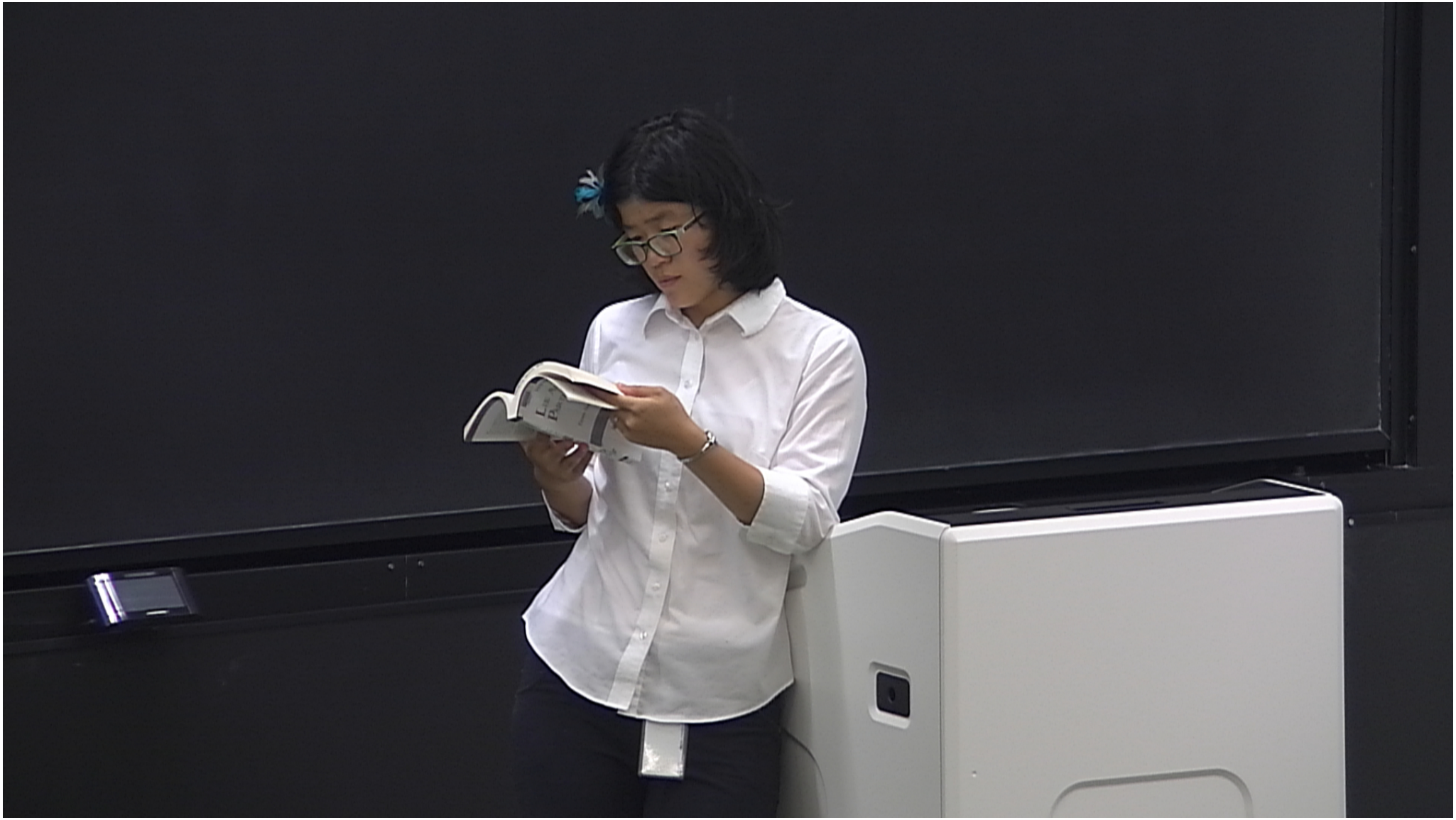
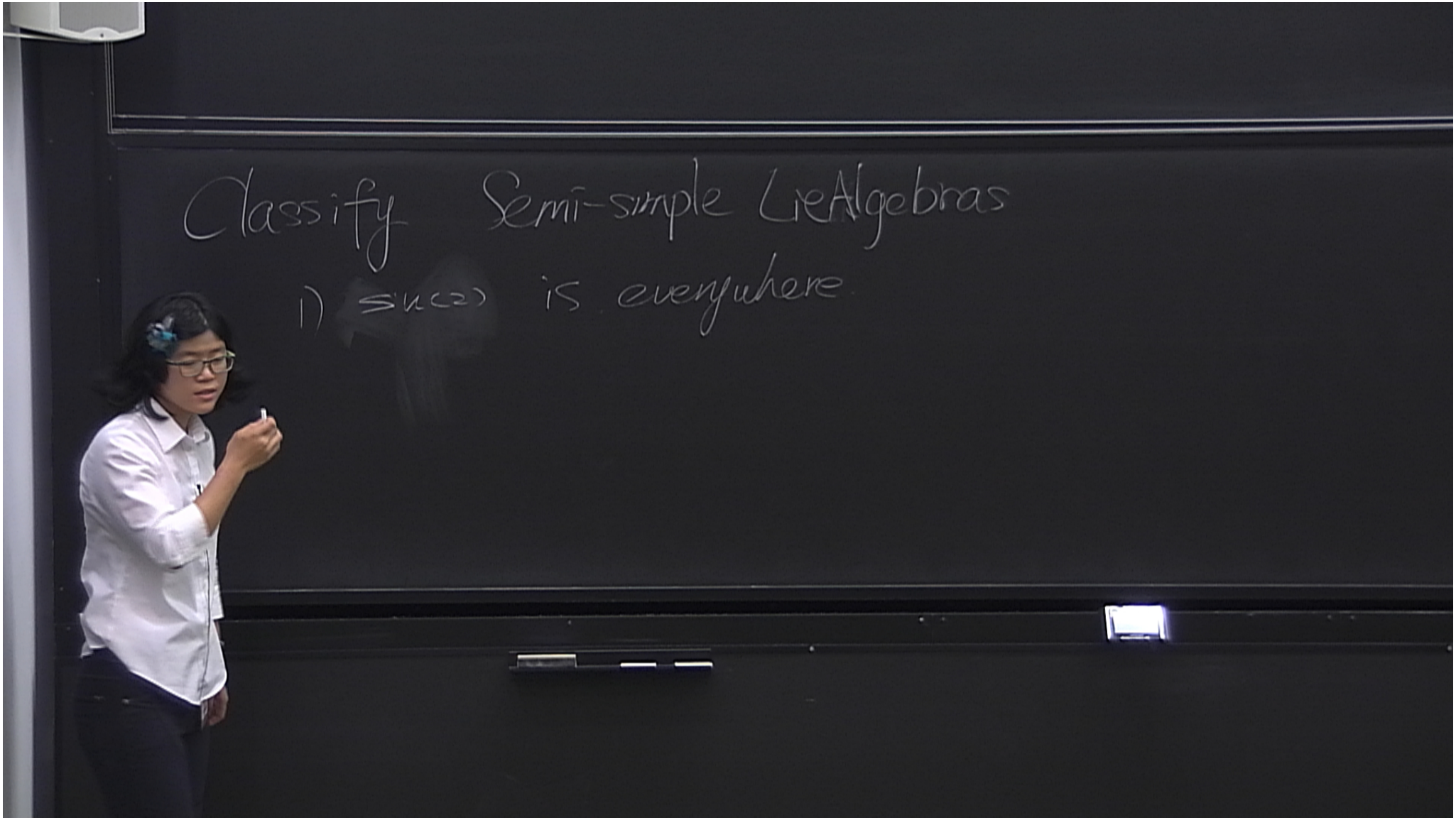
Title: PSI 2015/2016 Lie Groups and Algebras - Gang Xu -5

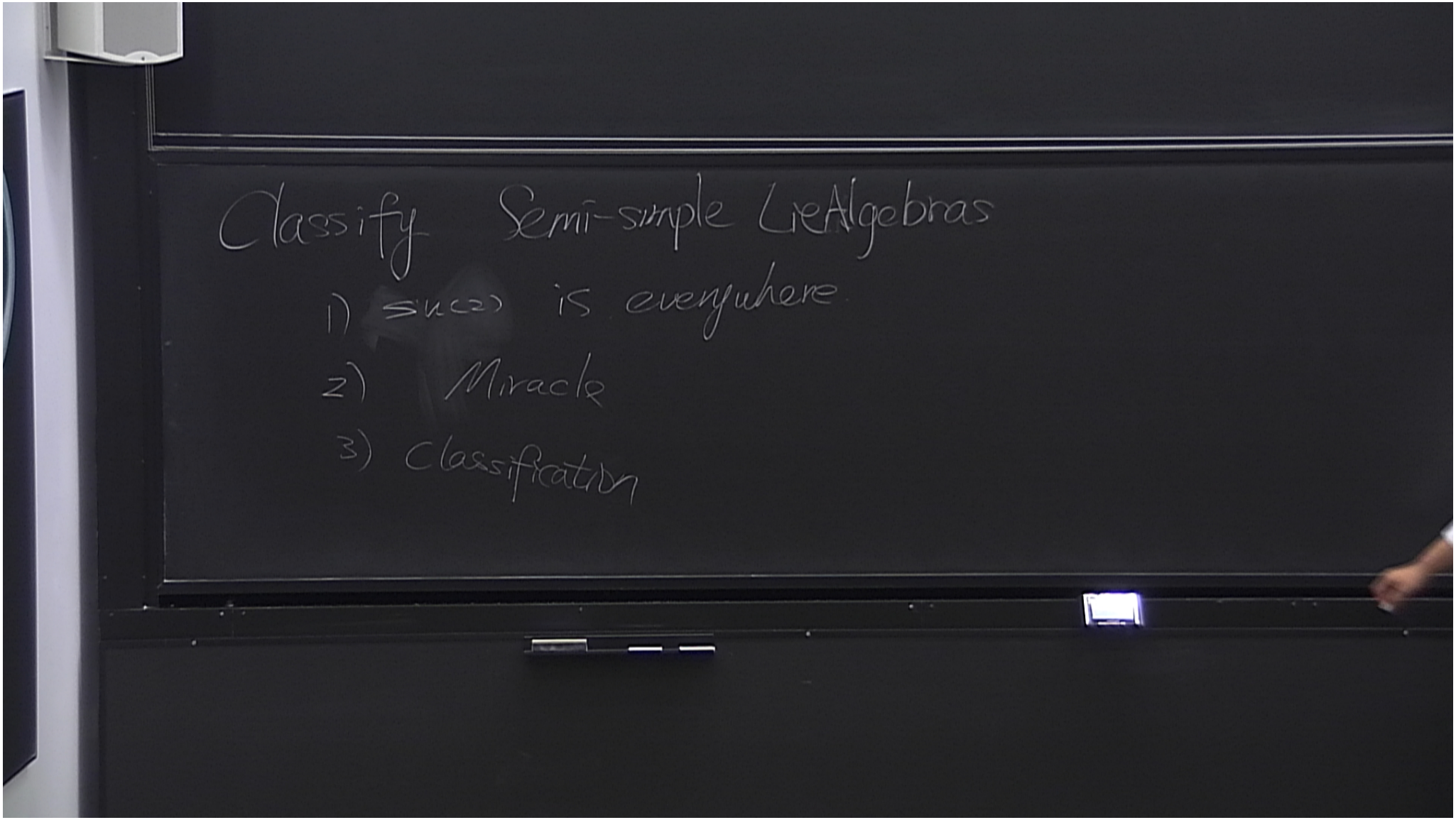
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URL: http://pirsa.org/15090013

Abstract:

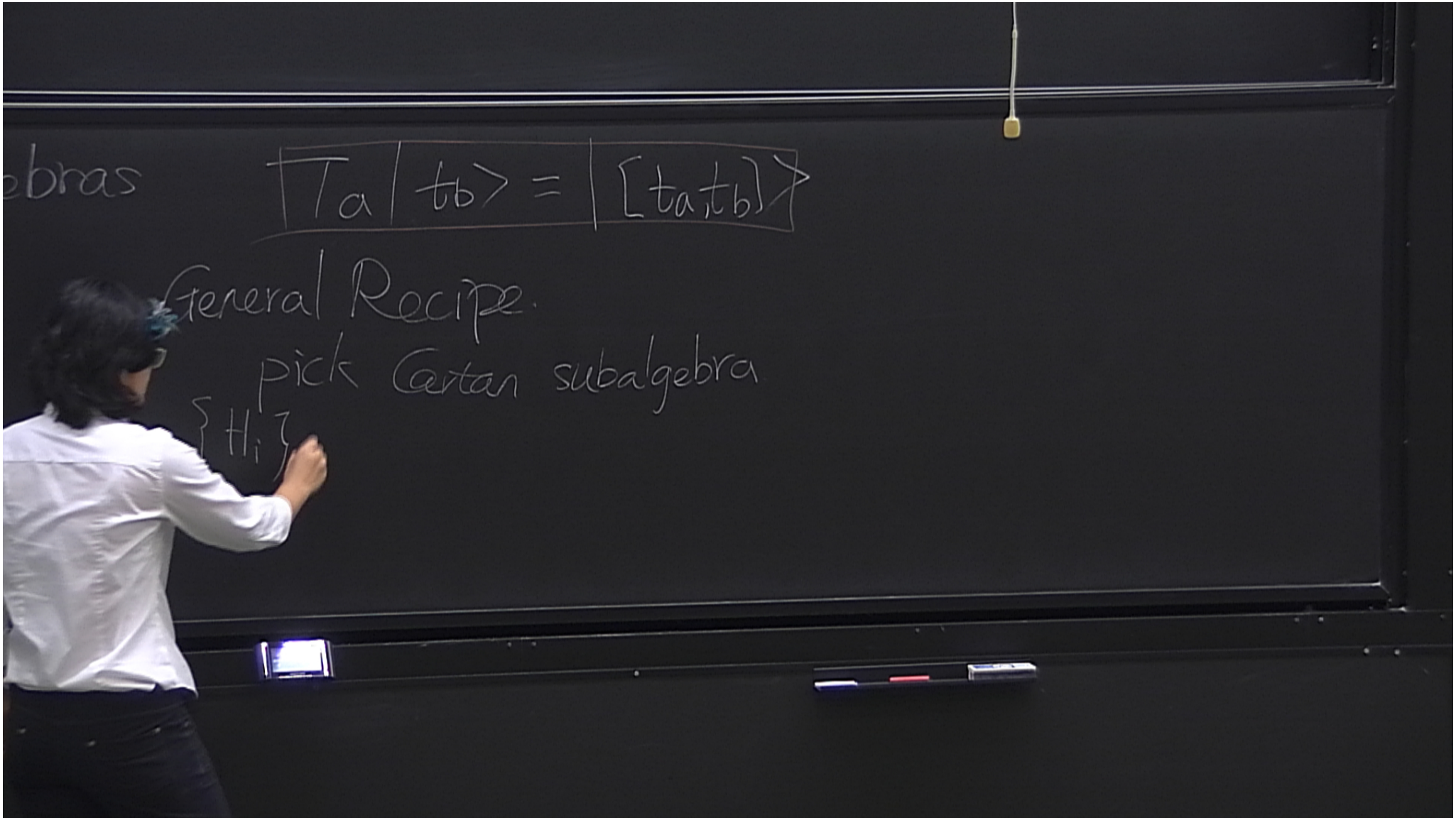


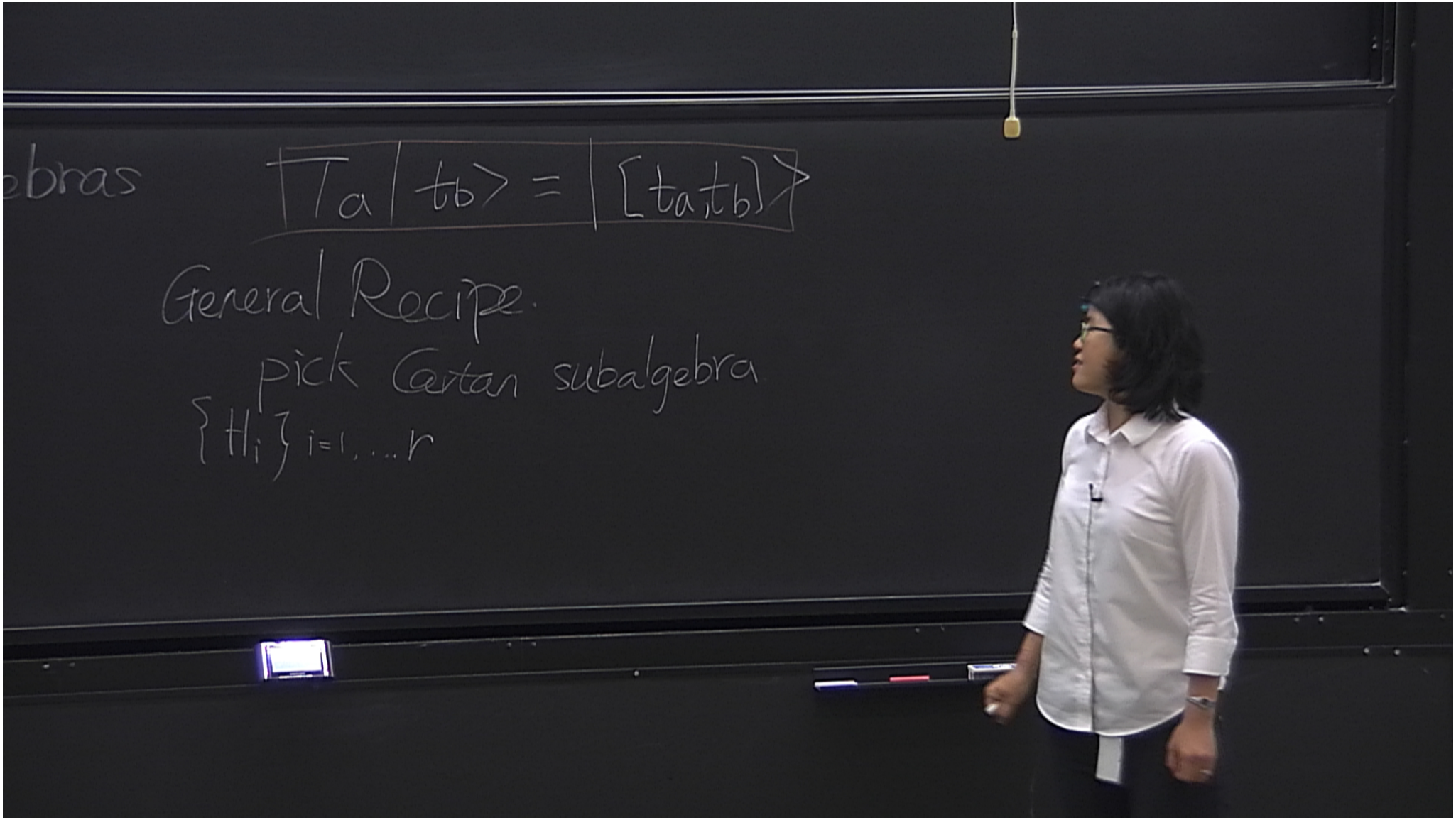

Classify Semi-simple LieAlgebras
1) su(2) is everywhere

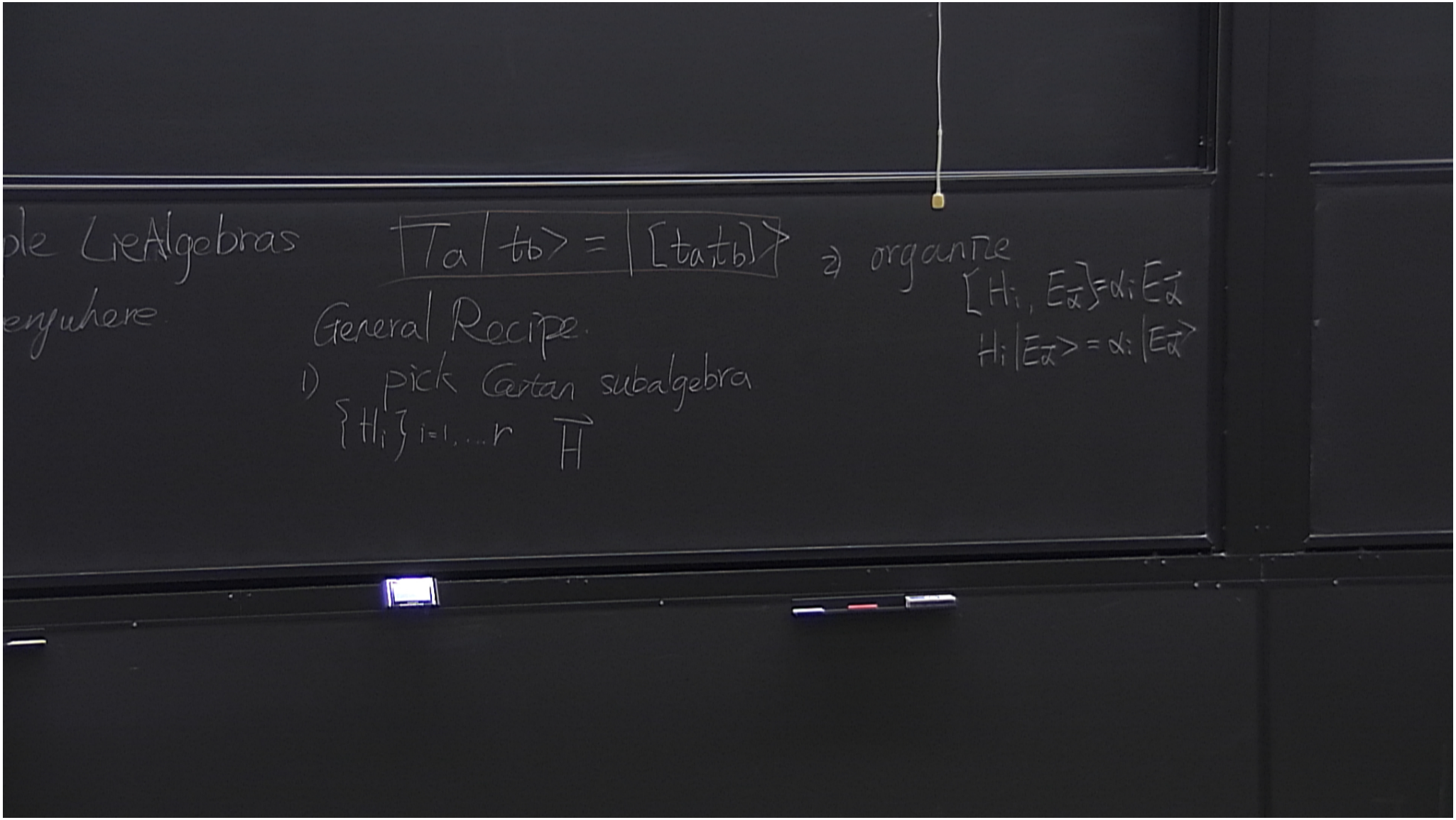

Classify Semi-simple LieAlgebras
1) su(2) is everywhere
2) Miracle
3) Classification

Classify Semi-simple LieAlgebras $T_a|t_b\rangle = |[t_a,t_b]\rangle$

1) su(2) is everywhere
2) Miracle
3) Classification


ebras
$T_a|t_b\rangle = |[t_a, t_b]\rangle$
General Recipe:
pick Cartan subalgebra
$\{H_i\}$


ebras
$T_a |t_b\rangle = |[t_a, t_b]\rangle$
General Recipe.
pick Cartan subalgebra
$\{H_i\}_{i=1,\ldots r}$


ple Lie Algebras
everywhere
$T_a|t_b\rangle = |[t_a,t_b]\rangle$
a organize
General Recipe
1) pick Cartan subalgebra
$\{H_i\}_{i=1,\dots r}$ $\vec{H}$
$[H_i, E_{\vec{\alpha}}] = \alpha_i E_{\vec{\alpha}}$
$H_i|E_{\vec{\alpha}}\rangle = \alpha_i |E_{\vec{\alpha}}\rangle$

$|T_a|t_b\rangle = |[t_a, t_b]\rangle$

General Recipe.

1) pick Cartan subalgebra

$\{H_i\}_{i=1,\ldots r}$ $\vec{H}$

2) organize

$[H_i, E_{\vec{\alpha}}] = \alpha_i E_{\vec{\alpha}}$

$H_i |E_{\vec{\alpha}}\rangle = \alpha_i |E_{\vec{\alpha}}\rangle$

↙ weights

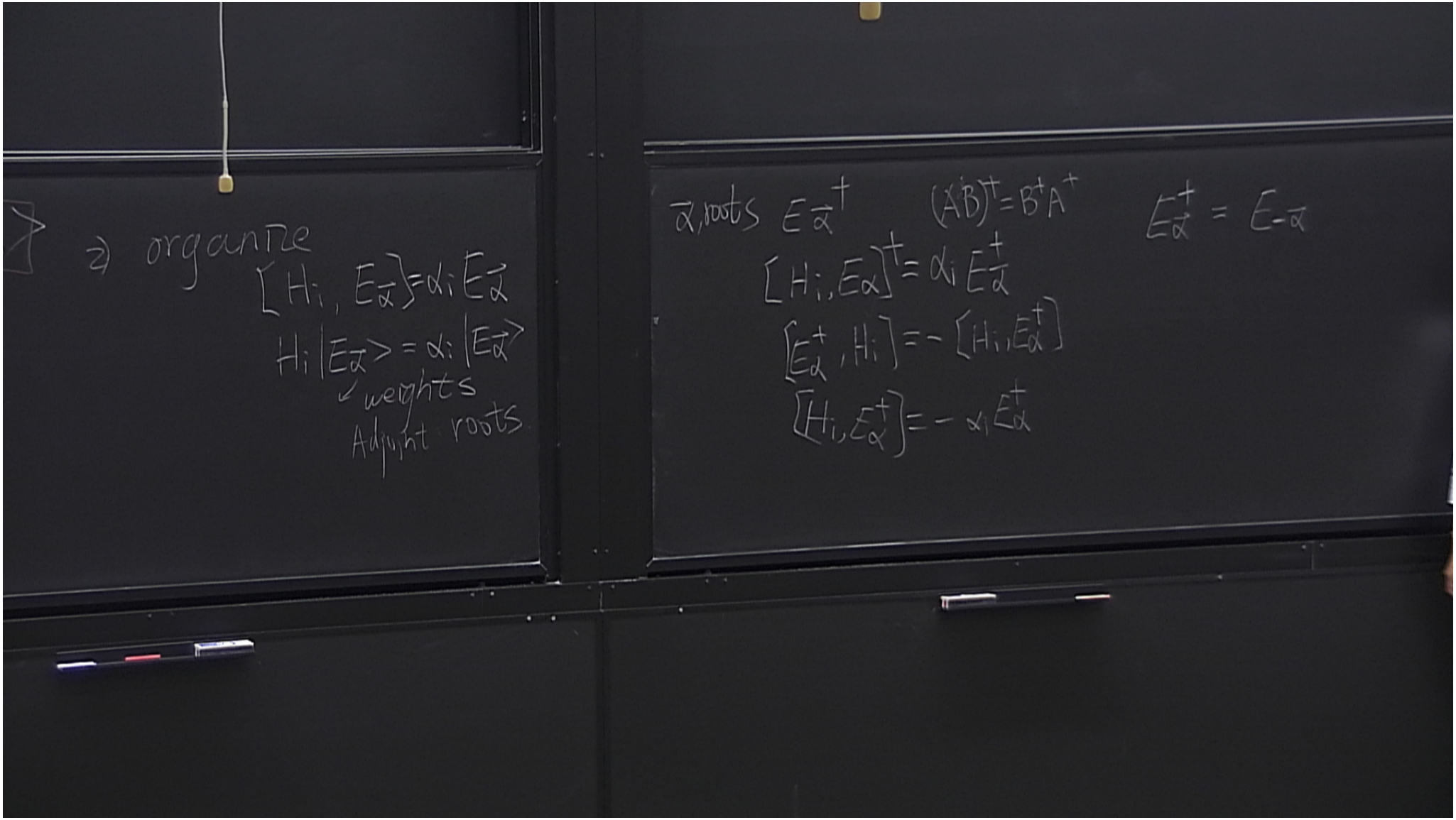
Adjoint: roots

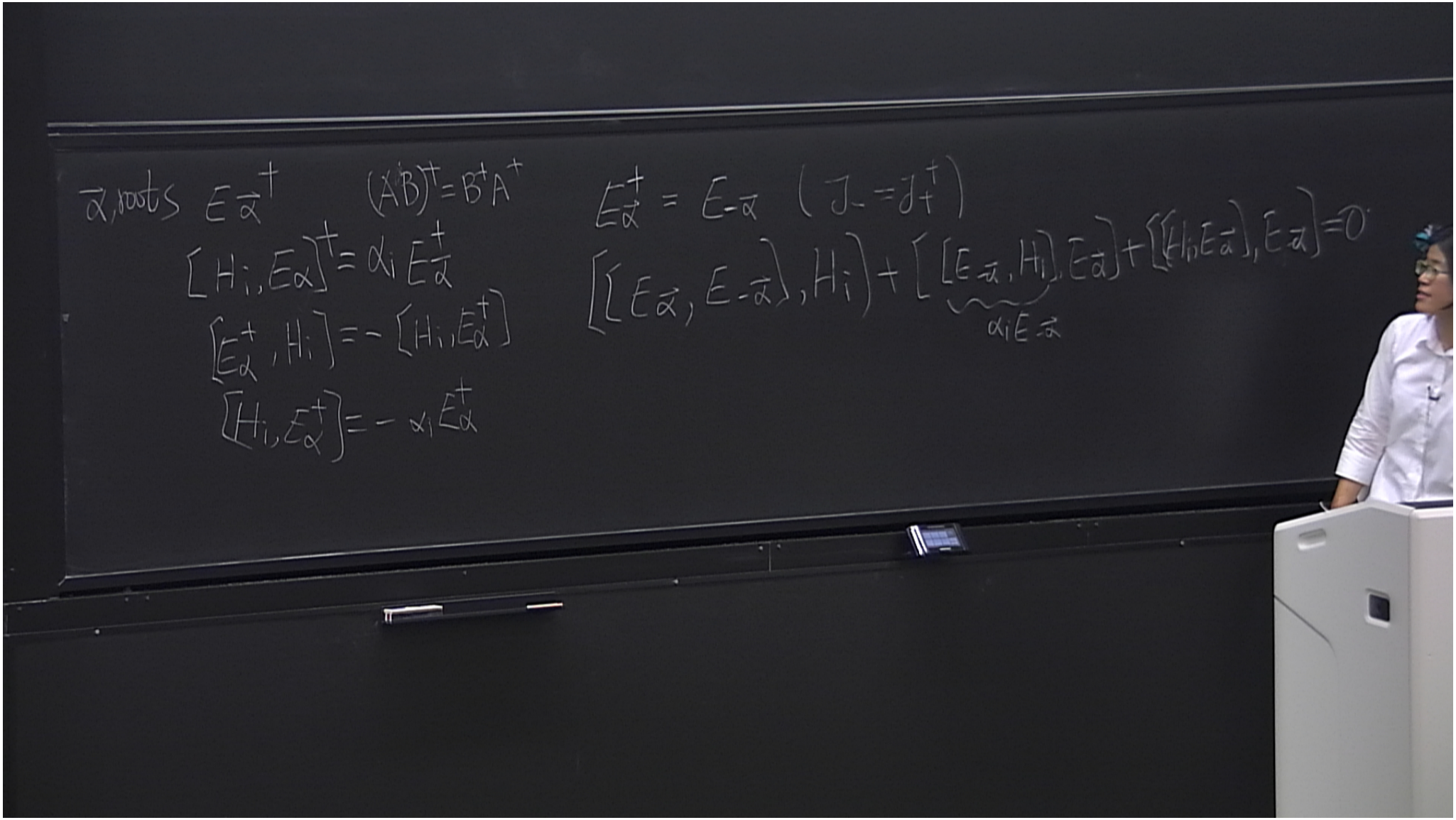
$E_{\vec{\alpha}}^{\dagger}$ $\qquad (AB)^{\dagger} = B^{\dagger} A^{\dagger}$

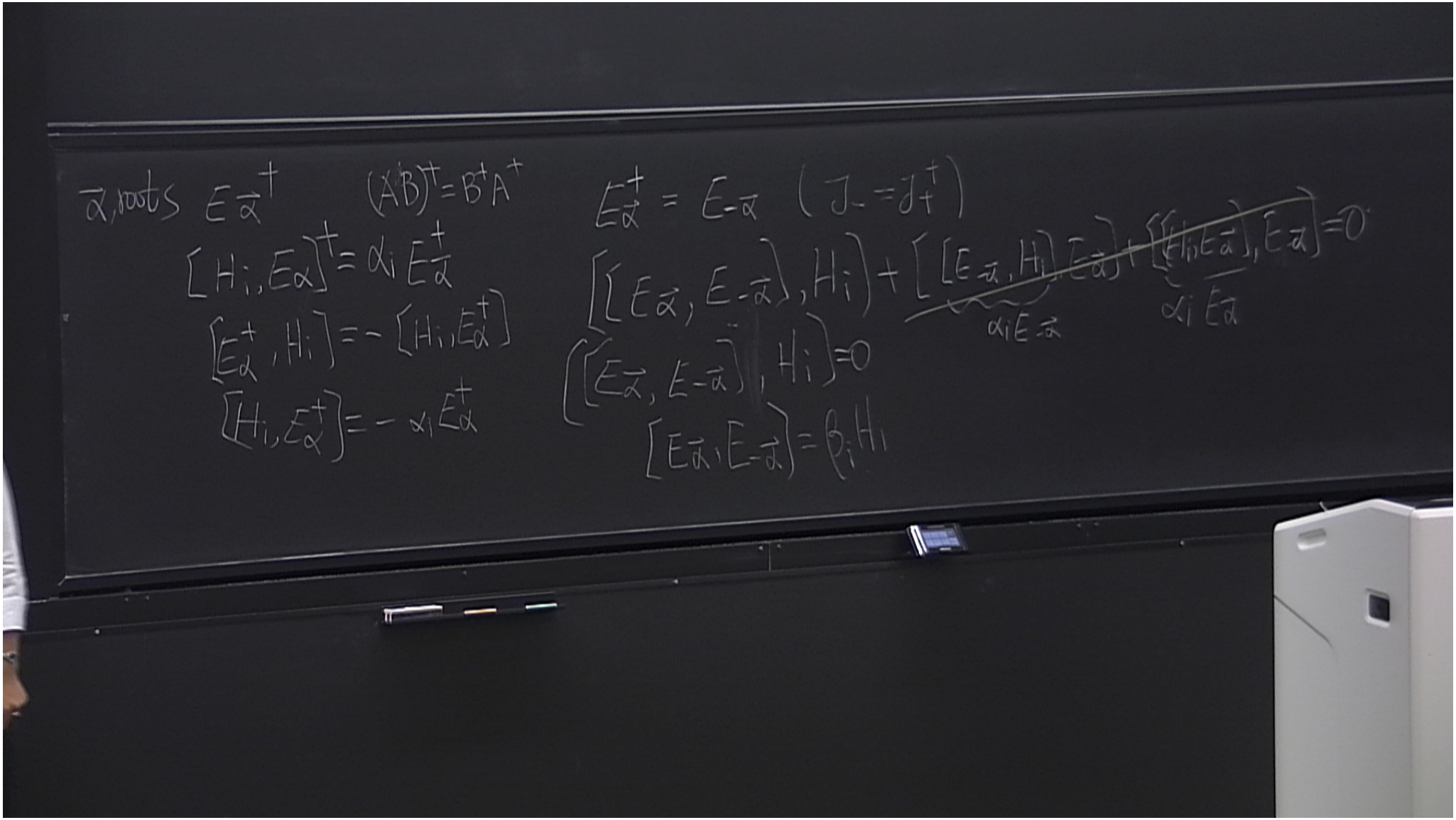
$$[H_i, E_\alpha]^{\dagger} = \alpha_i E_\alpha^{\dagger}$$

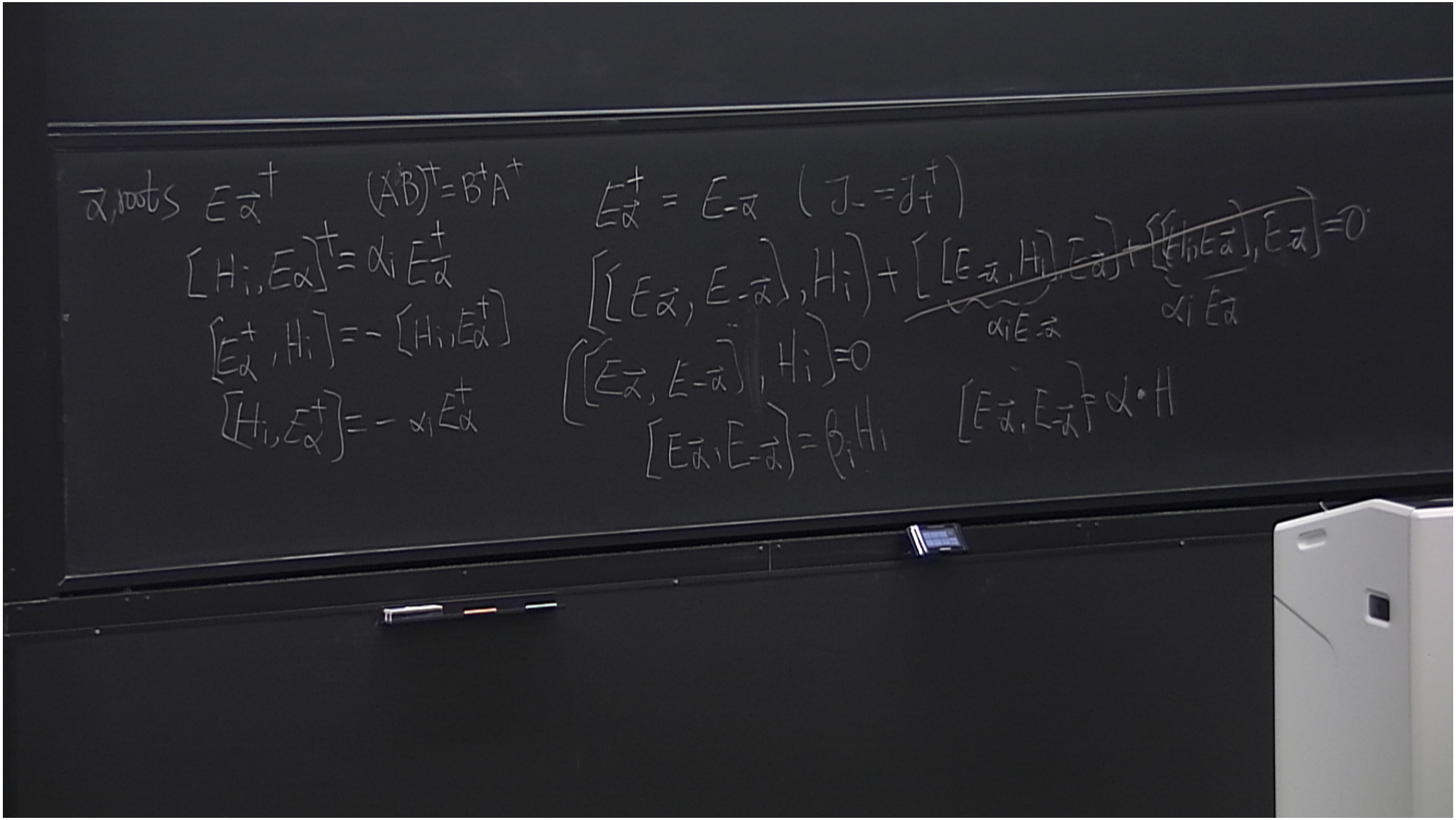
$$[E_\alpha^{\dagger}, H_i] = -[H_i, E_\alpha^{\dagger}]$$

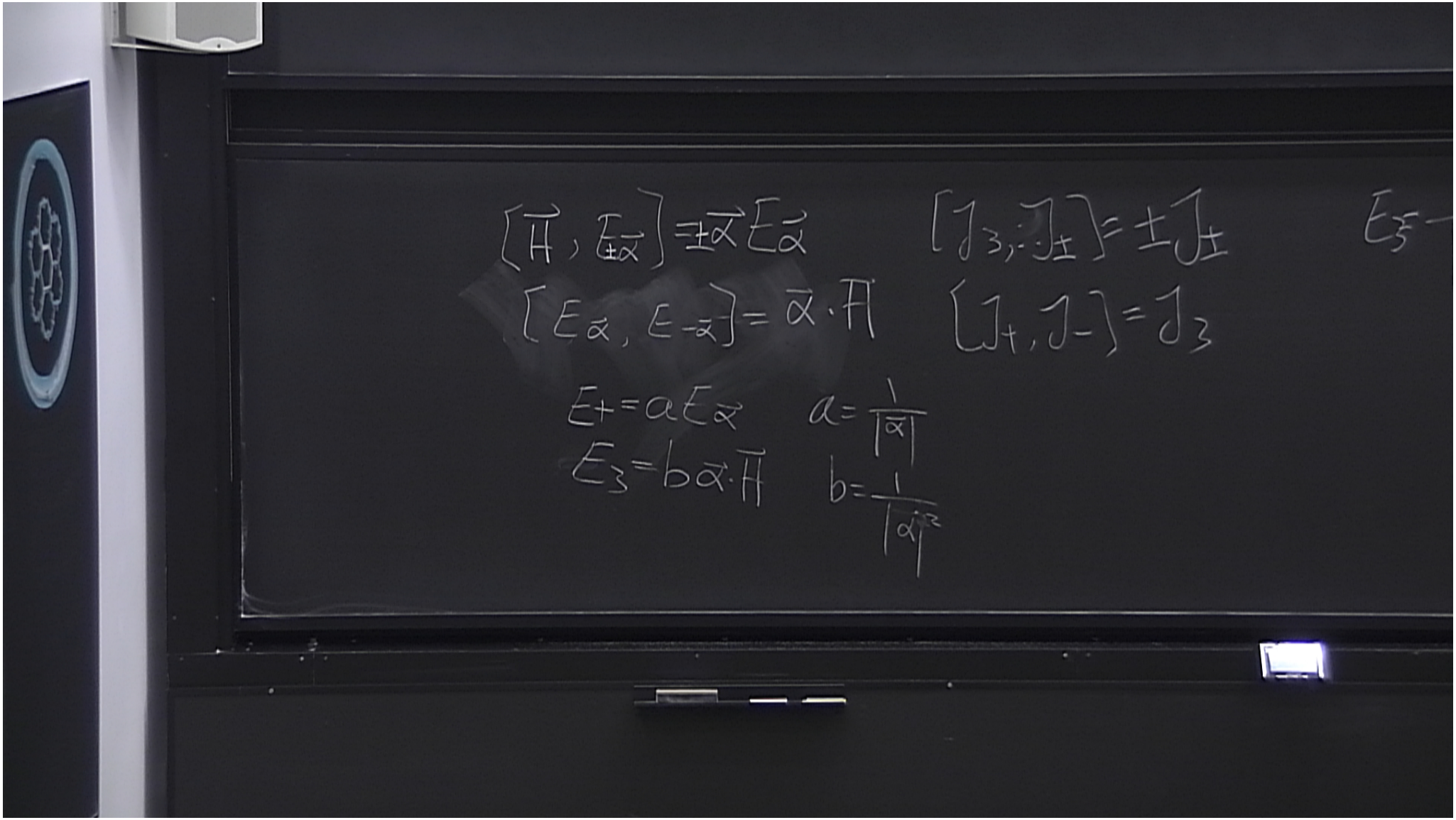
$$[H_i, E_\alpha^{\dagger}] = -\alpha_i E_\alpha^{\dagger}$$

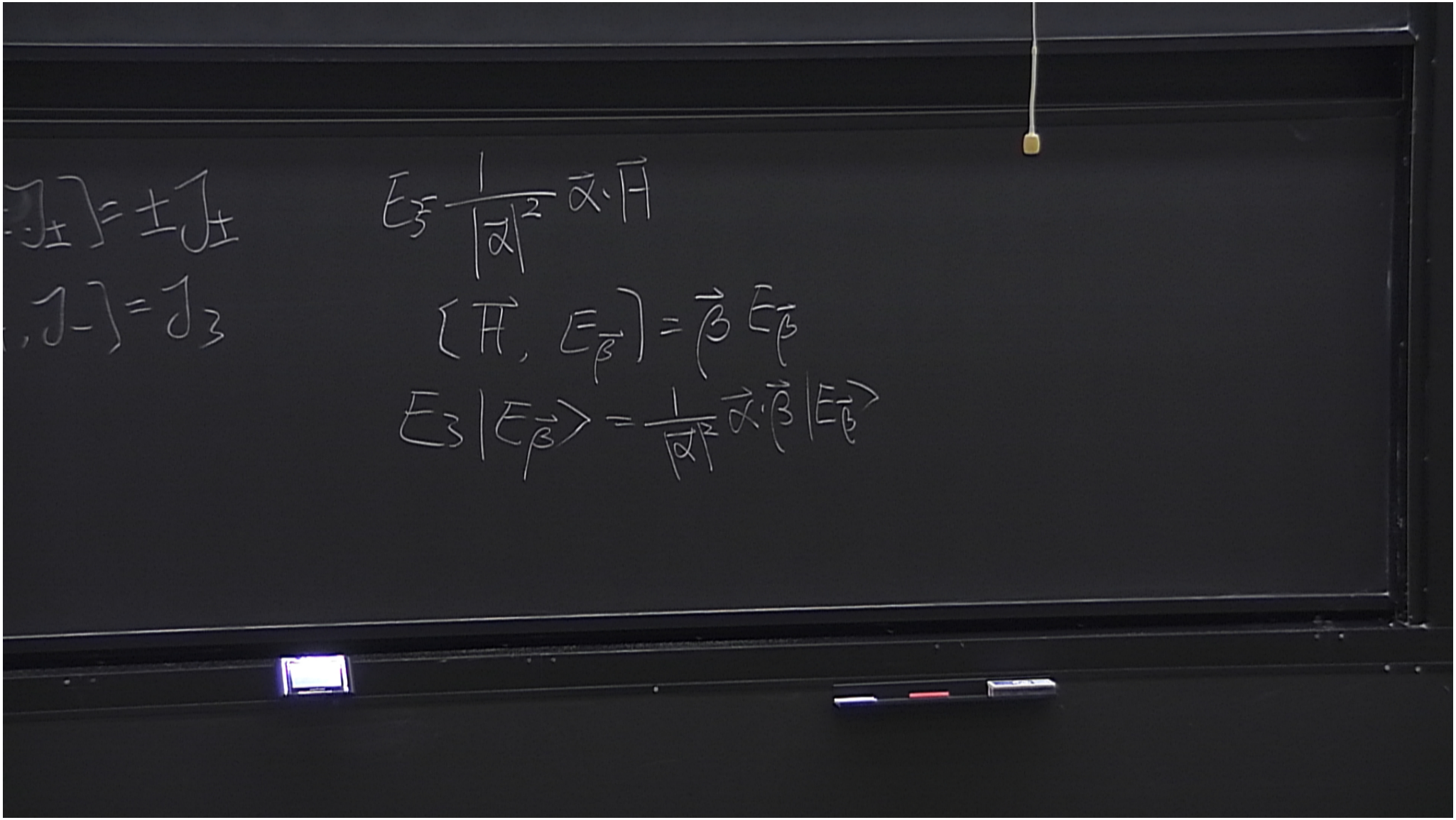

2) organize
$[H_i, E_{\vec{\alpha}}] = \alpha_i E_{\vec{\alpha}}$
$H_i |E_{\vec{\alpha}}\rangle = \alpha_i |E_{\vec{\alpha}}\rangle$
weights
Adjoint roots
$\vec{\alpha}$, roots $E_{\vec{\alpha}}^\dagger$
$(AB)^\dagger = B^\dagger A^\dagger$
$E_{\vec{\alpha}}^\dagger = E_{-\vec{\alpha}}$
$[H_i, E_\alpha]^\dagger = \alpha_i E_\alpha^\dagger$
$[E_\alpha^\dagger, H_i] = -[H_i, E_\alpha^\dagger]$
$[H_i, E_\alpha^\dagger] = -\alpha_i E_\alpha^\dagger$

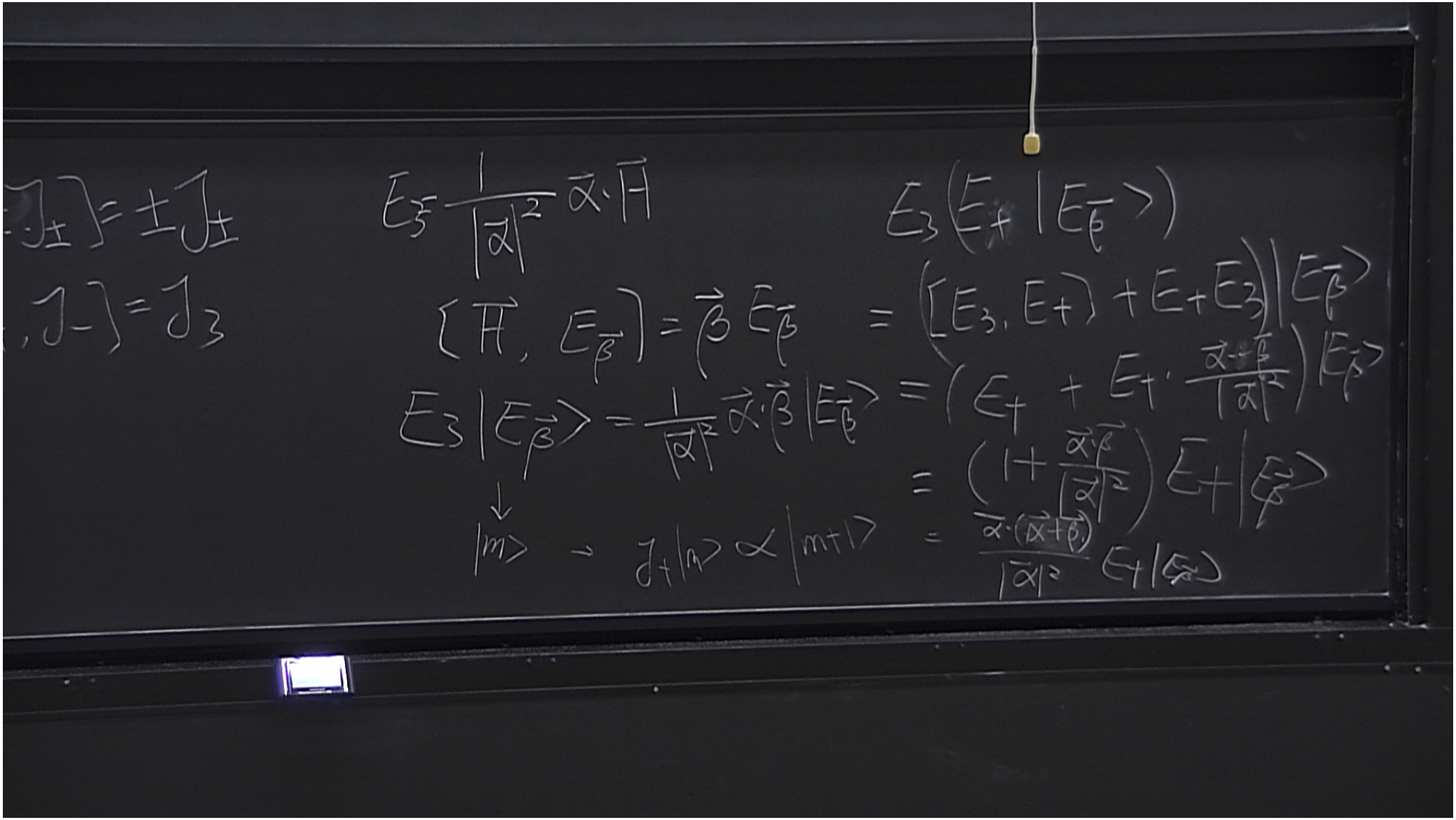

$\vec{\alpha}$, roots $E_{\vec{\alpha}}$ $(AB)^\dagger = B^\dagger A^\dagger$ $E_{\vec{\alpha}}^\dagger = E_{-\vec{\alpha}}$ $(J_- = J_+^\dagger)$
$[H_i, E_{\vec{\alpha}}]^\dagger = \alpha_i E_{\vec{\alpha}}^\dagger$
$[E_{\vec{\alpha}}^\dagger, H_i] = -[H_i, E_{\vec{\alpha}}^\dagger]$
$[H_i, E_{\vec{\alpha}}^\dagger] = -\alpha_i E_{\vec{\alpha}}^\dagger$
$[[E_{\vec{\alpha}}, E_{-\vec{\alpha}}], H_i] + [\underbrace{[E_{-\vec{\alpha}}, H_i]}_{\alpha_i E_{-\vec{\alpha}}}, E_{\vec{\alpha}}] + [[H_i, E_{\vec{\alpha}}], E_{-\vec{\alpha}}] = 0$

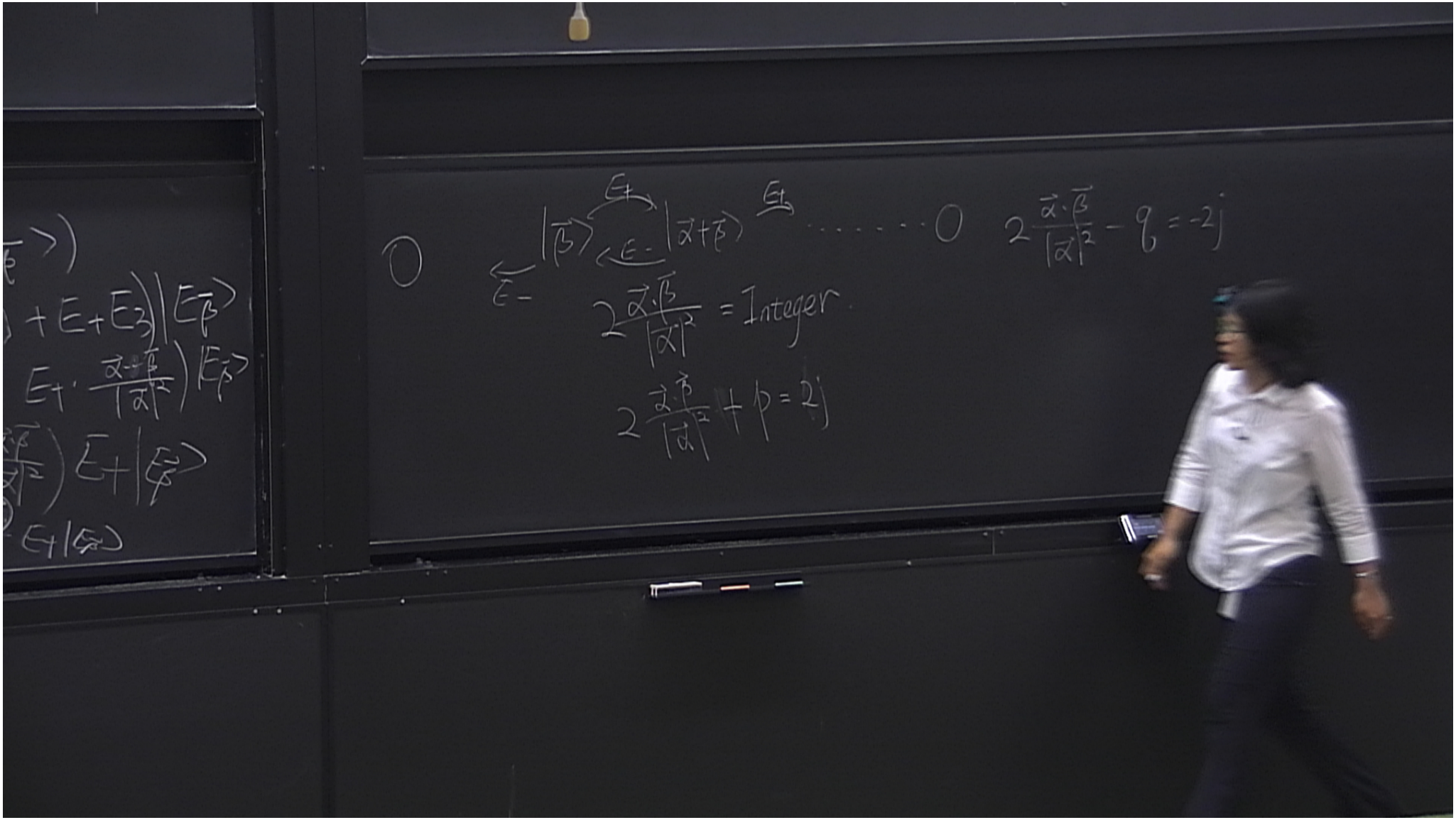

$\vec{\alpha}$, roots $E_{\vec{\alpha}}^{\dagger}$ $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$ $E_{\vec{\alpha}}^{\dagger} = E_{-\vec{\alpha}}$ $(J_{-} = J_{+}^{\dagger})$
$[H_i, E_{\vec{\alpha}}]^{\dagger} = \alpha_i E_{\vec{\alpha}}^{\dagger}$
$[E_{\vec{\alpha}}^{\dagger}, H_i] = -[H_i, E_{\vec{\alpha}}^{\dagger}]$
$[H_i, E_{\vec{\alpha}}^{\dagger}] = -\alpha_i E_{\vec{\alpha}}^{\dagger}$
$[[E_{\vec{\alpha}}, E_{-\vec{\alpha}}], H_i] + [[E_{-\vec{\alpha}}, H_i], E_{\vec{\alpha}}] + [[H_i, E_{\vec{\alpha}}], E_{\vec{\alpha}}] = 0$
$\alpha_i E_{-\vec{\alpha}}$
$\alpha_i E_{\vec{\alpha}}$
$[[E_{\vec{\alpha}}, E_{-\vec{\alpha}}], H_i] = 0$
$[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \beta_i H_i$

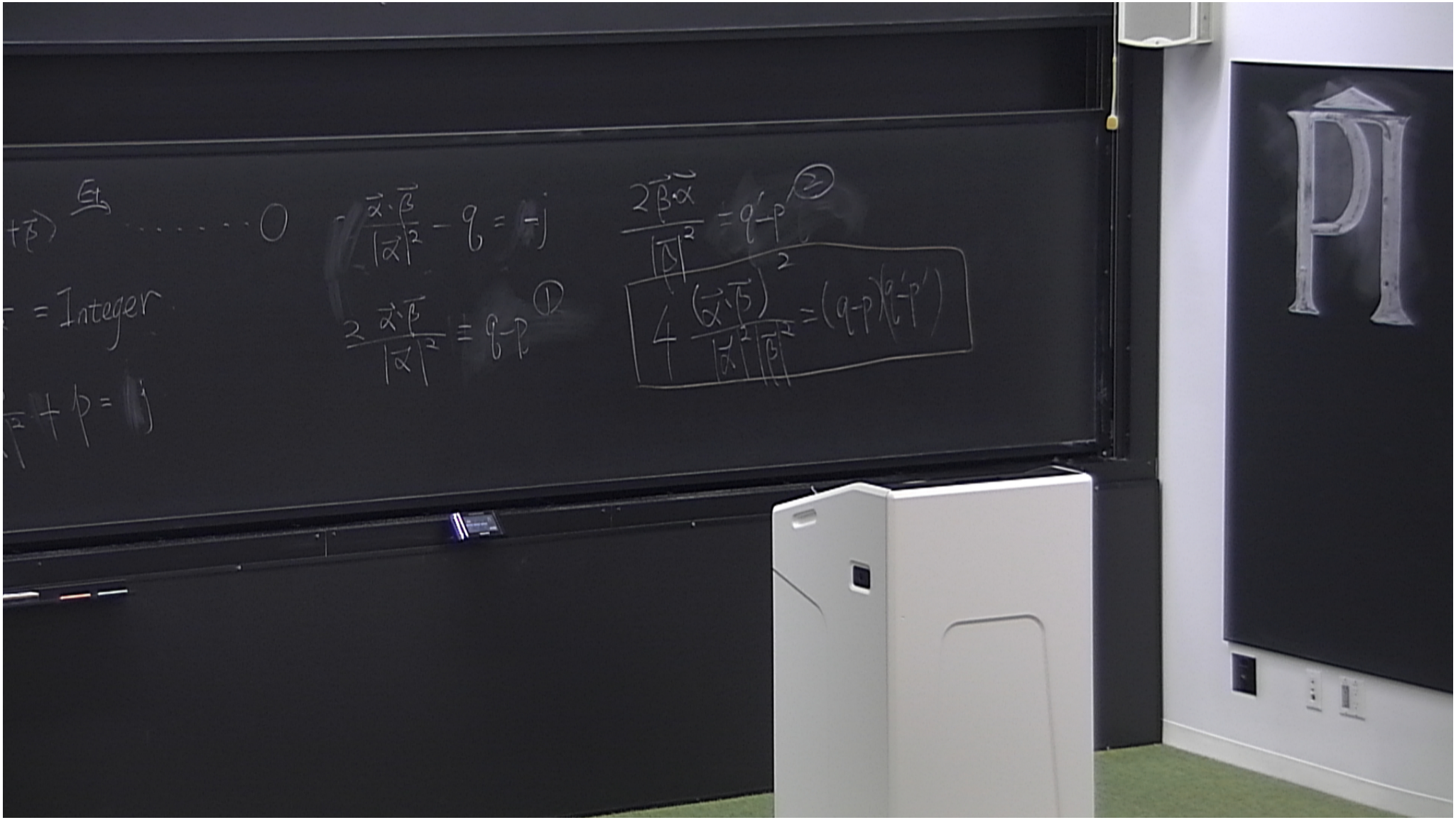

$\vec{\alpha}$, roots $E_{\vec{\alpha}}^{\dagger}$ $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$ $E_{\vec{\alpha}}^{\dagger} = E_{-\vec{\alpha}}$ $(J_{-} = J_{+}^{\dagger})$
$[H_i, E_{\vec{\alpha}}]^{\dagger} = \alpha_i E_{\vec{\alpha}}^{\dagger}$
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$[H_i, E_{\vec{\alpha}}^{\dagger}] = -\alpha_i E_{\vec{\alpha}}^{\dagger}$
$[[E_{\vec{\alpha}}, E_{-\vec{\alpha}}], H_i] + [[E_{-\vec{\alpha}}, H_i], E_{\vec{\alpha}}] + [[H_i, E_{\vec{\alpha}}], E_{-\vec{\alpha}}] = 0$
$\alpha_i E_{-\vec{\alpha}}$
$\alpha_i E_{\vec{\alpha}}$
$[[E_{\vec{\alpha}}, E_{-\vec{\alpha}}], H_i] = 0$
$[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \beta_i H_i$
$[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \alpha \cdot H$

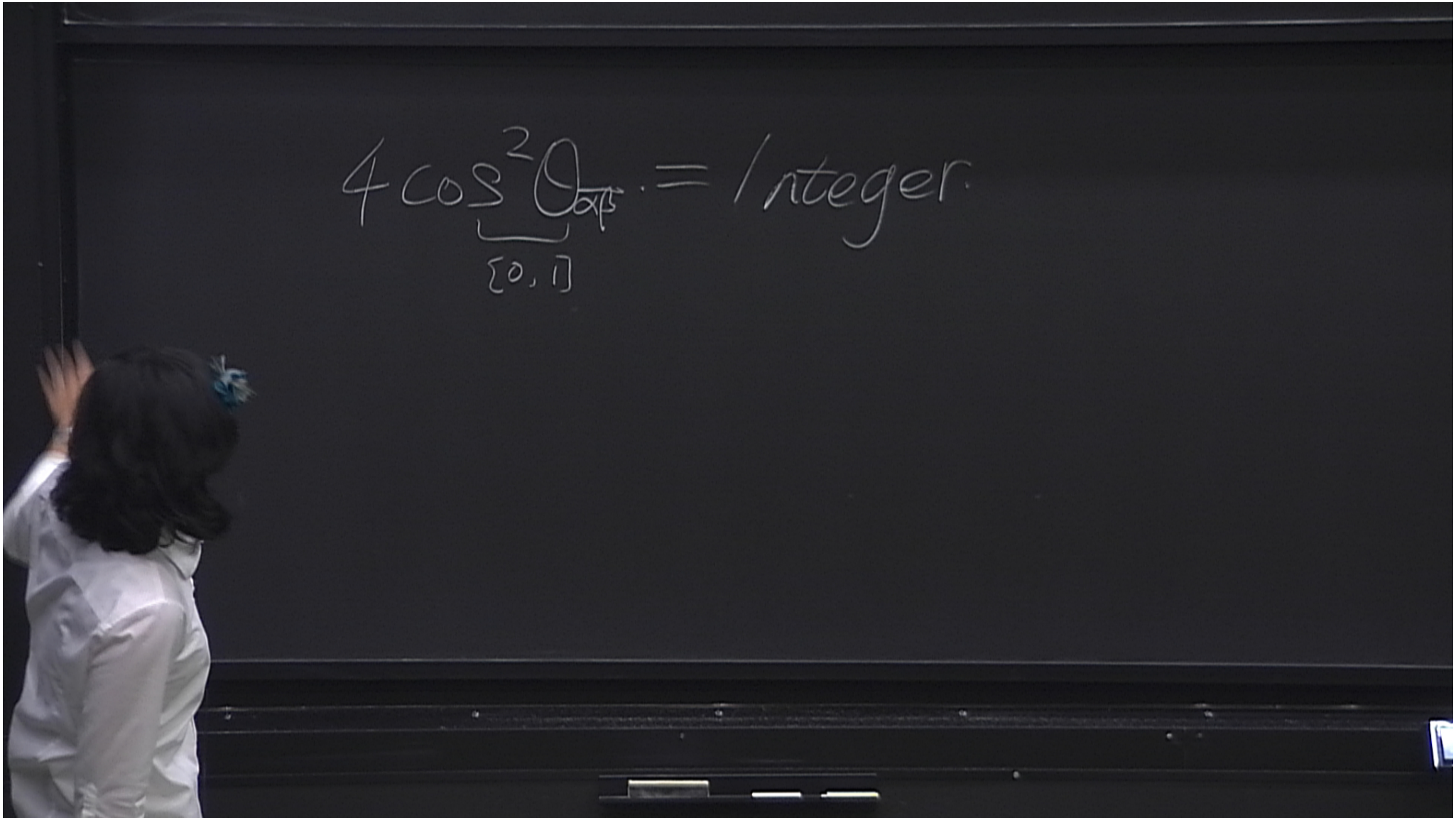

$[\vec{H}, E_{\pm\vec{\alpha}}] = \pm\vec{\alpha} E_{\vec{\alpha}}$
$[J_3, J_\pm] = \pm J_\pm$
$[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \vec{\alpha}\cdot\vec{H}$
$[J_+, J_-] = J_3$
$E_+ = a E_{\vec{\alpha}}$ $a = \frac{1}{|\vec{\alpha}|}$
$E_3 = b\vec{\alpha}\cdot\vec{H}$ $b = \frac{1}{|\vec{\alpha}|^2}$

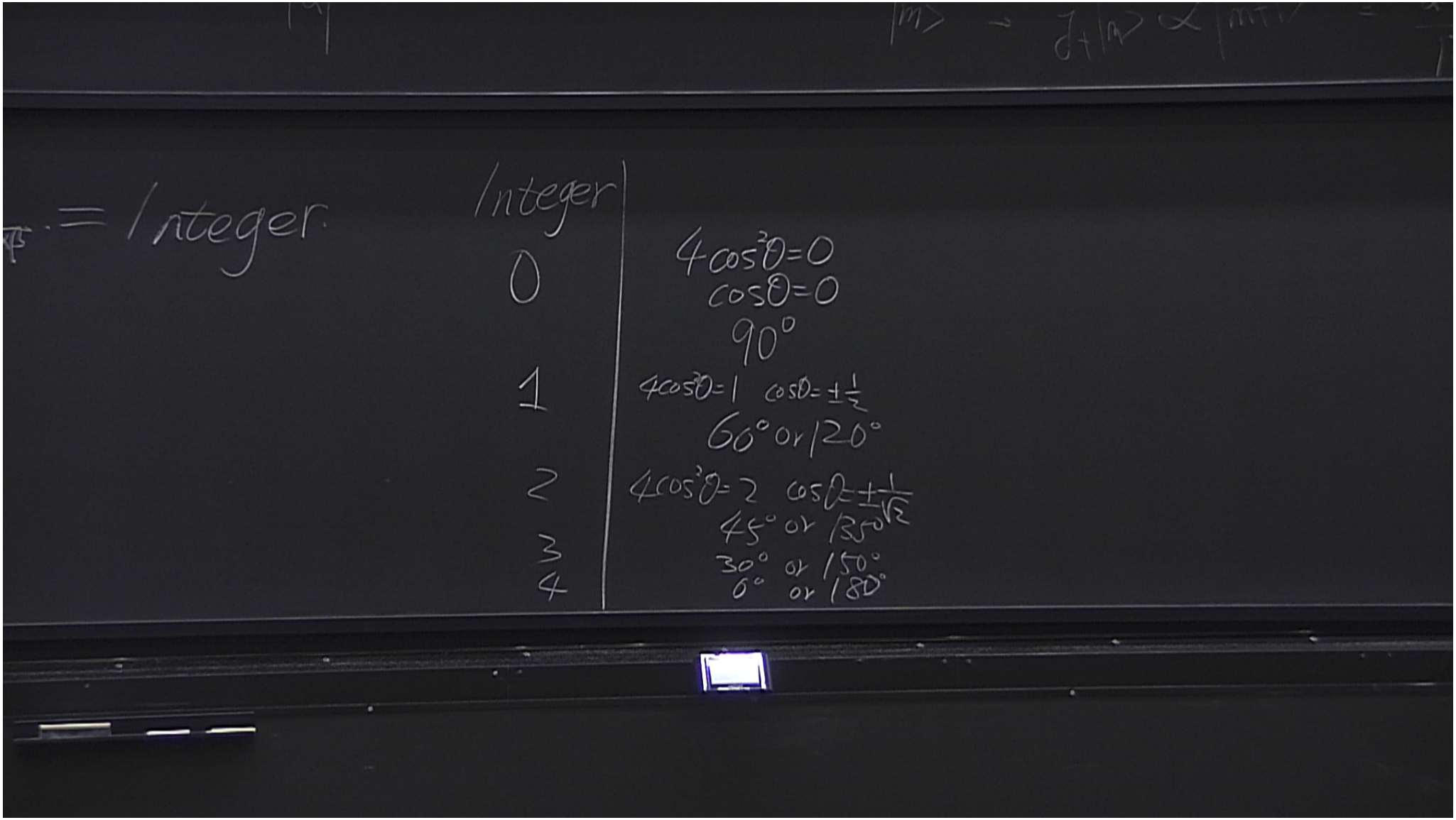

$[J_\pm] = \pm J_\pm$
$[J_+, J_-] = J_3$
$E_3 \frac{1}{|\vec{\alpha}|^2} \vec{\alpha} \cdot \vec{H}$
$[\vec{H}, E_{\vec{\beta}}] = \vec{\beta} E_{\vec{\beta}}$
$E_3 |E_{\vec{\beta}}\rangle = \frac{1}{|\vec{\alpha}|^2} \vec{\alpha} \cdot \vec{\beta} |E_{\vec{\beta}}\rangle$


$[J_3, J_\pm] = \pm J_\pm$
$[J_+, J_-] = J_3$
$E_3 = \frac{1}{|\alpha|^2} \vec{\alpha} \cdot \vec{H}$
$[\vec{H}, E_\beta] = \vec{\beta} E_\beta$
$E_3 |E_\beta\rangle = \frac{1}{|\alpha|^2} \vec{\alpha} \cdot \vec{\beta} |E_\beta\rangle$
$|m\rangle \rightarrow J_+ |m\rangle \propto |m+1\rangle$
$E_3 (E_+ |E_\beta\rangle)$
$= ([E_3, E_+] + E_+ E_3) |E_\beta\rangle$
$= \left(E_+ + E_+ \cdot \frac{\vec{\alpha} \cdot \vec{\beta}}{|\alpha|^2}\right) |E_\beta\rangle$
$= \left(1 + \frac{\vec{\alpha} \cdot \vec{\beta}}{|\alpha|^2}\right) E_+ |E_\beta\rangle$
$= \frac{\vec{\alpha} \cdot (\vec{\alpha} + \vec{\beta})}{|\alpha|^2} E_+ |E_\beta\rangle$

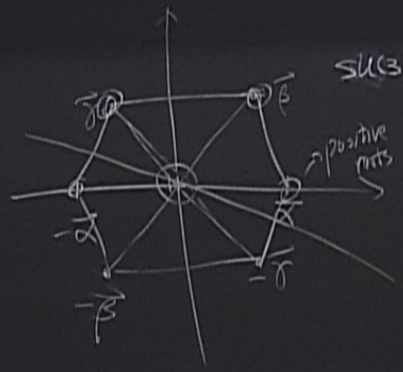

$|\beta\rangle$ E_+ E_- $|\alpha+\beta\rangle$ E_+
$2\frac{\vec{\alpha}\cdot\vec{\beta}}{|\alpha|^2} = \text{Integer}$
$2\frac{\vec{\alpha}\cdot\vec{\beta}}{|\alpha|^2} + p = 2j$
$2\frac{\vec{\alpha}\cdot\vec{\beta}}{|\alpha|^2} - q = -2j$


= Integer
PI


$4\cos^2\theta = \text{integer}$
$[0,1]$


= Integer
Integer
0
1
2
3
4
$4\cos^2\theta = 0$
$\cos\theta = 0$
90°
$4\cos^2\theta = 1$ $\cos\theta = \pm\frac{1}{2}$
60° or 120°
$4\cos^2\theta = 2$ $\cos\theta = \pm\frac{1}{\sqrt{2}}$
45° or 135°
30° or 150°
0° or 180°

(2)



$8 = 3^2 - 1$

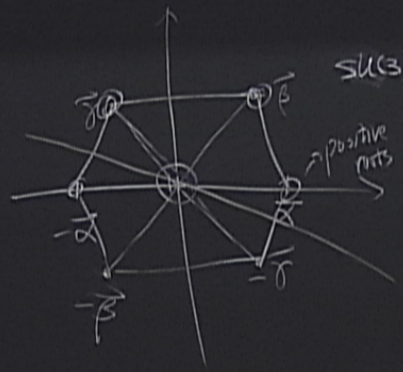
all roots come from simple roots

positive root

draw a r-1 plane that pass through origin.

simple roots are roots s.t other roots can be expressed by linear combination. positive integer coefficient

(2)



$8 = 3^2 - 1$

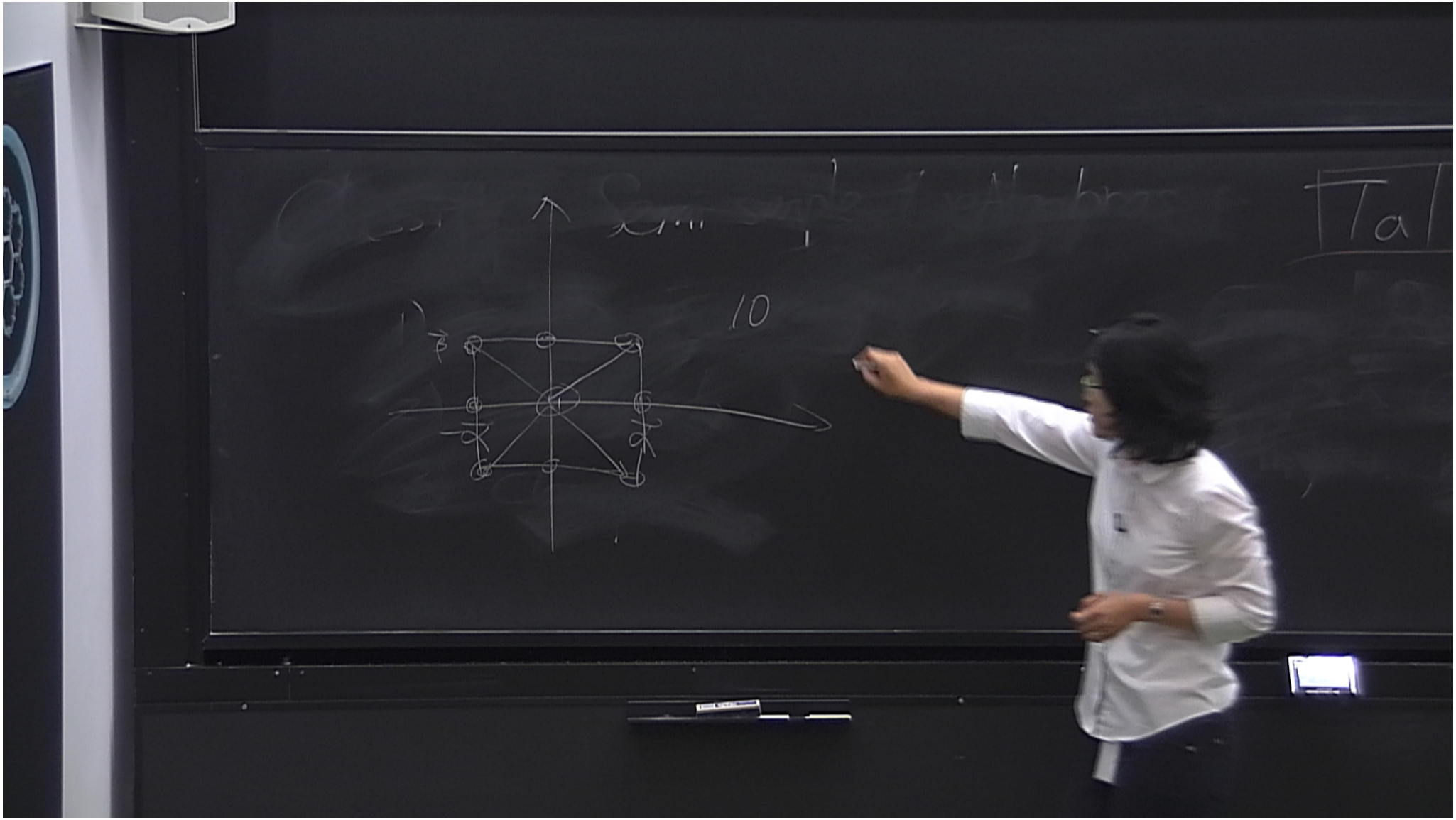
all roots come from simple roots

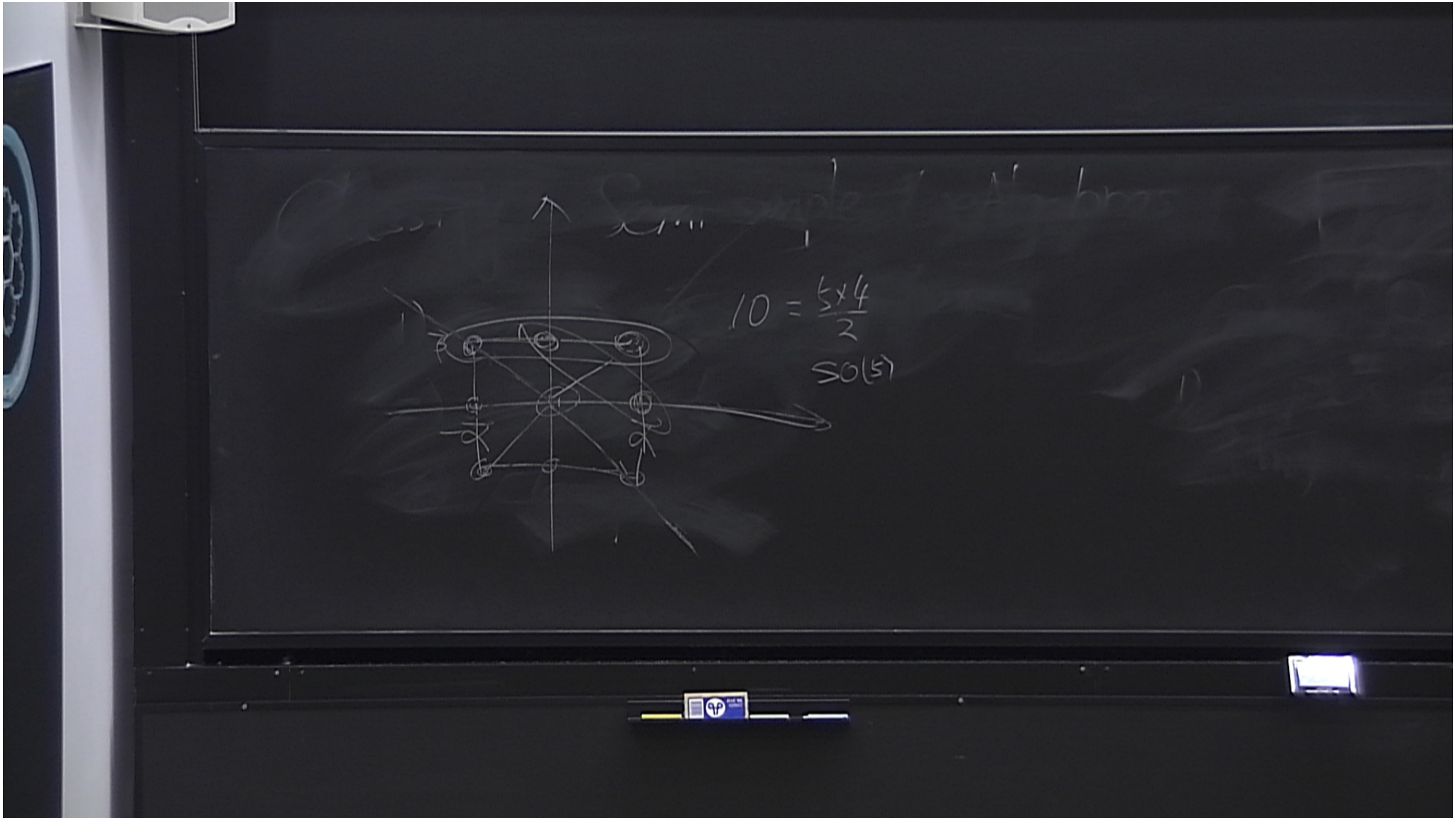
positive root

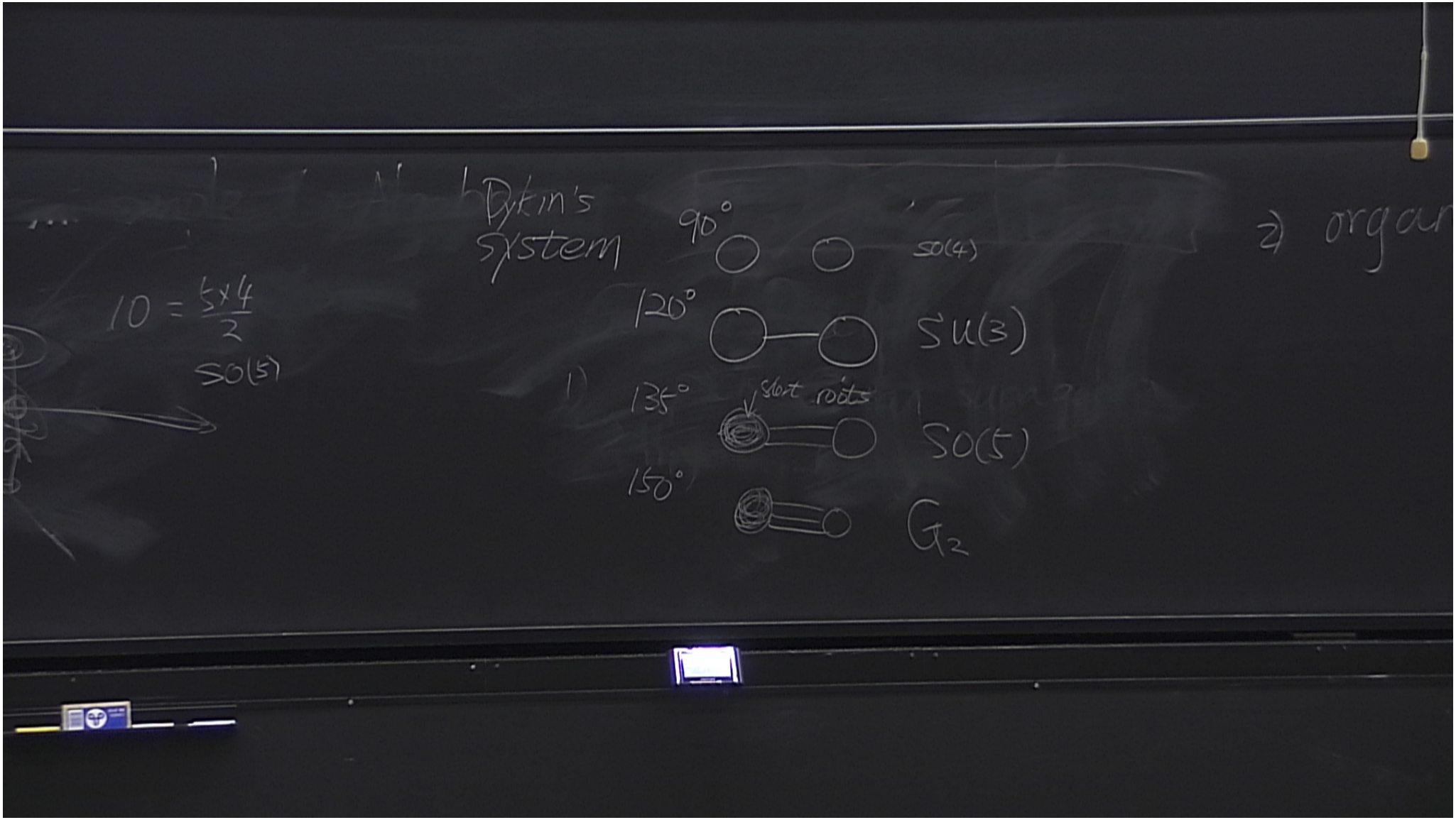
draw a r-1 plane that pass through origin.

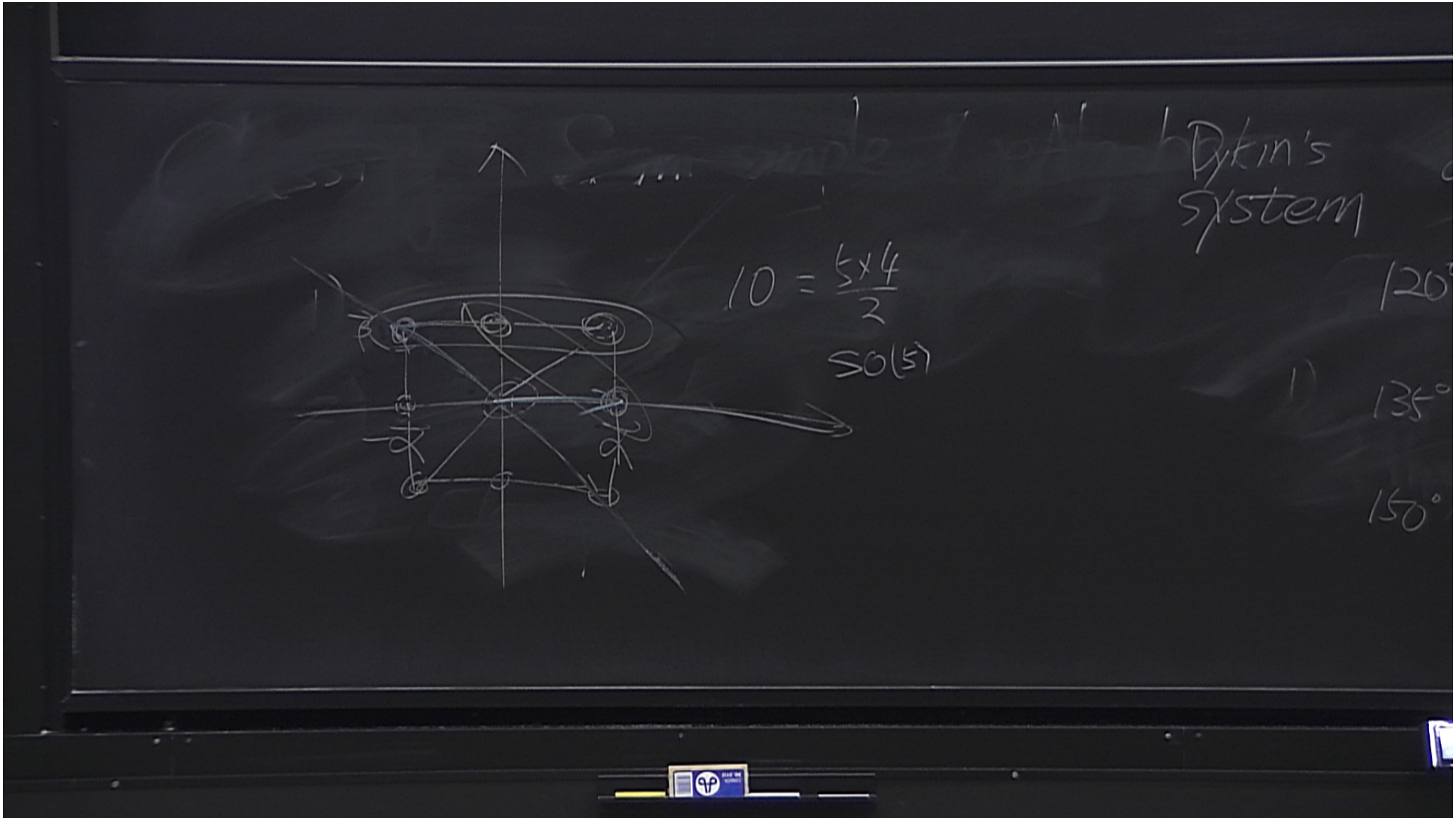
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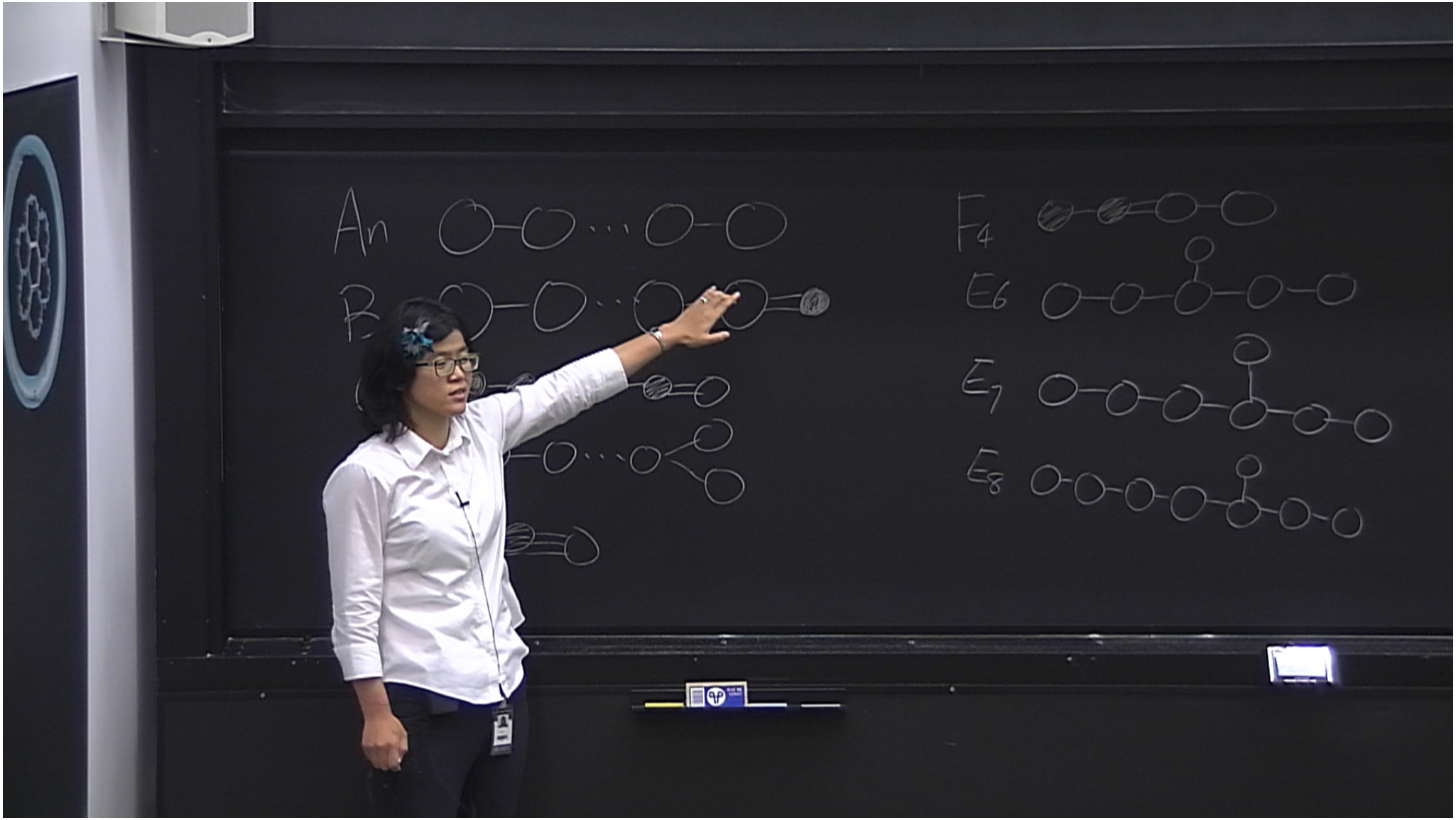
$\vec{\beta} = \vec{\alpha} + \vec{\gamma}$

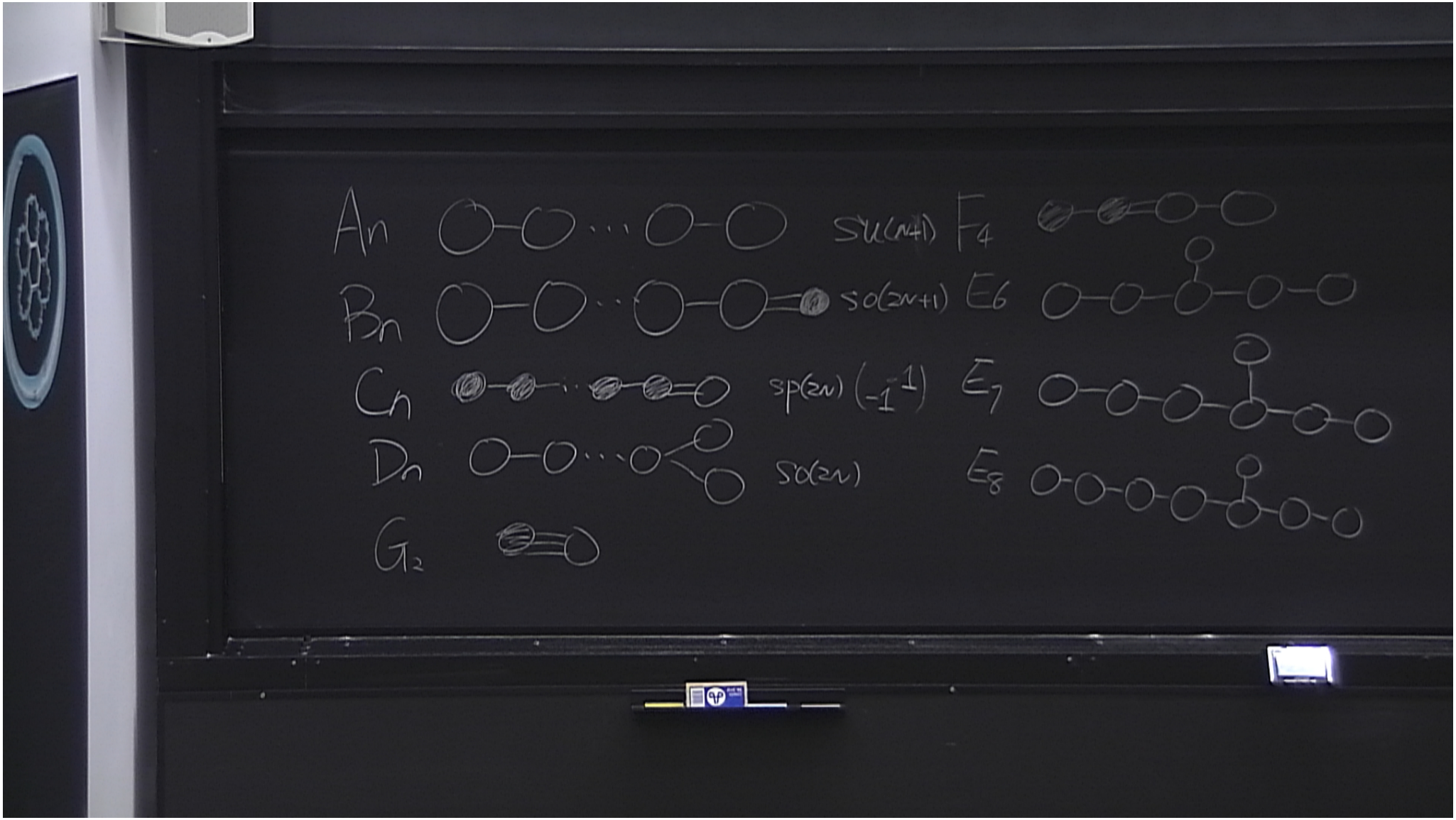

10

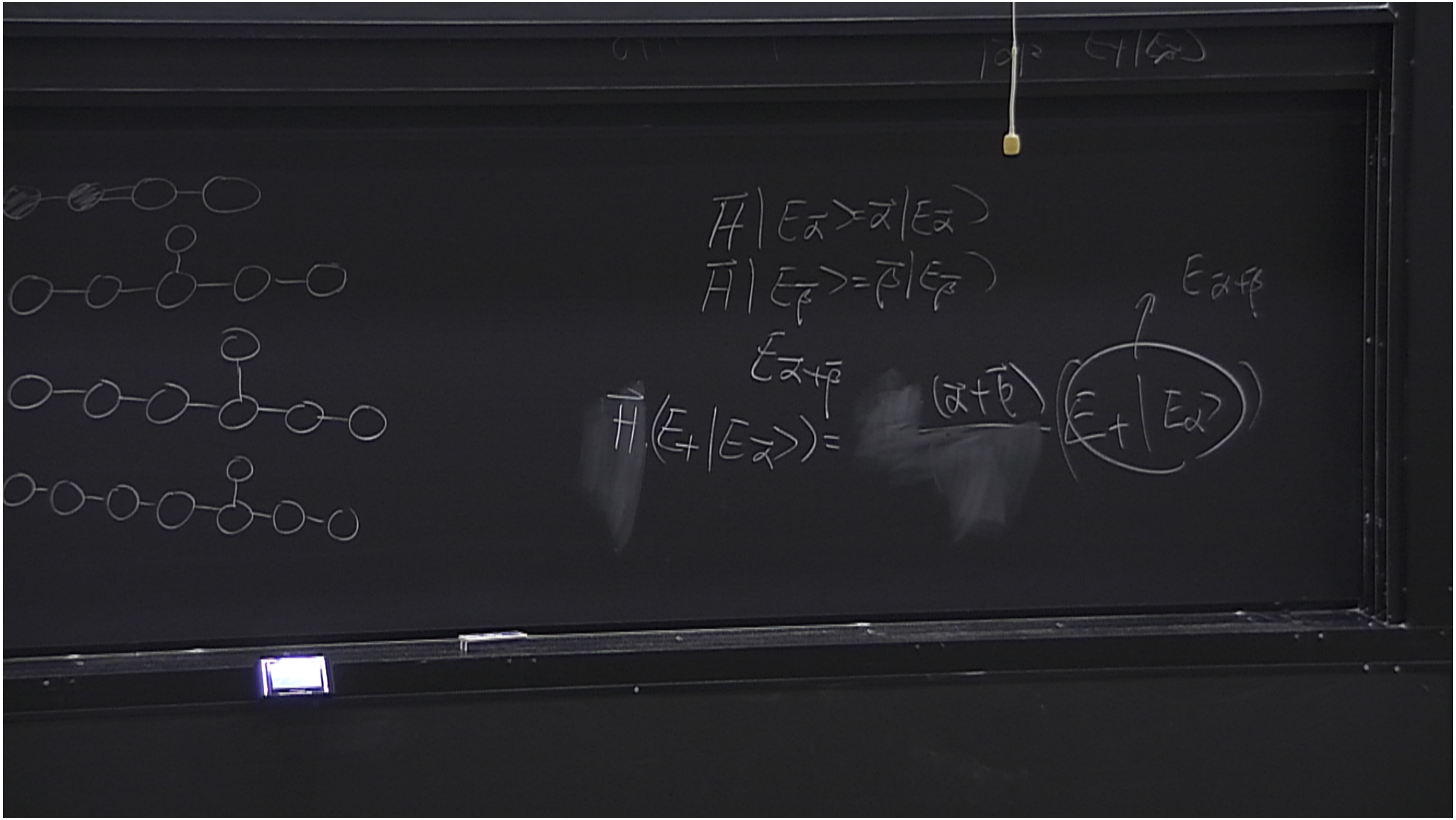

$10 = \frac{5 \times 4}{2}$
$SO(5)$

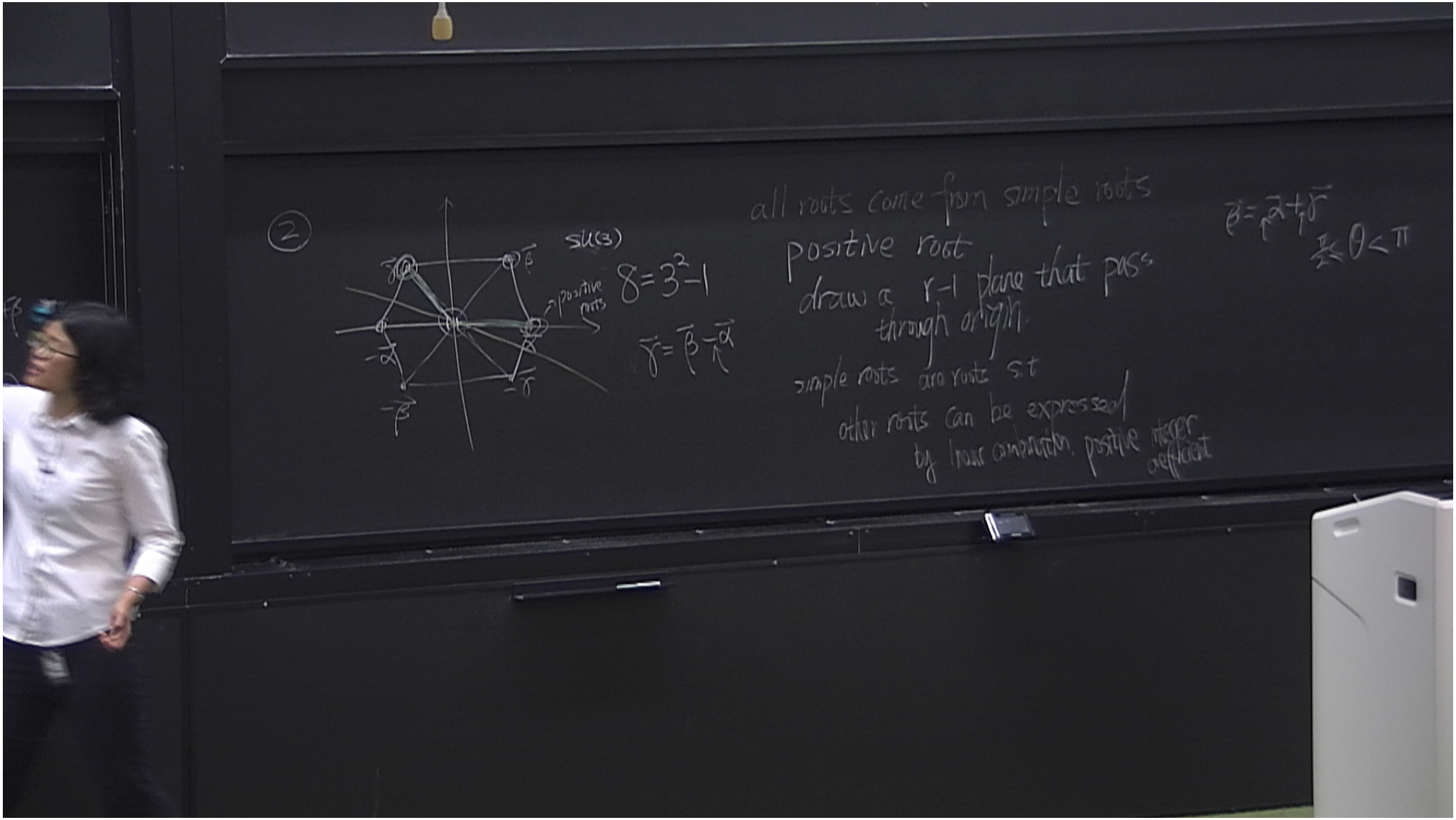

Dykin's system
90°
SO(4)
120°
SU(3)
135°
short roots
SO(5)
150°
G_2
$10 = \frac{5 \times 4}{2}$
SO(5)
2) organ

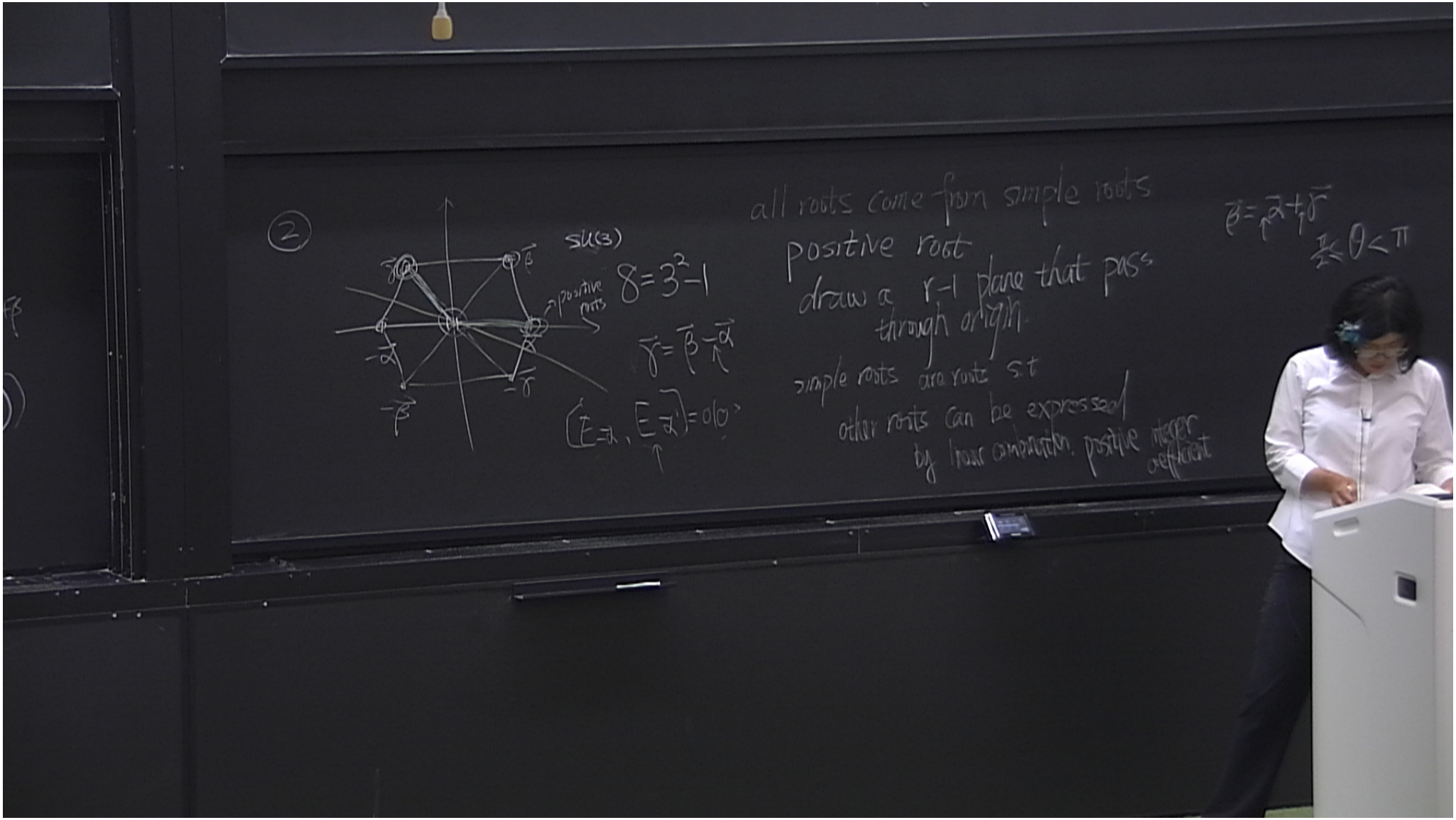

Dykin's
System
$10 = \frac{5 \times 4}{2}$
SO(5)
120°
135°
150°


An
F4
E6
E7
E8


An
SU(N+1)
Bn
SO(2N+1)
Cn
Sp(2N)
Dn
SO(2N)
G2
F4
E6
E7
E8


$\hat{H}|E_\alpha\rangle = \alpha|E_\alpha\rangle$
$\hat{H}|E_\beta\rangle = \beta|E_\beta\rangle$
$E_{\alpha+\beta}$
$\hat{H}(E_+|E_\alpha\rangle) =$
$(\alpha+\beta)$
$E_+|E_\alpha\rangle$
$E_{\alpha+\beta}$


(2)
SU(3)
positive roots
$8 = 3^2 - 1$
$\vec{\gamma} = \vec{\beta} - \vec{\alpha}$
all roots come from simple roots
positive root
draw a r-1 plane that pass through origin.
simple roots are roots s.t
other roots can be expressed by linear combination. positive integer coefficient
$\vec{\beta} = \vec{\alpha} + \vec{\gamma}$
$\frac{\pi}{3} \leq \theta < \pi$


(2)
SU(3)
positive roots
$8 = 3^2 - 1$
$\vec{\gamma} = \vec{\beta} - \vec{\alpha}$
$[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \alpha \cdot H$
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