

Title: PSI 2015/2016 Relativity - Neil Turok - Lecture 10

Date: Sep 21, 2015 09:00 AM

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Abstract:

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h^\lambda{}_\lambda$$

last time

harmonic gauge

$$k_\mu \bar{h}^{\mu\nu} = 0$$

: picked $k^\mu = (k, 0, 0, k)$

eliminates h_{0i} , sets $h_{ii} = \frac{1}{3} h^\lambda{}_\lambda$

residual g.t's $x'^\mu = x^\mu + \epsilon^\mu(x) : \square \epsilon^\mu = 0$

$\epsilon'_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + C_\mu k_\nu + C_\nu k_\mu$
→ elim

$\frac{1}{h_{11}} = 0$ } ie. removes 4 components
 $h_{11} + h_{22} = 0$

$\rightarrow$ eliminate h_{31}, h_{00} } removes another 4 components

leaves us with helicity ± 2 components of massive

Spin 1, 0. } these 4 components get "ganged away"

leaves us
with helicity ± 2 components of massless spin 2 field.

ments get "ganged away".

$$\delta E = 0$$

$$\epsilon'_{\mu\nu} \rightarrow \epsilon$$

linear order in $h^{\mu\nu}$

$$G_{\mu\nu}^{(1)} = -\frac{1}{2} \square \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

last time

harmonic gauge

residual g.t's

$$k_\mu \bar{h}^{\mu\nu} = 0$$

: picked $k^\mu = (k, 0, 0, k)$
 $\rightarrow$ eliminates h_{0i} , sets $h_{ii} = 0$
i.e. $h_{11} + h_{22} + h_{33} = 0$

$$x'^\mu = x^\mu + \epsilon^\mu(x) : \square \epsilon^\mu = 0$$

$\rightarrow \epsilon'_\mu \rightarrow \epsilon_\mu + C_\mu k_\nu + C_\nu k_\mu$
 $\rightarrow$ eliminate h_{3i}, h_{00}

Spin 1,

linear order in $h^{\mu\nu}$

$$G_{\mu\nu}^{(1)} = -\frac{1}{2} \square \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(both sides are divergenceless)

$$\square h_{\mu\nu} = -\frac{1}{2} \square h_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (\text{both sides are also gauge invariant})$$

$\Rightarrow$ solved, showed that

$$\bar{h}_{ke} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int \rho(t, \mathbf{x}) x^k x'^e d^3x' \right]_{t \rightarrow t-r}$$

(not unique, still have freedom to perform harmonic coordinate transformations.)

$$\left[\frac{d^2}{dt^2} \int \rho(t, \vec{x}) X^k X'^l d^3 X' \right]$$

harmonic coordinate transformations, to go to TT gauge)

→ solved, showed that

$$\bar{h}_{kl} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \right] \int \rho(t, \underline{x}) x^k x^l$$

(not unique, still have freedom to perform harmonic coordinate

write

$$\int \rho(t, \underline{x}') x'^k x'^l d^3x' = \frac{1}{3} Q^{kl}(t) + \frac{1}{3} \left(\int d^3x' \rho(t, \underline{x}') x'^2 \right) \delta^{kl}$$

$$\text{where } Q^{kl}(t) = \int d^3x' \left(3x'^k x'^l - \delta^{kl} x'^2 \right) \rho(t, \underline{x}')$$

$$\frac{1}{t^2} \int p(t, \underline{x}) x^k x'^l d^3 x'$$

$t \rightarrow t-r$

main coordinate transformations, to go to TT gauge)

$$p(t, \underline{x}') \delta^{kl}$$

: go to TT gauge $\rightarrow$ second term disappears.

$$p(t, \underline{x}')$$

transverse, traceless components of $\bar{h}_{\mu\nu}$

$\Rightarrow$ on the z axis,

$$\underbrace{h_{ij}^{TT}}_{\substack{h_{11}^{TT} = -h_{22}^{TT} \\ h_{12}^{TT}}} = -\frac{2G}{3r} \ddot{Q}_{ij}(t) \Big|_{t \rightarrow t-r}$$

Energy.

Energy-momentum carried by a gravitational wave:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$= -\frac{1}{2} \Box \bar{h}_{\mu\nu} + \text{second order in } h_{\mu\nu}.$$

$$\Rightarrow -\frac{1}{2} \Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\mu\nu}}_{E_{\mu\nu}} \quad ; \text{ again, } \partial_\mu E^{\mu\nu} = 0$$

$$\delta T_{\mu\nu}$$

$$\square \bar{h}_{\mu\nu} + \text{second order in } h_{\mu\nu} .$$

$$T_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\bar{E}_{\mu\nu}}}_{\bar{E}_{\mu\nu}} \quad ; \text{ again, } \partial_\mu \bar{E}^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$= -\frac{1}{2} \square \bar{h}_{\mu\nu} + \text{second order in } h_{\mu\nu}.$$

$$\Rightarrow -\frac{1}{2} \square \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\mu\nu}}_{E_{\mu\nu}} \quad ; \text{ again,}$$

$E_{\mu\nu}$ given in Weinberg, eqⁿs (7.6.15) and (10.3.1)

ed by a gravitational wave:

+ second order in $h_{\mu\nu}$.

$$8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\mu\nu}}_{E_{\mu\nu}} \quad ; \text{ again, } \partial_\mu E^{\mu\nu} = 0$$

einberg, eqⁿs (7.6.15) and (10.3.1) BUT change sign

ions in space and time, i.e. $\langle A \partial_\mu B \rangle \rightarrow -\langle \partial_\mu A B \rangle$

gravitational wave:

$h_{\mu\nu}$

gravitational energy $\bar{T}_{\mu\nu}$: again, $\partial_\mu \bar{T}^{\mu\nu} = 0$

(6.15) and (10.3.1) BUT change sign

and time, i.e. $\langle A \partial_\mu B \rangle \rightarrow - \langle \partial_\mu A B \rangle$

i.e. drop total derivatives

Energy-momentum carried by a gravitational wave:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ = -\frac{1}{2} \Box \bar{h}_{\mu\nu} + \text{second order in } h_{\mu\nu}.$$

$$\Rightarrow -\frac{1}{2} \Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\mu\nu}}_{E_{\mu\nu}} \quad ; \text{ again,}$$

$E_{\mu\nu}$ given in Weinberg, eqⁿs (7.6.15) and (10.3.1) BUT change s

If average $E_{\mu\nu}$ over oscillations in space and time, i.e. $\langle A \partial_\mu B \rangle$

$$G_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (\text{both sides are divergenceless})$$

then $E_{\mu\nu} \rightarrow -\frac{1}{4} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$

↑
average over
oscillations

and in TT gauge, rhs = $-\frac{1}{4} \left\langle \partial_\mu h^{\text{TT}}_{\rho\sigma} \partial_\nu h^{\text{TT}\rho\sigma} \right\rangle$

$$L_5 = -\frac{1}{4} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} \right\rangle$$

$$L_{\text{g-waves}} = -\frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} \right\rangle$$

$$G_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (\text{both sides are divergenceless})$$

then $E_{\mu\nu} \rightarrow -\frac{1}{4} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$

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average over
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and in TT gauge, rhs = $-\frac{1}{4} \left\langle \partial_\mu h^{\text{TT}}_{\rho\sigma} \partial_\nu h^{\text{TT}\rho\sigma} \right\rangle$

$\Rightarrow T_{\mu\nu}^{\text{g-waves}} = -\frac{1}{32\pi G} \left\langle \partial_\mu h^{\text{TT}}_{\rho\sigma} \partial_\nu h^{\text{TT}\rho\sigma} \right\rangle$

Energy-momentum carried by a gr

($c=1$)

GR.

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\underline{\partial_\mu T^{\mu\nu}} = 0$$

$$\Rightarrow \int d^3x T^{00} = \text{const}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$= -\frac{1}{2} \square \bar{h}_{\mu\nu} + \text{second order}$$

$$\Rightarrow -\frac{1}{2} \square \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu} + \text{"grav"}$$

$E_{\mu\nu}$ given in Weinberg, eqⁿs (7.6)

If average $E_{\mu\nu}$ over oscillations in space and

carried by a gravitational wave:

$$T_{\mu\nu}$$

$\bar{T}_{\mu\nu}$ + second order in $h_{\mu\nu}$.

$$T_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\text{"gravitational energy"}_{\mu\nu}}_{\substack{E_{\mu\nu} \text{ Landau-Lifshitz} \\ \text{pseudo-energy-momentum} \\ \text{tensor}}} : \text{again, } \boxed{\partial_\mu \bar{T}^{\mu\nu} = 0}$$

in Weinberg, eq's (7.6.15) and (10.3.1) BUT change sign

oscillations in space and time, i.e. $\langle A \partial_\mu B \rangle \rightarrow -\langle \partial_\mu A B \rangle$

(both sides are divergenceless)

then $E_{\mu\nu} \rightarrow +\frac{1}{4} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$

average over oscillations

and in TT gauge, rhs =

$\Rightarrow T_{\mu\nu}^{\text{g-waves}} = +\frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma}^{\text{TT}} \partial_\nu h^{\rho\sigma}_{\text{TT}} \right\rangle$

(both sides are divergenceless)

then $E_{\mu\nu} \rightarrow +\frac{1}{4} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$

↑
average over
oscillations

and in TT gauge, rhs = $+\frac{1}{4} \left\langle \partial_\mu h^{\text{TT}}_{\rho\sigma} \partial_\nu h^{\text{TT}\rho\sigma} \right\rangle$

$\Rightarrow T^{\text{g-waves}}_{\mu\nu} = +\frac{1}{32\pi G} \left\langle \partial_\mu h^{\text{TT}}_{\rho\sigma} \partial_\nu h^{\text{TT}\rho\sigma} \right\rangle$

$$= -\frac{1}{32\pi G} \left\langle \partial_0 \underset{\perp}{h_{ij}^{\text{TT}}} \partial_3 h_{ij}^{\text{TT}} \right\rangle$$

$$h_{ij}^{\text{TT}} = h_{ij}^{\text{TT}}(z-t)$$

$$\partial_0 h_{ij}^{\text{TT}} = -\partial_3 h_{ij}^{\text{TT}} = h_{ij}^{\text{TT}{'}}$$

$$= -\frac{1}{32\pi G} \left\langle \partial_0 h_{ij}^{\text{TT}} \partial_3 h_{ij}^{\text{TT}} \right\rangle$$

$$= \frac{1}{32\pi G} \left\langle h_{11}^{\text{TT}'}^2 + h_{22}^{\text{TT}'}^2 + 2h_{12}^{\text{TT}'}^2 \right\rangle$$

$$= \frac{1}{16\pi G} \left\langle \frac{1}{4}(\dot{h}_{11} - \dot{h}_{22})^2 + \dot{h}_{12}^2 \right\rangle$$

$$= \frac{G}{36\pi r^2} \left[\ddot{Q}_{12}^2 + \frac{1}{4}(\ddot{Q}_{11} - \ddot{Q}_{22})^2 \right]$$

$$h_{ij}^{\text{TT}} = h_{ij}^{\text{TT}}(z-t)$$

$$\partial_0 h_{ij}^{\text{TT}} = -\partial_3 h_{ij}^{\text{TT}} = h_{ij}^{\text{TT}'}$$

energy flux

$$(T^{gw})^{03} = -(T^{gw})_{03} = -\frac{1}{32\pi G} \langle \partial_0 h_{ij}^{\text{TT}} \partial_3 h_{ij}^{\text{TT}} \rangle$$

$$= \frac{1}{32\pi G} \langle h_{11}^{\text{TT}'}^2 + h_{22}^{\text{TT}'}^2 + 2h_{12}^{\text{TT}'}^2 \rangle$$

$$= \frac{1}{16\pi G} \langle \frac{1}{4}(\dot{h}_{11} - \dot{h}_{22})^2 + \dot{h}_{12}^2 \rangle$$

in TT gauge,
 $h_{22}^{\text{TT}} = -h_{11}^{\text{TT}}$

$$h_{ij}^{\text{TT}} \sim \frac{2G}{3r} \ddot{Q}_{ij}(t-z)$$

$$= \underbrace{\frac{6}{36\pi r^2}}_{\frac{4G^2}{9r^2} \frac{1}{16\pi G}} \left[\ddot{Q}_{12}^2 + \frac{1}{4}(\ddot{Q}_{11} - \ddot{Q}_{22})^2 \right]$$

rotational invariance.

(most general rotational invariant
made from Q_{ke} and n_k)
and second order in $\ddot{Q}$

just by rotational invariance.

$$n^l n^m + C \left(\ddot{Q}_{ke} n^k n^e \right)^2$$

(most general rotations
made from Q_{ke} and
and second order in $\ddot{Q}$)

$$\left(Q_{23}^2 + Q_{13}^2 \right) + B \left(Q_{13}^2 + Q_{23}^2 + Q_{33}^2 \right) + C Q_{33}^2$$

from this, we can infer the flux of energy in any direction, in

$$\text{In direction } n_i, (T^{gw})^{oi} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m +$$

$$\text{If } n_i = (0, 0, 1), \text{ this must equal } \ddot{Q}_{12}^2 + \frac{1}{4} (\ddot{Q}_{11} - \ddot{Q}_{22})^2$$

$$= A (\ddot{Q}_{11}^2 + \ddot{Q}_{22}^2 + \ddot{Q}_{33}^2 + 2(\ddot{Q}_{12}^2 + \ddot{Q}_{23}^2 + \ddot{Q}_{13}^2)) +$$

quate coefficients of Q_{11}, Q_{22}, Q_{12}

4 eqⁿs for 3 unknowns

$$A = \frac{1}{2}, B = -1, C = \frac{1}{4}.$$

$$\Rightarrow \left(\overset{\text{g-waves}}{T_{oi}} \right)_{hi} = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ke} \ddot{Q}_{ke} - \ddot{Q}_{ke} \ddot{Q}_{km} h^e h^m + \frac{1}{4} (\ddot{Q}_{ke} h^k h^e)^2 \right]$$

$$\Rightarrow \text{Power } P = \frac{dE}{dt} = r^2 \int d\Omega \left(\sum_i \dot{n}_i \right)$$

$$\int d\Omega = 4\pi$$

$$\int d\Omega n^k n^m = 4\pi \frac{\delta^{km}}{3}$$

$$\int d\Omega n^k n^l n^m n^n = \frac{4\pi}{15} (\delta^{kl} \delta^{mn} + \delta^{km} \delta^{ln} + \delta^{kn} \delta^{lm})$$

$$P = \frac{G}{45} \ddot{Q}_{ke} \ddot{Q}_{ke}$$

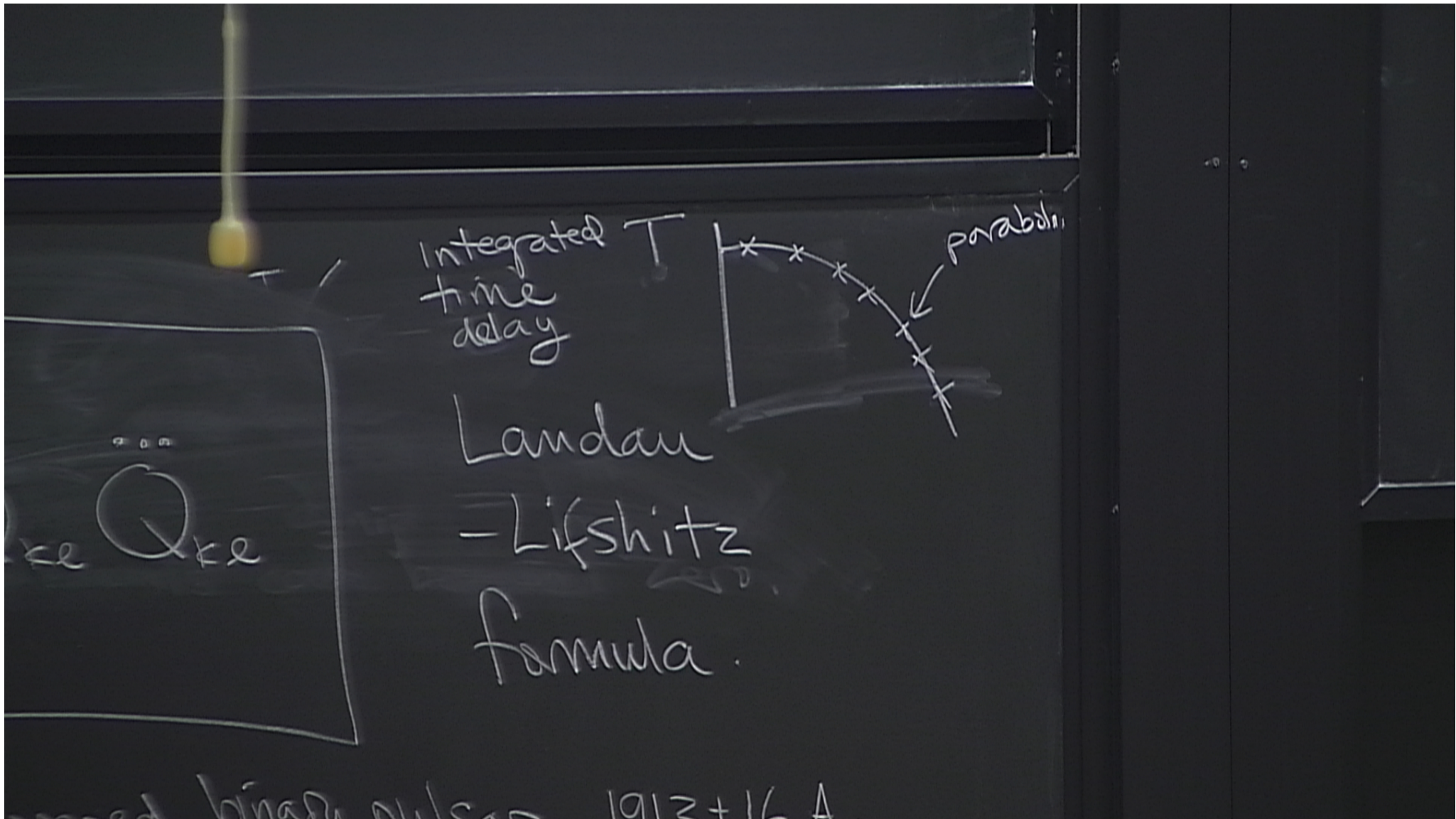
Landau
- Lifshitz
formula.

Hulse + Taylor : discovered binary pulsar 1913+16 A.



⇒ fit all parameters — separation, masses, angular momenta etc.

⇒ predict P . ⇒ predict how orbital period changes with time.

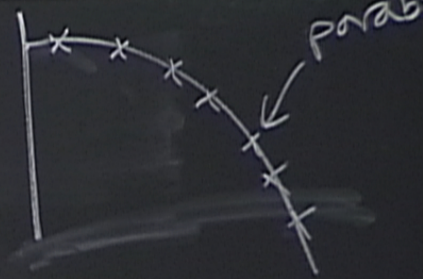


$$P = \frac{G}{45} \frac{\overset{\dots}{Q}_{ke} \overset{\dots}{Q}_{ke}}{c^5}$$

Integrated T
time delay

Landau
- Lifshitz
formula.

by dimensions



Hulse + Taylor : discovered binary pulsar 1913+16 A.



⇒ fit all parameters - separation, masses, angular momenta etc
⇒ predict $\dot{P}$ ⇒ predict how orbital period change



$$Q \sim ML^2$$

$$Q \sim \int \rho_{\text{mass}} (3x^k x^k - \delta^{kl} x^l x^l)$$

↑
mass

$$\sim \frac{dE}{dt}$$

$$P \sim \frac{W^6 G M^2 L^4}{c^5}$$

$$\sim \frac{W^6}{c^5} L^5 \left(\frac{GM^2}{L} \right)$$

$$\frac{GM^2}{L} = \text{grav. P.E.}$$

for

$$L = 1 \text{ m}$$

$$M = 1 \text{ kg}$$

$$\omega = 1 \text{ s}^{-1}$$

$$C = 3 \times 10^8$$

$$G = 6.7 \times 10^{-11}$$

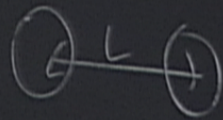
$$P \sim 10^{-52} \text{ Watts.}$$

Jupiter in orbit $\rightarrow 5 \text{ kW}$

$$\begin{array}{l} \text{for } L = 1\text{m} \\ M = 1\text{kg} \\ \omega = 1\text{s}^{-1} \\ C = 3 \times 10^8 \\ G = 6.7 \times 10^{-11} \end{array} \left. \vphantom{\begin{array}{l} L \\ M \\ \omega \\ C \\ G \end{array}} \right\} \underline{P \sim 10^{-52} \text{ watts.}}$$

Jupiter in orbit $\rightarrow 5\text{kW}$

- Pulsar binary $> 10^{48}\text{W.}$



10^{48} watts

10⁴⁸ W.

10⁴⁸ W.

$\Lambda \Rightarrow$ defines a length scale
happens to be the Hubble radius
 $= 3000 h^{-1} \text{ Mpc}$

galaxy $< 100 \text{ kpc}$
 $\frac{1}{30,000}$

$\sim 10^5$ watts

5kW

10^{48} W.

$\Lambda \Rightarrow$ defines a length scale
happens to be the Hubble radius
 $= 3000 h^{-1} \text{Mpc}$

galaxy $< 100 \text{ kpc}$

$\frac{1}{30,000}$

Dan Holz

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ed
ritational
waves

$$\Rightarrow \text{Power } P = \frac{dE}{dt} = r^2 \int d\Omega \left(T^{0i} n_i \right)$$

$$h \sim \sin kt \cos kx$$

$T_{\mu\nu}^{gw}$

$$\sim h^2 + 2h\partial_\mu h$$

$$\frac{\sin^2 kt}{\cos^2 kx}$$

$$\Rightarrow \frac{1}{2}$$

$$\underbrace{\sin kt \cos kt}_{\cos kx \sin kx}$$

$$\int d\Omega = 4\pi$$

$$\int d\Omega n^l n^m = 4\pi \frac{\delta^{lm}}{3}$$

$$\int d\Omega n^k n^l n^m n^n = \frac{4\pi}{15} (\delta^{kl} \delta^{mn} + \dots)$$