

Title: Understanding primordial physics in a finite universe

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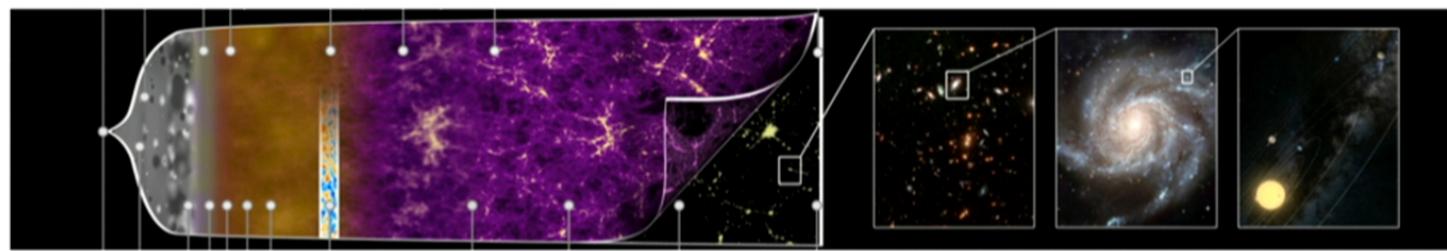
Abstract: <p>We hope to find clues about the particle physics of the primordial (inflationary?) universe in the statistics of the cosmological perturbations. However, if the fluctuations are non-Gaussian the statistics we observe can differ significantly from the global, mean predictions of an inflationary model. I will discuss how the conclusions we draw about the primordial degrees of freedom can be affected in interesting ways by the finiteness of our observable universe.</p>

Understanding primordial physics in a finite universe

Sarah Shandera
Penn State

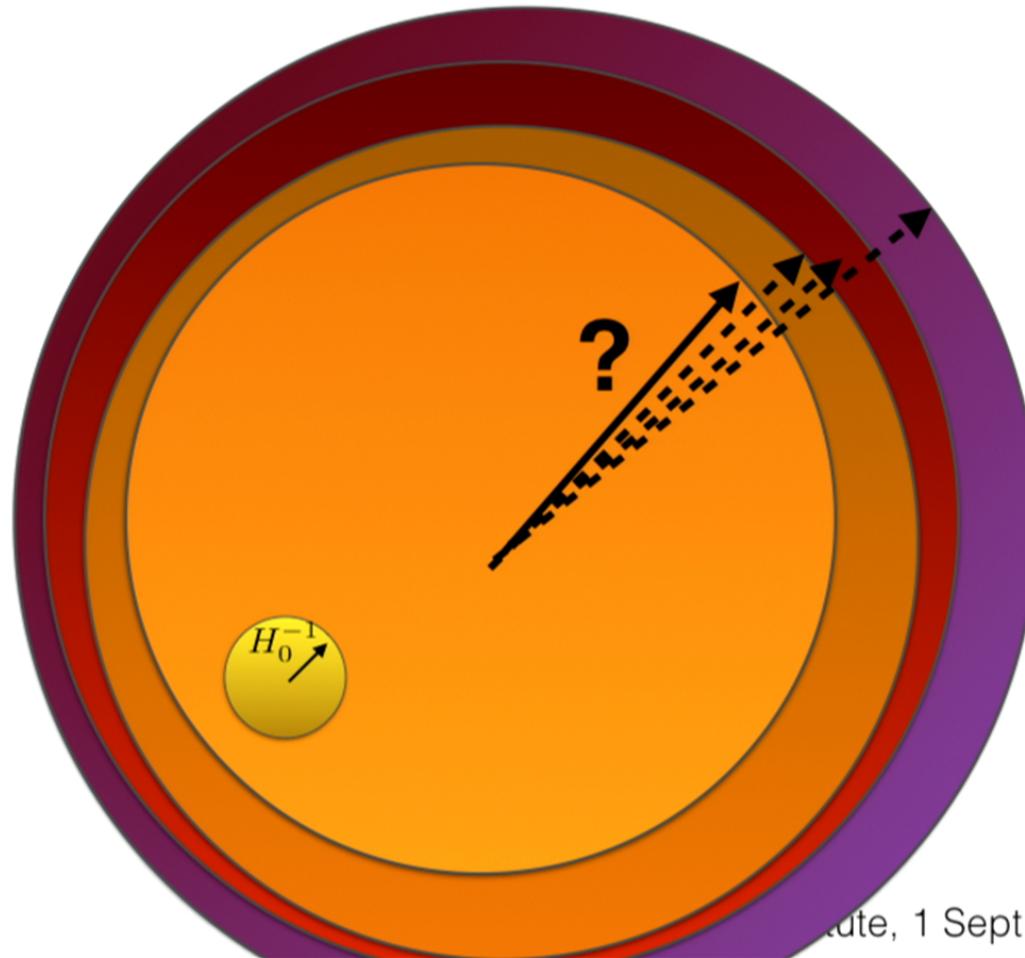
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What we are absolutely sure about:



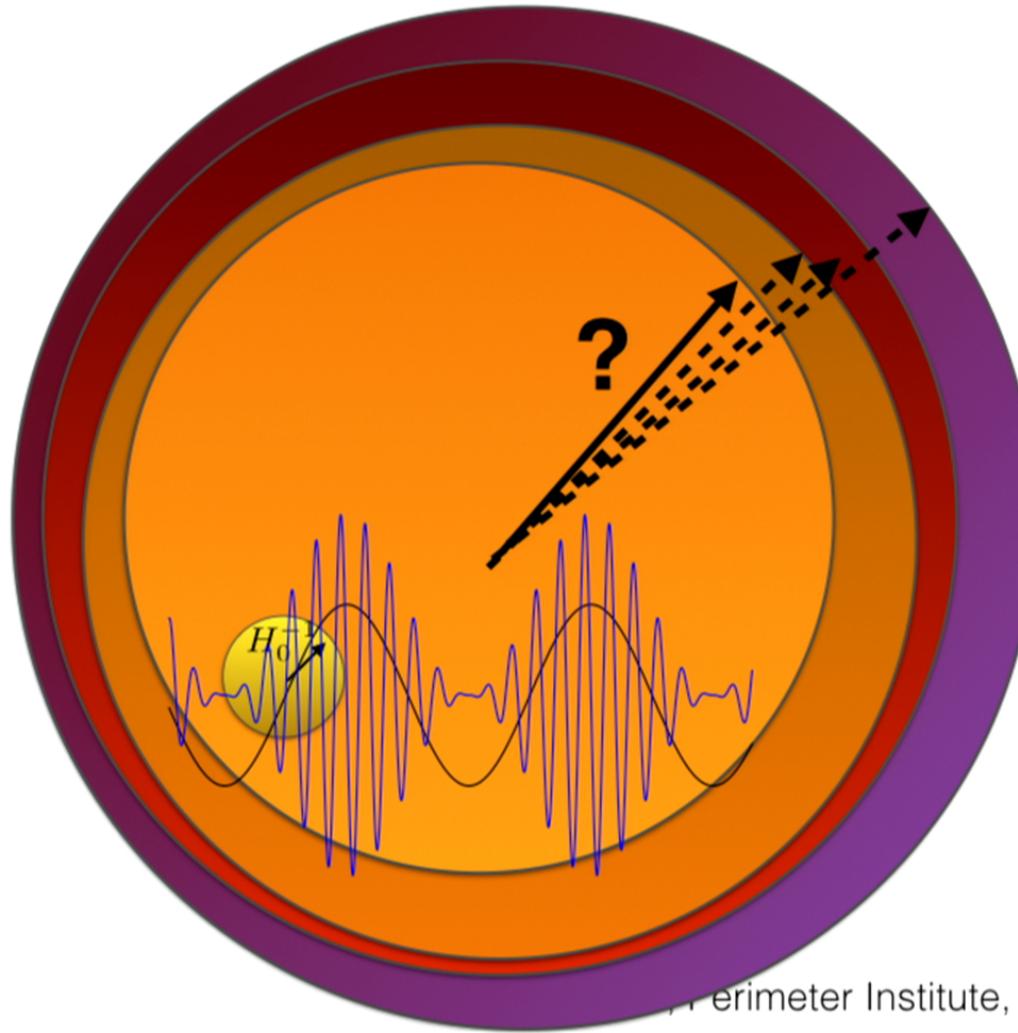
Planck satellite team image

Assuming our present Hubble volume is not of a special size:



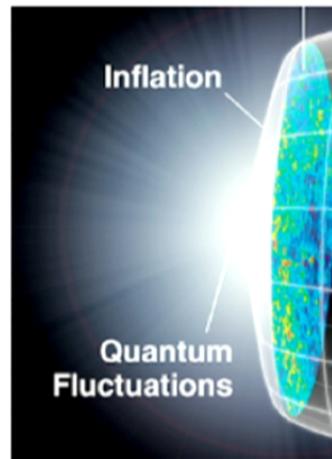
date, 1 Sept 2015

+ Fluctuations on all scales



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Inflationary picture



- possibly lots of dynamics between inflation and observable era (more than one light scalar, reheating, any process that transfers isocurvature modes into the adiabatic mode)
- not clear there are observables that will disentangle all that
- so, let's focus on the correlation functions after the dust settles (where only known physics is left)

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So, what can we learn about the physics of inflation?

(Can inflation “predict” absolutely anything?)

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So, what can we learn about the physics of inflation?

(Can inflation “predict” absolutely anything?)

- the most important qualitative distinction we know of right now: *are short wavelength modes coupled to long wavelength modes?*
 - single-clock (consistency relations): No
 - multiple fields: Yes

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Is non-Gaussianity relevant for our universe?

(How else can we learn more?)

Constraints are great:

$$f_{\text{NL}}^{\text{local}} = 0.8 \pm 5.0$$

$$f_{\text{NL}}^{\text{equil}} = -4 \pm 43$$

$$f_{\text{NL}}^{\text{ortho}} = -26 \pm 21$$

(68% C.L., T+E)

Paper XVII, 1502.0159

But they are not yet definitive.

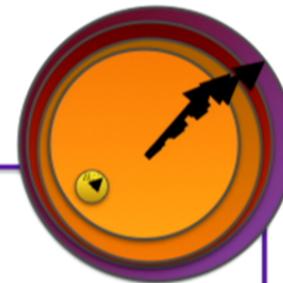
A minimum future goal:

Detailed discussion: **arXiv:**
1412.4671 “Testing Inflation
with Large Scale Structure:
Connecting Hopes with
Reality”

$$|f_{\text{NL}}^{\text{ANY}}| = \mathcal{O}(1)$$

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The Plan



Framework:

Consider general (Non-Gaussian) correlations
on a single time slice + sub-sampling (no
dynamics)

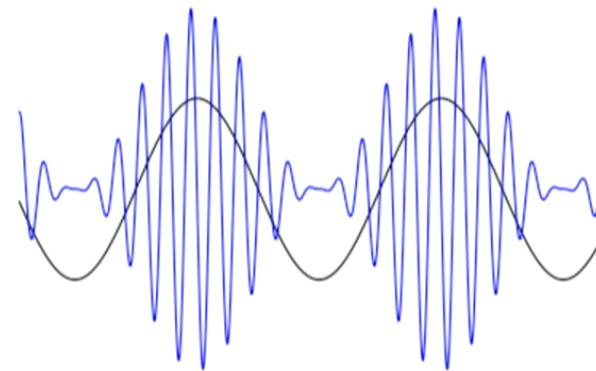
- Examples + Pictures of how correlations look in sub-volumes
- Calculations + generalities
- Connect/Contrast with the inflationary picture

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Super Cosmic Variance examples

Effects of long-short mode coupling

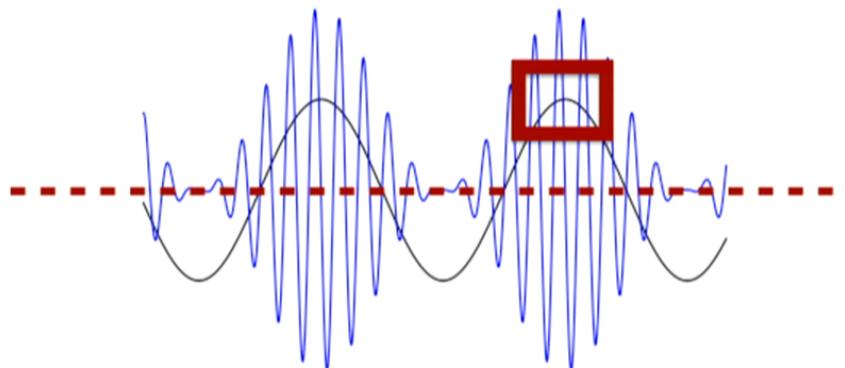
- Ex: the amplitude of fluctuations depends on the long-wavelength fluctuation



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Effects of long-short mode coupling

- Ex: the amplitude of fluctuations depends on the long-wavelength fluctuation



Bias: How different the local background
is from the mean

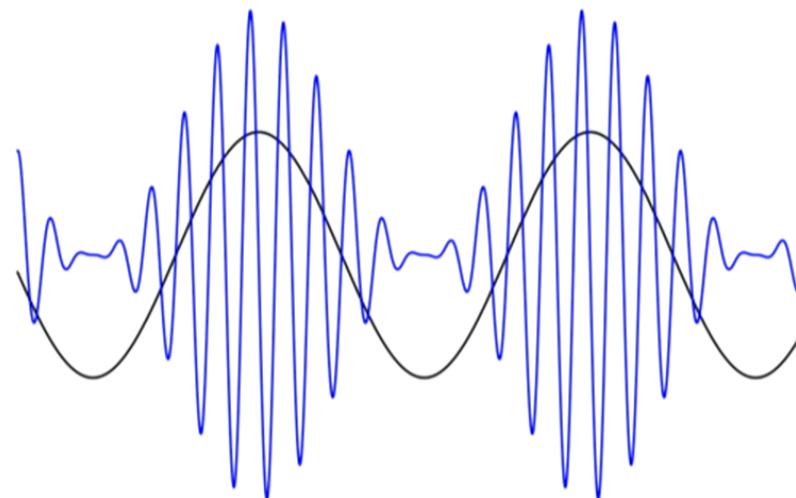
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How can Hubble volume statistics vary?

Power spectrum:

Amplitude

$$\Delta_{\zeta}^2 = A_0 \left(\frac{k}{k_0} \right)^{n_s - 1}$$



Nelson and Shandera (1212.4550)
LoVerde, Nelson, Shandera (1303.3549)
Bramante, Kumar, Nelson, Shandera (1307.5083)

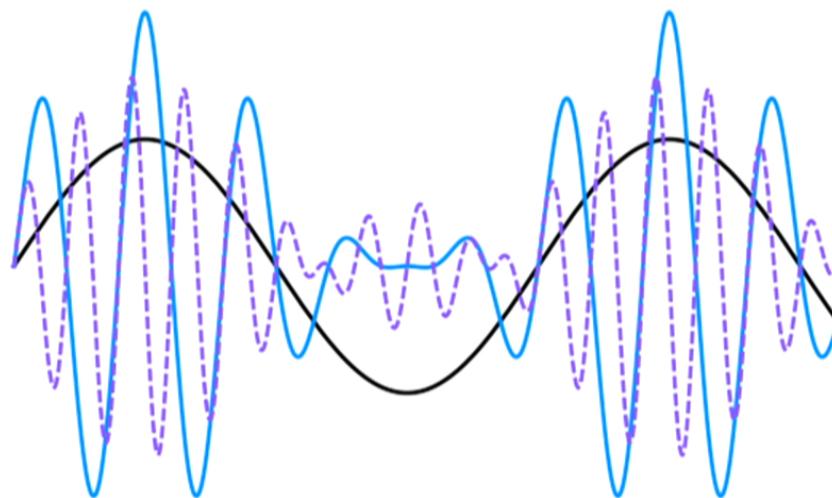
Bartolo et al; Byrnes et al; Thorsrud et
al; LoVerde; Nurmi, Sloth

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How can Hubble volume statistics vary?

Power spectrum:
Amplitude
Spectral index

$$\Delta_\zeta^2 = A_0 \left(\frac{k}{k_0} \right)^{n_s - 1}$$



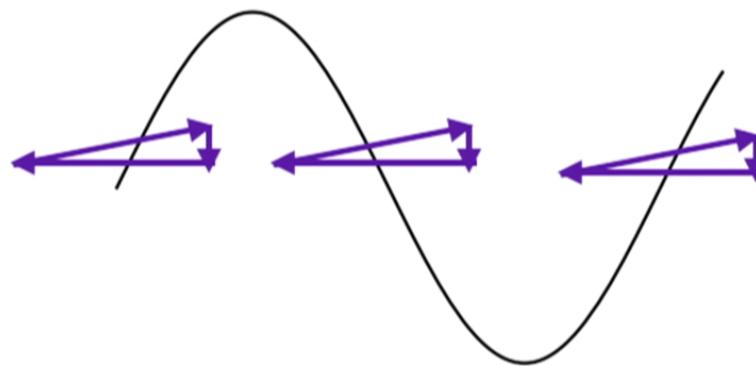
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How can Hubble volume statistics vary?

Bispectrum:



Nelson and Shandera (1212.4550)

LoVerde, Nelson, Shandera (1303.3549)

Baytas, Kesavan, Nelson, Park, Shandera (1502.01009)

Byrnes et al;

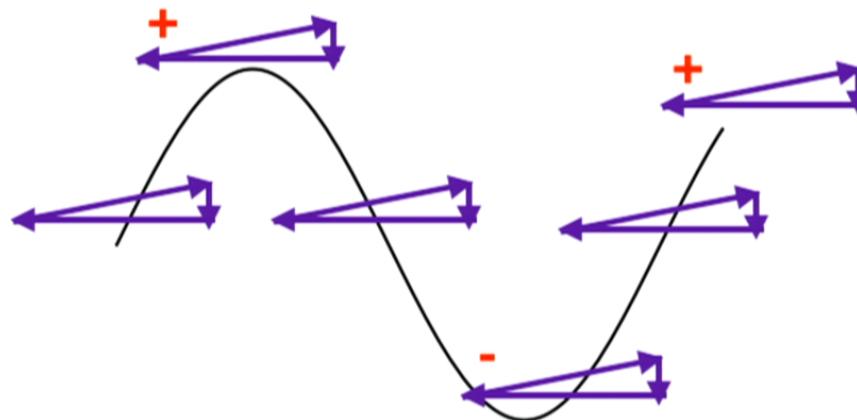
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How can Hubble volume statistics vary?

Bispectrum:

Amplitude

$$|f_{\text{NL}}^{\text{local}}|$$



Nelson and Shandera (1212.4550)

LoVerde, Nelson, Shandera (1303.3549)

Baytas, Kesavan, Nelson, Park, Shandera (1502.01009)

Byrnes et al;

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How can we tell if it matters for our universe?

- We detect a Super Cosmic Variance bispectrum



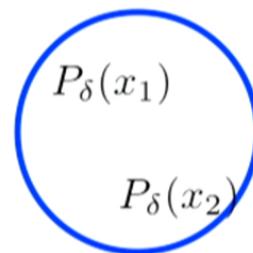
- We detect non-Gaussian halo bias

Dalal et al



- We detect a position-dependent power spectrum

Chiang et al



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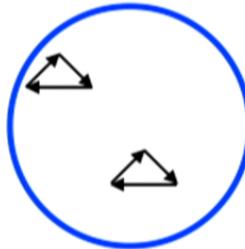
How can we tell if it matters for our universe?

- We detect a Super Cosmic Variance trispectrum



- We detect a position-dependent bispectrum

... (etc, at higher order)



Adhikari, Jeong, Shandera
in progress

- We detect deviations from homogeneity/isotropy

Adhikari, Shandera, Erickcek
1508.06489

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Super Cosmic Variance



If local modes are coupled to super horizon modes }

- and -

If our observed universe is a biased region of a larger inflationary volume }

Then observed statistics on all scales may differ from the mean statistics predicted by the model }

Knowable

+

Unknowable

=

Cosmic Variance

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Super Cosmic Variance Generalities

Doing the calculation

The non-Gaussian field:

$$\Phi[\phi(\mathbf{x})] = \phi(\mathbf{x}) + f_{\text{NL}}\Phi_2[\phi(\mathbf{x})] + g_{\text{NL}}\Phi_3[\phi(\mathbf{x})] + \dots$$

Gaussian

Or, in Fourier space:

$$\Phi(\mathbf{k}) = \phi(\mathbf{k}) + f_{\text{NL}}\Phi_2(\mathbf{k}) + g_{\text{NL}}\Phi_3(\mathbf{k}) + \dots$$

Note: at any order in the expansion, complete set of correlation functions

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Quadratic kernel

$$\Phi_2(\mathbf{k}) = \frac{1}{2!} \int \frac{d^3 p_1}{(2\pi)^3} \int d^3 p_2 [\phi(\mathbf{p}_1)\phi(\mathbf{p}_2) - \langle \phi(\mathbf{p}_1)\phi(\mathbf{p}_2) \rangle] \\ \times N_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \delta^3(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2)$$

Generates bispectrum

eg, Local ansatz: $N_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) = 2$

Equilateral bispectrum:

$$N_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) = -6 + 4 \left(\frac{p_1 + p_2}{k} \right) + 2 \left(\frac{p_1^2 + p_2^2}{k^2} \right) - 4 \left(\frac{p_1 p_2}{k^2} \right)$$

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Finding the sub-volume statistics: power spectrum

$$\left\langle \Phi\left(\mathbf{x} - \frac{\mathbf{r}}{2}\right) \Phi\left(\mathbf{x} + \frac{\mathbf{r}}{2}\right) \right\rangle = \int \frac{d^3 k}{(2\pi)^3} P_\Phi(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

Local ansatz for simplicity:

$$\Phi(\mathbf{k}) = \phi(\mathbf{k}) + f_{\text{NL}} \int \frac{d^3 q}{(2\pi)^3} [\phi(\mathbf{k} - \mathbf{q})\phi(\mathbf{q}) - \langle \phi(\mathbf{k} - \mathbf{q})\phi(\mathbf{q}) \rangle]$$

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Two-point, cont'd

$$\left\langle \Phi\left(\mathbf{x} - \frac{\mathbf{r}}{2}\right) \Phi\left(\mathbf{x} + \frac{\mathbf{r}}{2}\right) \right\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} P_\phi(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$+ f_{\text{NL}}$$

$$\{\propto \langle \phi(\mathbf{k}') \phi(\mathbf{p}) \phi(\mathbf{k} - \mathbf{p}) \rangle\}$$

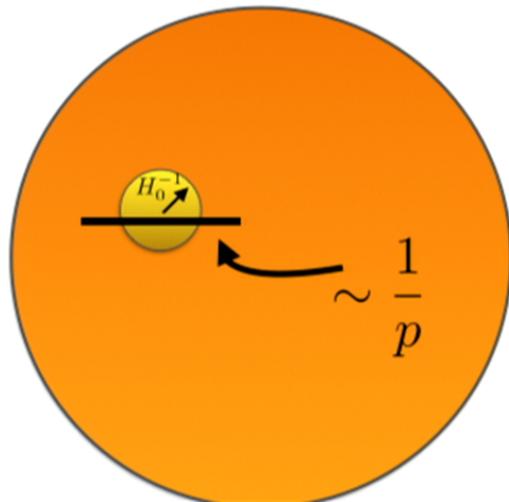
$$+\mathcal{O}(f_{\text{NL}}^2 \mathcal{P}_\phi^2)$$

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...which isn't zero when some modes are larger than the sub-volume:

$$\langle \phi(\mathbf{k}')\phi(\mathbf{k})\phi(\mathbf{p}) \rangle|_{\text{sub-volume}} = \phi(\mathbf{p})\langle \phi(\mathbf{k}')\phi(\mathbf{k}) \rangle|_{\text{sub-volume}}$$

Fixed value



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Two-point result

$$\left\langle \Phi\left(\mathbf{x} - \frac{\mathbf{r}}{2}\right) \Phi\left(\mathbf{x} + \frac{\mathbf{r}}{2}\right) \right\rangle_{\text{sub-volume}} = \int_{|\mathbf{k}| > k_{\min}} \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{r}} P_\phi(k)$$
$$\times \left[1 + 4f_{\text{NL}} \int_{|\mathbf{p}| < k_{\min}} \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[\phi(\mathbf{p}) \cos\left(\frac{\mathbf{p} \cdot \mathbf{r}}{2}\right) \right] e^{i\mathbf{p} \cdot \mathbf{x}} \right]$$

a sensible Fourier transform when
 $r \ll p$

Expand in sphere. harm.:
monopole
dipole (power asymmetry)
etc

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The upshot (monopole term)

$$P_\phi(k)|_{\text{obs}} = P_\phi(k) \left[1 + 4f_{\text{NL}} \int_{|\mathbf{p}| < k_{\min}} \frac{d^3 \mathbf{p}}{(2\pi)^3} \phi(\mathbf{p}) \right]$$

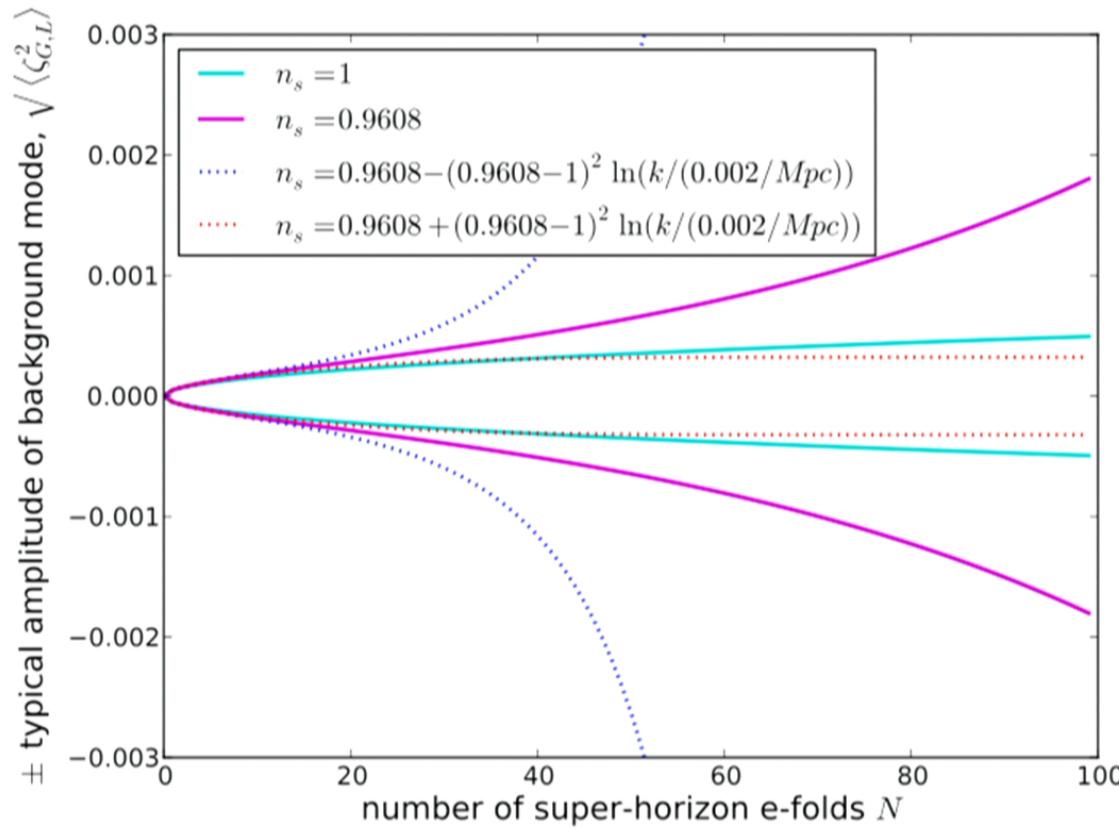
A number in each sub-volume $\equiv \Phi_{G,L}$

$$\langle \Delta P_{\text{cosm.var.}} \rangle_{\text{LargeVol.}} = 0$$

$$\begin{aligned} \langle (\Delta P_{\text{cosm.var.}})^2 \rangle_{\text{LargeVol.}} &\approx 16 f_{\text{NL}}^2 \int \frac{dk_\ell}{k_\ell} \mathcal{P}_\phi(k_\ell) \\ &\propto A_0 \left[\frac{1 - e^{-(n_s - 1)N_e}}{n_s - 1} \right] \end{aligned}$$

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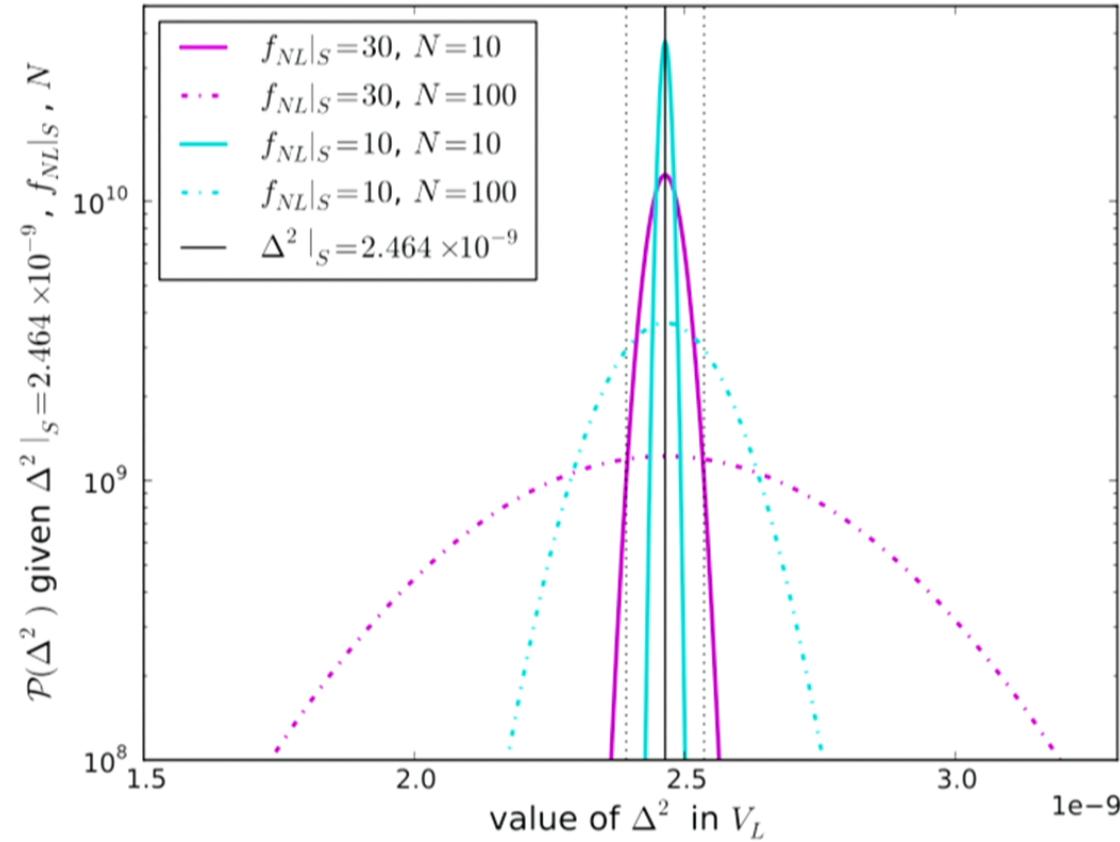
How big are the effects?



LoVerde, Nelson, Shandera, 1303.3549

Shandera, Perimeter Institute, 1 Sept 2015

Observed power spectrum



LoVerde, Nelson, Shandera, 1303.3549

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Finding the sub-volume statistics

$$\begin{aligned}\Phi(\mathbf{k}) \sim & \phi(\mathbf{k}) + f_{\text{NL}} \int \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) N_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \\ & + g_{\text{NL}} \int \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) \phi(\mathbf{p}_3) N_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k}) + \dots\end{aligned}$$

$$\begin{aligned}\Phi(\mathbf{k})|_{\text{sub-vol}} \sim & \phi(\mathbf{k}) \left[1 + f_{\text{NL}} \int_{p_1 \ll k} \phi(\mathbf{p}_1) N_2 \right. \\ & \left. + g_{\text{NL}} \int_{p_1, p_2 \ll k} \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) N_3 + \dots \right]\end{aligned}$$

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Finding the sub-volume statistics

$$\begin{aligned}\Phi(\mathbf{k}) \sim & \phi(\mathbf{k}) + f_{\text{NL}} \int \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) N_2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \\ & + g_{\text{NL}} \int \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) \phi(\mathbf{p}_3) N_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k}) + \dots\end{aligned}$$

$$\begin{aligned}\Phi(\mathbf{k})|_{\text{sub-vol}} \sim & \phi(\mathbf{k}) \left[1 + f_{\text{NL}} \int_{p_1 \ll k} \phi(\mathbf{p}_1) N_2 \right. \\ & \left. + g_{\text{NL}} \int_{p_1, p_2 \ll k} \phi(\mathbf{p}_1) \phi(\mathbf{p}_2) N_3 + \dots \right] \\ & + f_{\text{NL}} (\phi \star \phi)_{\mathbf{k}} \left[1 + g_{\text{NL}} \int_{p_1 \ll k} \phi(\mathbf{p}_1) N_3 + \dots \right] + \dots\end{aligned}$$

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Kernels weight IR modes

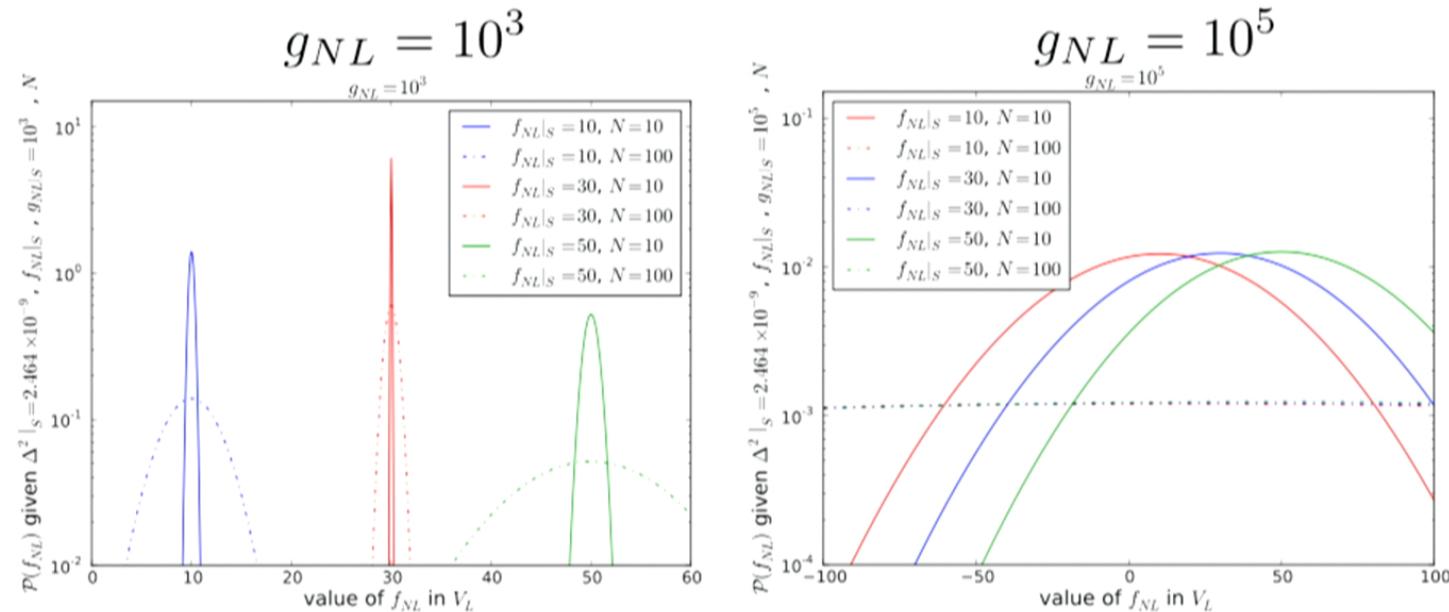
eg, equilateral case:

$$\chi_G(\mathbf{k}) = \phi_s(\mathbf{k}) \left[1 + f_{\text{NL}} \frac{1}{k^2} \int_{\Lambda}^{k_*} \frac{d^3 p}{(2\pi)^3} p^2 \phi(\mathbf{p}) \right]$$

Contribution from IR modes is negligible

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Local NG: Observed amplitude of bispectrum



LoVerde, Nelson, Shandera, 1303.3549

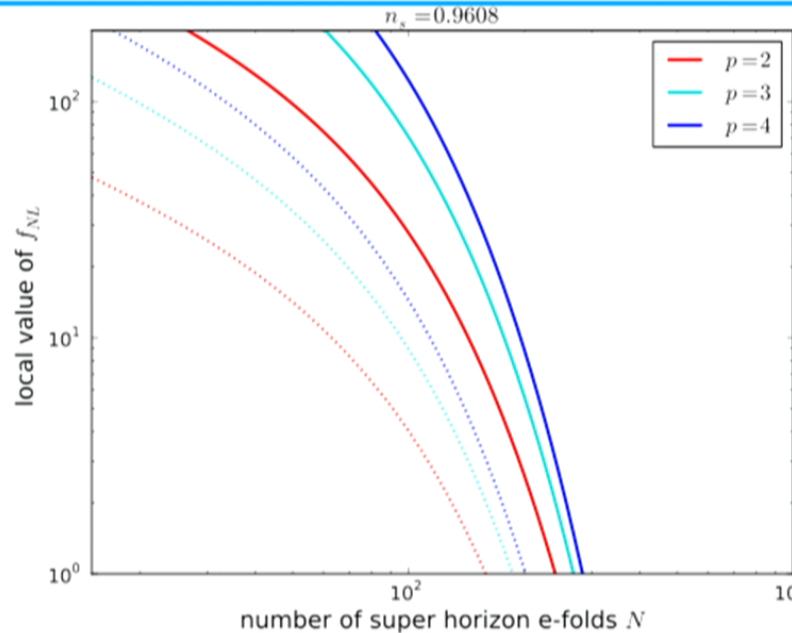
See also: Nurmi, Byrnes, Tasinato 1301.3128

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Strongly non-Gaussian in large Vol.

$$\zeta_{NG}(\mathbf{x}) = \zeta_G^p(\mathbf{x}) - \langle \zeta_G^p \rangle$$

$$\zeta_{NG}(\mathbf{x})|_S = p\zeta_{G,L}^{p-1}\zeta_{G,S}(\mathbf{x}) + \frac{p!}{2!(p-2)!}\zeta_{G,L}^{p-2}(\zeta_{G,S}^2(\mathbf{x}) - \langle \zeta_{G,S}^2 \rangle) + \frac{p!}{3!(p-3)!}\zeta_{G,L}^{p-3}\zeta_{G,S}^3 + \dots$$



Dashed:

$$\zeta_{G,L} = 3\sqrt{\langle \zeta_{G,L}^2 \rangle}$$

Solid:

$$\zeta_{G,L} = 5\sqrt{\langle \zeta_{G,L}^2 \rangle}$$

LoVerde, Nelson, Shandera, 1303.3549

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Local NG, generally

* Weakly non-Gaussian local ansatz is statistically natural

- One non-fundamental parameter (bias) can control size of all terms

$$\zeta_{NG}(\mathbf{x}) = f(\zeta_G(\mathbf{x})) - \langle f(\zeta_G) \rangle$$



biased sub-samples

$$\zeta_{NG}|_S = \chi_G(\mathbf{x}) + \frac{3}{5} f_{NL}|_S (\chi_G^2(\mathbf{x}) - \langle \chi_G^2 \rangle) + \frac{9}{25} g_{NL}|_S (\chi_G^3 - 3\langle \chi^2 \rangle \chi_G(\mathbf{x})) + \dots$$

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Local NG, generally

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Nelson, Shandera, 1212.4550 (PRL); Byrnes, Nurmi, Tasinato, Wands 1306.2370;

*

“Renormalization” is controlled by bias:
sub-volume size + rarity of fluctuations

Lyth (“curvature in a box”)

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So, given correlations, we can find the statistics in sub-volumes

To generalize beyond local type:
Given n -point, generate $(n+1)$ correlations
that will reduce to the previous set upon sub-sampling

Baytas, Kesavan, Nelson, Park, Shandera,
[1502.01009](#);
[Scoccimarro et al 1108.5512](#)

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Generating interesting correlations for sub-sampling

$$\Phi[\phi(\mathbf{x})] = \phi(\mathbf{x}) + f_{\text{NL}}\Phi_2[\phi(\mathbf{x})] + g_{\text{NL}}\Phi_3[\phi(\mathbf{x})] + \dots$$

With quadratic term: $\Phi_2(\mathbf{x}) = \partial^{\alpha_3}(\partial^{\alpha_2}\phi\partial^{\alpha_1}\phi)$

$$\partial^n \phi(\mathbf{x}) \equiv \int \frac{d^3 k}{(2\pi)^3} k^n \phi(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

With $\alpha_i \geq -2$ recover standard equilateral, orthogonal, local templates for bispectrum

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Add cubic terms:

$$\Phi_3(\mathbf{x}) = \partial^{\beta_5} (\partial^{\beta_4} \phi \partial^{\beta_3} (\partial^{\beta_2} \phi \partial^{\beta_1} \phi))$$

18 terms after imposing some restrictions:

- no UV div. loops
- scale invariance
- $\beta_i \geq -2$.

Baytas, Kesavan, Nelson, Park, Shandera,
1502.01009;

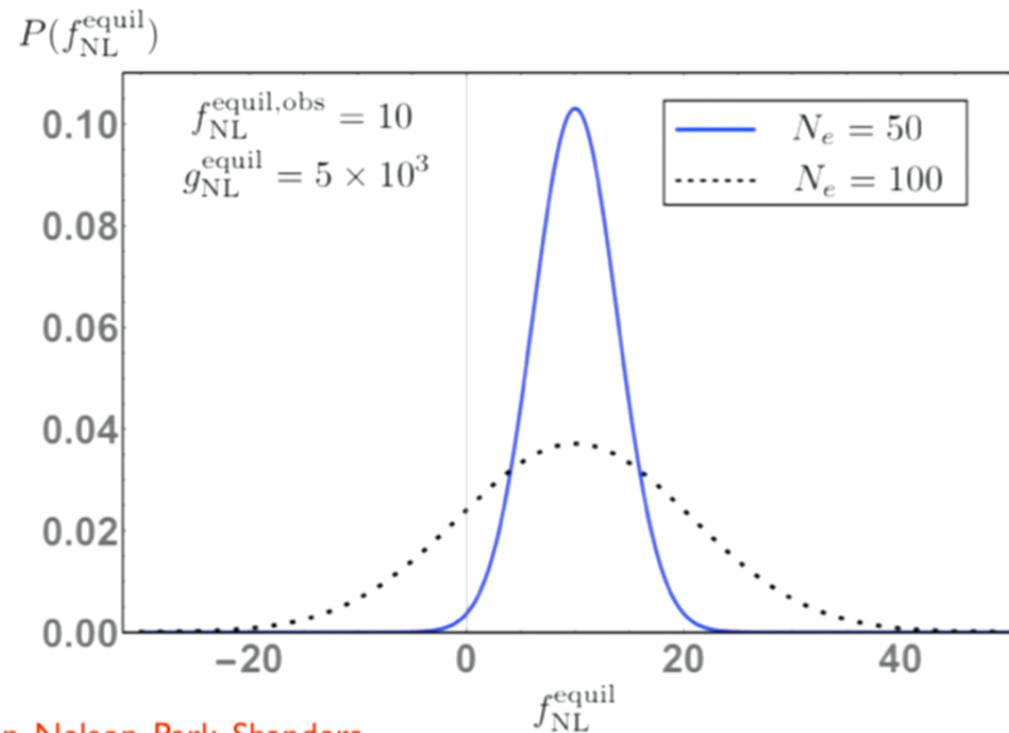
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In biased sub-volumes?

- 8 terms give IR divergent (local type) shift the power spectrum
- By construction, any of the quadratic terms can be induced in sub-volumes
- One trispectrum that generates equilateral bispectrum and shifts the power spectrum
- One trispectrum that generates equilateral bispectrum and but no shift to the power spectrum

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Probability of observing equilateral bispectrum from SCV



Baytas, Kesavan, Nelson, Park, Shandera,
1502.01009

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Comparing with
inflation model building

That is:
What can we infer about particle
physics from measurements in a
finite volume?

1. 2 field inflation model

“Quasi-single field” (Chen,Wang; Assassi et al)

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\Sigma - \frac{1}{2} \frac{(\partial\Phi)^2}{\Lambda} \Sigma - \frac{1}{2} \frac{\square\Phi}{\Lambda} \Sigma^2$$

Then in the fluctuations:

$$\mathcal{L}_{\text{int.}} = \left(\frac{\dot{\Phi}_0}{\Lambda} \right) \dot{\varphi} \sigma$$

If the extra field has self-interactions:



also: curvaton, Linde, Mukhanov; LoVerde

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Quasi-single field, cont'd

Mass of extra field determines which IR modes matter

In-in calculation agrees with phenom. calculation:

$$\Phi_{\mathbf{k},\mathbf{s}} = \phi_{G,\mathbf{k}} \left[1 + f_{\text{NL}}^{\text{eff}}(kM)^{\nu-3/2} \int_{L^{-1}}^{M^{-1}} \frac{d^3 p}{(2\pi)^3} (pM)^{3/2-\nu} \phi_{G,\mathbf{p}} \right]$$

$$\langle \phi_{G\ell}^2(x) \rangle = \frac{\Delta^2(M^{-1})[1 - e^{-N(3-2\nu+n_s-1)}]}{(3-2\nu)+(n_s-1)}$$

Massless limit:

$$\nu = \frac{3}{2}$$

Repeat the analysis as for local model, with series of self-interaction terms

work in progress with B. Bonga, S. Brahma, A.S. Deutsch

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2. Generating templates with no Super Cosmic Variance

Back to the quadratic term: $\Phi_2(\mathbf{x}) = \partial^{\alpha_3} (\partial^{\alpha_2} \phi \partial^{\alpha_1} \phi)$

Consider a larger range of terms: $\alpha_i \geq -7$

Collect bispectra with no significant SCV:

- how many templates?
- the same as the set of “natural” shapes proposed for single-clock inflation?

Behbahani et al 1407.7042

work in progress with B. Baytas

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3. Inflation model that generates an equilateral-inducing trispectrum?

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3. Inflation model that generates an equilateral-inducing trispectrum?
4. Other families of correlations with “statistically natural” limits in biased sub-volumes?
5. Contrast with EFT (or even EFT for the last 55 e-folds)

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Testing Inflation?

Derivations didn't depend on any dynamics to generate the fluctuations:

- * Good: can we find a “natural” pattern inflation does not predict? (If not, is that bad for inflation?)
- * Bad: a random pattern in a big universe may not look random to local observers (*inflationary perturbations cannot confirm our prejudices about high energy particle physics?*)

A familiar aspect of inflation, dressed up in different clothes: IC problem for our observable 60 e-folds region

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Two important points about observational constraints

- Non-Gaussianity: *anything* other than Gaussian (what to measure?? on what scales??)
- Measuring an all-sky average n -point function tells us *nothing* about higher order functions

equilateral bispectrum

\neq no super cosmic variance

\neq ?
single clock inflation

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Summary

- If local physics is coupled to background physics, new source of cosmic variance



- *Planck* bounds: no Non-Gaussianity? Or, a really big universe?
- We need further tests of non-Gaussianity

Target: $|f_{NL}| \lesssim \mathcal{O}(1)$ (and position-dependent spectra)

- Cosmological correlations are *not* collider data
- Consequences for inflation theory?

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