Title: A quantum kinematics for asymptotically flat gravity

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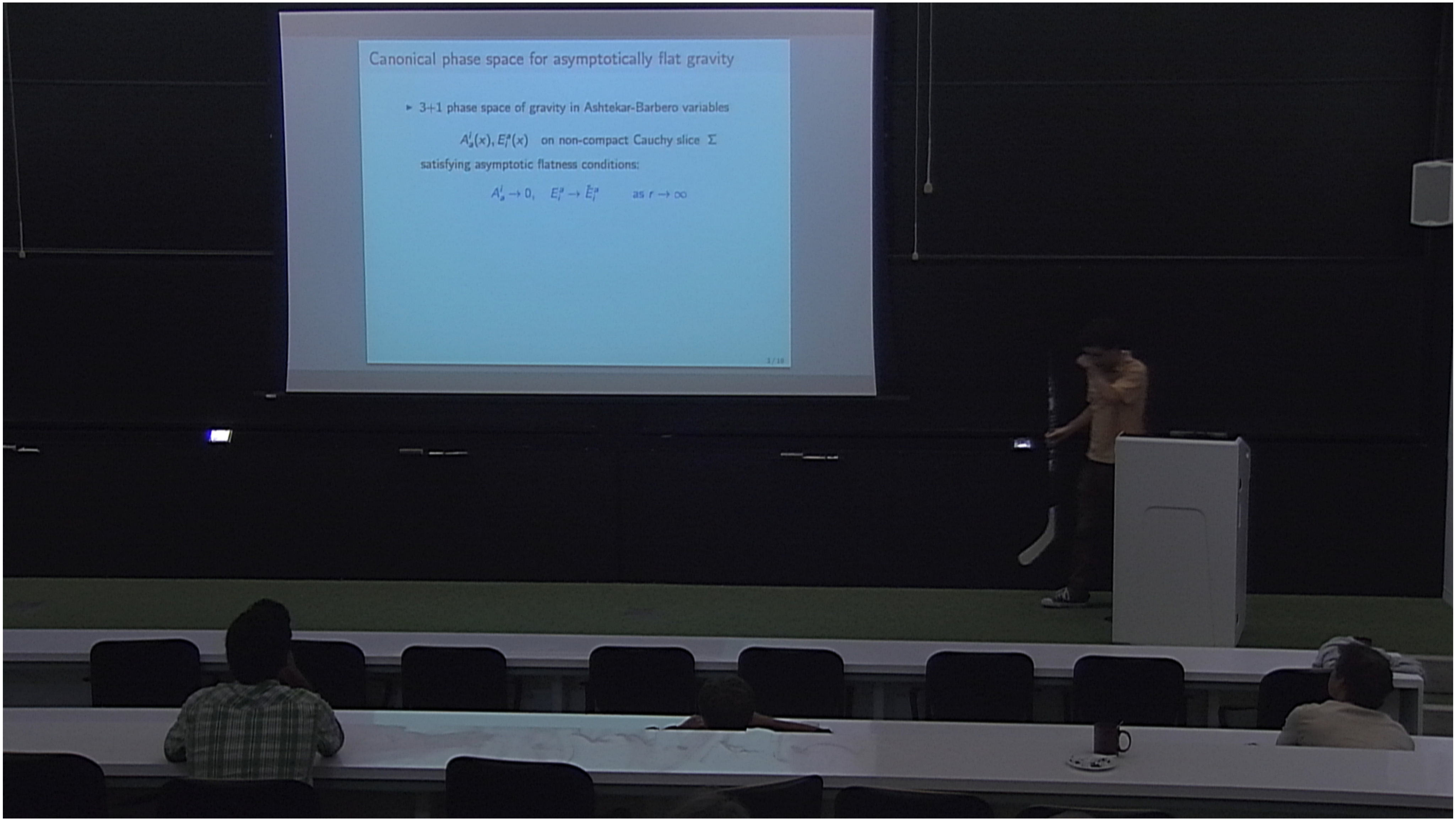
Abstract: <p>The kinematical framework of canonical loop quantum gravity has mostly been studied in the context of compact Cauchy slices. However many key physical notions such as total energy and momentum require the use of asymptotically flat boundary conditions (and hence non-compact slices). We present a quantum kinematics, based on the Koslowski-Sahlmann representation, that successfully incorporates such asymptotically flat boundary conditions. Based on joint work with Madhavan Varadarajan.</p>

Motivation

- In GR isolated systems are modeled by asymptotically flat spacetimes

$$g_{\mu\nu} \to \eta_{\mu\nu} \quad \text{as} \quad r \to \infty$$

(for canonical gravity: $r \to \infty$ in spatial direction)


Canonical phase space for asymptotically flat gravity
3+1 phase space of gravity in Ashtekar-Barbero variables
$A^i_a(x), E^a_i(x)$ on non-compact Cauchy slice Σ
satisfying asymptotic flatness conditions:
$A^i_a \to 0, \quad E^a_i \to \mathring{E}^a_i \qquad \text{as } r \to \infty$

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⇒ well-defined phase space

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- **how to capture these conditions at quantum level?**
- **can we have $\widehat{\text{energy}}$, $\widehat{\text{momentum}}$?**

Difficulties for an asym flat LQG

- non-compact $\Sigma \to$ 'infinitely large' spin networks
  Sahlmann-Thiemann-Winkler '01
  mathematically very difficult (infinite tensor product, ...)

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- one idea could be to redefine $\widehat{E}^a(x)$:

$$\widehat{E}^a_{\text{new}}(x) = \widehat{E}^a_{\text{old}}(x) + \mathring{E}^a \qquad (\star)$$

Arnsdorf-Gupta '00

- but in conflict with main driving pple in LQG: diffeomorphism covariance
- **KS representation**: makes $(\star)$ compatible with diffeo covariance

KS representation

Koslowski '07, Sahlmann '12

- spin-networks as excitations over a smooth background geometry h^{ab}

$$h^{ab} \sim \mathrm{Tr}[\mathbf{E}^a \mathbf{E}^b]$$

- background geometry $\mathbf{E}^a$ as new label of states:

$$|s\rangle \to |\mathbf{E}; s\rangle$$

- Fluxes acquire a contribution from the background geometry

$$\hat{F}_S|\mathbf{E}; s\rangle = \hat{F}_S^{\mathrm{LQG}}|\mathbf{E}; s\rangle + \Big(\int_S dS_a \mathbf{E}^a\Big)|\mathbf{E}; s\rangle$$

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- **does not spoil action of diffeos:**

$$|\mathbf{E}; s\rangle \xrightarrow{\phi} |\phi_* \mathbf{E}; \phi(s)\rangle$$

- seems well suited to incorporate asym flat conditions
- full kinematics: imposition of Diff and $SU(2)$-gauge invariance

strategy

1. Develop full KS kinematics (including imposition of Diff and $SU(2)$ gauge invariance) in case of compact Σ

2. Extend analysis to asym flat case

3. Applications: asymptotic translations and rotations

KS kinematics: underlying classical algebra

- KS states admit different 'background' **E**-labels:

$$|\psi\rangle = c_1|\mathbf{E}, s\rangle + c_2|\mathbf{E}', s'\rangle + \ldots$$

- different **E** labels $\Rightarrow$ 'background exponential' operators

(Varadarajan'13)

$$\widehat{\beta}_{\mathbf{f}}|\mathbf{E}, s\rangle = |\mathbf{E} + \mathbf{f}, s\rangle$$

- Such operators can be understood as quantization of classical phase-space function, "background exponential function"

$$\beta_{\mathbf{f}}[A] := e^{i \int_\Sigma \mathrm{Tr}[\mathbf{f}^a A_a]}$$

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- **KS representation can be understood as quantization of**

Fluxes,	Holonomies,	Background exponentials
F_S	h_γ	$\beta_{\mathbf{f}}$

Diffeos and $SU(2)$ gauge transformations in LQG

- Nice transformation law of holonomies under diffeos ϕ and $SU(2)$ gauge transformations g

$$
\begin{aligned}
\phi \cdot h_\gamma[A] &:= h_\gamma[\phi_*^{-1} A] \\
&= h_{\phi(\gamma)}[A]
\end{aligned}
$$

$$
\begin{aligned}
g \cdot h_\gamma[A] &:= h_\gamma[g^{-1} A g + g^{-1} dg] \\
&= g^{-1}(\gamma(1)) h_\gamma[A] g(\gamma(0))
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for $\gamma : [0,1] \to \Sigma$

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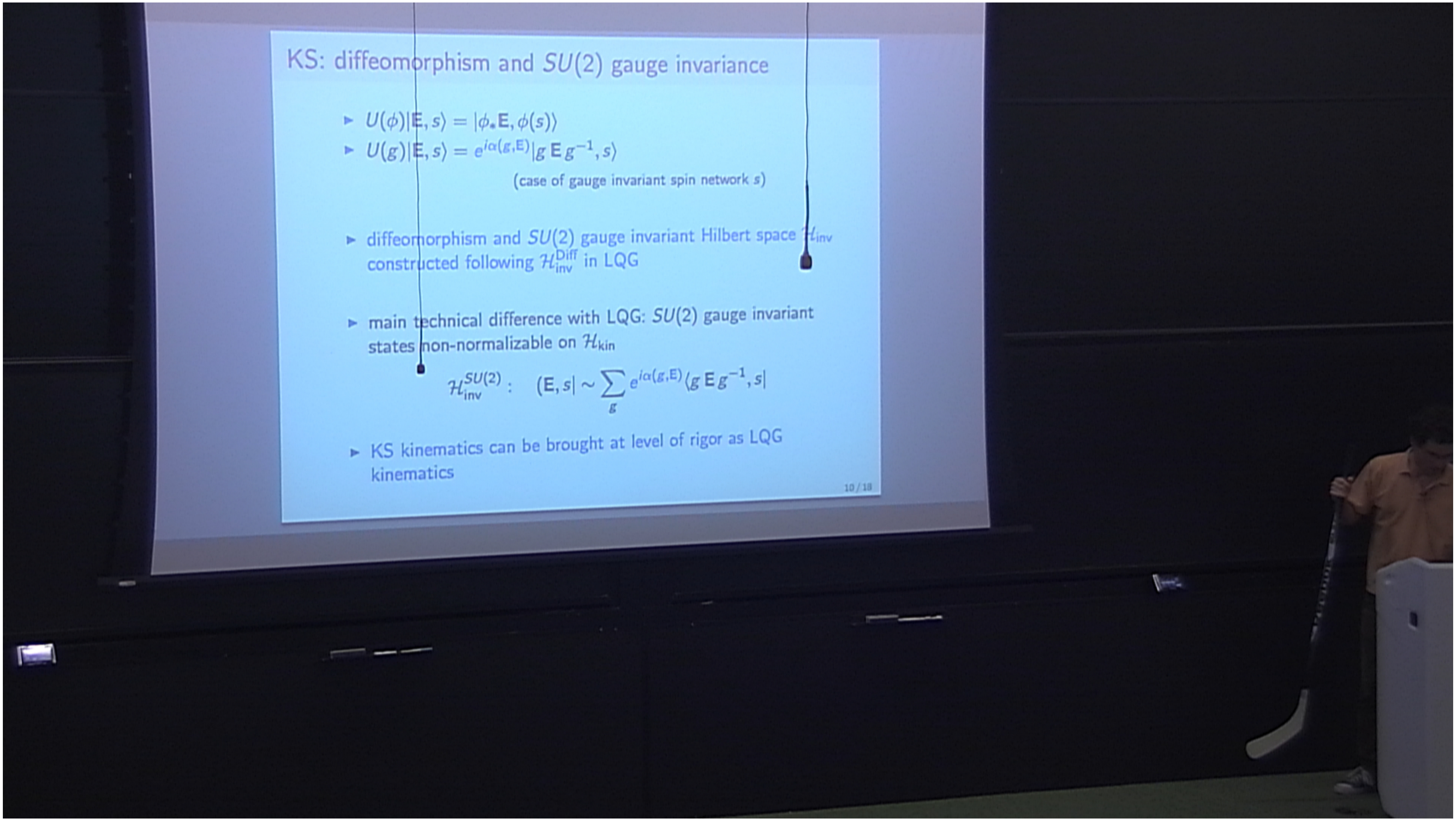
- $\mathcal{H}_{\text{inv}}^{SU(2)}$: gauge invariant spin-networks

- $\mathcal{H}_{\text{inv}}^{\text{Diff}}$: $\quad (s| \sim \sum_{\phi \in \text{diffeos}} \langle \phi(s)| \qquad$ (not normalizable on $\mathcal{H}_{\text{kin}}$)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(case of gauge invariant spin network s)
diffeomorphism and $SU(2)$ gauge invariant Hilbert space $\mathcal{H}_{\text{inv}}$ constructed following $\mathcal{H}^{\text{Diff}}_{\text{inv}}$ in LQG
main technical difference with LQG: $SU(2)$ gauge invariant states non-normalizable on $\mathcal{H}_{\text{kin}}$
$\mathcal{H}^{SU(2)}_{\text{inv}}: \quad (\mathbf{E}, s| \sim \sum_g e^{i\alpha(g,\mathbf{E})}(g\,\mathbf{E}\,g^{-1}, s|$
KS kinematics can be brought at level of rigor as LQG kinematics
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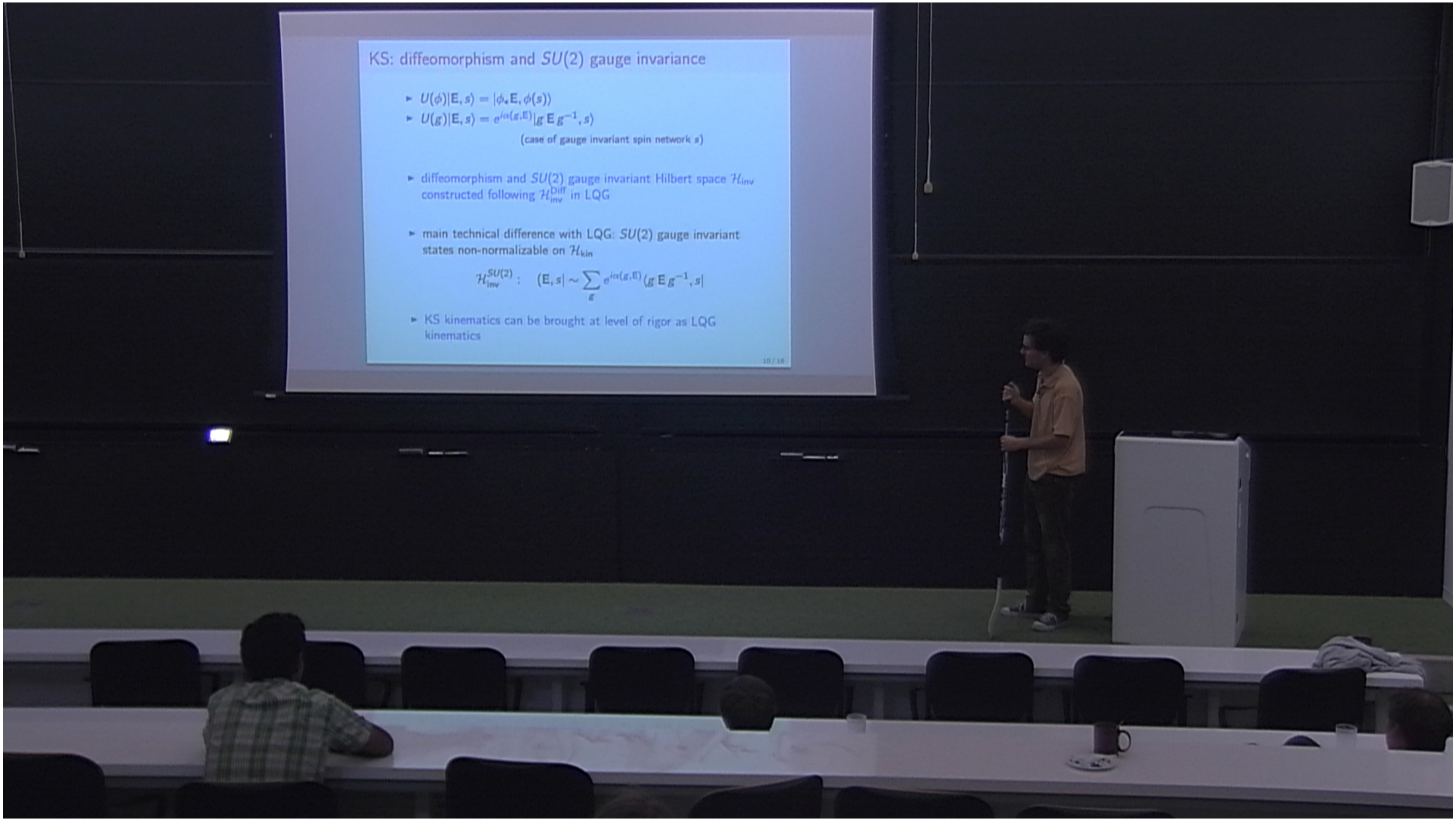
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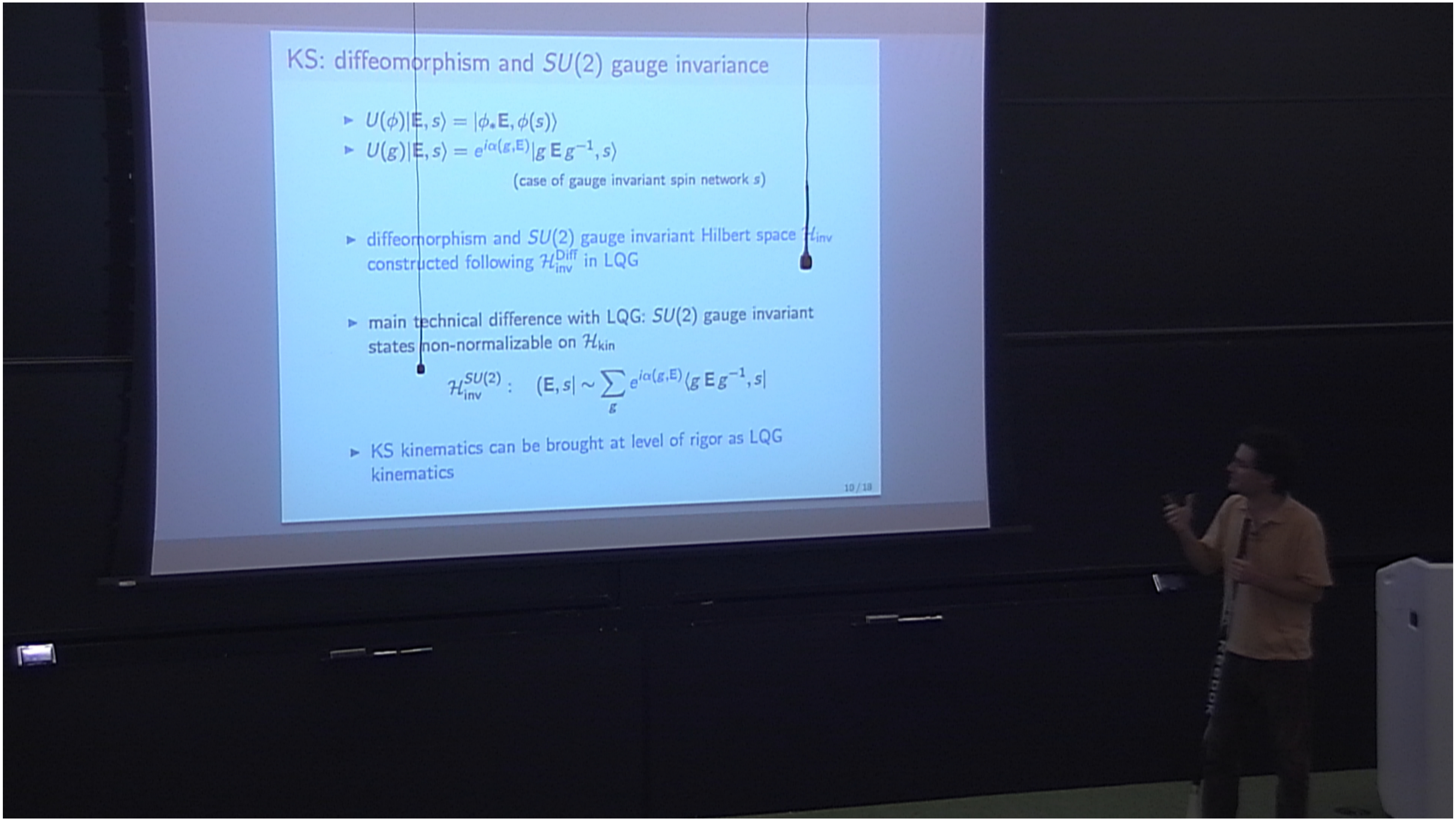
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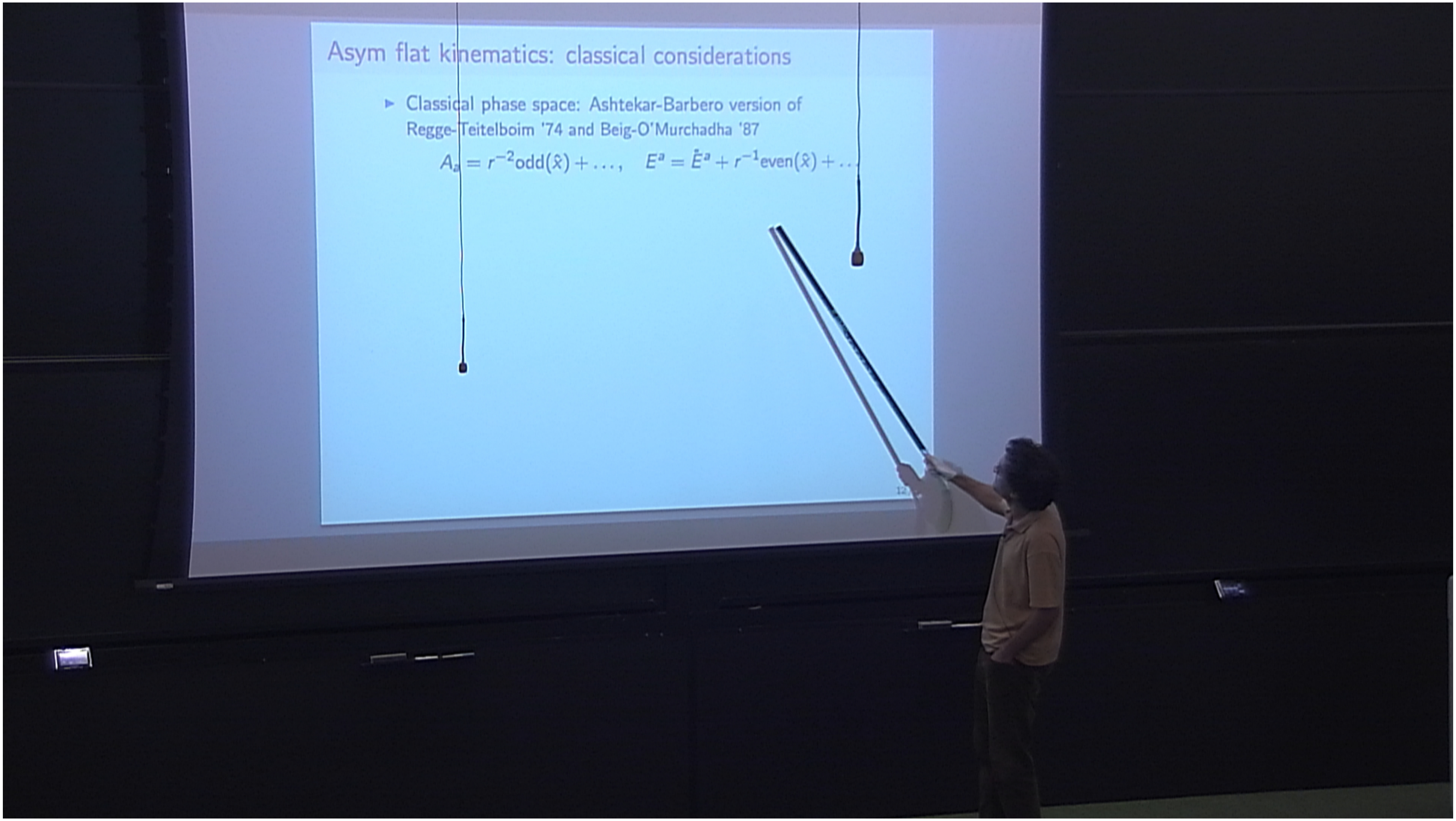
- since we are only treating spatial diffeomorphisms, we can only discuss Euclidean subgroup of Poincare group (spatial rotations and translations)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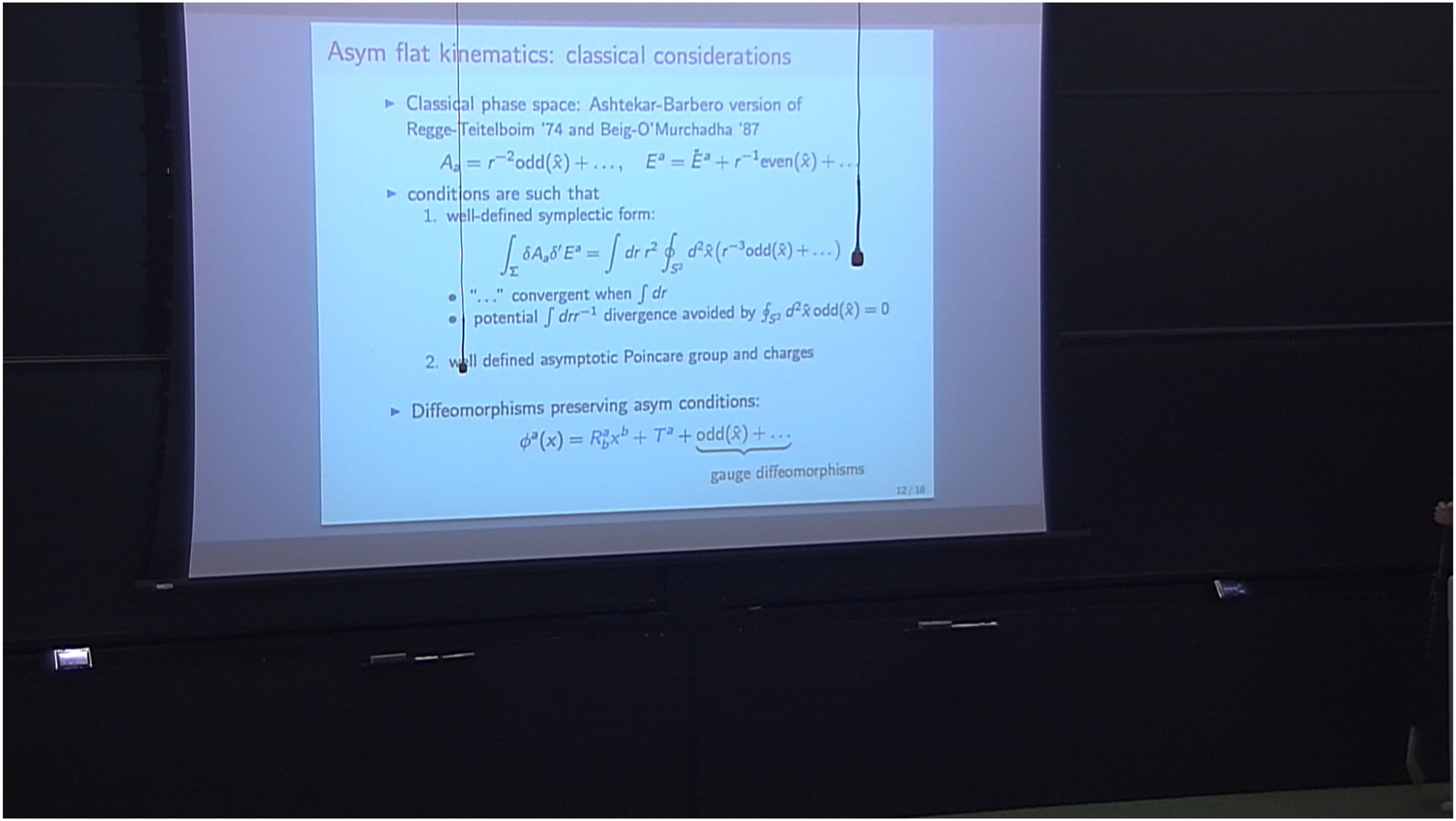
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conditions are such that
1. well-defined symplectic form:
$\int_\Sigma \delta A_a \delta' E^a = \int dr\, r^2 \oint_{S^2} d^2\hat{x} \left(r^{-3}\mathrm{odd}(\hat{x}) + \ldots\right)$
"..." convergent when $\int dr$
potential $\int drr^{-1}$ divergence avoided by $\oint_{S^2} d^2\hat{x}\, \mathrm{odd}(\hat{x}) = 0$
2. well defined asymptotic Poincare group and charges
Diffeomorphisms preserving asym conditions:
$\phi^a(x) = R^a_b x^b + T^a + \underbrace{\mathrm{odd}(\hat{x}) + \ldots}_{\text{gauge diffeomorphisms}}$
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classical functions

- Classical functions to be quantized

  Fluxes, Holonomies, Background exponentials

  F_S h_γ $\beta_{\mathbf{f}}$

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$$\mathbf{f}^a = r^{-1}(\text{even}) + \ldots$$

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$\alpha(g, \phi, \mathbf{f}) = \int \mathrm{Tr}[\phi_* \mathbf{f}^a g^{-1} \partial_a g] < \infty$

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quantum theory

For simplicity will focus on 'background labels'

- $\mathcal{H}_{\text{kin}}$ spanned by $|\mathbf{E}\rangle$ with

$$\mathbf{E}^a = \mathring{E}^a + r^{-1}\text{even}(\hat{x}) + \ldots$$

- well defined $\widehat{\beta}_{\mathbf{f}}$

$$\widehat{\beta}_{\mathbf{f}}|\mathbf{E}\rangle = |\mathbf{E} + \mathbf{f}\rangle$$

$U(g,\phi)$

- We are after $U(g,\phi)$ s.t.

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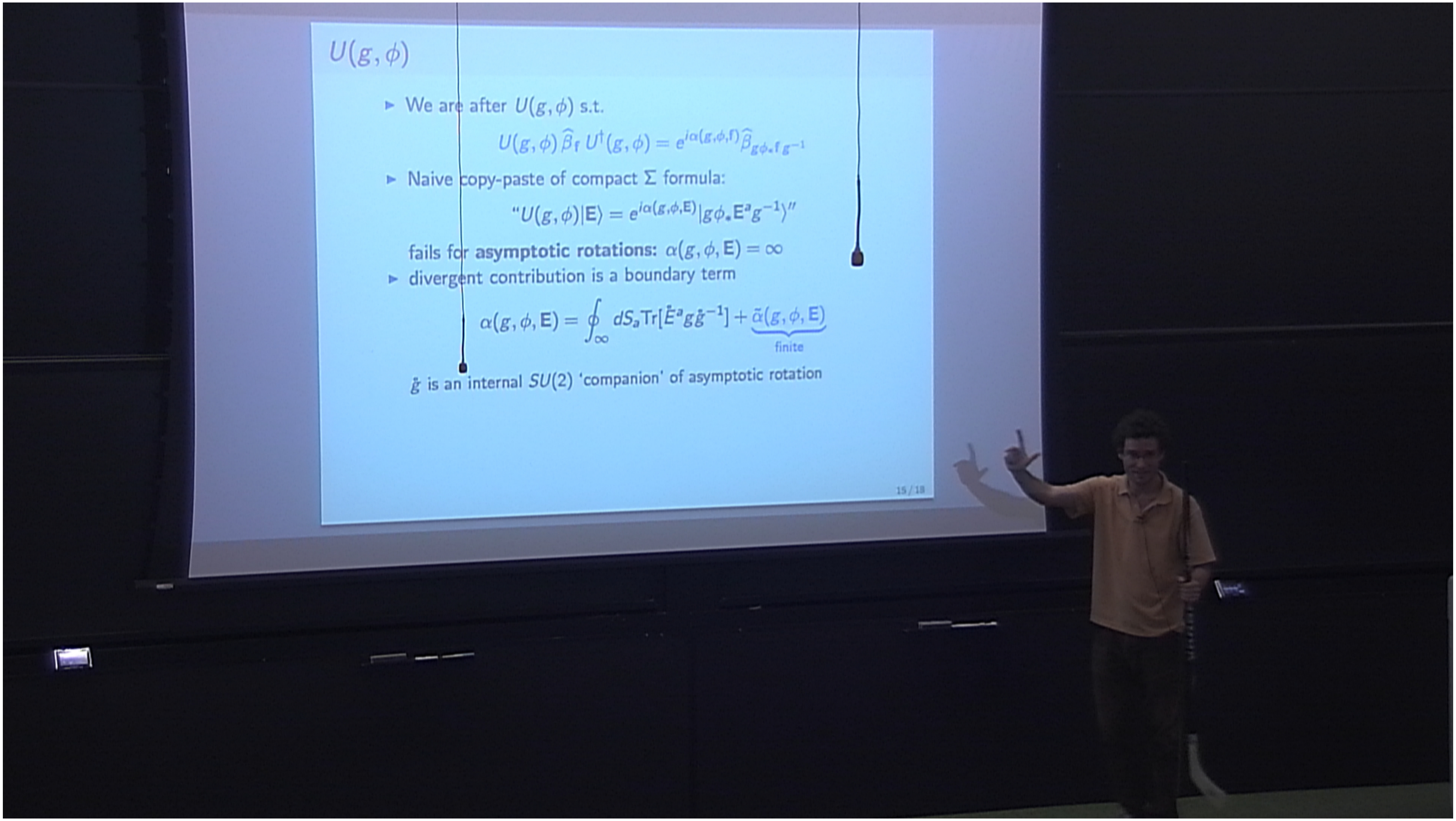
fails for **asymptotic rotations:** $\alpha(g, \phi, \mathbf{E}) = \infty$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divergent contribution is a boundary term
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$\mathring{g}$ is an internal $SU(2)$ ‘companion’ of asymptotic rotation

- improved guess:

$$U(g,\phi)|\mathbf{E}\rangle = e^{i\tilde{\alpha}(g,\phi,\mathbf{E})}|g\phi_*\mathbf{E}^a g^{-1}\rangle$$

works! $\Rightarrow$ unitary action of diffeos and $SU(2)$ gauge rotations

asym flat KS kinematics

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asym flat KS kinematics

- $\mathcal{H}_{\text{inv}}$ constructed by group averaging over **gauge** diffeos and $SU(2)$ transformations
- Unitary action of asymptotic Euclidean group on $\mathcal{H}_{\text{inv}}$
- As discussed by Friedman and Sorkin there are two possible asymptotic groups (depending on topology of Σ)

$$\mathbb{R}^3 \rtimes SO(3) \quad \text{or} \quad \mathbb{R}^3 \rtimes SU(2)$$

- In the latter case there are odd-spin states on $\mathcal{H}_{\text{inv}}$ as predicted by Friedman-Sorkin '80

asym flat KS kinematics

- $\mathcal{H}_{\text{inv}}$ constructed by group averaging over **gauge** diffeos and $SU(2)$ transformations

- Unitary action of asymptotic Euclidean group on $\mathcal{H}_{\text{inv}}$

- As discussed by Friedman and Sorkin there are two possible asymptotic groups (depending on topology of Σ)

$$\mathbb{R}^3 \rtimes SO(3) \quad \text{or} \quad \mathbb{R}^3 \rtimes SU(2)$$

- In the latter case there are odd-spin states on $\mathcal{H}_{\text{inv}}$ as predicted by Friedman-Sorkin '80

Comments

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- important part of technical work went into adapting Regge-Teitelboim/Beig-O'Murchadha conditions to setting used in LQG (finite diffeos and semianalytic category)

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Asym flat kinematics: classical considerations

- Classical phase space: Ashtekar-Barbero version of Regge-Teitelboim '74 and Beig-O'Murchadha '87

$$A_a = r^{-2}\text{odd}(\hat{x}) + \ldots, \quad E^a = \mathring{E}^a + r^{-1}\text{even}(\hat{x}) + \ldots$$

- conditions are such that
  1. well-defined symplectic form:

  $$\int_\Sigma \delta A_a \delta' E^a = \int dr\, r^2 \oint_{S^2} d^2\hat{x}\left(r^{-3}\text{odd}(\hat{x}) + \ldots\right)$$

     - "..." convergent when $\int dr$
     - potential $\int drr^{-1}$ divergence avoided by $\oint_{S^2} d^2\hat{x}\,\text{odd}(\hat{x}) = 0$

  2. well defined asymptotic Poincare group and charges

- Diffeomorphisms preserving asym conditions:

$$\phi^a(x) = R^a_b x^b + T^a + \underbrace{\text{odd}(\hat{x}) + \ldots}_{\text{gauge diffeomorphisms}}$$

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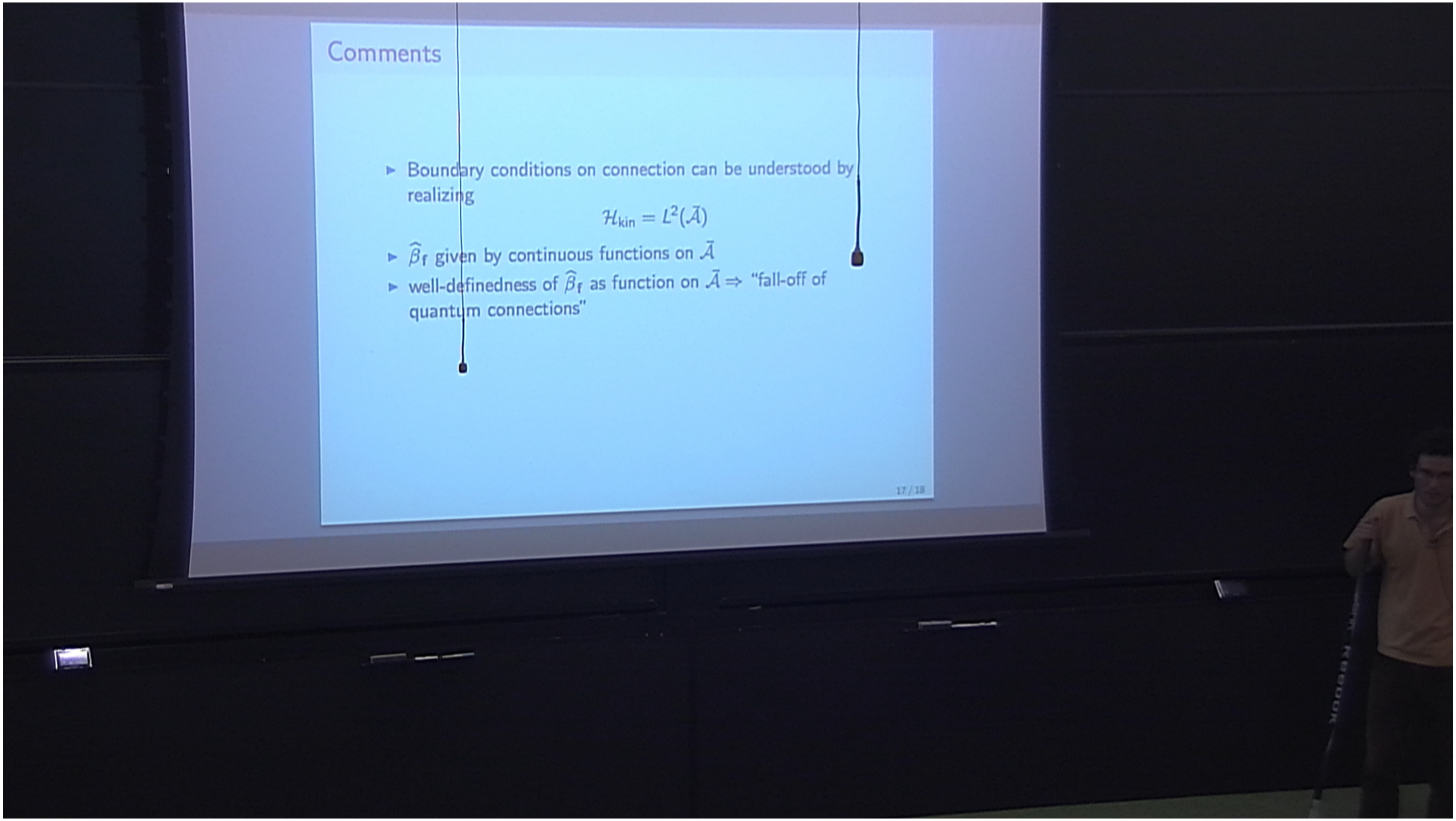
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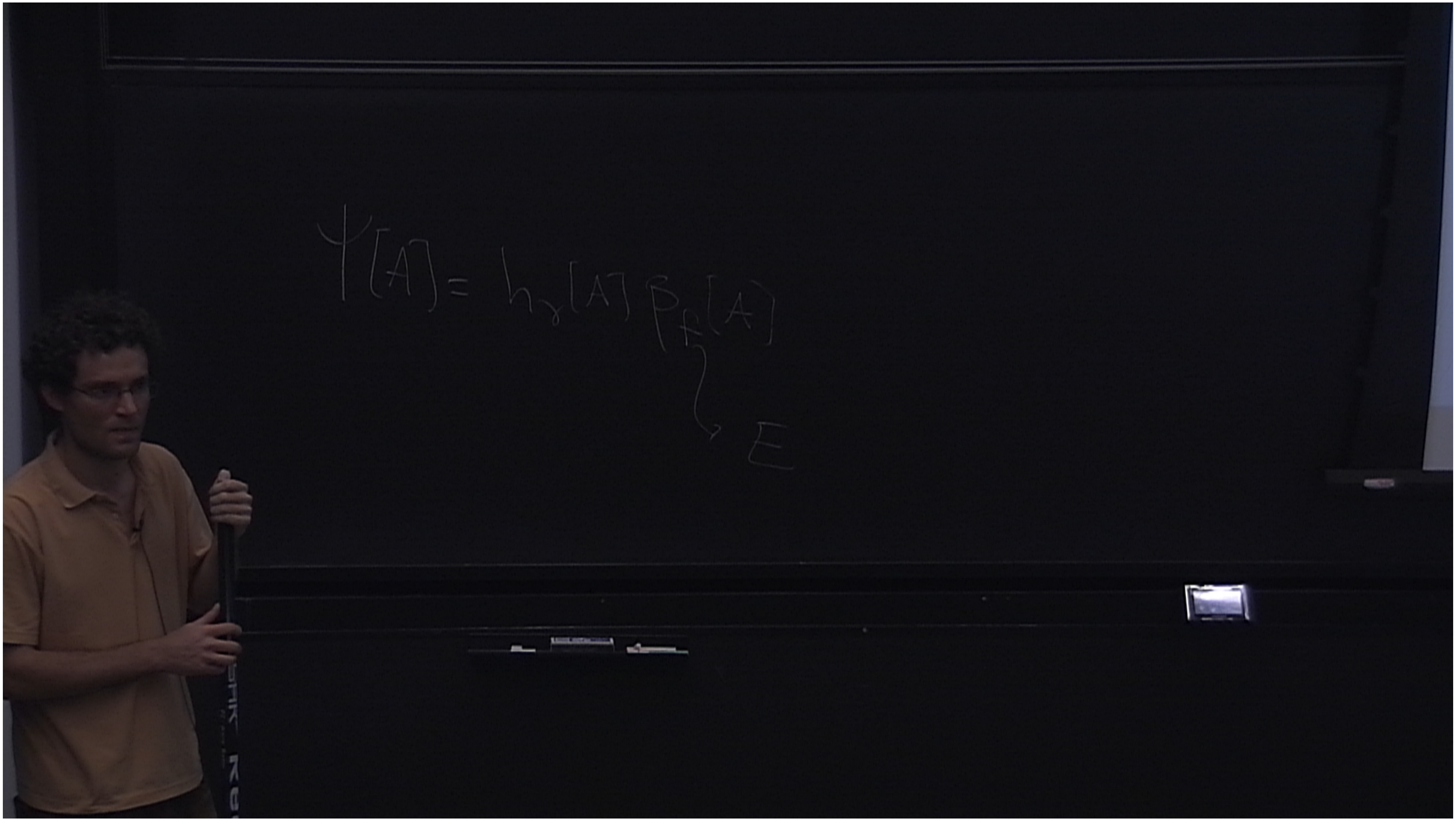
Ψ[A] = h_γ[A] β_f[A]
E