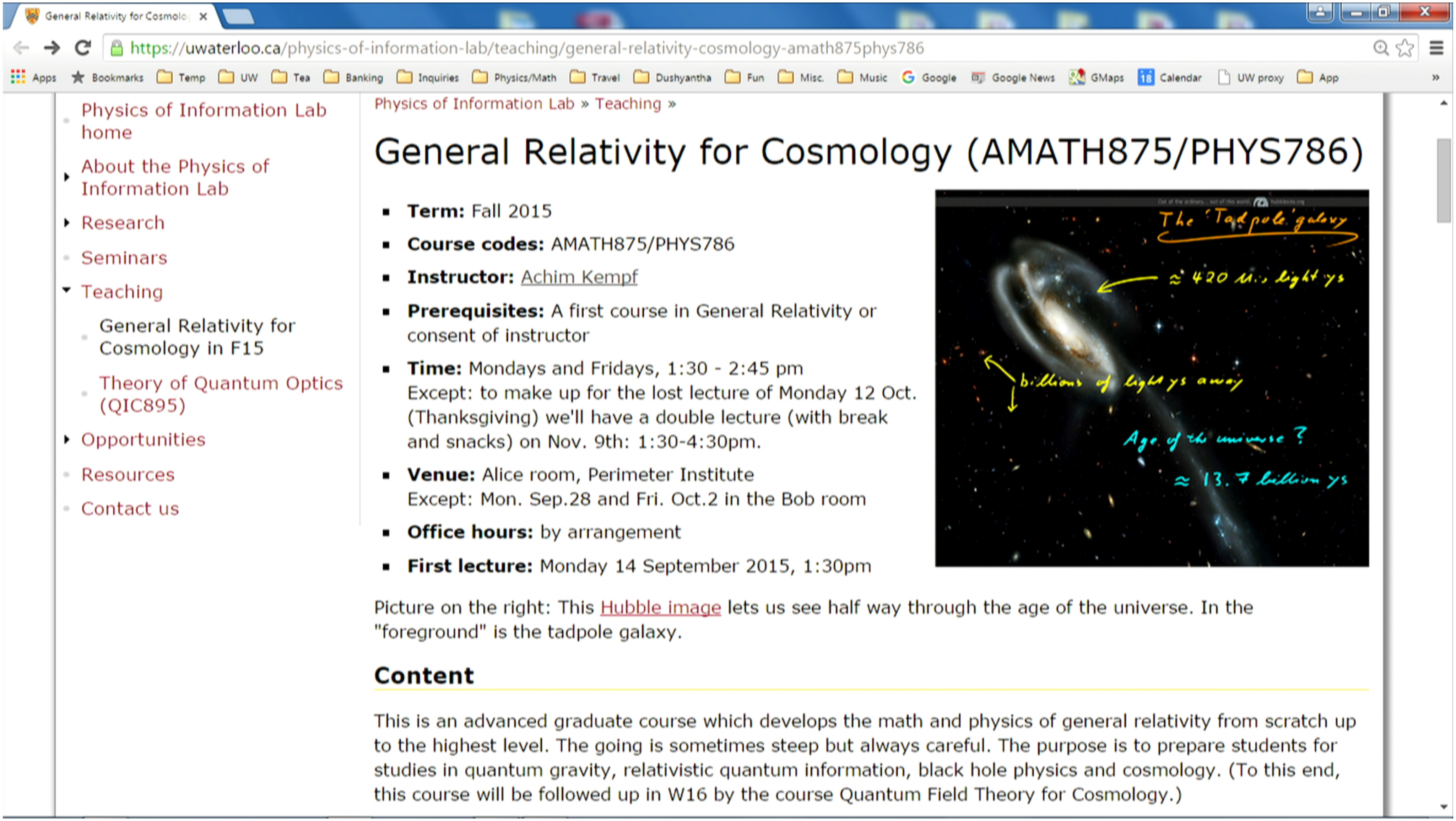
Title: AMATH 875/PHYS 786 - Fall 2015 - Lecture 1

Date: Sep 14, 2015 01:30 PM

URL: http://pirsa.org/15090002

Abstract: <p>Course Description coming soon.</p>



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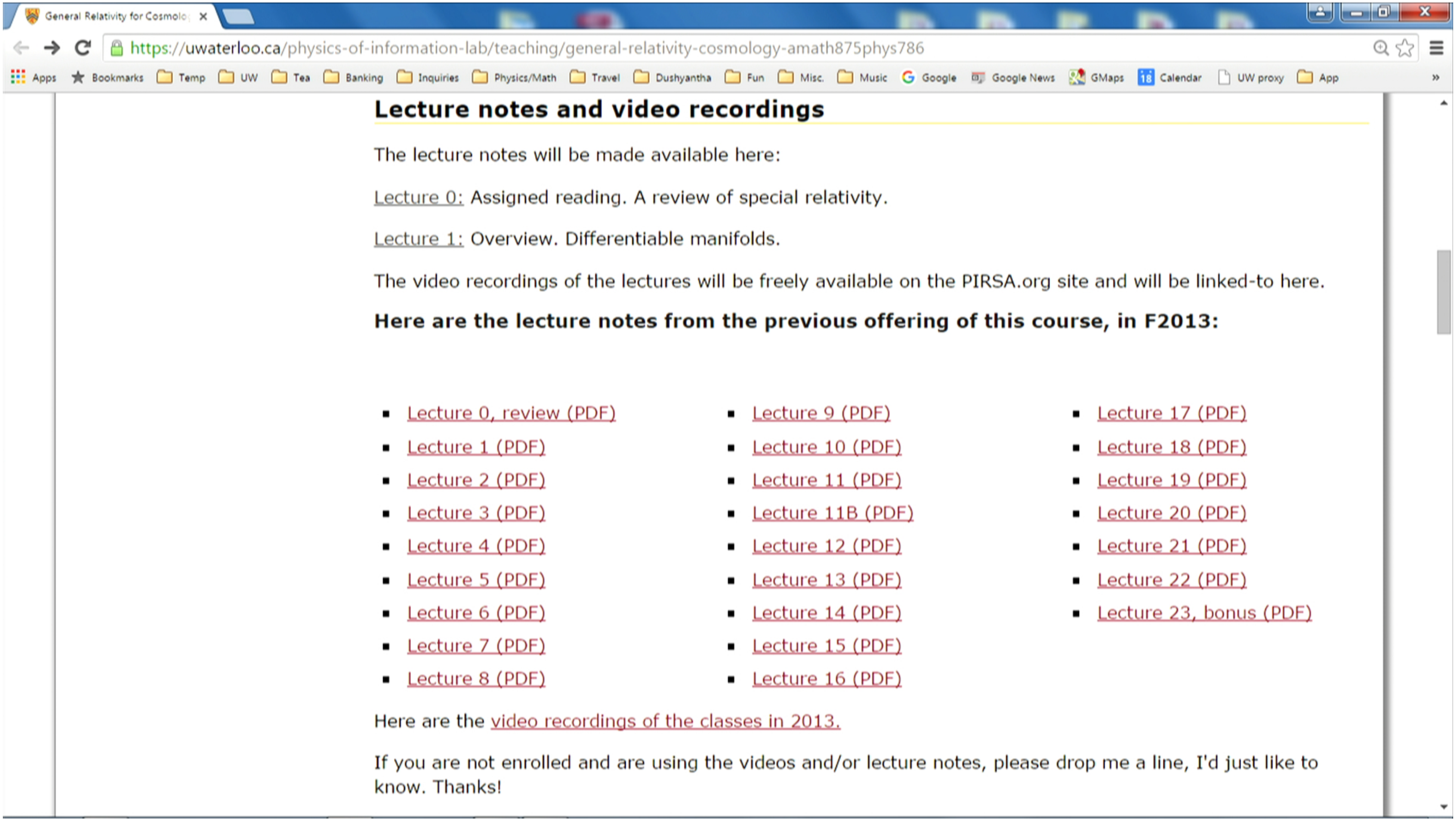
General Relativity for Cosmology (AMATH875/PHYS786)

- **Term:** Fall 2015
- **Course codes:** AMATH875/PHYS786
- **Instructor:** Achim Kempf
- **Prerequisites:** A first course in General Relativity or consent of instructor
- **Time:** Mondays and Fridays, 1:30 - 2:45 pm
Except: to make up for the lost lecture of Monday 12 Oct. (Thanksgiving) we'll have a double lecture (with break and snacks) on Nov. 9th: 1:30-4:30pm.
- **Venue:** Alice room, Perimeter Institute
Except: Mon. Sep.28 and Fri. Oct.2 in the Bob room
- **Office hours:** by arrangement
- **First lecture:** Monday 14 September 2015, 1:30pm

Picture on the right: This Hubble image lets us see half way through the age of the universe. In the "foreground" is the tadpole galaxy.

Content

This is an advanced graduate course which develops the math and physics of general relativity from scratch up to the highest level. The going is sometimes steep but always careful. The purpose is to prepare students for studies in quantum gravity, relativistic quantum information, black hole physics and cosmology. (To this end, this course will be followed up in W16 by the course Quantum Field Theory for Cosmology.)



Lecture notes and video recordings

The lecture notes will be made available here:

Lecture 0: Assigned reading. A review of special relativity.

Lecture 1: Overview. Differentiable manifolds.

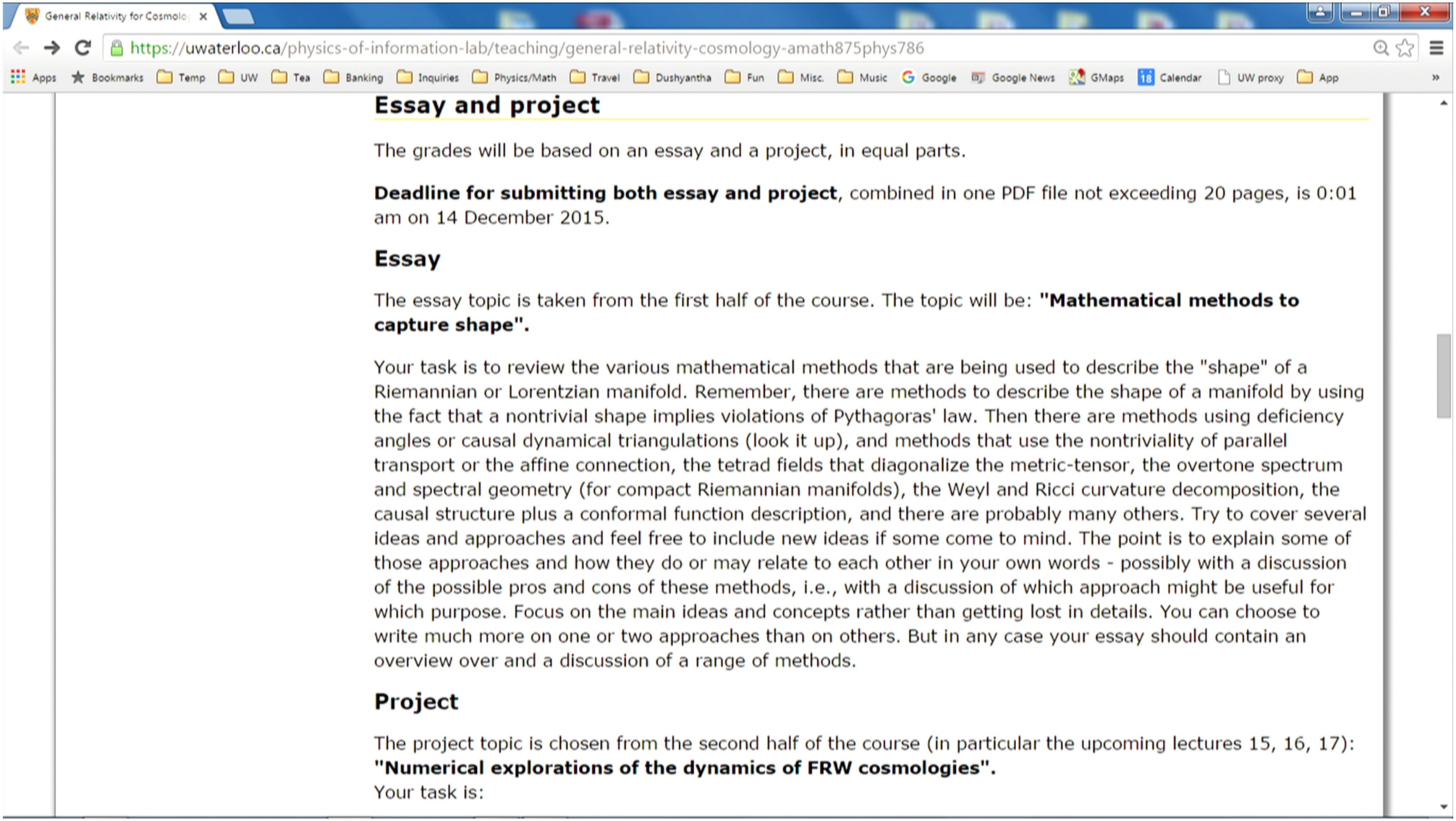
The video recordings of the lectures will be freely available on the PIRSA.org site and will be linked-to here.

Here are the lecture notes from the previous offering of this course, in F2013:

- Lecture 0, review (PDF)
- Lecture 1 (PDF)
- Lecture 2 (PDF)
- Lecture 3 (PDF)
- Lecture 4 (PDF)
- Lecture 5 (PDF)
- Lecture 6 (PDF)
- Lecture 7 (PDF)
- Lecture 8 (PDF)
- Lecture 9 (PDF)
- Lecture 10 (PDF)
- Lecture 11 (PDF)
- Lecture 11B (PDF)
- Lecture 12 (PDF)
- Lecture 13 (PDF)
- Lecture 14 (PDF)
- Lecture 15 (PDF)
- Lecture 16 (PDF)
- Lecture 17 (PDF)
- Lecture 18 (PDF)
- Lecture 19 (PDF)
- Lecture 20 (PDF)
- Lecture 21 (PDF)
- Lecture 22 (PDF)
- Lecture 23, bonus (PDF)

Here are the video recordings of the classes in 2013.

If you are not enrolled and are using the videos and/or lecture notes, please drop me a line, I'd just like to know. Thanks!



Essay and project

The grades will be based on an essay and a project, in equal parts.

Deadline for submitting both essay and project, combined in one PDF file not exceeding 20 pages, is 0:01 am on 14 December 2015.

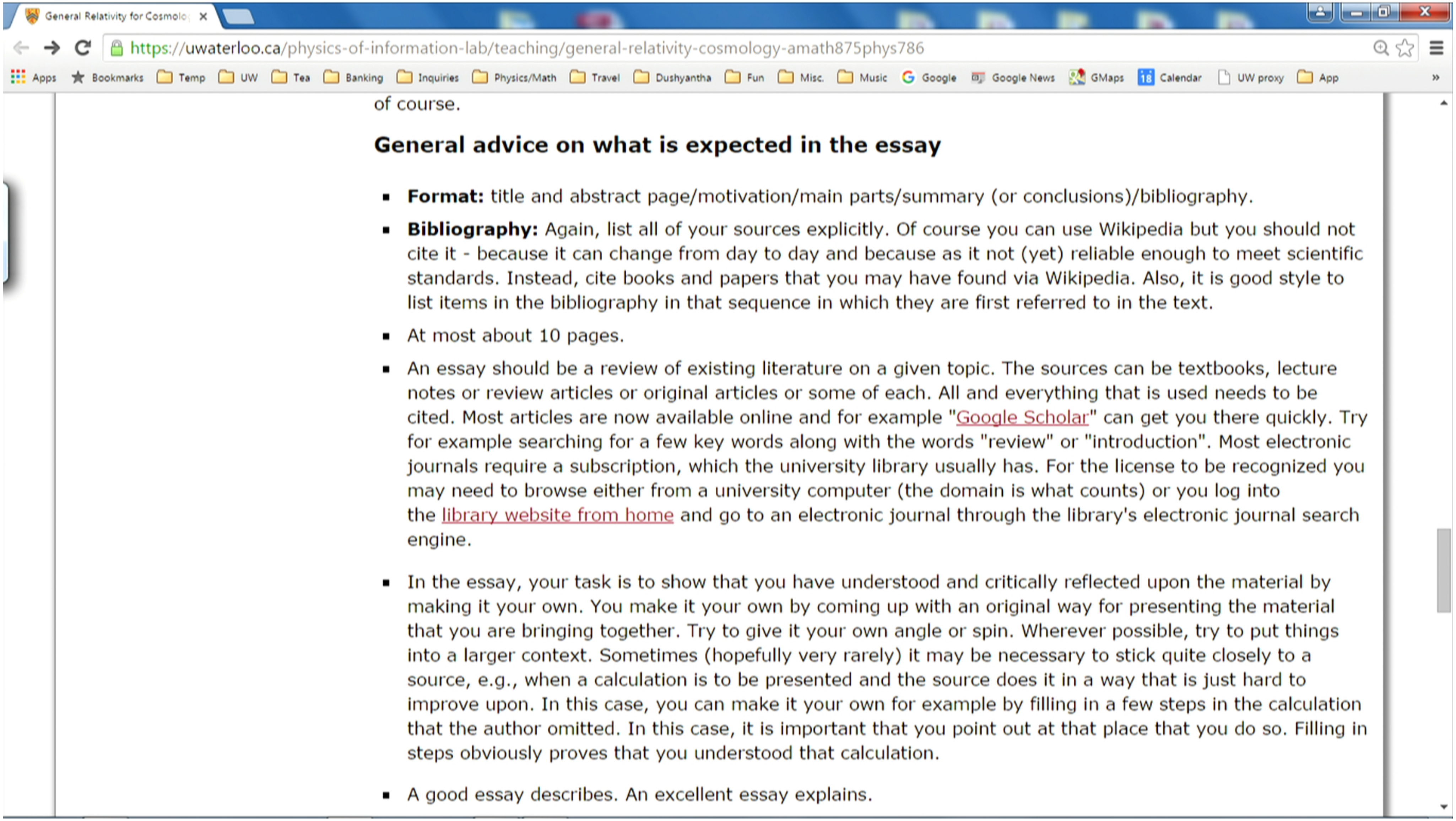
Essay

The essay topic is taken from the first half of the course. The topic will be: **"Mathematical methods to capture shape".**

Your task is to review the various mathematical methods that are being used to describe the "shape" of a Riemannian or Lorentzian manifold. Remember, there are methods to describe the shape of a manifold by using the fact that a nontrivial shape implies violations of Pythagoras' law. Then there are methods using deficiency angles or causal dynamical triangulations (look it up), and methods that use the nontriviality of parallel transport or the affine connection, the tetrad fields that diagonalize the metric-tensor, the overtone spectrum and spectral geometry (for compact Riemannian manifolds), the Weyl and Ricci curvature decomposition, the causal structure plus a conformal function description, and there are probably many others. Try to cover several ideas and approaches and feel free to include new ideas if some come to mind. The point is to explain some of those approaches and how they do or may relate to each other in your own words - possibly with a discussion of the possible pros and cons of these methods, i.e., with a discussion of which approach might be useful for which purpose. Focus on the main ideas and concepts rather than getting lost in details. You can choose to write much more on one or two approaches than on others. But in any case your essay should contain an overview over and a discussion of a range of methods.

Project

The project topic is chosen from the second half of the course (in particular the upcoming lectures 15, 16, 17): **"Numerical explorations of the dynamics of FRW cosmologies".**
Your task is:



of course.

General advice on what is expected in the essay

- **Format:** title and abstract page/motivation/main parts/summary (or conclusions)/bibliography.
- **Bibliography:** Again, list all of your sources explicitly. Of course you can use Wikipedia but you should not cite it - because it can change from day to day and because as it not (yet) reliable enough to meet scientific standards. Instead, cite books and papers that you may have found via Wikipedia. Also, it is good style to list items in the bibliography in that sequence in which they are first referred to in the text.
- At most about 10 pages.
- An essay should be a review of existing literature on a given topic. The sources can be textbooks, lecture notes or review articles or original articles or some of each. All and everything that is used needs to be cited. Most articles are now available online and for example "Google Scholar" can get you there quickly. Try for example searching for a few key words along with the words "review" or "introduction". Most electronic journals require a subscription, which the university library usually has. For the license to be recognized you may need to browse either from a university computer (the domain is what counts) or you log into the library website from home and go to an electronic journal through the library's electronic journal search engine.
- In the essay, your task is to show that you have understood and critically reflected upon the material by making it your own. You make it your own by coming up with an original way for presenting the material that you are bringing together. Try to give it your own angle or spin. Wherever possible, try to put things into a larger context. Sometimes (hopefully very rarely) it may be necessary to stick quite closely to a source, e.g., when a calculation is to be presented and the source does it in a way that is just hard to improve upon. In this case, you can make it your own for example by filling in a few steps in the calculation that the author omitted. In this case, it is important that you point out at that place that you do so. Filling in steps obviously proves that you understood that calculation.
- A good essay describes. An excellent essay explains.

Literature

We will be using mainly material from the following three texts:

- N. Straumann, General Relativity with Applications to Astrophysics, Springer (2004)
- J. Stewart, Advanced General Relativity, Cambridge (1991)
- S. Hawking, G.F.R. Ellis, The Large Scale Structure of Space-Time, Cambridge (1973)

Note: These three texts are available at the Davis Library.

Recommended general references are also:

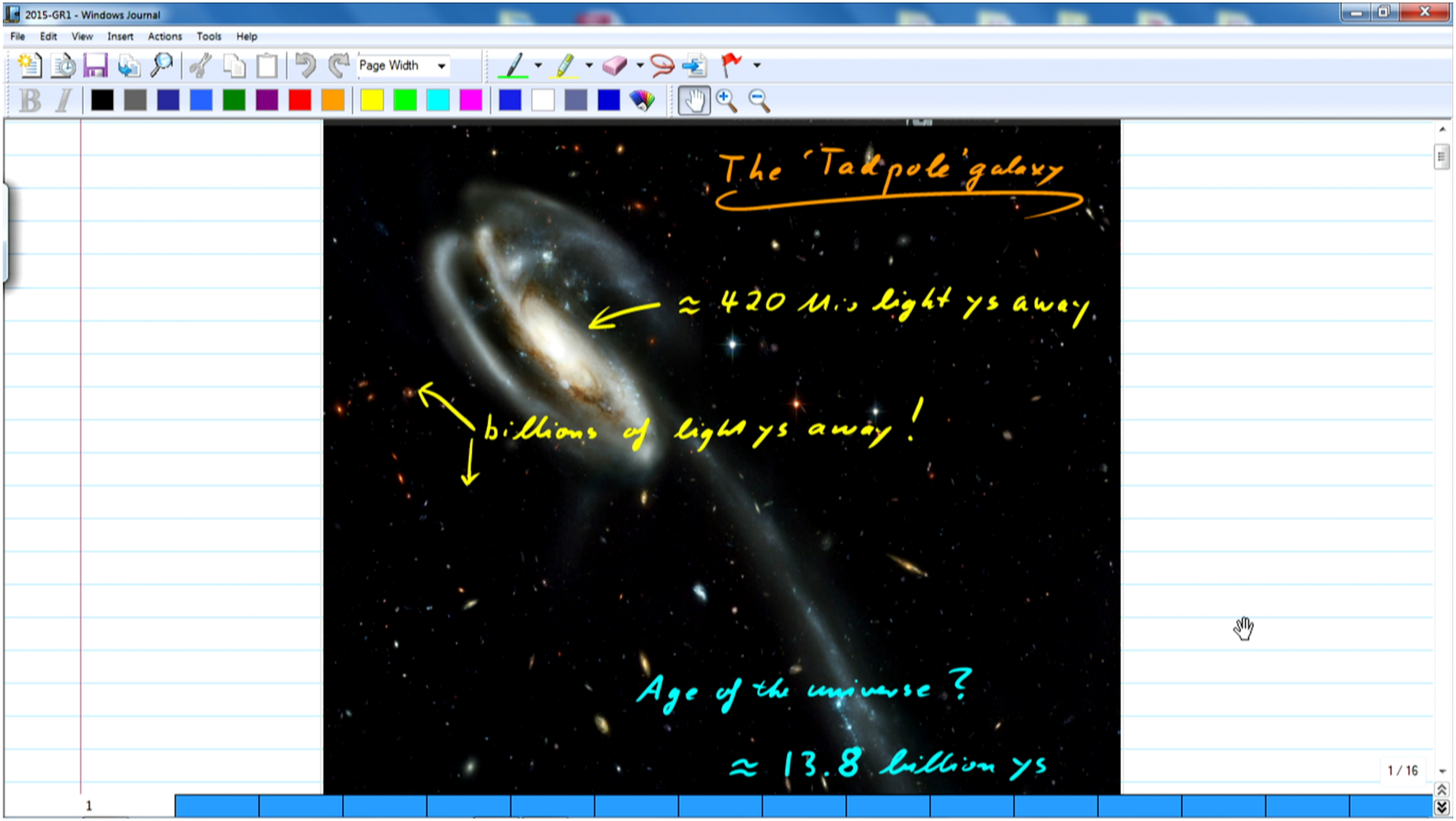
- Scott Dodelson, Modern Cosmology, Academic Press, San Diego, (2003)
- A.R. Liddle, D.H. Lyth, Cosmological Inflation and Large-Scale Structure, CUP (2000)
- G.F.R. Ellis and J. Wainwright, Dynamical Systems in Cosmology, CUP (1997)
- R. M. Wald, General Relativity, University of Chicago Press (1984)
- H. Stephani, General Relativity, Cambridge University Press (CUP) (1982)

We will cover Sakharov's "induced gravity" argument. Read the original (very short) paper here:

- Vacuum quantum fluctuations in curved space and the theory of gravitation (PDF)
- Sakharov's induced gravity: a modern perspective (PDF)

Here are links to general online reviews:

- INSPIRE-High Energy Physics (HEP) information system at Stanford University
- Living Reviews in Relativity



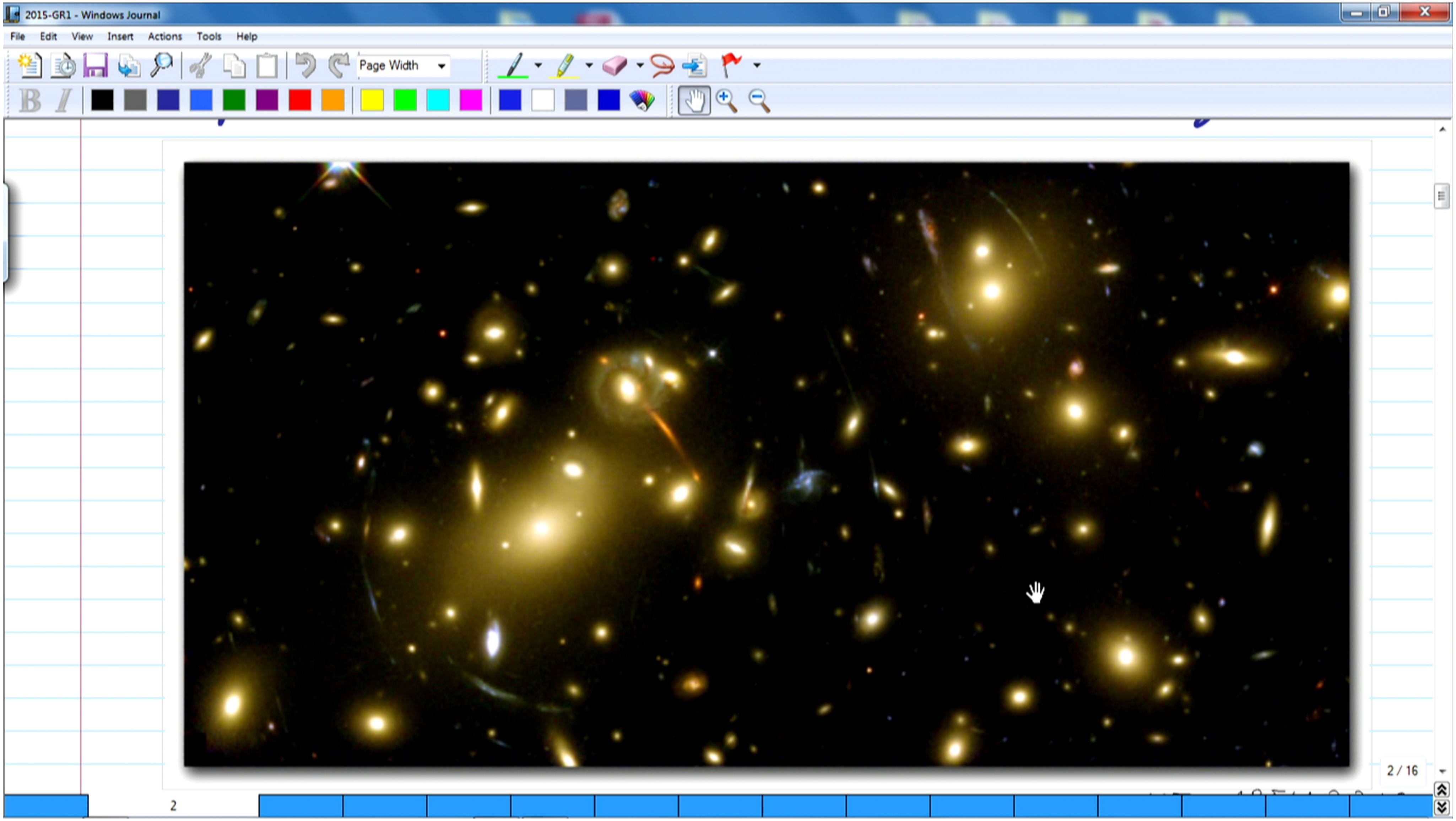
The 'Tadpole' galaxy

≈ 420 Mio light ys away

billions of light ys away!

Age of the universe?

≈ 13.8 billion ys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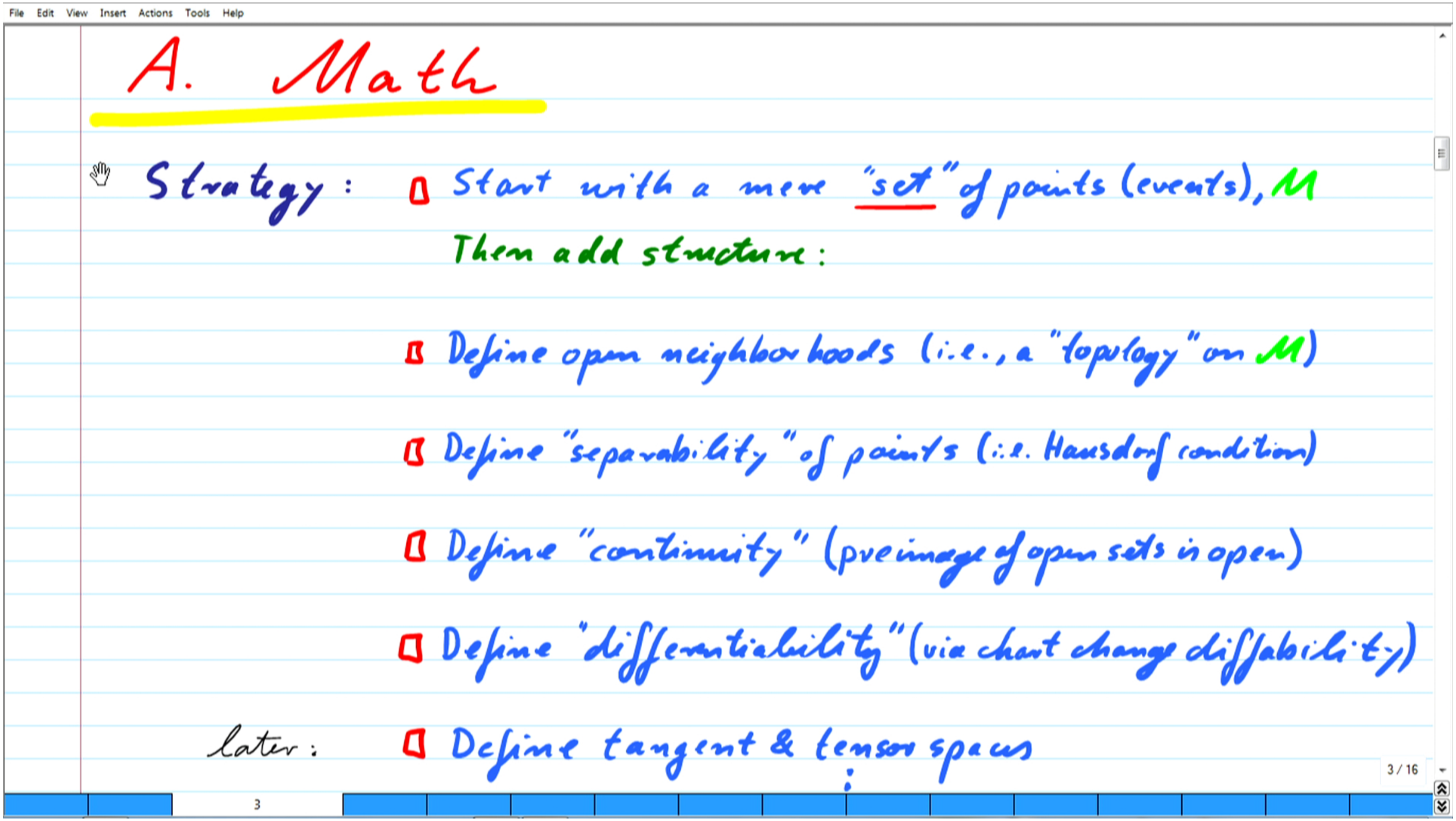


A. Math

Strategy: ☐ Start with a mere "set" of points (events), M

Then add structure:

☐ Define open neighborhoods (i.e., a "topology" on M)

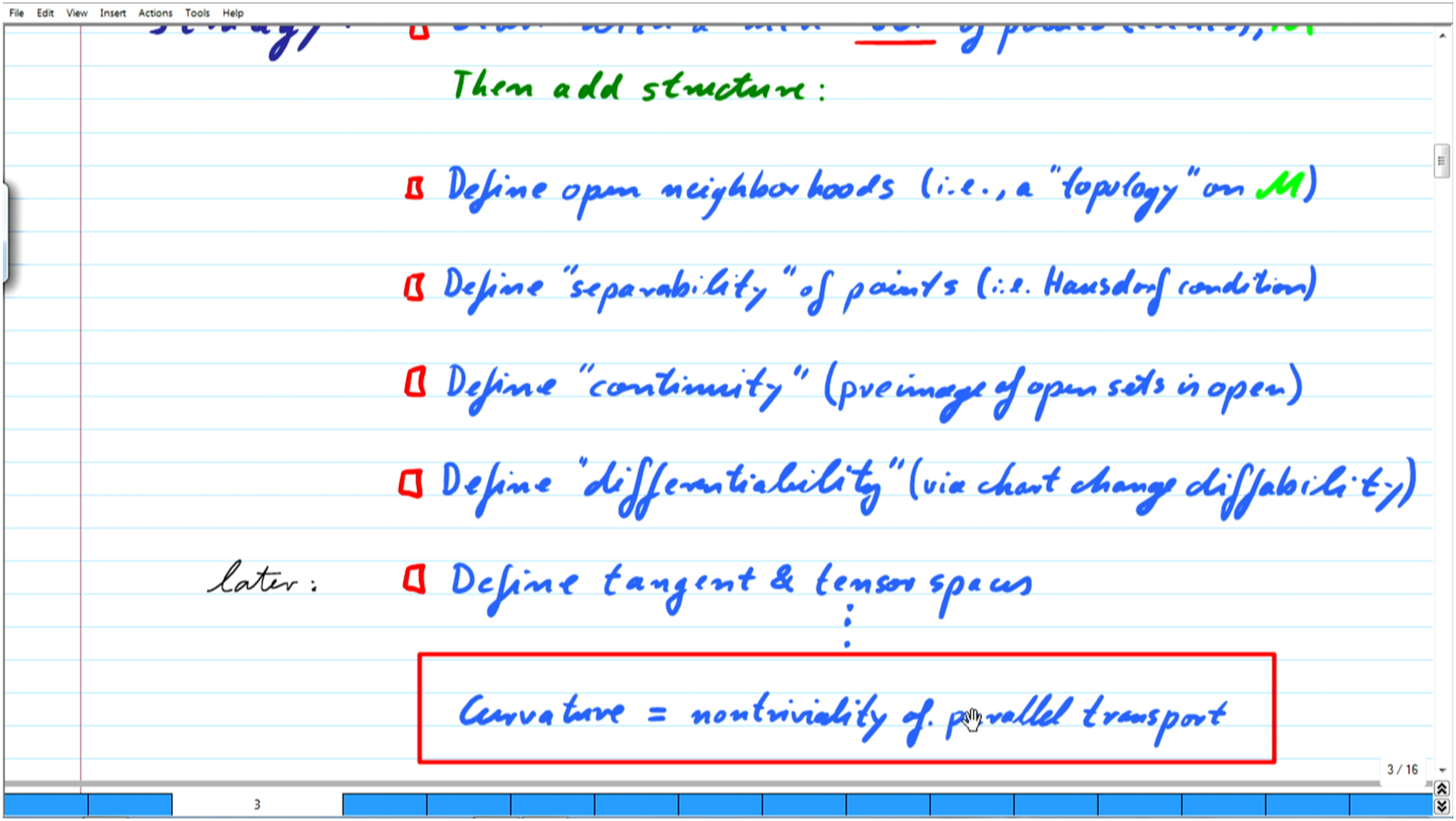
☐ Define "separability" of points (i.e. Hausdorf condition)

☐ Define "continuity" (preimage of open sets is open)

☐ Define "differentiability" (via chart change diffability)

later: ☐ Define tangent & tensor spaces

⋮



Then add structure:

- ☐ Define open neighborhoods (i.e., a "topology" on $\mathcal{M}$)
- ☐ Define "separability" of points (i.e. Hausdorf condition)
- ☐ Define "continuity" (preimage of open sets is open)
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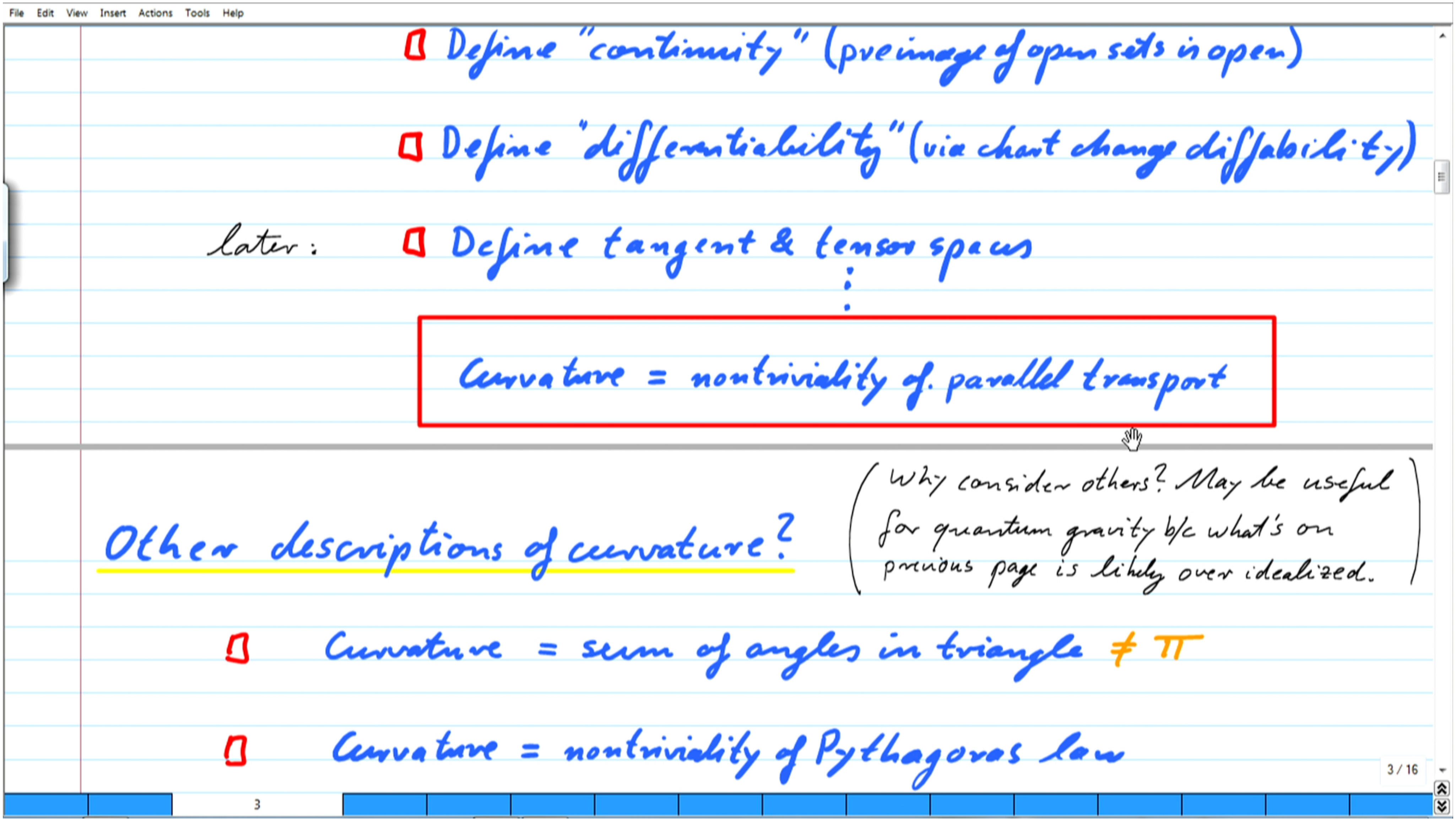
$$\boxed{\text{Curvature} = \text{nontriviality of parallel transport}}$$



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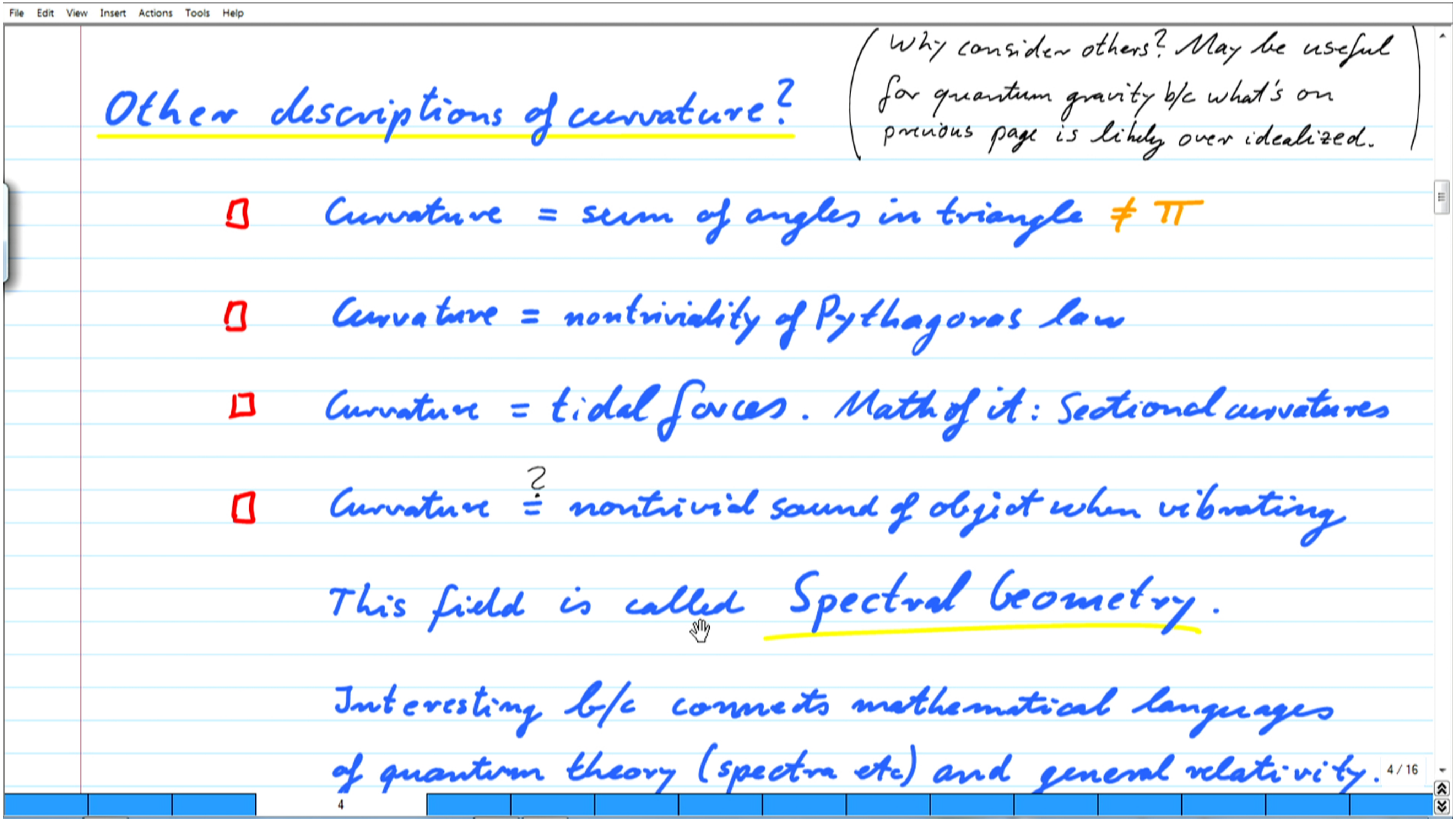
Curvature = nontriviality of parallel transport

Other descriptions of curvature?

(Why consider others? May be useful for quantum gravity b/c what's on previous page is likely over idealized.)

☐ Curvature = sum of angles in triangle $\neq \pi$

☐ Curvature = nontriviality of Pythagoras law



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(Why consider others? May be useful for quantum gravity b/c what's on previous page is likely over idealized.)

☐ Curvature = sum of angles in triangle $\neq \pi$

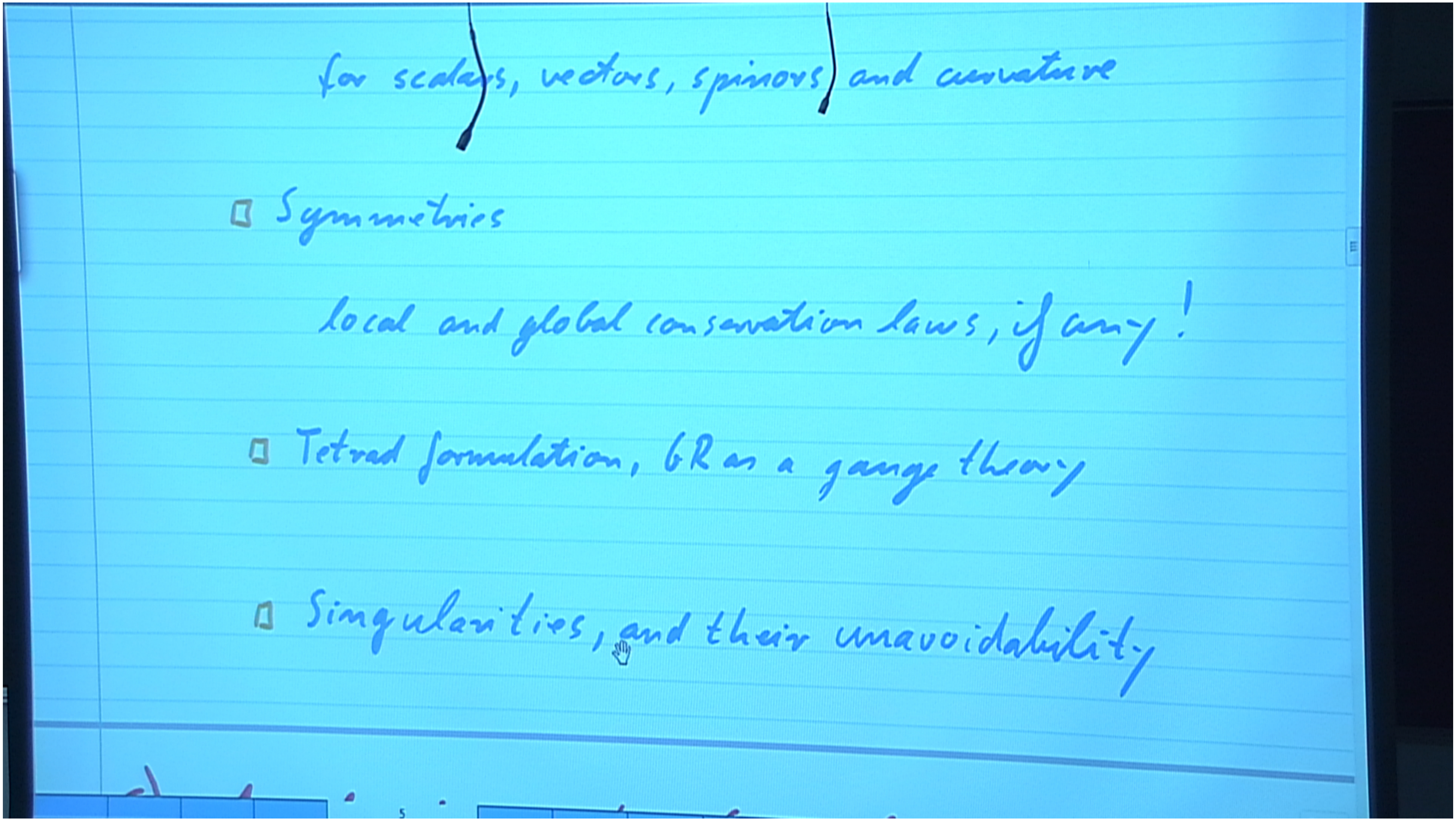
☐ Curvature = nontriviality of Pythagoras law

☐ Curvature = tidal forces. Math of it: Sectional curvatures

☐ Curvature $\overset{?}{=}$ nontrivial sound of object when vibrating

This field is called Spectral Geometry.

Interesting b/c connects mathematical languages of quantum theory (spectra etc) and general relativity.



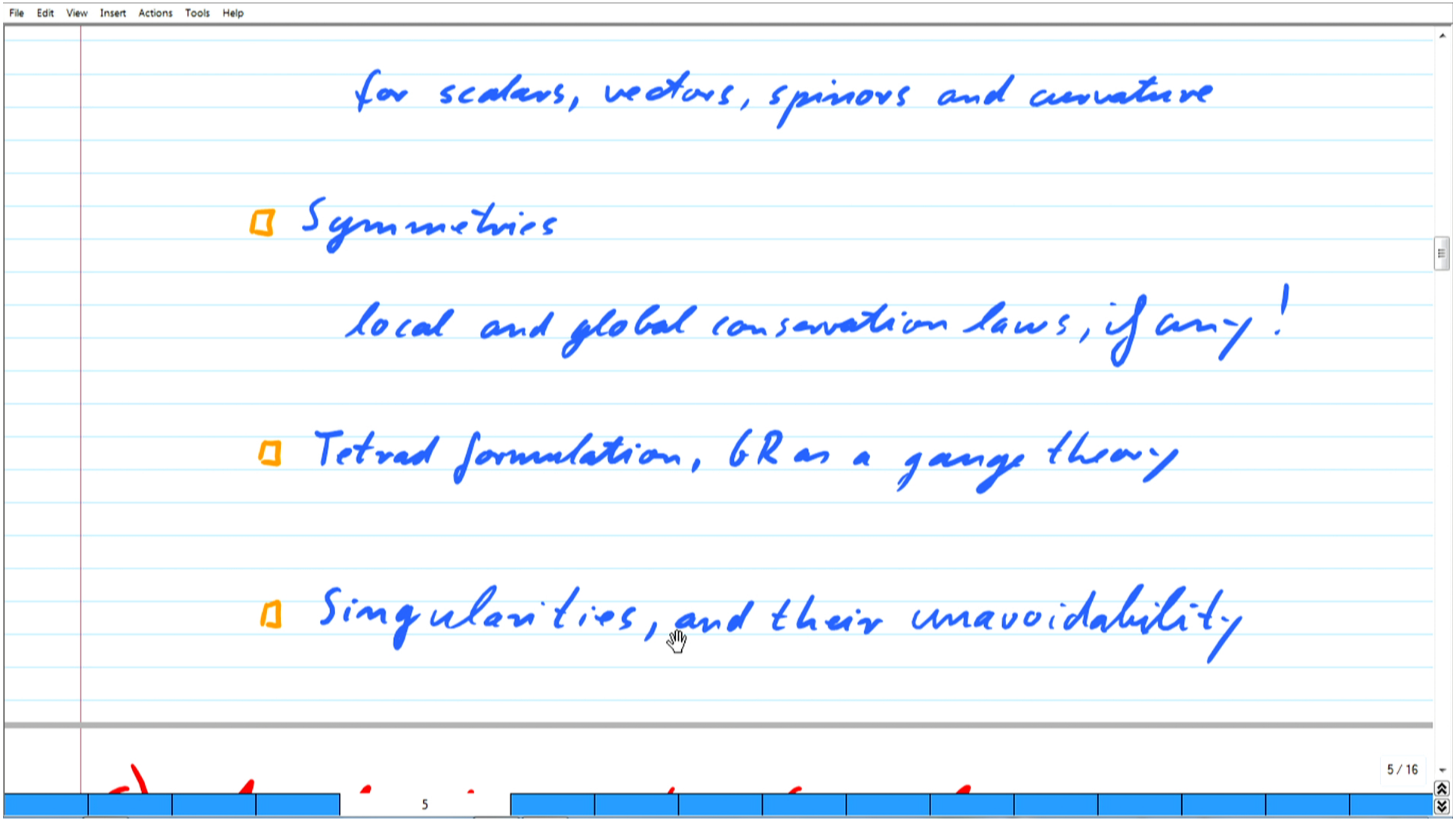
for scalars, vectors, spinors and curvature

- ☐ Symmetries

  local and global conservation laws, if any!

- ☐ Tetrad formulation, GR as a gauge theory

- ☐ Singularities, and their unavoidability



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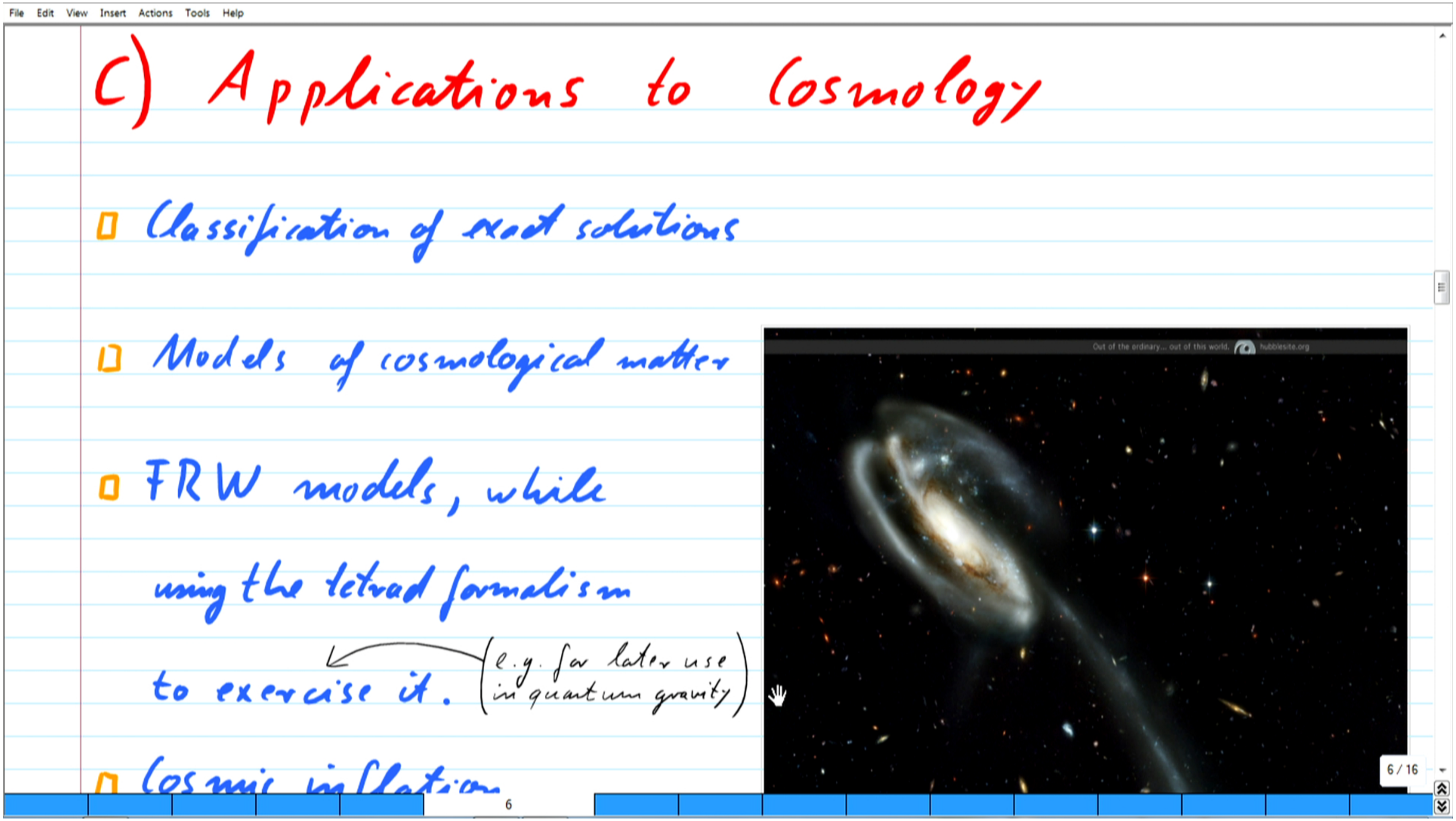
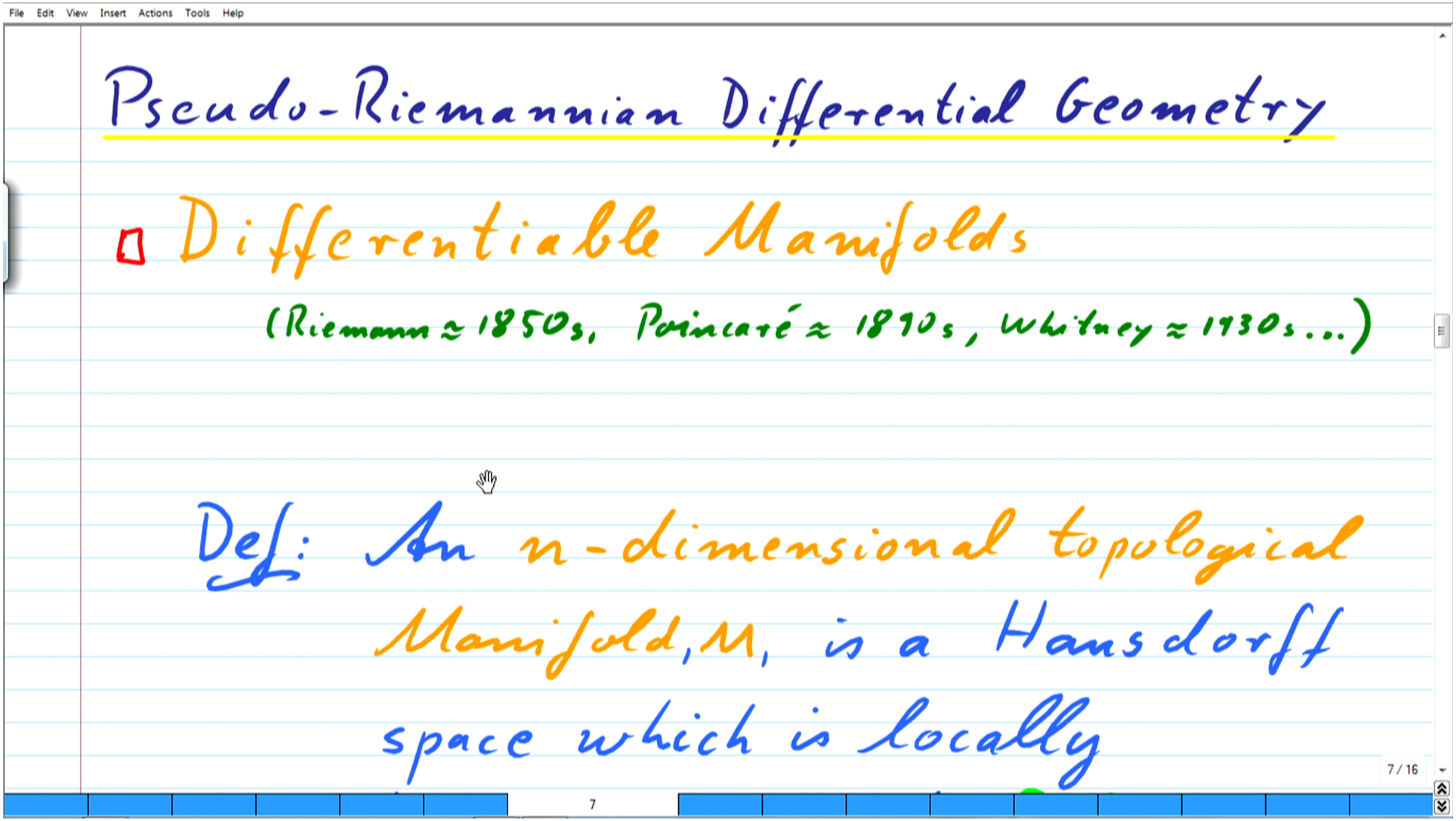


C) Applications to Cosmology

- ☐ Classification of exact solutions
- ☐ Models of cosmological matter
- ☐ FRW models, while using the tetrad formalism to exercise it. (e.g. for later use in quantum gravity)
- ☐ Cosmic inflation

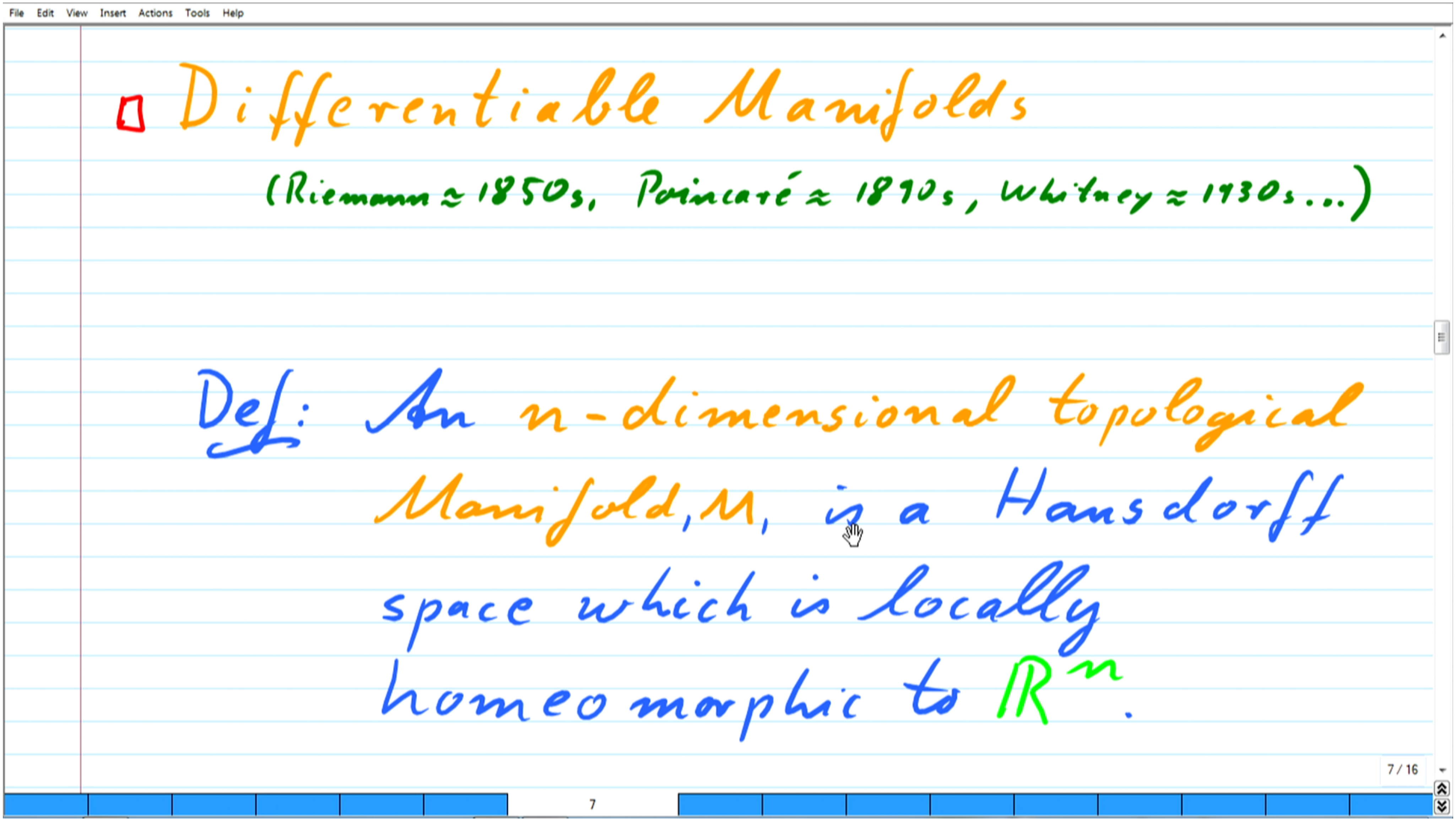


Pseudo-Riemannian Differential Geometry

□ Differentiable Manifolds

(Riemann ≈ 1850s, Poincaré ≈ 1890s, Whitney ≈ 1930s...)

Def: An n-dimensional topological Manifold, M, is a Hausdorff space which is locally



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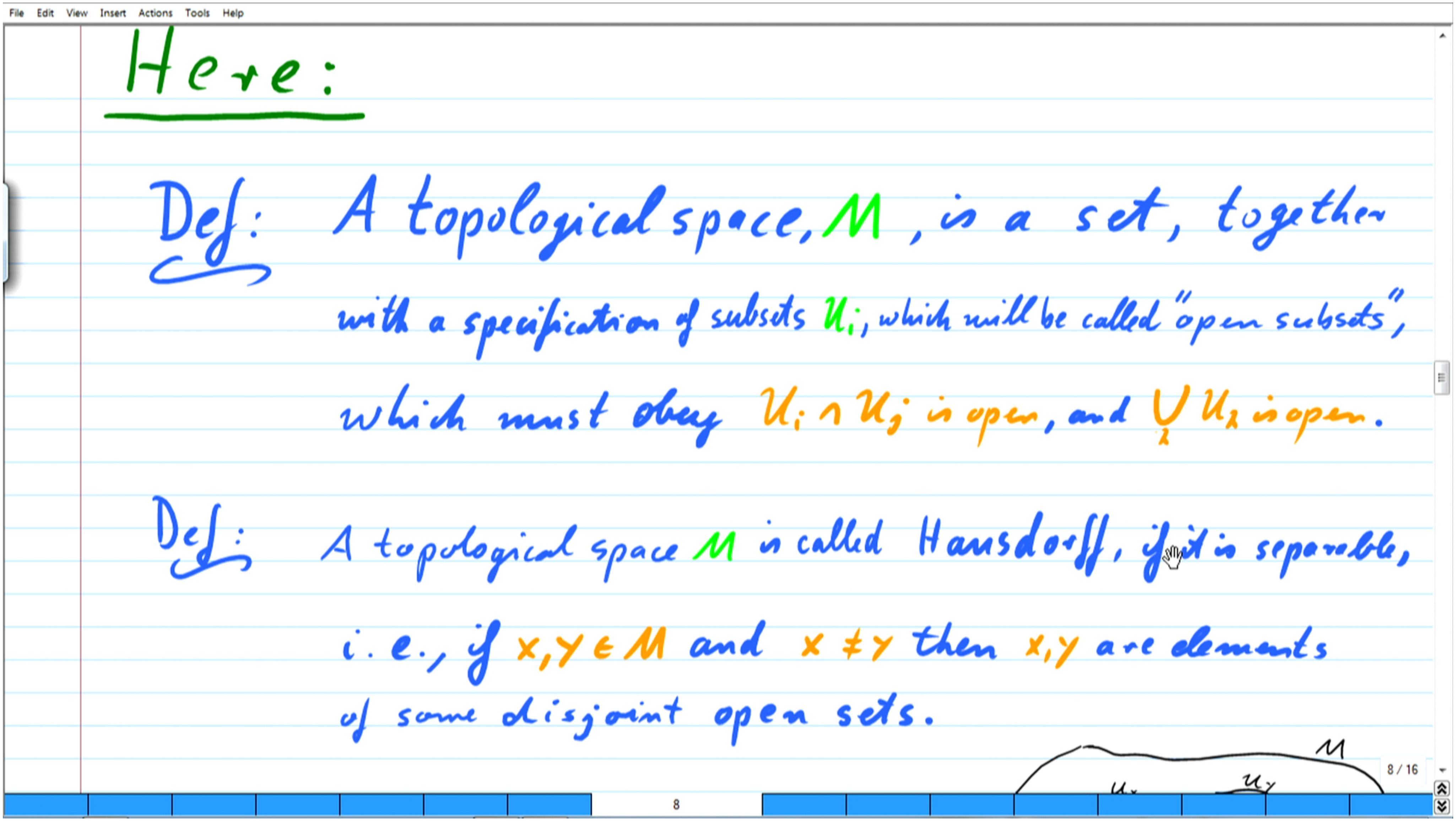
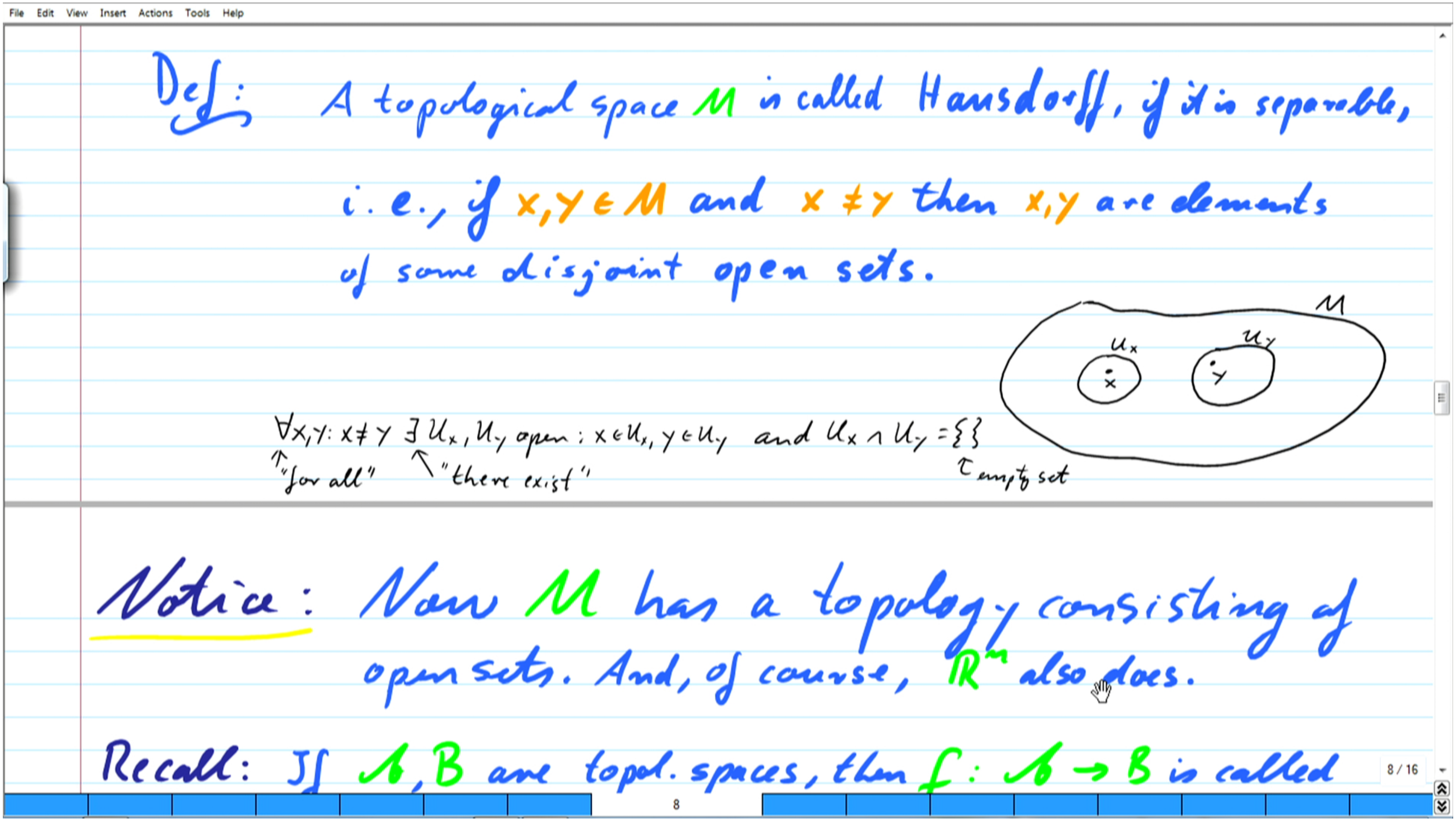


Here:

Def: A topological space, M, is a set, together with a specification of subsets U_i, which will be called "open subsets", which must obey $U_i \cap U_j$ is open, and $\bigcup_\alpha U_\alpha$ is open.

Def: A topological space M is called Hausdorff, if it is separable, i.e., if $x, y \in M$ and $x \neq y$ then x, y are elements of some disjoint open sets.



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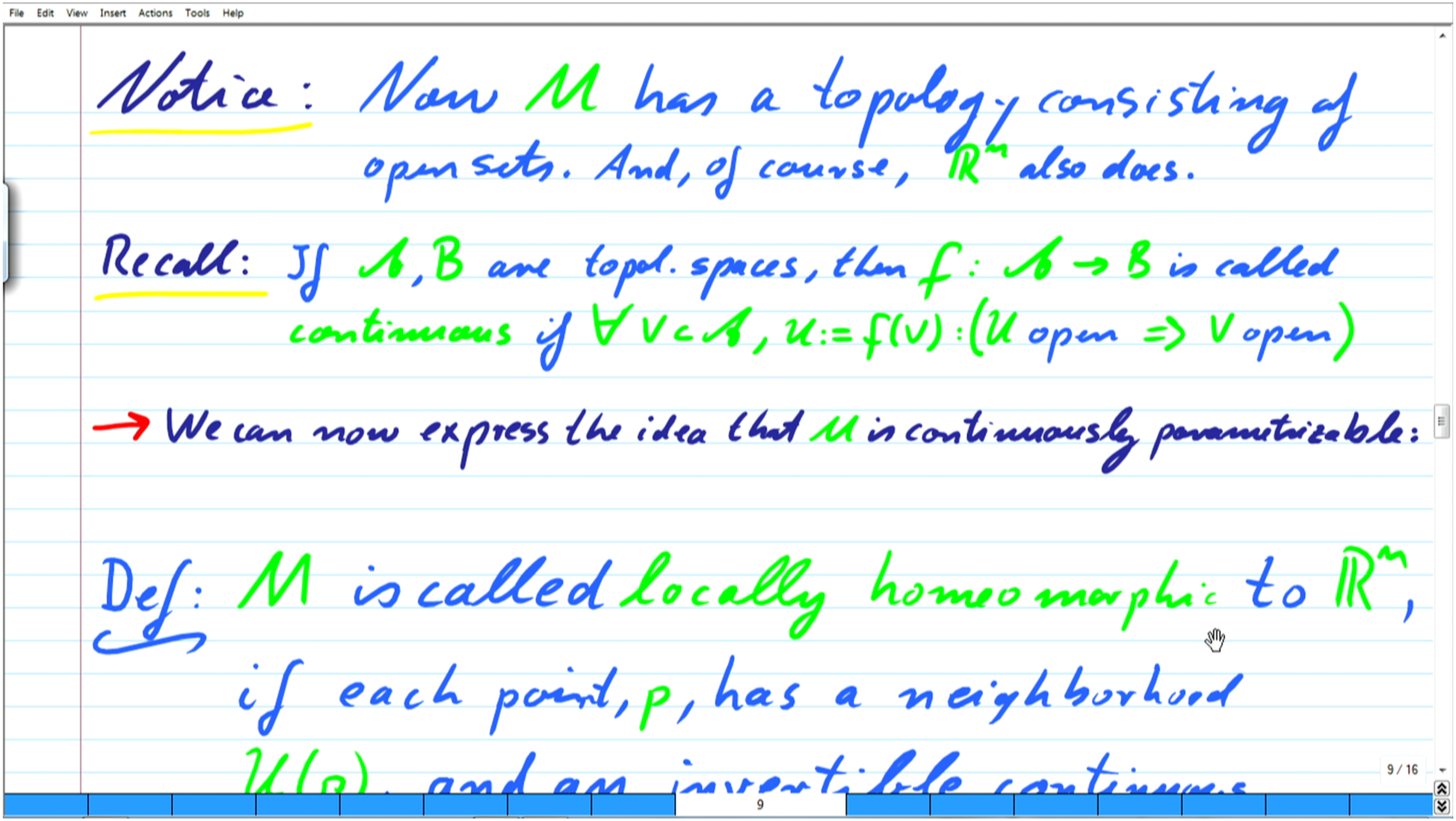
$\forall x,y: x \neq y \ \exists \, U_x, U_y$ open : $x \in U_x, y \in U_y$ and $U_x \cap U_y = \{\}$

↑ "for all" ↖ "there exist" ↖ empty set

Notice: Now M has a topology consisting of open sets. And, of course, $\mathbb{R}^n$ also does.

Recall: If A, B are topol. spaces, then $f: A \rightarrow B$ is called

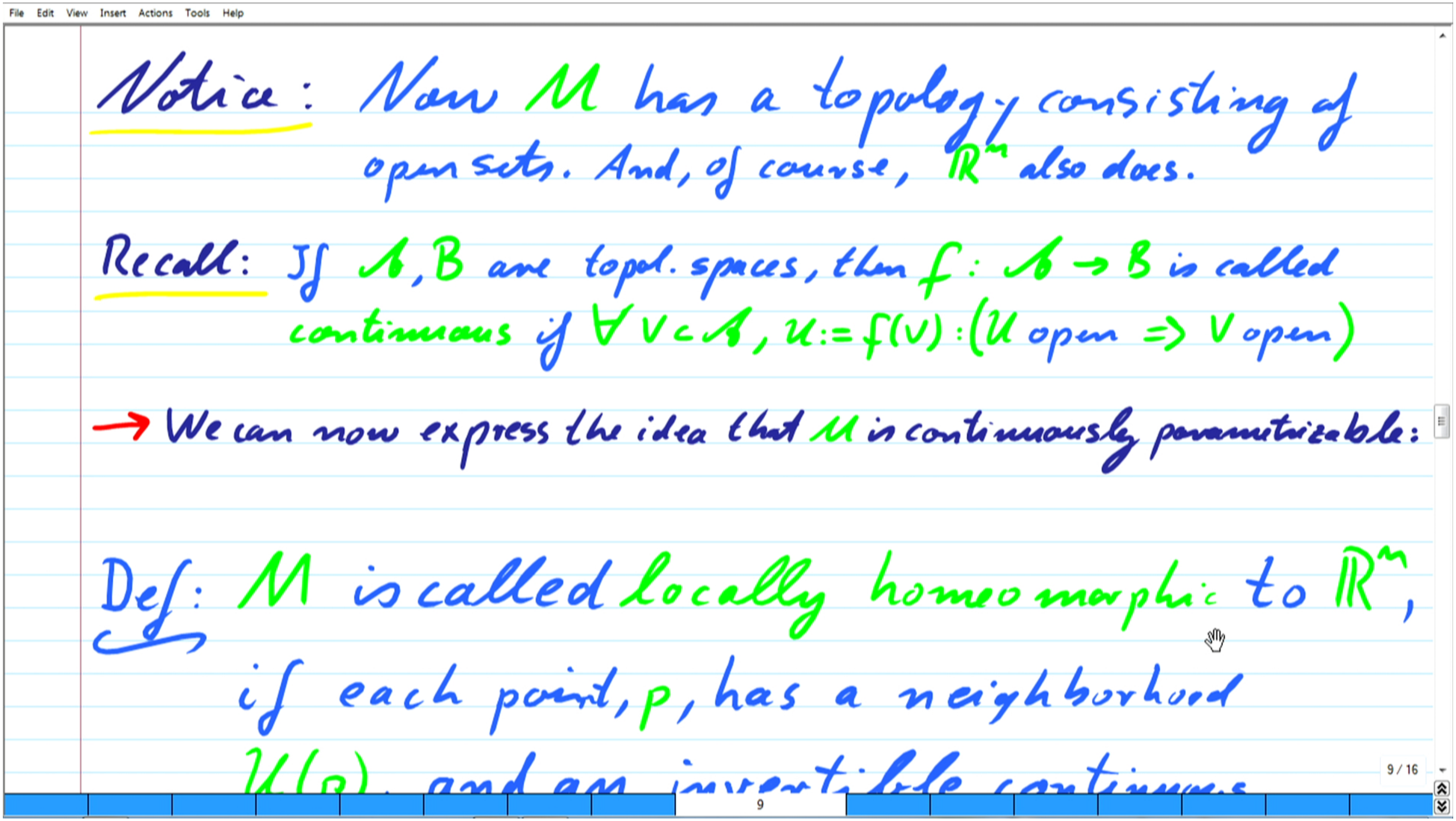


Notice: Now M has a topology consisting of open sets. And, of course, $\mathbb{R}^n$ also does.

Recall: If A, B are topol. spaces, then $f: A \to B$ is called continuous if $\forall V \subset A$, $U := f(V) : (U \text{ open} \Rightarrow V \text{ open})$

→ We can now express the idea that M is continuously parametrizable:

Def: M is called locally homeomorphic to $\mathbb{R}^n$, if each point, p, has a neighborhood $U(p)$, and an invertible continuous



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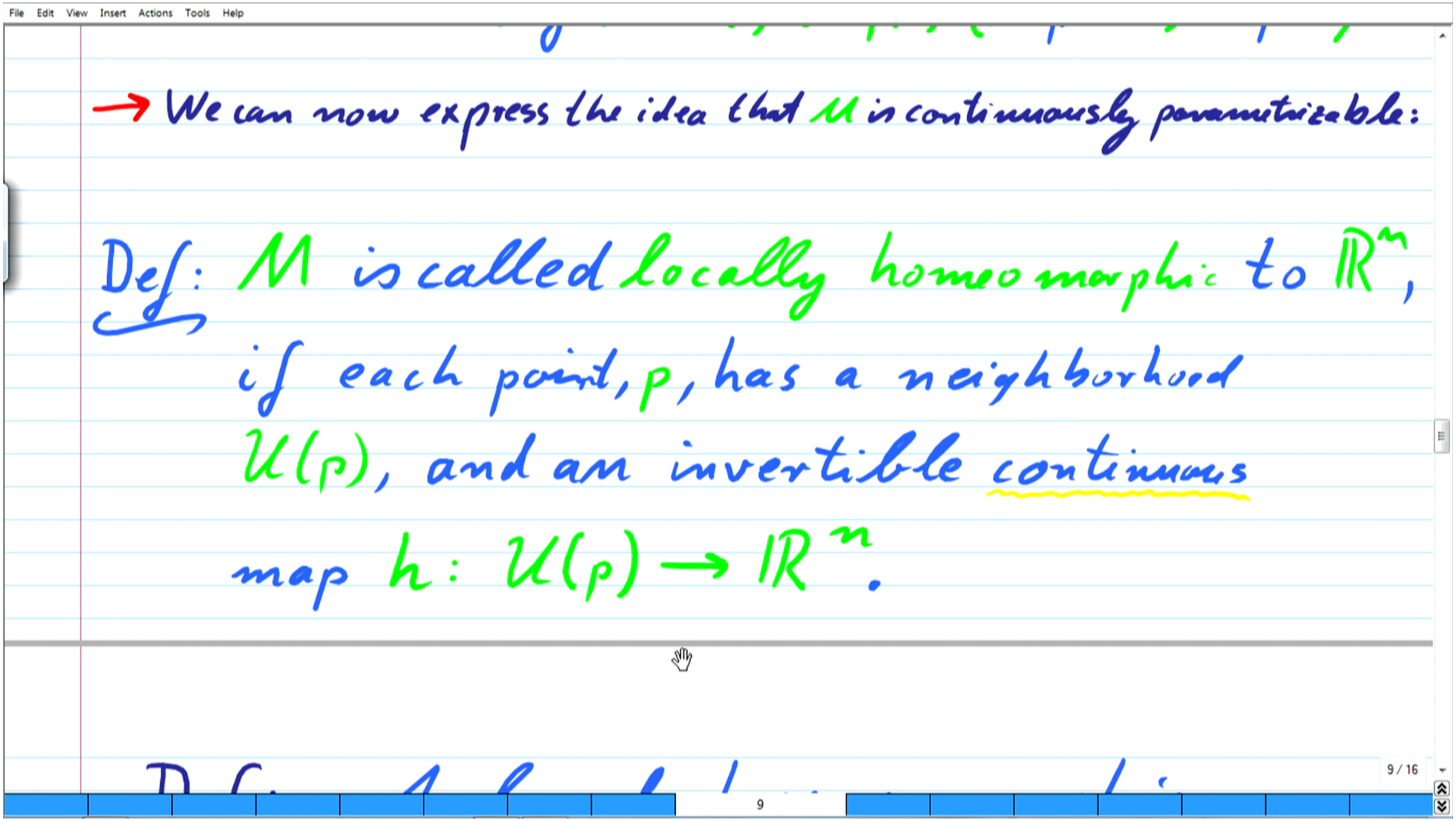
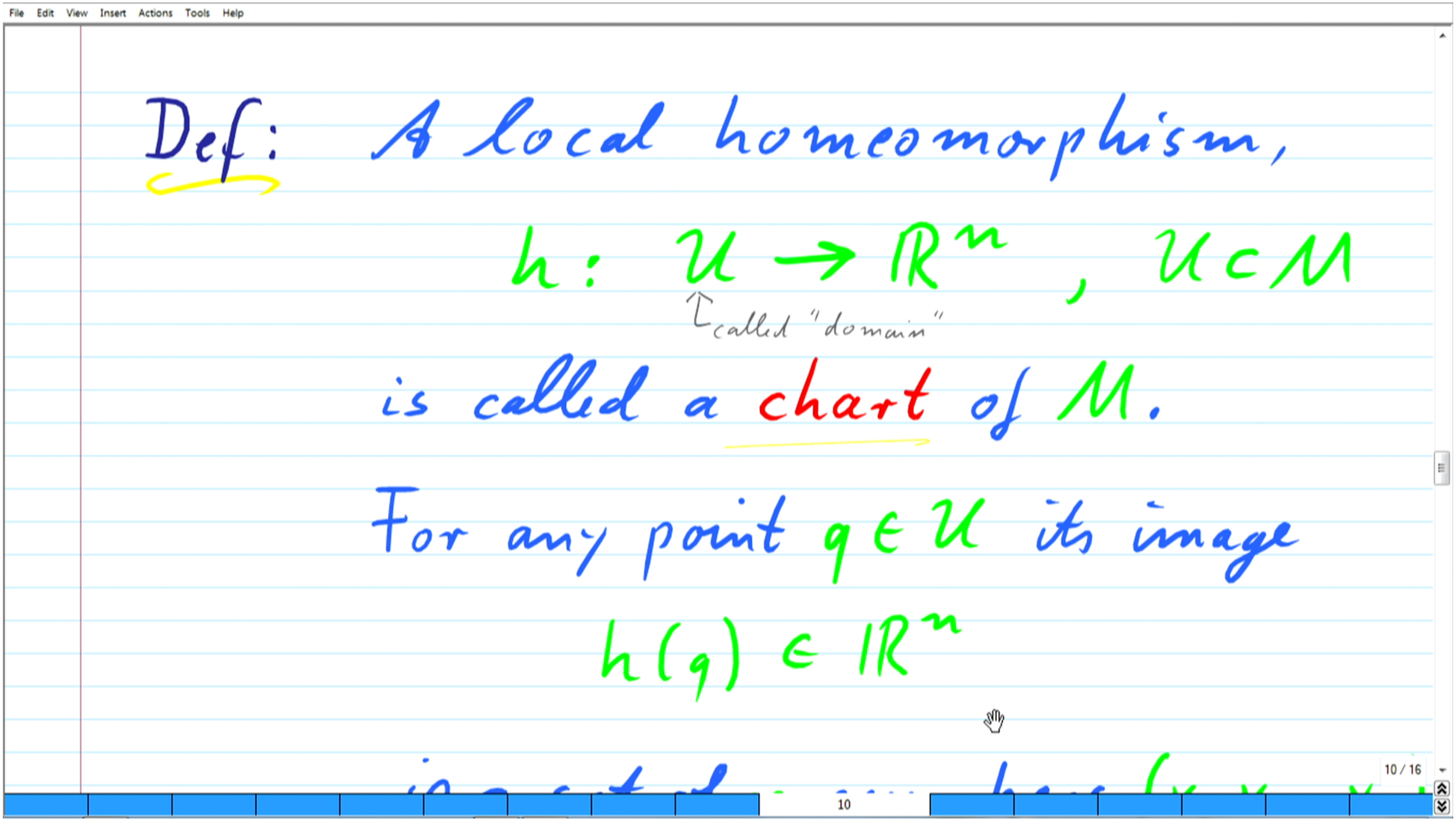


→ We can now express the idea that M is continuously parametrizable:

Def: M is called locally homeomorphic to $\mathbb{R}^n$, if each point, p, has a neighborhood $U(p)$, and an invertible continuous map $h: U(p) \to \mathbb{R}^n$.



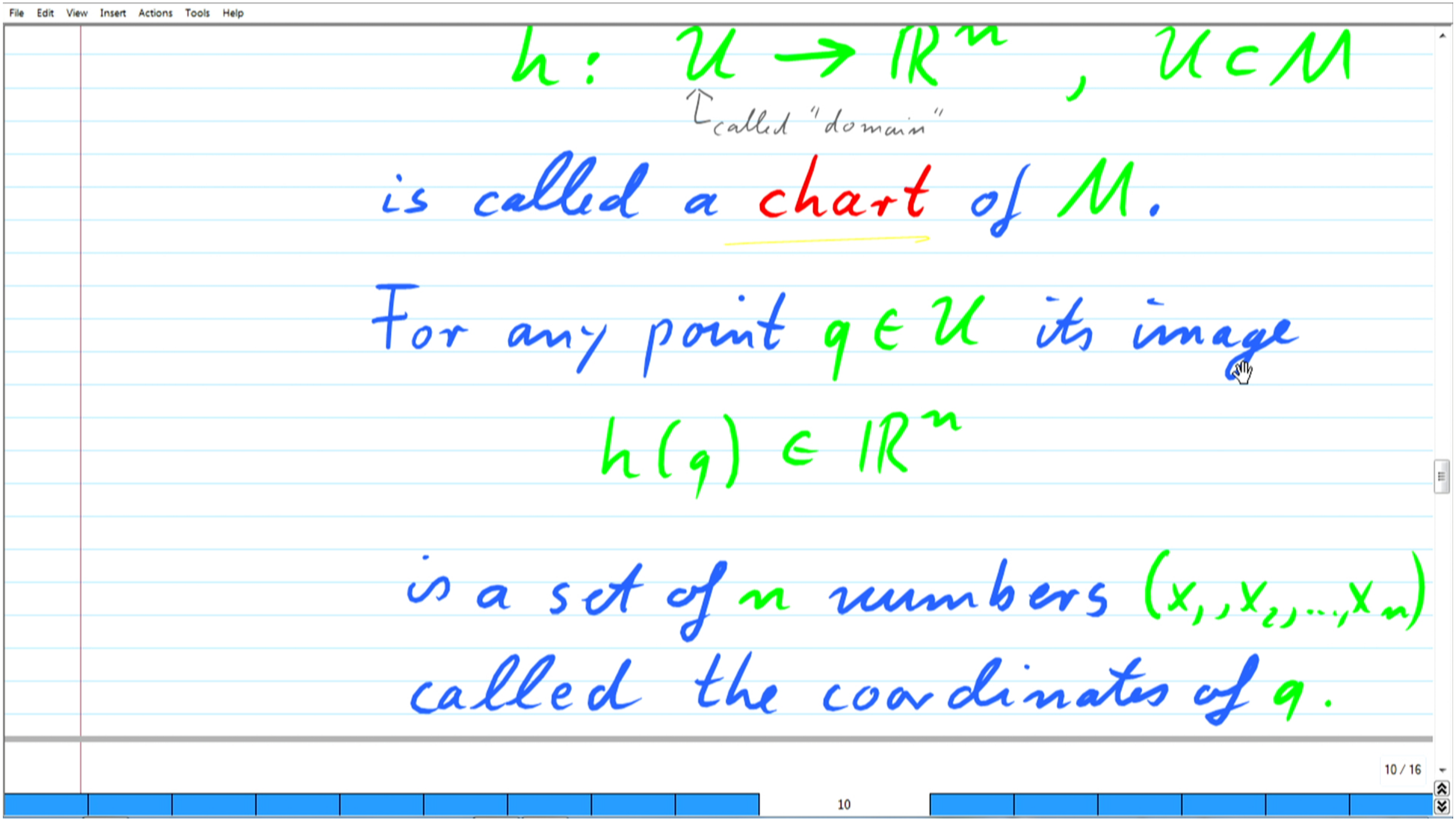
Def: A local homeomorphism,

$$h : \underset{\uparrow \text{called "domain"}}{U} \to \mathbb{R}^n, \quad U \subset M$$

is called a chart of M.

For any point $q \in U$ its image

$$h(q) \in \mathbb{R}^n$$



$$h: \mathcal{U} \to \mathbb{R}^n \quad , \quad \mathcal{U} \subset M$$

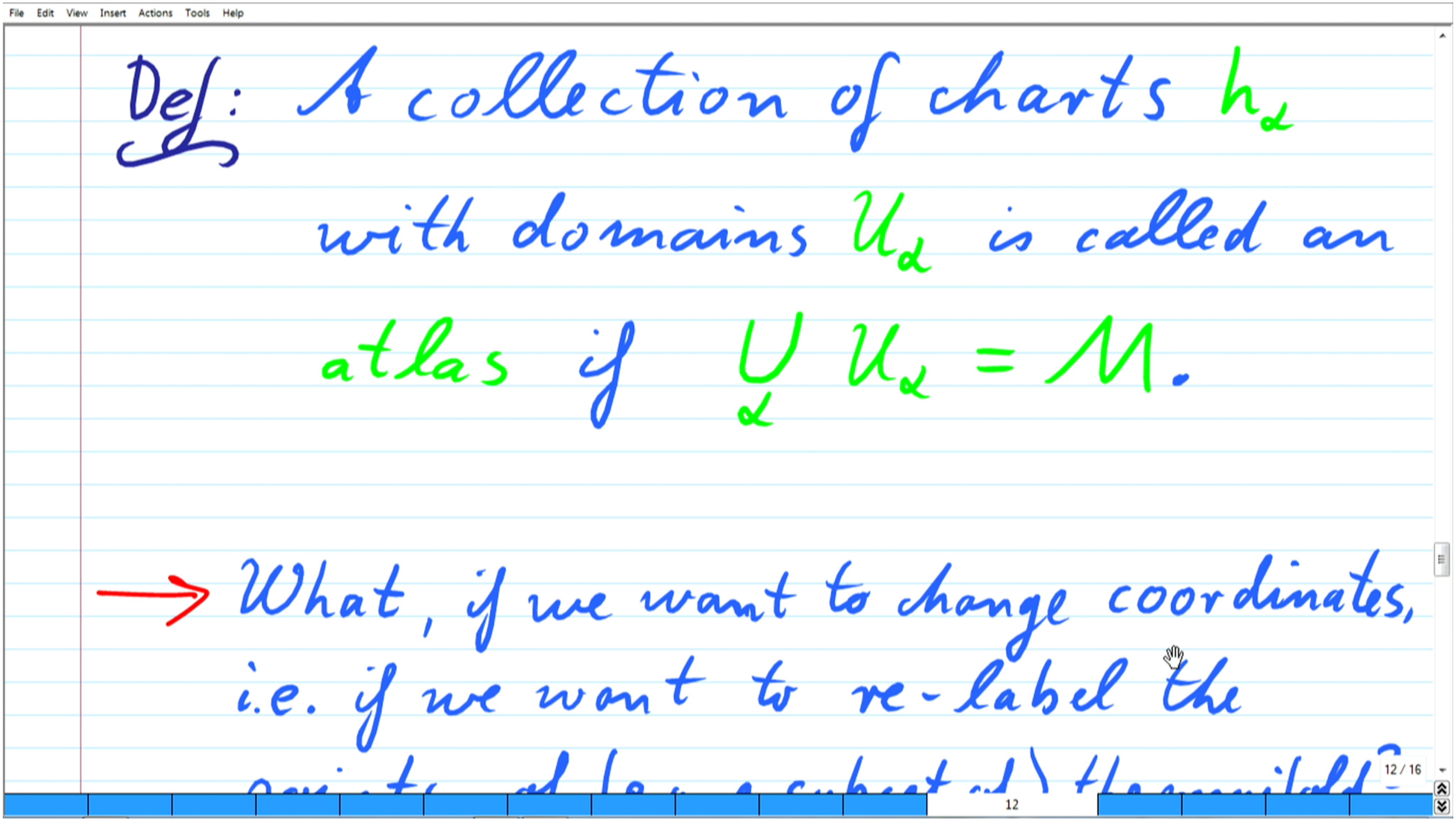
called "domain"

is called a chart of M.

For any point $q \in \mathcal{U}$ its image

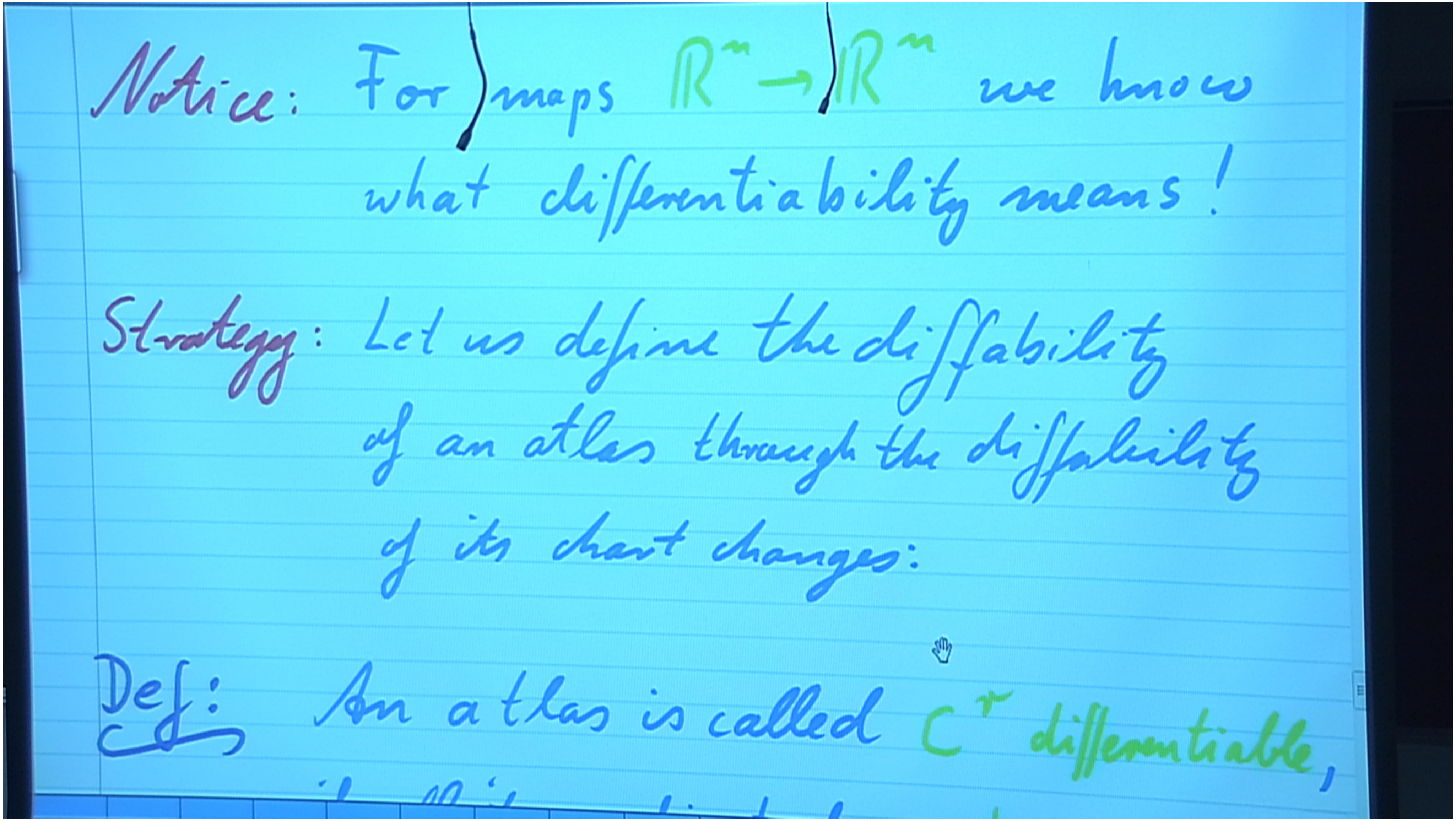
$$h(q) \in \mathbb{R}^n$$

is a set of n numbers $(x_1, x_2, \dots, x_n)$ called the coordinates of q.



Def: A collection of charts h_α with domains U_α is called an atlas if $\bigcup_\alpha U_\alpha = M$.

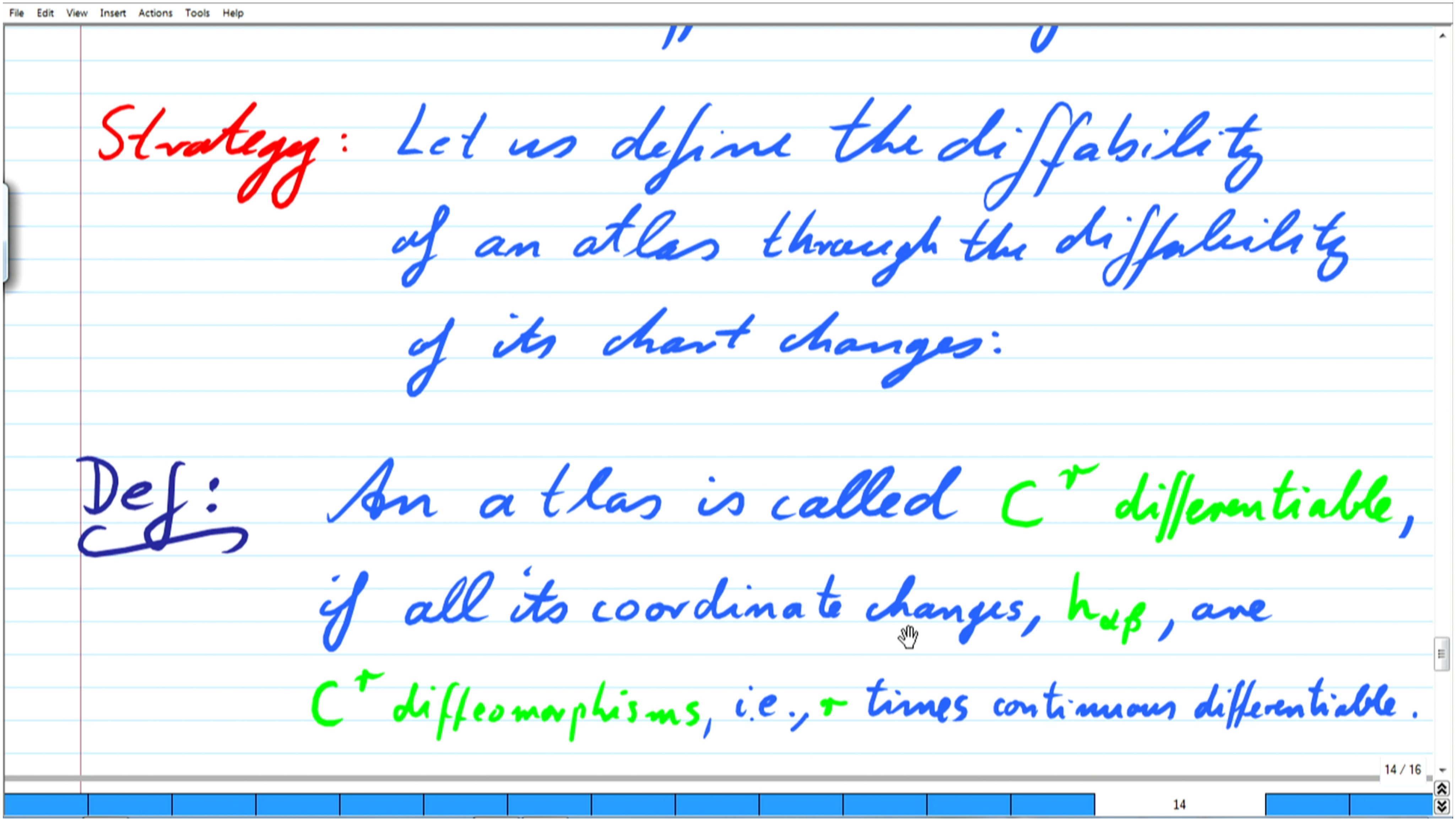
→ What, if we want to change coordinates, i.e. if we want to re-label the points of (or a subset of) the manifold?



Notice: For maps $\mathbb{R}^n \to \mathbb{R}^n$ we know what differentiability means!

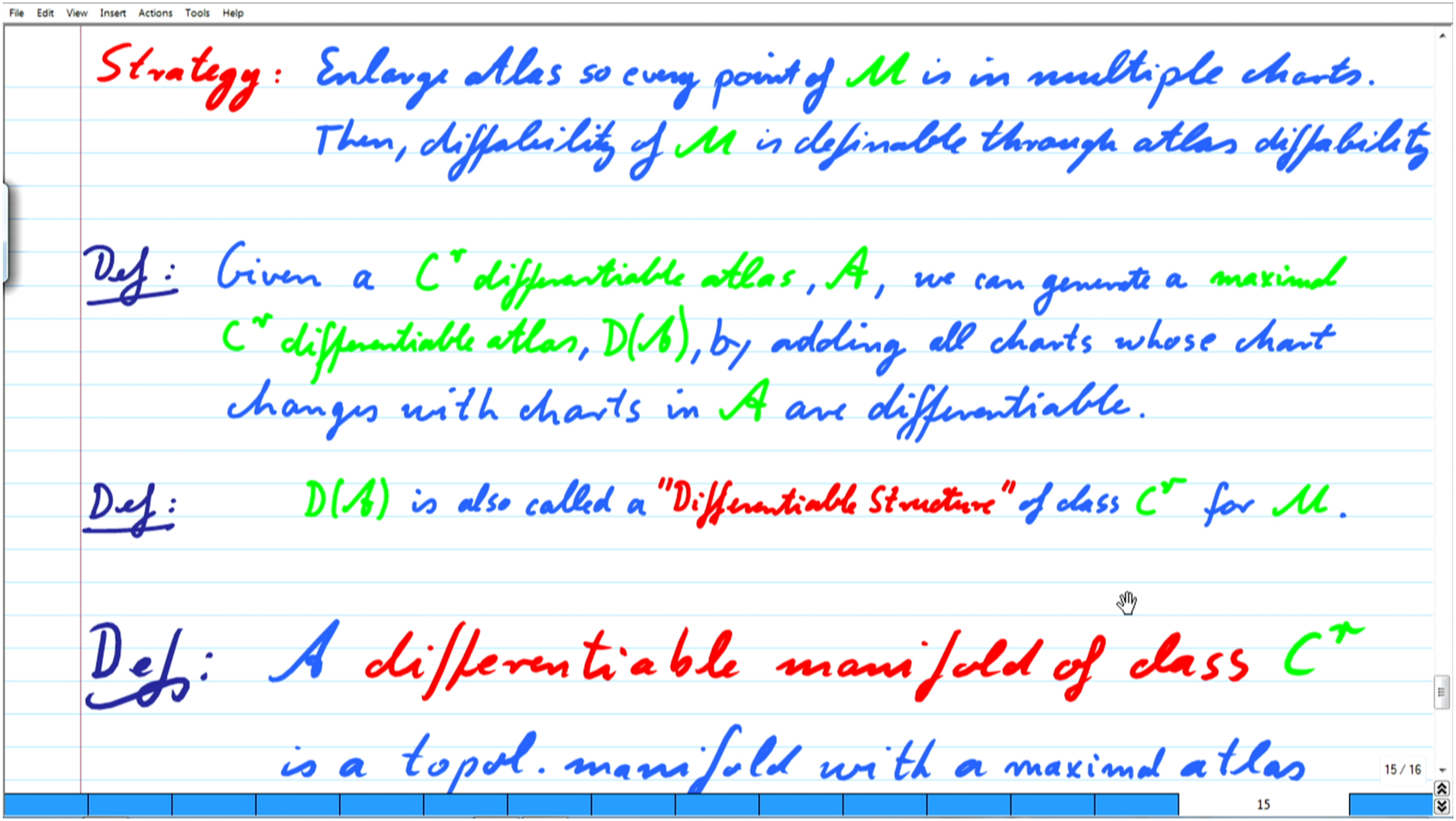
Strategy: Let us define the diff'ability of an atlas through the diff'ability of its chart changes:

Def: An atlas is called C^r differentiable,



Strategy: Let us define the diffability of an atlas through the diffability of its chart changes:

Def: An atlas is called C^r differentiable, if all its coordinate changes, $h_{\alpha\beta}$, are C^r diffeomorphisms, i.e., r times continuous differentiable.

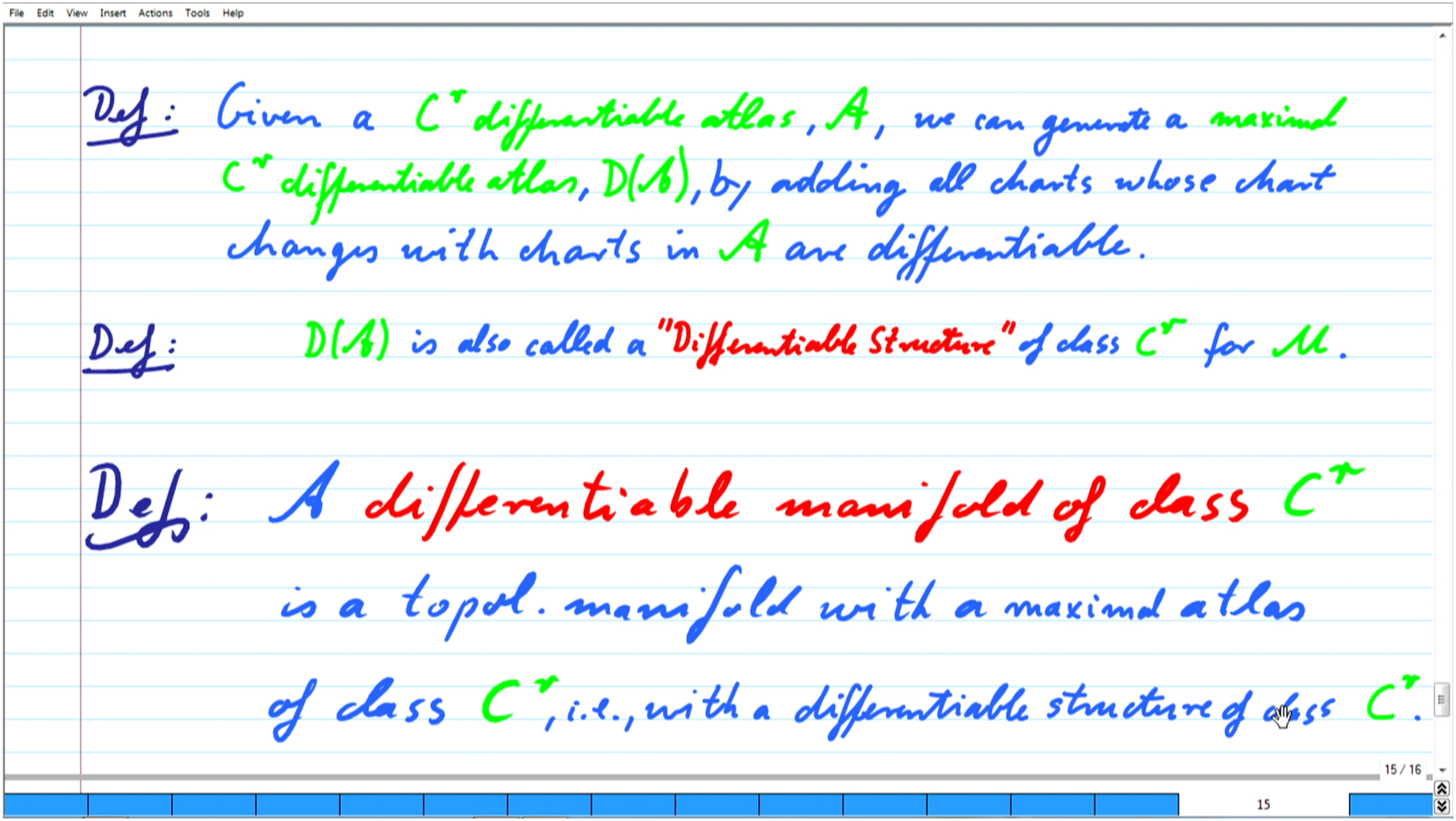


Strategy: Enlarge atlas so every point of $\mathcal{M}$ is in multiple charts.
Then, diffability of $\mathcal{M}$ is definable through atlas diffability

Def: Given a C^r differentiable atlas, $\mathcal{A}$, we can generate a maximal C^r differentiable atlas, $D(\mathcal{A})$, by adding all charts whose chart changes with charts in $\mathcal{A}$ are differentiable.

Def: $D(\mathcal{A})$ is also called a "Differentiable Structure" of class C^r for $\mathcal{M}$.

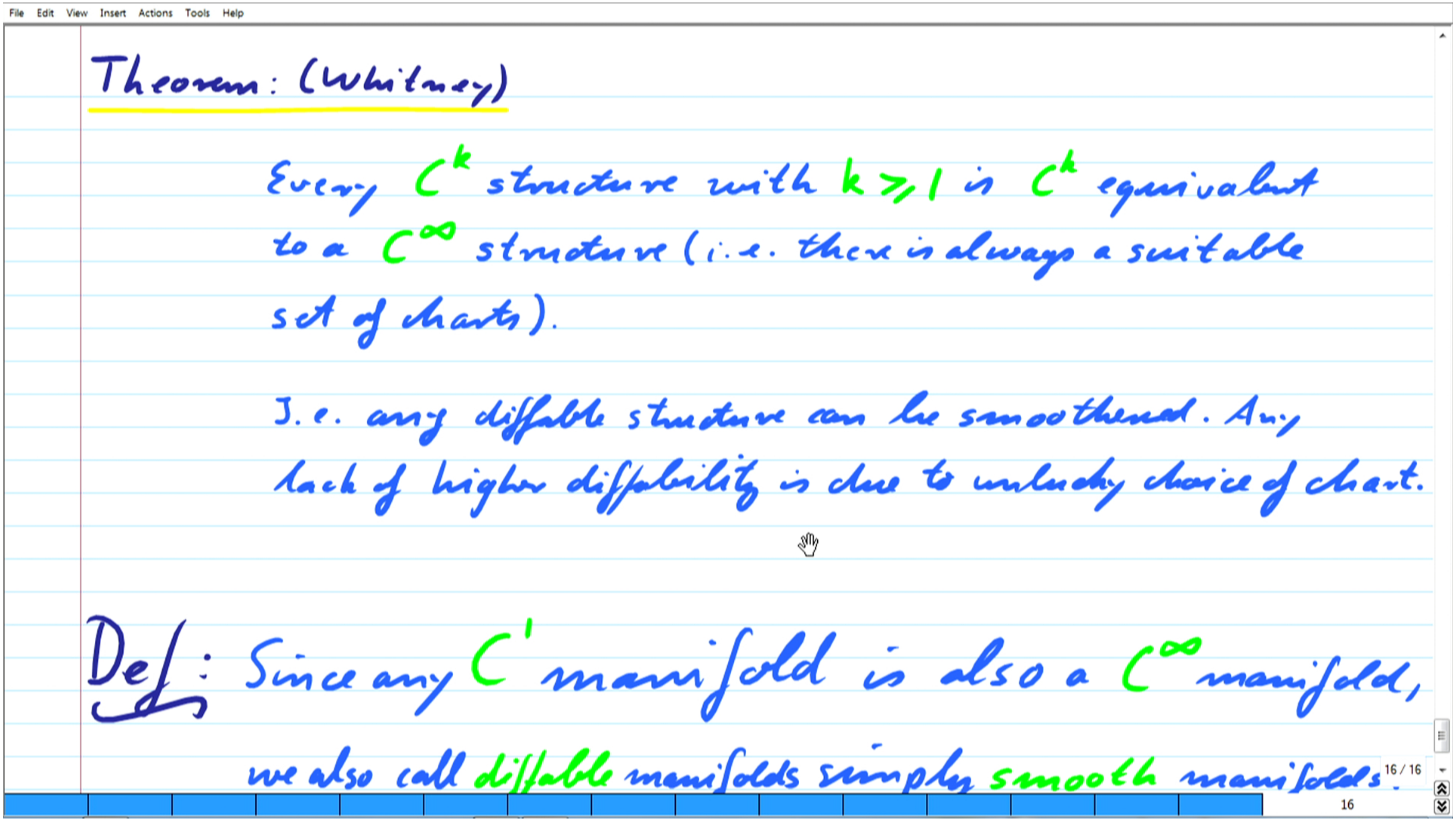
Def: A differentiable manifold of class C^r is a topol. manifold with a maximal atlas



Def: Given a C^r differentiable atlas, $\mathcal{A}$, we can generate a maximal C^r differentiable atlas, $D(\mathcal{A})$, by adding all charts whose chart changes with charts in $\mathcal{A}$ are differentiable.

Def: $D(\mathcal{A})$ is also called a "Differentiable Structure" of class C^r for $\mathcal{M}$.

Def: A differentiable manifold of class C^r is a topol. manifold with a maximal atlas of class C^r, i.e., with a differentiable structure of class C^r.



Theorem: (Whitney)

Every C^k structure with $k \geq 1$ is C^k equivalent to a C^∞ structure (i.e. there is always a suitable set of charts).

I.e. any diffable structure can be smoothened. Any lack of higher diffability is due to unlucky choice of chart.

Def: Since any C^1 manifold is also a C^∞ manifold, we also call diffable manifolds simply smooth manifolds.