Title: PSI 2015/2016 Relativity - Neil Turok - Lecture 3

Date: Sep 10, 2015 09:00 AM

URL: http://pirsa.org/15090030

Abstract:

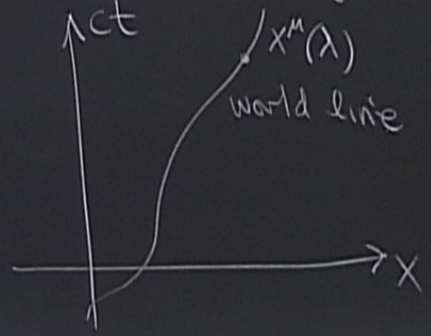


2nd argument natural to unify energy and momentum into a 4-vector

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right)$$

$-c^2\left(\frac{dt}{d\lambda}\right)^2 + \left(\frac{dx}{d\lambda}\right)^2$

Consider a particle moving in spacetime



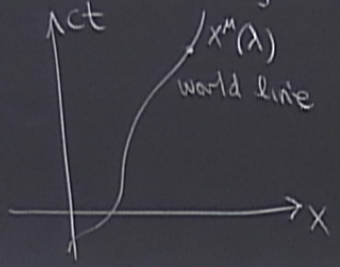
world line is timelike if $\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} < 0$

null " " $= 0$

spacelike " " > 0

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Consider a particle moving in spacetime



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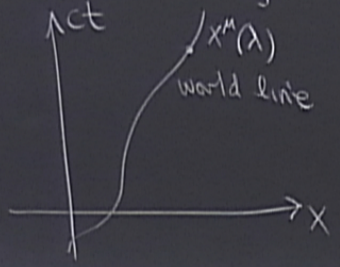
spacelike " " > 0 $\Rightarrow$ "tachyons"

Massive particles: we can define proper time along worldline $d\tau^2 = dt^2 - \frac{dx^2}{c^2} = \eta_{\mu\nu}\frac{dx^\mu dx^\nu}{c^2}$ $v = \frac{dx}{dt}$

$\Rightarrow d\tau = dt\sqrt{1 - \left(\frac{dx}{dt}\right)^2/c^2} = dt\sqrt{1 - v^2/c^2}$

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4-velocity $u^\mu \equiv \frac{dx^\mu}{d\tau}$, $\eta_{\mu\nu}\frac{u^\mu u^\nu}{c^2} = -1$

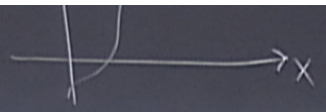
4-momentum $p^\mu = m_0 u^\mu$, m_0 = "rest mass"

$\eta_{\mu\nu}p^\mu p^\nu = -m_0^2 c^2$

$= -p^{0^2} + \underline{p}^2$

If we identify p^0 with E/c,

then $E = \sqrt{\underline{p}^2c^2 + m_0^2c^4} \approx$



spacelike " " > 0 $\Rightarrow$ "tachyons"

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extra term!

If $|\underline{p}| \ll m_0 c$ familiar Newtonian KE

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If we identify p^0 with E/c,

then $E = \sqrt{\underline{p}^2 c^2 + m_0^2 c^4} \geq m_0 c^2$ ("rest mass energy")

extra term!

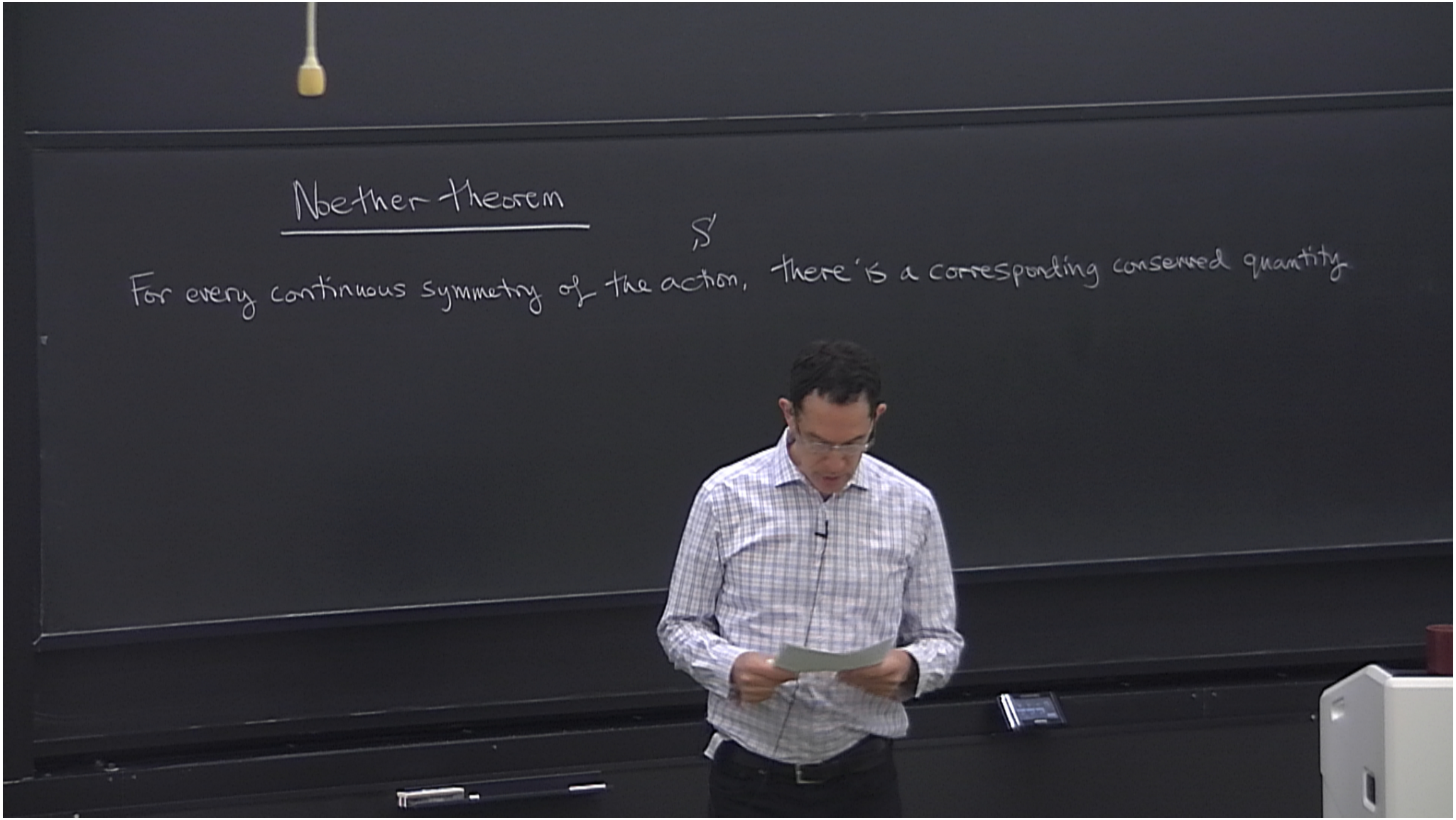
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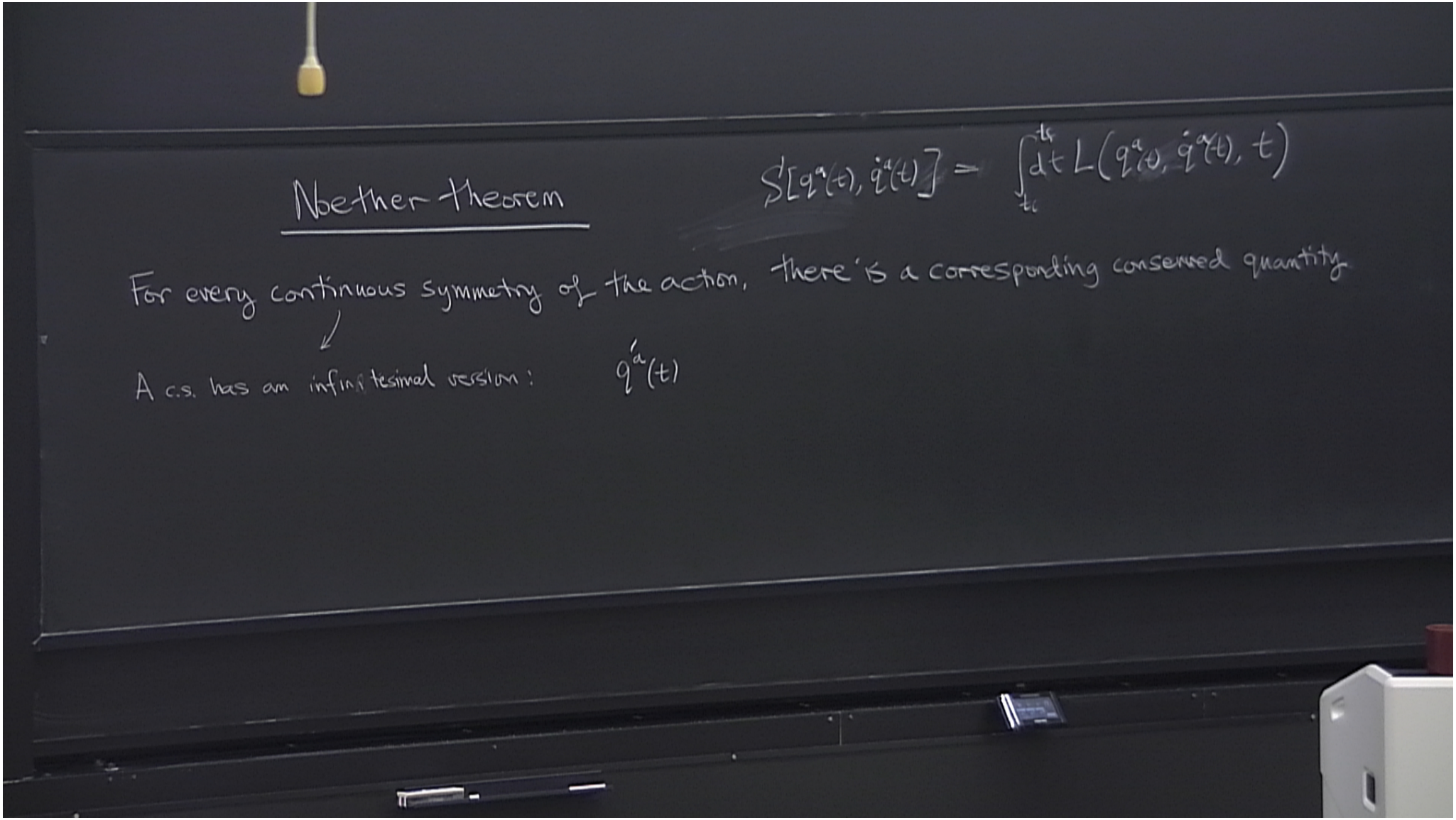
$\approx m_0 c^2 + \frac{1}{2} m_0 \left(\frac{\underline{p}}{m_0}\right)^2 +$ higher order

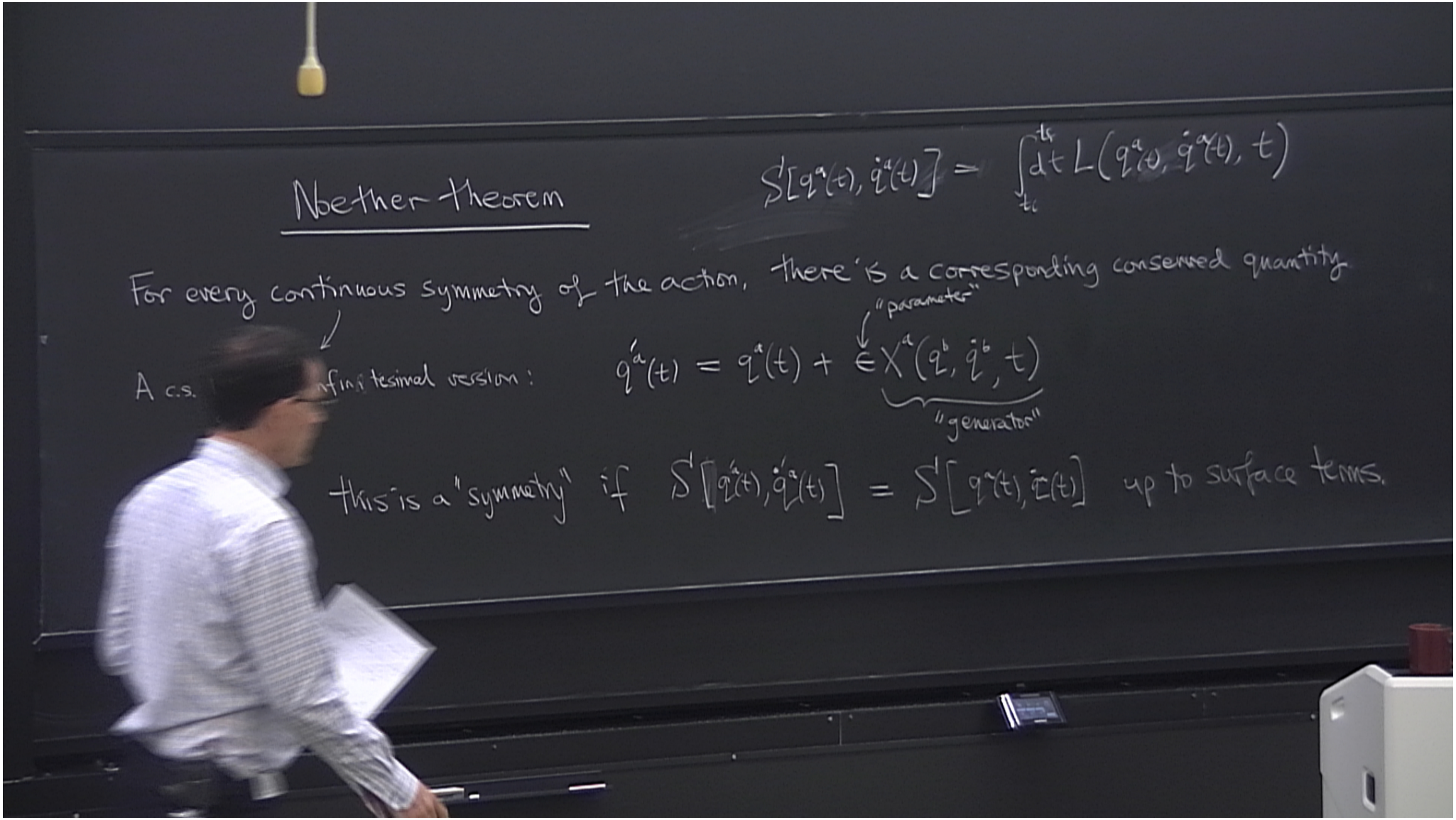
in general, at any v, $\underline{v} = \frac{d\underline{x}}{dt} = \frac{\underline{p}}{p^0/c} = \frac{c\underline{p}}{\sqrt{\underline{p}^2 + m_0^2 c^2}}$

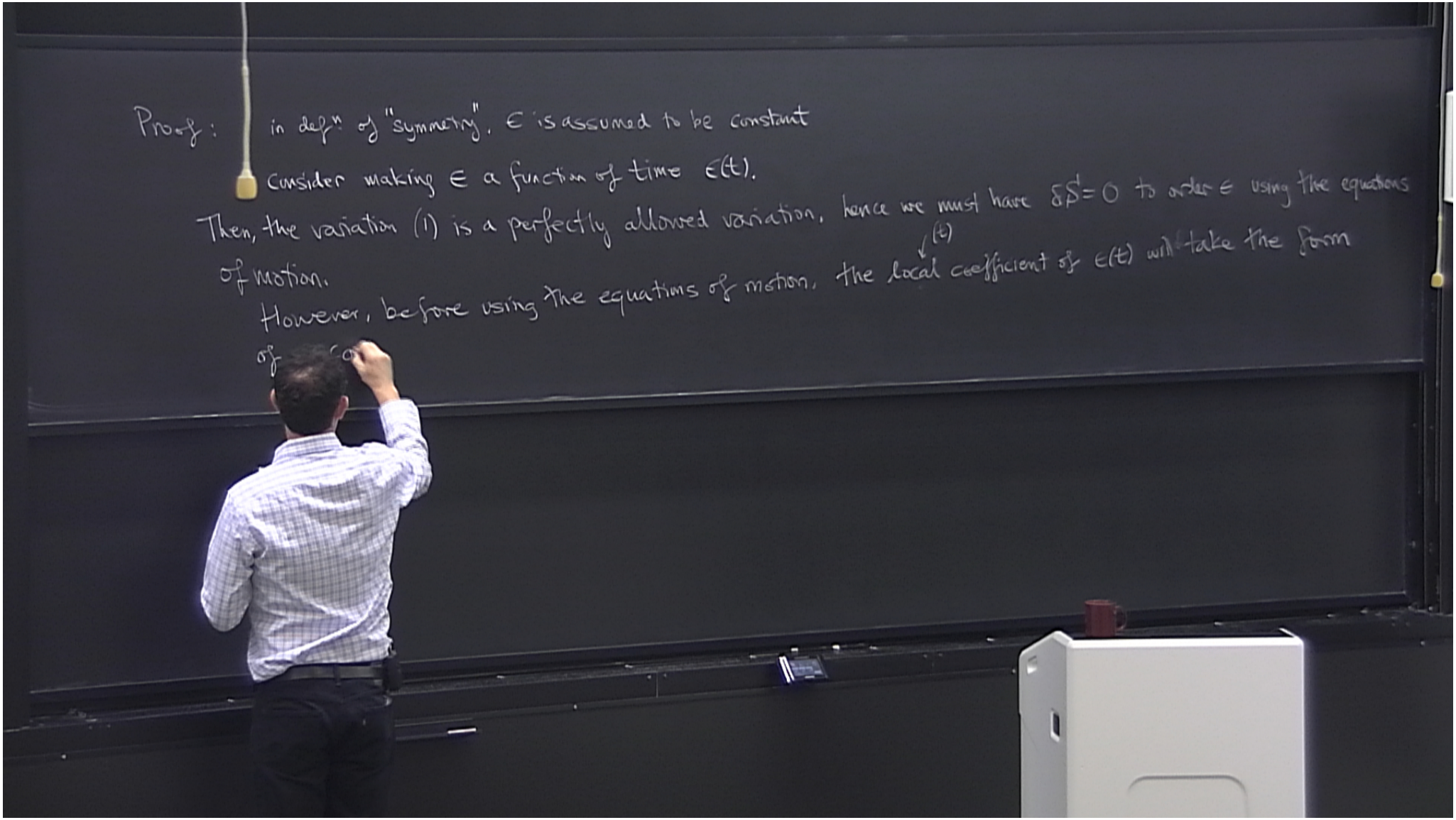
if apply a force to increase $|\underline{p}|$, as $|\underline{p}| \to \infty$, $|\underline{v}| \to c$.

Sim'y, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{\sqrt{\underline{p}^2 + m_0^2 c^2}}{m_0 c} \Rightarrow \boxed{E = \gamma m_0 c^2}$


Noether theorem
For every continuous symmetry of the action, there is a corresponding conserved quantity


Noether Theorem
$S[q^a(t), \dot{q}^a(t)] = \int_{t_i}^{t_f} dt\, L(q^a(t), \dot{q}^a(t), t)$
For every continuous symmetry of the action, there is a corresponding conserved quantity
A c.s. has an infinitesimal version: $q'^a(t)$


Noether theorem
$S[q^a(t), \dot{q}^a(t)] = \int_{t_i}^{t_f} dt\, L(q^a(t), \dot{q}^a(t), t)$
For every continuous symmetry of the action, there is a corresponding conserved quantity.
A c.s. infinitesimal version:
$q'^a(t) = q^a(t) + \epsilon X^a(q^b, \dot{q}^b, t)$
"parameter"
"generator"
this is a "symmetry" if $S[q'^a(t), \dot{q}'^a(t)] = S[q^a(t), \dot{q}^a(t)]$ up to surface terms.


Proof: in def^n of "symmetry", ϵ is assumed to be constant
consider making ϵ a function of time $\epsilon(t)$.
Then, the variation (1) is a perfectly allowed variation, hence we must have $\delta S' = 0$ to order ϵ using the equations
of motion.
However, before using the equations of motion, the local coefficient of $\epsilon(t)$ will take the form
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Then the variation (1) is a perfectly allowed variation, hence we must have $\delta S = 0$ to order ϵ using the equations of motion.

However, before using the equations of motion, the local coefficient of $\epsilon(t)$ will take the form of a conservation law.

Only possibilities for the variation are

$$\delta S = \int dt \left(\dot{\epsilon} J(q, \dot{q}, t) + \epsilon \frac{dK}{dt} \right)$$

$\dot{\epsilon} \to 0$ if $\epsilon =$ const

a surface term if $\epsilon =$

$$= \int dt\, \epsilon \frac{d}{dt}(K - J) \quad + \text{surface terms.}$$

$$\forall \epsilon(t) \Rightarrow \frac{d}{dt}(K - J) = 0 \quad \text{using the equations of}$$

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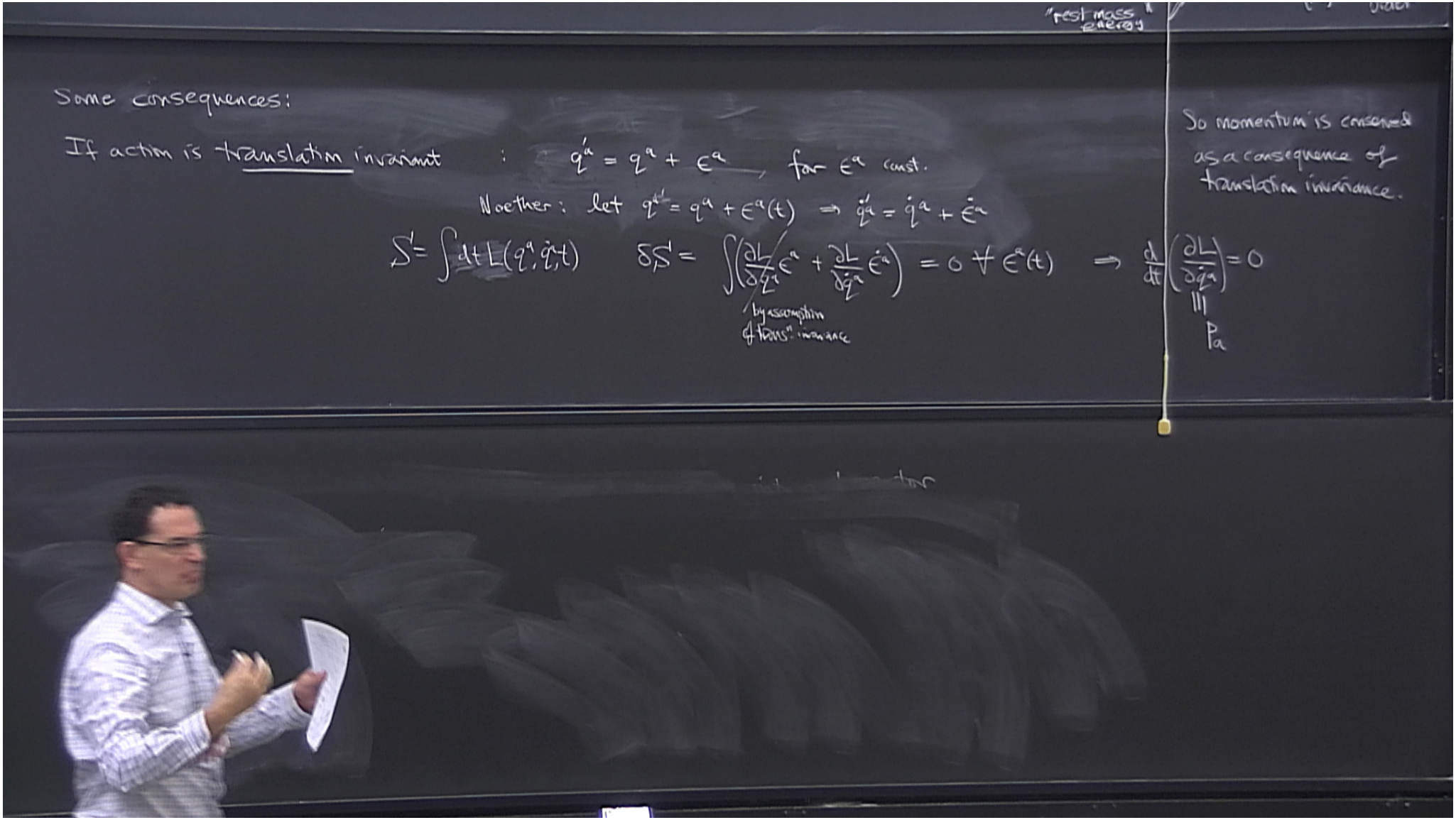
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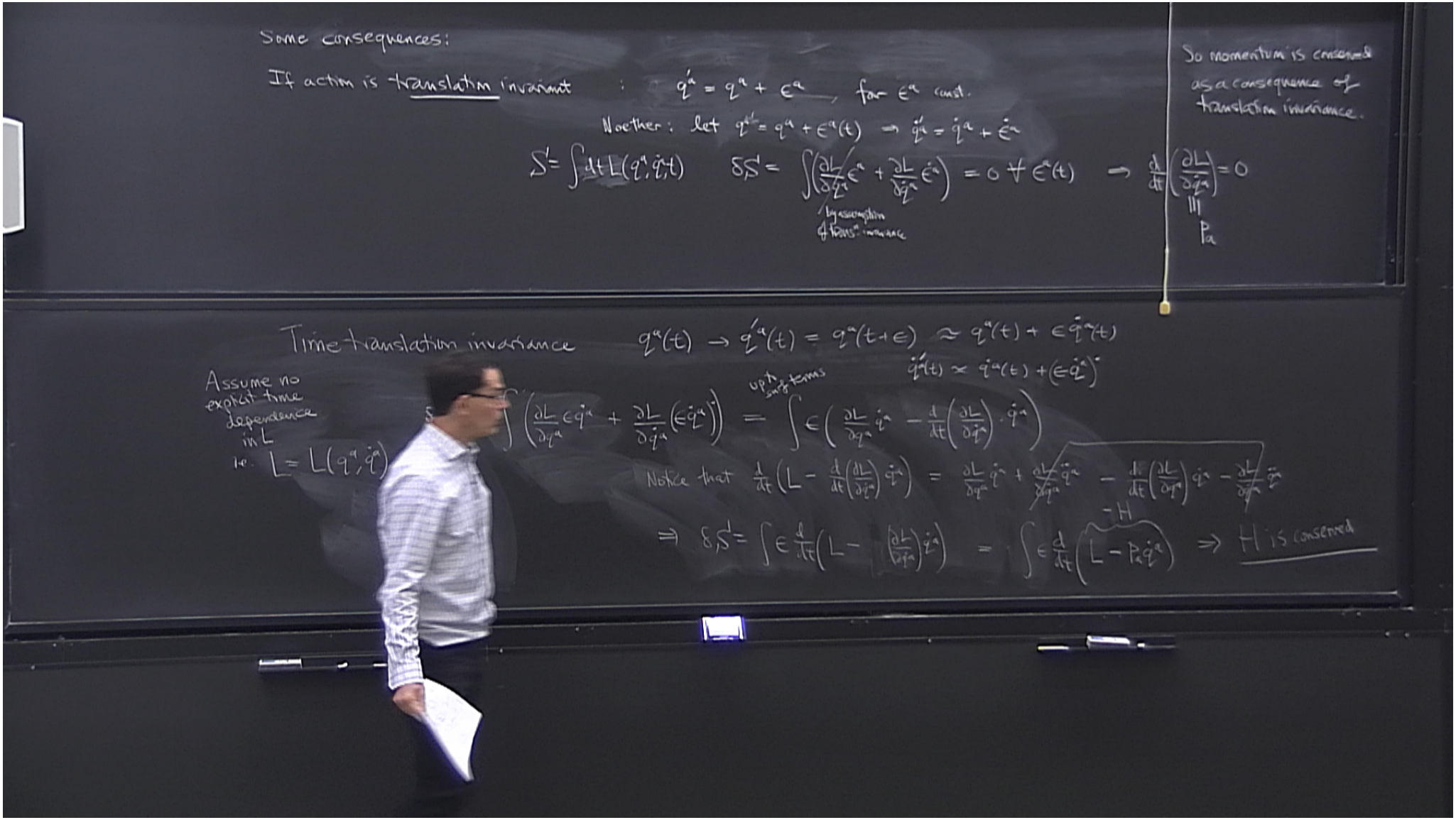
$\dot{\epsilon} \to 0$ if $\epsilon = \text{const}$

$\epsilon \frac{dK}{dt}$ a surface tem if $\epsilon = \text{const}$

$$= \int dt \, \epsilon \frac{d}{dt}(K - J) \quad + \text{surface terms.}$$

$\forall \epsilon(t) \Rightarrow \frac{d}{dt}(K - J) = 0$ using the equations of motion.


"rest mass" energy
Some consequences:
If action is translation invariant : $q'^a = q^a + \epsilon^a$ for ϵ^a const.
Noether: let $q'^a = q^a + \epsilon^a(t) \Rightarrow \dot{q}'^a = \dot{q}^a + \dot{\epsilon}^a$
$S' = \int dt\, L(q', \dot{q}', t)$ $\delta S' = \int \left(\frac{\partial L}{\partial q^a} \epsilon^a + \frac{\partial L}{\partial \dot{q}^a} \dot{\epsilon}^a \right) = 0 \; \forall \epsilon^a(t) \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right) = 0$
by assumption of trans. invariance
$\equiv p_a$
So momentum is conserved as a consequence of translation invariance.


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Time translation invariance $q^a(t) \to q'^a(t) = q^a(t+\epsilon) \approx q^a(t) + \epsilon \dot{q}^a(t)$
$\dot{q}'^a(t) \propto \dot{q}^a(t) + (\epsilon \dot{q}^a)\dot{}$
Assume no explicit time dependence in L
i.e. $L = L(q^a, \dot{q}^a)$
up to surface terms
$\int \left(\frac{\partial L}{\partial q^a}\epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}(\epsilon \dot{q}^a)\dot{}\right) = \int \epsilon \left(\frac{\partial L}{\partial q^a}\dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right)$
Notice that $\frac{d}{dt}\left(L - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right) = \frac{\partial L}{\partial q^a}\dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial q^a}\right)\dot{q}^a - \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a$
$\Rightarrow \delta S' = \int \epsilon \frac{d}{dt}\left(L - \left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right) = \int \epsilon \frac{d}{dt}\left(L - p_a \dot{q}^a\right) \Rightarrow$ H is conserved
$-H$

Some consequences:

If action is translation invariant : $q'^a = q^a + \epsilon^a$, for ϵ^a const.

So momentum is conserved as a consequence of translation invariance.

Noether: let $q'^a = q^a + \epsilon^a(t) \Rightarrow \dot{q}'^a = \dot{q}^a + \dot{\epsilon}^a$

$$S' = \int dt\, L(q^a, \dot{q}^a, t) \qquad \delta S' = \int \left(\frac{\partial L}{\partial q^a}\epsilon^a + \frac{\partial L}{\partial \dot{q}^a}\dot{\epsilon}^a \right) = 0 \;\forall\, \epsilon^a(t) \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right) = 0$$

(by assumption of transl. invariance)

$$\frac{\partial L}{\partial \dot{q}^a} = p_a$$

Time translation invariance $\quad q^a(t) \to q'^a(t) = q^a(t+\epsilon) \approx q^a(t) + \epsilon \dot{q}^a(t)$

$$\dot{q}'^a(t) \propto \dot{q}^a(t) + (\epsilon \dot{q}^a)^{\cdot}$$

Assume no explicit time dependence in L i.e. $L = L(q^a, \dot{q}^a)$

$$\delta S' = \int \left(\frac{\partial L}{\partial q^a}\epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}(\epsilon \dot{q}^a)^{\cdot} \right) \overset{\text{up to surface terms}}{=} \int \epsilon \left(\frac{\partial L}{\partial q^a}\dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a \right)$$

Notice that $\frac{d}{dt}\left(L - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right) = \frac{\partial L}{\partial q^a}\dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial q^a}\right)\dot{q}^a - \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a$

$$\Rightarrow \delta S' = \int \epsilon \frac{d}{dt}\underbrace{\left(L - \left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right)}_{-H} = \int \epsilon \frac{d}{dt}\left(L - p_a \dot{q}^a\right) \Rightarrow H \text{ is conserved}$$

If action is translation invariant : $q^a = q^a + \epsilon^a$, for ϵ^a const.

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Noether: let $q^{a\prime} = q^a + \epsilon^a(t) \Rightarrow \dot{q}^{a\prime} = \dot{q}^a + \dot{\epsilon}^a$

$$S' = \int dt\, L(q', \dot{q}', t) \qquad \delta S' = \int \left(\frac{\partial L}{\partial q^a}\epsilon^a + \frac{\partial L}{\partial \dot{q}^a}\dot{\epsilon}^a \right) = 0 \;\forall\, \epsilon^a(t) \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right) = 0$$

(by assumption of trans^n invariance)

$\| \; P_a$

Time-translation invariance $\quad q^a(t) \to q'^a(t) = q^a(t+\epsilon) \approx q^a(t) + \epsilon \dot{q}^a(t)$

$\dot{q}'^a(t) \approx \dot{q}^a(t) + (\epsilon \dot{q}^a)^{\cdot}$

Assume no explicit time dependence in L i.e. $L = L(q^a, \dot{q}^a)$

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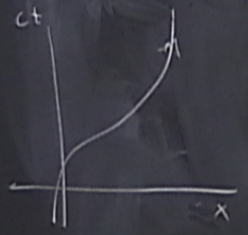
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$\forall \epsilon(t) \Rightarrow \frac{d}{dt}(K-J) = 0$ using the equations of motion.

Relativity

massive particle:



$S = \text{const} \times$ invariant length of world line $= \text{const} \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \, d\lambda = \text{const} \int \sqrt{\dot{t}^2 - \dot{\underline{x}}^2} \, d\tau = \int$

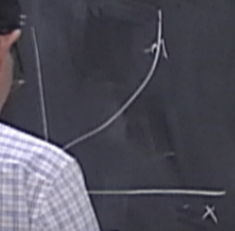
$x^\mu(\lambda)$ ... $\dot{t} = \frac{dt}{d\tau}$

reparameterisation invart. $\lambda \to \tilde{\lambda}(\lambda)$

$\forall \epsilon(t) \Rightarrow \frac{d}{dt}(K \cdot J) = 0$ using the equations of motion.

Relativity

massive particle



$S = \text{const} \times$ invariant length of world line $= \text{const} \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \cdot d\lambda = \text{const} \int \sqrt{\dot{t}^2 - \dot{\underline{x}}^2}\, d\tau$ OR $= \text{const} \int \sqrt{1 - \dot{\underline{x}}^2}\, dt$

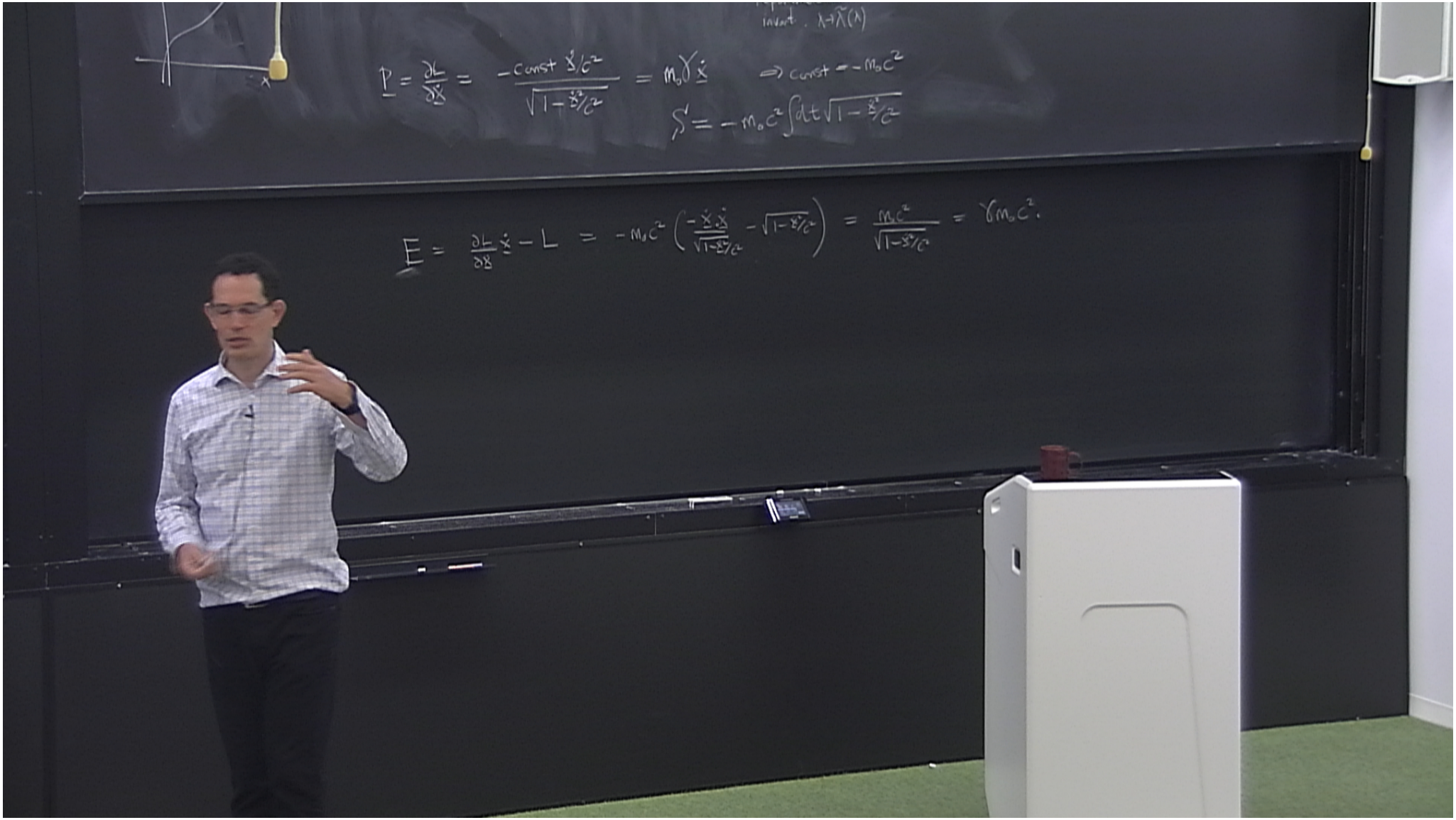
$x^\mu(\lambda)$ . .

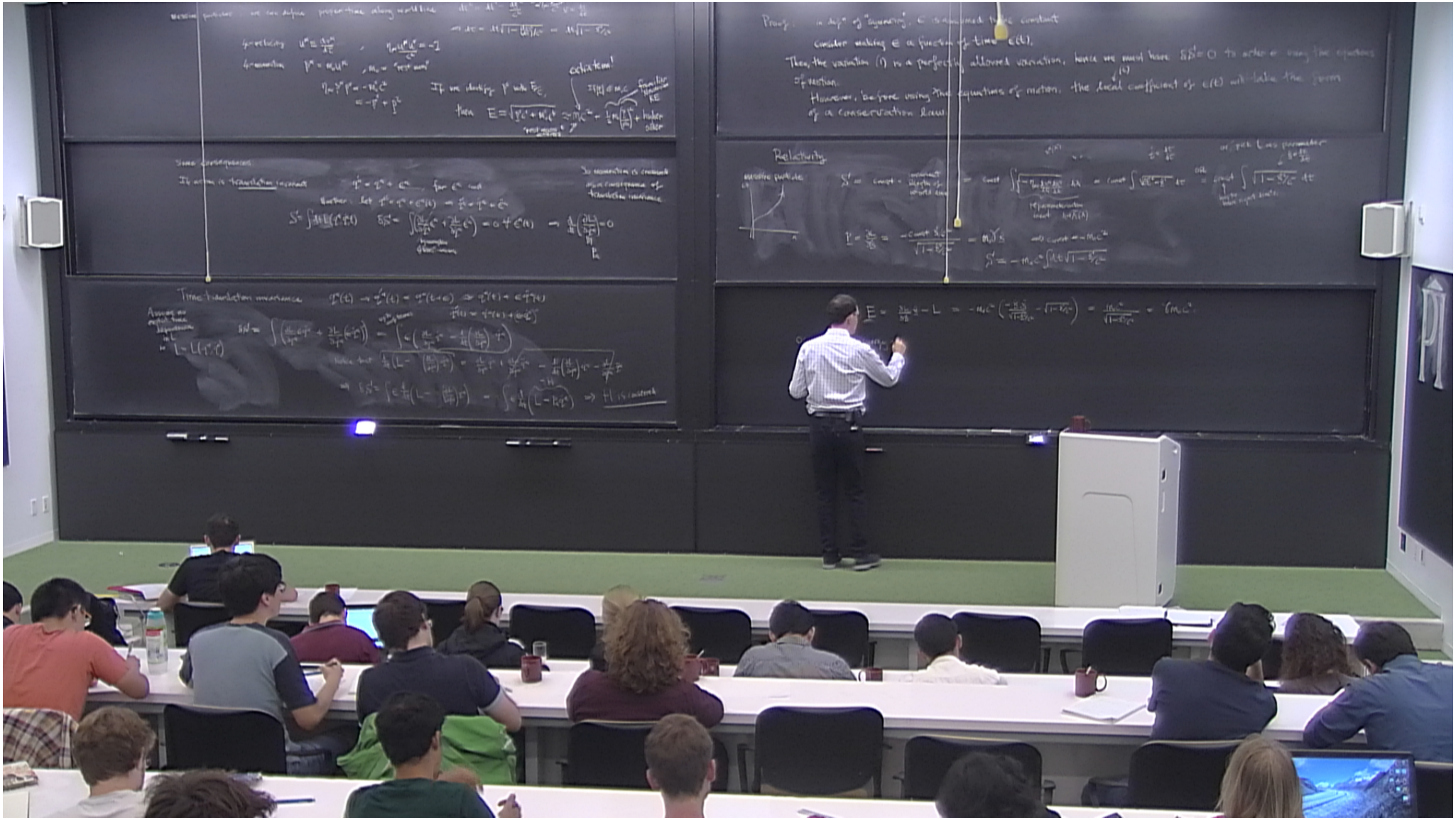
$\dot{t} = \frac{dt}{d\tau}$

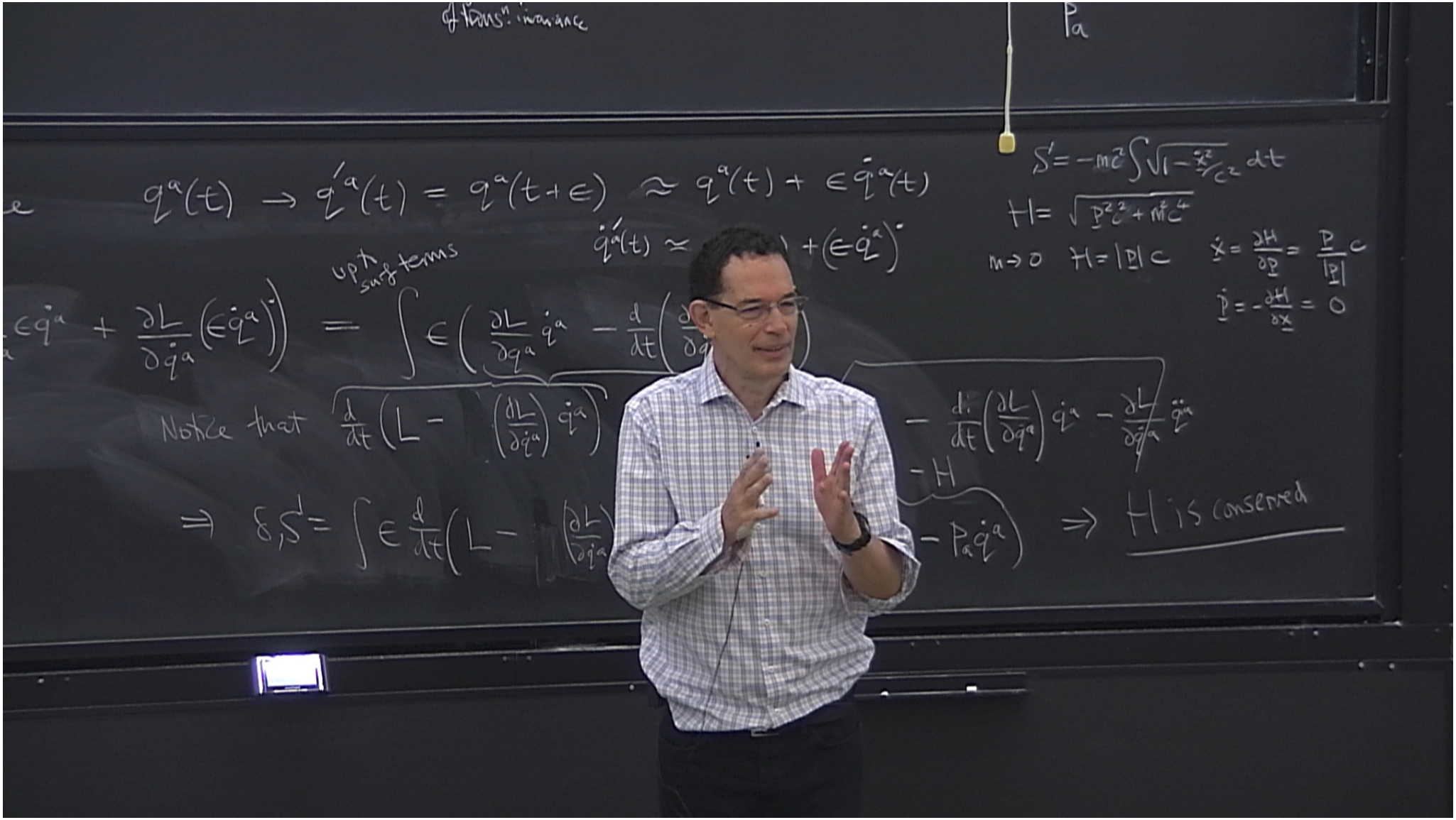
or, pick t as parameter $\dot{\underline{x}} = \frac{d\underline{x}}{dt}$

reparameterisation invart. $\lambda \to \tilde{\lambda}(\lambda)$

has to have right dim's.


$\underline{p} = \frac{\partial L}{\partial \dot{\underline{x}}} = \frac{-\text{const}\,\dot{x}/c^2}{\sqrt{1-\dot{x}^2/c^2}} = m_0\gamma\dot{\underline{x}}$
$\Rightarrow \text{const} = -m_0c^2$
$S' = -m_0c^2\int dt\sqrt{1-\dot{x}^2/c^2}$
$E = \frac{\partial L}{\partial \dot{x}}\dot{x} - L = -m_0c^2\left(\frac{-\dot{x}\cdot\dot{x}}{\sqrt{1-\dot{x}^2/c^2}} - \sqrt{1-\dot{x}^2/c^2}\right) = \frac{m_0c^2}{\sqrt{1-\dot{x}^2/c^2}} = \gamma m_0c^2.$


Relativity


$q^a(t) \to q'^a(t) = q^a(t+\epsilon) \approx q^a(t) + \epsilon \dot{q}^a(t)$
$S' = -mc^2 \int \sqrt{1 - \frac{\dot{x}^2}{c^2}}\, dt$
$H = \sqrt{p^2c^2 + m^2c^4}$
$m \to 0 \quad H = |p|c$
$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{|p|}c$
$\dot{p} = -\frac{\partial H}{\partial x} = 0$
up to surface terms
Notice that
$\Rightarrow \delta S' =$
$-H$
$\Rightarrow$ H is conserved

by assumption of trans^{n}. invariance

$= p_a$

$S' = -mc^2 \int \sqrt{1 - \dot{x}^2/c^2}\, dt$

$H = \sqrt{p^2c^2 + m^2c^4}$

$m \to 0 \quad H = |p|c \qquad \dot{x} = \frac{\partial H}{\partial p} = \frac{p}{|p|}c$

$\dot{p} = -\frac{\partial H}{\partial x} = 0$

ance $\quad q^a(t) \to q'^a(t) = q^a(t+\epsilon) \approx q^a(t) + \epsilon \dot{q}^a(t)$

$\dot{q}'^a(t) \approx \dot{q}^a(t) + (\epsilon \dot{q}^a)^{\cdot}$

$$\left(\frac{\partial L}{\partial q^a}\epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}(\epsilon \dot{q}^a)^{\cdot}\right) \overset{\text{up to surf. terms}}{=} \int \epsilon \left(\frac{\partial L}{\partial q^a}\dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\cdot \dot{q}^a\right)$$

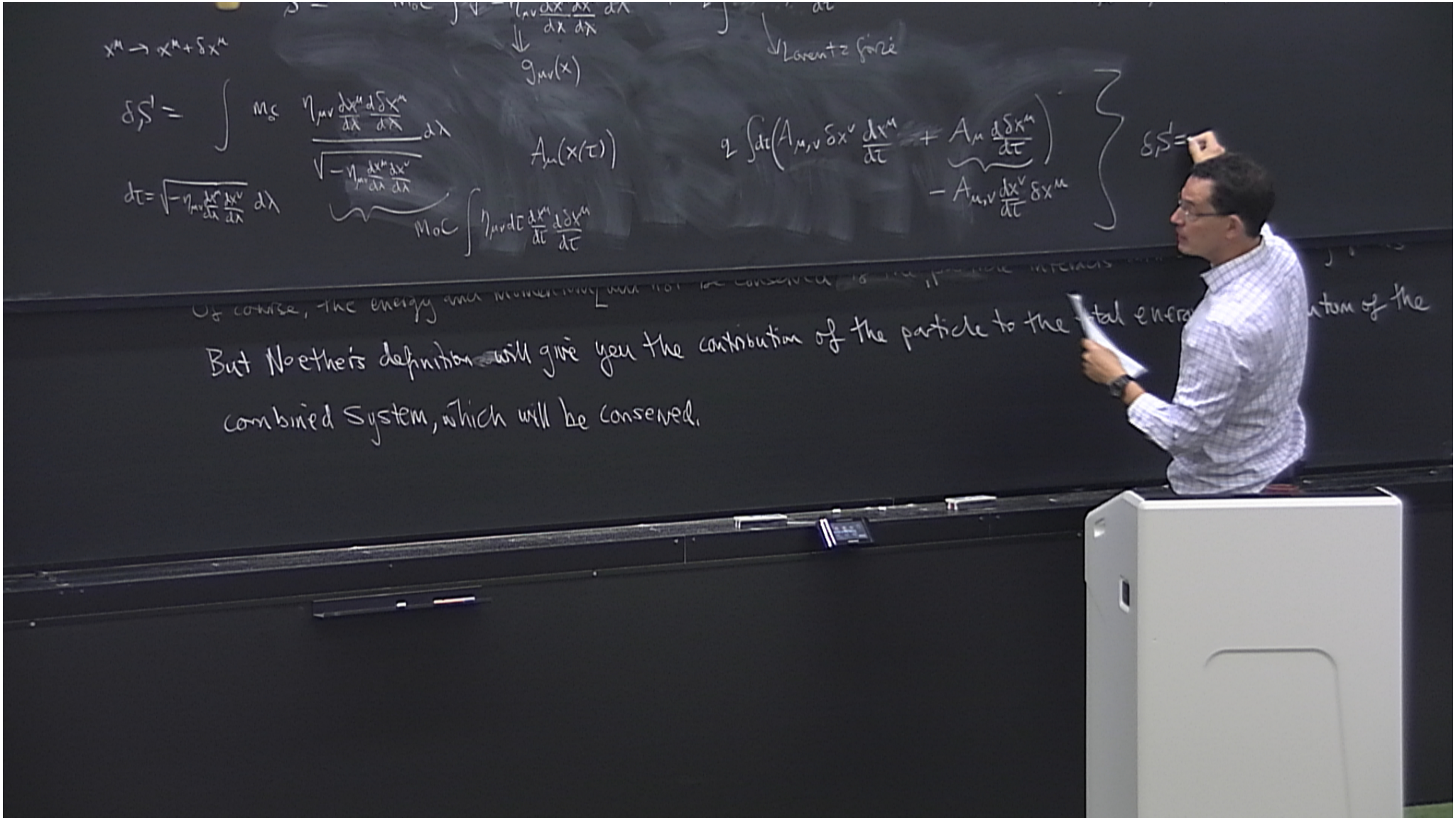
Notice that $\frac{d}{dt}\left(L - \left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right) = \frac{\partial L}{\partial q^a}\dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a - \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a$

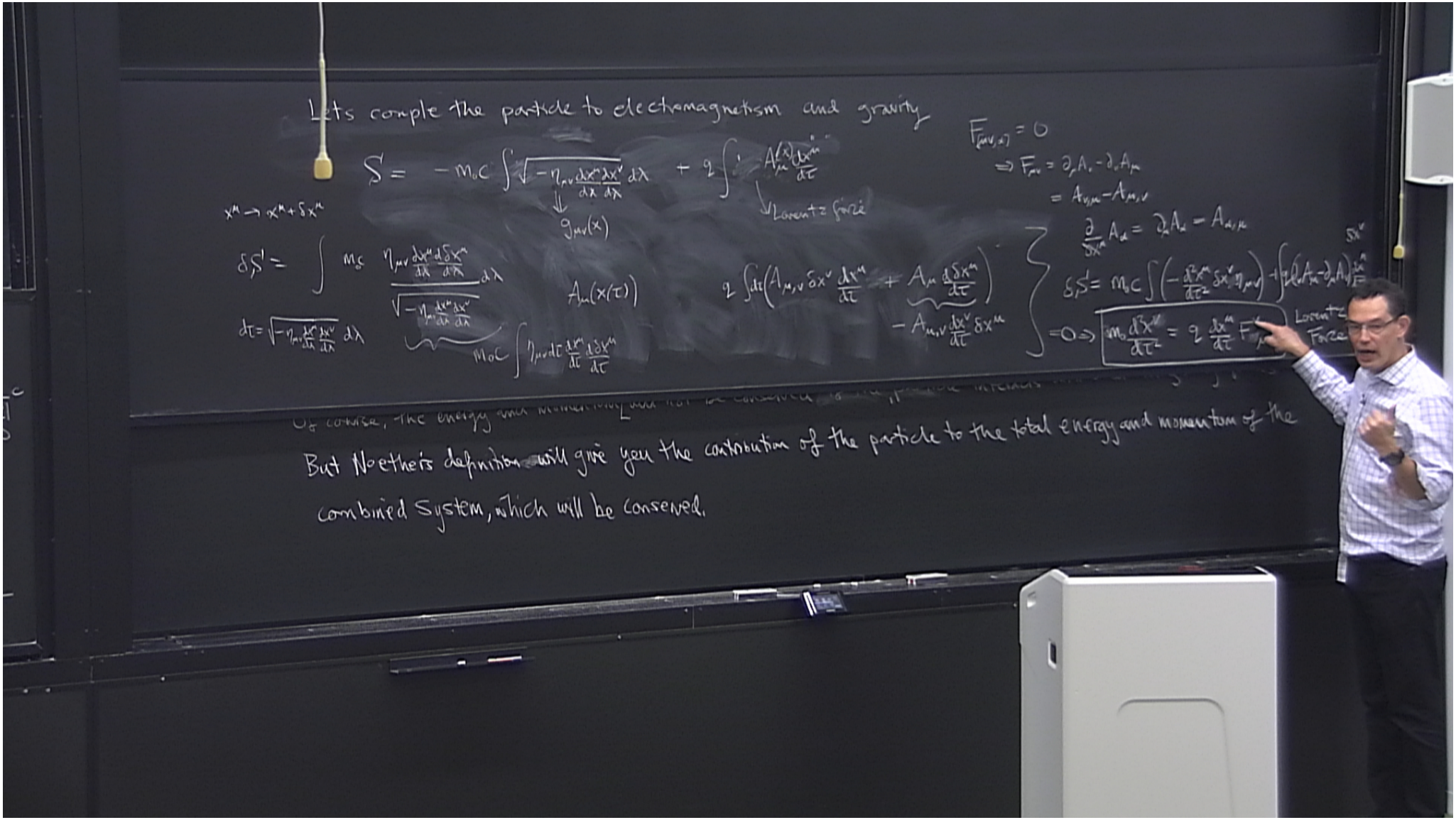
$\Rightarrow \delta S' = \int \epsilon \frac{d}{dt}\left(L - \left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right) = \int \epsilon \frac{d}{dt}\left(\overbrace{L - p_a \dot{q}^a}^{-H}\right) \Rightarrow$ H is conserved

Then, the variation (1) is a perfectly allowed variation, hence we must have ...
of motion.
However, before using the equations of motion, the local coefficient of $\epsilon(t)$ will take the form of a conservation law.

Lets couple the particle to electromagnetism and gravity

$$S = -m_0 c^2 \int \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$$


But Noether's definition will give you the contribution of the particle to the
combined system, which will be conserved.


Lets couple the particle to electromagnetism and gravity
$S = -m_0 c \int \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\,d\lambda + q\int A_\mu(x)\frac{dx^\mu}{d\tau}$
$g_{\mu\nu}(x)$
Lorentz force
$F_{[\mu\nu,\sigma]} = 0$
$\Rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$= A_{\nu,\mu} - A_{\mu,\nu}$
$x^\mu \to x^\mu + \delta x^\mu$
$\frac{\partial}{\partial x^\mu} A_\alpha = \partial_\mu A_\alpha = A_{\alpha,\mu}$
$\delta S = \int m_0 c \frac{\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{d\delta x^\mu}{d\lambda}}{\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}}d\lambda$
$A_\mu(x(\tau))$
$q\int d\tau\left(A_{\mu,\nu}\delta x^\nu\frac{dx^\mu}{d\tau} + A_\mu\frac{d\delta x^\mu}{d\tau}\right)$
$-A_{\mu,\nu}\frac{dx^\nu}{d\tau}\delta x^\mu$
$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\,d\lambda$
$m_0 c\int \eta_{\mu\nu}d\tau\frac{dx^\mu}{d\tau}\frac{d\delta x^\mu}{d\tau}$
$\delta S = m_0 c\int\left(-\frac{d^2x^\mu}{d\tau^2}\delta x^\nu\eta_{\mu\nu}\right) + \int q(\partial_\mu A_\nu - \partial_\nu A_\mu)\frac{dx^\mu}{d\tau}\delta x^\nu$
$= 0 \Rightarrow \boxed{m_0\frac{d^2x^\mu}{d\tau^2} = q\frac{dx^\nu}{d\tau}F^{\mu}{}_{\nu}}$
Lorentz Force
But Noether's definition will give you the contribution of the particle to the total energy and momentum of the combined system, which will be conserved.


Lets couple the particle to electromagnetism and gravity
But Noether's definition will give you the conservation of the
combined system, which will be conserved.


Relativity
Let's couple the particle to electromagnetism and gravity.
But Noether's theorem will give you the conservation of the particle's total energy and momentum of the combined system, which will be conserved.