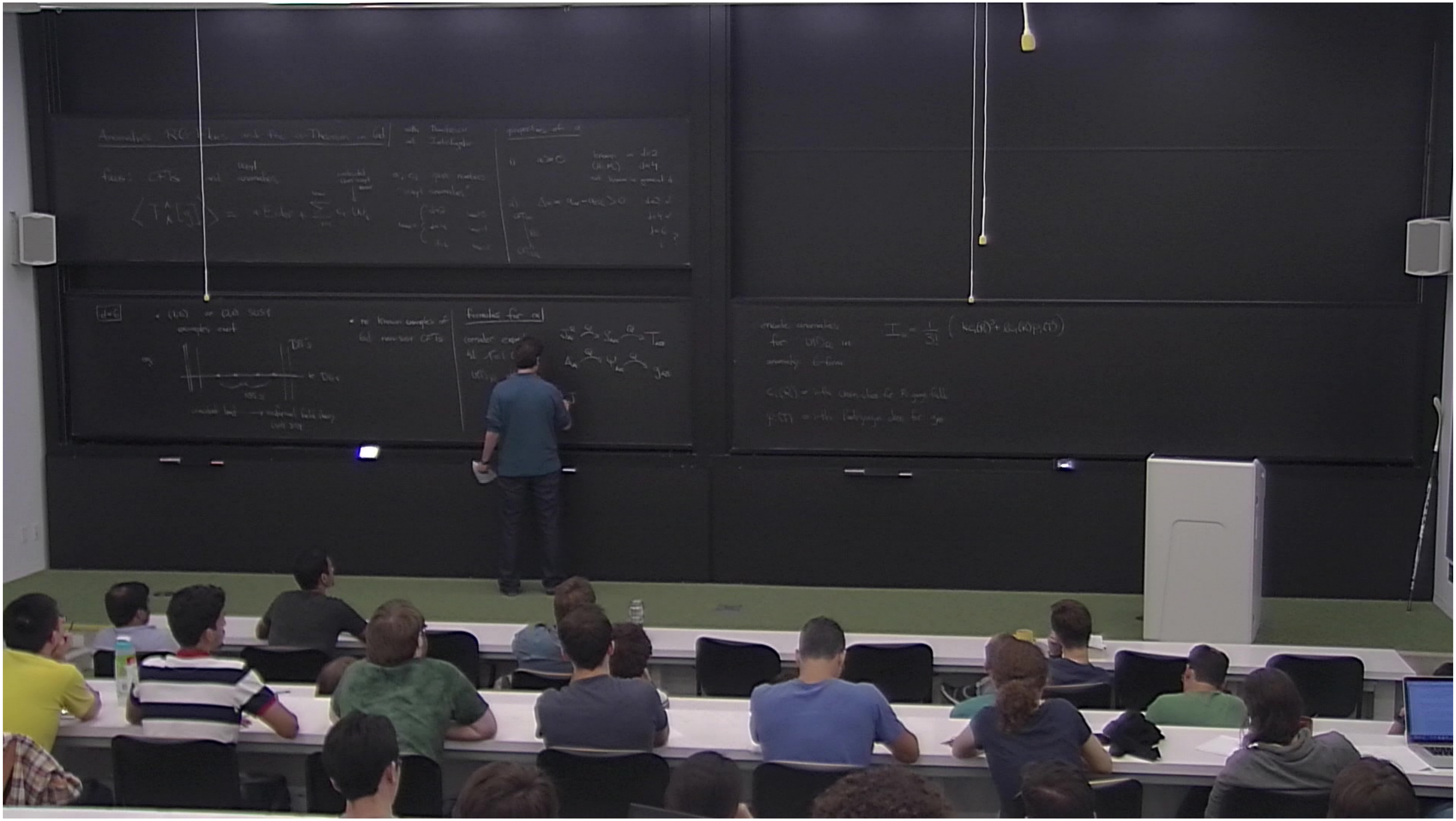
Title: Anomalies, RG Flows, and the a-Theorem in 6d

Date: Sep 29, 2015 02:00 PM

URL: http://pirsa.org/15090018

Abstract: <p>I will overview recent progress in understanding Weyl anomalies and the a-theorem in supersymmetric six-dimensional field theories.</p>



$\to T_{AB}$

$\to g_{AB}$

encode anomalies for $U(1)_R$ in anomaly 6-form

$$I_6 = \frac{1}{3!}\left(k c_1(R)^3 + \ell c_1(R) p_1(T)\right)$$

$c_i(R)$ = i-th Chern class for R gauge field

$p_i(T)$ = i-th Pontryagin class for g_{AB}

- no known examples of 6d non-susy CFTs

formulas for a!

consider example

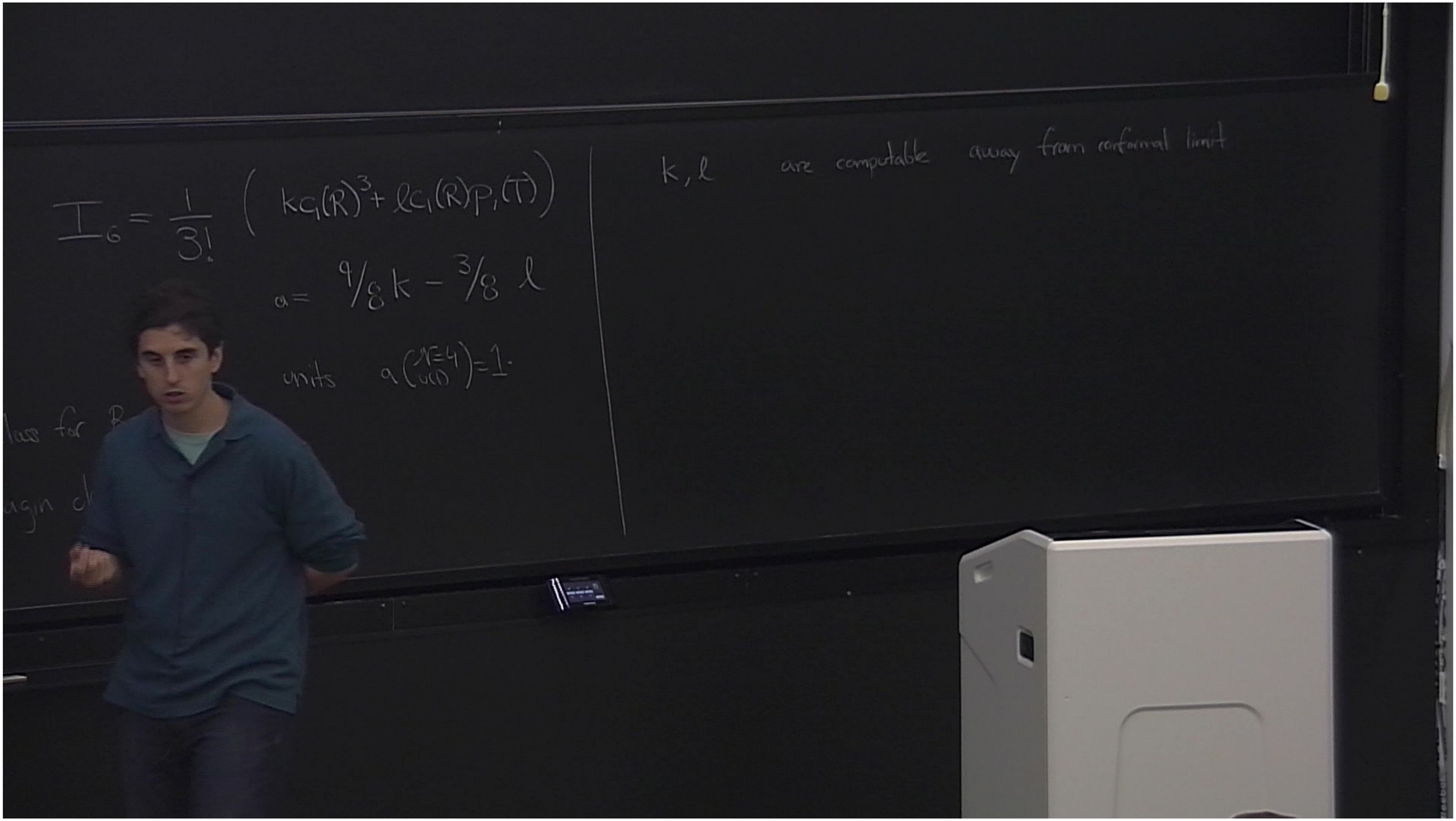
4d $\mathcal{N}=1$ CFTs

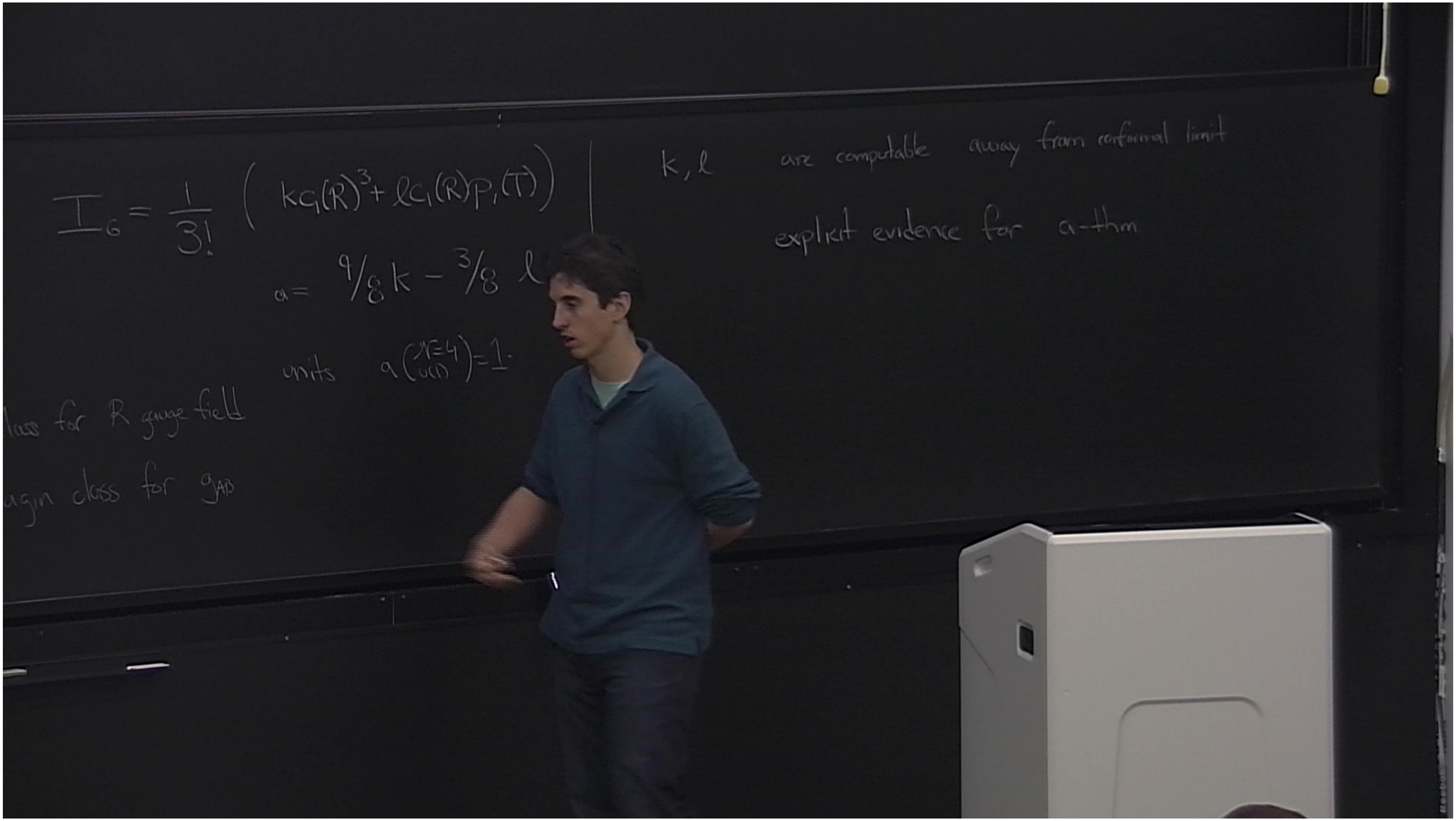
$U(1)_R$ global sym

$$J_A^R \xrightarrow{Q} S_{A\alpha} \xrightarrow{Q} T_{AB}$$

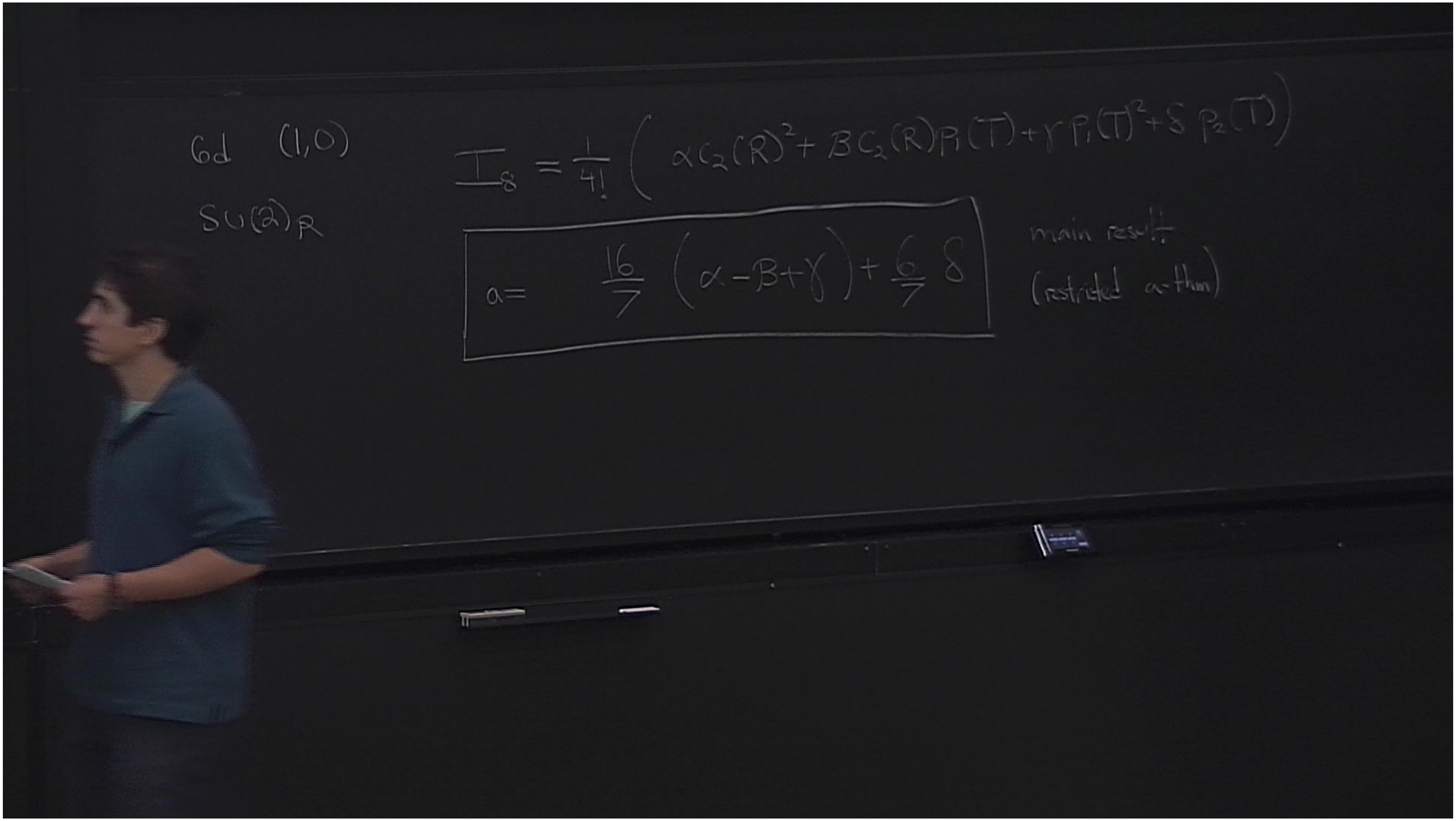
$$A_A \xrightarrow{Q} \psi_{A\alpha} \xrightarrow{Q} g_{AB}$$

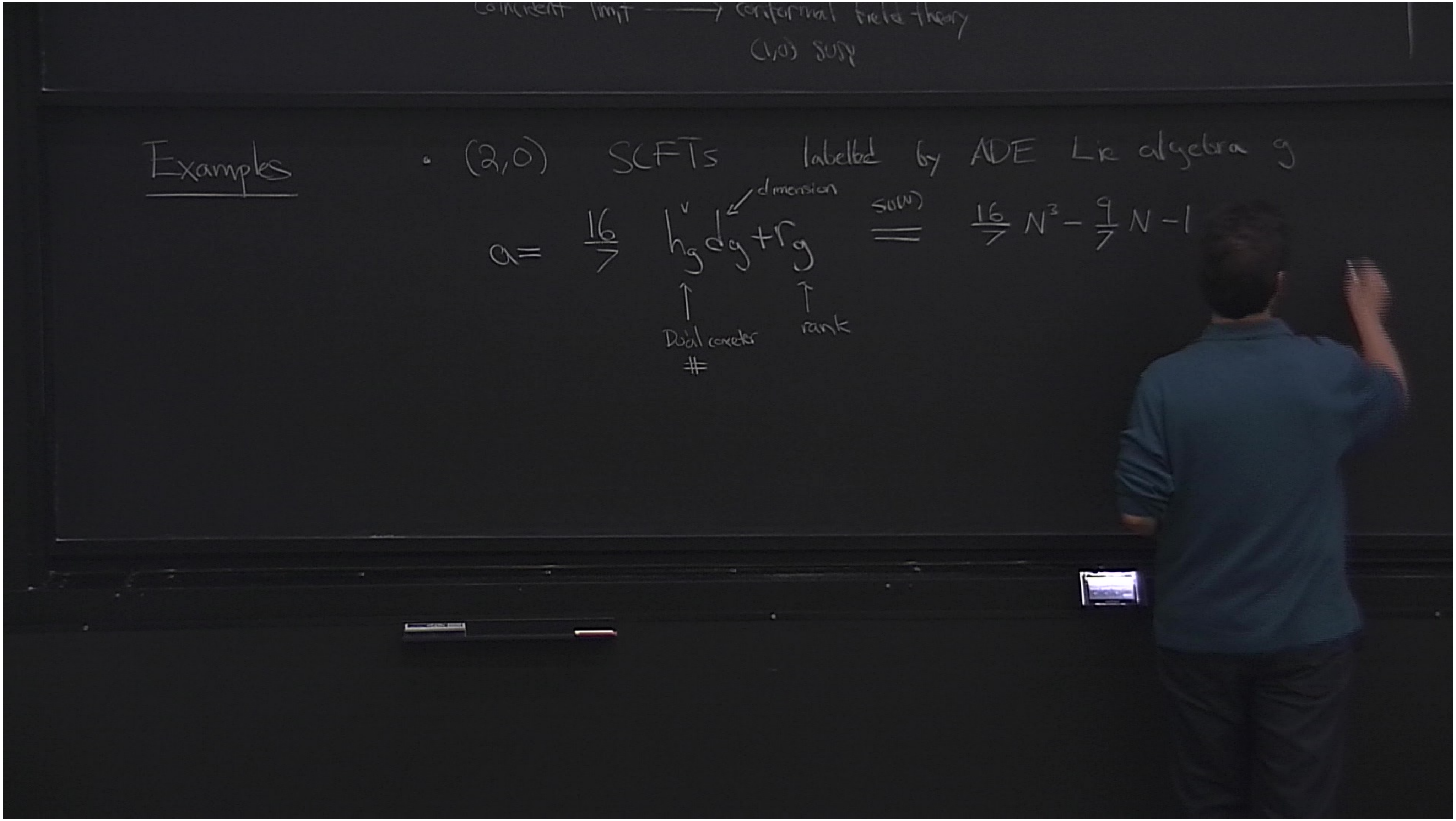
$$\partial^A J_A \overset{Q}{\longleftrightarrow} T^A_A$$

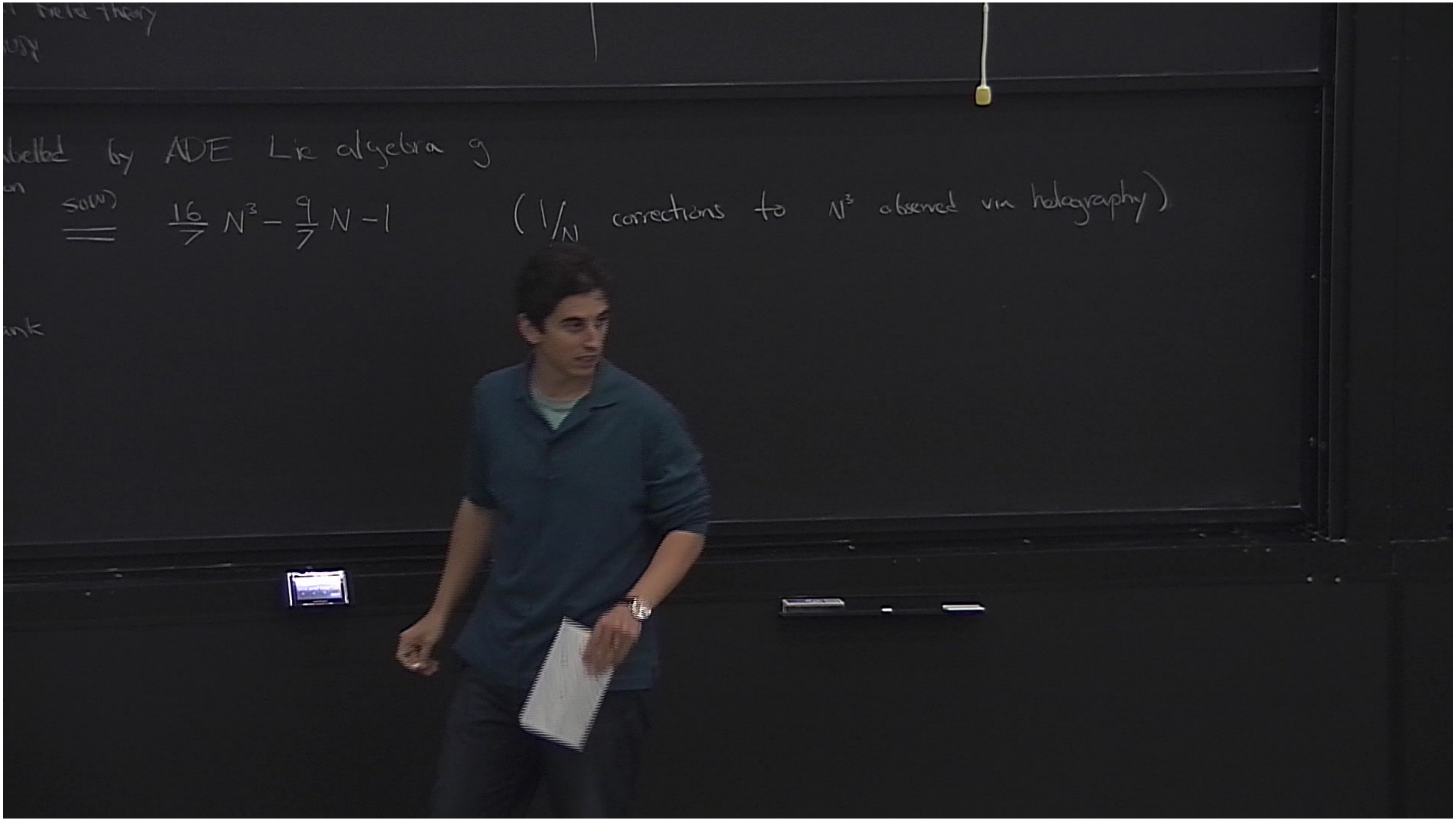

I_6 = 1/3! (k c_1(R)^3 + l c_1(R) p_1(T))
a = 9/8 k - 3/8 l
units a(N=4, U(1)) = 1.
k, l are computable away from conformal limit

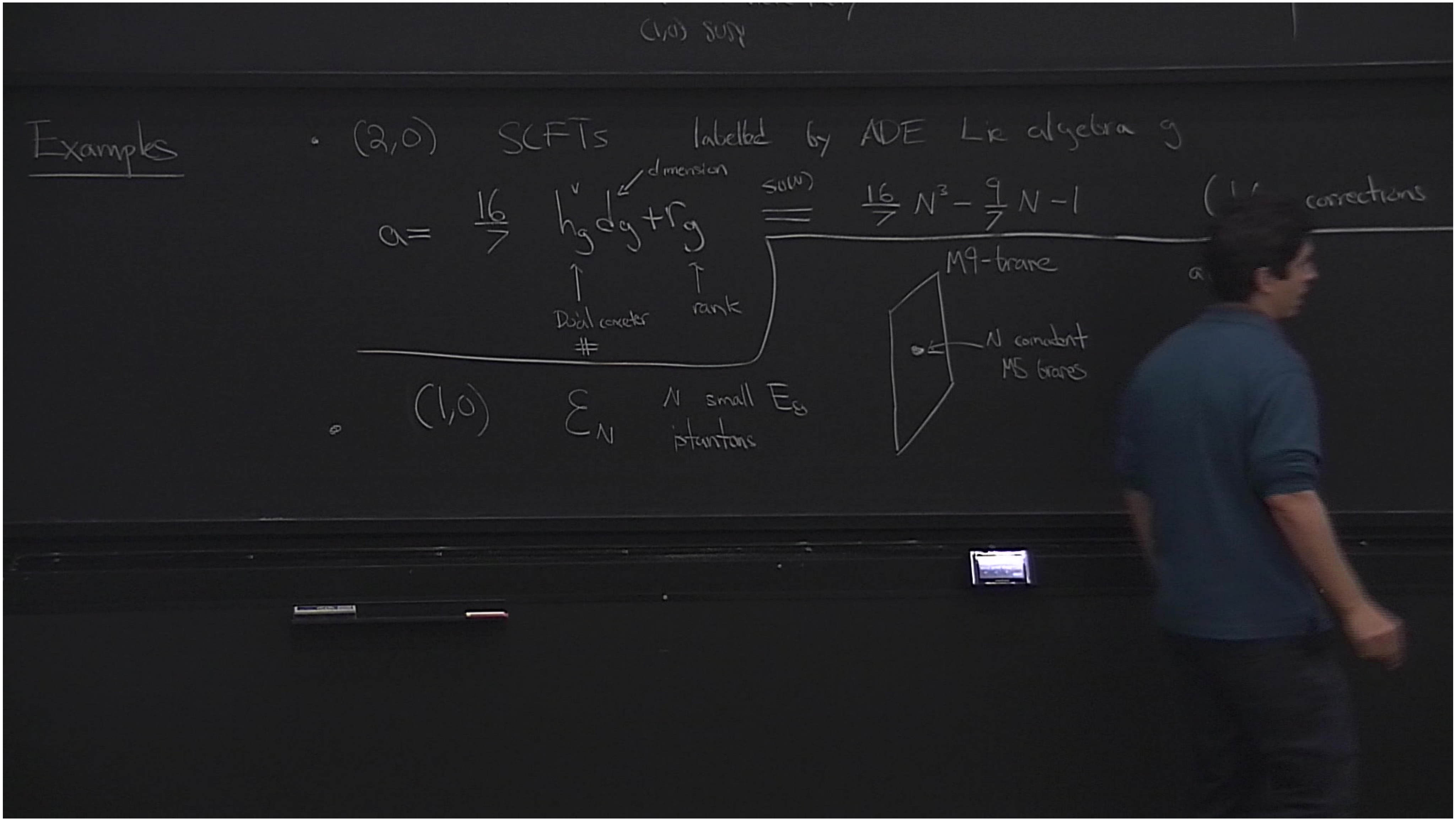


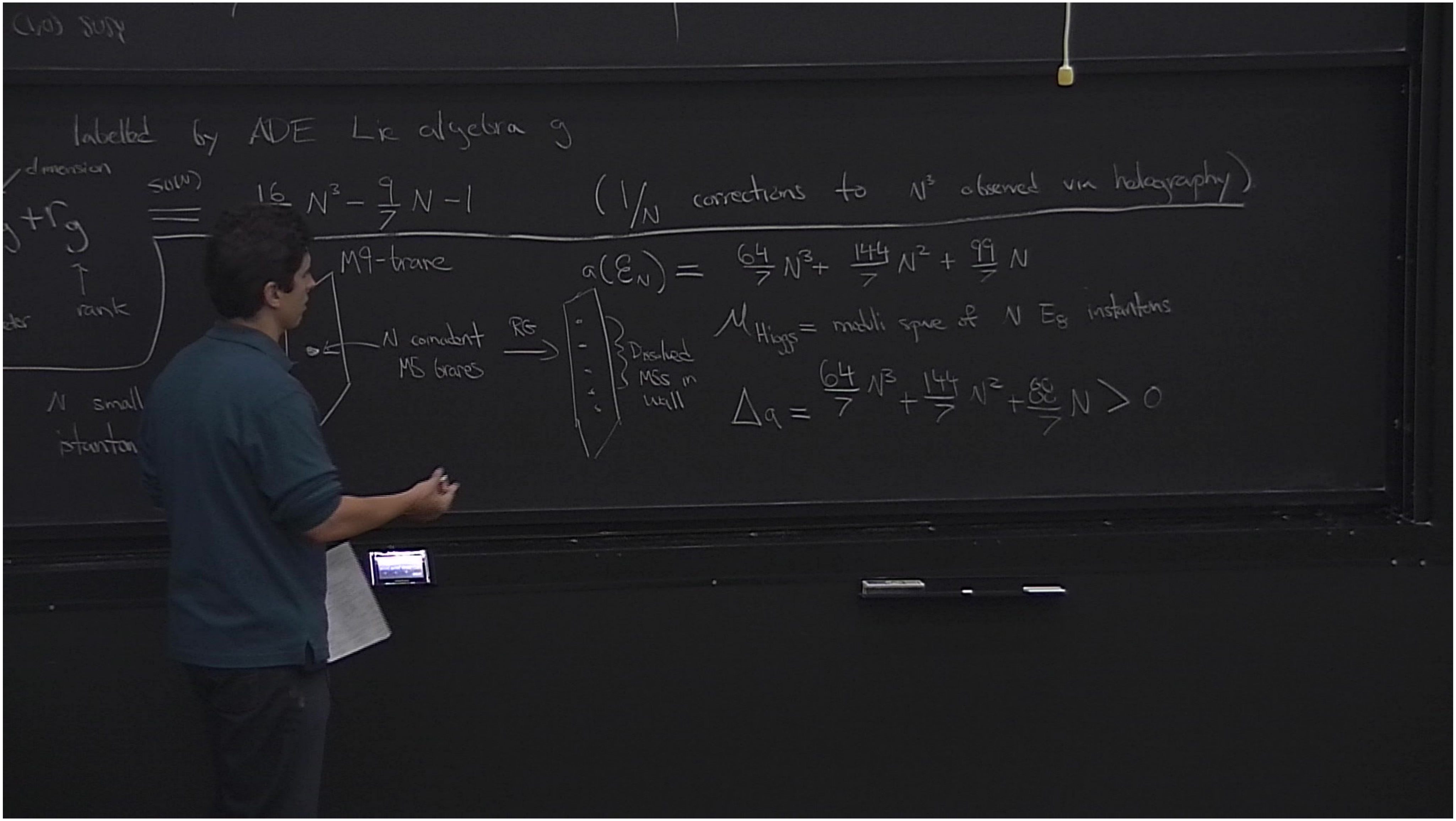
I_6 = 1/3! (k c_1(R)^3 + l c_1(R) p_1(T))
a = 9/8 k - 3/8 l
units a(N=4 U(1)) = 1
lass for R gauge field
ugin class for g_AB
k, l are computable away from conformal limit
explicit evidence for a-thm

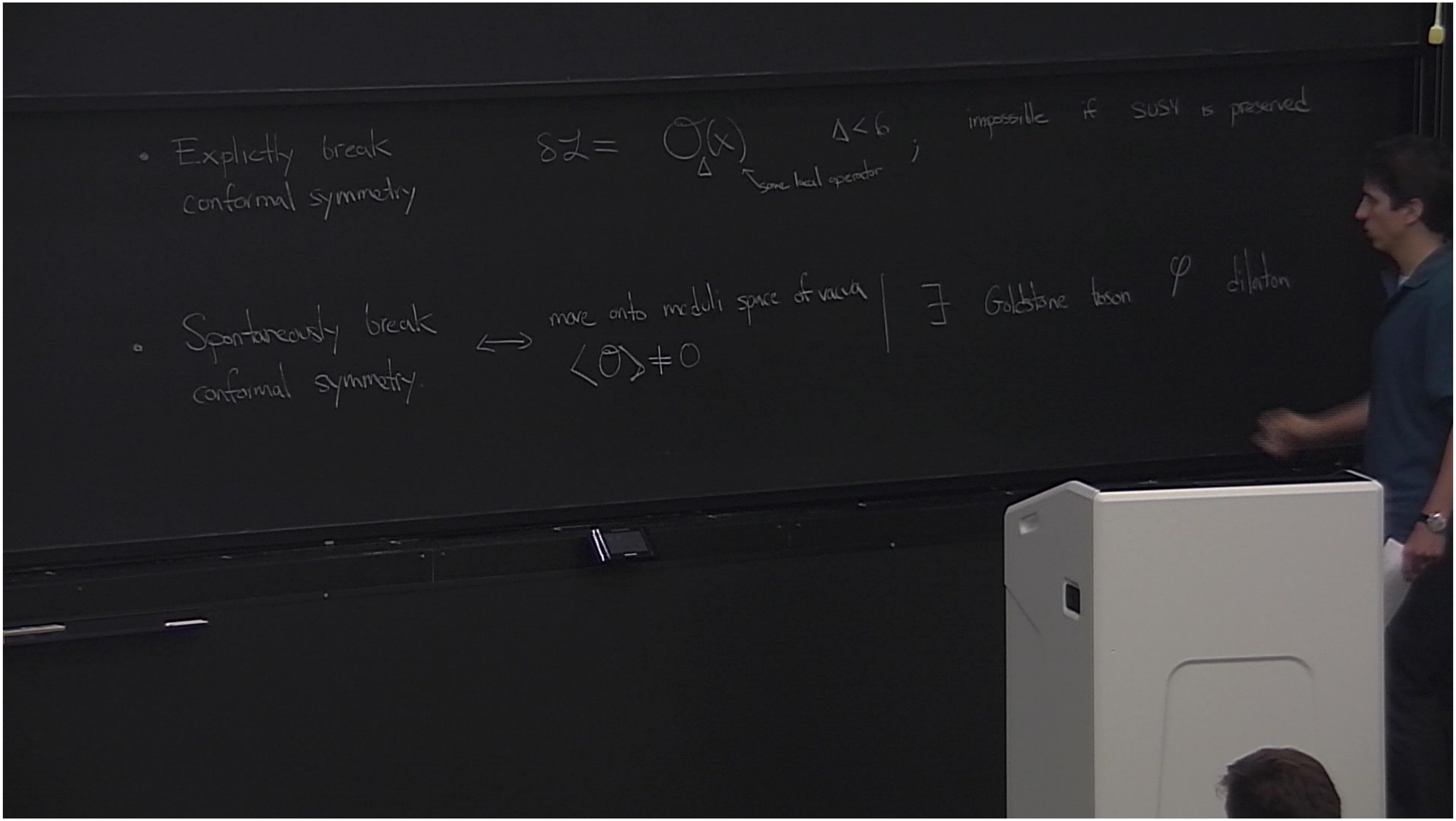

6d (1,0)
$SU(2)_R$
$I_8 = \frac{1}{4!}\left(\alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)\right)$
$\boxed{a = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta}$
main result!
(restricted a-thm)


coincident limit → conformal field theory
(1,0) susy
Examples
• (2,0) SCFTs labelled by ADE Lie algebra g
dimension
$a = \frac{16}{7} h^\vee_g d_g + r_g \overset{SU(N)}{=} \frac{16}{7} N^3 - \frac{9}{7} N - 1$
Dual coxeter #
rank


by ADE Lie algebra g
16/7 N^3 − 9/7 N − 1
(1/N corrections to N^3 observed via holography)


(1,0) SUSY
Examples
(2,0) SCFTs labelled by ADE Lie algebra g
dimension
a = 16/7 h_g d_g + r_g
SU(N)
16/7 N^3 − 9/7 N − 1
corrections
Dual coxeter #
rank
M9-brane
N coincident M5 branes
(1,0) E_N N small E8 instantons


(1,0) SUSY
labelled by ADE Lie algebra g
dimension
SU(N)
16/7 N^3 - 9/7 N - 1
(1/N corrections to N^3 observed via holography)
M9-brane
a(E_N) = 64/7 N^3 + 144/7 N^2 + 99/7 N
rank
N coincident M5 branes
RG
Dissolved M5s in wall
M_Higgs = moduli space of N E_8 instantons
Δa = 64/7 N^3 + 144/7 N^2 + 66/7 N > 0
N small
instanton


• Explictly break conformal symmetry
$\delta \mathcal{L} = \mathcal{O}_\Delta(x)$
some local operator
$\Delta < 6$;
impossible if SUSY is preserved
• Spontaneously break conformal symmetry
$\longleftrightarrow$ move onto moduli space of vacua
$\langle \mathcal{O} \rangle \neq 0$
$\exists$ Goldstone boson φ dilaton

- Higgs branch $\cancel{SU(2)_R}$

$\varphi \in$ hypermultiplet

4 scalars $= \underset{\substack{\uparrow \\ \text{goldstones} \\ \text{for } SU(2)_R}}{3} + \underset{\substack{\uparrow \\ \text{dilaton}}}{1}$

- Tensor branch $\boxed{SU(2)_R}$

$\varphi \in$ tensor multiplet

$\varphi \, , \, \psi^{i}_{\alpha}$ (i: doublet of $SU(2)_R$)

$dB_{BC} = H_{ABC} = * H$

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial\varphi)^2 + b(\partial\varphi)^4 + \underbrace{\Delta a(\partial\varphi)^6}_{\text{why?}} + \dots$$

$\Delta a = a_{UV} - a_{IR}$

WZ term makes up a mismatch

not Weyl invariant

$$S \supseteq \Delta a \int \log(\varphi)\,\mathrm{Euler}(g) + \dots + (\partial\varphi)^6$$

on even in flat space $g = \eta$

$$\frac{\delta S}{\delta \sigma} = \Delta a \int \sigma\, \mathrm{Euler}(g)$$

$$\underbrace{\Delta a (\partial\varphi)^6}_{\text{why?}} + \dots \qquad \Delta a = a_{UV} - a_{IR}$$

$\dots + (\partial\varphi)^6$ ... n even in ... flat space ... $g = \eta$

Need to SUSY complete

$$\mathcal{L} = \mathcal{L}_{\substack{\text{Free} \\ (1,0)\ \text{tensor} \\ \text{multiplet}}} + Q^8(\mathcal{O})$$

8 is total # of Q's

eg) 6 term $\quad Q^8(\varphi^4) \supset (\partial\varphi)^4 + H^4 + \dots$

Anomalies, RG Flows, and the a-Theorem in 6d

with Dumitrescu and Intriligator

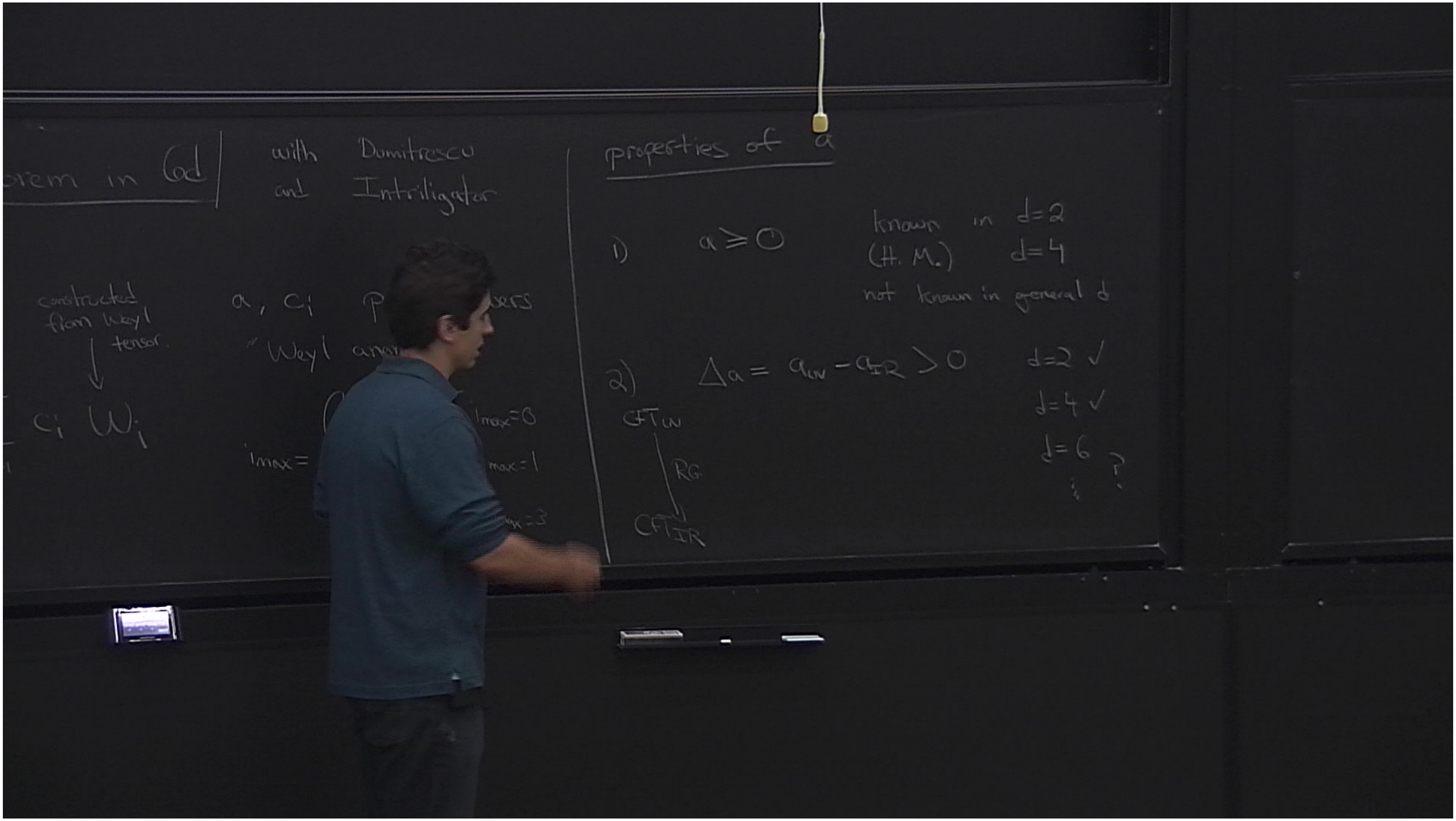
focus: CFTs and Weyl anomalies.

$$\langle T^A_A[g]\rangle = a\,\mathrm{Euler} + \sum_{i=1}^{i_{max}} c_i \underbrace{W_i}_{\text{constructed from Weyl tensor}}$$

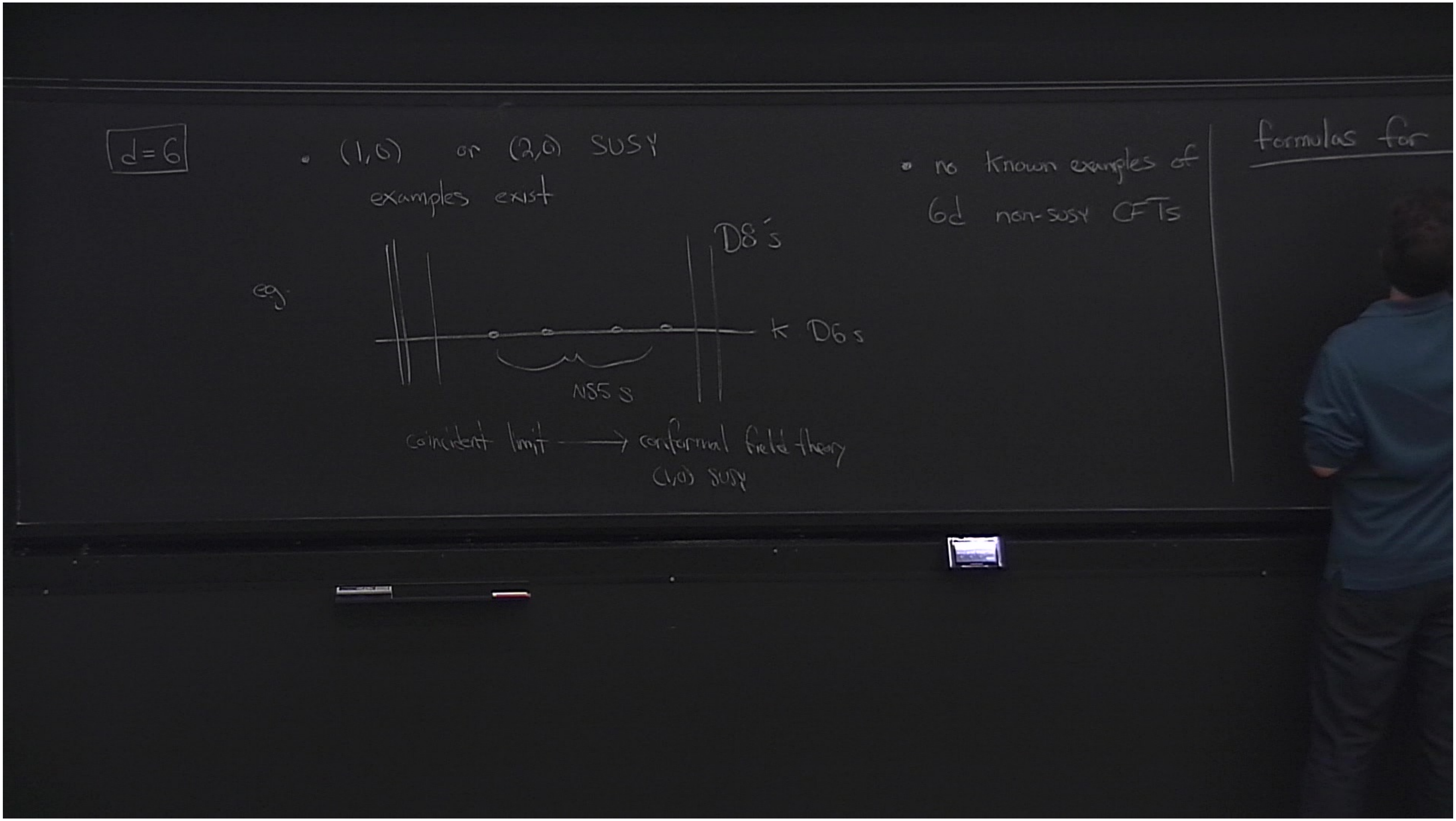
a, c_i pure numbers

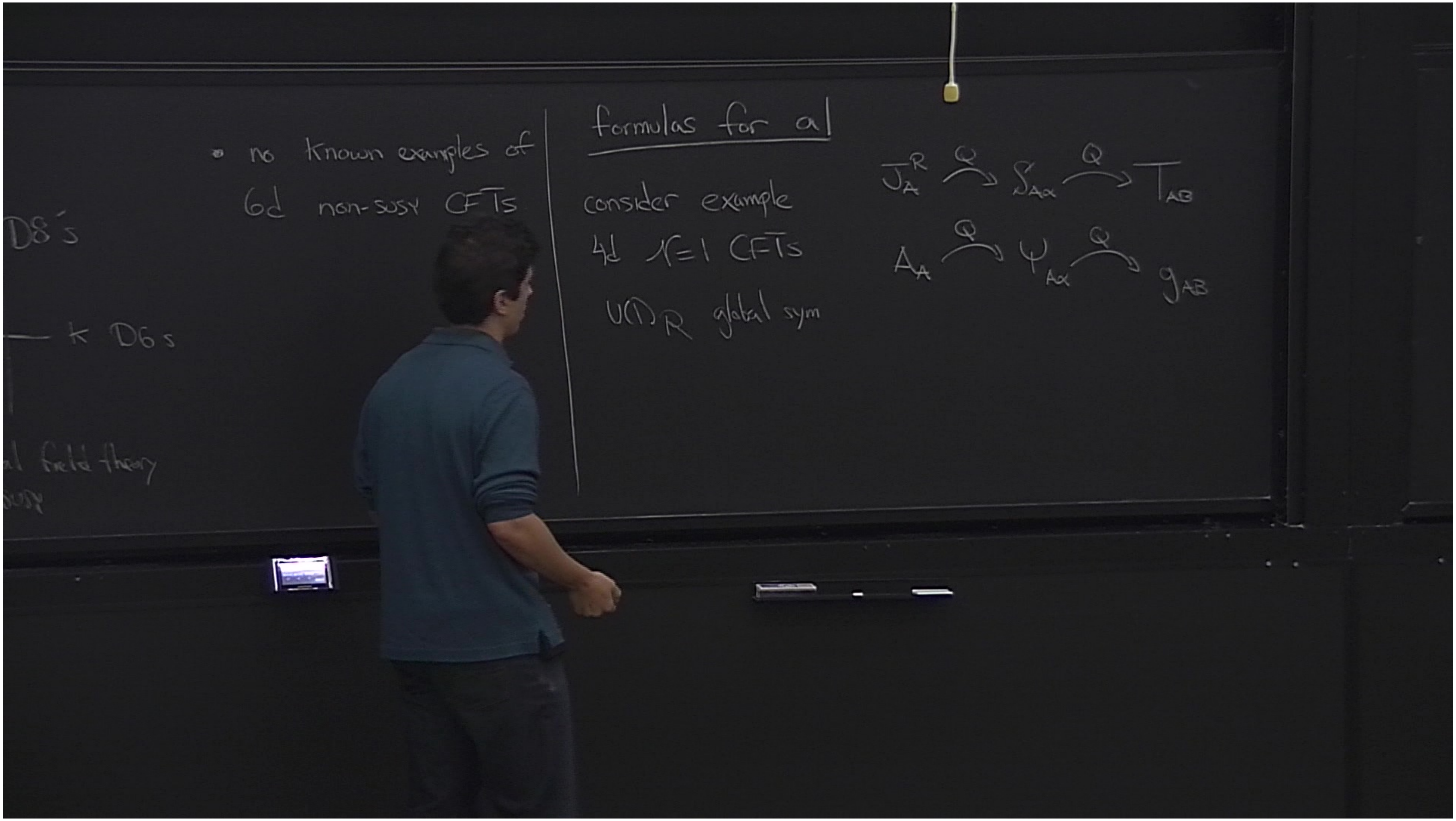
"Weyl anomalies"

$$i_{max} = \begin{cases} d=2 & i_{max}=0 \\ d=4 & i_{max}=1 \\ d=6 & i_{max}=3 \end{cases}$$



orem in 6d
with Dumitrescu
and Intriligator
constructed from Weyl tensor
a, c_i
"Weyl an
$c_i\, W_i$
properties of a
1) $a \geq 0$
known in d=2
(H. M.) d=4
not known in general d
2) $\Delta a = a_{UV} - a_{IR} > 0$
d=2 ✓
d=4 ✓
d=6 ?
CFT_{UV}
RG
CFT_{IR}


d=6
• (1,0) or (2,0) SUSY
examples exist
eg.
D8's
k D6s
NS5 s
coincident limit ⟶ conformal field theory
(1,0) SUSY
• no known examples of
6d non-SUSY CFTs
formulas for


no known examples of
6d non-susy CFTs
D8's
k D6s
field theory
formulas for a!
consider example
4d N=1 CFTs
U(1)_R global sym
J_A^R → Q → S_Aα → Q → T_AB
A_A → Q → ψ_Aα → Q → g_AB

$(\partial\varphi)^6$ does not arise in $Q^8(\mathcal{O})$ for any $\mathcal{O}$?!

$$Q^8 \left(\partial_A\varphi\,\partial^A\varphi\ \varphi^4\right) \overset{\text{EOM}}{\sim} Q^8\, \partial_A\left(\partial^A\varphi\ \varphi^4\right)$$

no change in action

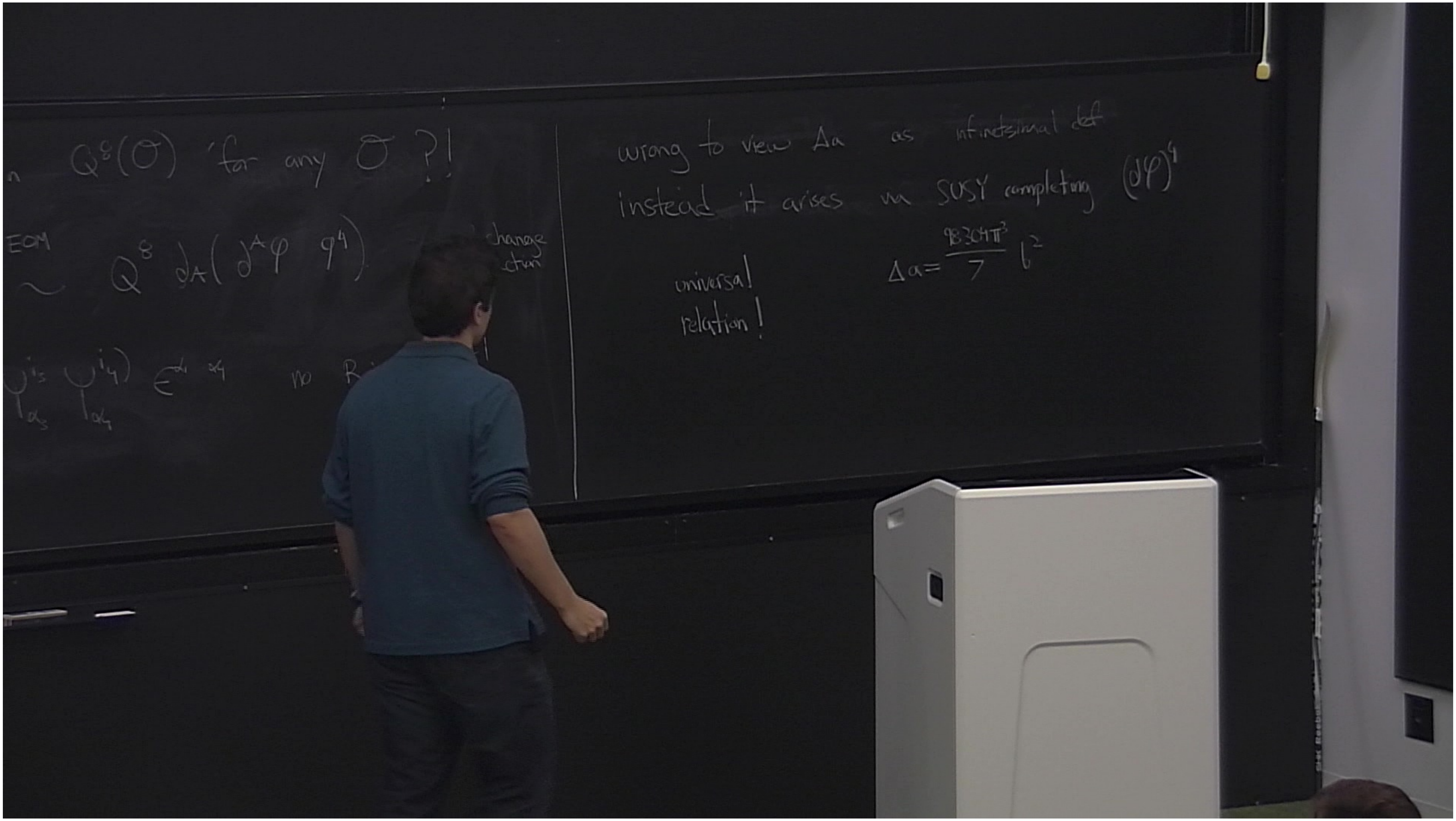
$$\mathcal{O} = \varphi^2\ \psi^{i_1}_{\alpha_1}\ \psi^{i_2}_{\alpha_2}\ \psi^{i_3}_{\alpha_3}\ \psi^{i_4}_{\alpha_4})\ \epsilon^{\alpha_1 \cdots \alpha_4}$$

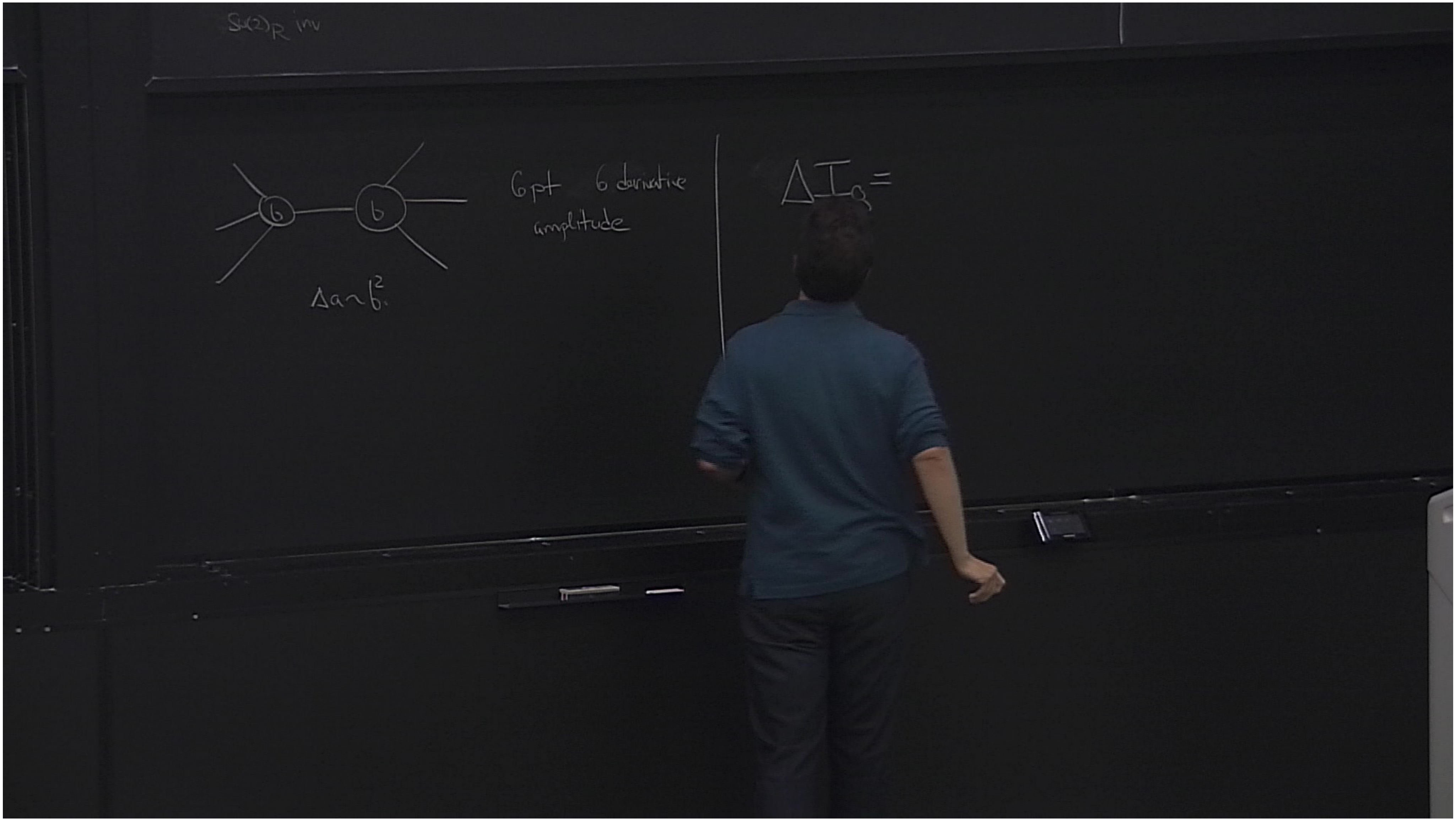
no R inv component!

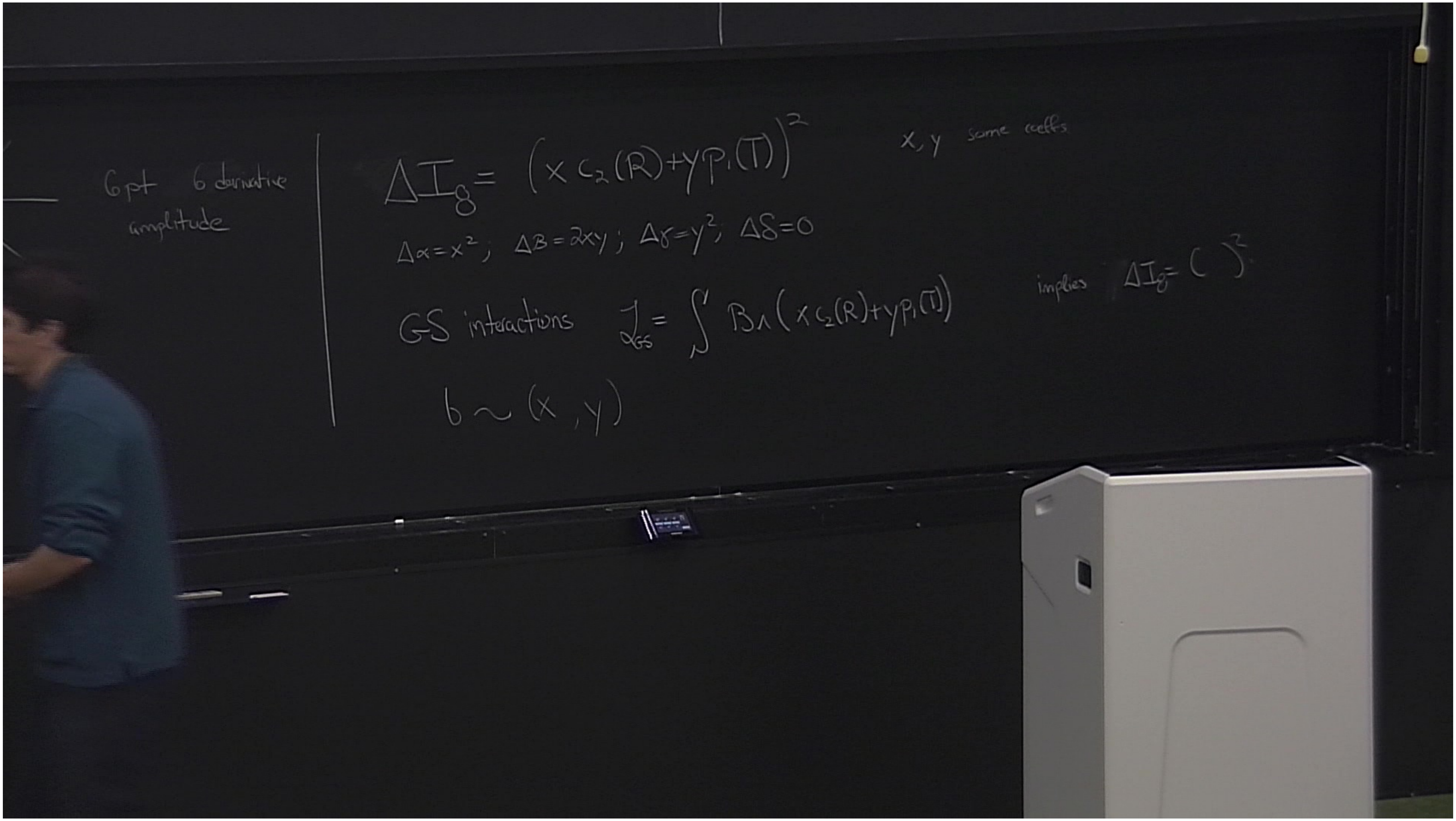
$\uparrow$

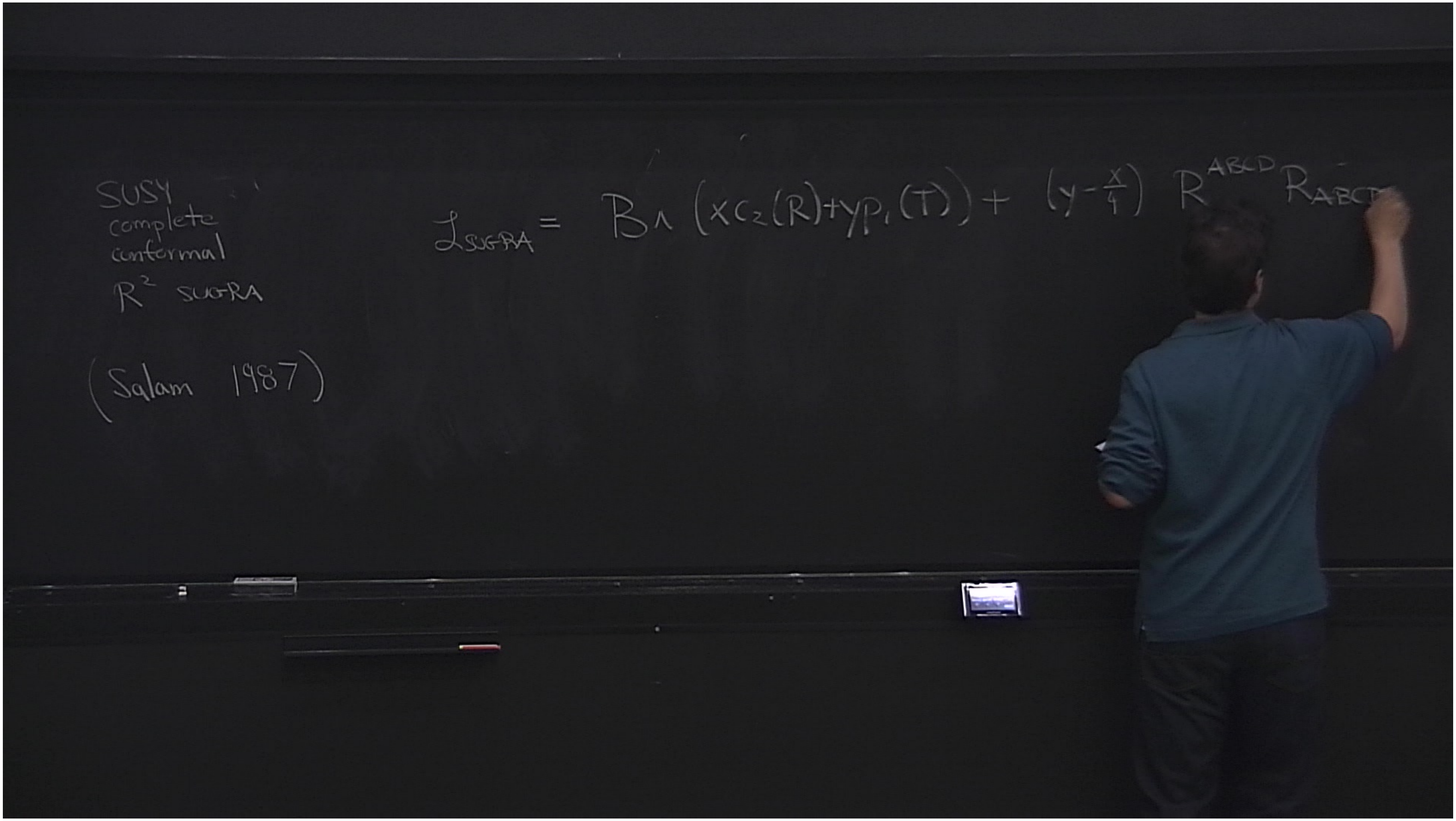
Lorentz scalar

$SU(2)_R$ inv


Q^δ(O) 'for any O ?!
EOM
Q^δ ∂_A(∂^A φ φ^4)
change
wrong to view Δa as infinitesimal def
instead it arises in SUSY completing (∂φ)^4
universal
relation!
Δa = 98304π^3/7 b^2


SU(2)_R inv
b
b
6pt 6 derivative
amplitude
Δa ~ b²
ΔI_g =


6pt 6 derivative amplitude
$\Delta I_8 = (x\, c_2(R) + y\, p_1(T))^2$
x, y some coeffs
$\Delta\alpha = x^2;\ \Delta\beta = 2xy;\ \Delta\gamma = y^2;\ \Delta\delta = 0$
implies $\Delta I_8 = (\ )^2$
GS interactions $\mathcal{L}_{GS} = \int B \wedge (x\, c_2(R) + y\, p_1(T))$
$b \sim (x, y)$

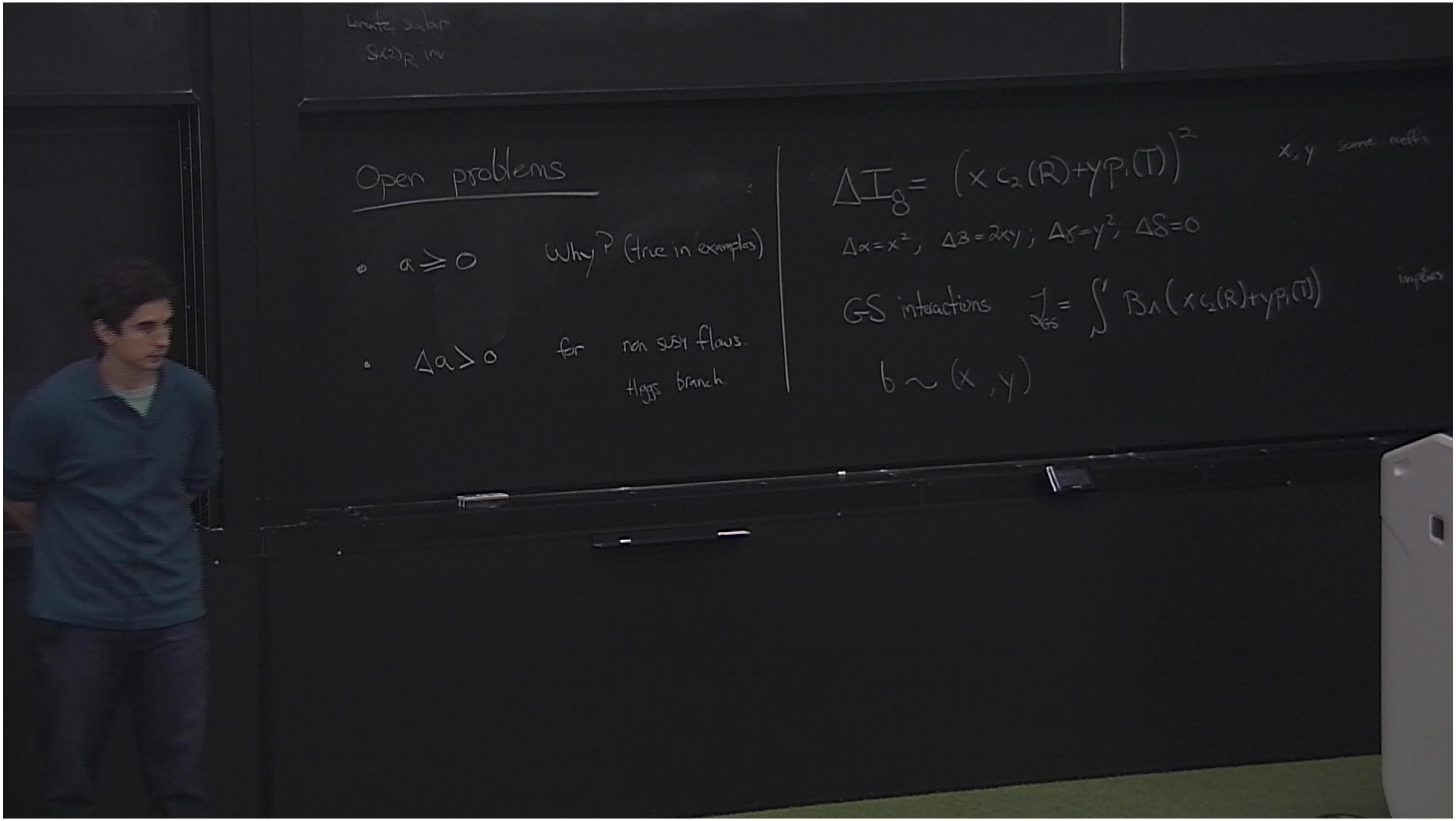

SUSY
complete
conformal
R² SUGRA
(Salam 1987)
L_SUGRA = B ∧ (x c₂(R) + y p₁(T)) + (y − x/4) R^ABCD R_ABCD

$\varphi \quad , \quad \varphi_\alpha \qquad dB = H_{ABC} = * H_{ABC}$

$$\mathcal{L}_{SUGRA} = B \wedge \left(x c_2(R) + y p_1(T) \right) + \underbrace{\left(y - \frac{x}{4} \right) R^{ABCD} R_{ABCD} + \frac{3}{2} x \, R_{[AB}{}^{AB} R_{CD]}{}^{CD}}_{\text{restrict to conformally flat}}$$

$(x-y)(\partial\varphi)^4$

$$b = (x-y) \Rightarrow \boxed{\Delta a = \frac{16}{7}\left(\Delta\alpha - \Delta\beta + \Delta\gamma\right)} \Longrightarrow a = \frac{16}{7}\left(\alpha - \beta + \gamma\right) + \frac{6}{7}\delta$$


Lorentz scalar
SU(2)R inv
Open problems
a ≥ 0 Why? (true in examples)
Δa > 0 for non susy flows.
Higgs branch
ΔI8 = (x c2(R) + y p1(T))^2
x, y some coeffs
Δα = x^2; Δβ = 2xy; Δγ = y^2; Δδ = 0
GS interactions
implies
b ~ (x, y)