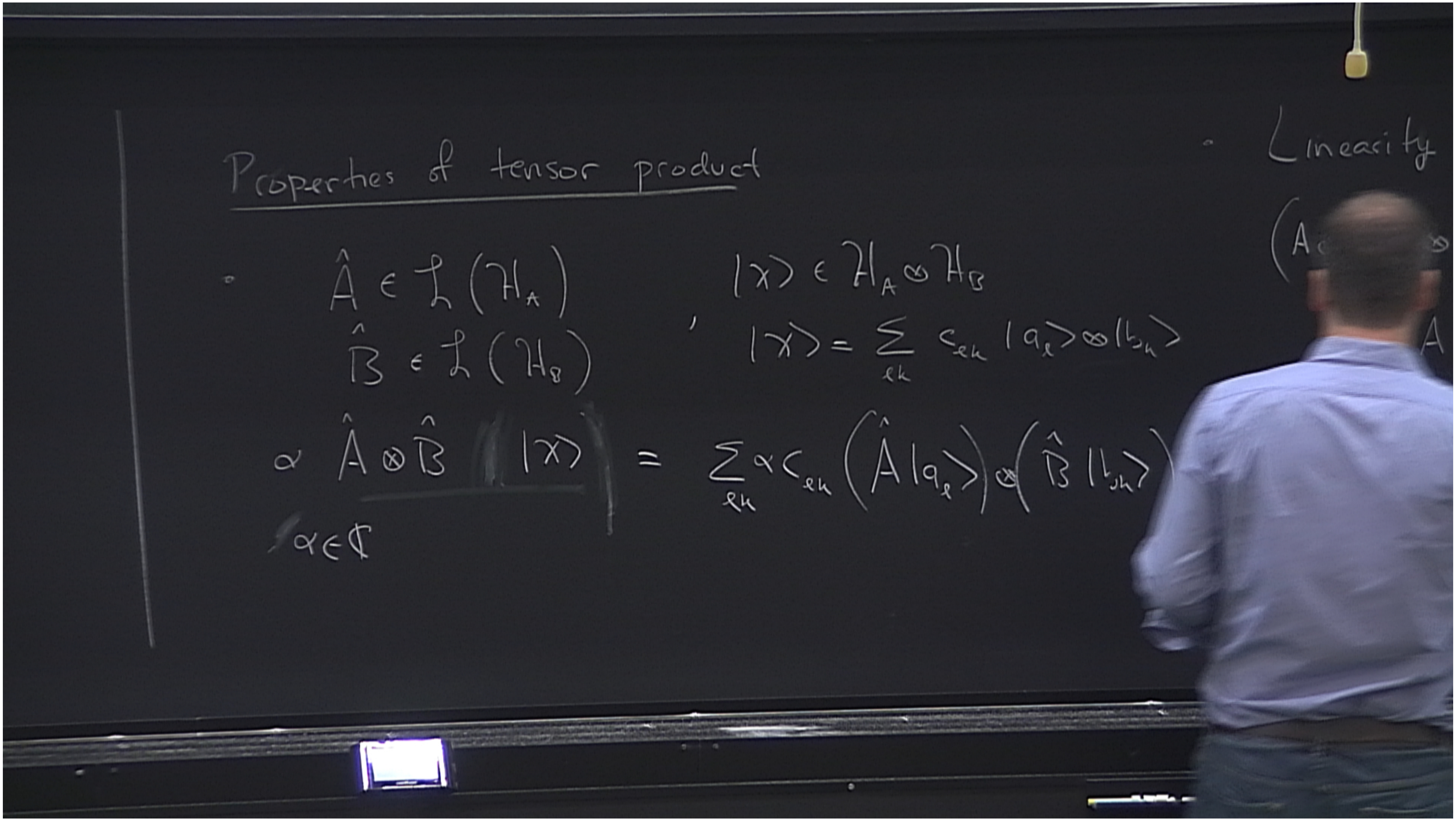
$\vec{r} \in \mathbb{R}^3 = \mathbb{R}^2 \oplus \mathbb{R}^1$

$\vec{r}_{xy} \in \mathbb{R}^2$



$\vec{p}^A = (p_x^A, p_y^A, p_z^A)$

$\vec{p}^B = (p_x^B, p_y^B, p_z^B)$


Properties of tensor product
$\hat{A} \in \mathcal{L}(\mathcal{H}_A)$, $|\chi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$
$\hat{B} \in \mathcal{L}(\mathcal{H}_B)$, $|\chi\rangle = \sum_{lk} c_{lk} |a_l\rangle \otimes |b_k\rangle$
$\alpha \, \hat{A} \otimes \hat{B} \, (|\chi\rangle) = \sum_{lk} \alpha c_{lk} (\hat{A} |a_l\rangle) \otimes (\hat{B} |b_k\rangle)$
$\alpha \in \mathbb{C}$
Linearity

$\left.\right] / \left[|c_2|^2 + |c_3|^2 \right] \Big|$

...perties of tensor product

$\hat{A} \in \mathcal{L}(\mathcal{H}_A)$

$\hat{B} \in \mathcal{L}(\mathcal{H}_B)$

$|\chi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$,

$|\chi\rangle = \sum_{lk} c_{lk} \, |a_l\rangle \otimes |b_k\rangle$

$$\alpha \, \hat{A} \otimes \hat{B} \, |\chi\rangle = \sum_{lk} \alpha c_{lk} \left(\hat{A} |a_l\rangle \right) \otimes \left(\hat{B} |b_k\rangle \right)$$

$\alpha \in \mathbb{C}$

- Linearity

$$\left(A \otimes B + C \otimes D \right) \left(|\psi\rangle \otimes |\phi\rangle \right) = A|\psi\rangle \otimes B|\phi\rangle + C|\psi\rangle \otimes D|\phi\rangle$$

- Linearity for states

$$\left(\alpha|\psi_1\rangle + \beta|\psi_2\rangle \right) \otimes \left(\gamma|\phi_1\rangle + \delta|\phi_2\rangle \right) = \alpha\gamma|\psi_1\rangle \otimes |\phi_1\rangle + \beta\gamma|\psi_2\rangle \otimes |\phi_1\rangle + \dots$$

- Dual vectors

$$|\chi\rangle = \alpha|\psi\rangle_A \otimes |\phi\rangle_B$$

$$\langle\chi| = \bar{\alpha}\langle\psi|_A \otimes \langle\phi|_B$$

- Inner product

$\langle\psi_1| \in \mathcal{H}_A^\dagger$

$|\psi_2\rangle \in \mathcal{H}_A$

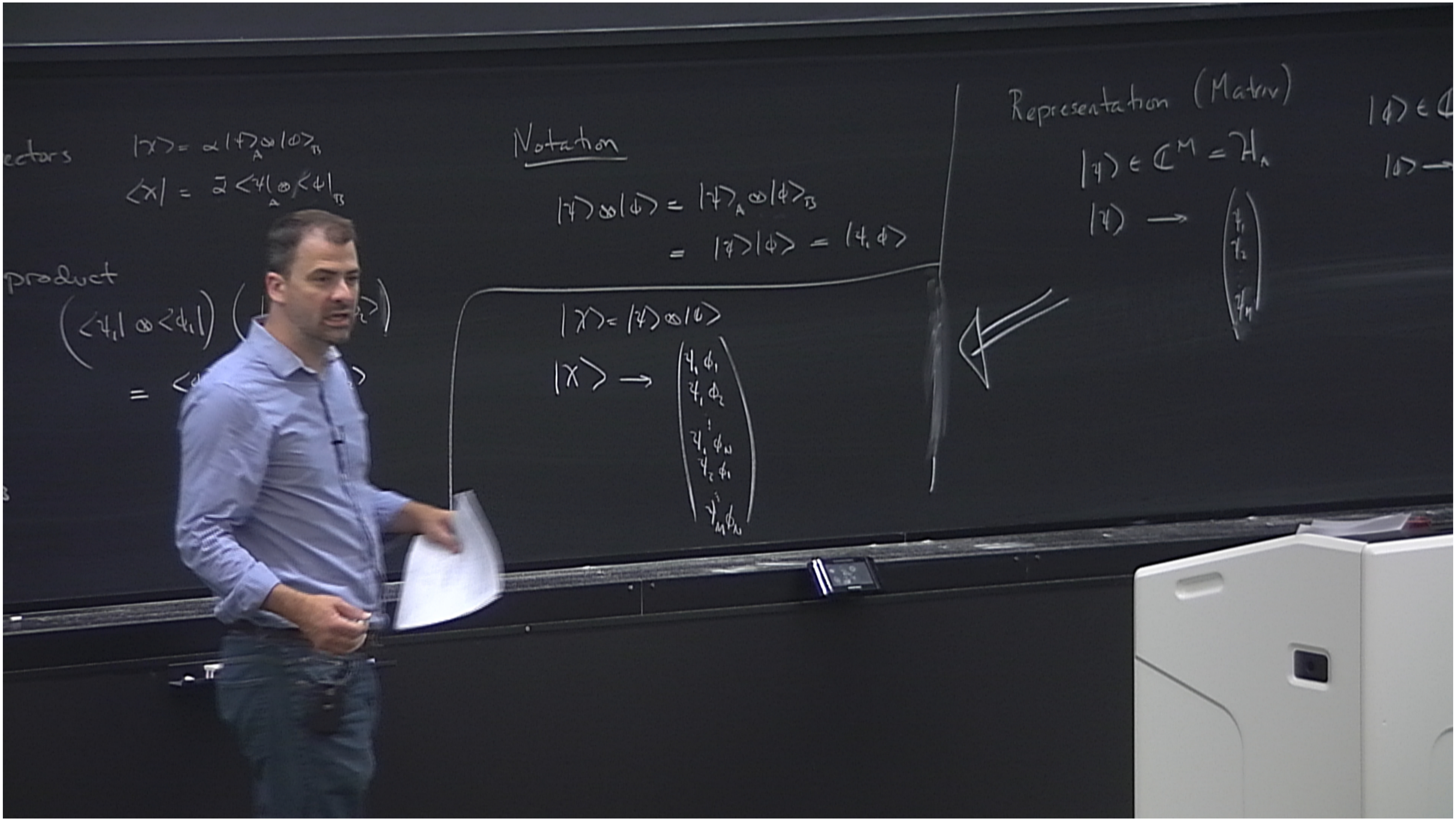
$\langle\phi_1| \in \mathcal{H}_B^\dagger$

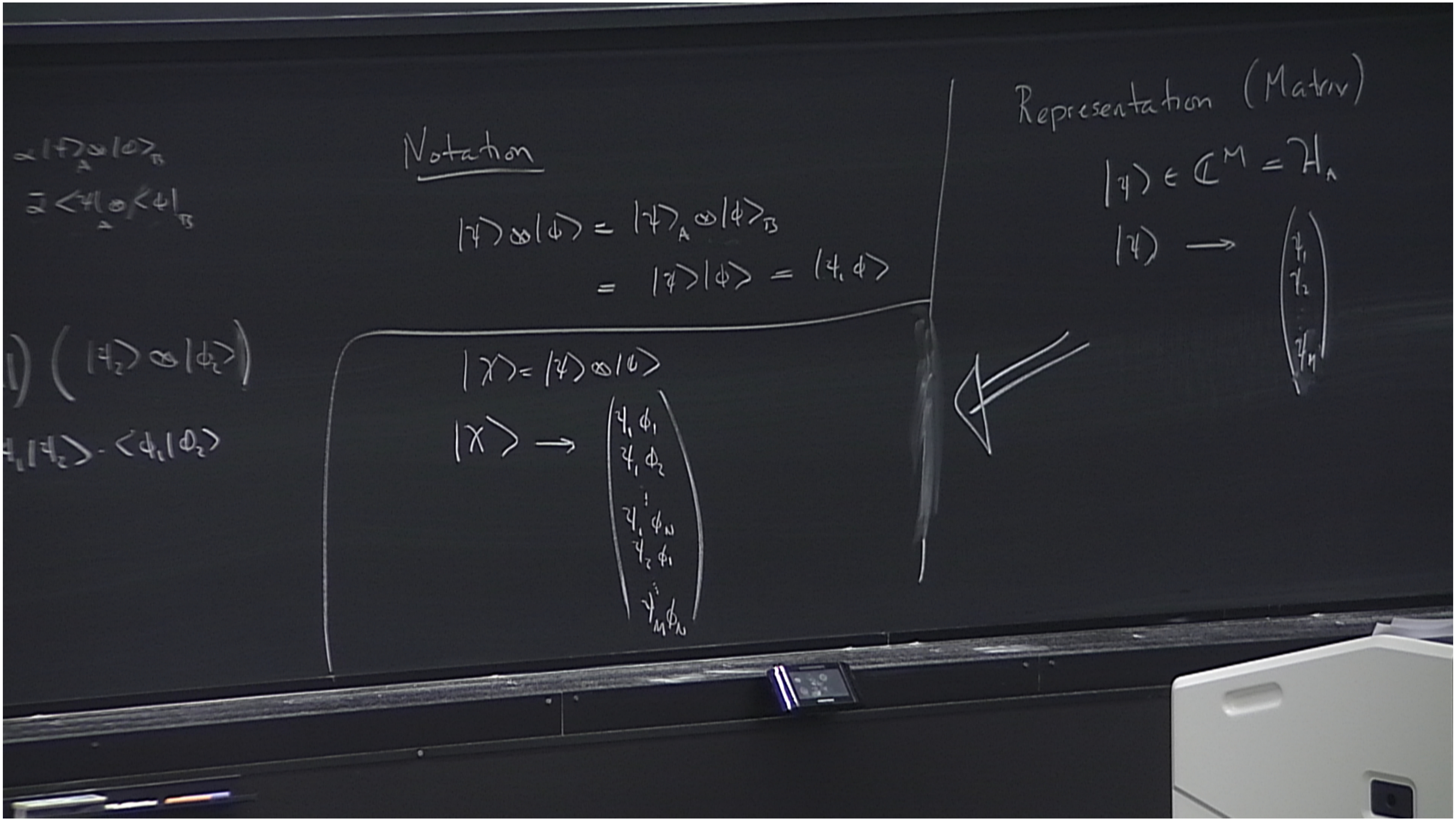
$|\phi_2\rangle \in \mathcal{H}_B$

$$\left(\langle\psi_1| \otimes \langle\phi_1|\right)\left(|\psi_2\rangle \otimes |\phi_2\rangle\right) = \langle\psi_1|\psi_2\rangle \cdot \langle\phi_1|\phi_2\rangle$$

Notation

$$|\psi\rangle \otimes |\phi\rangle = |\psi\rangle_A \otimes |\phi\rangle_B = |\psi\rangle|\phi\rangle = |\psi, \phi\rangle$$

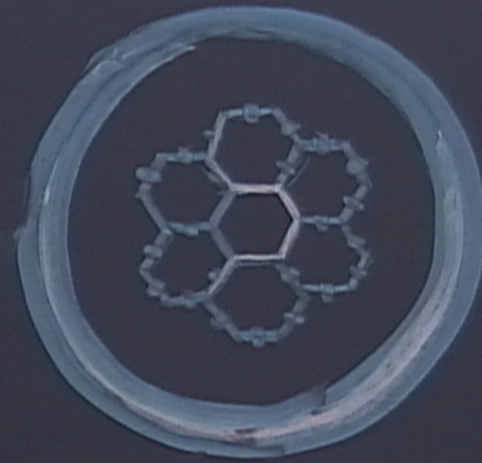

Notation
Representation (Matrix)


Notation
$|\psi\rangle \otimes |\phi\rangle = |\psi\rangle_A \otimes |\phi\rangle_B$
$= |\psi\rangle|\phi\rangle = |\psi, \phi\rangle$
$|\chi\rangle = |\psi\rangle \otimes |\phi\rangle$
$|\chi\rangle \longrightarrow \begin{pmatrix} \psi_1\phi_1 \\ \psi_1\phi_2 \\ \vdots \\ \psi_1\phi_N \\ \psi_2\phi_1 \\ \vdots \\ \psi_M\phi_N \end{pmatrix}$
Representation (Matrix)
$|\psi\rangle \in \mathbb{C}^M = \mathcal{H}_A$
$|\psi\rangle \longrightarrow \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_M \end{pmatrix}$

$$O = A \otimes B$$

$$A \rightarrow \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1M} \\ \vdots & & & \\ A_{M1} & & \cdots & A_{MM} \end{bmatrix}$$

$$B \rightarrow \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ \vdots & & & \\ B_{N1} & & \cdots & B_{NN} \end{bmatrix}$$



$$A \otimes B \rightarrow \begin{bmatrix} A_{11}[B] & A_{12}[B] & \cdots \\ \vdots & & \\ & & \end{bmatrix}$$

$$= A_{11}B_{11} \quad A_{11}B_{12} \quad \cdots$$
$$A_{11}B_{21} \quad \cdots$$
$$\vdots$$