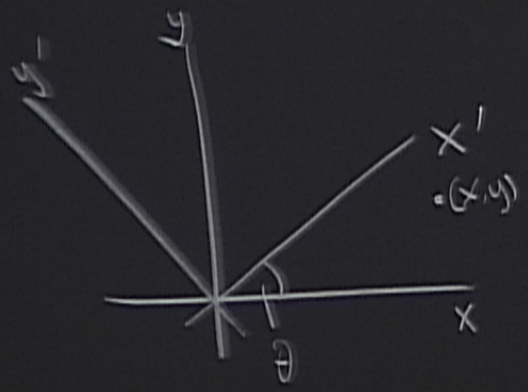
Title: PSI 2015/2016 Relativity - Neil Turok - Lecture 2

Date: Sep 09, 2015 09:00 AM

URL: http://pirsa.org/15090029

Abstract:

2d rotation



$$x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$
$$x'^2 + y'^2 = x^2 + y^2$$

ie. $x'_i = O_{ij} x_j$

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$O_{ij} O_{lm} \delta_{jm} = \delta_{il}$$

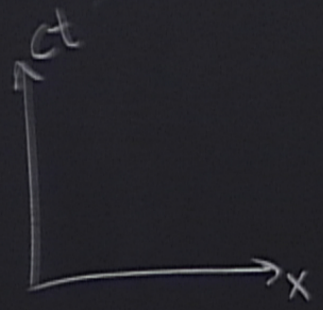
$$O O^T = \mathbb{1}.$$

2d Lorentz boost

$$x' = \Lambda x$$

$$\Lambda = \begin{pmatrix} \cosh\theta & -\sinh\theta \\ -\sinh\theta & \cosh\theta \end{pmatrix}$$

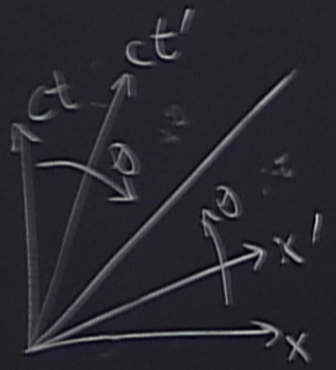
$$\Lambda \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Lambda^T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$ct' = ct\cosh\theta - x\sinh\theta$$

$$x' = x\cosh\theta - ct\sinh\theta$$

2d Lorentz boost



$$x' = \Lambda x$$

$$\Lambda = \begin{pmatrix} \cosh\theta & -\sinh\theta \\ -\sinh\theta & \cosh\theta \end{pmatrix}$$

$$\Lambda \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Lambda^T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$ct' = ct\cosh\theta - x\sinh\theta$$

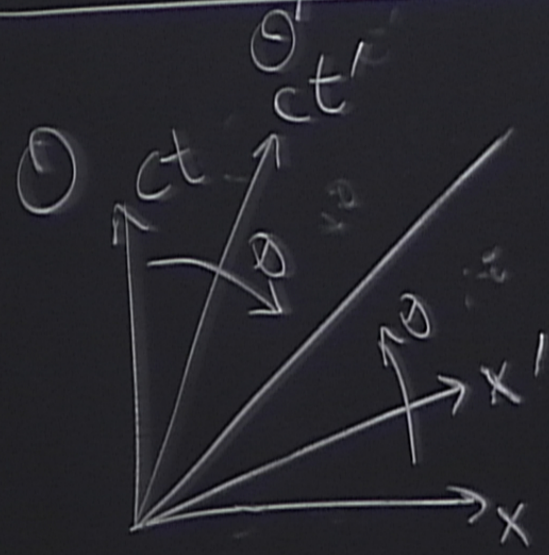
$$x' = x\cosh\theta - ct\sinh\theta$$

$\Rightarrow$ ct' axis is given by

$$ct = \coth\theta\, x$$

x' axis $ct = \tanh\theta\, x$

2d Lorentz boost



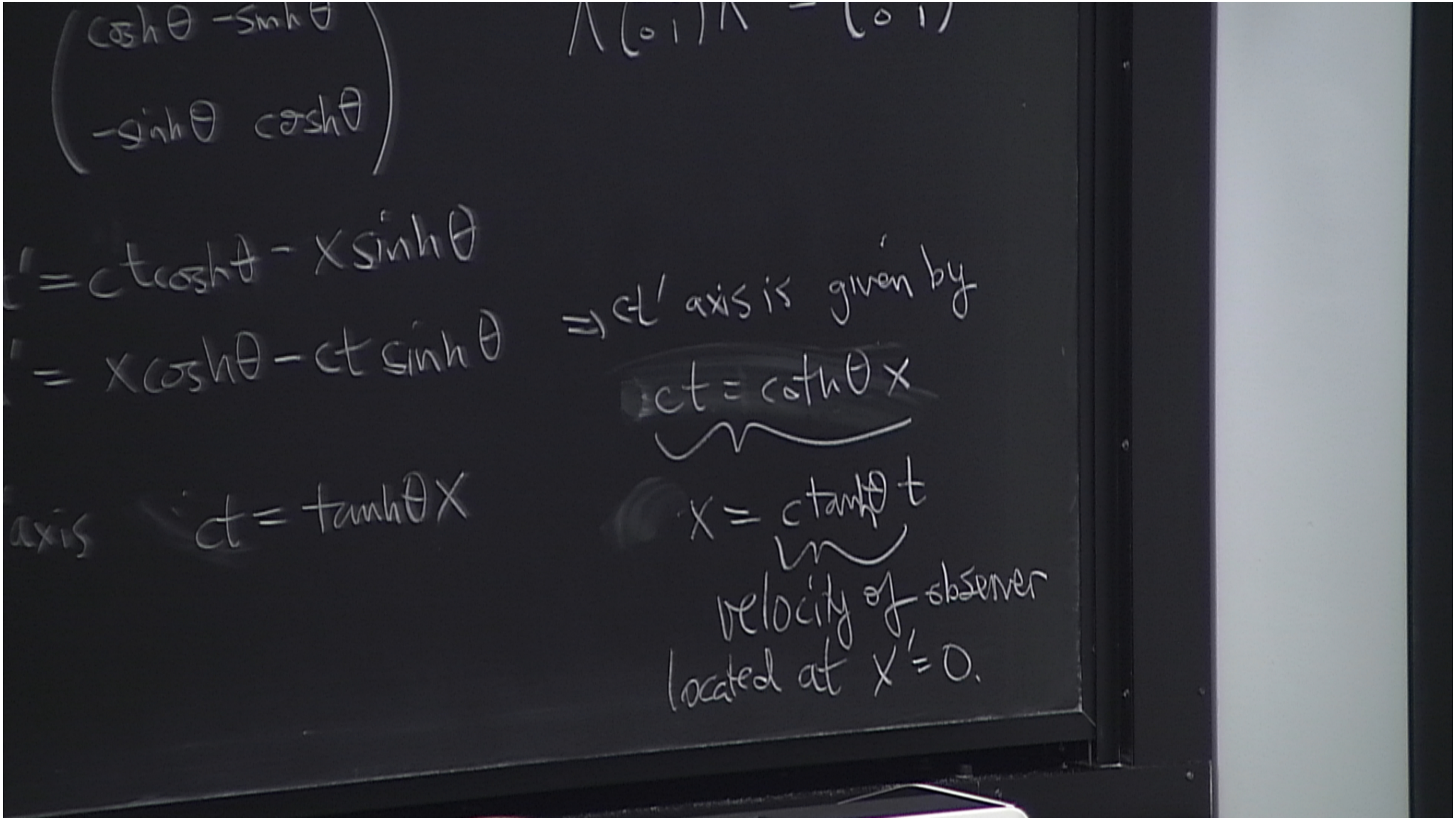
$$x' = \Lambda x$$

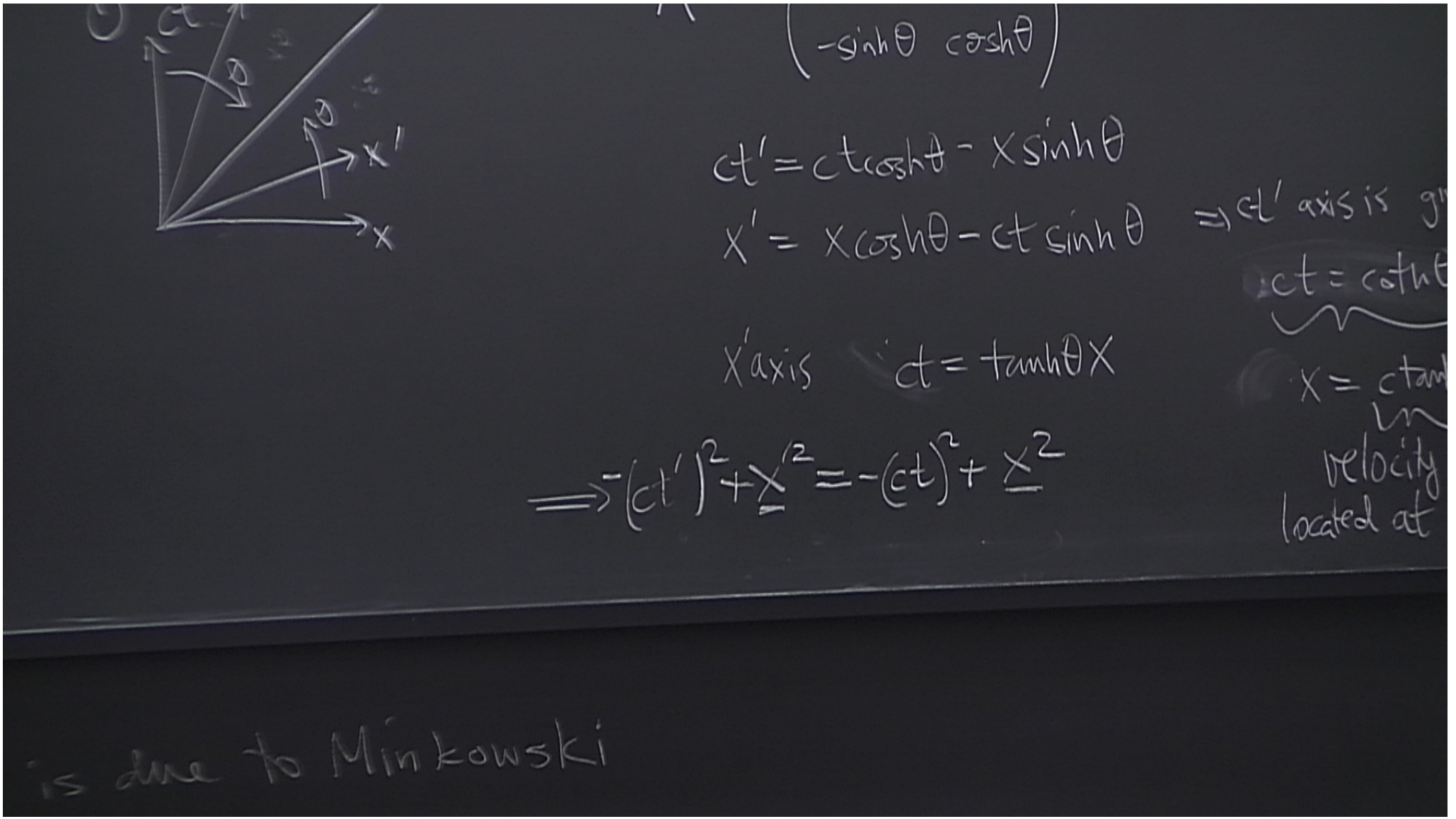
$$\Lambda = \begin{pmatrix} \cosh\theta & -\sinh\theta \\ -\sinh\theta & \cosh\theta \end{pmatrix}$$

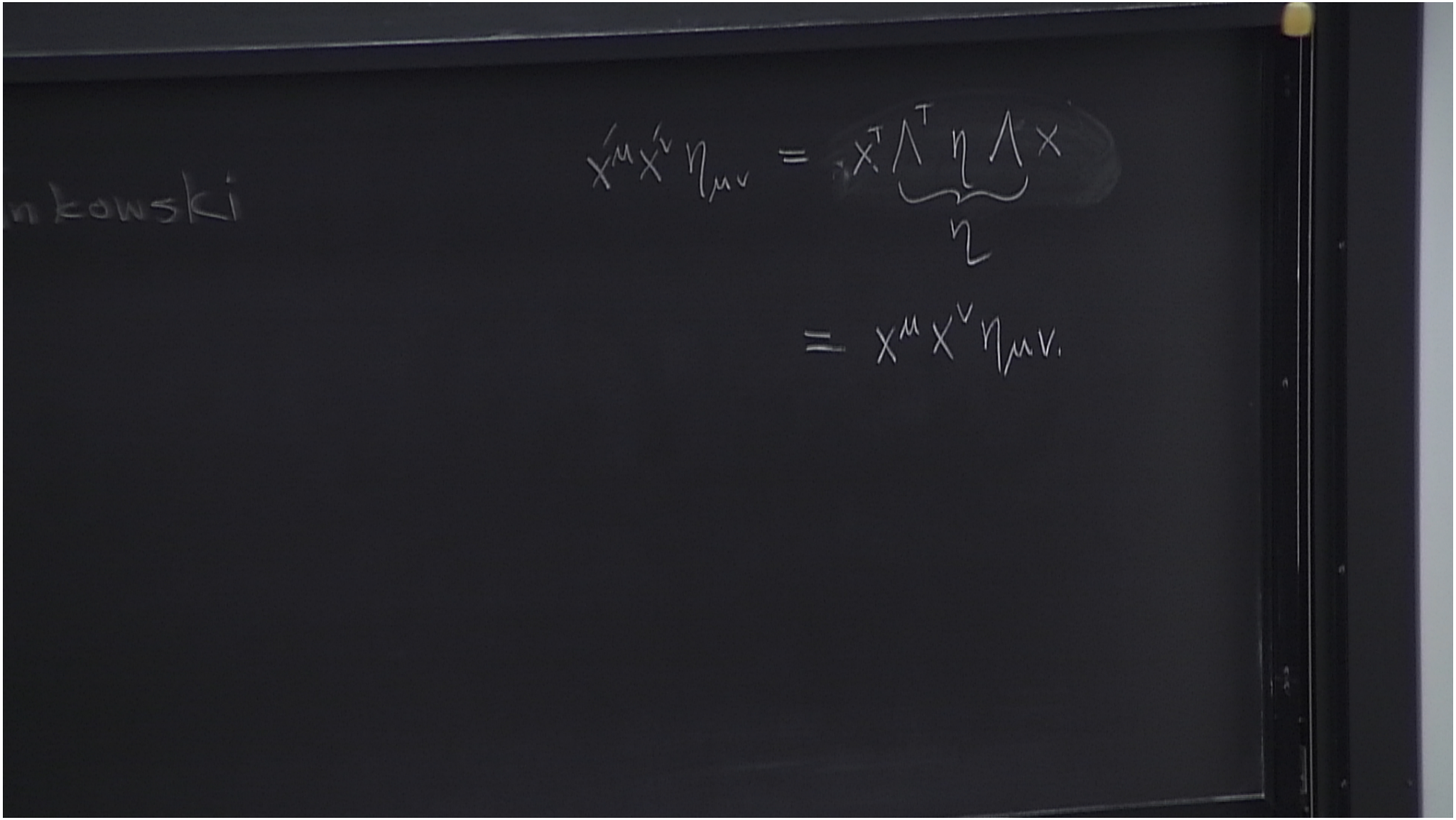
$$ct' = ct\cosh\theta - x\sinh$$

$$x' = x\cosh\theta - ct\sin$$

x' axis $\quad ct = \tanh\theta$


$\begin{pmatrix} \cosh\theta & -\sinh\theta \\ -\sinh\theta & \cosh\theta \end{pmatrix}$
$\Lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$t' = ct\cosh\theta - x\sinh\theta$
$x' = x\cosh\theta - ct\sinh\theta$
$\Rightarrow ct'$ axis is given by
$ct = \coth\theta\, x$
axis $ct = \tanh\theta\, x$
$x = c\tanh\theta\, t$
velocity of observer
located at $x' = 0$.


ct
θ
θ
x'
x
(-sinhθ coshθ)
ct' = ct coshθ - x sinhθ
x' = x coshθ - ct sinhθ
⇒ ct' axis is g
ct = cothθ
x'axis ct = tanhθ x
x = ctan
velocity
located at
⟹ -(ct')² + x'² = -(ct)² + x²
is due to Minkowski


nkowski
$x'^\mu x'^\nu \eta_{\mu\nu} = x^T \Lambda^T \eta \Lambda x$
$= x^\mu x^\nu \eta_{\mu\nu}$

nkowski

$$x'^{\mu} x'^{\nu} \eta_{\mu\nu} = x^T \Lambda^T \eta \Lambda x$$

$$= \eta$$

$$= x^{\mu} x^{\nu} \eta_{\mu\nu}.$$

(noticed that $ct = ict$ then get usual Pythagoras rule)

Note : $v = c\tanh\theta$ is the relative velocity of Θ and Θ' so nature

$$\cosh\theta = \frac{1}{\sqrt{1-\tanh^2\theta}} = \frac{1}{\sqrt{1-v^2/c^2}} \equiv \gamma$$

$$\sinh\theta = \tanh\theta \cdot \cosh\theta = \gamma v/c$$

LT. is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad (1)$$

$$x' = \gamma(x - vt) \quad (2)$$

inverse

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad (1')$$

$$x = \gamma(x' + vt') \quad (2')$$

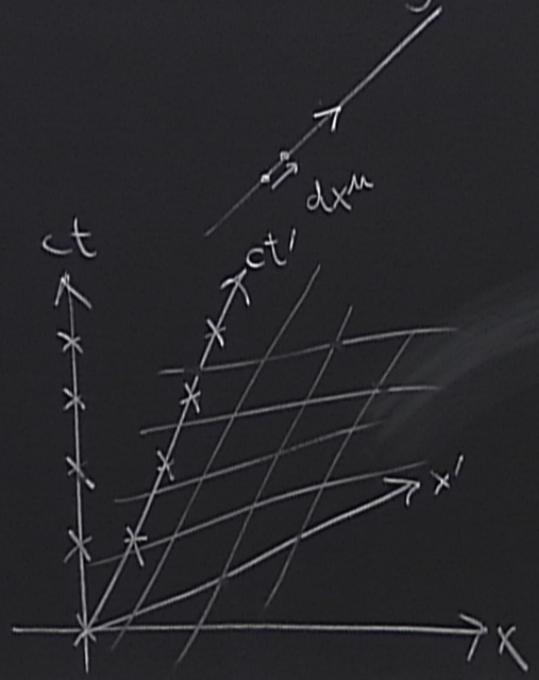
$= \pm ct$, mapped to $x' = \pm ct'$

$x^\nu = 0$, then $\eta_{\mu\nu} dx'^\mu dx'^\nu = 0$

LTs map light rays to light rays

clock. $x' = 0$ so (1) $\Rightarrow t = \gamma t'$ TIME DILATION

Note that light rays $x = \pm ct$, mapped to $x' = \pm ct'$

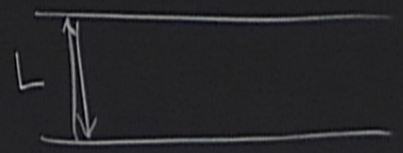


If $\eta_{\mu\nu} dx^\mu dx^\nu = 0$, then $\eta_{\mu\nu} dx'^\mu dx'^\nu = 0$

CLOCKS

Moving clock. $x' = 0$ so (1) $\Rightarrow t = \gamma t'$

$x = \gamma_v t' = v$



ight rays $x = \pm ct$, mapped to $x' = \pm ct'$

If $\eta_{\mu\nu} dx^\mu dx^\nu = 0$, then $\eta_{\mu\nu} dx'^\mu dx'^\nu = 0$

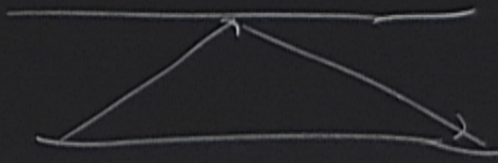
L.Ts map light rays

Moving clock. $x' = 0$ so (1) $\Rightarrow t = \gamma t'$ TIME DILATION

$x = \gamma v t' = vt.$



$t = \frac{2L}{c}$



, mapped to $x' = \pm ct'$

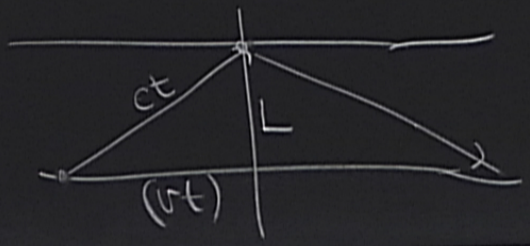
LTs map light rays to light rays

, then $\eta_{\mu\nu} dx'^{\mu} dx'^{\nu} = 0$

$x' = 0$ So (1) $\Rightarrow t = \gamma t'$ TIME DILATION

$x = \gamma v t' = vt.$

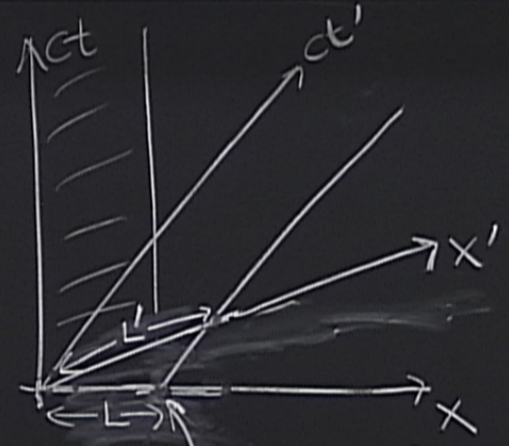
$t = \frac{L}{c}$



$(vt)^2 + L^2 = (ct)^2$

$t^2 = \frac{L^2}{c^2 - v^2} \Rightarrow t = \frac{L}{c\sqrt{1 - v^2/c^2}}.$

RULERS



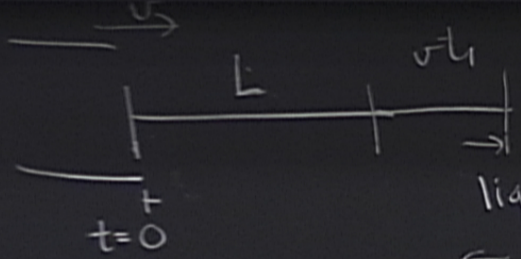
$t=0$ ① $\quad x'=L'$ ②

$$t' = -\frac{vL'}{c^2} \Rightarrow x = \gamma\left(L' - \frac{v^2}{c^2}L'\right) = \frac{L'}{\gamma}$$

$$= L$$

$= \frac{L'}{\gamma}$

$L = \frac{L'}{\gamma}$ $\Rightarrow$ can fit large car into a small garage just by going very fast.



light hits end at $t_1 = \frac{L + v t_1}{c}$ $\quad t_1 = \frac{L}{c(1 - v/c)}$

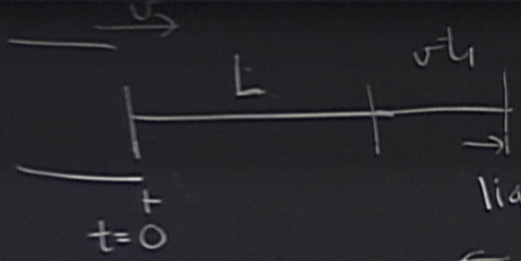
← light comes back

← light hits the end after a time interval $t_2 = \frac{L - v t_2}{c} \rightarrow t_2 = \frac{L}{c(1 + v/c)}$

total time $T = \frac{L}{c}\left(\frac{1}{1 - v/c} + \frac{1}{1 + v/c}\right) = \frac{2L}{c}\frac{1}{1 - v^2/c^2}$

but $T = \gamma T'$

$= \gamma \frac{2L'}{c}$ $\Rightarrow L = L'/\gamma$



light hits end at $t_1 = \frac{L + v t_1}{c}$ $\quad t_1 = \frac{L}{c(1 - v/c)}$

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Energy and Momentum in an EM field

Energy density $\rho_{em} = \frac{1}{2}\epsilon_0 \underline{E}^2 + \frac{1}{2}\underline{B}^2/\mu_0$

Momentum density $\underline{J}_{mom} = \frac{1}{\mu_0 c}\left(\underline{E} \wedge \underline{B}\right)$ Poynting

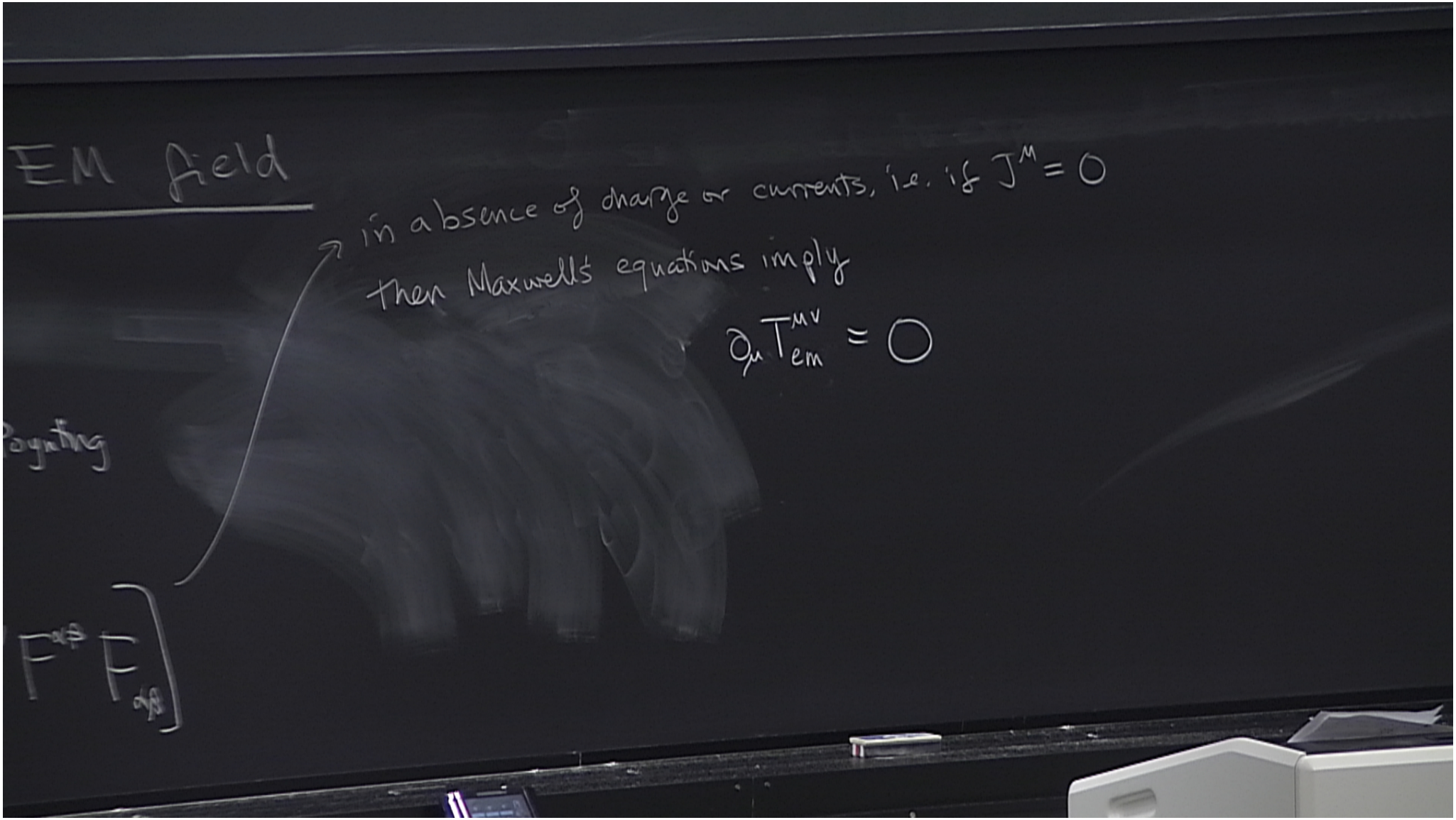
Energy and Momentum in an EM field

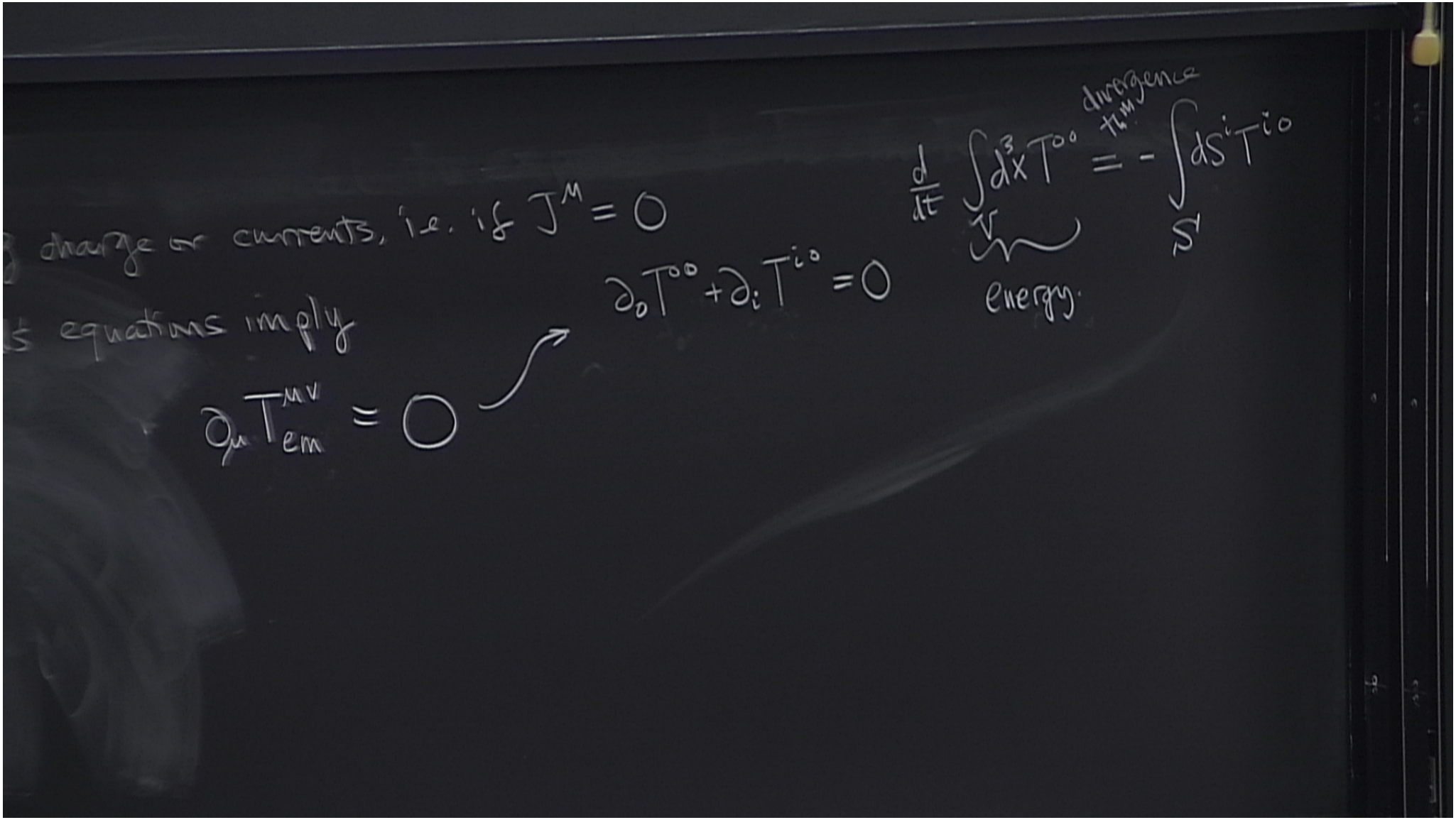
Energy density $\rho_{em} = \frac{1}{2}\epsilon_0 \underline{E}^2 + \frac{1}{2}\underline{B}^2/\mu_0$

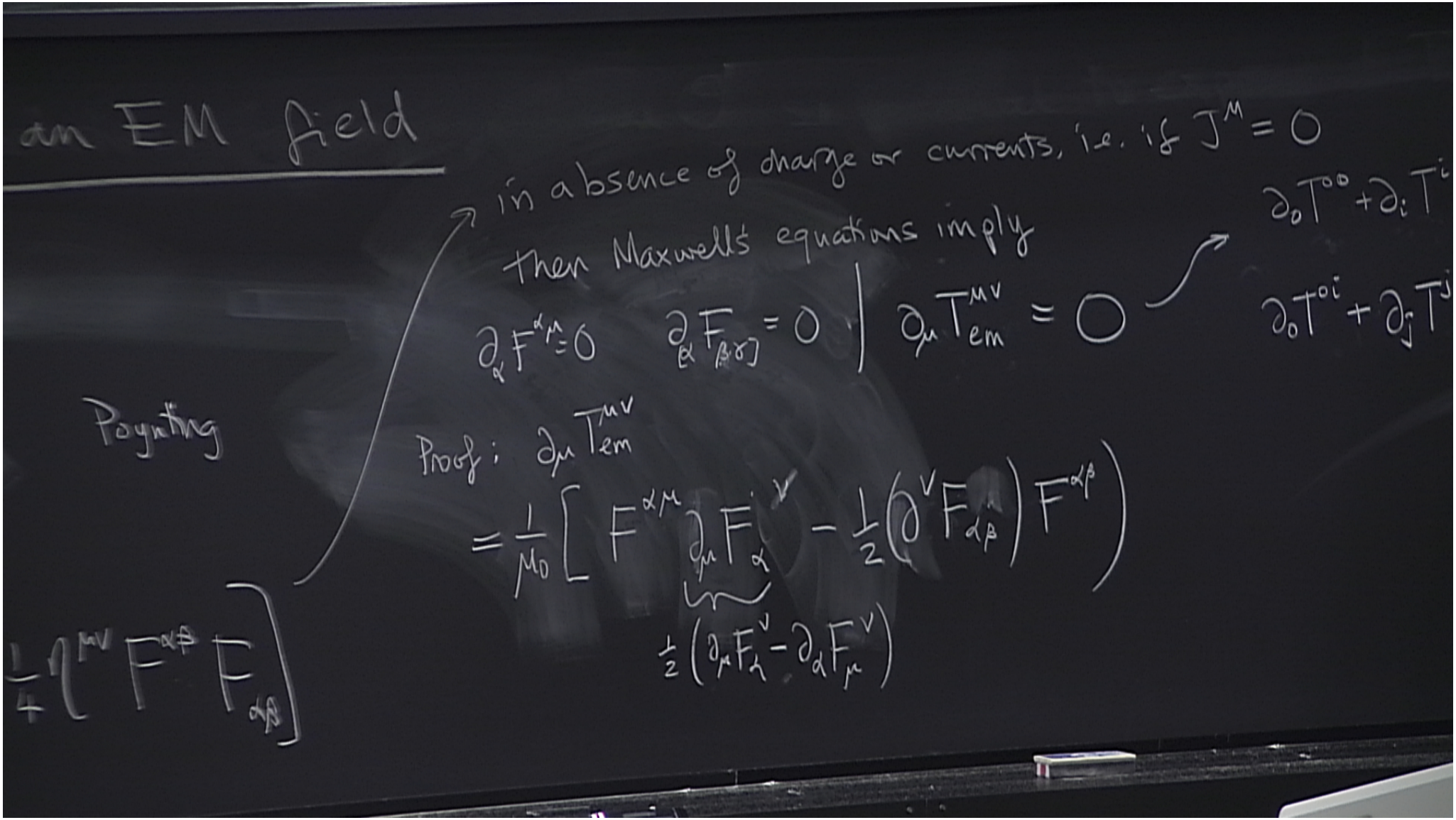
Momentum density $\underline{J}_{mom} = \frac{1}{\mu_0 c}(\underline{E} \wedge \underline{B})$ Poynting

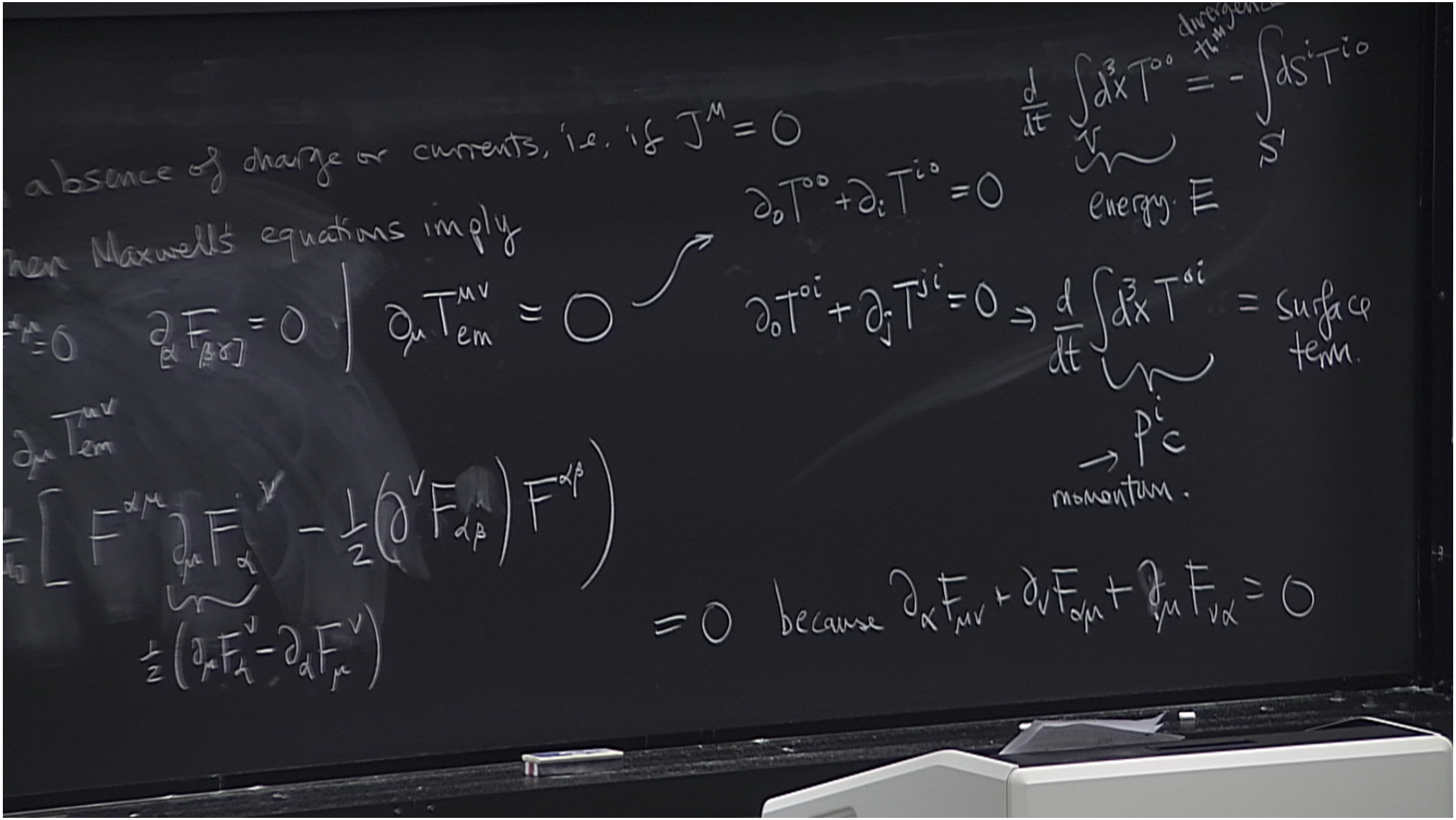
Combined into energy-momentum tensor

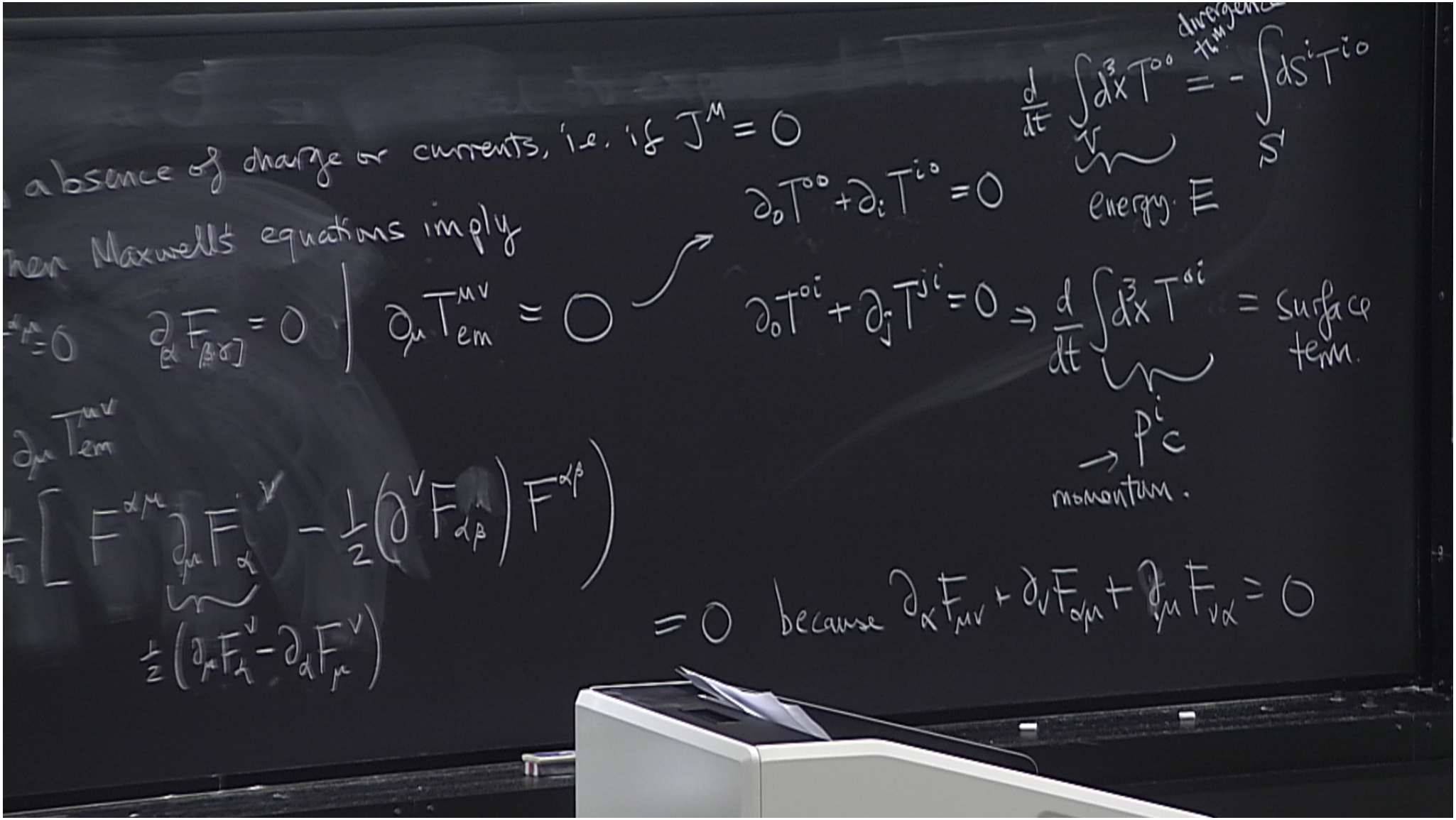
$$T^{\mu\nu}_{em} = \frac{1}{\mu_0}\left[F^{\alpha\mu}F^{\nu}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right]$$


EM field
in absence of charge or currents, i.e. if $J^\mu = 0$
then Maxwell's equations imply
$\partial_\mu T^{\mu\nu}_{em} = 0$
Poynting
$F^{\alpha\beta} F_{\alpha\beta}$


charge or currents, i.e. if $J^\mu = 0$
equations imply
$\partial_\mu T^{\mu\nu}_{em} = 0$
$\partial_0 T^{00} + \partial_i T^{i0} = 0$
$\frac{d}{dt}\int_V d^3x\, T^{00} = -\int_S dS^i T^{i0}$
divergence thm.
energy.


an EM field
in absence of charge or currents, i.e. if $J^\mu = 0$
Then Maxwell's equations imply
$\partial_\alpha F^{\alpha\mu} = 0 \quad \partial_{[\alpha} F_{\beta\gamma]} = 0$
$\partial_\mu T^{\mu\nu}_{em} = 0$
$\partial_0 T^{00} + \partial_i T^{i}$
$\partial_0 T^{0i} + \partial_j T^{j}$
Poynting
Proof: $\partial_\mu T^{\mu\nu}_{em}$
$= \frac{1}{\mu_0}\left[F^{\alpha\mu} \partial_\mu F_\alpha{}^\nu - \frac{1}{2}\left(\partial^\nu F_{\alpha\beta}\right) F^{\alpha\beta} \right)$
$\frac{1}{2}\left(\partial_\mu F_\alpha{}^\nu - \partial_\alpha F_\mu{}^\nu\right)$
$\frac{1}{4}\eta^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}\Big]$


absence of charge or currents, i.e. if $J^\mu = 0$
Then Maxwell's equations imply
$\partial_{[\alpha} F_{\beta\gamma]} = 0$
$\partial_\mu T^{\mu\nu}_{em} = 0$
$\partial_0 T^{00} + \partial_i T^{i0} = 0$
$\partial_0 T^{0i} + \partial_j T^{ji} = 0 \Rightarrow \frac{d}{dt}\int d^3x\, T^{0i} = \text{surface term.}$
$\frac{d}{dt}\int_V d^3x\, T^{00} \overset{\text{divergence } T^{\mu\nu}}{=} -\int_S dS^i T^{i0}$
energy. E
P^i_c
momentum.
$\partial_\mu T^{\mu\nu}_{em}$
$= 0$ because $\partial_\alpha F_{\mu\nu} + \partial_\nu F_{\alpha\mu} + \partial_\mu F_{\nu\alpha} = 0$


absence of charge or currents, i.e. if $J^\mu = 0$
Then Maxwell's equations imply
$\partial_{[\alpha} F_{\beta\gamma]} = 0$
$\partial_\mu T^{\mu\nu}_{em} = 0$
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$\partial_0 T^{0i} + \partial_j T^{ji} = 0 \Rightarrow \frac{d}{dt}\int d^3x\, T^{0i} = \text{surface term}$
$\frac{d}{dt}\int_V d^3x\, T^{00} = -\int_S dS^i T^{i0}$
divergence theorem
energy E
P^i_{EM}
momentum.
$\partial_\mu T^{\mu\nu}_{em}$
$= 0$ because $\partial_\alpha F_{\mu\nu} + \partial_\nu F_{\alpha\mu} + \partial_\mu F_{\nu\alpha} = 0$

note: momentum density $= \frac{\text{energy flux}}{c}$

e.g. pulse of light



$\underline{E} = (0, 0, f(x-ct))$

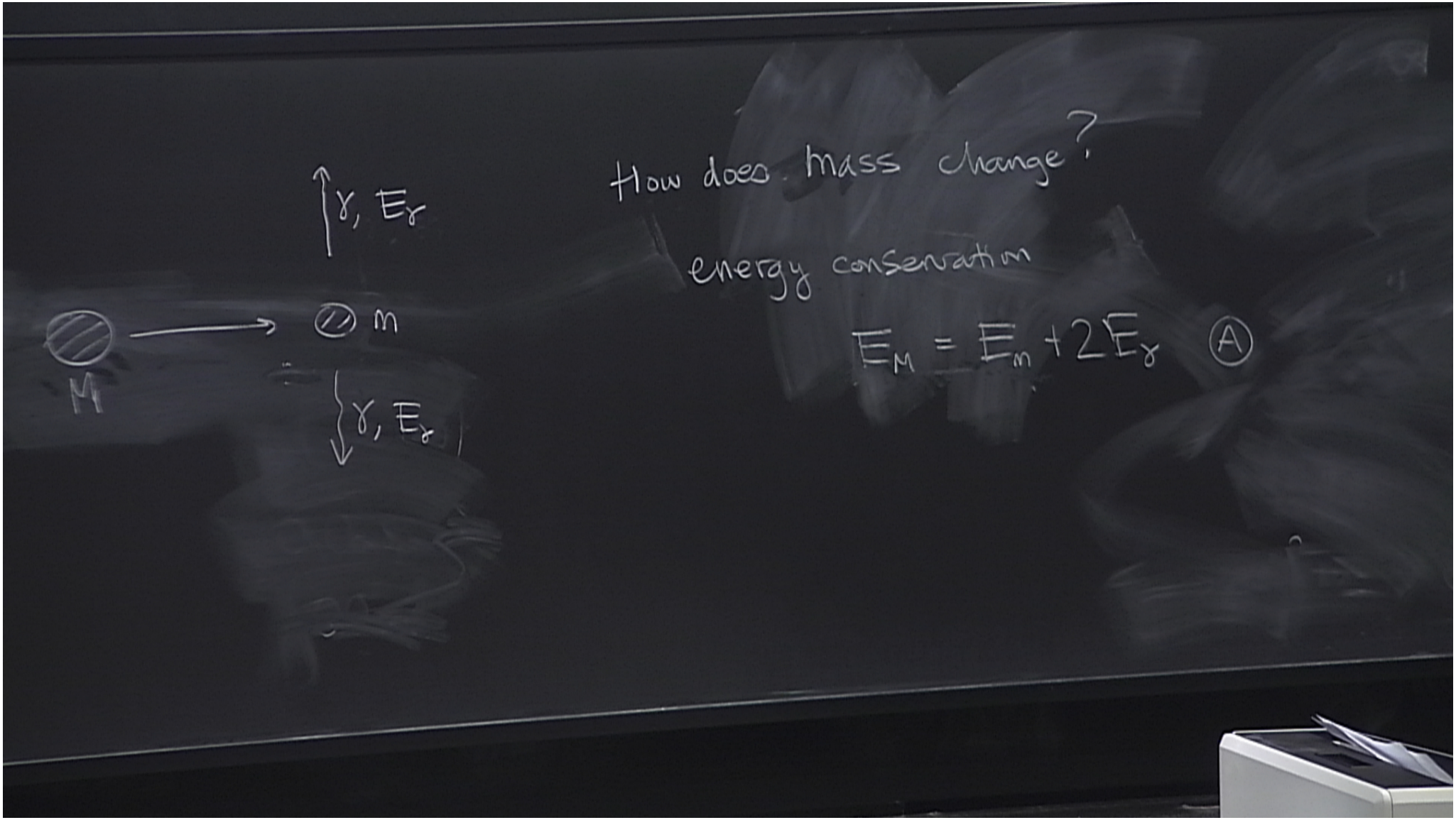
$\underline{B} = (0, -\frac{f(x-ct)}{c}, 0)$

$\int dy\,dz$ (↓)

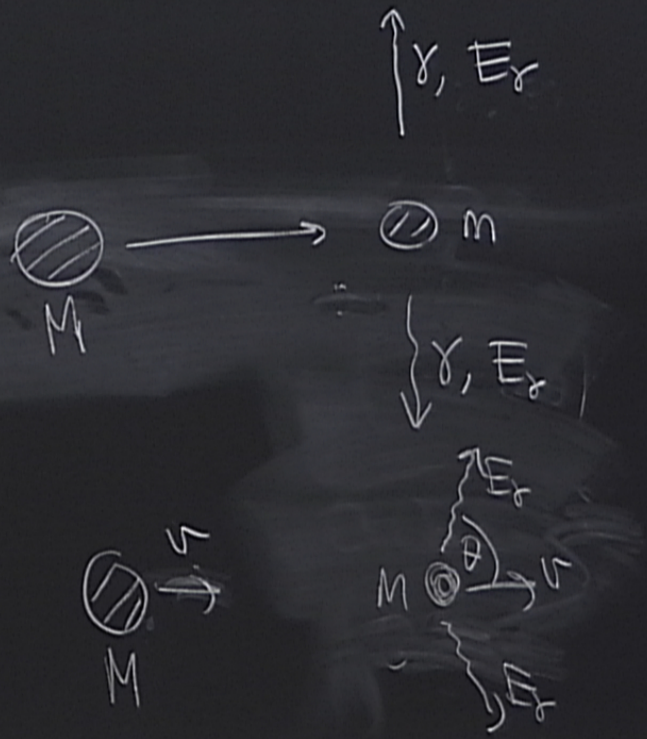
$E = A \int dx\, \epsilon_0 f^2(x-ct)$

$P_z c = A \int dx\, \frac{f^2(x-ct)}{M_0 c^2}$ (↑ z)

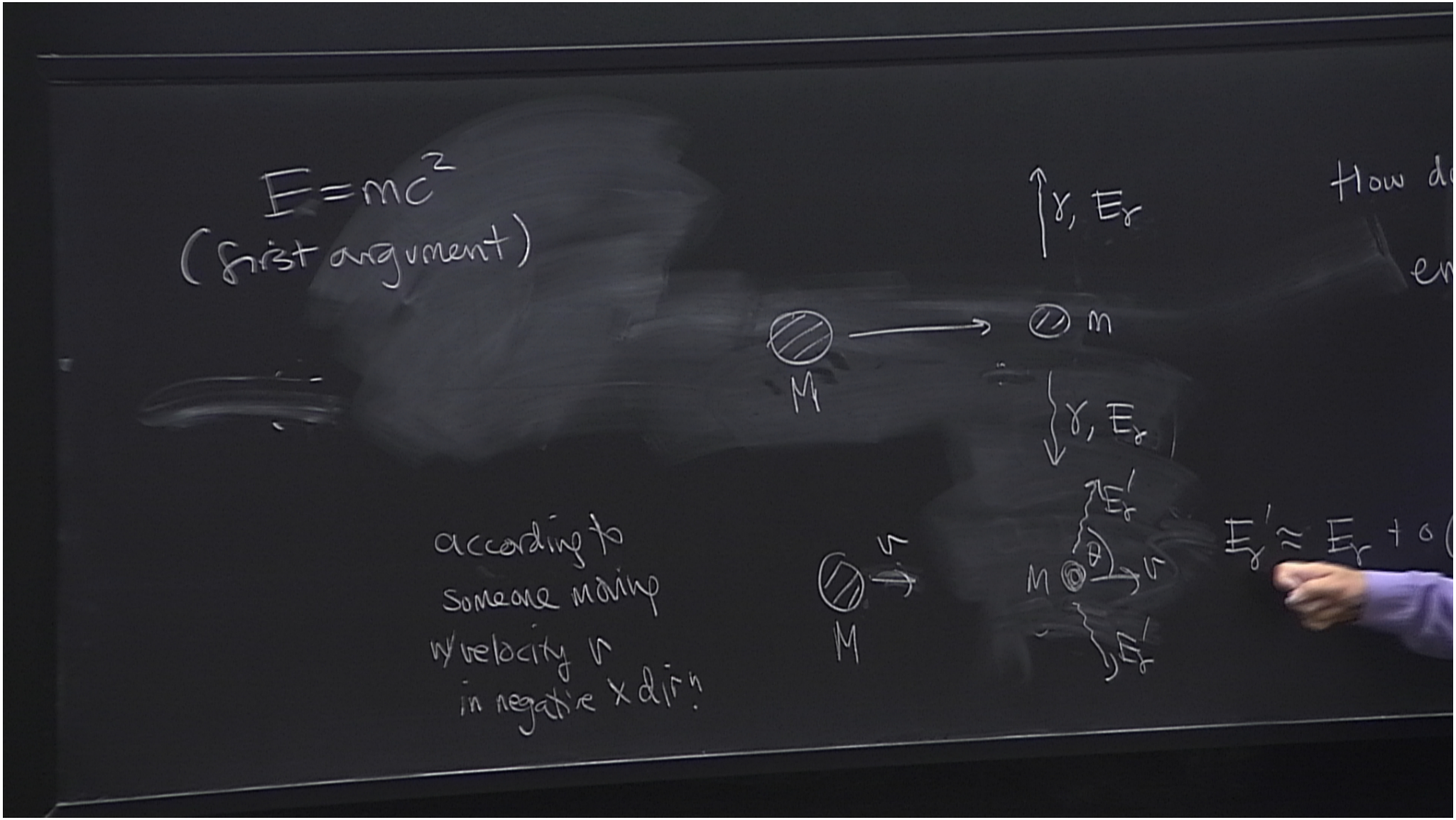
$\Rightarrow$ for a pulse of light, $|\underline{P}|c = E$

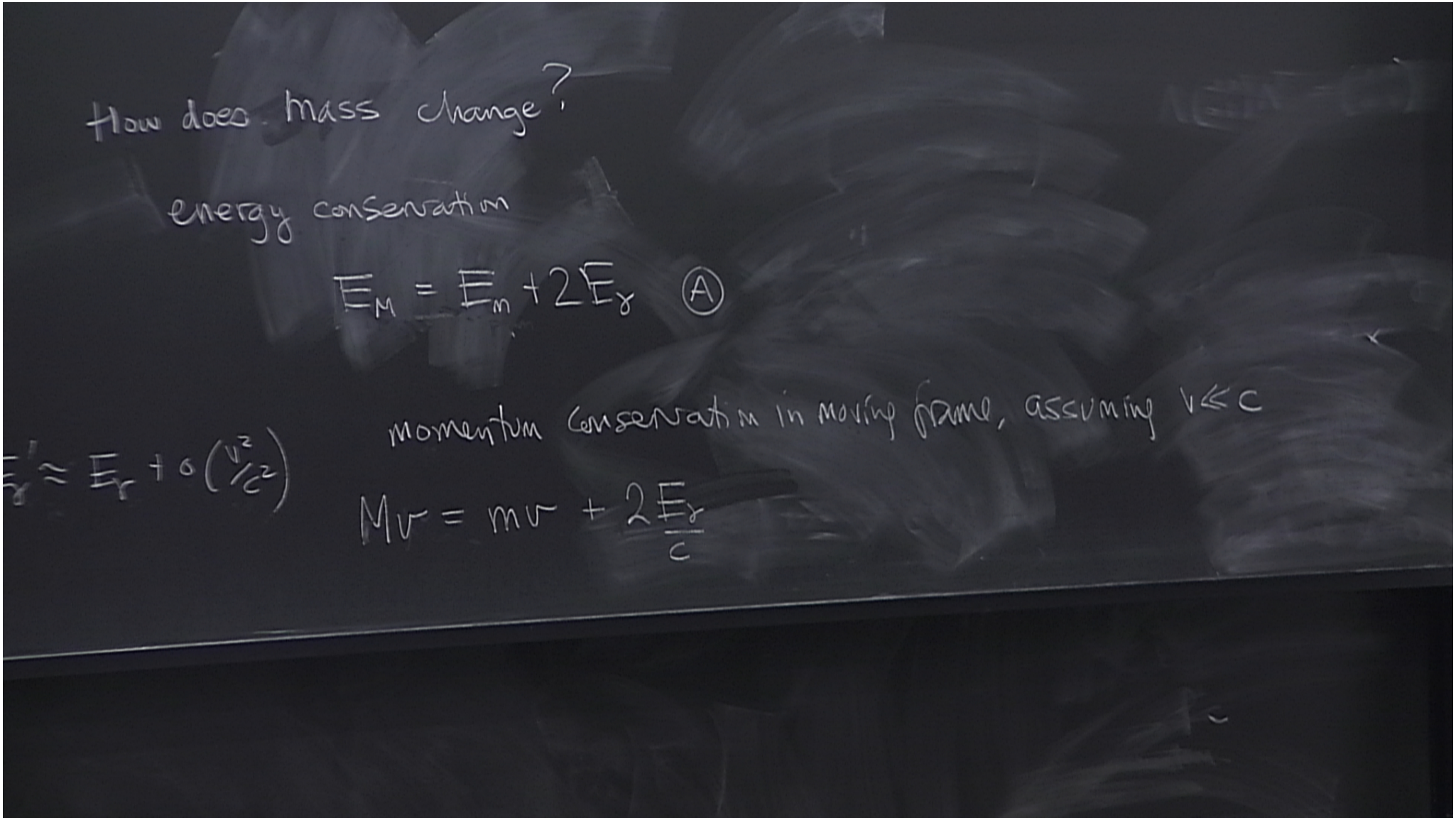

How does mass change?
energy conservation
$E_M = E_m + 2E_\gamma$ (A)
M
m
γ, E_γ
γ, E_γ

$E = mc^2$
(first argument)



according to
someone moving
w/ velocity v
in negative x dir^n


$E=mc^2$
(first argument)
γ, E_γ
How d
en
M
m
γ, E_γ
according to
someone moving
w/ velocity v
in negative x dir'n
M
v
E_γ'
θ
M
v
E_γ'
$E_\gamma' \approx E_\gamma$ + o


How does mass change?
energy conservation
$E_M = E_n + 2E_\gamma$ (A)
momentum conservation in moving frame, assuming $v \ll c$
$E_\gamma' \approx E_\gamma + o(v^2/c^2)$
$Mv = mv + \frac{2E_\gamma}{c}$

s change?

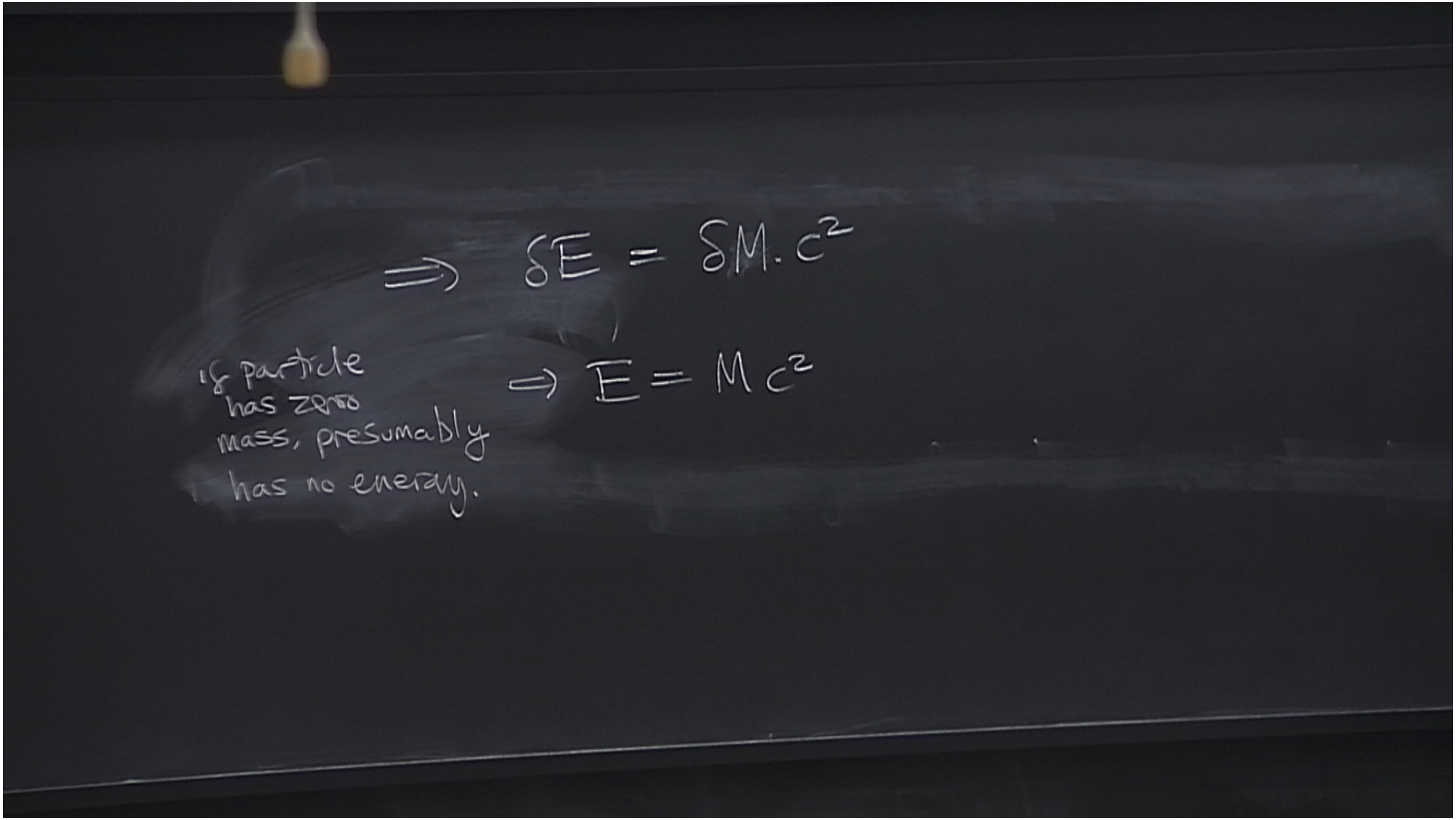
servation

$E_M = E_m + 2E_\gamma$ (A)

momentum conservation in moving frame, assuming $v \ll c$

$Mv = mv + \frac{2E_\gamma}{c}\cos\theta$ (v/c)

(B) $\Rightarrow \frac{2E_\gamma}{c^2} = (M-m) \overset{(A)}{=} \frac{(E_M - E_m)}{c^2}$


$\Rightarrow \delta E = \delta M . c^2$
If particle has zero mass, presumably has no energy.
$\Rightarrow E = M c^2$