

Title: PSI 2015/2016 Relativity - Neil Turok - Lecture 1

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Abstract:

Special Relativity

1860's Maxwell
laws of E&M

Gauss

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

charge density

$$\nabla \cdot \underline{B} = 0$$

Faraday

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Ampere

$$\nabla \wedge \underline{B} = \mu_0 \underline{J}$$

electric current
density

Special Relativity

60's Maxwell
laws of E&M

Gauss

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$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

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electric current
density

$$+ \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

Maxwell's displacement current

$\frac{\partial}{\partial t}$ Maxwell's displacement current

Franklin $\frac{\partial \rho}{\partial t} = -\underline{\nabla} \cdot \underline{J}$

electric
charge $Q = \int d^3x \rho(t, \underline{x})$
 $\frac{dQ}{dt} = \int d^3x \frac{\partial \rho}{\partial t} =$

Maxwell's displacement current

electric
charge

$$Q = \int d^3x \rho(t, \underline{x})$$

$$\frac{dQ}{dt} = \int_V d^3x \frac{\partial \rho}{\partial t} =$$

$$- \int_V d^3x \nabla \cdot \underline{J}$$

divergence
th. m.

$$= - \int_{S'} \underline{J} \cdot d\underline{S}$$



Franklin $\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}$

local charge conservation

Maxwell discovered that $\ast$ was needed for consistency.

Franklin $\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}$ (5)

local charge conservation

Maxwell discovered that * was needed for consistency

Easy to see that (2) and (3) are consistent:

$\nabla \cdot (3)$

lhs = $\nabla \cdot (\nabla \times \underline{B})$

rhs = $-\frac{\partial}{\partial t} \nabla \cdot \underline{B}$

$\nabla \cdot (4)$: lhs = 0 rhs = $\mu_0 \nabla \cdot \underline{J} = -\mu_0 \frac{\partial \rho}{\partial t}$ } cancel

Maxwell's term $+ \mu_0 \epsilon_0 \frac{\partial}{\partial t} \nabla \cdot \underline{E} \stackrel{(1)}{=} + \mu_0 \frac{\partial \rho}{\partial t}$

Amazing consequence: ①-④ have wave solutions even when $\rho, \mathbf{J}$ are zero.

$$\frac{\partial^2 \underline{\mathbf{E}}}{\partial t^2} \stackrel{(4)}{=} \frac{1}{\mu_0 \epsilon_0} \underline{\nabla} \wedge \frac{\partial \underline{\mathbf{B}}}{\partial t} \stackrel{(3)}{=} -\frac{1}{\mu_0 \epsilon_0} \underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{\mathbf{E}}) \stackrel{\underline{\nabla} \cdot \underline{\mathbf{E}} = 0}{=} +\frac{1}{\mu_0 \epsilon_0} \underline{\nabla}^2 \underline{\mathbf{E}}$$

$$\Rightarrow \underline{\mathbf{E}} \text{ obeys } \underbrace{\left(\frac{\partial^2}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \underline{\nabla}^2 \right)}_{\text{wave operator or d'Alembertian}} \underline{\mathbf{E}} = 0$$

$$(\text{also have } \underline{\nabla} \cdot \underline{\mathbf{E}} = \underline{\nabla} \cdot \underline{\mathbf{B}} = 0)$$

→ solutions
like

$$\underline{E} = (0, 0, f(x-ct))$$

c = speed of wave obeys $c^2 = \frac{1}{\mu_0 \epsilon_0}$

→ solutions
like

$$\nabla \cdot \underline{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\underline{E} = (0, 0, f(x-ct))$$

$$\underline{B} = (0, -\frac{f(x-ct)}{c}, 0)$$

c = speed of light

$$\underline{E} = (0, 0, f(x-ct))$$

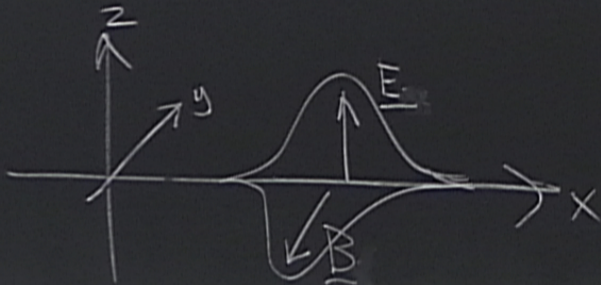
$$\underline{B} = (0, -\frac{f(x-ct)}{c}, 0)$$

$$\Rightarrow \nabla \cdot \underline{E} = (0, -\partial_x f, 0)$$

$$\frac{\partial B}{\partial t} = (0, +\partial_x f, 0)$$

c = speed of wave obeys $c^2 = \frac{1}{\epsilon_0 \mu_0}$

, likewise can check $\nabla \cdot \underline{B} = \frac{1}{c^2} \frac{\partial E}{\partial t}$



$$\frac{d\rho}{dt}$$

ns even when $\rho, \underline{I}$ are zero.

$$\underline{E} = 0 + \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$$

rtian

SI units	Coulombs
	Amps
	Tesla

$$\mu_0 \equiv 4\pi \times 10^{-7}$$

$$\epsilon_0 \approx \frac{1}{4\pi} \frac{1}{9 \times 10^9}$$

$$\left. \begin{matrix} \mu_0 \\ \epsilon_0 \end{matrix} \right\} \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \text{ m s}^{-1}$$

gradual realisation that electromagnetism has a set of incredible symmetries

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- Special relativity
- gauge symmetry
- conformal symmetry

gradual realisation that electromagnetism has a set of incredible symmetries

- Special relativity
- gauge symmetry
- conformal symmetry
- electromagnetic duality

gradual realisation that electromagnetism has a set of incredible symmetries

- Special relativity
- gauge symmetry
- conformal symmetry
- electromagnetic duality (if $\rho = \underline{J} = 0$)

Special Relativity :

$$x^\alpha = (ct, \underline{x}) = (ct, x_i)$$

$$\begin{aligned} \bar{i} &= 1, 2, 3 \\ \alpha &= 0, 1, 2, 3 \end{aligned}$$

Special Relativity :

$$x^\alpha = (ct, \underline{x}) = (ct, x_i)$$

$$\bar{i} = 1, 2, 3$$

$$\alpha = 0, 1, 2, 3$$

$$\partial_\alpha = \frac{\partial}{\partial x^\alpha} = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right) = \left(\frac{1}{c} \partial_t, \partial_i \right)$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{inverse } \eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(t, x_i)

$\bar{i} = 1, 2, 3$

$\alpha = 0, 1, 2, 3$

$$(\partial, \underline{\nabla}) = \left(\frac{1}{c} \partial_t, \partial_i \right)$$

inverse $\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Einstein summation convention:
repeated indices are summed.

$$\eta^{\alpha\beta} \partial_\alpha \partial_\beta = -\frac{1}{c^2} \partial_t^2 + \underline{\nabla}^2$$

$$\epsilon_{123} = 1$$

or $F_{0i} = -\frac{E_i}{c}$

$$F_{ij} = \epsilon_{ijk} B_k$$

$$F_{2A} = -F_{2B}$$

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Try:

$$F_{2B} =$$

$$\begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ +\frac{E_x}{c} & 0 & B_z & -B_y \\ +\frac{E_y}{c} & -B_z & 0 & B_x \\ +\frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

can use $\eta^{\alpha\beta}$ to raise indices

$$F^{\alpha\beta} = \eta^{\alpha\mu} \eta^{\beta\nu} F_{\mu\nu} = \begin{pmatrix} 0 & \\ & \end{pmatrix}$$

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can use $\eta^{\alpha\beta}$ to raise indices

$$F^{\alpha\beta} = \eta^{\alpha\mu} \eta^{\beta\nu} F_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{c} \vec{E} \\ (-) & 0 \end{pmatrix}$$

Defin 4-current $J^\alpha = (, \underline{J})$

can use $\eta^{\alpha\beta}$ to raise indices

$$F^{\alpha\beta} = \eta^{\alpha\mu} \eta^{\beta\nu} F_{\mu\nu} = \begin{pmatrix} 0 & E_x \\ & 0 \\ & & 0 \\ (-) & & & 0 \end{pmatrix}$$

Define 4-current $J^\alpha = (\rho c, \mathbf{J})$

then $\Leftrightarrow \partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
 ①, ④

CHECK : $\beta=0 \rightarrow \partial_i F^{i0} =$

$$\begin{pmatrix} +\frac{E_x}{c} & 0 & B_z & -B_y \\ +\frac{E_y}{c} & -B_z & 0 & B_x \end{pmatrix}$$

$$F_{ij} = \epsilon_{ijk} B_k$$

$$F^{\mu\nu} = \eta^{\mu\alpha} \eta^{\beta\nu} F_{\alpha\beta} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ 0 & 0 & B_z & -B_y \\ (-) & 0 & B_x & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

CHECK : $\beta=0 \rightarrow$ $\partial_i F^{i0} = -\frac{\nabla \cdot \underline{E}}{c}$ $\xrightarrow{\text{lhs}} -\mu_0 \rho c$ $\left. \begin{matrix} \text{lhs} \\ \text{rhs} \end{matrix} \right\} \nabla \cdot \underline{E} = \mu_0 \epsilon^2 \rho = \frac{1}{\epsilon_0} \rho \quad \checkmark$

$$\beta=i \quad \partial_0 F^{0i} + \partial_j F^{ji} = \partial_0 \frac{E_i}{c} + \partial_j \epsilon^{jik} B_k = \frac{1}{c^2} \partial_t E_i - (\nabla \wedge \underline{B})_i \stackrel{?}{=} -\mu_0 J^i \quad \checkmark$$

$\frac{1}{c^2} \xrightarrow{\mu_0 \epsilon_0} \frac{1}{\epsilon_0 \mu_0}$

can use $\eta^{\alpha\beta}$ to raise indices

$$F^{\alpha\beta} = \eta^{\alpha\mu} \eta^{\beta\nu} F_{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ 0 & 0 & B_z & -B_y \\ (-) & 0 & 0 & B_x \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Define 4-current $J^\alpha = (\rho c, \underline{J})$

then $\Leftrightarrow \boxed{\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta}$
 ①, ④

CHECK : $\beta=0 \rightarrow \partial_i F^{i0} = -\frac{\nabla \cdot \underline{E}}{c} \quad \text{lhs} \quad \text{rhs} \quad \left. \begin{array}{l} -\mu_0 \rho c \end{array} \right\}$
 $\beta=i \quad \partial_0 F^{0i} + \partial_j F^{ji} = \partial_0 \frac{E_i}{c} + \partial_j \epsilon^{jik} B_k$

Equations (2) and (3) can be written as

$$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$$

$$\left(\partial_{[\gamma} F_{\alpha\beta]} = 0 \right)$$

$$\epsilon^{0123} = +1$$

even perms are +1
odd " " -1

$$\begin{array}{ccc} 2 & 1 & 0 & 3 \\ & \times & & 1 \\ 0 & 1 & 2 & 3 \end{array}$$

$$\Leftrightarrow \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

$$F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$$

$$\partial_\epsilon F_{\alpha\beta} = 0$$

CHECK

these give (2), (3)

NEED TO USE

$$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\epsilon_{ijk}\epsilon_{jkm} = 2\delta_{im}$$

ins ② and ③ can be written as $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$
 $\Downarrow$
 $(\partial_{[\alpha} F_{\beta\gamma]} = 0)$

$$\Leftrightarrow \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

Consequence of this equation is that $F_{\alpha\beta}$ can be written as the gradient of a potential

$$A^\mu = (\phi, \underline{A})$$

CHECK
these give ②, ③

NEED TO USE

$$\epsilon_{ijk}\epsilon_{lmn} = \delta_{il}\delta_{jm}\delta_{kn} - \delta_{im}\delta_{jn}\delta_{kl} - \delta_{in}\delta_{jl}\delta_{km} + \delta_{in}\delta_{jm}\delta_{kl} + \delta_{il}\delta_{kn}\delta_{jm} - \delta_{im}\delta_{kn}\delta_{jl} - \delta_{in}\delta_{km}\delta_{jl} + \delta_{im}\delta_{jn}\delta_{kl}$$

$$\epsilon_{ijk}\epsilon_{jlm} = 2\delta_{il}\delta_{km} - \delta_{im}\delta_{kl}$$

Equations (2) and (3) can be written as $\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
 $\left(\partial_{[\alpha} F_{\beta\gamma]} = 0 \right)$

$$\epsilon^{0123} = +1$$

even perms are +1

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$$\begin{array}{ccc} 2 & 1 & 0 & 3 \\ & \times & & 1 \\ 0 & 1 & 2 & 3 \end{array}$$

electrostatics

$$\underline{E} = -\underline{\nabla}\phi$$

magnetostatics

$$\underline{B} = \underline{\nabla} \times \underline{A}$$

$$\Leftrightarrow \boxed{\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0}$$

Consequence of this equation is that $F_{\alpha\beta}$ can be written as the

$$A^\mu = \left(\phi, \underline{A} \right), \quad A_\mu = \left(-\frac{\phi}{c}, \underline{A} \right) : F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Try: $F_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ +\frac{E_x}{c} & 0 & B & -B \\ & & & \end{pmatrix}$ or $F_{0i} = -\frac{E_i}{c}$

and ③ can be written as $\partial_\alpha F_{\alpha\beta} + \partial_\beta F_{\beta\alpha} + \partial_\alpha F_{\beta\alpha} = 0$

$$\left(\partial_\alpha F_{\alpha\beta} \right) = 0$$

$$\Leftrightarrow \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

consequence of this equation is that $F_{\alpha\beta}$ can be written as the gradient of a potential

$$A^\mu = \left(\frac{\phi}{c}, \underline{A} \right), \quad A_\mu = \left(-\frac{\phi}{c}, \underline{A} \right) : F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Rightarrow \begin{aligned} \underline{B} &= \nabla \wedge \underline{A} \\ \underline{E} &= -\nabla \phi - \frac{\partial \underline{A}}{\partial t} \end{aligned}$$

CHECK
these give ②, ③

NEED TO USE

$$\epsilon_{ijk}\epsilon_{lmn} = \delta_{il}\delta_{jm}\delta_{kn} - \delta_{im}\delta_{jn}\delta_{kl} + \dots$$

$$\epsilon_{ijk}\epsilon_{jkm} = 2\delta_{im}$$

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

all of
electromagnetism:

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$$

tensor equations, therefore

$$m\ddot{\mathbf{x}}_i = -\nabla_i V$$

$$x_i \rightarrow x'_i = O_{ij} x_j$$

ations, therefore invariant under certain coordinate transformations on

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2.

↙ constant matrix

$$X'^{\alpha} = \Lambda^{\alpha}_{\gamma} X^{\gamma}$$

ations, therefore invariant under certain coordinate transformations on

2. $\swarrow$ constant matrix

$$x'^{\alpha} = \Lambda^{\alpha}_{\gamma} x^{\gamma}$$

$$\frac{\partial}{\partial x^{\alpha}} = \Lambda^{\beta}_{\alpha} \frac{\partial}{\partial x'^{\beta}}$$

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} = \Lambda^{\alpha}_{\gamma} \underbrace{\frac{\partial x^{\gamma}}{\partial x^{\beta}}}_{\delta^{\gamma}_{\beta}}$$

$$J^{\alpha} = \Lambda^{\alpha}_{\gamma} J^{\gamma}$$

$$F'^{\alpha\beta} = \Lambda^{\alpha}_{\gamma} \Lambda^{\beta}_{\delta} F^{\gamma\delta}$$

etc.

ditions, therefore invariant under certain coordinate transformations on

constant matrix

$$x'^{\alpha} = \Lambda^{\alpha}_{\gamma} x^{\gamma}$$

$$J'^{\alpha} = \Lambda^{\alpha}_{\gamma} J^{\gamma}$$

$$F'^{\alpha\beta} = \Lambda^{\alpha}_{\gamma} \Lambda^{\beta}_{\delta} F^{\gamma\delta}$$

etc.

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} = \Lambda^{\alpha}_{\gamma} \underbrace{\frac{\partial x^{\gamma}}{\partial x^{\beta}}}_{\delta^{\gamma}_{\beta}} = \Lambda^{\alpha}_{\beta}$$

$$\frac{\partial}{\partial x^{\alpha}} = \Lambda^{\beta}_{\alpha} \frac{\partial}{\partial x^{\beta}}$$

$$\Rightarrow \frac{\partial}{\partial x'^{\alpha}} = \underbrace{\tilde{\Lambda}^{\beta}_{\alpha}}_{\text{inverse of } \Lambda} \frac{\partial}{\partial x^{\beta}}, \text{ where}$$

$$\Lambda^{\alpha}_{\beta} \tilde{\Lambda}^{\beta}_{\gamma} = \delta^{\alpha}_{\gamma}$$

$$\text{Sim! } \Lambda^{\beta}_{\alpha} \tilde{\Lambda}^{\alpha}_{\beta} = \delta^{\beta}_{\beta}$$

...ions, therefore invariant under certain coordinate transformations on

constant matrix

$$x'^{\alpha} = \Lambda^{\alpha}_{\gamma} x^{\gamma}$$

$$J'^{\alpha} = \Lambda^{\alpha}_{\gamma} J^{\gamma}$$

$$F'^{\alpha\beta} = \Lambda^{\alpha}_{\gamma} \Lambda^{\beta}_{\delta} F^{\gamma\delta}$$

etc.

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} = \Lambda^{\alpha}_{\gamma} \underbrace{\frac{\partial x^{\gamma}}{\partial x^{\beta}}}_{\delta^{\gamma}_{\beta}} = \Lambda^{\alpha}_{\beta}$$

$$\frac{\partial}{\partial x^{\alpha}} = \Lambda^{\beta}_{\alpha} \frac{\partial}{\partial x^{\beta}} \Rightarrow \frac{\partial}{\partial x'^{\alpha}} = \underbrace{\tilde{\Lambda}^{\beta}_{\alpha}}_{\text{inverse of } \Lambda} \frac{\partial}{\partial x^{\beta}}, \text{ where } \Lambda^{\alpha}_{\beta} \tilde{\Lambda}^{\beta}_{\gamma} = \delta^{\alpha}_{\gamma}$$

Sim! $\Lambda^{\beta}_{\alpha} \tilde{\Lambda}^{\alpha}_{\beta} = \delta^{\beta}_{\beta}$

Rule: upstairs indices transform with Λ
 downstairs " " " $\tilde{\Lambda}$

Quantities with indices contracted eg. $A^i B_i = \underbrace{\Lambda^i_r \tilde{\Lambda}^s_r}_{\delta^s_r} A^r B_s$
 $= A^r B_r.$

Quantities with indices contracted eg. $A'^{\alpha} B'_{\alpha} = \underbrace{\Lambda^{\alpha}_r \tilde{\Lambda}^{\beta}_s}_{\delta^s_r} A^r B_s$
 $= A^r B_r.$

In order for M.F.'s to be invariant, must impose that η is invariant, i.e. $\eta'^{\alpha\beta} = \Lambda^{\alpha}_r \Lambda^{\beta}_s \eta^{rs}$

downstairs " " "

$$= \underbrace{\Lambda^r_\gamma \tilde{\Lambda}^\delta_\alpha}_{\delta^\gamma_\alpha} A^\gamma B_\delta$$

$$= A^r B_r.$$

η is invariant, i.e. $\eta'^{\alpha\beta} = \Lambda^\alpha_\gamma \Lambda^\beta_\delta \eta^{\gamma\delta} = \eta^{\alpha\beta}$
to the Lorentz transformations.

(η is an invariant tensor)

$\Rightarrow$ reduces set of Λ 's to the Lore

$$\Lambda \eta \Lambda^T = \eta \Rightarrow (\det \Lambda)^2 = 1 \Rightarrow \det \Lambda = \pm 1$$

$$\epsilon'^{\alpha\beta\gamma\delta} = \Lambda^\alpha_\mu \Lambda^\beta_\nu \Lambda^\gamma_\rho \Lambda^\delta_\sigma \epsilon^{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma}$$

set of Λ s to the Lorentz transformations.

$$= \pm 1$$

$$\epsilon_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \det \Lambda \Rightarrow \epsilon \text{ is invariant up to a sign.}$$

set of Λ s to the Lorentz transformations.

$$= \pm 1$$

$$\epsilon^{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \det \Lambda \Rightarrow \epsilon \text{ is invariant up to a sign.}$$

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$$\epsilon^{\mu\nu\rho\sigma} = \epsilon^{\alpha\beta\gamma\delta} \det \Lambda \Rightarrow \epsilon \text{ is invariant up to a sign.}$$

magnetostatics $A = \left(\frac{\Phi}{c}, \underline{A} \right)$, $A_\mu = \left(\frac{1}{c}, \underline{A} \right)$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\underline{B} = \nabla \wedge \underline{A}$$

Examples of Lorentz transformations

rotation
about
z axis by θ

$$\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} x' &= x \cos\theta + y \sin\theta \\ y' &= y \cos\theta - x \sin\theta \end{aligned}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Examples of Lorentz transformations

rotation
about
z axis by θ

$$\Lambda_{\beta}^{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} x' &= x \cos\theta + y \sin\theta \\ y' &= y \cos\theta - x \sin\theta \end{aligned}$$

$$X'^{\alpha} = (ct, x, y, z) = \Lambda_{\beta}^{\alpha} X^{\beta}$$

boost, involves
time

$$\Lambda_{\beta}^{\alpha} = \begin{pmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c & -s \\ -s & c \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c & -s \\ -s & c \end{pmatrix} = \begin{pmatrix} -c^2 + s^2 & 0 \\ 0 & c^2 - s^2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left. \begin{aligned} ct' &= ct \cosh\theta - x \sinh\theta \\ x' &= x \cosh\theta - ct \sinh\theta \end{aligned} \right\} \text{Lorentz boost}$$

of Λ to the Lorentz transformations. (invariant tensor)

$$\pm 1$$

$$x'^{\alpha} = x^{\alpha}(x^{\beta})$$

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}}$$

what is the most gen. c-trans s.t.

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial x'^{\gamma}}{\partial x^{\delta}} \eta^{\beta\delta} = \eta^{\alpha\gamma}$$

$$\epsilon^{\mu\nu\rho\sigma} = \epsilon^{\alpha\beta\gamma\delta} \det \Lambda \Rightarrow \epsilon \text{ is invariant} \Rightarrow x'^{\alpha} = \underbrace{\Lambda^{\alpha}_{\gamma} x^{\gamma}}_{\text{constants}} + a^{\alpha}$$

of Λ to the Lorentz transformations. (invariant tensor)

$$\pm 1$$

$$x'^{\alpha} = \Lambda^{\alpha}_{\beta} x^{\beta}$$

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}}$$

what is the most gen. c. trans s.t.

$$\frac{\partial x'^{\alpha}}{\partial x^{\beta}} \frac{\partial x'^{\gamma}}{\partial x^{\delta}} \eta^{\beta\delta} = \eta^{\alpha\gamma}$$

$$\epsilon^{\mu\nu\rho\sigma} = \epsilon^{\alpha\beta\gamma\delta} \det \Lambda \Rightarrow \epsilon \text{ is invariant} \Rightarrow x'^{\alpha} = \underbrace{\Lambda^{\alpha}_{\beta}}_6 x^{\beta} + \underbrace{a^{\alpha}}_4$$

constants.

Λ, a parametrise
the Poincaré group