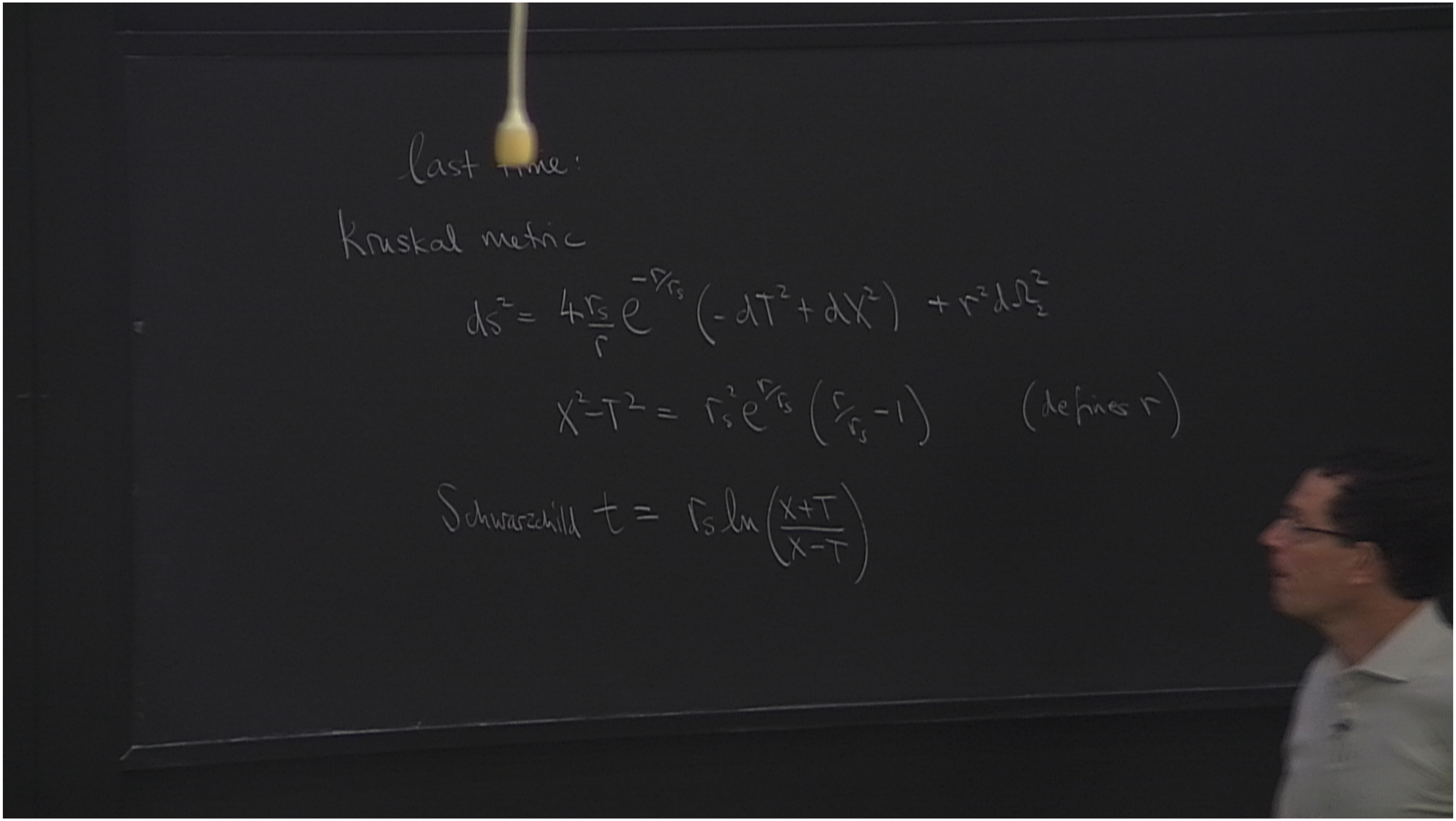
Title: PSI 2015/2016 Relativity - Neil Turok - Lecture 14

Date: Sep 25, 2015 09:00 AM

URL: http://pirsa.org/15090041

Abstract:



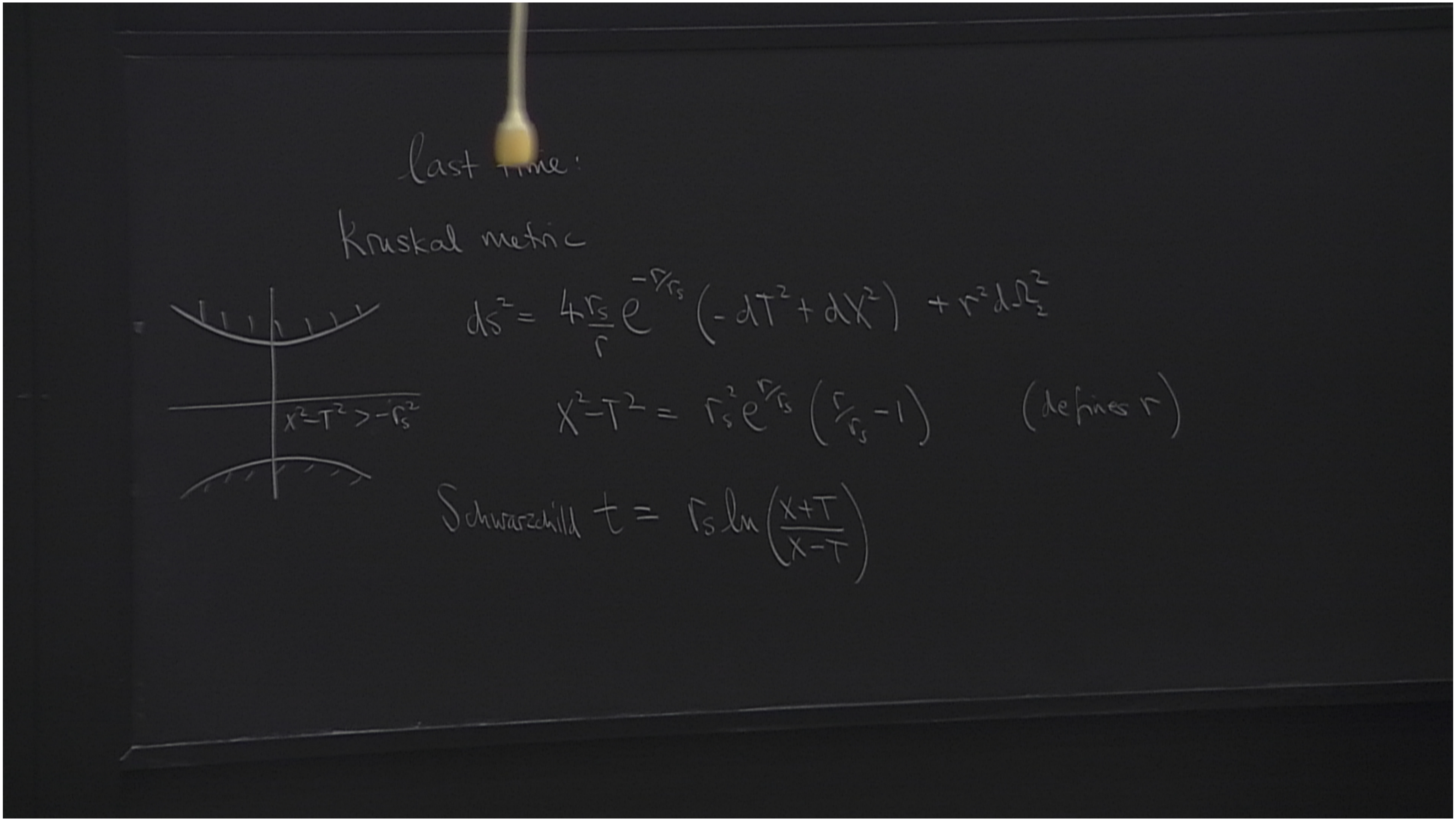
last time:

Kruskal metric

$$ds^2 = 4\frac{r_s}{r} e^{-r/r_s}\left(-dT^2 + dX^2\right) + r^2 d\Omega_2^2$$

$$X^2 - T^2 = r_s^2 e^{r/r_s}\left(r/r_s - 1\right) \qquad \text{(defines } r\text{)}$$

Schwarzschild $t = r_s \ln\left(\frac{X+T}{X-T}\right)$



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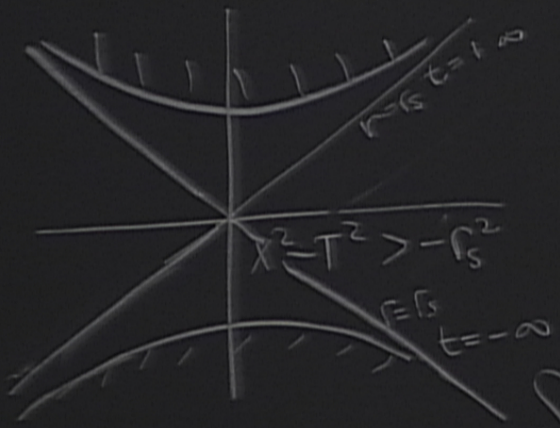

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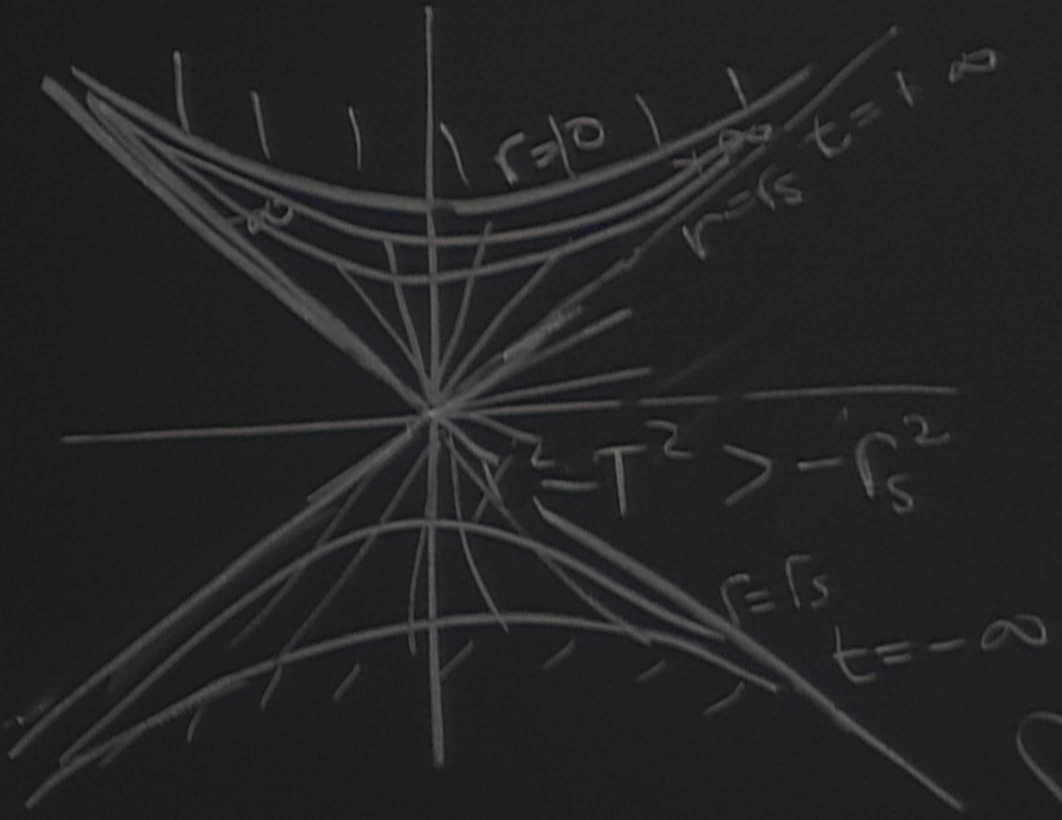
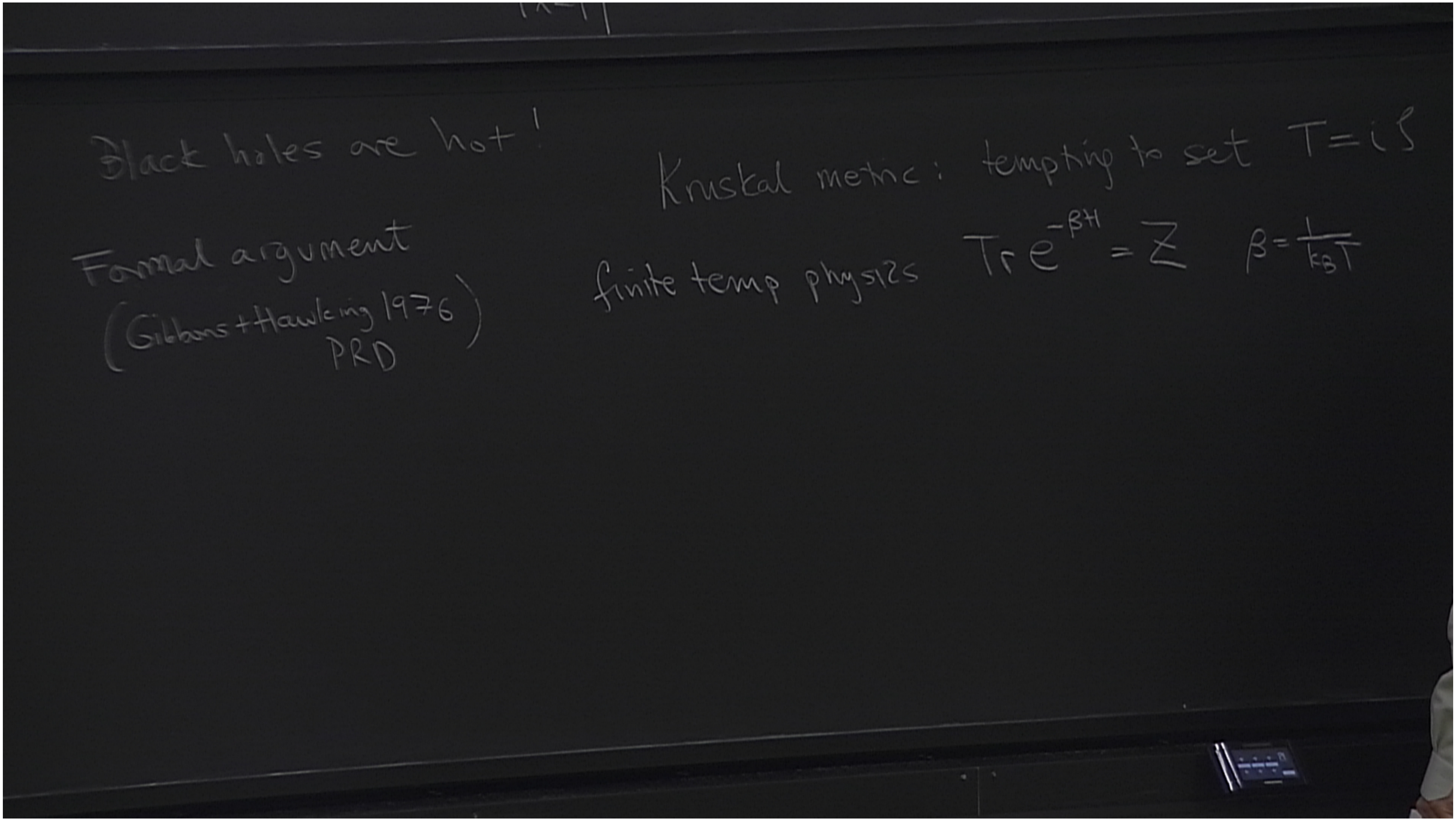
Kruskal metric



$$ds^2 = 4\frac{r_s}{r}e^{-}$$

$$X^2 - T^2$$

Schwarzchild $t =$



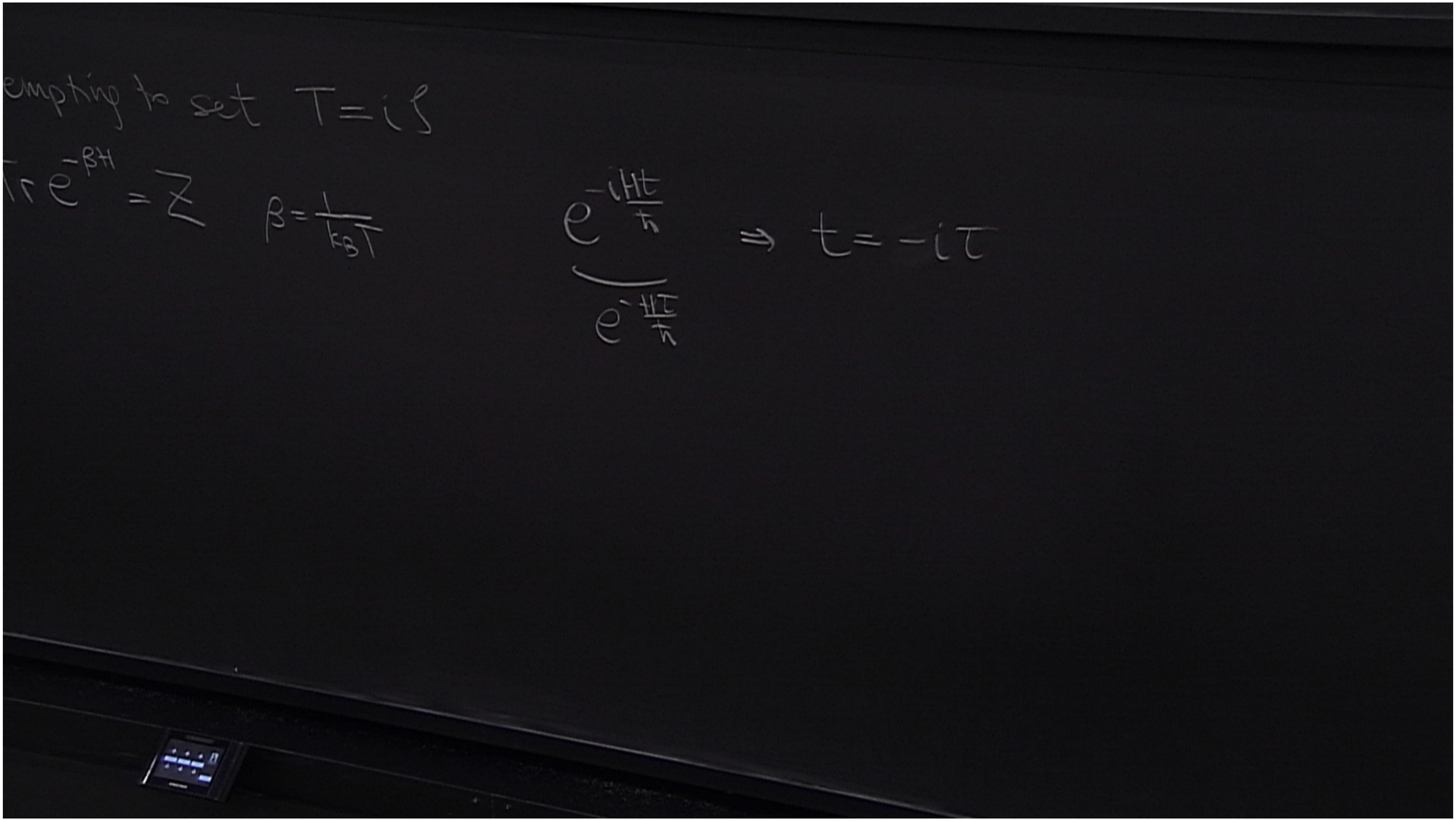
Black holes are hot!

Formal argument

(Gibbons + Hawking 1976 PRD)

Kruskal metric: tempting to set $T = i\zeta$

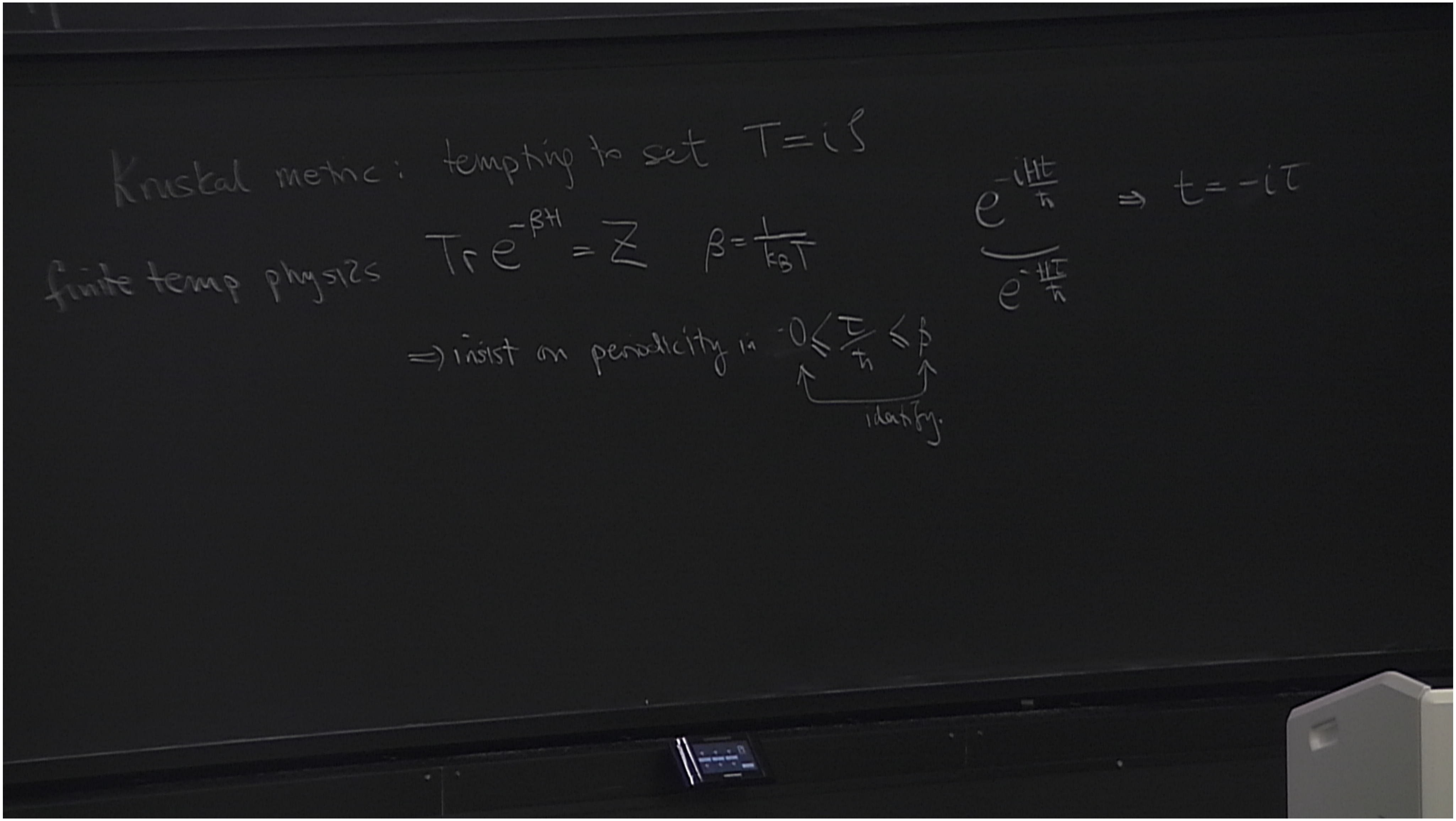
finite temp physics $\mathrm{Tr}\, e^{-\beta H} = Z$ $\beta = \frac{1}{k_B T}$



empting to set $T = i\zeta$

$\mathrm{Tr}\, e^{-\beta H} = Z \qquad \beta = \frac{1}{k_B T}$

$$\underbrace{e^{-\frac{iHt}{\hbar}}}_{e^{-\frac{H\tau}{\hbar}}} \Rightarrow t = -i\tau$$



Kruskal metric: tempting to set $T = i\zeta$

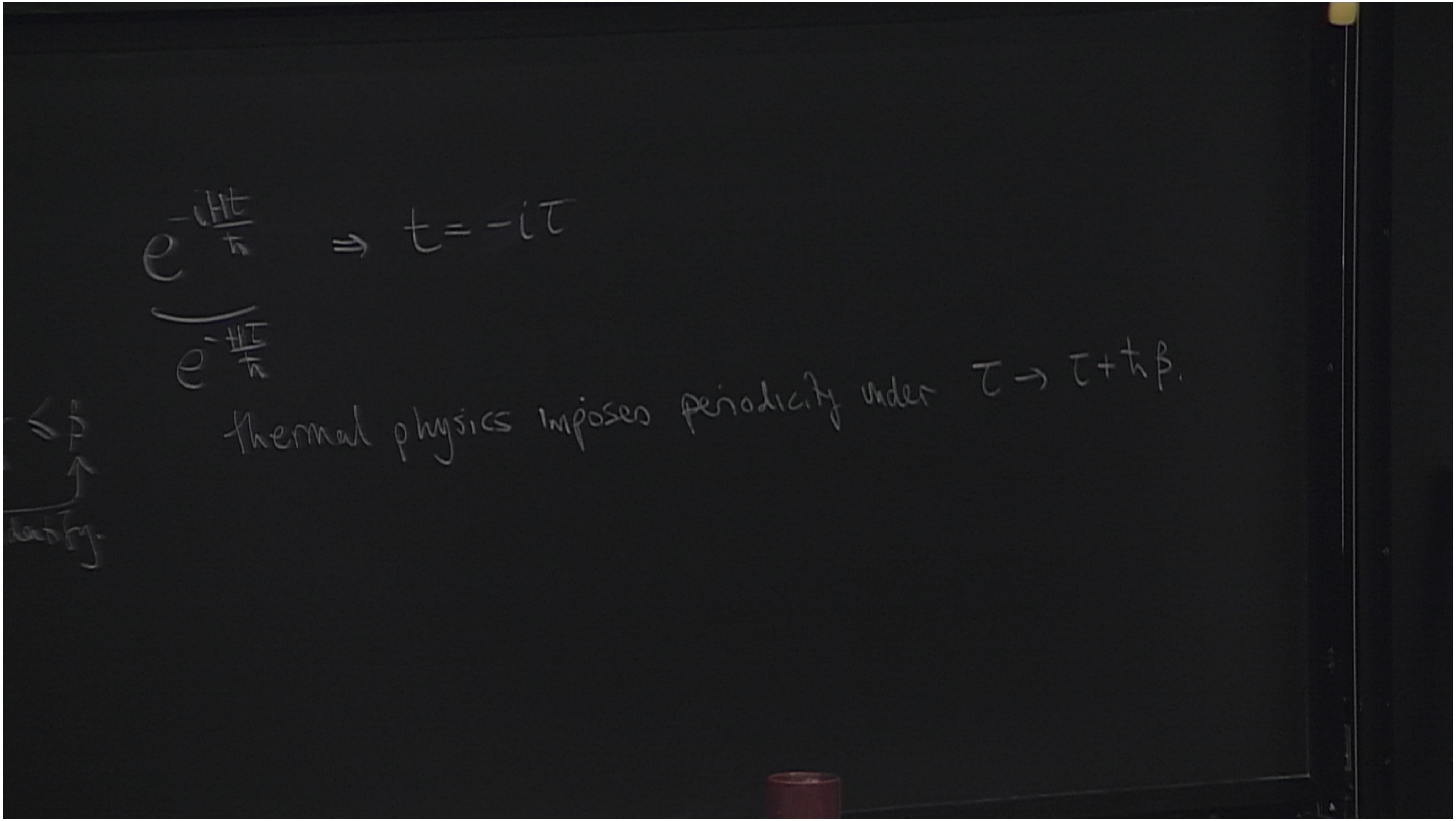
finite temp physics $\mathrm{Tr}\, e^{-\beta H} = Z \quad \beta = \frac{1}{k_B T}$

$\Rightarrow$ insist on periodicity in $0 \leqslant \frac{\tau}{\hbar} \leqslant \beta$

identify.

$e^{-\frac{iHt}{\hbar}} \Rightarrow t = -i\tau$

$e^{-\frac{H\tau}{\hbar}}$



$$\underbrace{e^{-\frac{iHt}{\hbar}}}_{e^{-\frac{H\tau}{\hbar}}} \Rightarrow t = -i\tau$$

thermal physics imposes periodicity under $\tau \to \tau + \hbar\beta$.

Formal argum

finite temp phys

(Gibbons + Hawking 1976
PRD)

$$e^{iS/\hbar}$$

$$\underbrace{-dt^2+d\underline{x}^2}_{t=-i\tau}$$

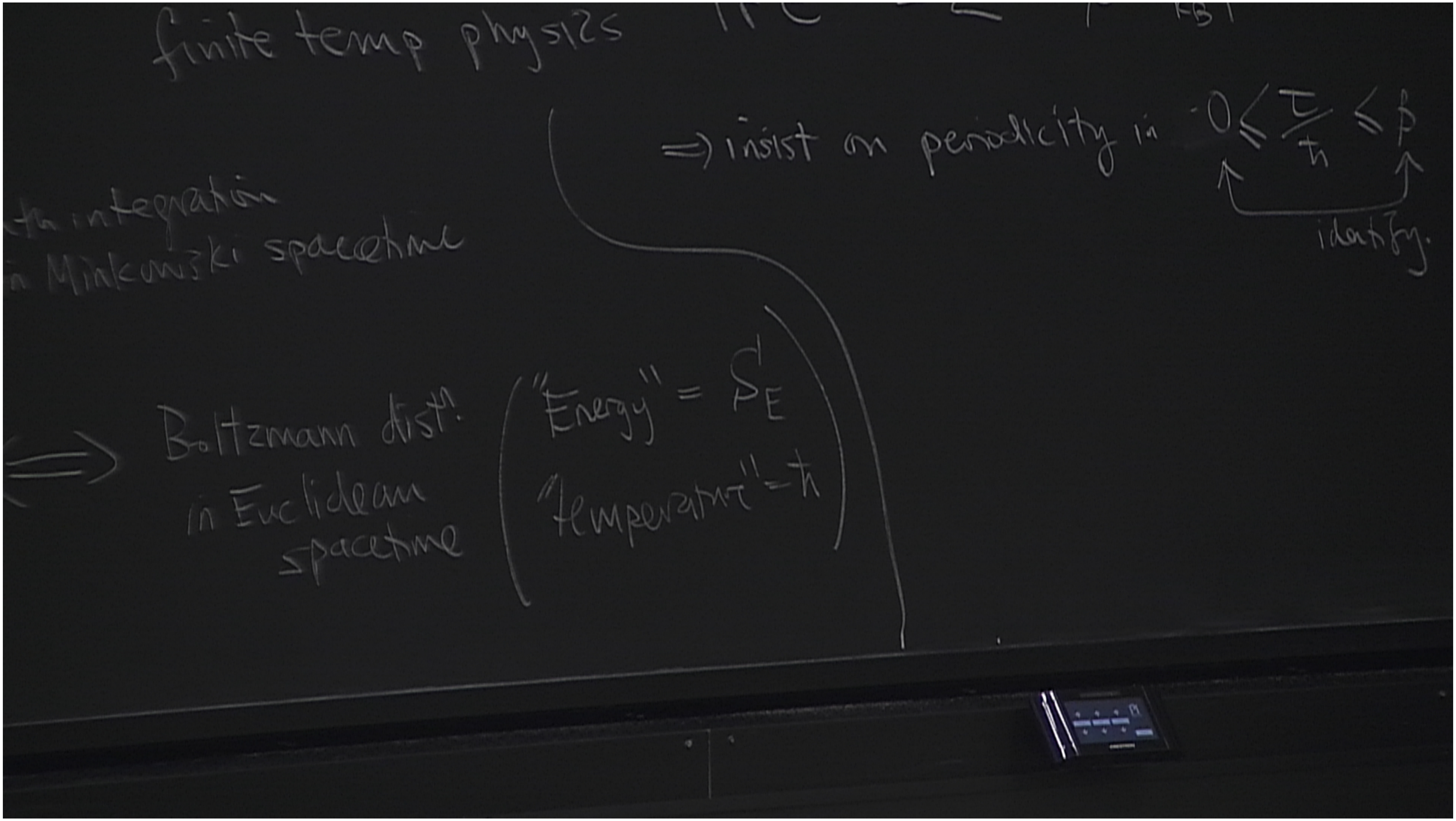
$$d\tau^2+d\underline{x}^2$$

$$\frac{iS}{\hbar} \Rightarrow \frac{i}{\hbar}\int dt\, L(t)$$

$$= \frac{1}{\hbar}\int d\tau \left(L(\tau) \right) = - \frac{S_E}{\hbar}$$

path integration
in Minkowski spacetime

$\Longleftrightarrow$ Boltzmann dist^{n}
in Euclidean
spacetime



finite temp physics

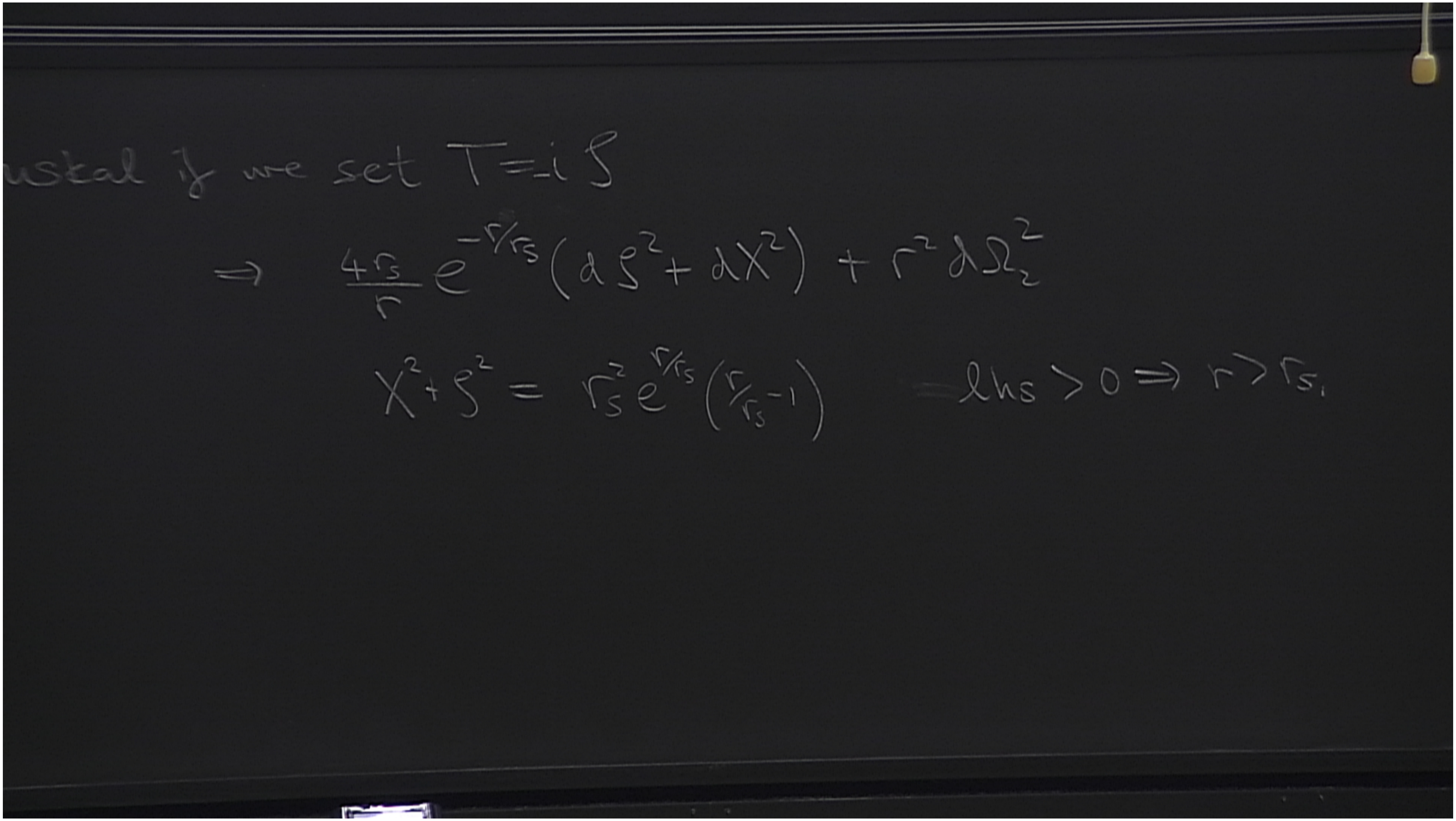
$\Longrightarrow$ insist on periodicity in $0 \leq \frac{\tau}{\hbar} \leq \beta$

identify.

th integration
in Minkowski spacetime

$\Longleftrightarrow$ Boltzmann distn.
in Euclidean
spacetime

("Energy" = S_E
"temperature" = $\hbar$)



uskal if we set $T = -i\rho$

$$\Rightarrow \quad \frac{4r_s}{r} e^{-r/r_s}\left(d\rho^2 + dX^2\right) + r^2 d\Omega_2^2$$

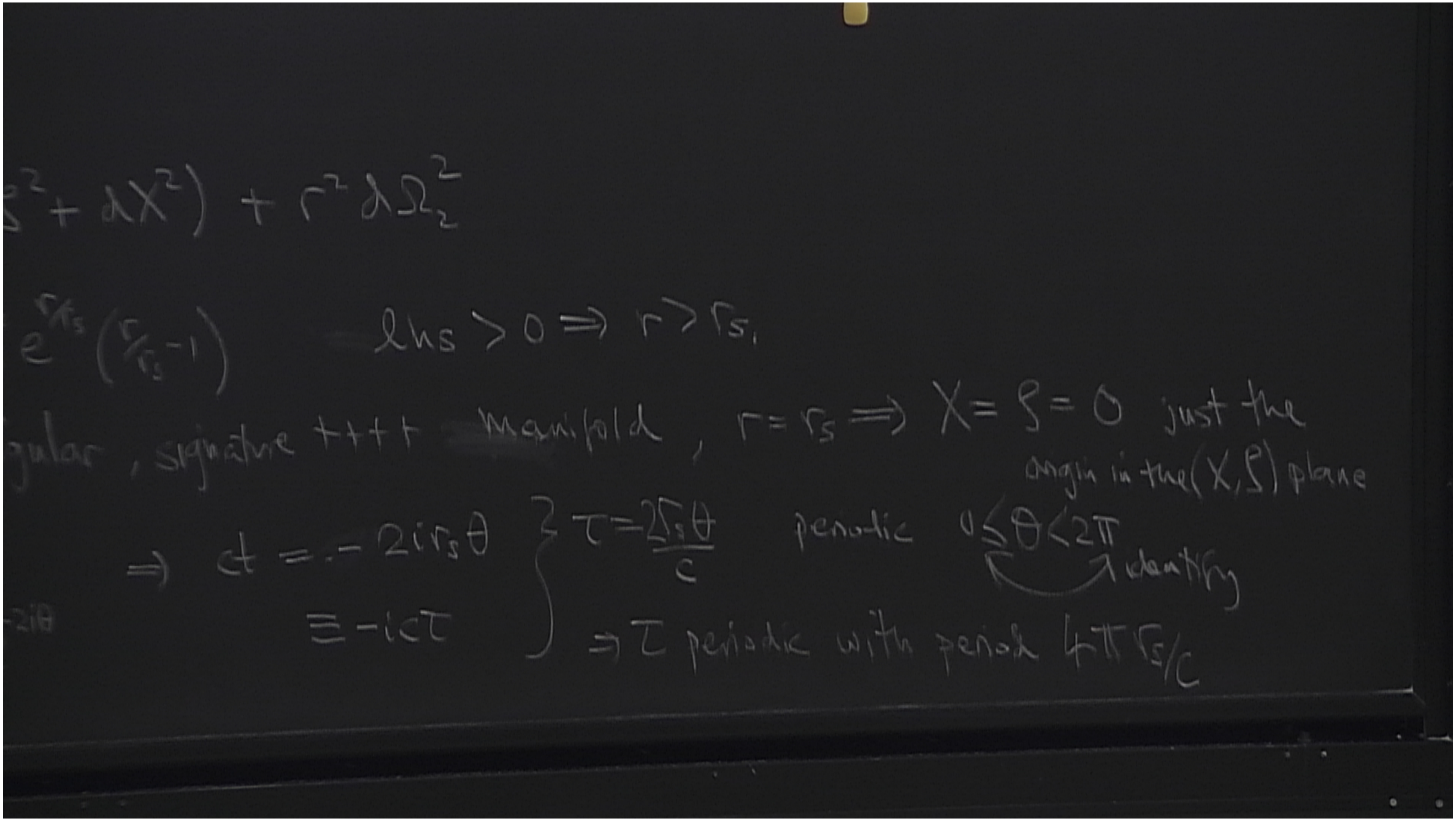
$$X^2 + \rho^2 = r_s^2 e^{r/r_s}\left(r/r_s - 1\right) \quad \Rightarrow lhs > 0 \Rightarrow r > r_s.$$

$= i\rho$

$^{r/r_s}(d\rho^2 + dX^2) + r^2 d\Omega_2^2$

$= r_s^2 e^{r/r_s}\left(\frac{r}{r_s} - 1\right)$ lhs $> 0 \Rightarrow r > r_s$.

y nonsingular, signature ++++ manifold, $r = r_s \Rightarrow X = \rho = 0$ just the origin in the (X, ρ) plane



$^2 + dX^2) + r^2 d\Omega_2^2$

$e^{r/r_s}\left(r/r_s - 1\right)$ $\qquad$ lhs $> 0 \Rightarrow r > r_s$.

gular, signature $++++$ manifold, $r = r_s \Rightarrow X = \rho = 0$ just the origin in the (X, ρ) plane

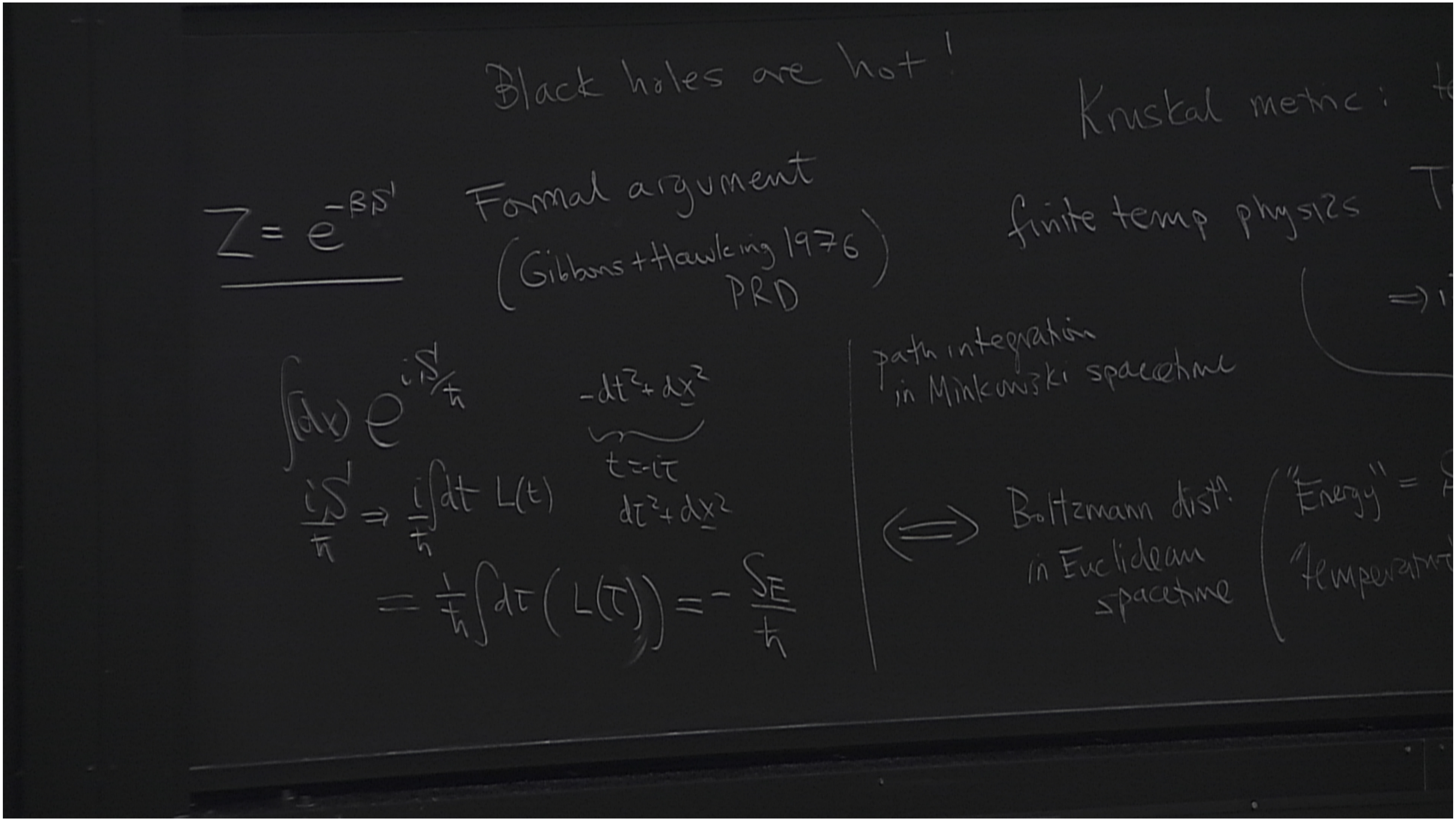
$\Rightarrow ct = -2ir_s\theta$ $\quad \Big\}$ $\tau = \frac{2r_s\theta}{c}$ $\quad$ periodic $0 \leq \theta < 2\pi$ identify

$-2i\theta$

$\equiv -ic\tau$ $\quad$ $\Rightarrow \tau$ periodic with period $4\pi r_s/c$

$$\Rightarrow \quad T = \frac{\hbar c}{k_B} \frac{1}{4\pi r_s} = \frac{\hbar c^3}{8\pi G M} \equiv T_{\text{Hawking}}$$

(tiny for real black holes)



Black holes are hot!

Kruskal metric:

$$\underline{Z = e^{-\beta S'}}$$

Formal argument
(Gibbons + Hawking 1976
PRD)

finite temp physics

$$\int (dx)\, e^{iS'/\hbar}$$

$$\underbrace{-dt^2 + d\underline{x}^2}_{t=-i\tau}$$

$$d\tau^2 + d\underline{x}^2$$

$$\frac{iS'}{\hbar} \Rightarrow \frac{i}{\hbar}\int dt\, L(t)$$

$$= \frac{1}{\hbar}\int d\tau \left(L(\tau) \right) = -\frac{S_E}{\hbar}$$

path integration
in Minkowski spacetime

$\Longleftrightarrow$ Boltzmann distn.
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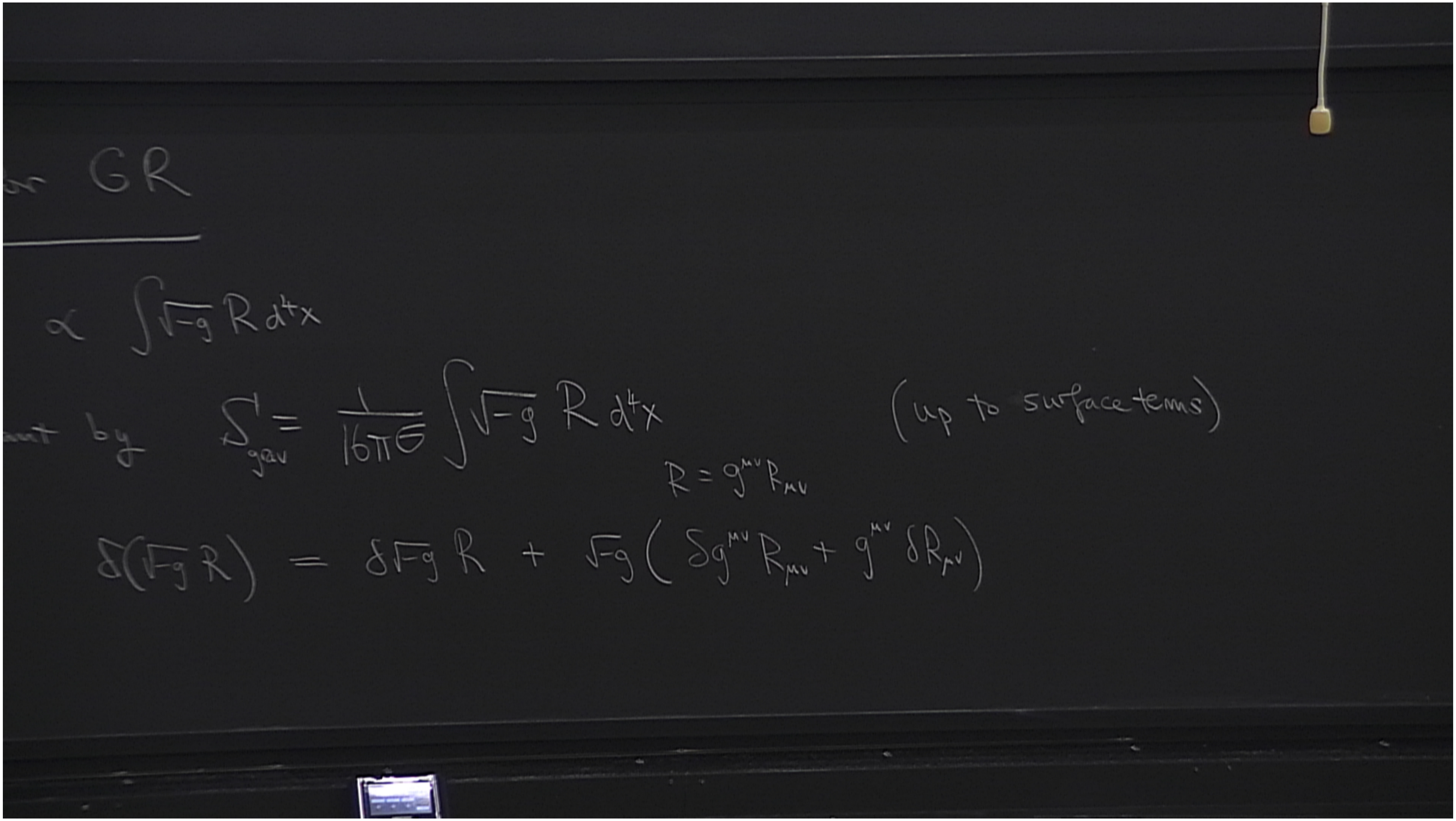
"Energy" =

"Temperat



or GR

$$\propto \int \sqrt{-g}\, R\, d^4x$$

nt by $S_{grav} = \frac{1}{16\pi G}\int \sqrt{-g}\, R\, d^4x$ (up to surface terms)

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$\delta(\sqrt{-g}\, R) = \delta\sqrt{-g}\, R + \sqrt{-g}\left(\delta g^{\mu\nu} R_{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}\right)$$

$$g = g_{\alpha\beta}\,\overset{\swarrow \text{cofactors}}{\tilde{g}^{\alpha\beta}}$$

$$\delta g = \delta g_{\alpha\beta}\,\tilde{g}^{\alpha\beta} = \delta g_{\alpha\beta}\, g^{\alpha\beta}\, g$$

(up to surface terms)

$$= g^{\mu\nu} R_{\mu\nu}$$

$$\delta g^{\mu\nu} R_{\mu\nu} + g^{\mu\nu}\,\delta R_{\mu\nu} \big)$$

Variation of inverse metric

$$\delta g_{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\mu} = \delta^{\alpha}_{\beta}$$

Vary:

$$\delta g^{\alpha\beta} g_{\beta\mu} + g^{\alpha\beta}\,\delta g_{\beta\mu} = 0$$

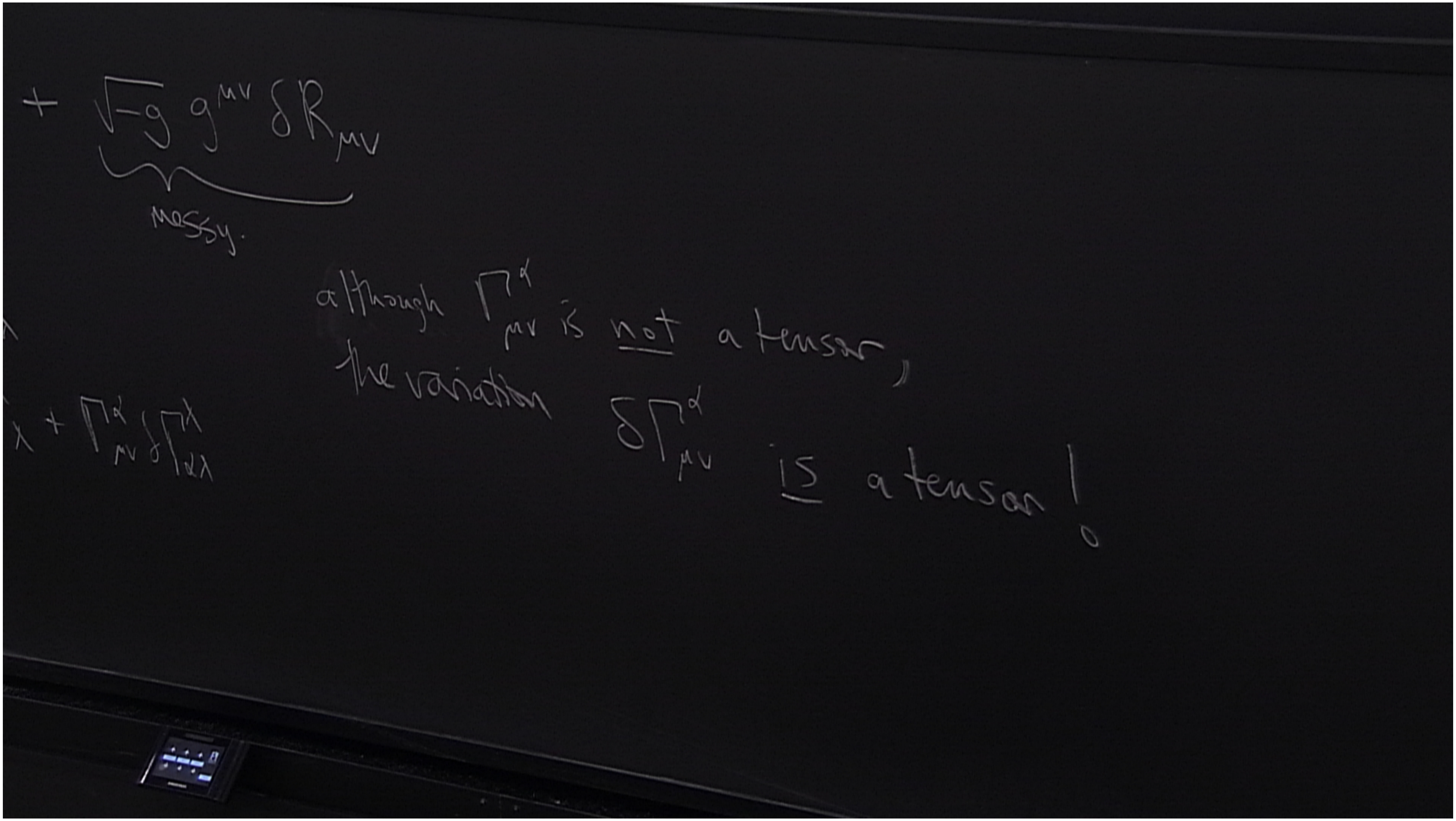
$$\Rightarrow \delta g^{\alpha\beta} = -g^{\alpha\nu}\,\delta g_{\nu\mu}\, g^{\mu\beta}$$

$$\delta\sqrt{-g} = \frac{1}{2}\frac{1}{\sqrt{-g}}\left(-\delta g\right) = \underline{\frac{1}{2}\sqrt{-g}\;\delta g_{\alpha\beta}\, g^{\alpha\beta}}$$

$$\delta(\sqrt{-g}\,R) = \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu} R\, \delta g_{\mu\nu} - R^{\mu\nu} \delta g_{\mu\nu}\right) + \underbrace{\sqrt{-g}\; g}_{\text{messy}}$$

$$R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}$$

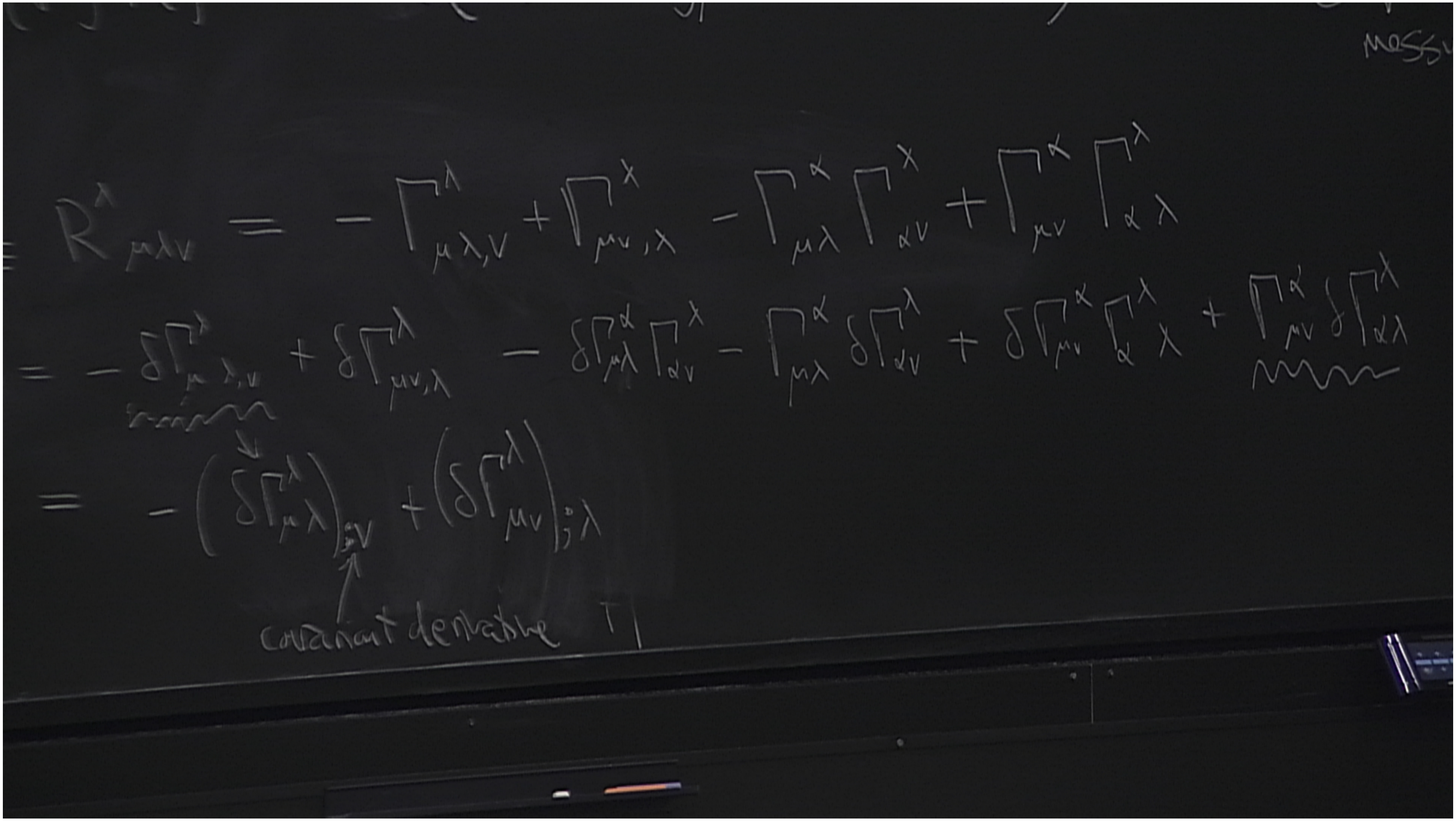
$$\delta R_{\mu\nu} = -\delta\Gamma^{\lambda}_{\mu\lambda,\nu} + \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\delta\Gamma^{\lambda}_{\alpha\nu} + \delta\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda} + \Gamma^{\alpha}_{\mu\nu}\delta\Gamma^{\lambda}_{\alpha\lambda}$$



$$+ \underbrace{\sqrt{-g}\, g^{\mu\nu}\, \delta R_{\mu\nu}}_{\text{messy.}}$$

$$\lambda + \Gamma^{\alpha}_{\mu\nu}\,\delta\Gamma^{\lambda}_{\alpha\lambda}$$

although $\Gamma^{\alpha}_{\mu\nu}$ is <u>not</u> a tensor, the variation $\delta\Gamma^{\alpha}_{\mu\nu}$ <u>is</u> a tensor!



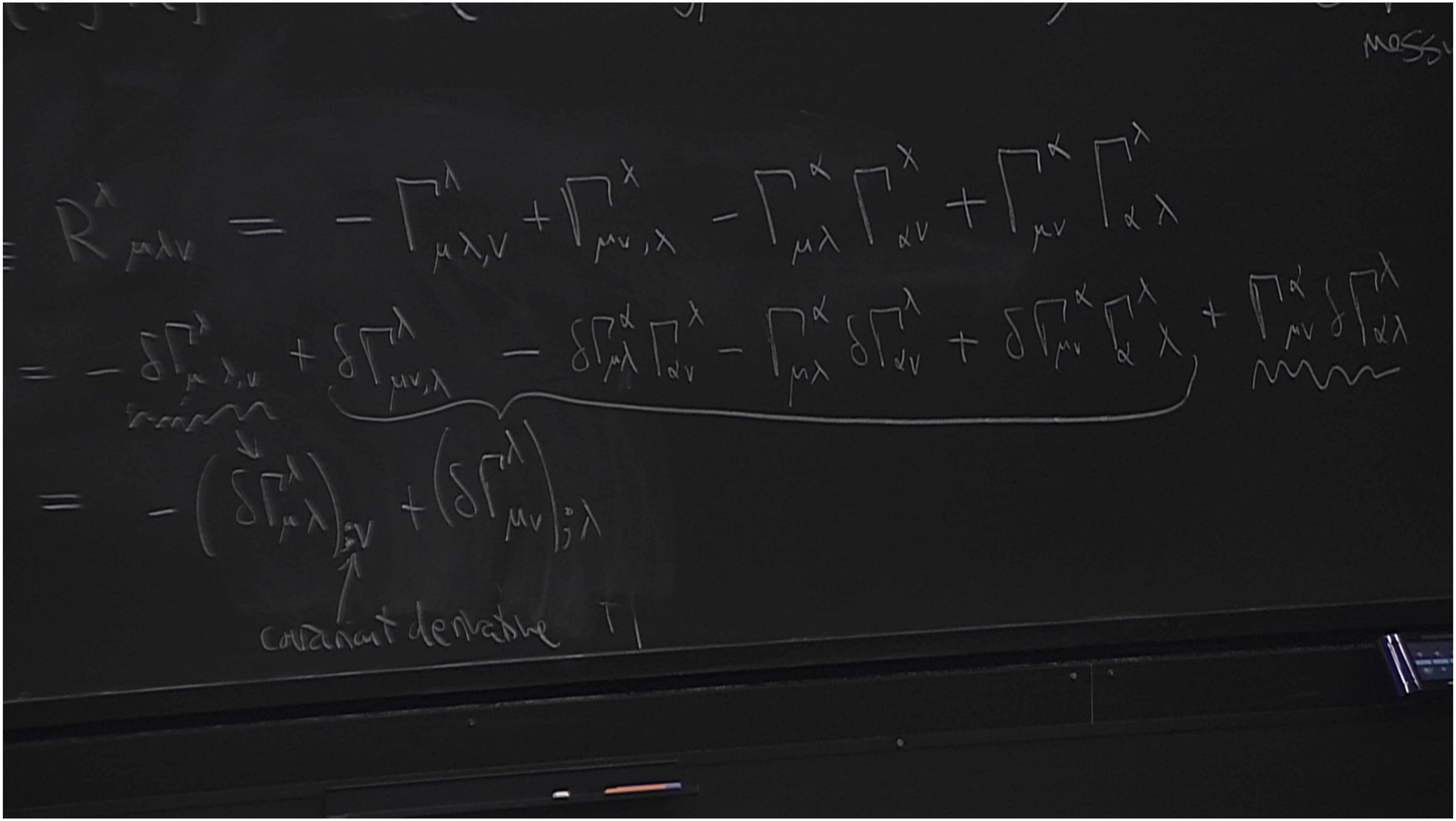
messu

$$R^{\lambda}{}_{\mu\lambda\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\delta\Gamma^{\lambda}_{\mu\lambda,\nu} + \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\delta\Gamma^{\lambda}_{\alpha\nu} + \delta\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda} + \Gamma^{\alpha}_{\mu\nu}\delta\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\left(\delta\Gamma^{\lambda}_{\mu\lambda}\right)_{;\nu} + \left(\delta\Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda}$$

covariant derivative



messu

$$R^{\lambda}{}_{\mu\lambda\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\delta\Gamma^{\lambda}_{\mu\lambda,\nu} + \underbrace{\delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\delta\Gamma^{\lambda}_{\alpha\nu} + \delta\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}} + \Gamma^{\alpha}_{\mu\nu}\delta\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\left(\delta\Gamma^{\lambda}_{\mu\lambda}\right)_{;\nu} + \left(\delta\Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda}$$

covariant derivative

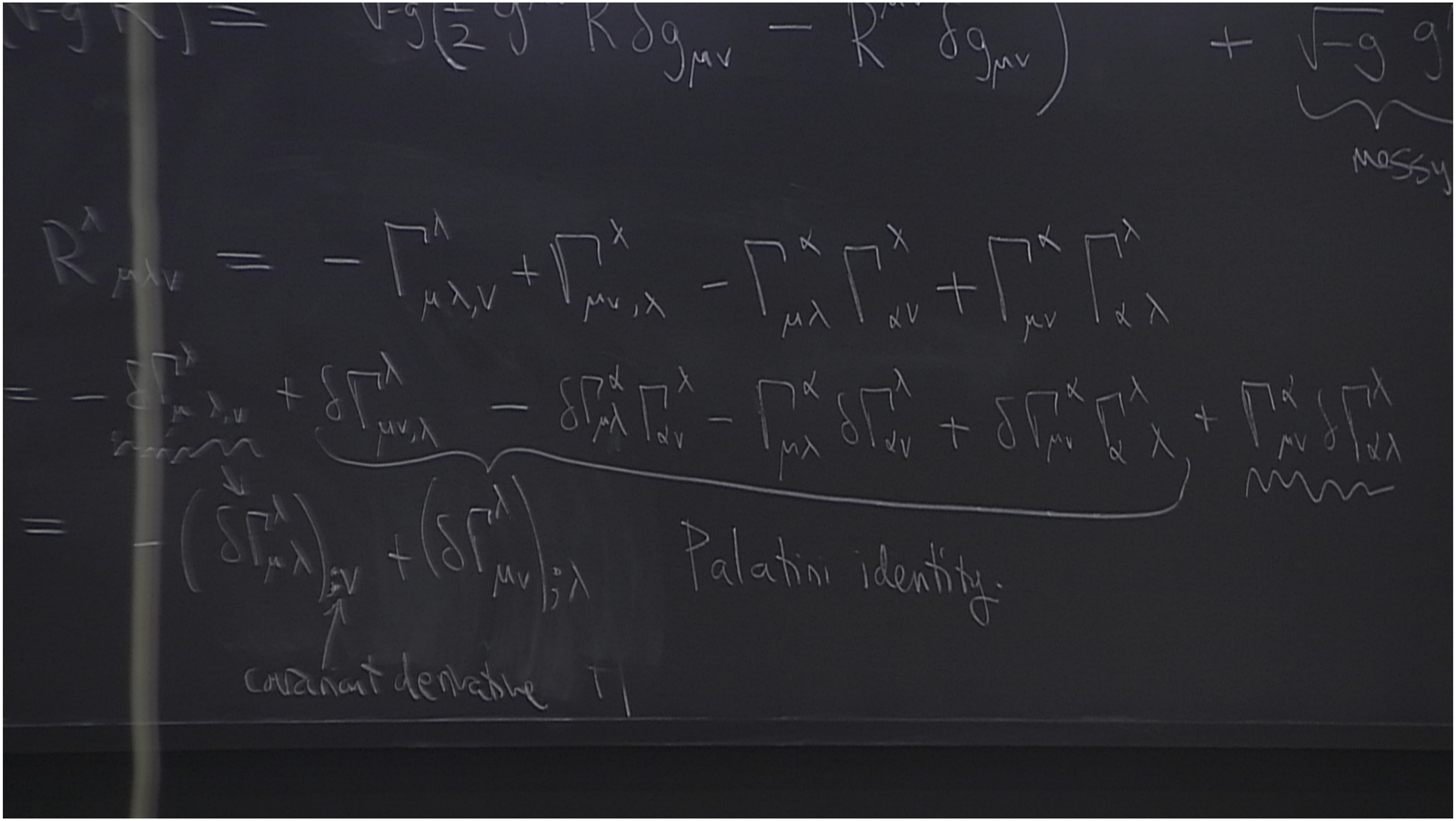
$$= R^\lambda{}_{\mu\lambda\nu} = -\Gamma^\lambda_{\mu\lambda,\nu} + \Gamma^\lambda_{\mu\nu,\lambda} - \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\lambda_{\alpha\lambda}$$

$$= -\delta\Gamma^\lambda_{\mu\lambda,\nu} + \underbrace{\delta\Gamma^\lambda_{\mu\nu,\lambda} - \delta\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\alpha\nu} - \Gamma^\alpha_{\mu\lambda}\delta\Gamma^\lambda_{\alpha\nu} + \delta\Gamma^\alpha_{\mu\nu}\Gamma^\lambda_{\alpha\lambda}} + \Gamma^\alpha_{\mu\nu}\delta\Gamma^\lambda_{\alpha\lambda}$$

$$= -\left(\delta\Gamma^\lambda_{\mu\lambda}\right)_{;\nu} + \left(\delta\Gamma^\lambda_{\mu\nu}\right)_{;\lambda}$$

Palatini identity.

covariant derivative



$$(\sqrt{-g}\,R) = \sqrt{-g}\left(\tfrac{1}{2}g\;R\,\delta g_{\mu\nu} - R\;\delta g_{\mu\nu}\right) + \underbrace{\sqrt{-g}\,g}_{\text{messy}}$$

$$R^{\lambda}{}_{\mu\lambda\nu} = -\Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\delta\Gamma^{\lambda}_{\mu\lambda,\nu} + \underbrace{\delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\delta\Gamma^{\lambda}_{\alpha\nu} + \delta\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\lambda}} + \Gamma^{\alpha}_{\mu\nu}\delta\Gamma^{\lambda}_{\alpha\lambda}$$

$$= -\left(\delta\Gamma^{\lambda}_{\mu\lambda}\right)_{;\nu} + \left(\delta\Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda}$$

Palatini identity.

covariant derivative

$$ {}_{;\nu} + \left(\delta\Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda} \Big) + $$

$$ {}_{;\nu} + \left(g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda} \Big] $$

divergences $$\underbrace{\nabla_\nu V^\nu}_{\partial_\nu V^\nu - \Gamma^{\lambda}_{\lambda\nu} V^\nu} = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,V^\mu\right)$$

$$\Gamma^{\lambda}_{\lambda\nu} = \frac{1}{2} g^{\lambda\alpha}\Big(g_{\lambda\alpha,\nu} + \underbrace{\cancel{g_{\nu\alpha,\lambda}} - \cancel{g_{\lambda\nu,\alpha}}}_{\text{antisym } \lambda\leftrightarrow\alpha}\Big) = \frac{1}{2} g^{\lambda\alpha} g_{\lambda\alpha,\nu}$$

$$\underbrace{- g_{\lambda\nu,\alpha}}_{\text{tisym } \lambda\leftrightarrow\alpha}) = \frac{1}{2} g^{\lambda\alpha} g_{\lambda\alpha,\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \left(\sqrt{-g}\right)$$

$$R_{\mu\nu} = \int \sqrt{-g}\, g^{\mu\nu} \left(-(\delta \Gamma_{\mu\lambda})_{;\nu} + (\delta \Gamma_{\mu\nu})_{;\lambda} \right)$$

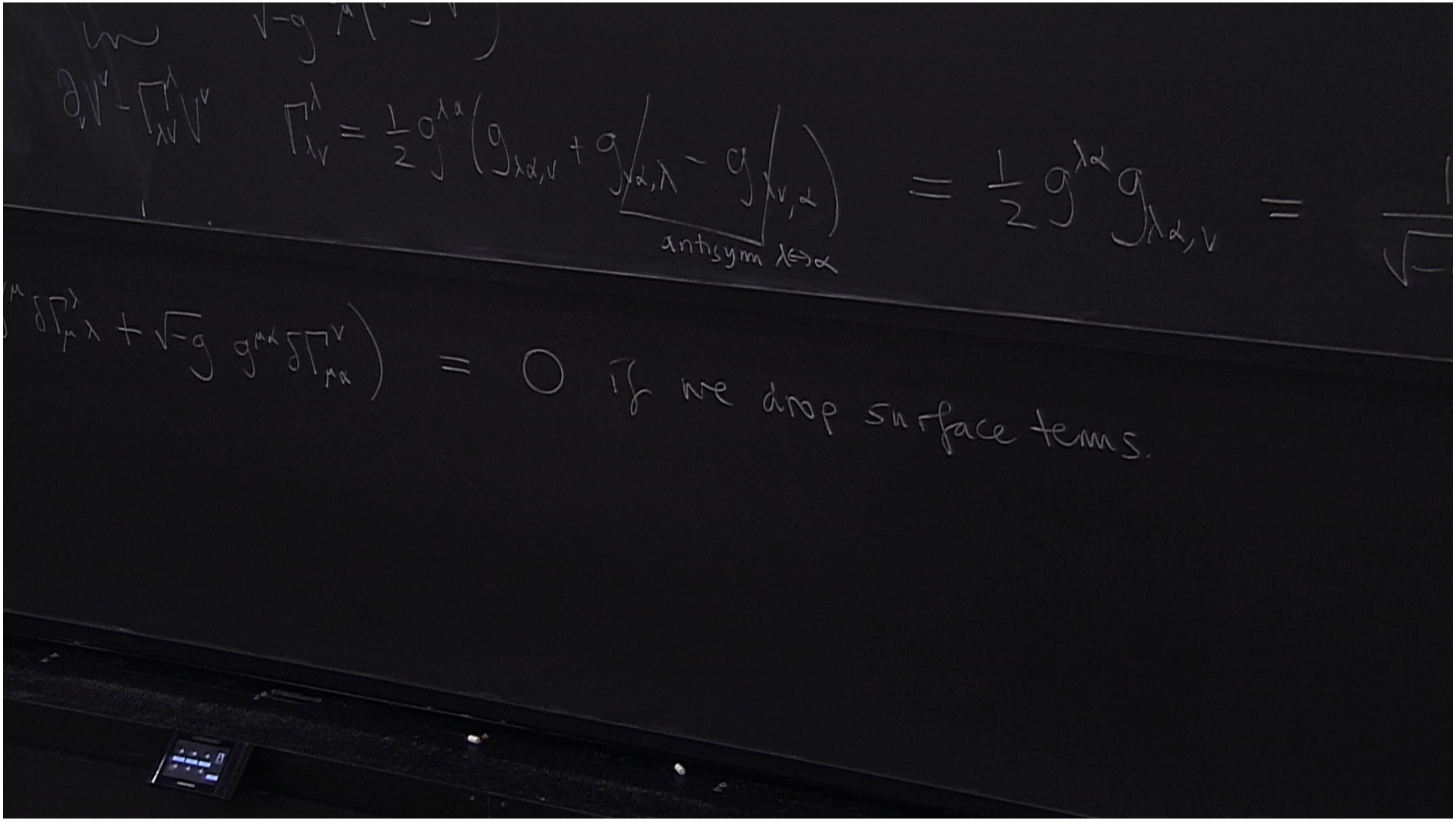
use metricity

$$\sqrt{-g} \left[-\left(g^{\nu;\mu} \delta \Gamma^{\lambda}_{\mu\lambda}\right)_{;\nu} + \left(g^{\mu\nu} \delta \Gamma^{\lambda}_{\mu\nu}\right)_{;\lambda} \right]$$

– both are divergences $\underbrace{\nabla_\nu V^\nu}_{\partial_\nu V^\nu - \Gamma^{\lambda}_{\lambda\nu} V^\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, V^\mu \right)$

$$\Gamma^{\lambda}_{\lambda\nu} = \frac{1}{2} g^{\lambda\alpha} \left(g_{\lambda\alpha,\nu} + \underbrace{g_{\nu\alpha,\lambda} - g_{\lambda\nu,\alpha}}_{\text{antisym } \lambda \Leftrightarrow \alpha} \right) =$$

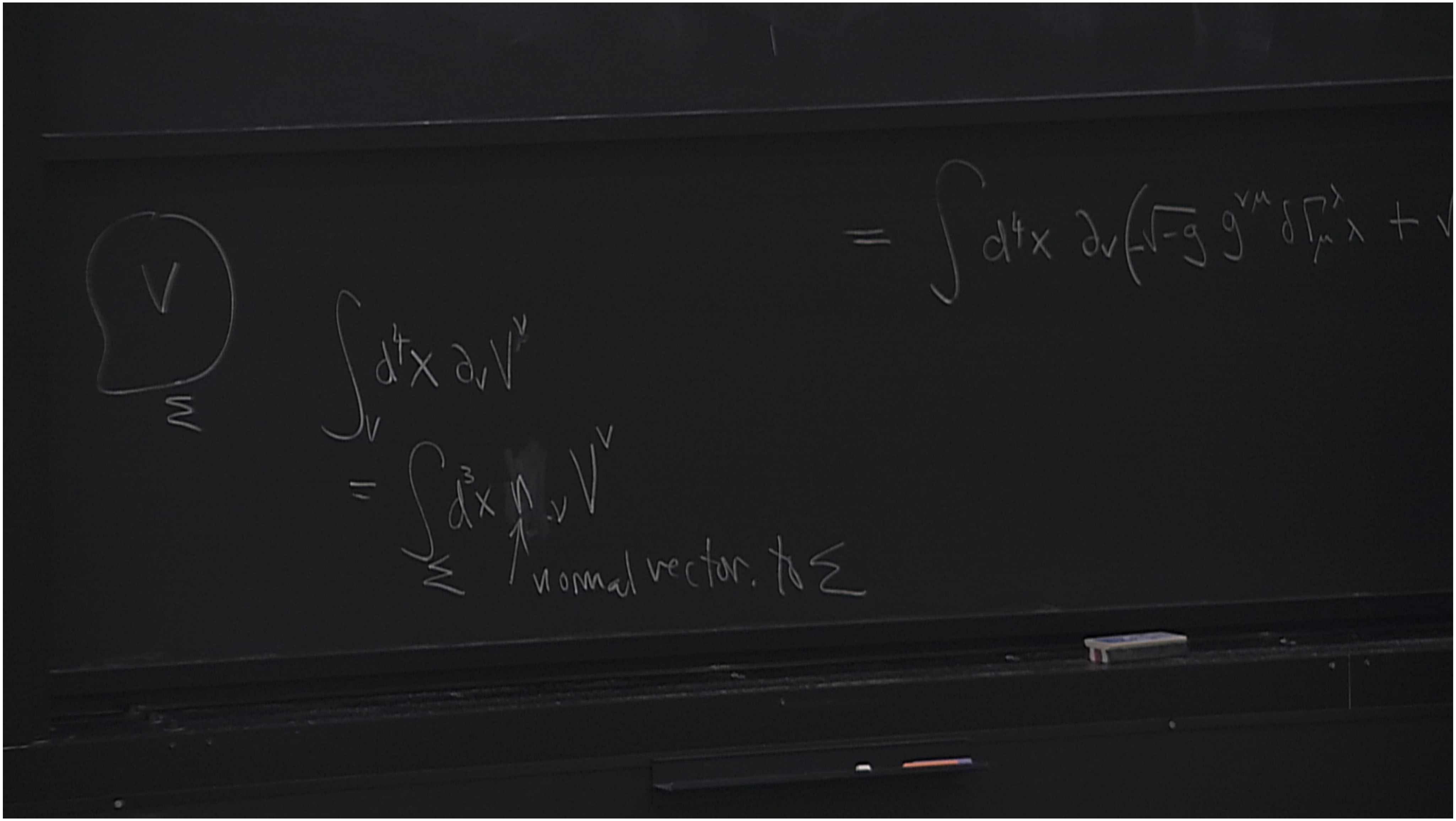
$$= \int d^4x\, \partial_\nu \left(-\sqrt{-g}\, g^{\nu\mu} \delta \Gamma^{\lambda}_{\mu}{}_{\lambda} + \sqrt{-g}\, g^{\mu\alpha} \delta \Gamma^{\nu}_{\mu\alpha} \right)$$



$\partial_\nu V^\nu - \Gamma^\lambda_{\lambda\nu} V^\nu$

$$\Gamma^\lambda_{\lambda\nu} = \frac{1}{2} g^{\lambda\alpha}\left(g_{\lambda\alpha,\nu} + \underbrace{\cancel{g_{\nu\alpha,\lambda}} - \cancel{g_{\lambda\nu,\alpha}}}_{\text{antisym } \lambda\leftrightarrow\alpha}\right) = \frac{1}{2} g^{\lambda\alpha} g_{\lambda\alpha,\nu} = \frac{1}{\sqrt{-}}$$

$\delta\Gamma^\lambda_{\mu\lambda} + \sqrt{-g}\; g^{\mu\alpha}\delta\Gamma^\nu_{\mu\alpha}\Big) = 0$ if we drop surface terms.

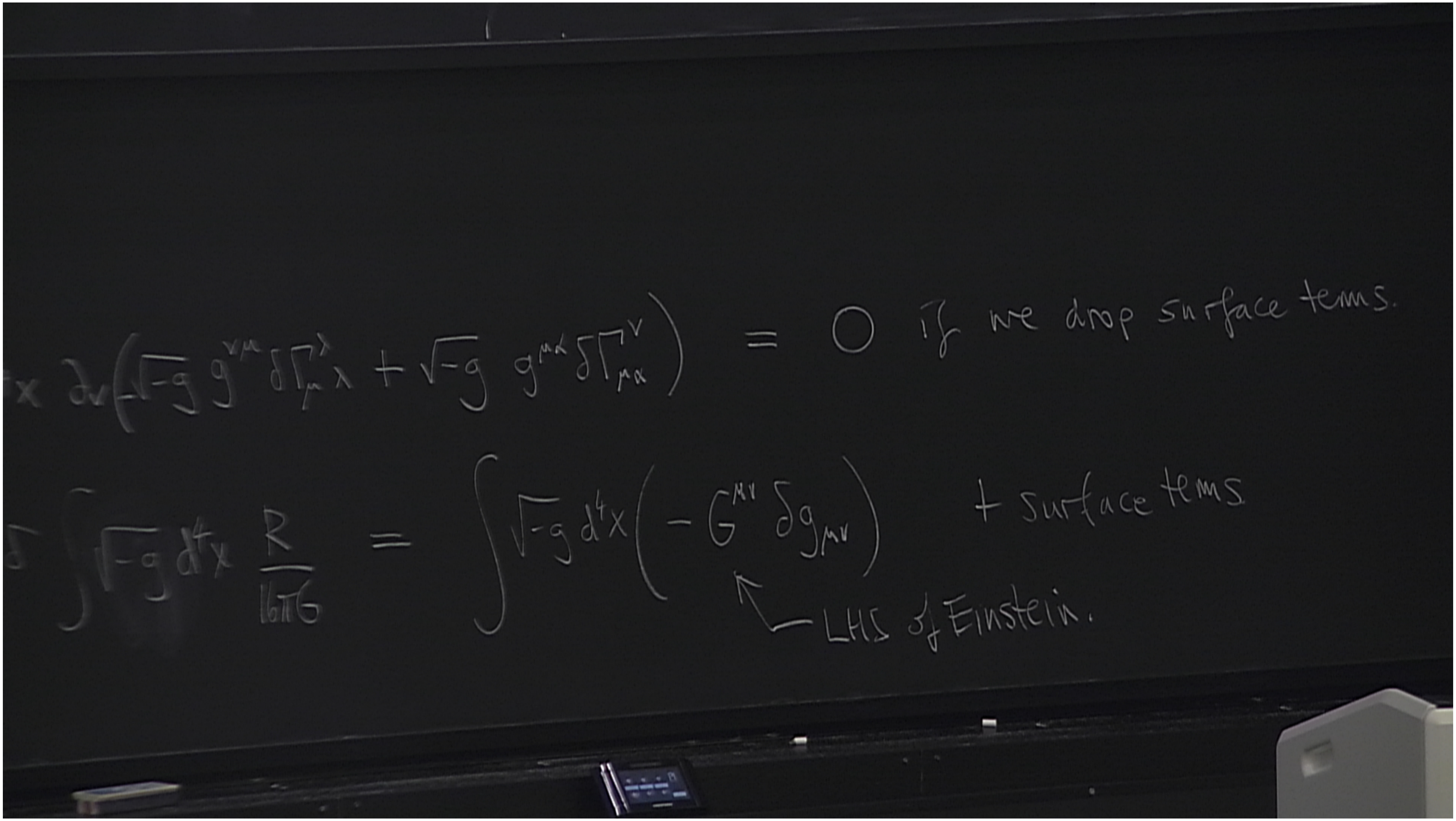


$$= \int d^4x\, \partial_\nu \left(-\sqrt{-g}\, g^{\nu\mu}\, \delta\Gamma^{\lambda}_{\mu\lambda} + \right.$$

$$\int_V d^4x\, \partial_\nu V^\nu$$

$$= \int_\Sigma d^3x\, n_\nu V^\nu$$

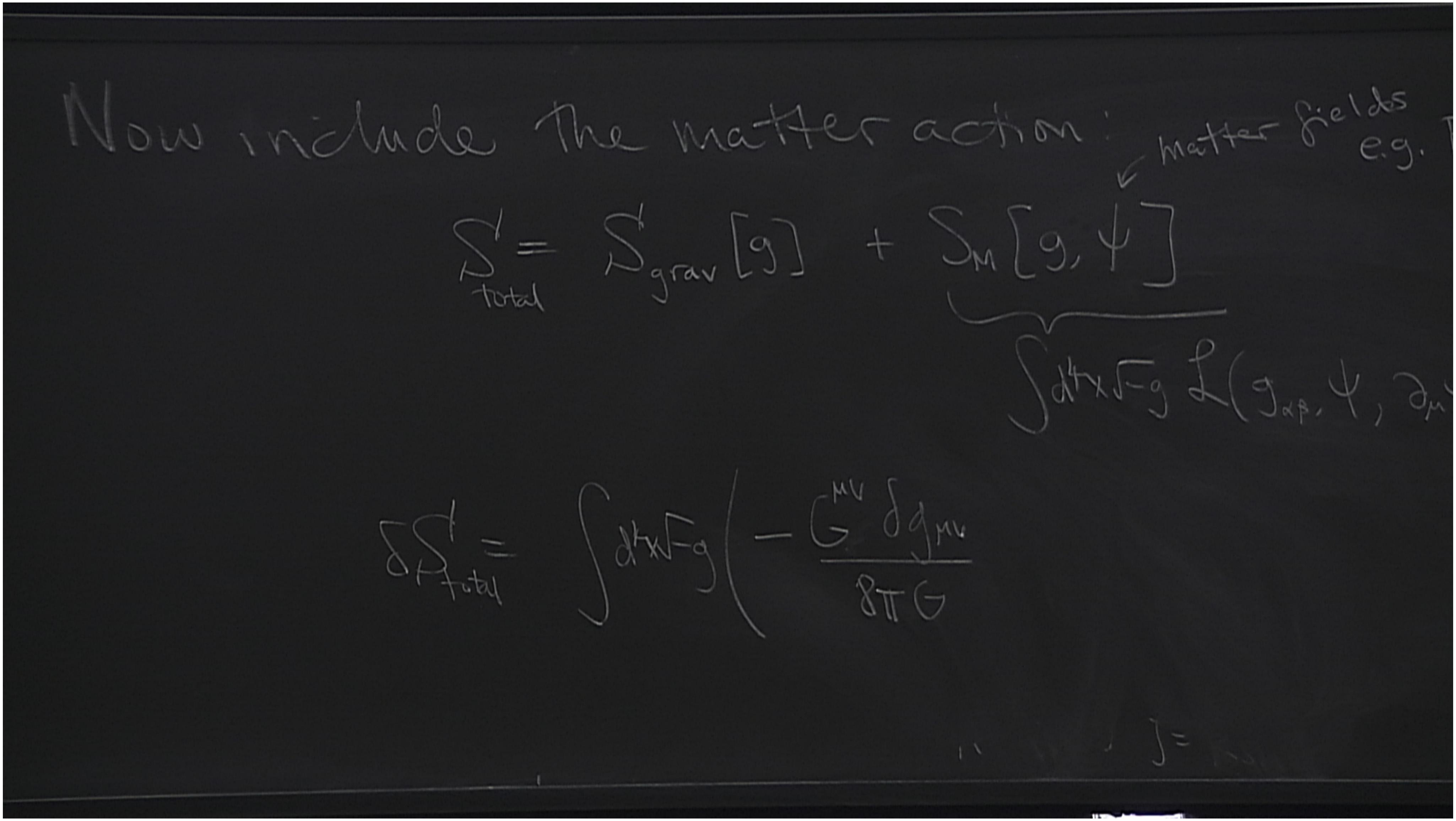
$\uparrow$ normal vector. to Σ



$$x\ \partial_\nu\left(-\sqrt{-g}\,g^{\nu\mu}\,\delta\Gamma^\lambda_{\mu\lambda}+\sqrt{-g}\;g^{\mu\alpha}\,\delta\Gamma^\nu_{\mu\alpha}\right)\;=\;0$$ if we drop surface tems.

$$\delta\int\sqrt{-g}\,d^4x\,\frac{R}{16\pi G}\;=\;\int\sqrt{-g}\,d^4x\left(-G^{\mu\nu}\,\delta g_{\mu\nu}\right)\quad+\text{ surface tems}$$

$\uparrow$ LHS of Einstein.



Now include the matter action:

matter fields e.g.

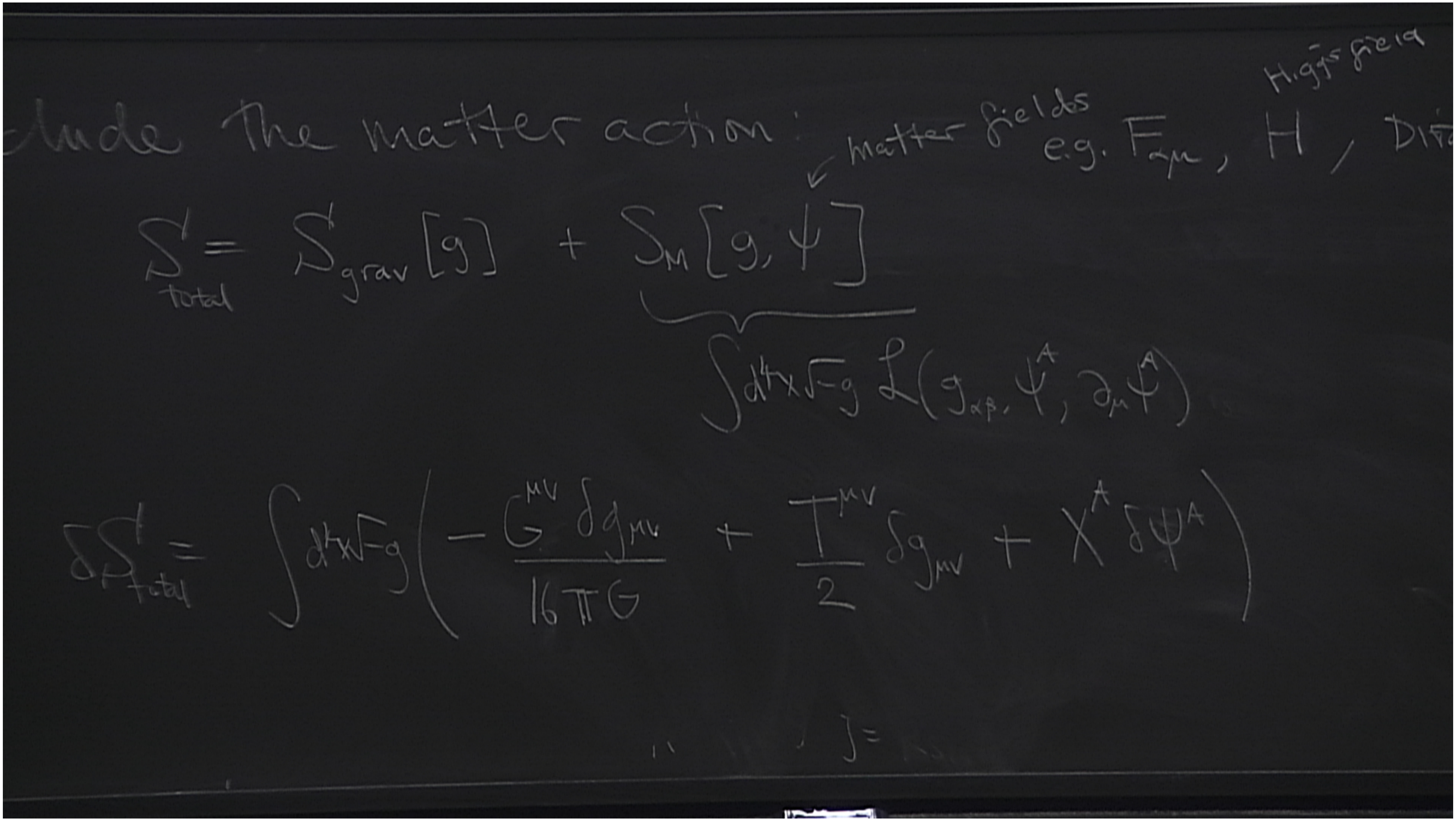
$$S_{total} = S_{grav}[g] + \underbrace{S_M[g, \psi]}_{\int d^4x \sqrt{-g}\, \mathcal{L}(g_{\alpha\beta}, \psi, \partial_\mu}$$

$$\delta S_{total} = \int d^4x \sqrt{-g} \left(- \frac{G^{\mu\nu} \delta g_{\mu\nu}}{8\pi G} \right.$$

$$+\sqrt{-g}\; g^{\mu\alpha}\,\delta\Gamma^{\nu}_{\mu\alpha}\Big) = 0$$ if we drop surface terms.

$$= \int \sqrt{-g}\, d^4x \left(-\frac{G^{\mu\nu}}{16\pi G}\,\delta g_{\mu\nu} \right) \quad + \text{surface terms}$$

LHS of Einstein.



lude the matter action:

matter fields

e.g. $F_{\alpha\mu}$, H, Dir

Higgs field

$$S_{total} = S_{grav}[g] + \underbrace{S_M[g,\psi]}_{\int d^4x\sqrt{-g}\,\mathcal{L}(g_{\alpha\beta},\,\psi^A,\,\partial_\mu\psi^A)}$$

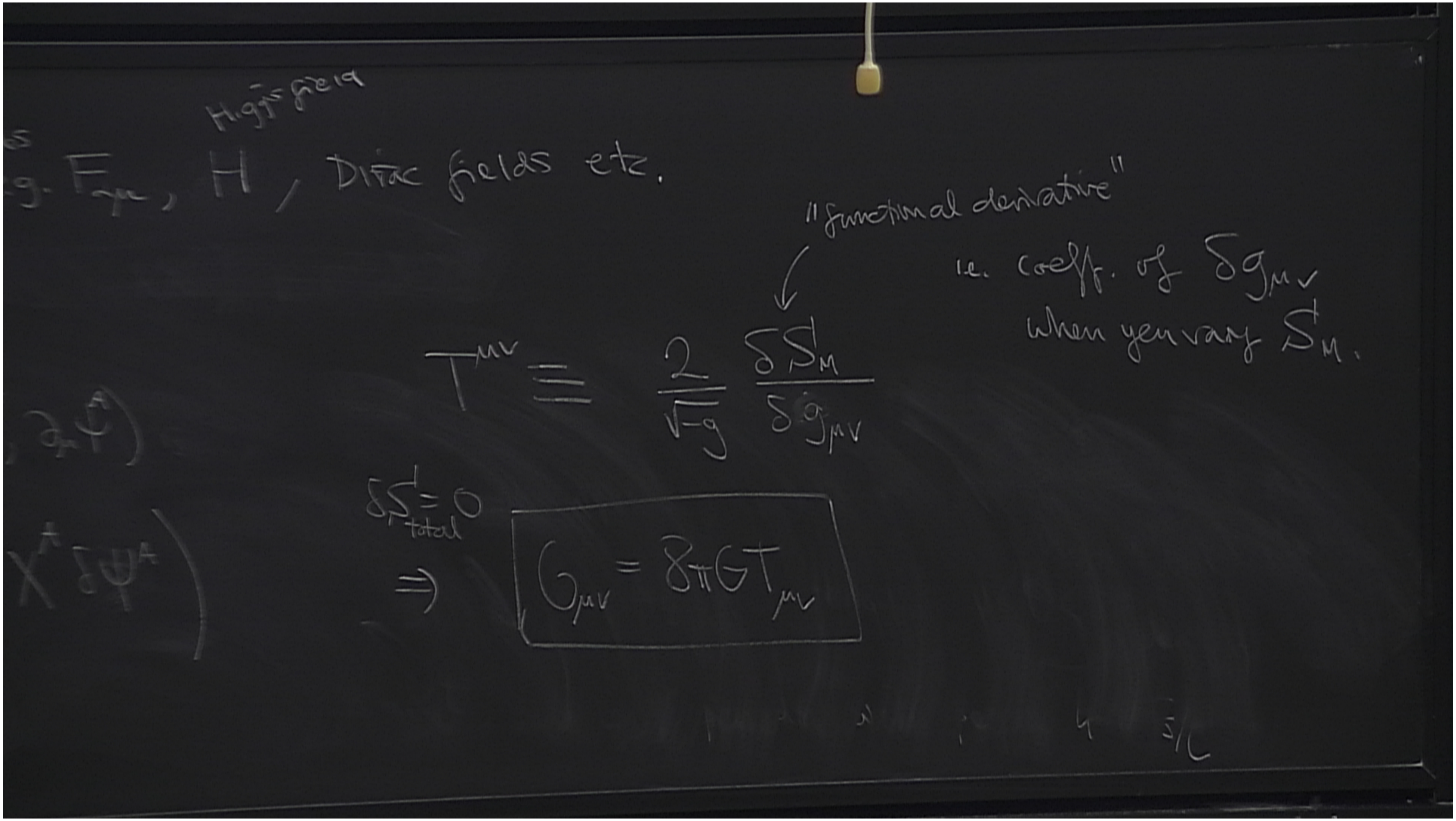
$$\delta S_{total} = \int d^4x\sqrt{-g}\left(-\frac{G^{\mu\nu}\delta g_{\mu\nu}}{16\pi G} + \frac{T^{\mu\nu}}{2}\delta g_{\mu\nu} + X^A\delta\psi^A\right)$$

Higgs field

lude the matter action: matter fields e.g. $F_{\alpha\mu}$, H, Dira

$$S_{\text{total}} = S_{\text{grav}}[g] + \underbrace{S_M[g,\psi]}_{\int d^4x\sqrt{-g}\,\mathcal{L}(g_{\alpha\beta},\,\psi^A,\,\partial_\mu\psi^A)}$$

$$\delta S_{\text{total}} = \int d^4x\sqrt{-g}\left(-\frac{G^{\mu\nu}\,\delta g_{\mu\nu}}{16\pi G} + \underbrace{\frac{T^{\mu\nu}}{2}}_{\text{by def}^n}\delta g_{\mu\nu} + X^A\,\delta\psi^A\right)$$



es

Higgs field

g. $F_{\mu\nu}$, H, Dirac fields etc.

"functional derivative"

i.e. coeff. of $\delta g_{\mu\nu}$ when you vary S_M.

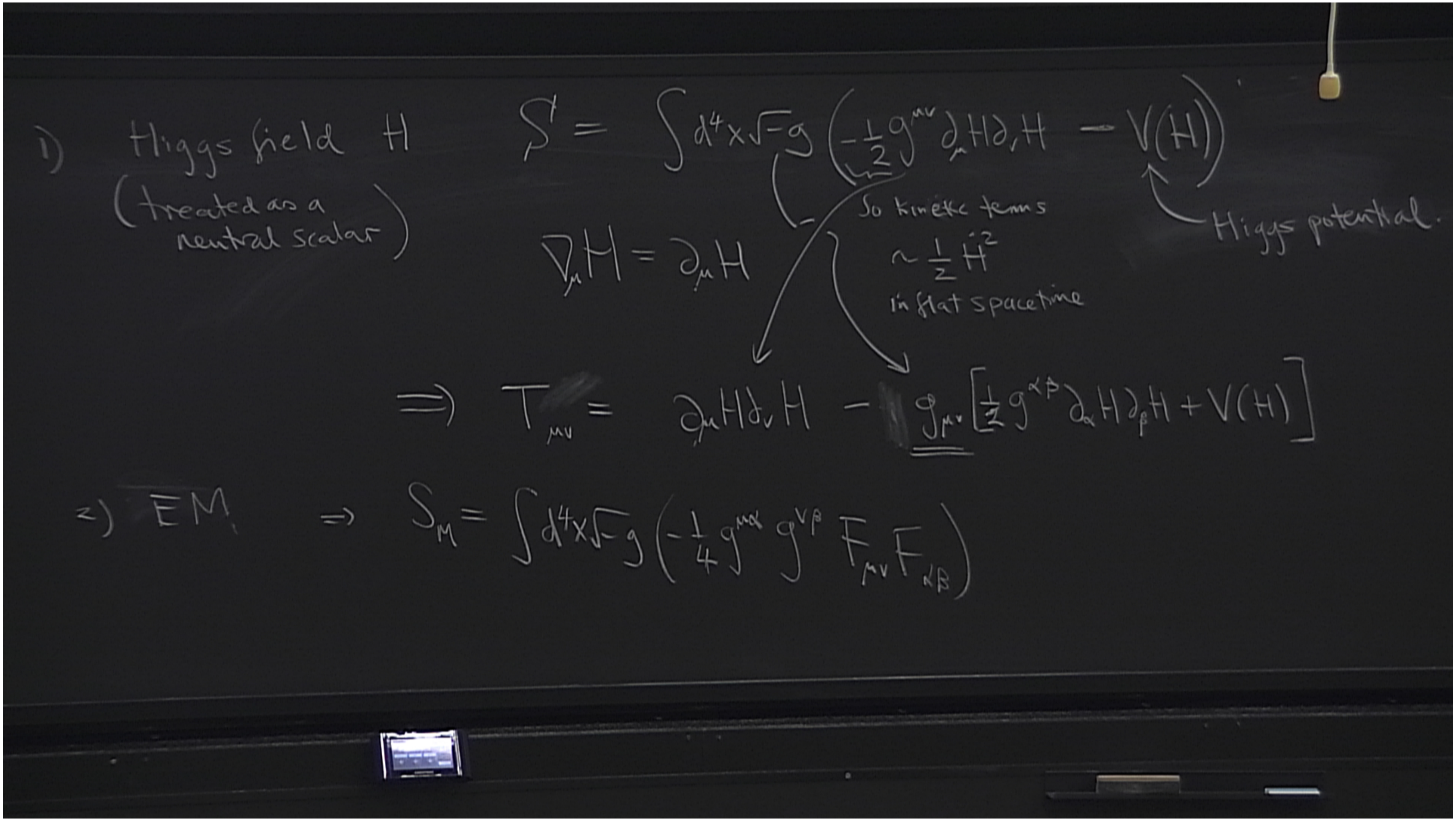
$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$$

$\partial_\mu \psi^A)$

$X^A \delta\psi^A)$

$\delta S_{total} = 0$

$$\Rightarrow \boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$



1) Higgs field H

(treated as a neutral scalar)

$$S' = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu H \partial_\nu H - V(H) \right)$$

Higgs potential.

$$\nabla_\mu H = \partial_\mu H$$

So kinetic terms $\sim \frac{1}{2} \dot{H}^2$ in flat spacetime

$$\Longrightarrow T_{\mu\nu} = \partial_\mu H \partial_\nu H - \underline{\underline{g_{\mu\nu}}} \left[\frac{1}{2} g^{\alpha\beta} \partial_\alpha H \partial_\beta H + V(H) \right]$$

2) EM $\Rightarrow$ $S_M = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \right)$

$$= \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu H\partial_\nu H - V(H)\right)$$

Higgs potential.

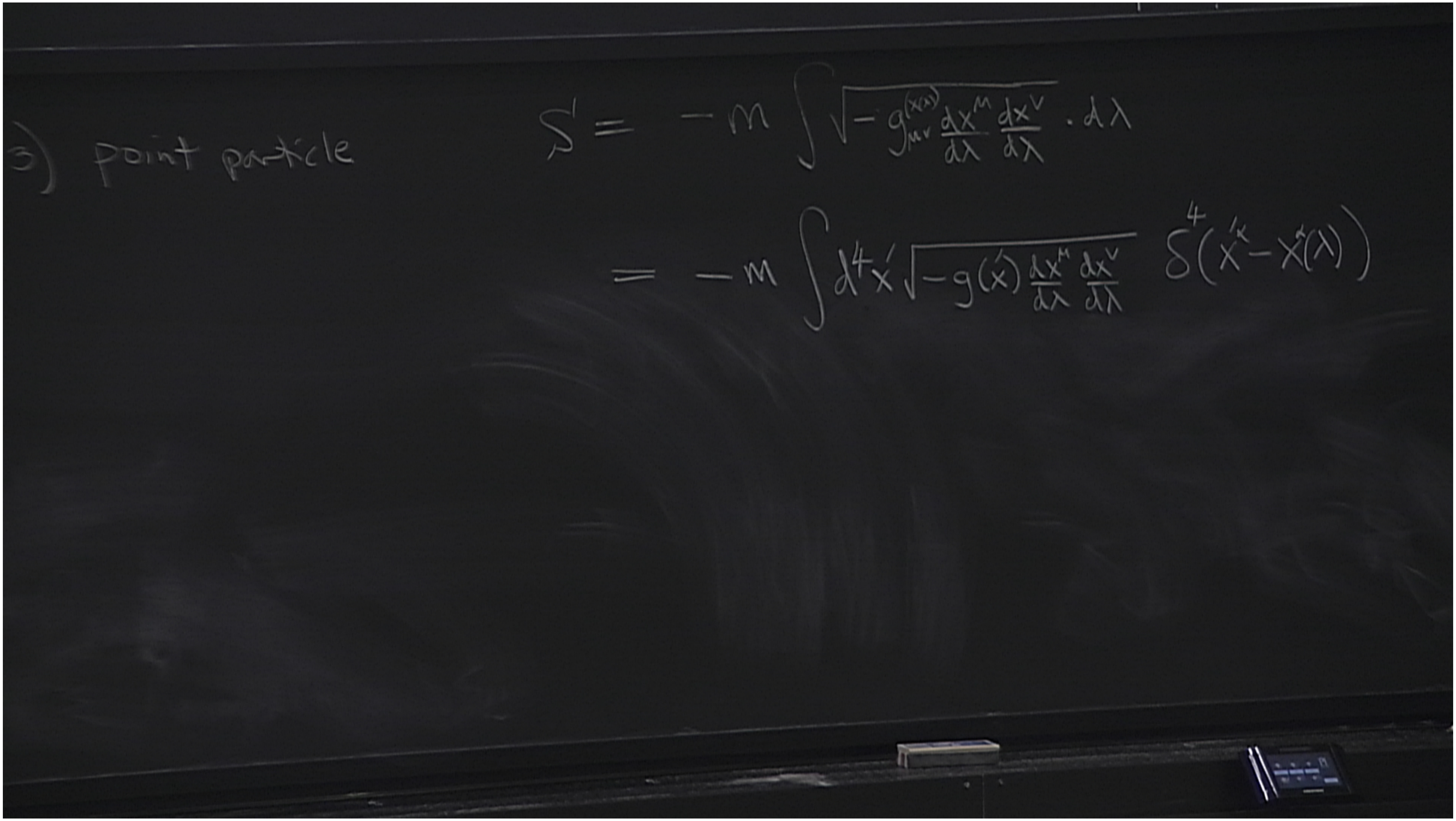
So kinetic terms $\sim \frac{1}{2}\dot{H}^2$ in flat spacetime

$$H = \partial_\mu H$$

$$= \partial_\mu H \partial_\nu H - \underline{\underline{g_{\mu\nu}}}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha H\partial_\beta H + V(H)\right]$$

$$\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right) \Longrightarrow$$ Calculate $T_{\mu\nu}$ (Ex)

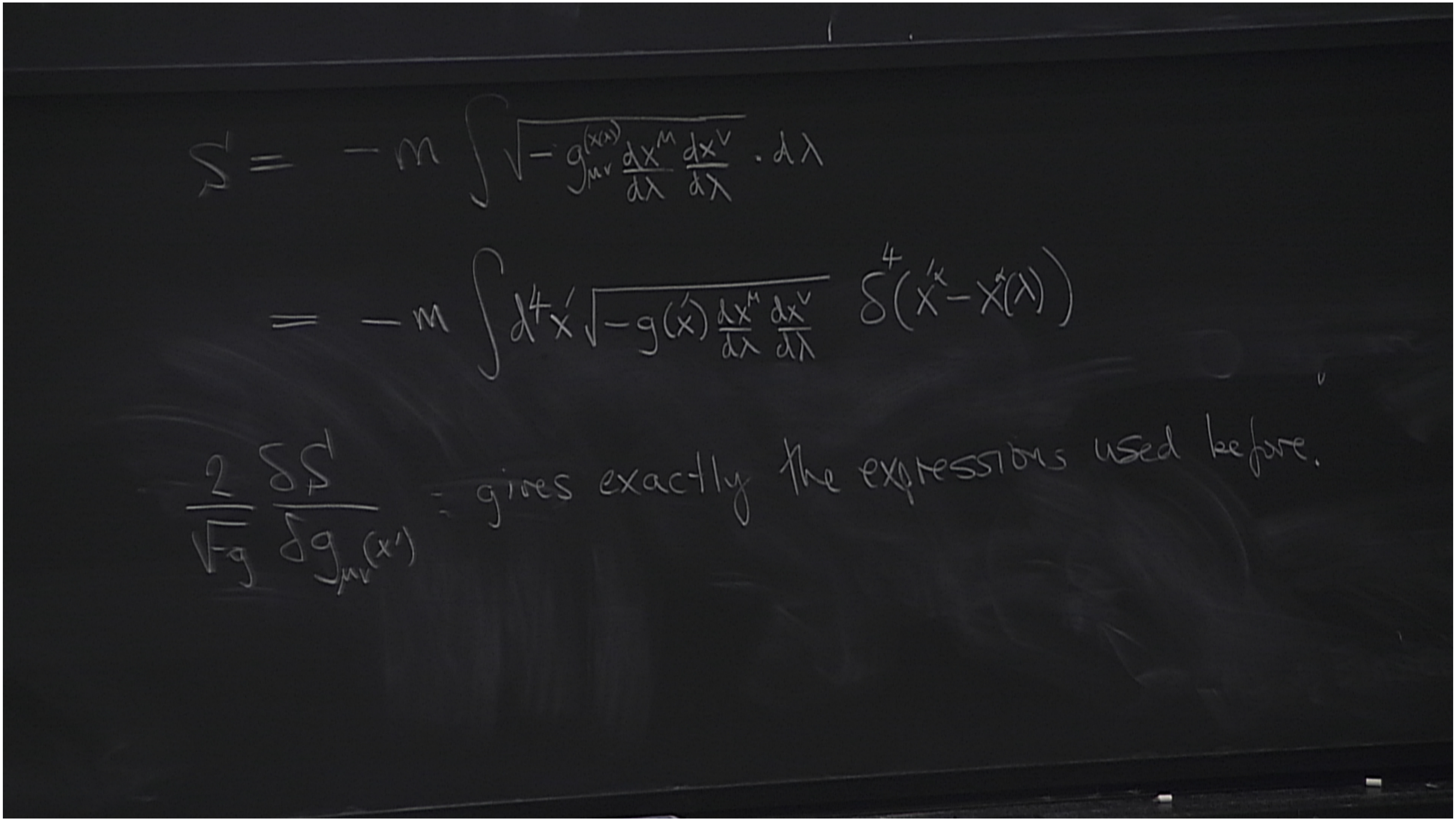
– recover exactly the form given earlier.



3) point particle

$$S' = -m \int \sqrt{-g_{\mu\nu}^{(x(\lambda))} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}} \cdot d\lambda$$

$$= -m \int d^4x' \sqrt{-g(x') \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}} \; \delta^4\left(x'^{\mu} - x^{\mu}(\lambda)\right)$$



$$S' = -m \int \sqrt{-g_{\mu\nu}^{(x(\lambda))} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \cdot d\lambda$$

$$= -m \int d^4x' \sqrt{-g(x') \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \; \delta^4(x'^\alpha - x^\alpha(\lambda))$$

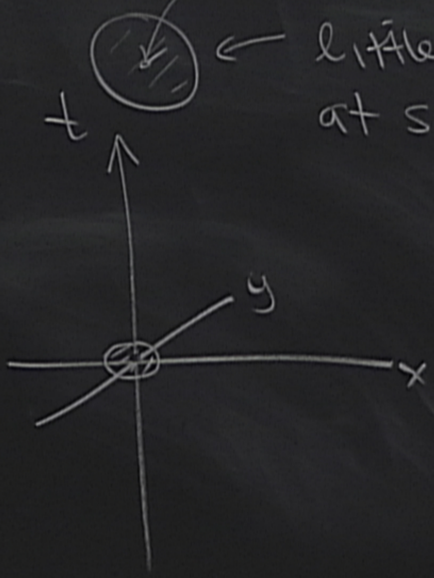
$\dfrac{2}{\sqrt{-g}} \dfrac{\delta S'}{\delta g_{\mu\nu}(x')} =$ gives exactly the expressions used before.

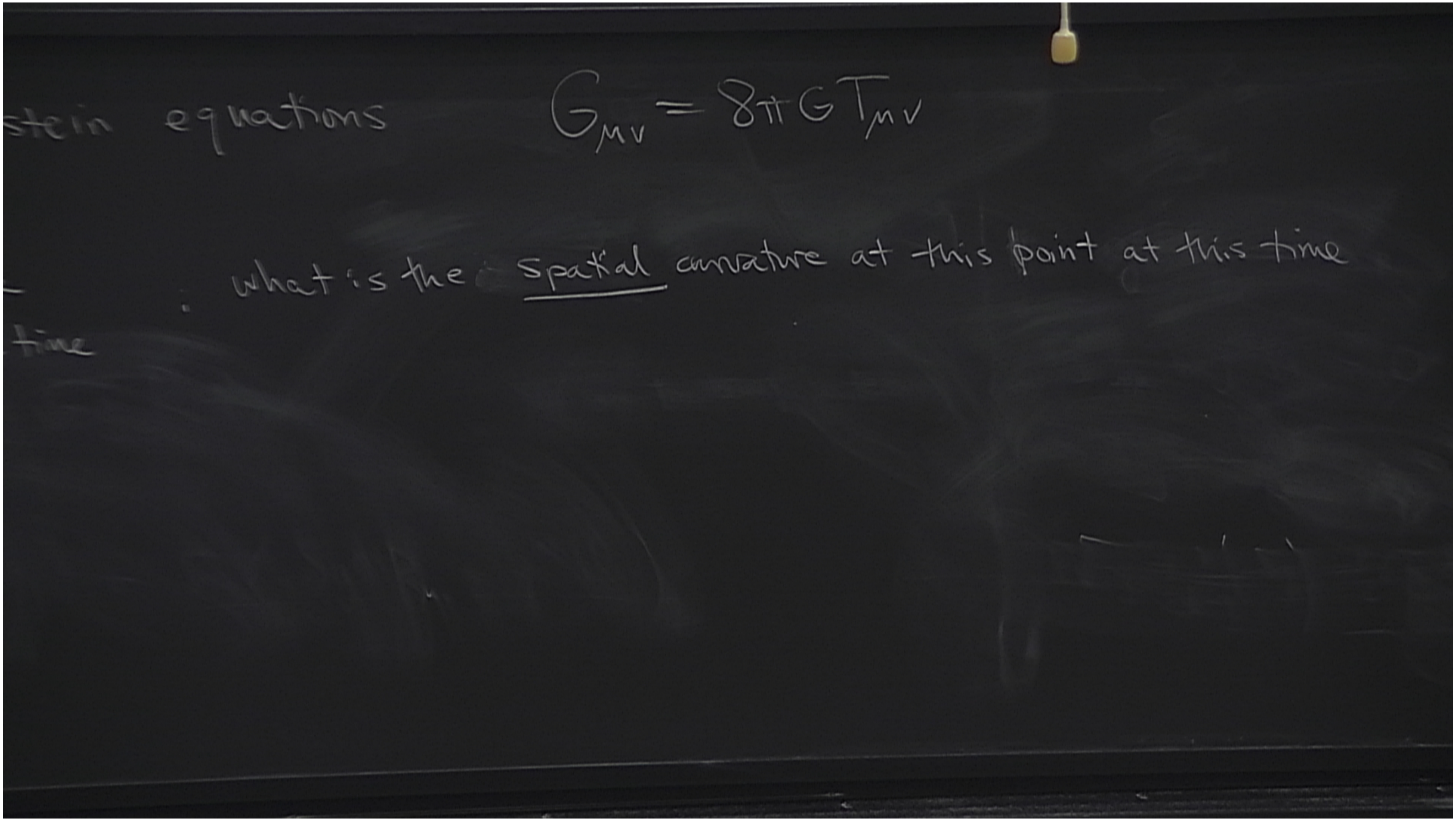
Curious argument: meaning of the Einstein

(in Feynman's lectures on gravity)

T^{00} = energy density

← little ball in space at some moment of time

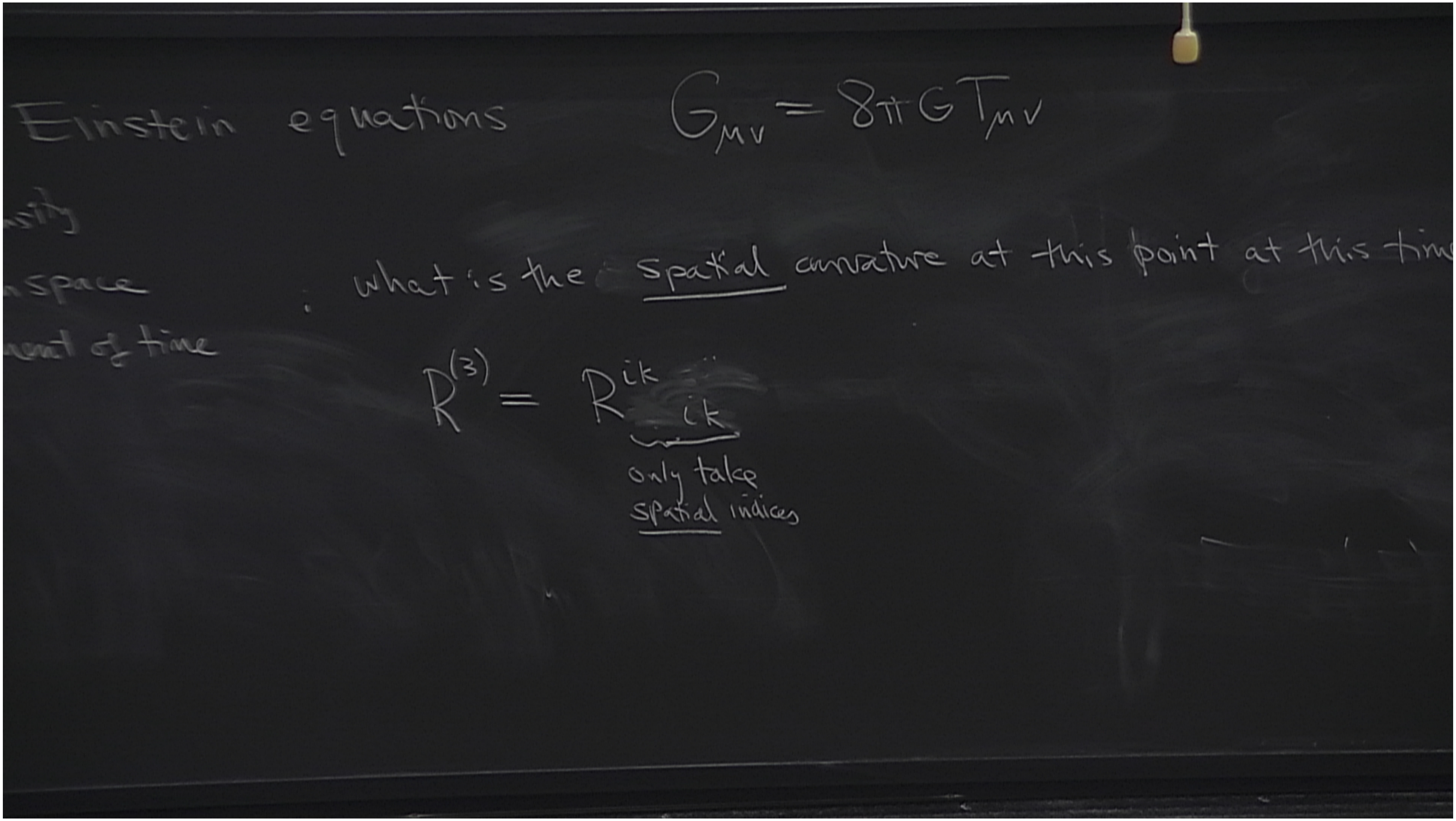




stein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

what is the spatial curvature at this point at this time

time



Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

nsity

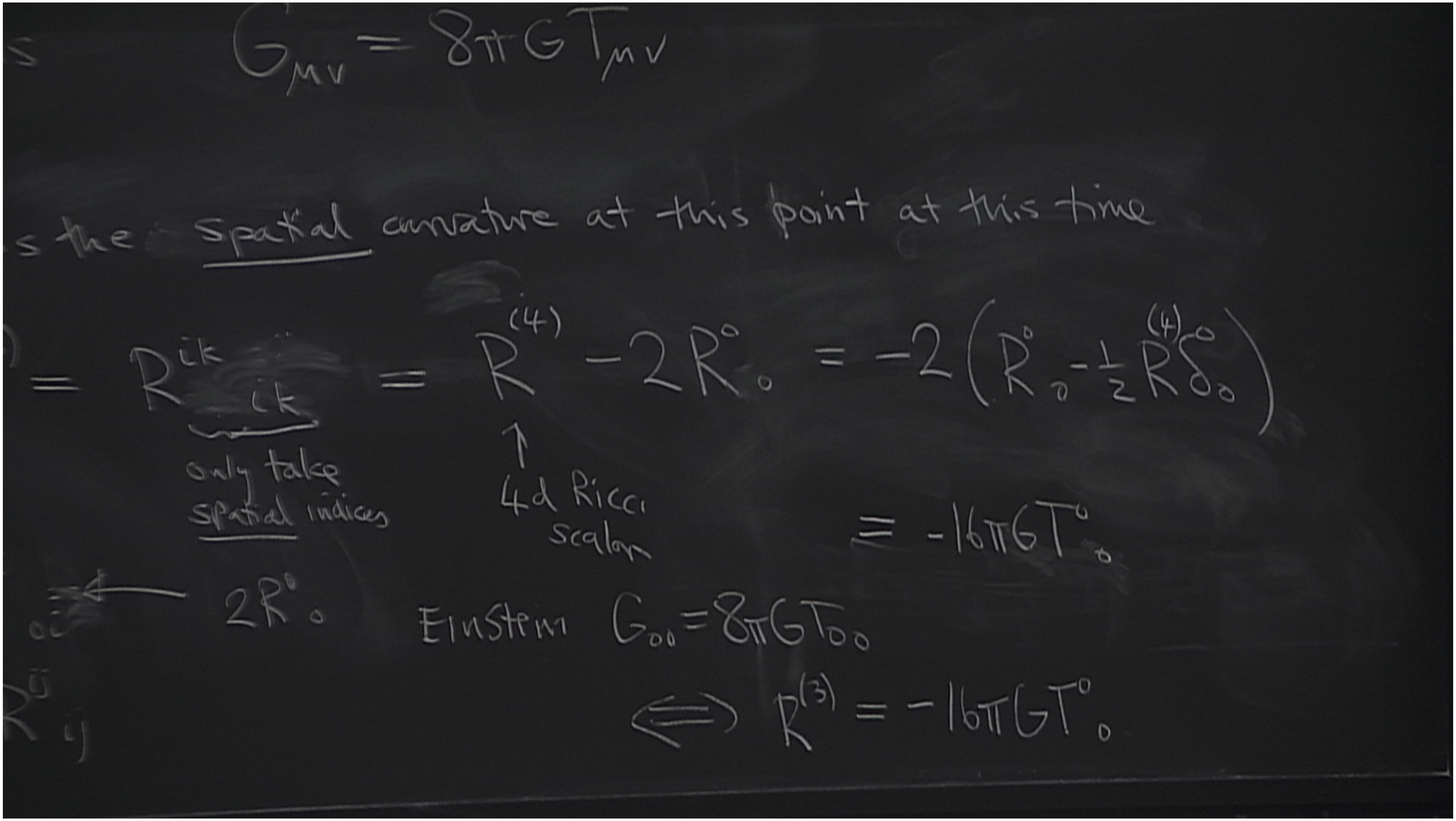
n space

ment of time

what is the spatial curvature at this point at this tim

$$R^{(3)} = \underbrace{R^{ik}{}_{ik}}$$

only take spatial indices



s

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

s the spatial curvature at this point at this time

$$= \underbrace{R^{ik}{}_{ik}}_{\text{only take spatial indices}} = \underset{\substack{\uparrow \\ \text{4d Ricci} \\ \text{scalar}}}{R^{(4)}} - 2R^{0}{}_{0} = -2\left(R^{0}{}_{0} - \frac{1}{2}R^{(4)}\delta^{0}{}_{0}\right)$$

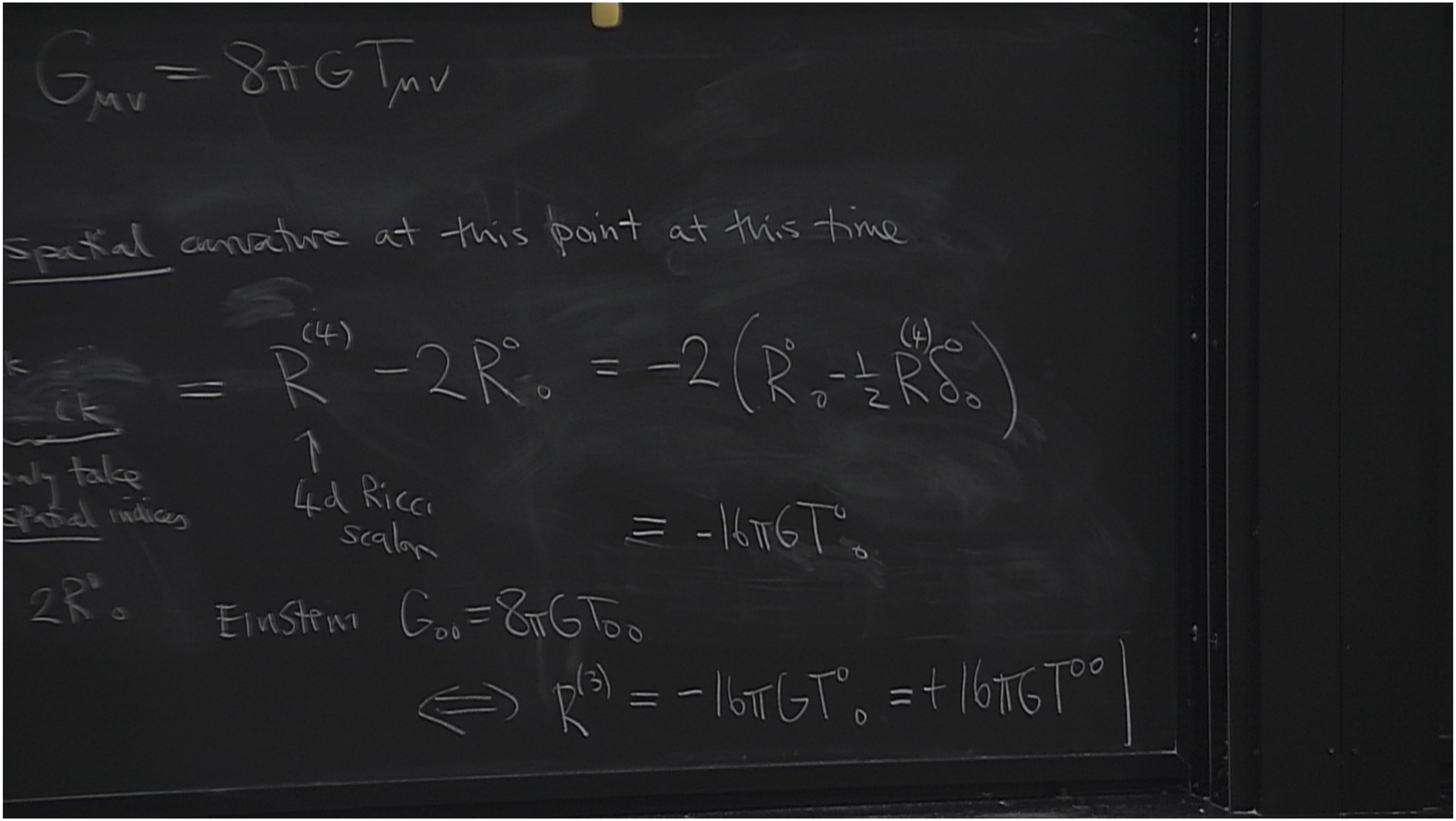
$$= -16\pi G T^{0}{}_{0}$$

$$_{0i} \longleftarrow 2R^{0}{}_{0}$$

Einstein $G_{00} = 8\pi G T_{00}$

$$\iff R^{(3)} = -16\pi G T^{0}{}_{0}$$

$$R^{ij}{}_{ij}$$



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

spatial curvature at this point at this time

k

ik

nly take
spatial indices

$2R^{0}{}_{0}$

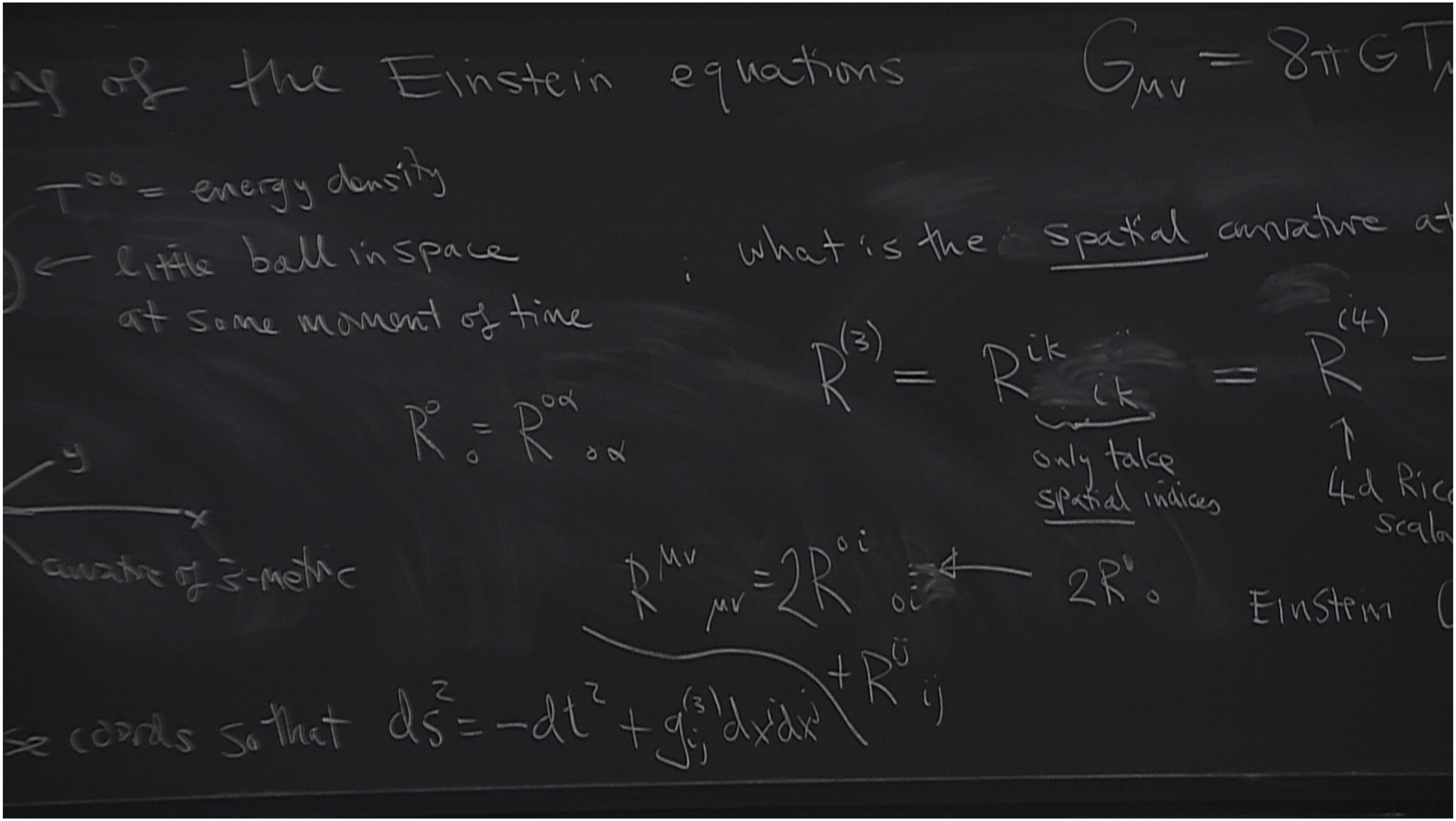
$$= R^{(4)} - 2R^{0}{}_{0} = -2\left(R^{0}{}_{0} - \tfrac{1}{2}R^{(4)}\delta^{0}{}_{0}\right)$$

↑
4d Ricci
scalar

$$= -16\pi G T^{0}{}_{0}$$

Einstein $G_{00} = 8\pi G T_{00}$

$$\Longleftrightarrow R^{(3)} = -16\pi G T^{0}{}_{0} = +16\pi G T^{00}$$



ng of the Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu}$

T^{00} = energy density

← little ball in space at some moment of time

; what is the spatial curvature at

$$R^{(3)} = R^{ik}{}_{ik} = R^{(4)} -$$

only take spatial indices

↑ 4d Ricci scalar

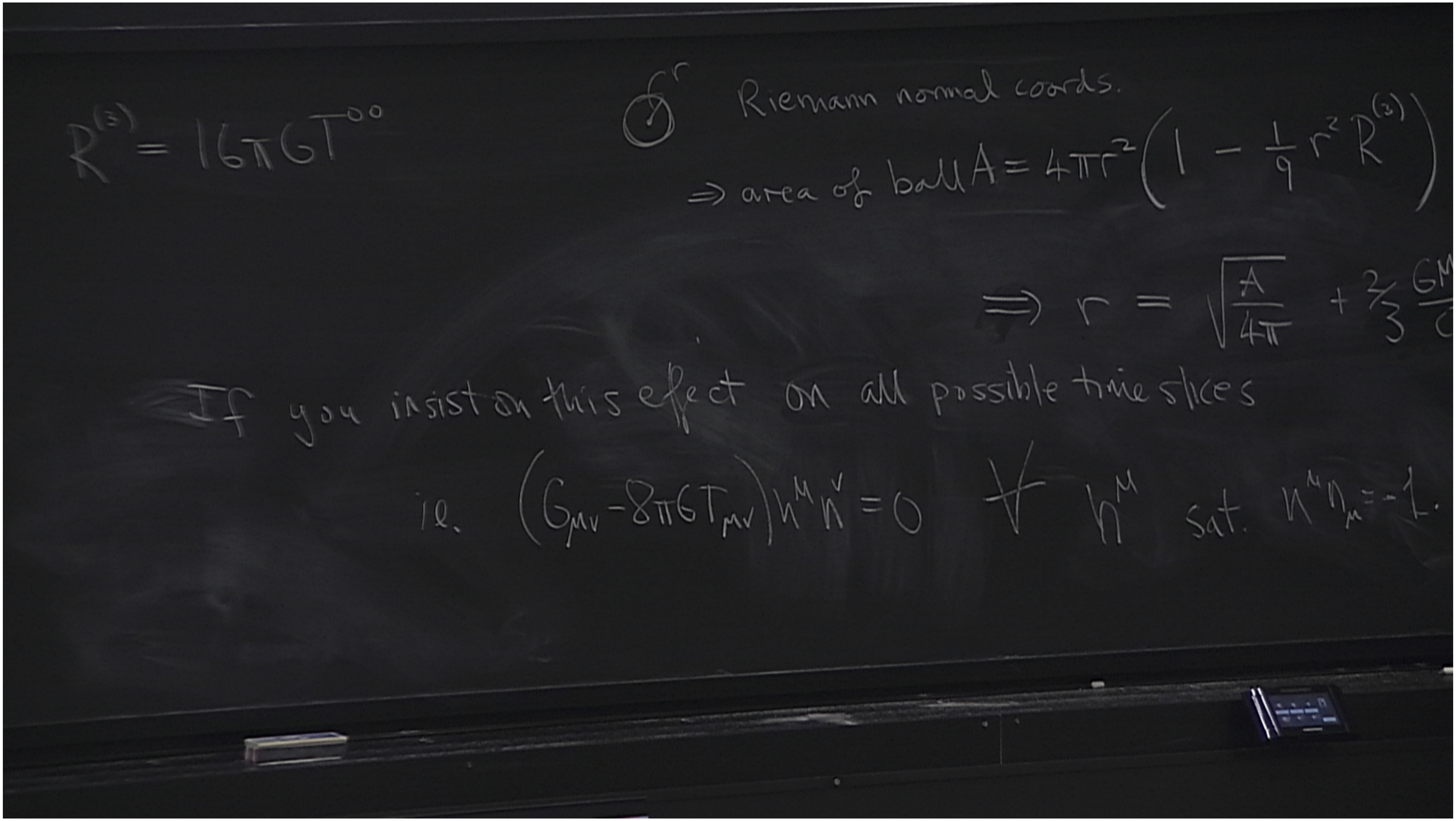
$$R^{0}{}_{0} = R^{0\alpha}{}_{0\alpha}$$

curvature of 3-metric

$$R^{\mu\nu}{}_{\mu\nu} = 2R^{0i}{}_{0i} + R^{ij}{}_{ij} \leftarrow 2R^{0}{}_{0}$$

Einstein (

se coords so that $ds^2 = -dt^2 + g^{(3)}_{ij}\, dx^i dx^j$



$$R^{(3)} = 16\pi G T^{00}$$

Riemann normal coords.

$\Rightarrow$ area of ball $A = 4\pi r^2\left(1 - \frac{1}{9} r^2 R^{(3)}\right)$

$$\Rightarrow r = \sqrt{\frac{A}{4\pi}} + \frac{2}{3}\frac{GM}{c}$$

If you insist on this effect on all possible time slices

i.e. $(G_{\mu\nu} - 8\pi G T_{\mu\nu}) n^\mu n^\nu = 0 \quad \forall \; n^\mu$ sat. $n^\mu n_\mu = -1$.

ations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

hat is the <u>spatial</u> curvature at this point at this time

$$R^{(3)} = \underbrace{R^{ik}{}_{ik}}_{\text{only take spatial indices}} = \underset{\substack{\uparrow \\ \text{4d Ricci scalar}}}{R^{(4)}} - 2R^{0}{}_{0} = -2\left(R^{0}{}_{0} - \tfrac{1}{2}\overset{(4)}{R}\,\delta^{0}{}_{0}\right)$$

$$= -16\pi G T^{0}{}_{0}$$

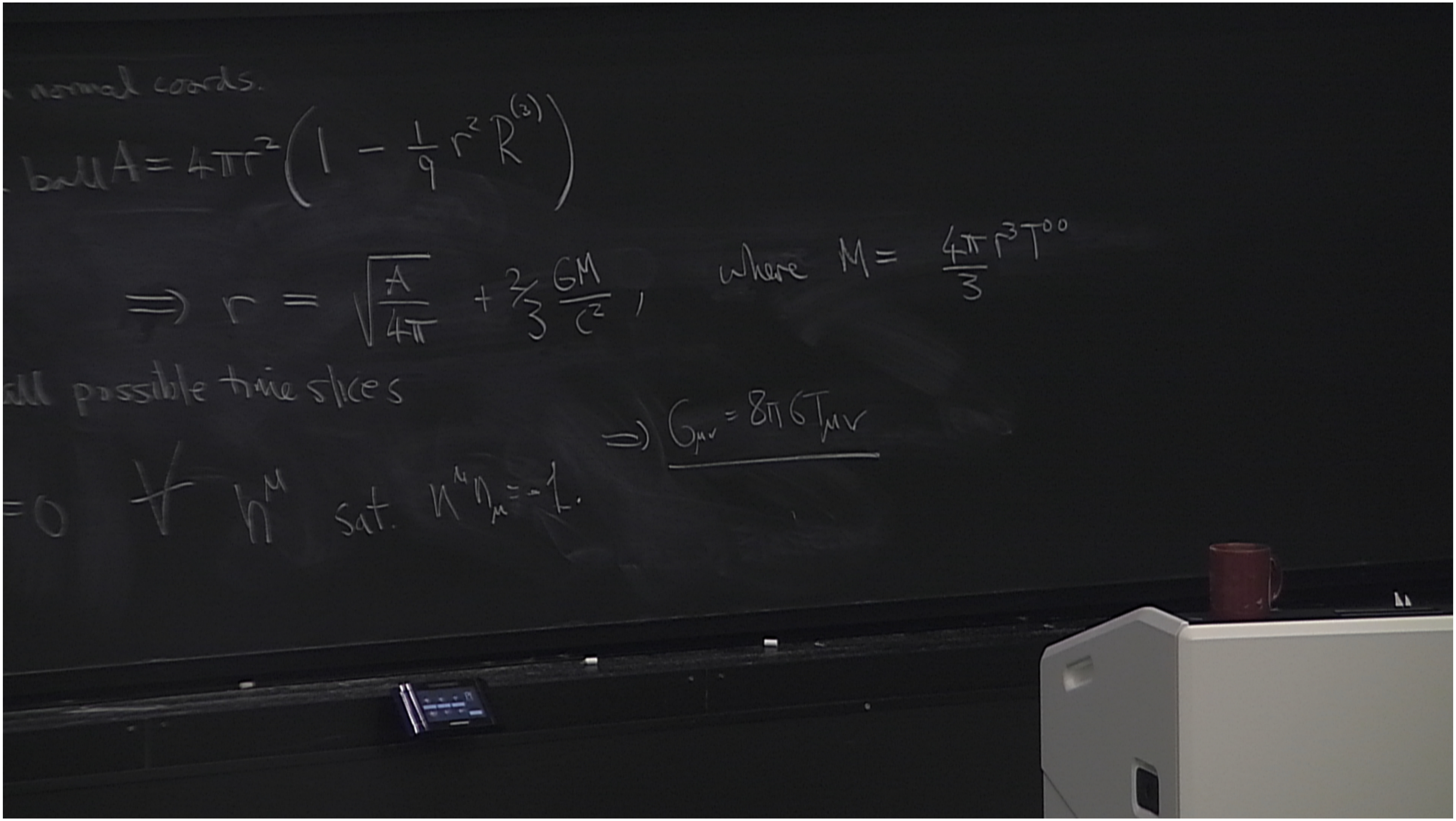
$$=2R^{0i}{}_{0i} \longleftarrow 2R^{0}{}_{0}$$

$$+R^{ij}{}_{ij}$$

kdx

Einstein $\boxed{G_{00} = 8\pi G T_{00}}$

$$\Longleftrightarrow R^{(3)} = -16\pi G T^{0}{}_{0} = +16\pi G \underline{\underline{T}}^{00}$$



normal coords.

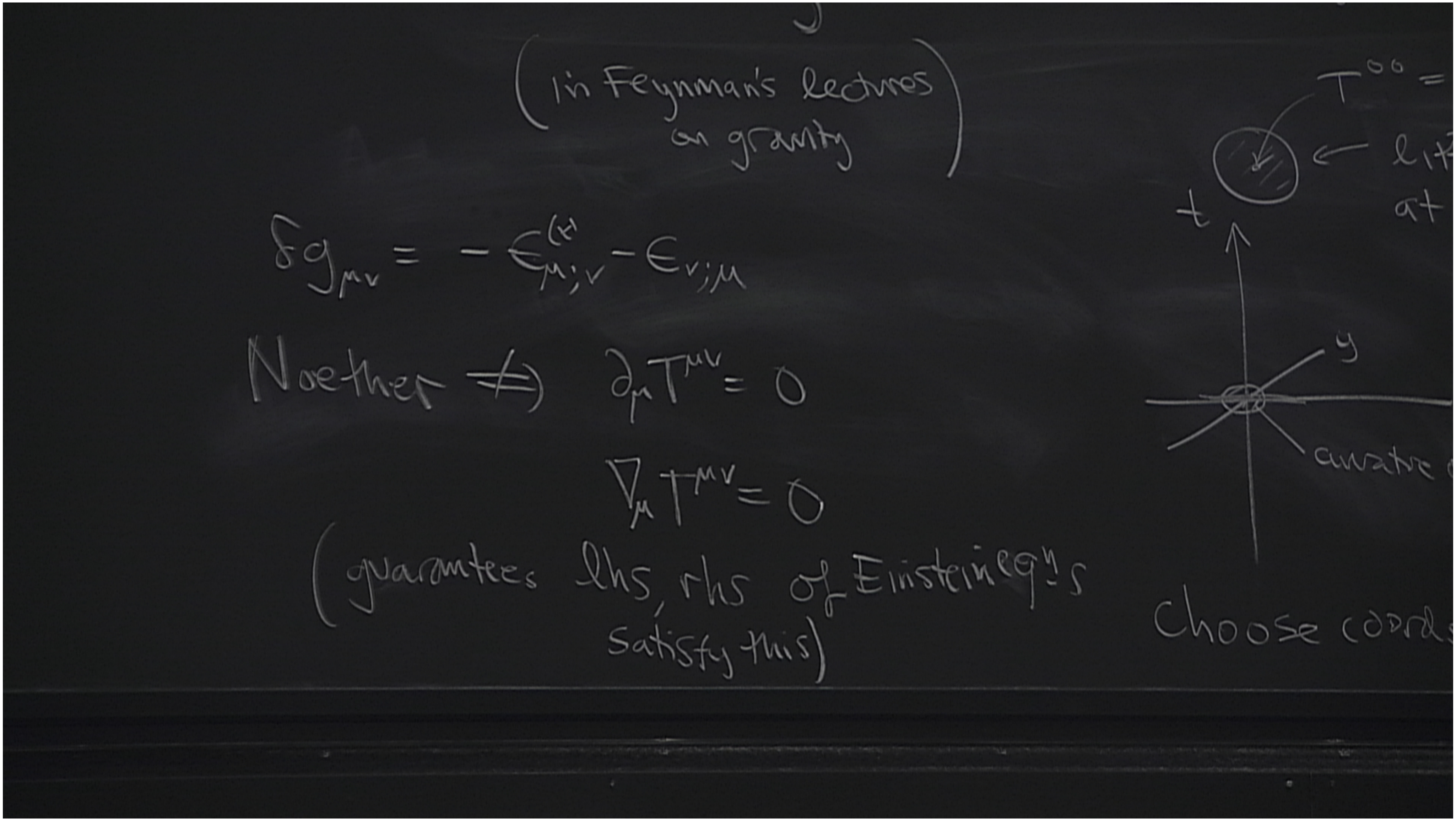
ball $A = 4\pi r^2 \left(1 - \frac{1}{9} r^2 R^{(3)}\right)$

$$\Rightarrow r = \sqrt{\frac{A}{4\pi}} + \frac{2}{3}\frac{GM}{c^2}, \quad \text{where } M = \frac{4\pi}{3} r^3 T^{00}$$

ll possible time slices

$$\Rightarrow \underline{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$= 0 \quad \forall \; n^\mu$ sat. $n^\mu n_\mu = -1$.



(1 in Feynman's lectures on gravity)

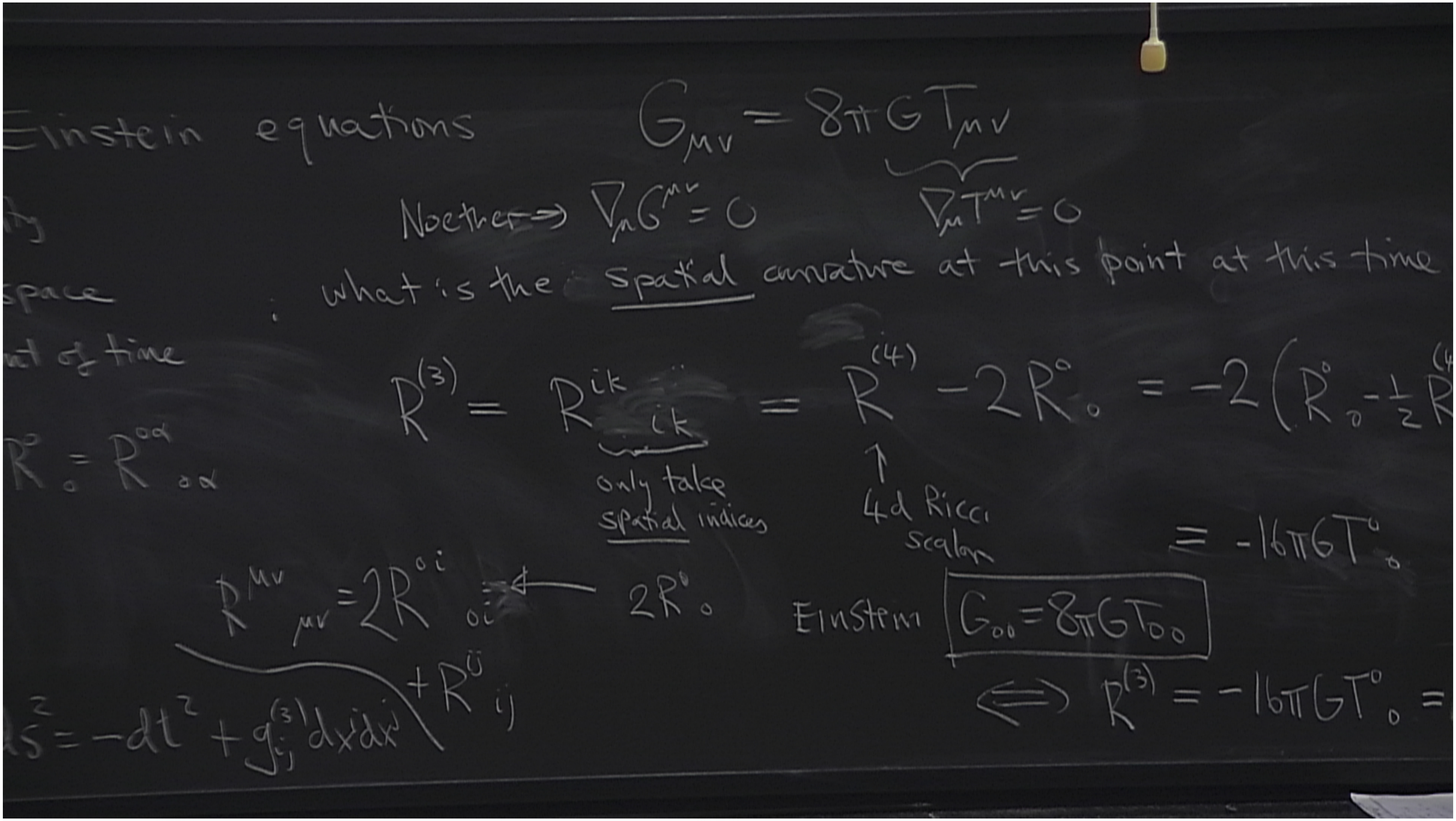
$$\delta g_{\mu\nu} = -\epsilon^{(x)}_{\mu;\nu} - \epsilon_{\nu;\mu}$$

Noether $\not\Rightarrow$ $\partial_\mu T^{\mu\nu} = 0$

$$\nabla_\mu T^{\mu\nu} = 0$$

(guarantees lhs, rhs of Einstein eqn's satisfy this)

Choose coord



Einstein equations $G_{\mu\nu} = 8\pi G \underbrace{T_{\mu\nu}}$

ity

Noether $\Rightarrow \nabla_\mu G^{\mu\nu} = 0$ $\qquad \nabla_\mu T^{\mu\nu} = 0$

space

: what is the spatial curvature at this point at this time

nt of time

$$R^{(3)} = \underbrace{R^{ik}{}_{ik}} = R^{(4)} - 2R^0{}_0 = -2\left(R^0{}_0 - \frac{1}{2}R^{(4)}\right.$$

only take spatial indices

↑ 4d Ricci scalar

$$= -16\pi G T^0{}_0$$

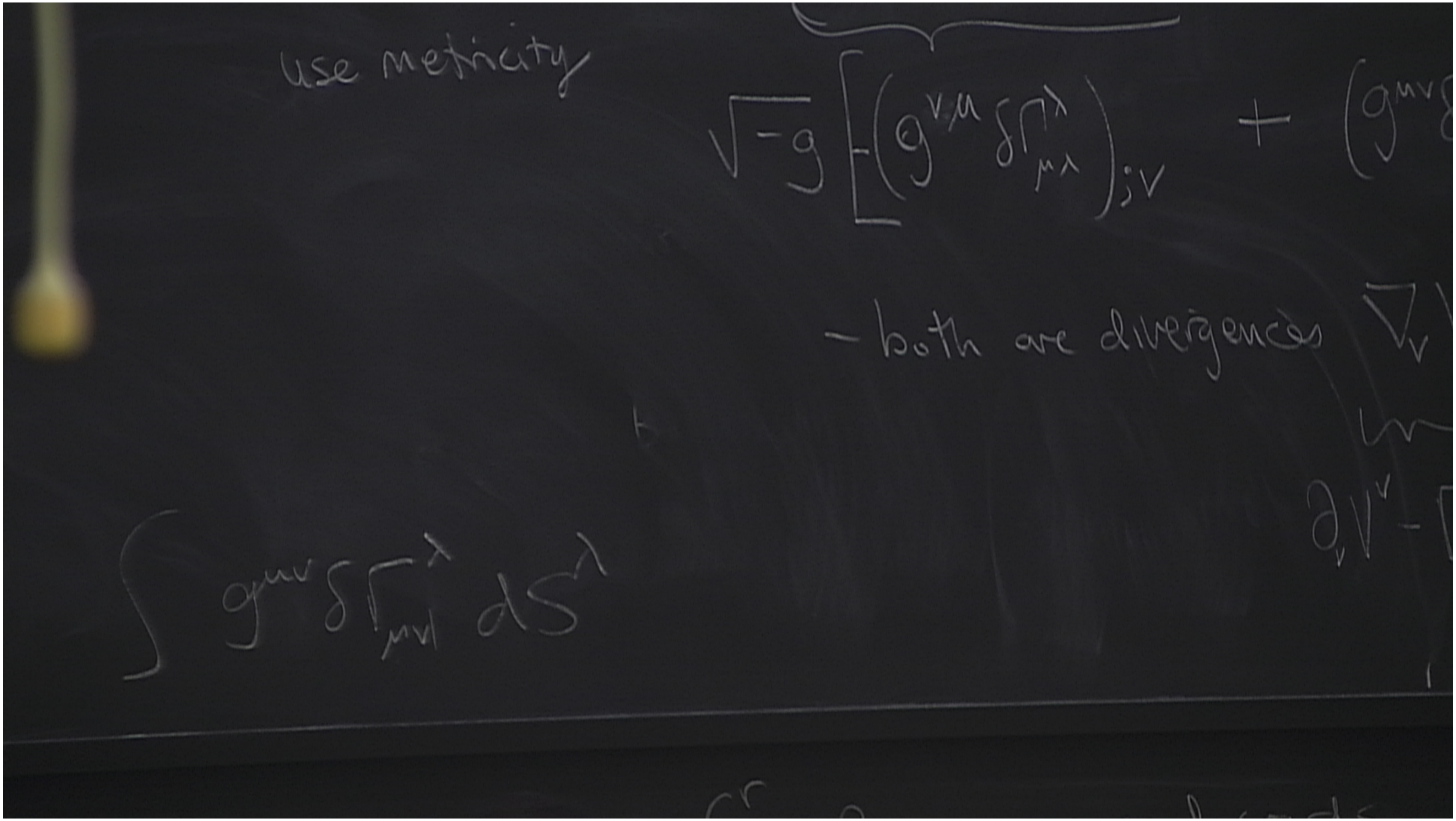
$$R^0{}_0 = R^{0\alpha}{}_{0\alpha}$$

$$R^{\mu\nu}{}_{\mu\nu} = 2R^{0i}{}_{0i} + R^{ij}{}_{ij} \quad = \longleftarrow 2R^0{}_0$$

Einstein $\boxed{G_{00} = 8\pi G T_{00}}$

$$\Longleftrightarrow R^{(3)} = -16\pi G T^0{}_0 =$$

$$ds^2 = -dt^2 + g^{(3)}_{ij} dx^i dx^j$$



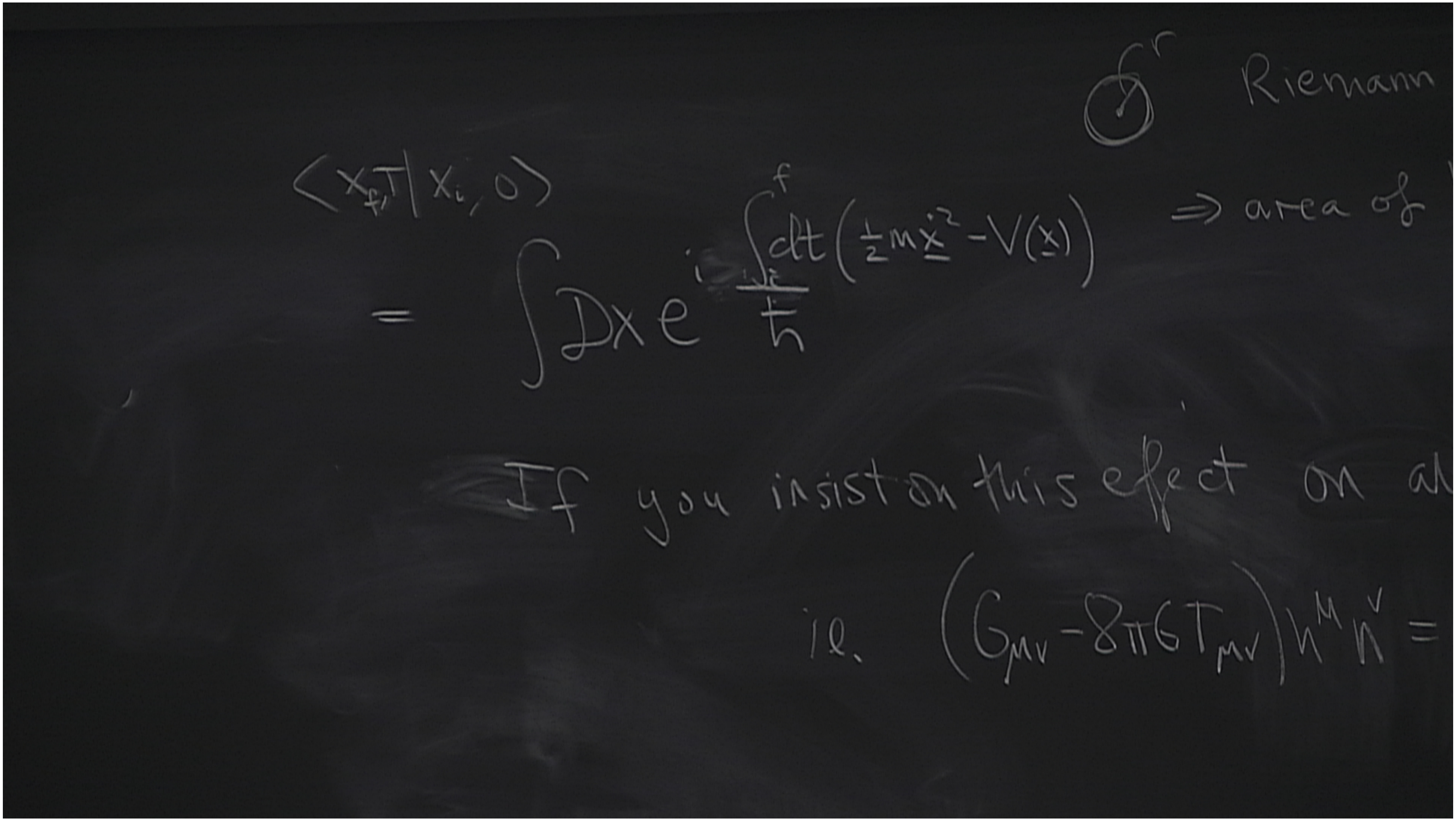
use metricity

$\sqrt{-g}\,\Big[-\big(g^{\nu\mu}\delta\Gamma^{\lambda}_{\mu\lambda}\big)_{;\nu} + \big(g^{\mu\nu}$

– both are divergences ∇_ν

$\partial_\nu V^\nu -$

$\int g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu}\, dS^{\lambda}$



Riemann

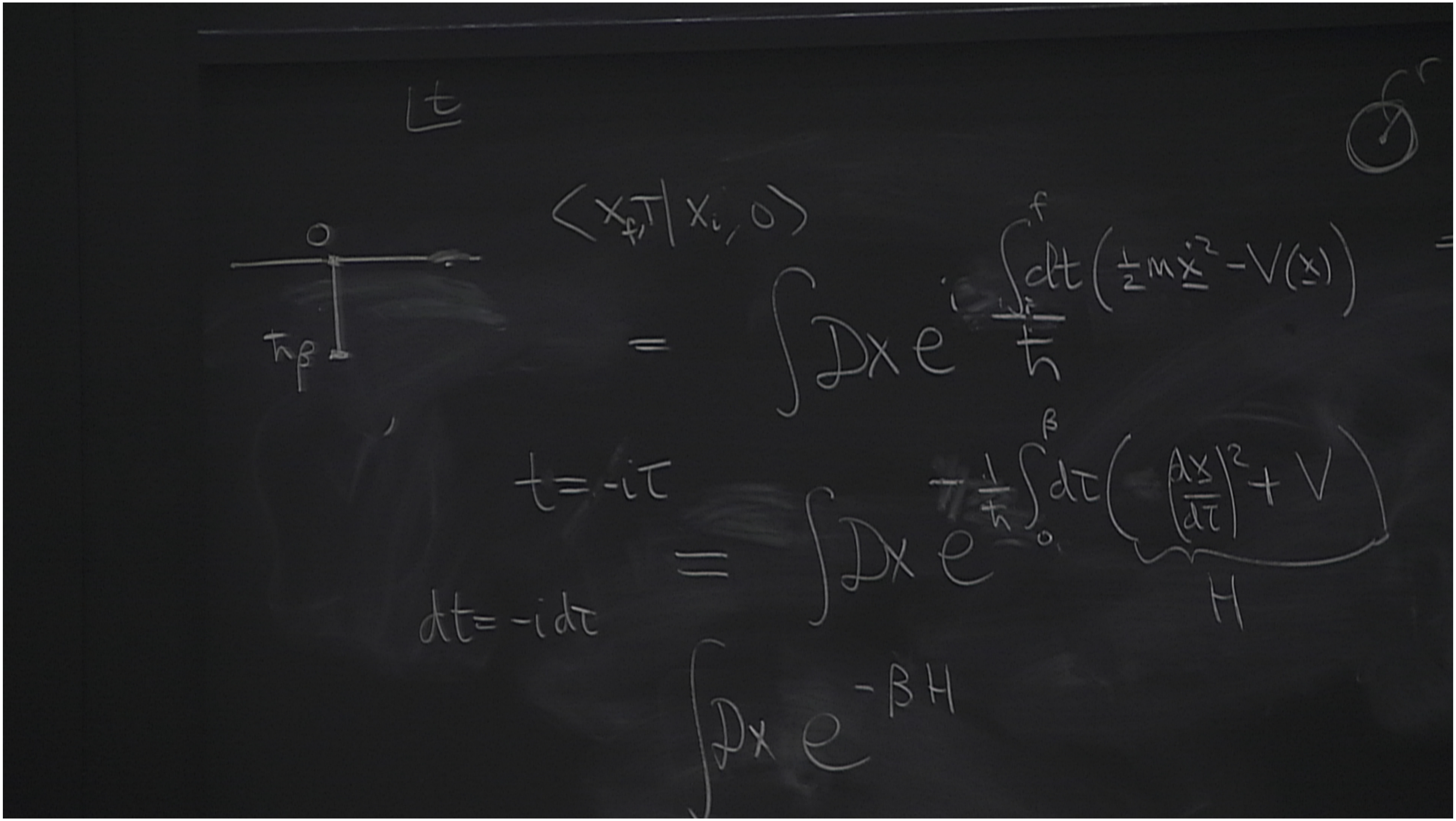
$$\langle x_f, T | x_i, 0 \rangle$$

$$= \int \mathcal{D}x \, e^{i \frac{\int_i^f dt \left(\frac{1}{2} m \dot{\underline{x}}^2 - V(\underline{x}) \right)}{\hbar}}$$

$\Rightarrow$ area of

If you insist on this effect on al

i.e. $\left(G_{\mu\nu} - 8\pi G T_{\mu\nu} \right) n^\mu n^\nu =$



$$\langle x_f, T | x_i, 0 \rangle = \int \mathcal{D}x \, e^{\frac{i}{\hbar}\int_i^f dt\left(\frac{1}{2}m\dot{\underline{x}}^2 - V(\underline{x})\right)} =$$

$t = -i\tau$

$dt = -i\,d\tau$

$$= \int \mathcal{D}x \, e^{-\frac{1}{\hbar}\int_0^\beta d\tau \underbrace{\left(\left(\frac{dx}{d\tau}\right)^2 + V\right)}_{H}}$$

$$\int \mathcal{D}x \, e^{-\beta H}$$