Title: PolyHEgon scattering and strings

Date: Sep 28, 2015 11:00 AM

URL: http://pirsa.org/15090025

Abstract: <p>String theory is starting to provide novel all-loop precision tools for the computation of scattering amplitudes in the high energy (HE) limit of N=4 SYM theory. After a review of some key insights and results for hexagon amplitudes, I will describe ongoing developments addressing higher numbers of external gluons.</p>

GATIS
Gauge Theory as an Integrable System

DESY

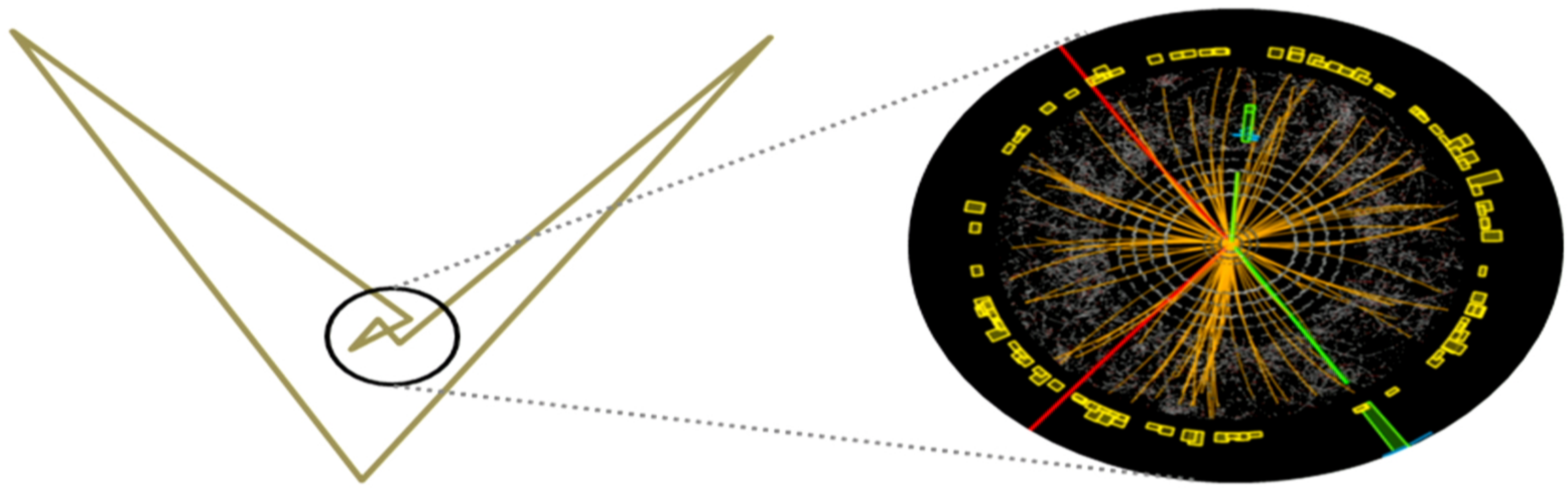
PolyHEgon Scattering and Strings

PI, Sep 28, 2015

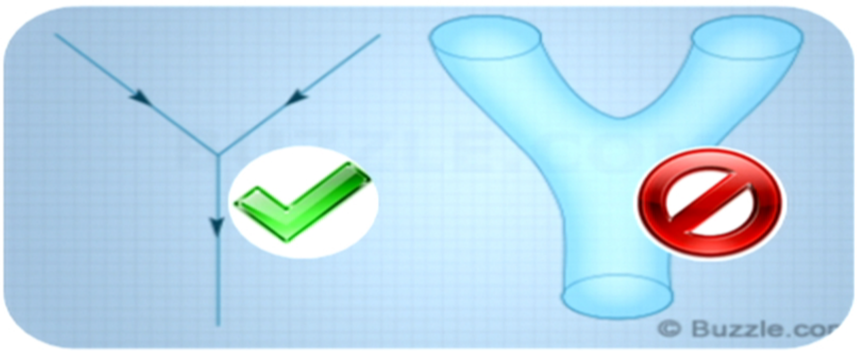
Volker Schomerus

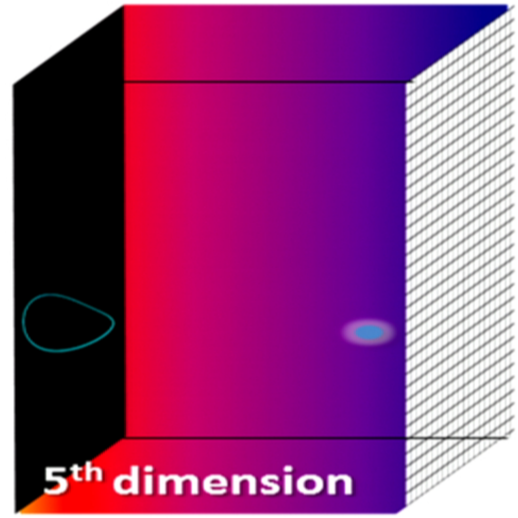
Based on work with J. Bartels, J. Kotanski, M. Sprenger; T. Bargheer, G. Papathanasiou *[in progress]*

Introduction: PolyHEgons



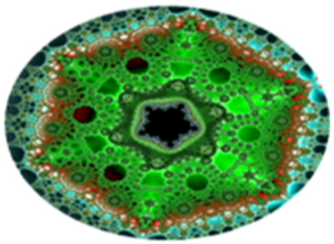
Introduction: Strings & HE Scattering





Strings provide precision results for HE scattering

Introduction: Plan



The hexagon – a review

HE scattering from weak to strong coupling

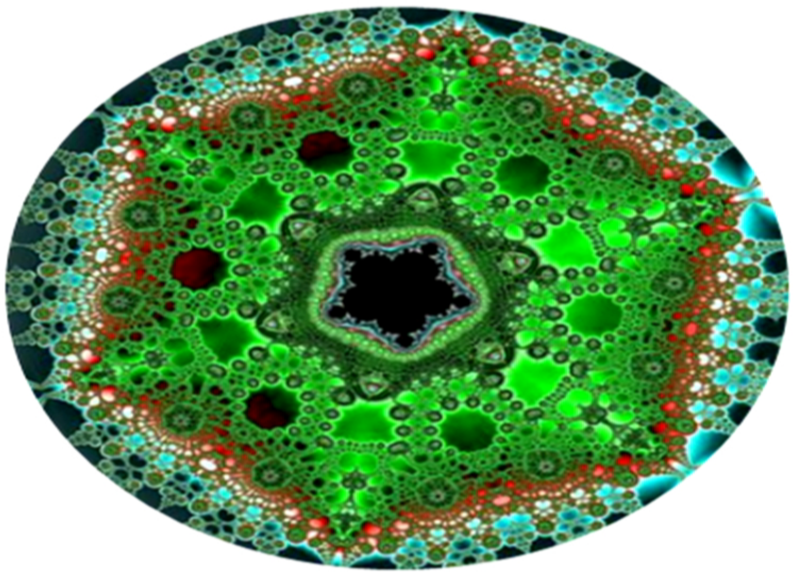
Multi-Regge kinematics

Multi-Regge limit & Mandelstam regions

Beyond the hexagon

HE scattering at weak and strong coupling

The Hexagon

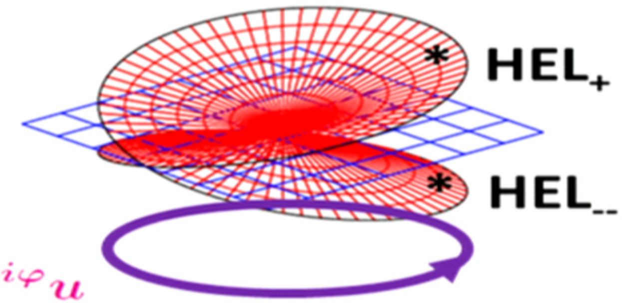


HE-limit of 2-loop Remainder

n=6 gluons: $u_j \longrightarrow u, w, w^*$

HEL: $|s| \gg -t \iff u \longrightarrow 1$

$$\frac{1}{2\pi i} R_6^{(2)\,\mathrm{HEL}_+} \sim 1$$



$$u(\varphi) = e^{i\varphi} u$$

From [Goncharov et al] formula for 6-gluon at 2-loops:

[Lipatov,Prygarin]

$$\frac{1}{2\pi i} R_6^{(2)\,\mathrm{HEL}_-} \sim \frac{a^2}{4} \ln(1-u) \ln|1+w|^2 \ln\left|1+\frac{1}{w}\right|^2 +$$

$$a = 2g^2 \sim \lambda \qquad + \frac{a^2}{4} \ln|w|^2 \ln^2|1+w|^2 - \frac{a^2}{6} \ln^3|1+w|^2$$

$$Li_2\left(1-\frac{1}{u'}\right) = -2\pi i \log\left(1-\frac{1}{u}\right) + Li_2\left(1-\frac{1}{u}\right) \sim_{\mathrm{HEL}} -2\pi i \log(1-u) + 2\pi^2$$

Remainder Function in HE-limit

$$\left(e^{R_6+i\delta}\right)^{\text{HEL}_-} \sim \sum_{k=-\infty}^{\infty} (-1)^k \left(\frac{w}{w^*}\right)^{\frac{k}{2}} \int \frac{d\nu}{\nu^2+\frac{k^2}{4}} |w|^{2i\nu} \Phi(\nu,k) \left((u-1)\frac{|w|}{|1+w|^2}\right)^{-\omega(\nu,k)}$$

where to LLA: $a \sim \lambda$ [Bartels,Lipatov,Sabio Vera]

$$\omega(\nu,k) = 2a\psi(1) - a\psi\left(1+i\nu+\frac{|k|}{2}\right) - a\psi\left(1-i\nu+\frac{|k|}{2}\right) + \frac{a}{2}\frac{|k|}{\nu^2+\frac{k^2}{4}} + O(a^2)$$

Eigenvalue of SL(2) Heisenberg magnet 

→ Explicit formulas for R in (N)LLA to high loops

using SVHP [Dixon,Duhr,Pennington]

to $\text{N}^{2/3}$LLA using symbology

Interpolation

BFKL eigenvalue ω(ν,k) and impact factor φ (ν,k) are now known to all-log order [Basso,Caron-Huot,Sever]

$$\omega(u,k) = -\int_0^\infty \frac{dt}{t}\left(\frac{Q(-t)+Q(t)}{2}\cos(ut)e^{-|k|t/2} - Q(t)\right)$$

$$\nu(u,k) = u + \int_0^\infty \frac{dt}{t}\frac{Q(-t)-Q(t)}{4}\sin(ut)e^{-|k|t/2}$$

Q = solution of BES eq. = charge density of GKP string

→ ω(ν,k) degenerates strong coupling: ω(ν,k) = ω^{∞}

Remainder Function in HE-limit

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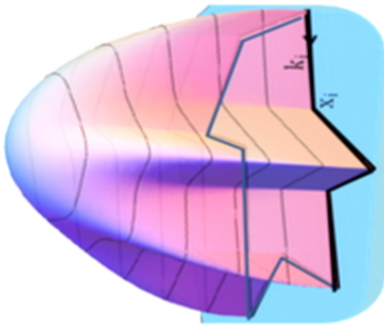

Strong Coupling

R = energy of 1D multiparticle QS

[Alday,Gaiotto,Maldacena,Sever,Vieira]

$$R_n^{\infty}(u) \sim \mathcal{E}(m,\mu)$$ ← solution of TBA

- **Mandelstam cuts in GT ↔ excitations in 1D QS**

[Bartels,Kotanski,VS]

- **HE limit in GT ↔ large mass/IR limit in 1D QS**

$$(e^{R_6^{\infty}+i\delta})_{-}^{\mathrm{HEL}} \sim \left((u-1)\frac{|w|}{|1+w|^2}\right)^{-\omega^{\infty}}$$

$$\omega^{\infty} = \frac{\sqrt{\lambda}}{2\pi}\left(\sqrt{2}-\log(1+\sqrt{2})\right)$$

[Bartels, Sprenger, VS]

solution of Bethe Ansatz eqs

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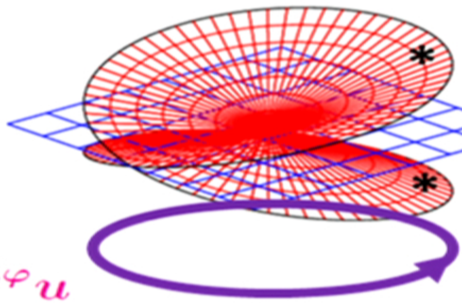

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leading log

[Lipatov,Prygarin]

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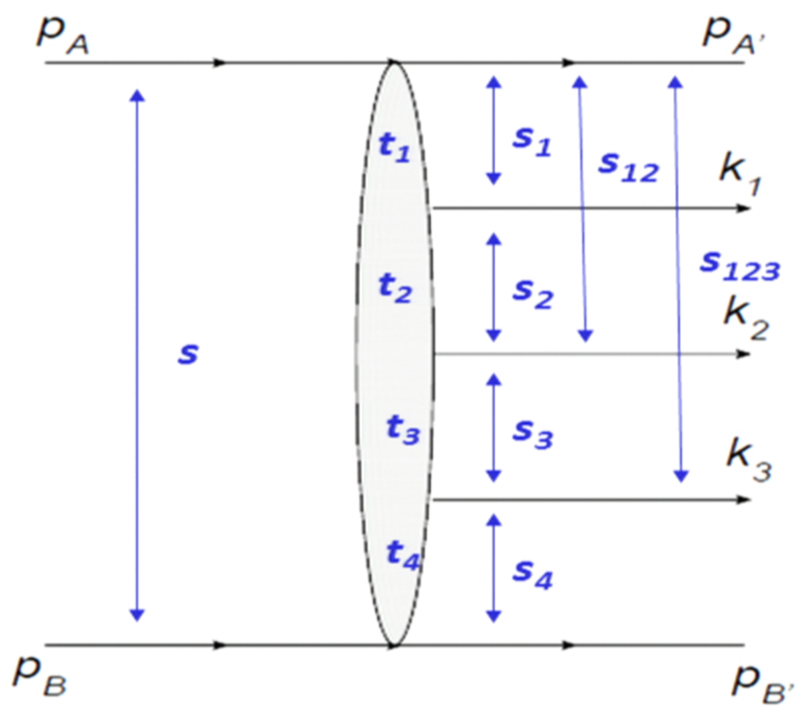
Kinematics

Kinematical Invariants



½ (n^2 -3n)

Mandelstam

invariants

2 → n – 2 = 5 ***production amplitude***

Kinematical Invariants

$$p_i = x_i - x_{i+1}$$

$$x_{ij}^2 = (x_i - x_j)^2$$

$$\begin{bmatrix} 0 & 0 & t_1 & t_2 & t_3 & t_4 & 0 \\ 0 & 0 & 0 & s_1 & s_{12} & s_{123} & s \\ t_1 & 0 & 0 & 0 & s_2 & s_{23} & s_{234} \\ t_2 & s_1 & 0 & 0 & 0 & s_3 & s_{34} \\ t_3 & s_{12} & s_2 & 0 & 0 & 0 & s_4 \\ & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 \end{bmatrix}$$

Kinematics: Cross Ratios

½ (n² -5n) basic cross ratios (tiles)

from Gram det

$$\begin{bmatrix} 0 & 0 & x^2_{13} & x^2_{14} & x^2_{15} & x^2_{16} & 0 \\ 0 & 0 & 0 & x^2_{24} & x^2_{25} & x^2_{26} & x^2_{27} \\ x^2_{13} & 0 & 0 & 0 & x^2_{35} & x^2_{36} & x^2_{37} \\ x^2_{14} & x^2_{24} & 0 & 0 & 0 & x^2_{46} & x^2_{47} \\ x^2_{15} & x^2_{25} & x^2_{35} & 0 & 0 & 0 & x^2_{57} \\ & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 \end{bmatrix}$$

u_{31} u_{32} u_{11} u u_{12} u_{21} u_{22}

3(n-5) fundamental cross ratios

$$\begin{matrix} x^2_{i,j} & x^2_{i,j+1} \\ x^2_{i+1,j} & x^2_{i+1,j+1} \end{matrix} \rightarrow \frac{x^2_{i,j+1} x^2_{i+1,j}}{x^2_{i,j} x^2_{i+1,j+1}} =: U_{ij}$$

Kinematics: Multi-Regge Limit

$-t_i << |s_j|$

$$\begin{bmatrix} 0 & 0 & x_{13}^2 & x_{14}^2 & x_{15}^2 & x_{16}^2 & 0 \\ 0 & 0 & 0 & x_{24}^2 & x_{25}^2 & x_{26}^2 & x_{27}^2 \\ x_{13}^2 & 0 & 0 & 0 & x_{35}^2 & x_{36}^2 & x_{37}^2 \\ x_{14}^2 & x_{24}^2 & 0 & 0 & 0 & x_{46}^2 & x_{47}^2 \\ x_{15}^2 & x_{25}^2 & x_{35}^2 & 0 & 0 & 0 & x_{57}^2 \\ & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 \end{bmatrix}$$

small

large

larger

$x_{ij}^2 \sim s_{i-1} \cdots s_{j-3}$

$$\frac{u_{2\sigma}}{1-u_{1\sigma}} = \frac{1}{|1+w_\sigma|^2} \qquad \frac{u_{3\sigma}}{1-u_{1\sigma}} = \frac{|w_\sigma|^2}{|1+w_\sigma|^2}$$

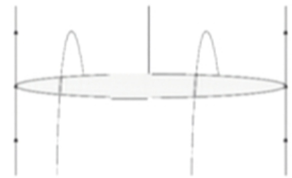
Multi-Regge Regions

2^{n-4} regions depending on the sign of $p_{i0} = E_i$

$$\varrho = (\mathrm{sgn}(E_i))$$

e.g.

$\varrho = (+ + +)$



$\varrho = (- + -)$

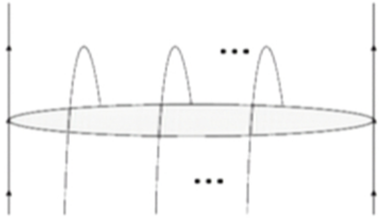
$u_{2\sigma} > 0$	$u_{3\sigma} > 0$	$u_{2\sigma} < 0$	$u_{3\sigma} < 0$
$s_1 > 0$	$s_4 > 0$	$s_1 < 0$	$s_4 < 0$
$s_{12} > 0$	$s_{34} > 0$	$s_{12} > 0$	$s_{34} > 0$
$s_{123} > 0$	$s_{234} > 0$	$s_{123} > 0$	$s_{234} < 0$

Continuation between Regions

To reach Multi-Regge region ϱ continue tile cross ratios U_{ij} along path with winding no

$$n_{ij} = \frac{1}{4}(\varrho_{i+1} - \varrho_{i+2})(\varrho_j - \varrho_{j+1})$$

e.g.



Only $U_{2,n-1}$ has non-vanishing winding number $n_{2,n-1} = -1$

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$u_{2\sigma} > 0$	$u_{3\sigma} > 0$	$u_{2\sigma} < 0$	$u_{3\sigma} < 0$
$s_1 > 0$	$s_4 > 0$	$s_1 < 0$	$s_4 < 0$
$s_{12} > 0$	$s_{34} > 0$	$s_{12} > 0$	$s_{34} > 0$
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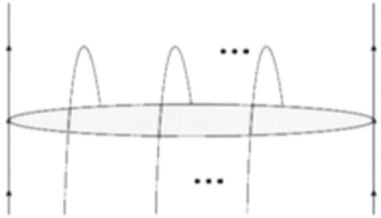


Only $U_{2,n-1}$ has non-vanishing winding number $n_{2,n-1} = -1$

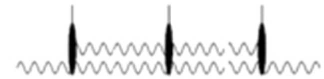
Beyond the Hexagon

Cut contributions in MRL - form

$|p_\sigma - p_{\sigma+1}| \le 1 \quad , \quad p_\sigma \ge 1$

Cut contributions are labeled by $P = (p_1, p_2, \ldots, p_m)$

$m = n - 5$



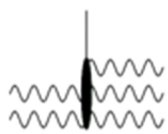
BFKL eigenvalue

$$\mu(\nu, n; w) = (-1)^n \left(\frac{w}{\bar{w}}\right)^{\frac{n}{2}} |w|^{2i\nu} \qquad F_p(n, \nu; u, w) = \omega_p(\nu, n) \left(\log(1-u) + \frac{1}{2}\log\frac{|w|^2}{|1+w|^4}\right)$$

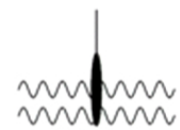
$$C_P(u_\sigma, w_\sigma) = i\frac{a}{2}\sum_{n_\sigma}\int\frac{d^m\nu_\sigma}{(2\pi)^m}\prod_{\sigma=1}^{m}\mu(\nu_\sigma, n_\sigma; w_\sigma)e^{-F_{p_\sigma}(\nu_\sigma, n_\sigma; u_\sigma, w_\sigma)}\times$$

$$\times\Phi_{1p_1}(\nu_1, n_1)\left(\prod_{\sigma=1}^{m-1}\Phi_{p_\sigma p_{\sigma+1}}(\nu_\sigma, n_\sigma|\nu_{\sigma+1}, n_{\sigma+1})\right)\Phi_{p_m 1}(\nu_m, n_m)$$

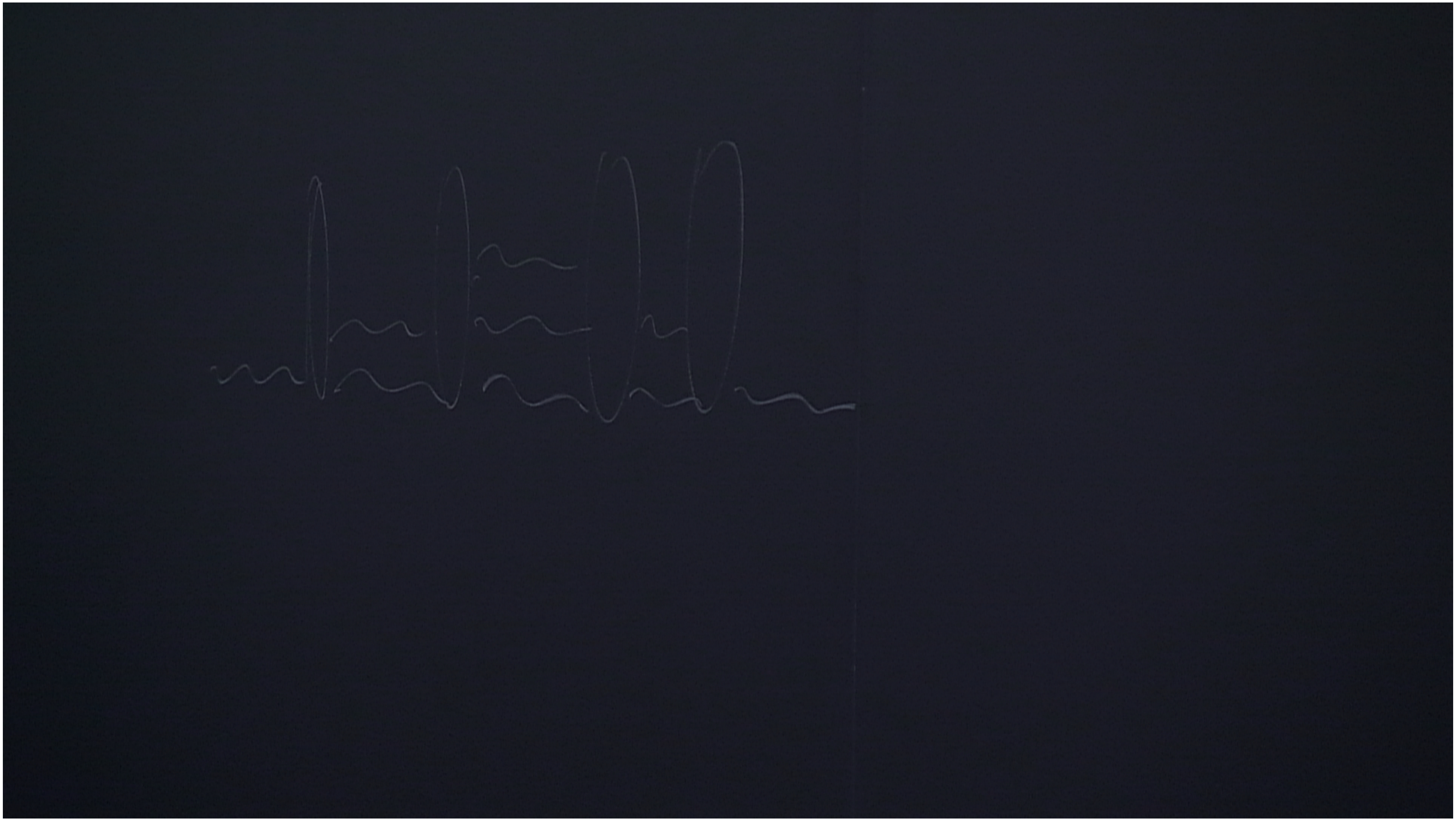
Impact factors **production vertices**



$\Phi_{2,3}$



$\Phi_{2,2}$



Results on BFKL Eigenvalue ω_p

LLA $\omega_p(\nu,n)$ ~ lowest EV of BFKL Hamiltonian on open SL(2) spin chain of length p in SL(2) rep (ν,n)

$$\omega_p(\nu, k) \sim \sum_{i=1}^{p-1} \mathrm{Re}\left(\psi(1+\frac{|k_i|}{2}+i\nu_i)-\psi(1)\right)$$

[Lipatov]
[Derkachov]

Hamiltonian known in NLLA [Bartels,Lipatov,Fadin,Vacca]

Recall: ω_2 known to all-log orders

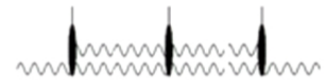
[Fadin,Lipatov],[Dixon et al.],[Basso,Caron-Huot,Sever]

Cut contributions in MRL - form

$|p_\sigma - p_{\sigma+1}| \le 1 \quad , \quad p_\sigma \ge 1$

Cut contributions are labeled by $P = (p_1, p_2, \ldots, p_m)$

$m = n - 5$



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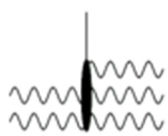
$$C_P(u_\sigma, w_\sigma) \;=\; i\frac{a}{2}\sum_{n_\sigma}\int \frac{d^m\nu_\sigma}{(2\pi)^m}\prod_{\sigma=1}^{m}\mu(\nu_\sigma, n_\sigma; w_\sigma)e^{-F_{p_\sigma}(\nu_\sigma, n_\sigma; u_\sigma, w_\sigma)}\times$$

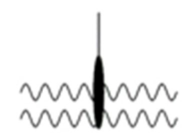
$$\times\Phi_{1p_1}(\nu_1, n_1)\left(\prod_{\sigma=1}^{m-1}\Phi_{p_\sigma p_{\sigma+1}}(\nu_\sigma, n_\sigma|\nu_{\sigma+1}, n_{\sigma+1})\right)\Phi_{p_m 1}(\nu_m, n_m)$$

Impact factors

production vertices



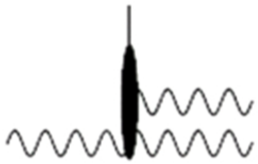
$\Phi_{2,3}$



$\Phi_{2,2}$

Results on Building Blocks ϕ

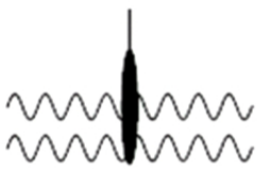
Only simplest impact factor known to all-log order



[Bartels,Lipatov,Sabio-Vera], N^3:[Dixon et al.]

[Basso,Caron-Huot,Sever]

Only simplest production vertex is known to LLA



[Bartels,Kormilitzin,Lipatov, Prygarin]

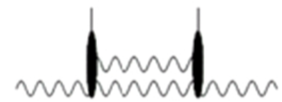
Sufficient to construct MRL of R in all regions, n = 7 to LLA

Building the Remainder

Cut (and pole) contributions used to build Remainder

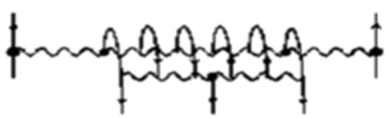
$$(e^{R_6+i\delta})^{\mathrm{HEL}-} \sim \sum_{k=-\infty}^{\infty} (-1)^k \left(\frac{w}{w^*}\right)^{\frac{k}{2}} \int \frac{d\nu}{\nu^2+\frac{k^2}{4}} |w|^{2i\nu}\Phi(\nu,k)\left((u-1)\frac{|w|}{|1+w|^2}\right)^{-\omega(\nu,k)}$$

$\varrho = (--)$



$P = (2)$

Similarly for n=7, for example:



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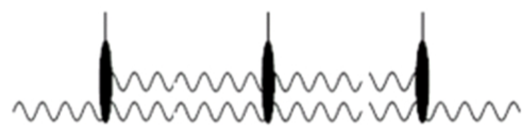


$P = (2,2)$

[Bartels,Kormilitzin, Lipatov, Prygarin]

In general several cuts *P* may contribute to one region ρ

Can be constructed case by case from locality [Bartels..]

Input from Symbology

Symbol $S[R_n^{(2)}]$ of the 2-loop remainder is known to NLLA for any number n of gluons [Caron-Huot]

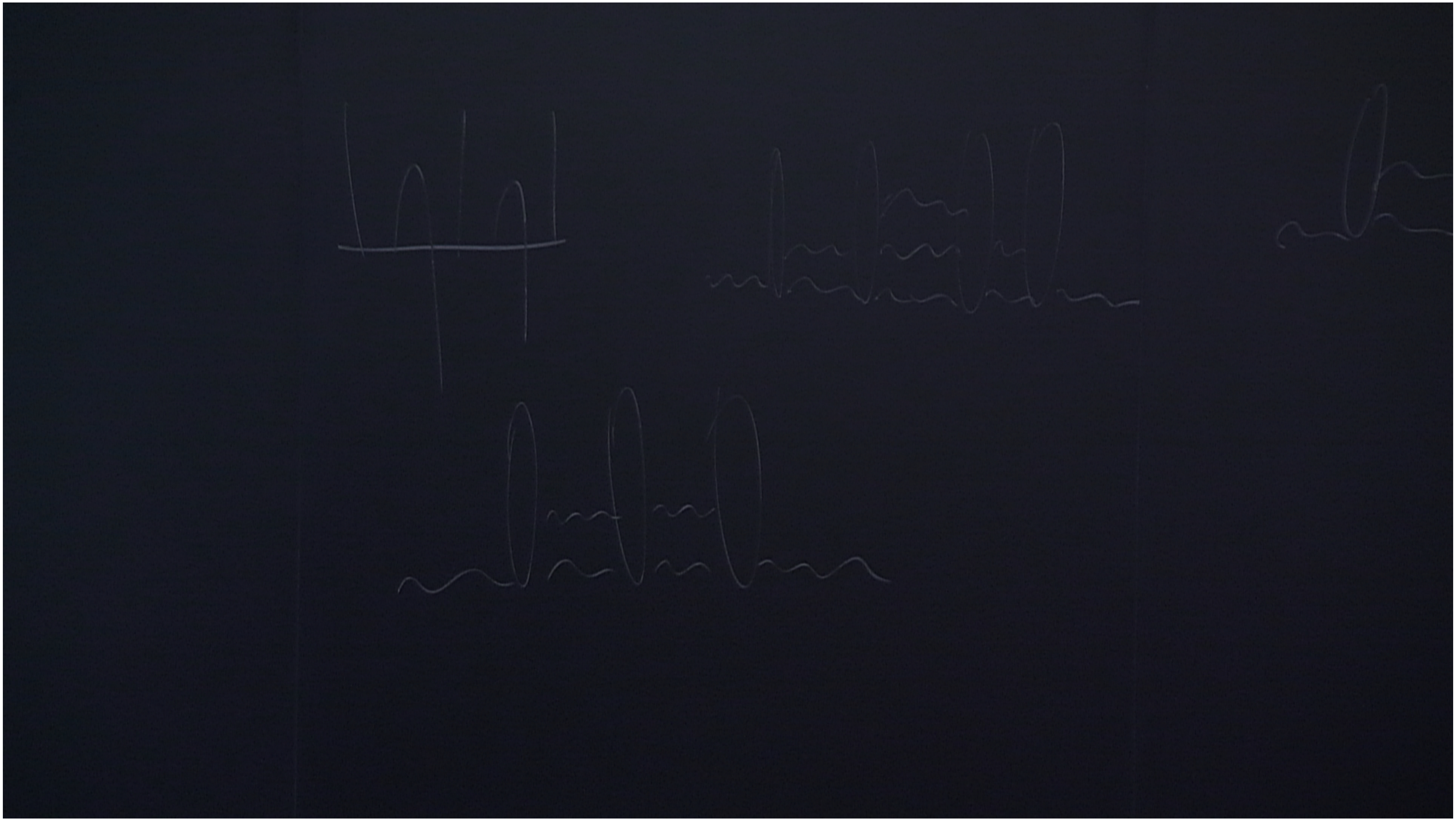
Symbol $S[R_{n,\varrho}^{(2)\mathrm{MRL}}]$ of cut contributions to 2-loop remainder can be computed for all regions ρ

[Prygarin et al.] .... [Bargheer,Papathanasiou,VS]

→ linear relations, for example

$$S[R_{7,(-+-)}^{(2)\mathrm{MRL}}] = S[R_{7,(---)}^{(2)\mathrm{MRL}}] - S[R_{7,(--+)}^{(2)\mathrm{MRL}}] - S[R_{7,(+--)}^{(2)\mathrm{MRL}}]$$

↔ P=(2,2) ↔ P=(2,1) ↔ P=(1,2)

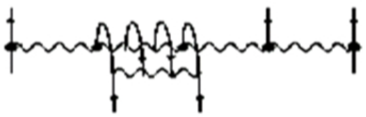


Strong Coupling

Now computed for three of four non-trivial regions

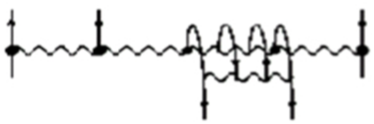
expressed through $$\mathcal{R}^\infty(u,w) = \omega^\infty\left(\log(1-u) + \frac{1}{2}\log\frac{|w|^2}{|1+w|^4}\right)$$

same appears in n=6



$\varrho_1 = (- - +)$

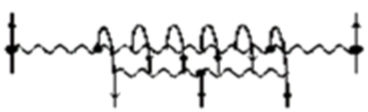
$$R^{\infty\,\mathrm{MRL}}_{7,\varrho_1} + i\pi\delta_{7,\varrho_1} = \mathcal{R}^\infty(u_{a1})$$



$\varrho_2 = (+ - -)$

$$R^{\infty\,\mathrm{MRL}}_{7,\varrho_2} + i\pi\delta_{7,\varrho_2} = \mathcal{R}^\infty(u_{a2})$$

$$R^{\infty\,\mathrm{MRL}}_{7,\varrho_3} + i\pi\delta_{7,\varrho_3} = \mathcal{R}^\infty(u_{a1}) + \mathcal{R}^\infty(u_{a2})$$



$\varrho_3 = (- - -)$

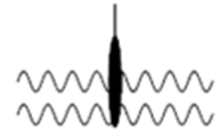
Consistent with factorization in N=4 SYM!

[Bartels,VS,Sprenger]

Conclusion and Outlook

What is strong coupling prediction for $R_{7,\,(-+-)}^{\infty\,\mathrm{MRL}}$?

To complete n=7 determine the vertex



Symbol → NLLA

At n=8 new `BFKL eigenvalue' $\omega_3\,(\nu,k)$ appears

↔ ω_2 in LLA; Symbol→ NLLA; compute ω_3^{∞}

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Full & explicit solution of high energy scattering

in planar N=4 SYM appears as realistic goal