Title: Generalized BMS symmetry and subleading soft graviton theorem

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Abstract: <p>I will describe a proposal for a generalization of the BMS group in which the conformal isometries of the sphere (Lorentz group) are replaced by arbitrary sphere diffeomorphisms. I describe the computation of canonical charges and show that the associated Ward identities are equivalent to the Cachazo-Strominger subleading soft graviton formula. Based on joint work with Alok Laddha.</p>

Generalized BMS symmetry and subleading soft graviton theorem

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Work in collaboration with Alok Laddha (Chennai Mathematical Institute) Phys. Rev. D **90** (2014) 12 and JHEP **1504**, 076 (2015)

Motivation

- Over past $\sim$ 2 years revival of BMS group initiated by Strominger and collaborators
- New insight: BMS is a (spontaneously broken) symmetry of the gravitational S-matrix
- Resulting constraints on the S-matrix were known in a different disguise: (Poincare invariance and)
  - Weinberg's (leading) soft graviton theorem
- These ideas extend to groups larger than BMS
  $\Rightarrow$ further constraints on the S-matrix:
  - Cachazo-Strominger (subleading) soft graviton theorem

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  $\Rightarrow$ further constraints on the S-matrix:
  • Cachazo-Strominger (subleading) soft graviton theorem
- We will argue that CS soft theorem arises from a novel (spontaneously broken) Diff(S^2) symmetry

Outline of the talk

1. Review:
   - BMS group and BMS charges
   - BMS in S-matrix and soft gravitons
   - CS soft theorem $\Rightarrow$ Virasoro $\subset$ 'extended BMS'
2. CS soft theorem $\Leftrightarrow$ Diff(S^2) $\subset$ 'generalized BMS'
   - Generalized BMS group
   - Diff(S^2) charges
   - Equivalence
3. Final comments

Asymptotically flat spacetimes at null infinity (BMS 1962)

- $1/r$ expansion of spacetime metric off null infinity in Bondi coordinates (r, u, x^A):

$$ds^2 = -du^2 - 2dudr + [r^2 \mathring{q}_{AB} + r\, C_{AB}(u, \hat{x})]dx^A dx^B + \dots$$

$\mathring{q}_{AB} \equiv$ metric of the unit sphere
Bondi gauge: $g_{rr} = g_{rA} = 0, \quad \sqrt{\det g_{AB}} = r^2 \sqrt{\det \mathring{q}_{AB}}$


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- $C_{AB}(u, \hat{x})$ **unconstrained by Einstein equations** $\Rightarrow$ **"free data"**
- "…" determined by $C_{AB}(u, \hat{x})$ through Einstein equations

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- "$\ldots$" determined by $C_{AB}(u, \hat{x})$ through Einstein equations

- $\mathring{q}^{AB} C_{AB} = 0 \Rightarrow 2$ independent components $\leftrightarrow$ 2 polarizations of gravitational waves

BMS group (Bondi, van der Burg, Metzner; Sachs 1962)

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$$\xi^a_f = f(\hat{x})\partial_u + \ldots,$$

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Translation subgroup T $\subset$ ST: $f(\hat{x}) = 1, \hat{x}_1, \hat{x}_2, \hat{x}_3$

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2) $V^A(\hat{x})$ CKV vector fields $\Rightarrow$ Conf(S^2) $\approx$ Lorentz group
(later to be generalized)

$\Rightarrow$ asymptotic isometries $\equiv$ ST $\rtimes$ Conf(S^2) $=:$ BMS

(compare with Poincare $\equiv$ T $\rtimes$ Lorentz)

BMS charges (Ashtekar, Streubel 1981)

- on space of free data at $\mathcal{I}$

$$\Gamma^{(\mathring{q})} = \{C_{AB}(u, \hat{x}) : \mathring{q}^{AB} C_{AB} = 0\}$$

there is a natural symplectic form Ω
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$$\delta Q_\xi = \Omega(\delta, \delta_\xi) \, \forall \delta$$

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- For supertranslations ξ^a_f:

$$\delta_f C_{AB} = f \partial_u C_{AB} - 2 D_A D_B f \Rightarrow \boxed{Q_f = Q_f^{\text{hard}} + Q_f^{\text{soft}}}$$

Q_f^{hard} = quadratic in C_{AB} Q_f^{soft} = linear in C_{AB}

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BMS as symmetry of the S-matrix (Strominger 2013)

- 'Diagonal' subgroup of $BMS^{+} \times BMS^{-}$ a symmetry of the classical gravitational scattering problem

- Quantum version of diagonal ST symmetry:

$$[\hat{Q}_f, \mathcal{S}] = \hat{Q}_f^{+}\mathcal{S} - \mathcal{S}\hat{Q}_f^{-} = 0 \qquad (\star)$$

Subleading soft graviton theorem and extended BMS

- CS subleading soft graviton formula (Cachazo, Strominger 2014)

$$\lim_{\lambda \to 0} \partial_\lambda \lambda \mathcal{M}(\lambda q; p_1, \dots, p_n) = \sum_{i=1}^{n} (q.p_i)^{-1} \epsilon_{\mu\nu} p_i^\mu q_\rho J_i^{\rho\nu} \mathcal{M}(p_1, \dots, p_n)$$

- Motivated by 'extended BMS': (Barnich, Troessaert 2010)
  allow poles in BMS $V^A \to$ infinitely many local CKV's $\tilde{V}$
  in $(z, \bar{z})$ coordinates on S^2 where $\mathring{q}_{AB} \propto dz d\bar{z}$

$$\tilde{V} = \tilde{V}^z(z)\partial_z + \tilde{V}^{\bar{z}}(\bar{z})\partial_{\bar{z}}$$

-

$$\text{CS formula } \Rightarrow [\hat{Q}_{\tilde{V}}, \mathcal{S}] = 0$$

Kapec, Lysov, Pasterski, Strominger (2014)

$$\text{CS formula } \Leftarrow \textbf{?}$$

can we obtain a 1-to-1 correspondence as in
Weinberg formula $\Leftrightarrow [Q_f, \mathcal{S}] = 0$?

Virasoro Ward identities

Kapek et.al.'14

Follow same procedure as in derivation of ST Ward identities:

1. Fock space of asymptotic gravitons:
$$|\vec{p}_1 h_1, \dots \vec{p}_m h_m\rangle = a^\dagger_{h_1}(\vec{p}_m)\dots a^\dagger_{h_m}(\vec{p}_m)|0\rangle$$
2. Write $\hat{Q}_{\tilde{V}}$ in terms of Fock operators
$$C_{zz}(u,\hat{x}) \sim \int dE\, a_+(\vec{p} = E\hat{x})e^{-iEu}$$
3. Evaluate matrix element of
$$[\hat{Q}_{\tilde{V}}, \mathcal{S}] = 0$$
between
$\langle \text{out}| = \langle \vec{p}_1 h_1, \dots \vec{p}_m h_m|, \qquad |\text{in}\rangle = |\vec{p}_{m+1} h_{m+1}, \dots \vec{p}_n h_n\rangle$

Resulting Ward Id takes the form:

" $\int d^2z\, \partial_z^3 \tilde{V}^z \times (\vec{q}(z,\bar{z})$ CS formula) "

CS formula $\Leftarrow$?

- By reverse engineering one finds CS formula seems to arise from a Ward Id associated to a **non-CKV**

$$V = (z - w)^{-1}(\bar{z} - \bar{w})^2 \partial_{\bar{z}}$$

- Can we make sense of non-CKV V^A?

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$$V = (z-w)^{-1}(\bar{z}-\bar{w})^2\partial_{\bar{z}}$$

- Can we make sense of non-CKV V^A?
- First hint of positive answer: If we replace

$$\tilde{V}^z(z) \to V^z(z,\bar{z}), \quad \tilde{V}^{\bar{z}}(z) \to V^{\bar{z}}(z,\bar{z})$$

in expression of Virasoro charges

$$Q_{\tilde{V}} \to Q_V$$

then all steps in "CS formula $\Rightarrow$ Virasoro Ward Id" go through !
- Only place CKV condition used is in the derivation of the charges $Q_{\tilde{V}}$

Generalized BMS group $\mathcal{G}$

- Asymptotically flat spacetimes in 'non-Bondi frame':

Barnich and Troessaert 2010

$$ds^2 = -\mathcal{R}/2du^2 - 2dudr + (r^2 q_{AB} + r\, C_{AB})dx^A dx^B + \ldots \quad (\star)$$

- Still Bondi gauge but q_{AB} arbitrary (u-independent) sphere metric
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- Vector fields that preserve form of $(\star)$ **allowing** $\delta q_{AB} \neq 0$:

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as in BMS but **no restriction on** V^A

$$\Rightarrow \mathrm{ST} \rtimes \mathrm{Diff}(S^2) =: \mathcal{G}$$

- $\delta_V q_{AB} \equiv \mathcal{L}_V q_{AB} - (D\cdot V)q_{AB} = 0 \Leftrightarrow V^A$ CKV

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- spacetime characterization: $\nabla_a \xi^a \to 0$

Action of $\mathcal{G}$

- Under action of V^A:

$$\Gamma^{(\mathring{q}_{AB})} \to \Gamma^{(\mathring{q}_{AB}+\delta_V \mathring{q}_{AB})}$$

- Need to work in larger space:

$$\Gamma := \{(q_{AB}, C_{AB}) : q^{AB} C_{AB} = 0\}$$

(with $\partial_u q_{AB} = 0$ and $\sqrt{q} = \sqrt{\mathring{q}}$)

- Well-defined action of $\mathcal{G}$ on Γ. But Γ not a symplectic space!

- Idea: try to find symplectic product on Γ from covariant phase space Ω^{cov}

Computation of Diff(S^2) charges

- We seek symplectic product from covariant phase space:

$$\Omega^{\text{cov}}(\delta,\delta') = \frac{1}{2}\int_\Sigma dS_a\big(\delta(\sqrt{g}g^{ab})\delta'\Gamma^c_{cb} - \delta(\sqrt{g}g^{bc})\delta'\Gamma^a_{bc} - \delta \leftrightarrow \delta'\big)$$

by taking $\Sigma \to \mathcal{I}$ with δg_{ab} in terms of δq_{AB} and δC_{AB}

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- For $\delta q_{AB} = 0$ procedure known to reproduce Ω on $\Gamma^{(\mathring{q}_{AB})}$

  (Ashtekar, Magnon 1982)

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- For $\delta q_{AB} = 0$ procedure known to reproduce Ω on $\Gamma^{(\mathring{q}_{AB})}$
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- well defined $\Sigma \to \mathcal{I}$ limit for defining equation of Q_V:

$$\delta Q_V = \Omega^{\text{cov}}(\delta, \delta_V)\ \forall \delta \in \Gamma^{(\mathring{q}_{AB})} \qquad (\star)$$

$$\delta_V = (\delta_V q_{AB}, \delta_V C_{AB}), \qquad \delta_V C_{AB} = \delta^{\text{lin}}_V C_{AB} - uD_A D_B D\cdot V$$

- can solve $(\star)$ for $Q_V \Rightarrow Q_V = Q^{\text{hard}}_V + Q^{\text{soft}}_V$

Diff(S^2) Charges

- $$Q_V^{\text{hard}} = \frac{1}{4}\int_{\mathcal{I}} \partial_u C^{AB} \delta_V^{\text{lin}} C_{AB}$$

  same expression as Virasoro $Q_{\tilde{V}}^{\text{hard}}$ (with $\tilde{V} \to V$) ✓

- $$Q_V^{\text{soft}} = \frac{1}{2}\int_{\mathcal{I}} C^{AB} s_{AB}$$

  with

  $$s_{AB} = D_A D_B D \cdot V - \frac{1}{2} D_{(A} D^C \delta_V q_{B)C} + D_{(A} V_{B)}$$

- going to $(z, \bar{z})$ coords:

  $$\boxed{s_{zz} = \partial_z^3 V^z, \quad s_{\bar{z}\bar{z}} = \partial_{\bar{z}}^3 V^{\bar{z}}}$$

  $\Rightarrow$ Q_V^{soft} matches with with Virasoro $Q_{\tilde{V}}^{\text{soft}}$! ✓

this explains why naive replacement $Q_{\tilde{V}} \to Q_V$ worked

Diff(S^2) Ward Identities

Follow same steps as Virasoro case:

1. Write quantum charges in terms of Fock operators

$$\hat{Q}_V^{\text{hard}}|\vec{p}, h\rangle = -i\mathbf{J}_V^{\vec{p},h}|\vec{p}, h\rangle$$

$$\hat{Q}_V^{\text{soft}} = \frac{1}{4\pi i}\lim_{E\to 0}\partial_E E\int d^2z\left[\partial_{\bar{z}}^3 V^{\bar{z}} a_+(E,z) + \partial_z^3 V^z a_-(E,z)\right]$$

2. Evaluate matrix element of $[\hat{Q}_V, \mathcal{S}] = 0$:

$$\langle\vec{p}_1 h_1;\ldots|[\hat{Q}_V^{\text{soft}}, \mathcal{S}]|\ldots,\vec{p}_n h_n\rangle = -\langle\vec{p}_1 h_1,\ldots|[\hat{Q}_V^{\text{hard}}, \mathcal{S}]|\ldots,\vec{p}_n h_n\rangle$$

$$\boxed{\lim_{E\to 0}\partial_E E\int\frac{d^2z}{4\pi}\left(\partial_{\bar{z}}^3 V^{\bar{z}}\mathcal{M}_{n+1}^+ + \partial_z^3 V^z\mathcal{M}_{n+1}^-\right) = \sum_{i=1}^n \mathbf{J}_V^{\vec{p}_i,h_i}\mathcal{M}_n}$$

$$\mathcal{M}_n \equiv \langle\vec{p}_1 h_1;\ldots|\mathcal{S}|\ldots,\vec{p}_n h_n\rangle$$

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Diff(S^2) Ward Id $\Leftrightarrow$ CS soft theorem

$$\lim_{E\to 0}\partial_E E\int \frac{d^2z}{4\pi}\left(\partial_{\bar z}^3 V^{\bar z}\mathcal{M}_{n+1}^{+} + \partial_z^3 V^z \mathcal{M}_{n+1}^{-}\right) = \sum_{i=1}^{n} \mathbf{J}_V^{\vec p_i, h_i}\mathcal{M}_n$$

- Take

$$V = (z-w)^{-1}(\bar z - \bar w)^2\partial_{\bar z} =: K_{(w,\bar w)}$$

- On LHS use:

$$\partial_{\bar z}^3 K^{\bar z}_{(w,\bar w)} = 4\pi\delta^{(2)}(z-w)$$

- On RHS use:

$$\mathbf{J}^{\vec p, h}_{K_{(w,\bar w)}} = (q.p)^{-1}\epsilon^{+}_{\mu\nu}p^\mu q_\rho J^{\rho\nu}$$

$$q^\mu = (1, \hat q(w,\bar w))$$

- CS formula is recovered!

$$\lim_{E\to 0}\partial_E E\mathcal{M}_{n+1}^{+} = \sum_{i=1}^{n}(q.p_i)^{-1}\epsilon^{+}_{\mu\nu}p_i^\mu q_\rho J_i^{\rho\nu}\mathcal{M}_n$$

Space of vacua and symmetry breaking: Diff(S^2)

- Larger space of vacua: $\{(q_{AB}, N_{AB} = 0)\} \in \Gamma$
- one can show that under $\mathcal{G}$:

$$(q_{AB}, N_{AB} = 0) \rightarrow (q'_{AB}, N_{AB} = 0)$$

- Each classical vacuum invariant under a Poincare $\subset \mathcal{G}$
- In quantum theory $\mathcal{G}$ broken down to Poincare:

$$\mathcal{G} \rightarrow \text{BMS} \rightarrow \text{Poincare}$$

- 'Subleading' soft gravitons as Goldstone modes of
  $\mathcal{G} \rightarrow$ BMS (Diff(S^2) $\rightarrow$ Conf(S^2))
- 'Leading' soft gravitons as Goldstone modes
  BMS $\rightarrow$ Poincare (ST $\rightarrow$ T)

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Conclusions

- Group of asymptotic symmetries

$$\mathcal{G} = \mathrm{ST} \rtimes \mathrm{Diff}(S^2)$$

accounts for $\boxed{\mathcal{M}_{n+1} = \left(E_s^{-1} S^{(0)} + S^{(1)}\right) \mathcal{M}_n} + O(E_s)$

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- $\mathcal{G}$ spontaneously broken to Poincare: soft gravitons as associated Goldstone modes
- Analysis can be extended to massive particles
- To do
  - Symmetries in IR finite S-matrix [Kulish-Faddeev (1970), Ashtekar (1981), Ware-Saotome-Akhoury (2013)]
  - Better understanding of classical conservation law [ST: "conservation of energy at every angle" Strominger (2013)]
  - Higher dimensions ? [ST: Kapec, Lysov, Pasterski Strominger (2015)]
  - ...

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