Title: Bekenstein-Hawking entropy and strange metals

Date: Aug 20, 2015 05:00 PM

URL: http://pirsa.org/15080122

Abstract:

# Bekenstein-Hawking entropy and strange metals

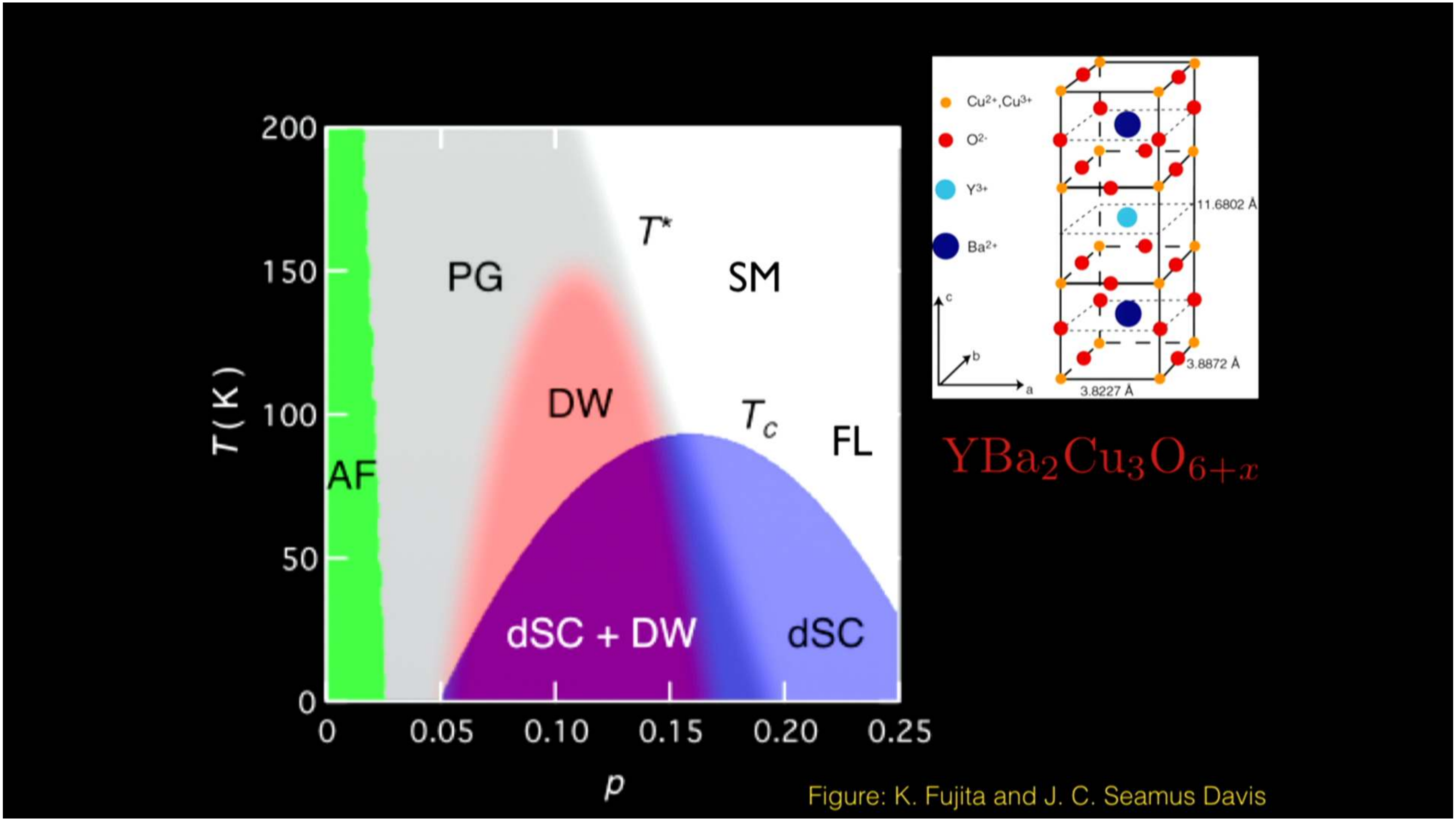
Quantum Information in Quantum Gravity II
Perimeter Institute, Waterloo
August 20, 2015

Subir Sachdev

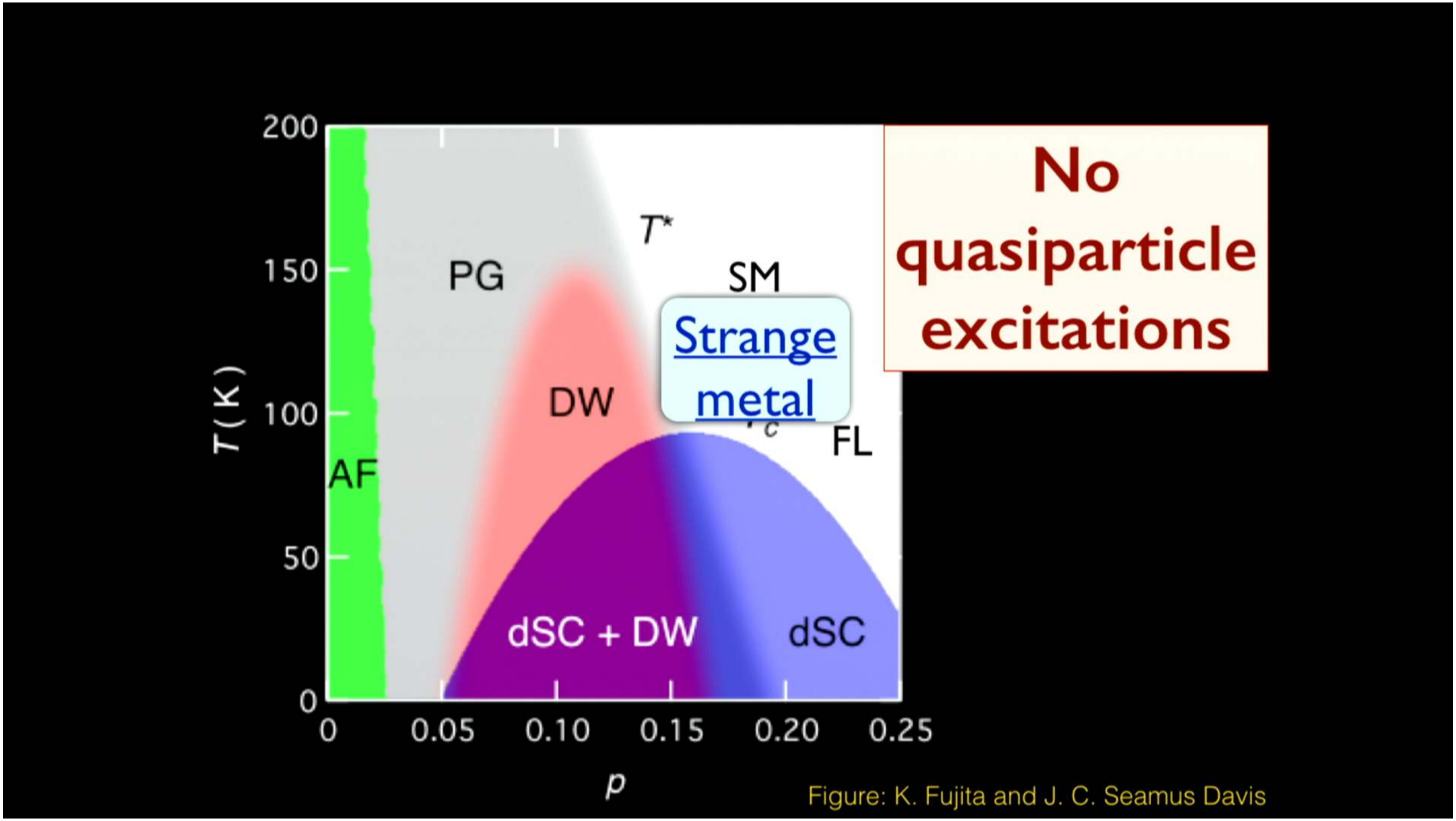
Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS
VE RI TAS
HARVARD

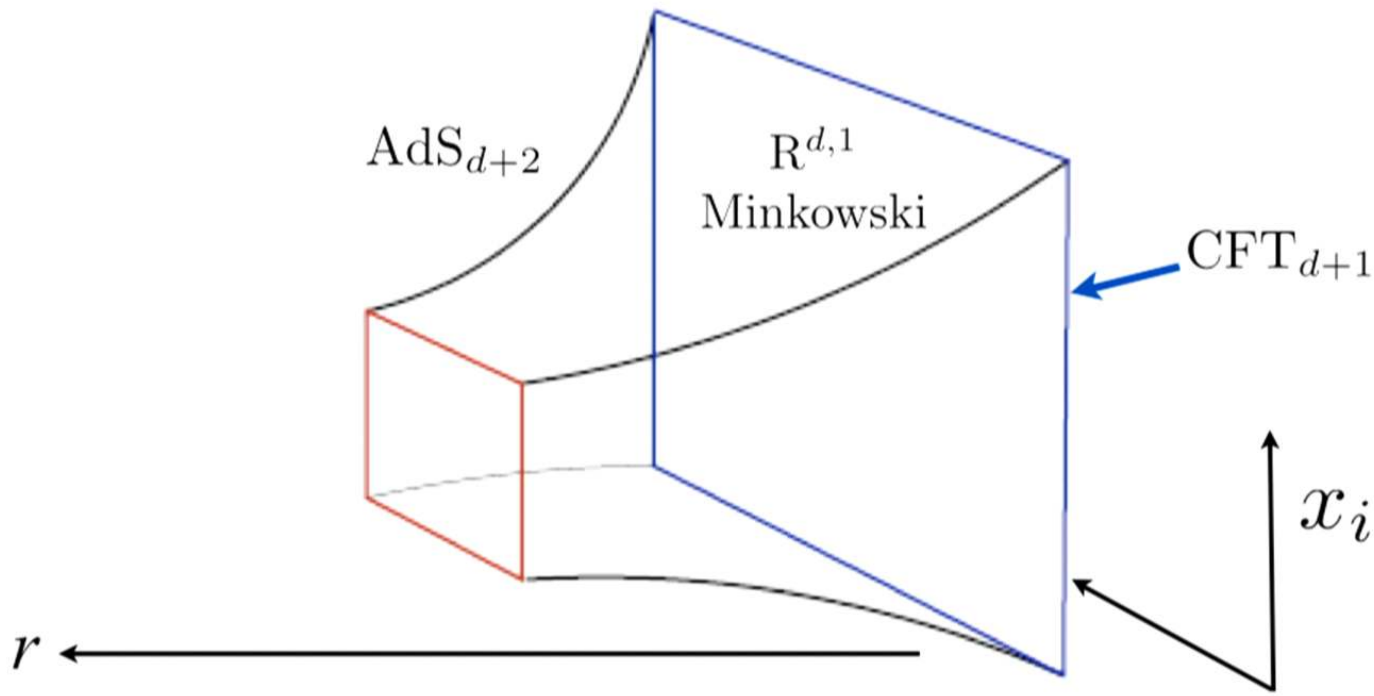


200
150
100
50
0
T (K)
0
0.05
0.10
0.15
0.20
0.25
p
AF
PG
T*
SM
DW
T_c
FL
dSC + DW
dSC
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å
YBa2Cu3O6+x
Figure: K. Fujita and J. C. Seamus Davis


200
150
100
50
0
T (K)
0
0.05
0.10
0.15
0.20
0.25
p
AF
PG
DW
T*
SM
Strange metal
FL
dSC + DW
dSC
No quasiparticle excitations
Figure: K. Fujita and J. C. Seamus Davis

# AdS/CFT correspondence at zero temperature

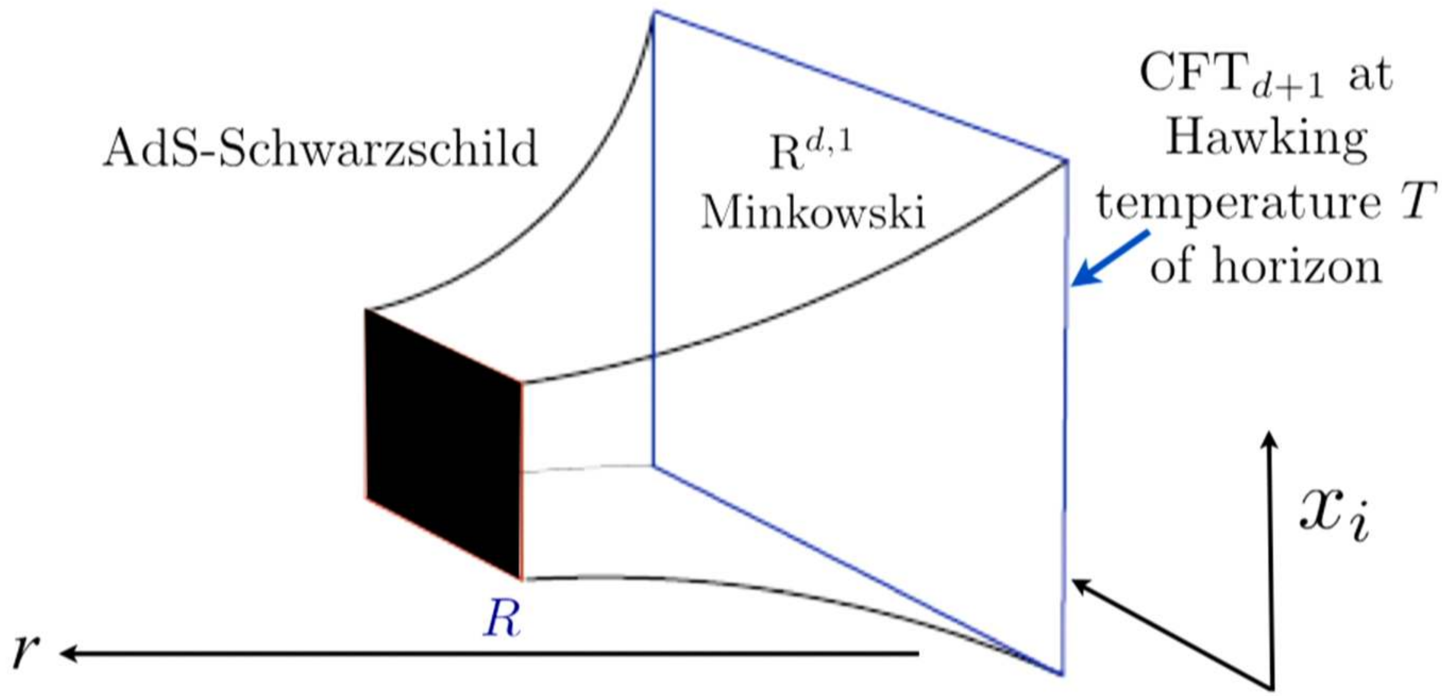
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}+\frac{d(d+1)}{L^2}\right)\right]$




$$ds^2 = \left(\frac{L}{r}\right)^2\left[dr^2 - dt^2 + d\vec{x}^2\right]$$

# AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$

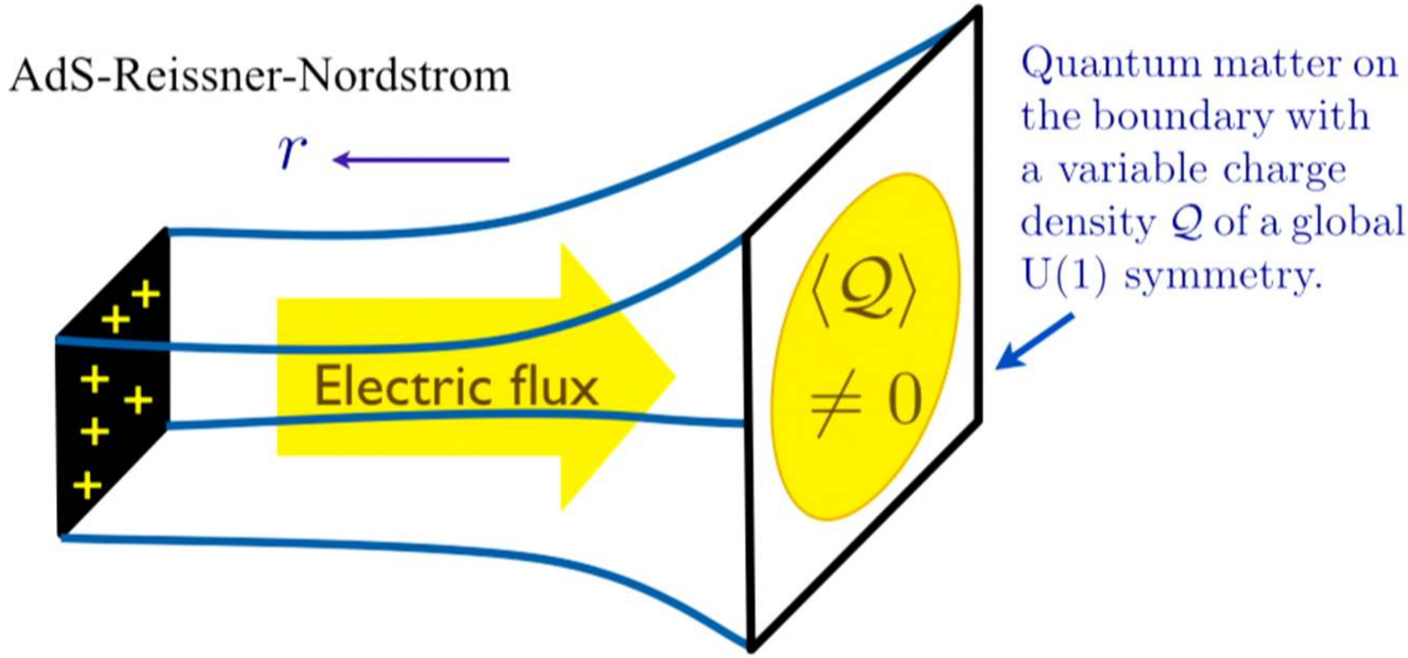



$$ds^2 = \left(\frac{L}{r}\right)^2\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

with $f(r) = 1 - (r/R)^{d+1}$ and $T = (d+1)/(4\pi R)$.

# Charged black branes

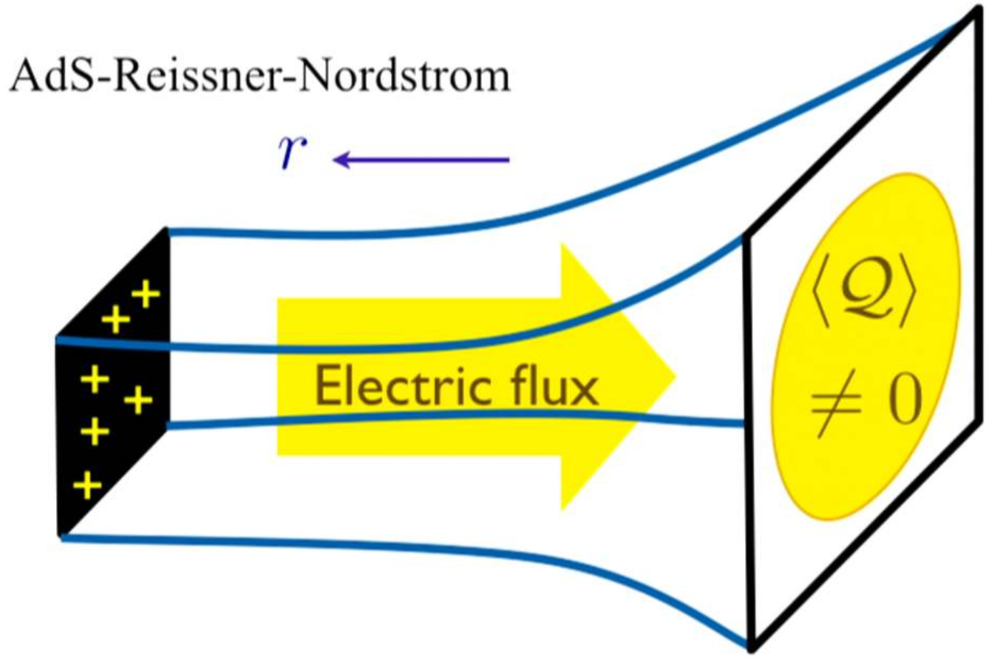
Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$




A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

## Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

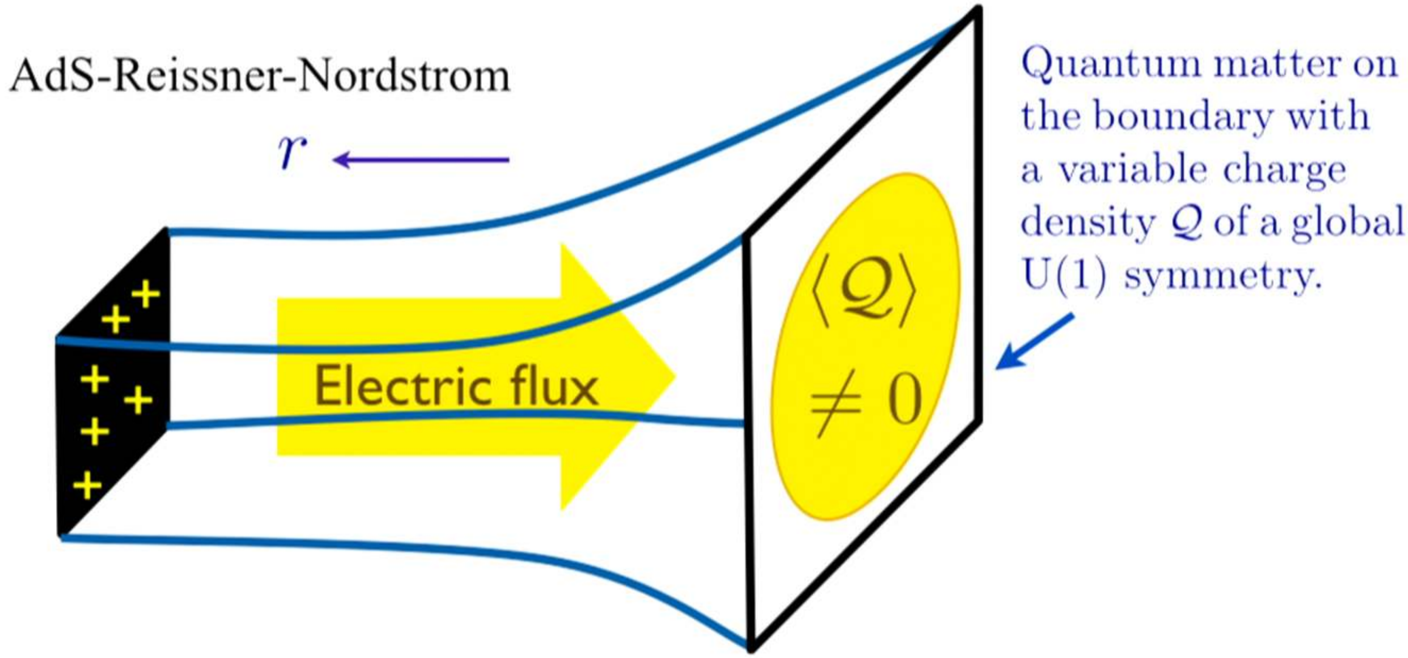



Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

## Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

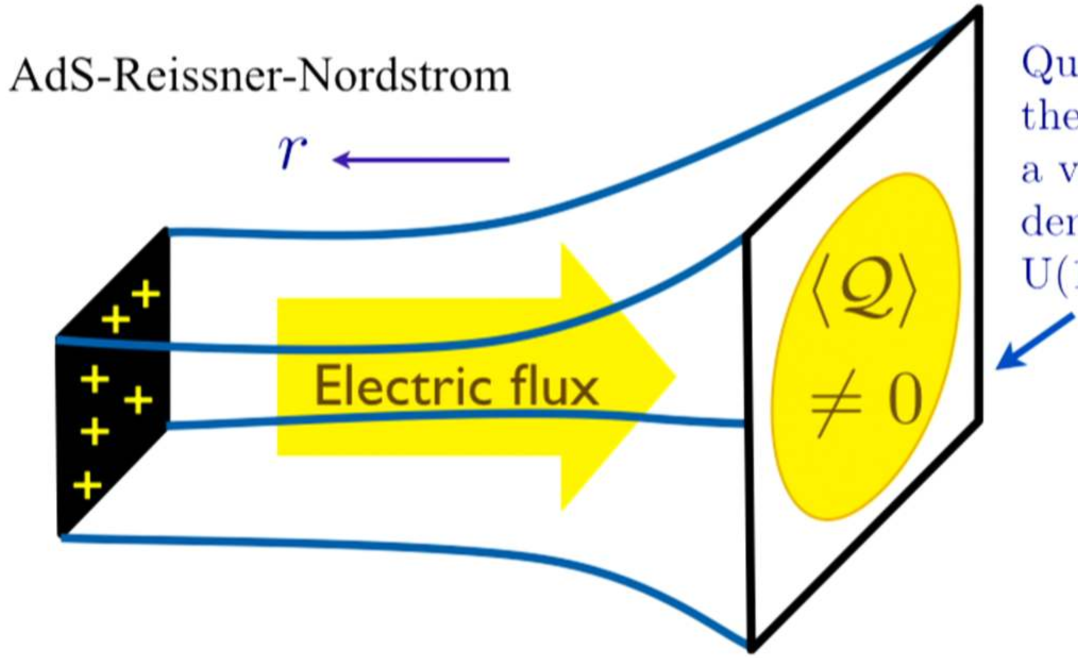



Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

## Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

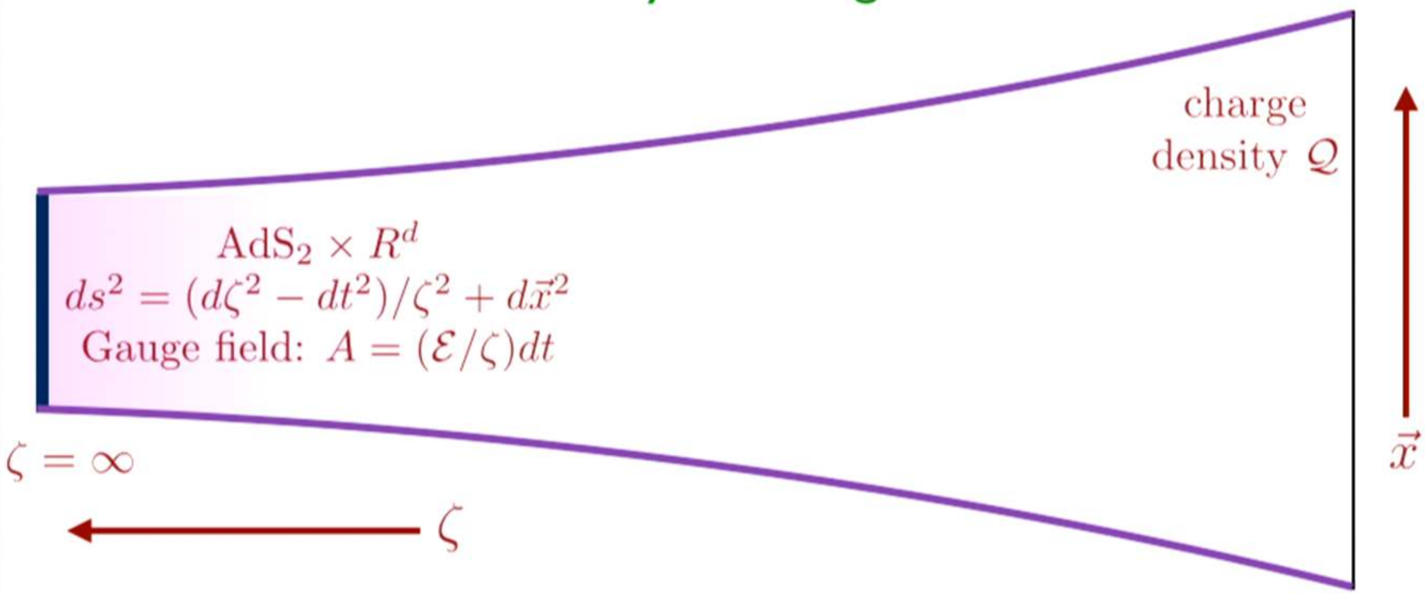



Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

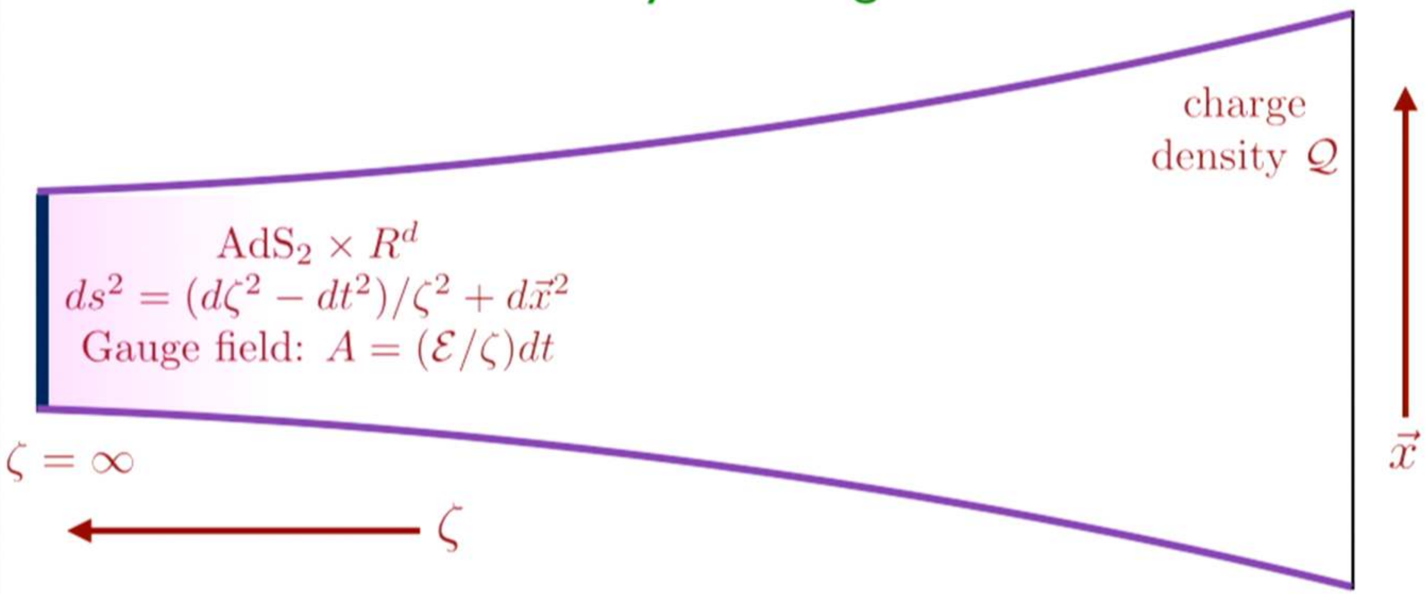
A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

# General Relativity of charged black branes




- Near-horizon metric is $\mathrm{AdS}_2$, with near-horizon electric field $\mathcal{E}$.

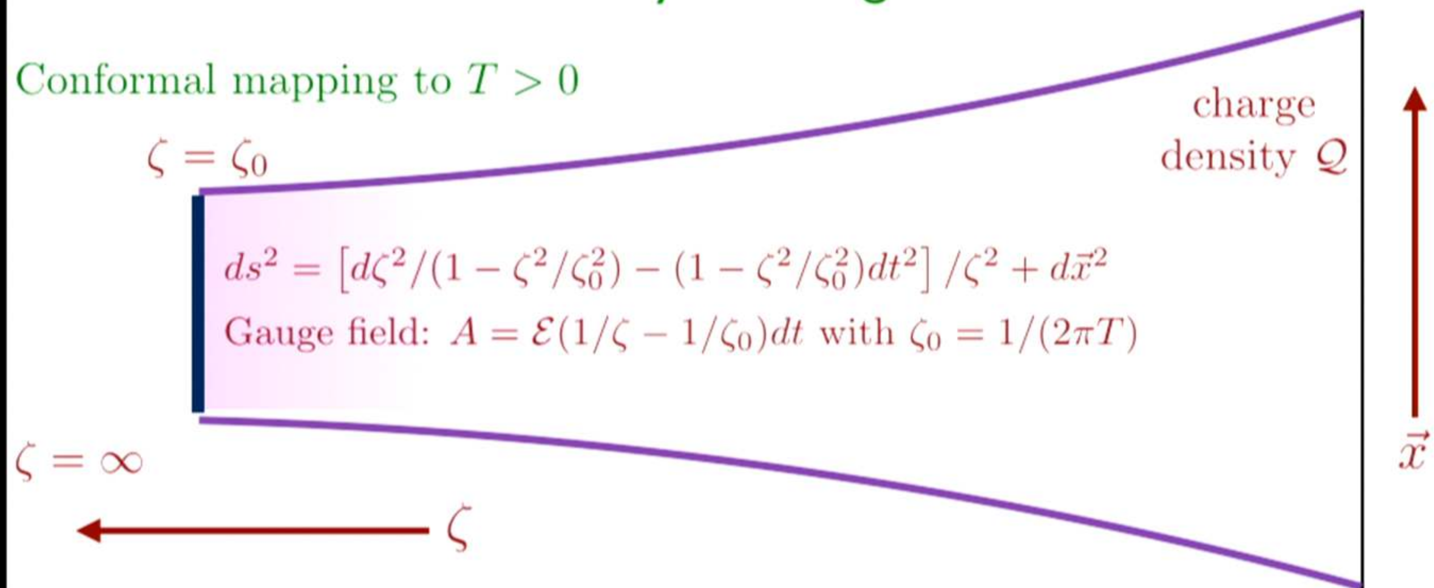
# General Relativity of charged black branes




- Near-horizon metric is $\mathrm{AdS}_2$, with near-horizon electric field $\mathcal{E}$.
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{\mathrm{BH}}$
- Both $\mathcal{E}$ and $\mathcal{S}_{\mathrm{BH}}$ are determined by $\mathcal{Q}$, and both vanish as $\mathcal{Q} \to 0$.

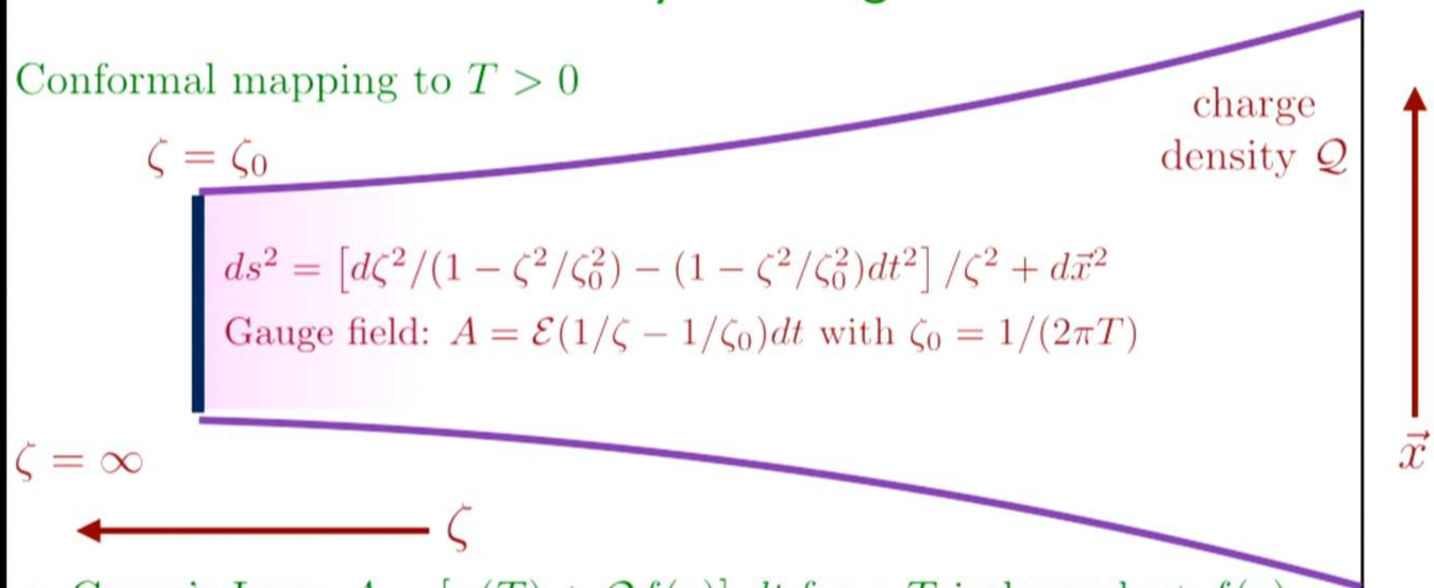
# General Relativity of charged black branes

Conformal mapping to $T > 0$



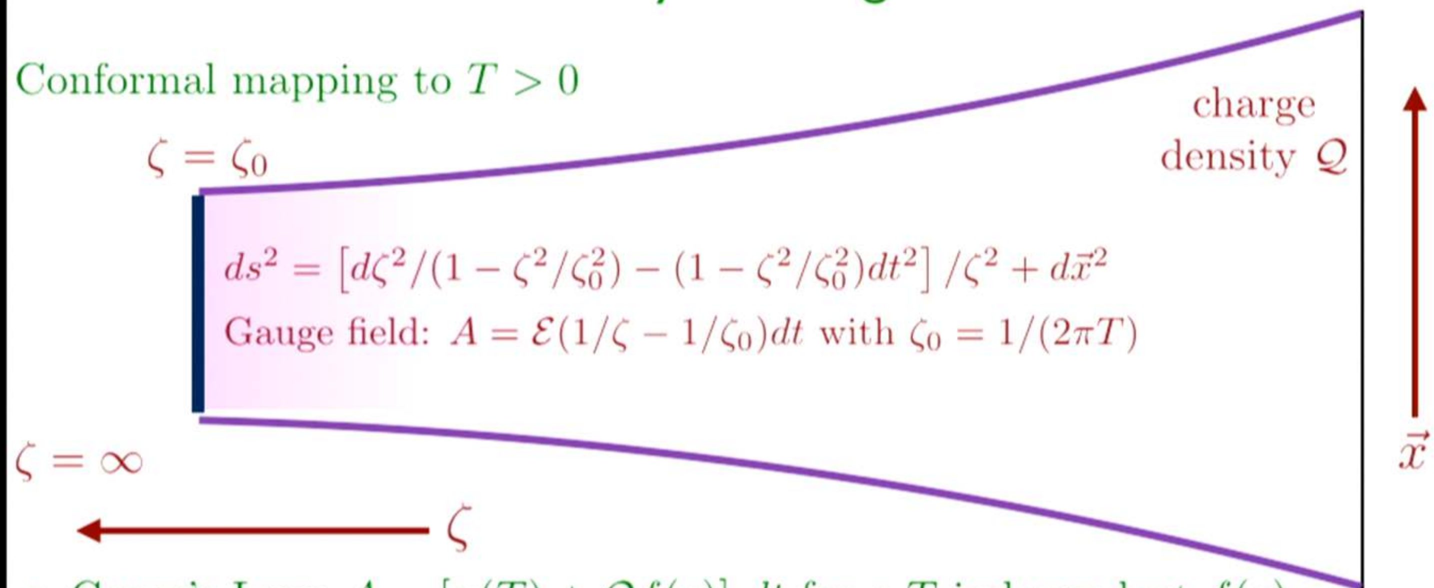
## General Relativity of charged black branes

Conformal mapping to $T > 0$




- Gauss's Law: $A = [\mu(T) + \mathcal{Q}f(r)]\, dt$ for a $T$-independent $f(r)$.

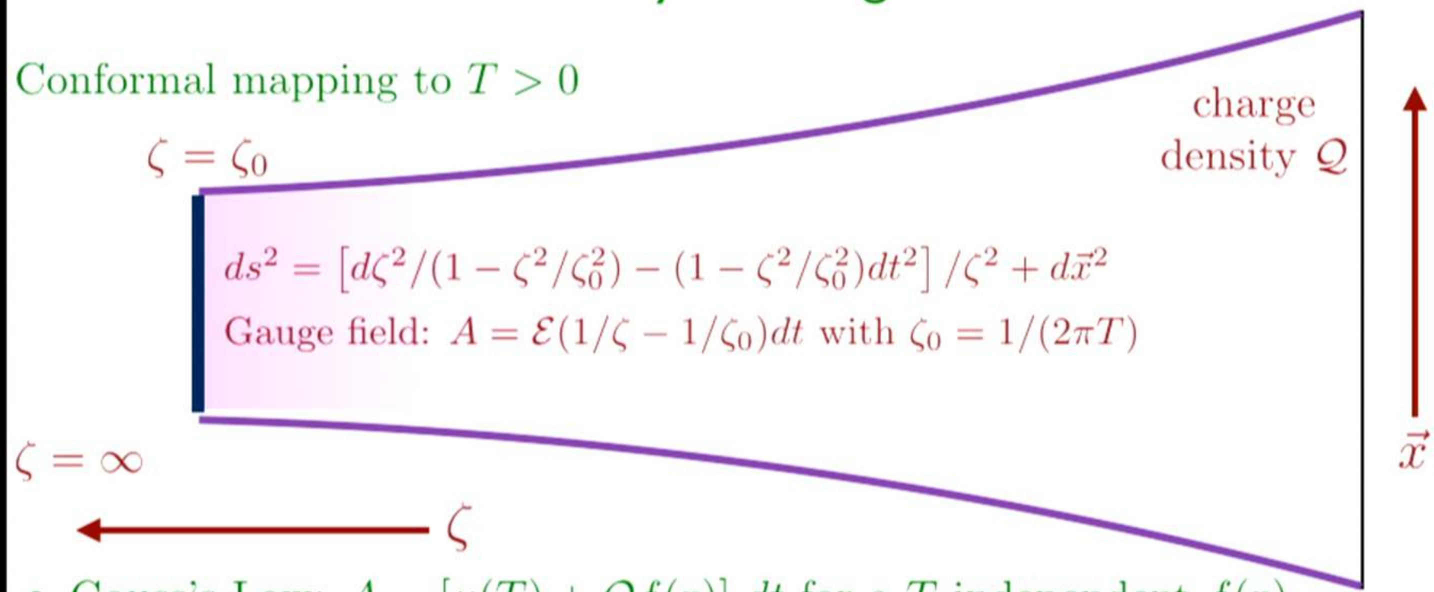
# General Relativity of charged black branes




- Gauss's Law: $A = [\mu(T) + \mathcal{Q} f(r)]\, dt$ for a $T$-independent $f(r)$.
- Comparing with the near-horizon: $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.

# General Relativity of charged black branes

Conformal mapping to $T > 0$



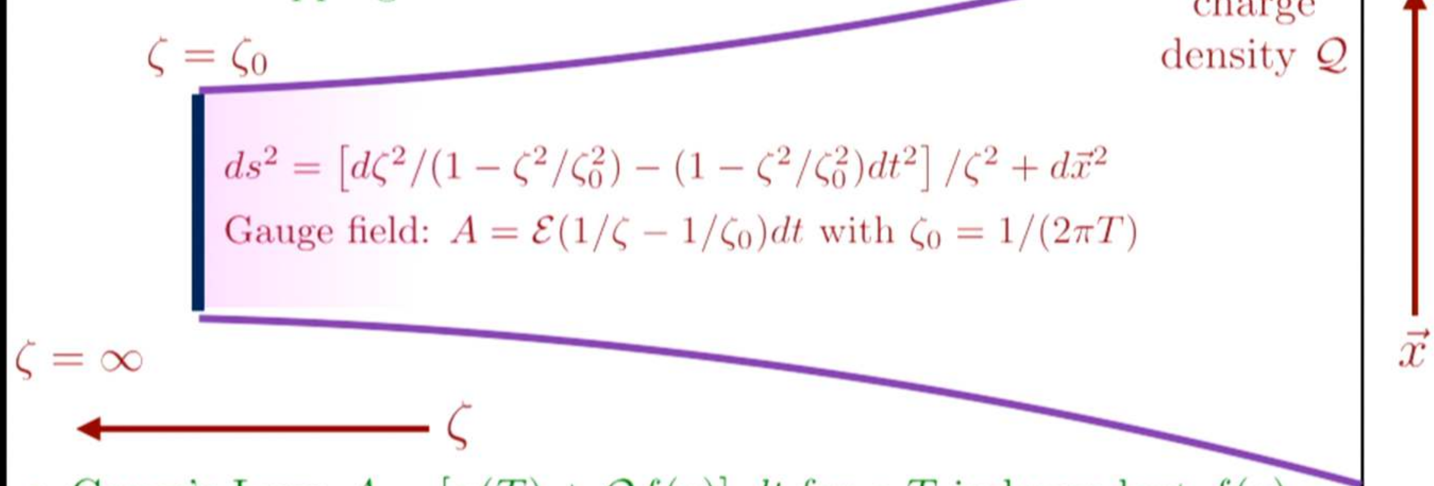

- Gauss's Law: $A = [\mu(T) + \mathcal{Q}f(r)]\, dt$ for a $T$-independent $f(r)$.
- Comparing with the near-horizon: $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

# General Relativity of charged black branes

Conformal mapping to $T > 0$



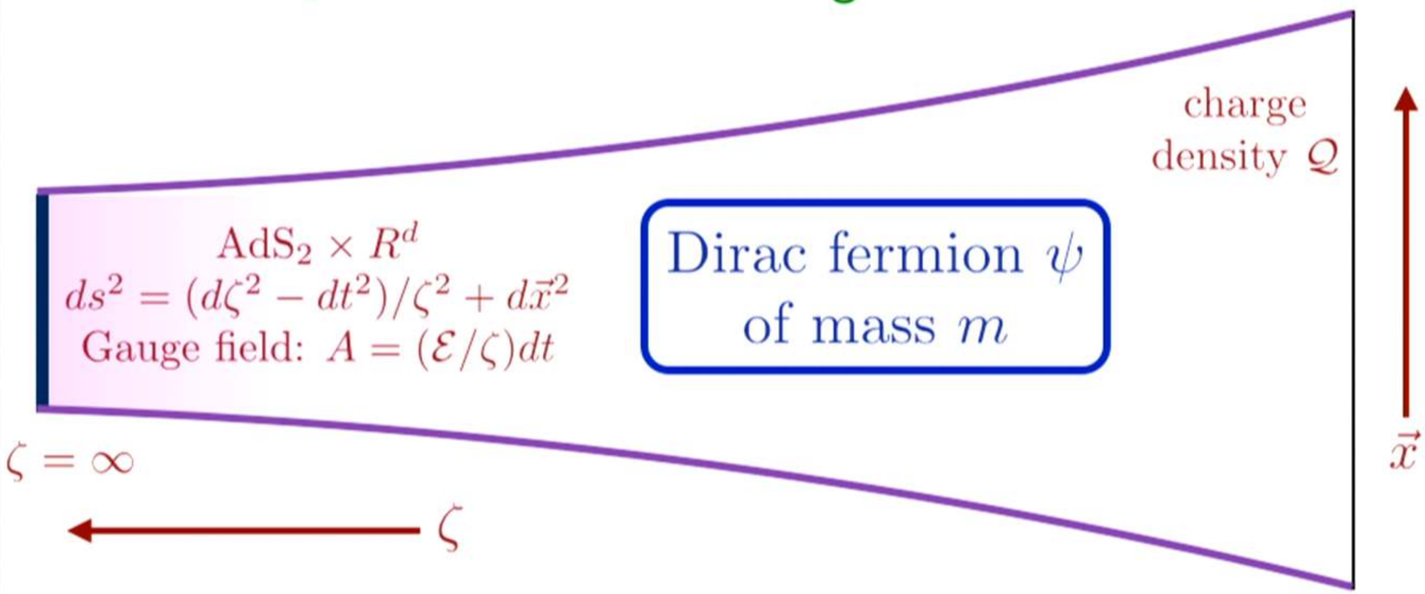

- Gauss's Law: $A = [\mu(T) + \mathcal{Q}f(r)]\, dt$ for a $T$-independent $f(r)$.
- Comparing with the near-horizon: $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

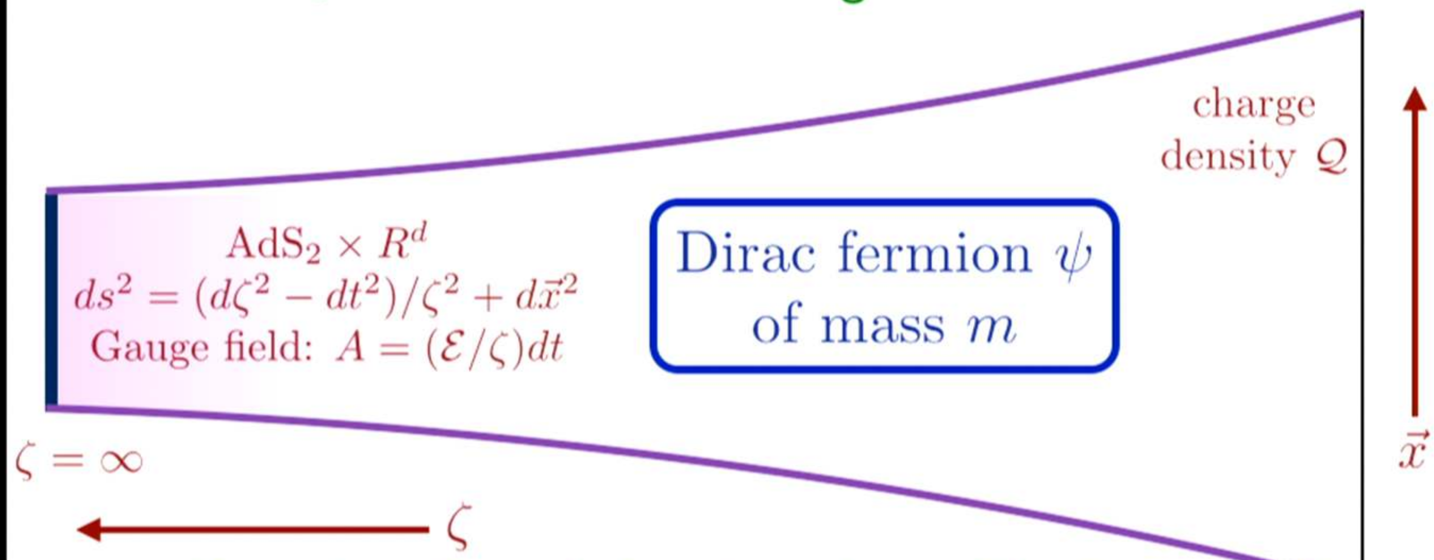
- Also obeyed by the Wald entropy in higher derivative gravity.

## Quantum fields on charged black branes




T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

# Quantum fields on charged black branes




Boundary Green's function of $\psi$ at $T = 0$

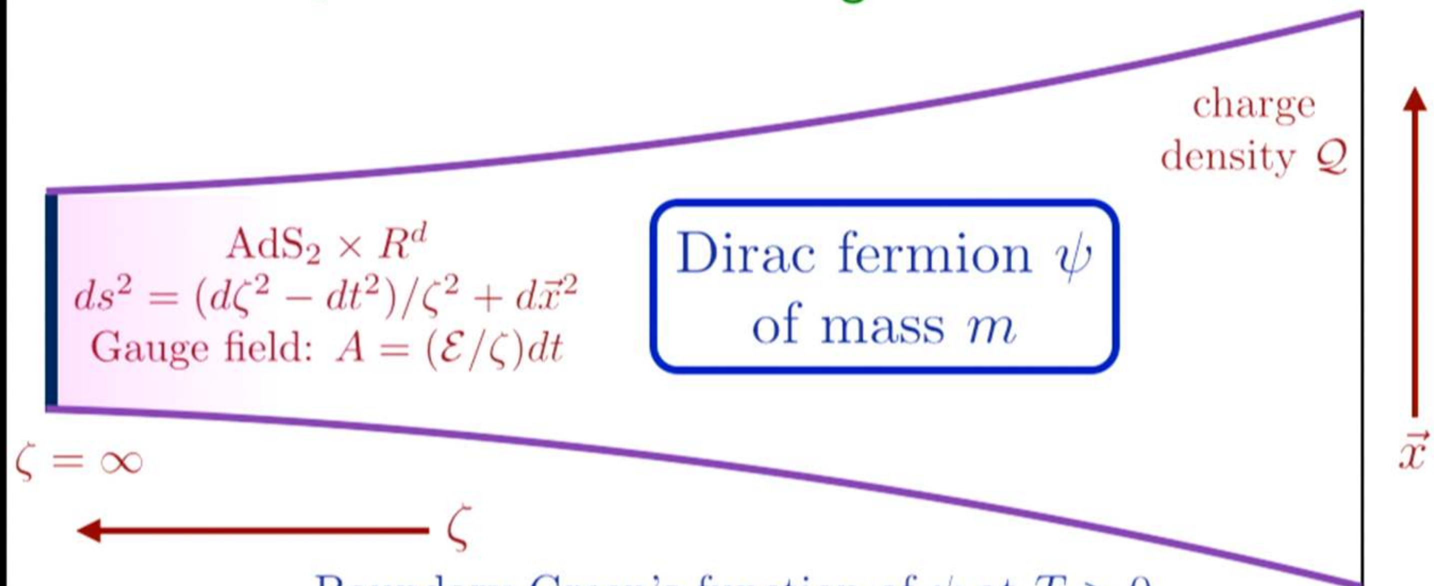
$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \omega < 0. \end{cases}$$

where the fermion scaling dimension $\Delta$ is a function of $m$

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

# Quantum fields on charged black branes




Boundary Green's function of $\psi$ at $T > 0$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

## What is a possible quantum theory on the boundary ?

## What is a possible quantum theory on the boundary ?

A critical strange metal state with infinite-range interactions obtained in

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

# Infinite-range strange metals

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^\dagger_{j\beta} + c^\dagger_{j\beta} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^\dagger_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

$J_{ij}$ are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# Infinite-range strange metals

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^\dagger c_{i\beta} c_{j\beta}^\dagger c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c_{j\beta}^\dagger + c_{j\beta}^\dagger c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} = \mathcal{Q}$$

$J_{ij}$ are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$

$N \to \infty$ at $M = 2$ yields spin-glass ground state.

$N \to \infty$ and then $M \to \infty$ yields critical strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

**OR**

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields same critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, arXiv:1506.05111

# Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# Infinite-range strange metals

At frequencies $\ll J$, the equations for $G$ and $\Sigma$ can be written as

$$\int d\tau_2 G(\tau_1, \tau_2)\widetilde{\Sigma}(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = -J^2 \left[G(\tau_1, \tau_2)\right]^2 G(\tau_2, \tau_1)$$

These equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

These equations and invariances have similarities to those of the large $N$ limit of quantum spins at the spatial boundary of a CFT2 (multi-channel Kondo problems)

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
PRB **58**, 3794 (1998)

## Infinite-range strange metals

At frequencies $\ll J$, the equations for $G$ and $\Sigma$ can be written as

$$\int d\tau_2 G(\tau_1, \tau_2)\widetilde{\Sigma}(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = -J^2 \left[G(\tau_1, \tau_2)\right]^2 G(\tau_2, \tau_1)$$

These equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

These equations and invariances have similarities to those of the large $N$ limit of quantum spins at the spatial boundary of a CFT2 (multi-channel Kondo problems)

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
PRB **58**, 3794 (1998)

## Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

## Infinite-range strange metals

From these expressions we obtain the Green's function

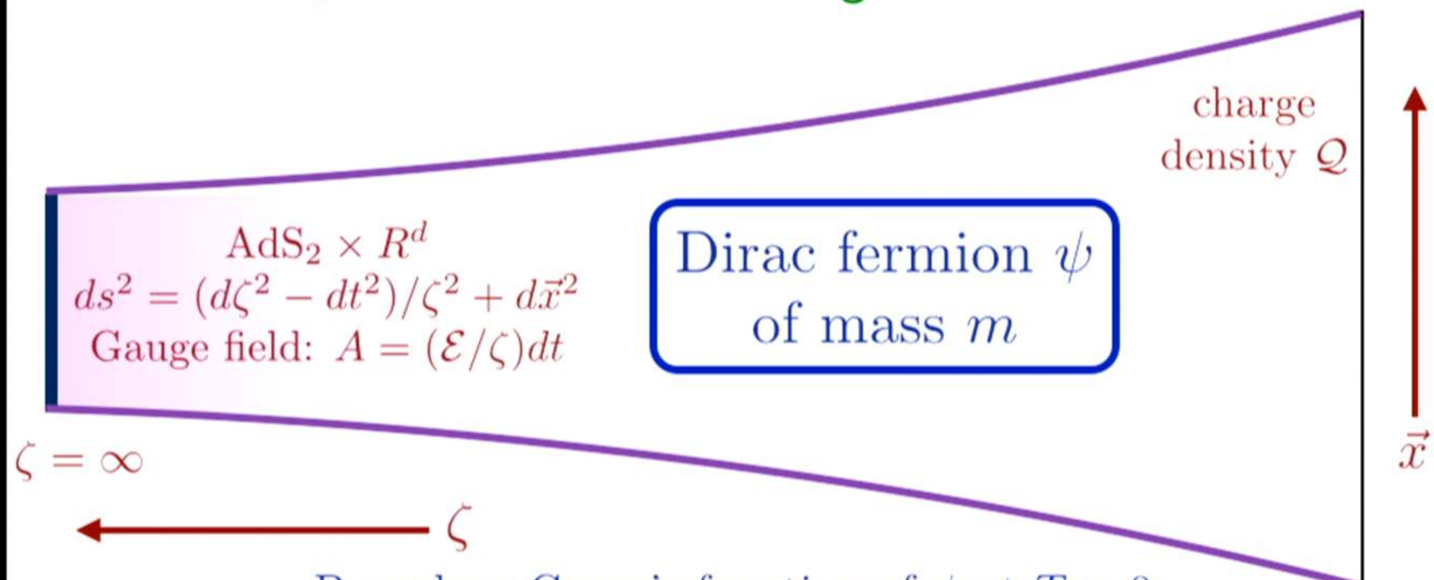
$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$

and $\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right)$.


S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

# Quantum fields on charged black branes




Boundary Green's function of $\psi$ at $T > 0$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

# Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and is similar to universal boundary entropy of the Kondo problem.

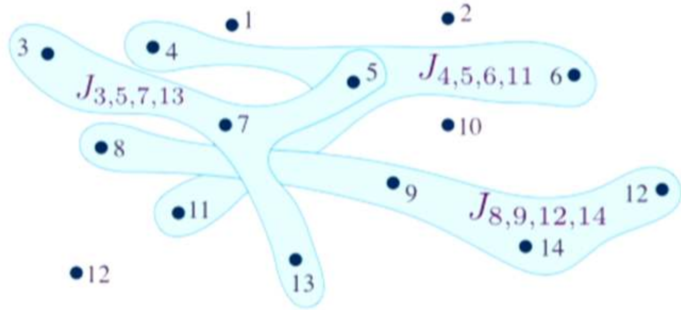
N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).

This entropy obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

# An infinite-range model of a strange metal

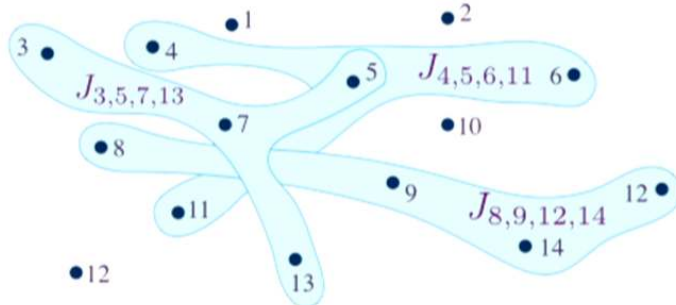
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

$c_i c_j + c_j c_i = 0$

$c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$

$J_{ij;k\ell}$ independent random numbers

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

$$c_i c_j + c_j c_i = 0$$

$$c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

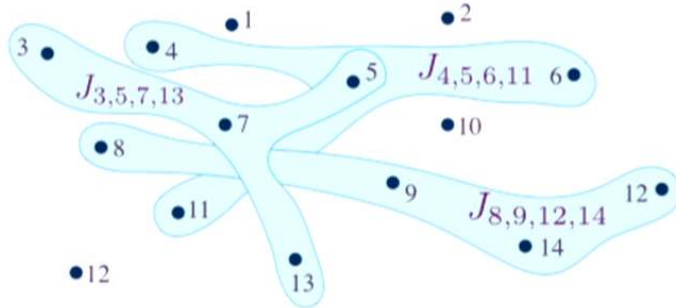
$J_{ij;k\ell}$ independent random numbers

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$$c_i c_j + c_j c_i = 0$$

$$c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$J_{ij;k\ell}$ independent random numbers




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

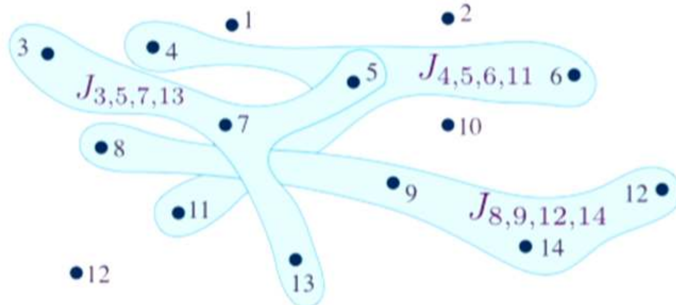
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \; \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right)$$

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

$$c_i c_j + c_j c_i = 0$$

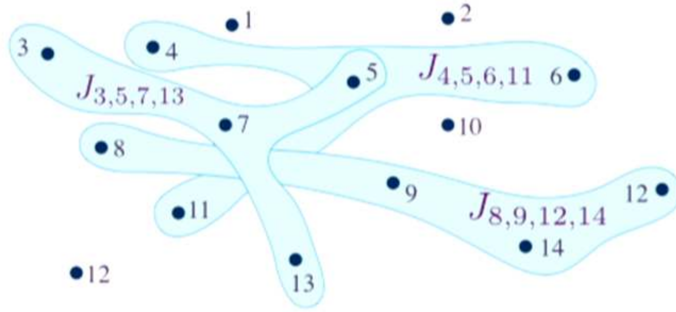
$$c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$J_{ij;k\ell}$ independent random numbers

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
Phys. Rev. B **58**, 3794 (1998)

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N}\sum_i \left\langle c_i^\dagger c_i \right\rangle.$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

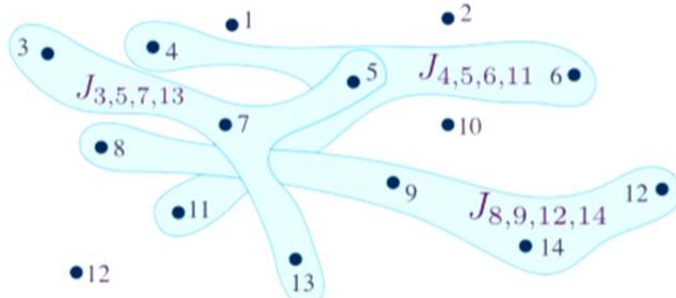
Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

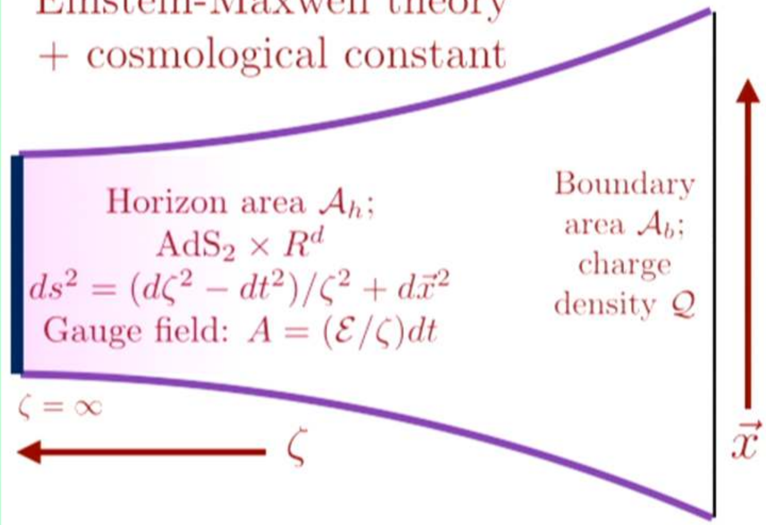
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

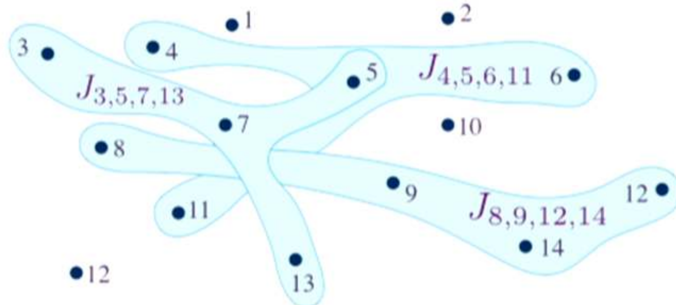
$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant




A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers
Phys. Rev. D **60**, 064018 (1999)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

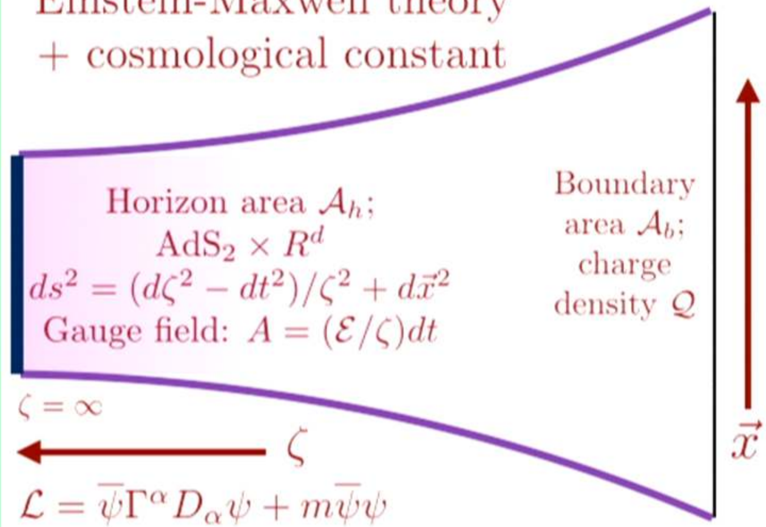
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant



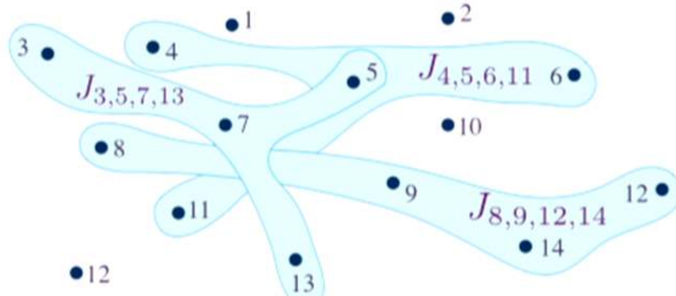

$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh
Phys. Rev. D **83**, 125002 (2011)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

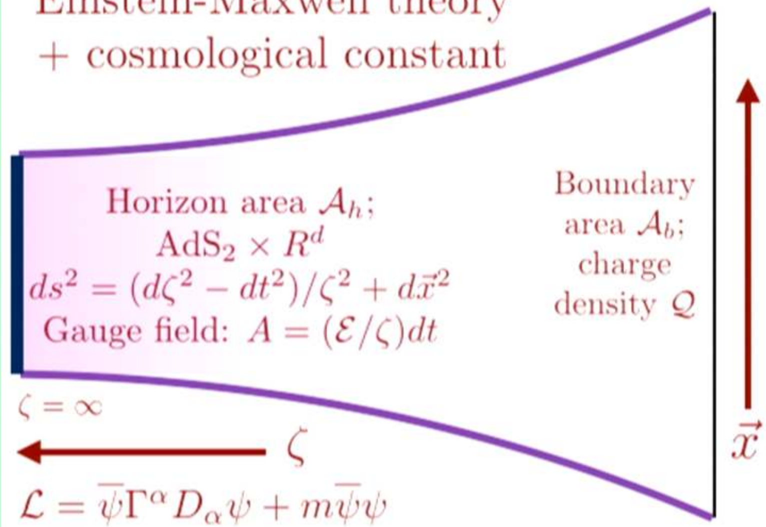
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant




$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the $\mathrm{AdS}_2$

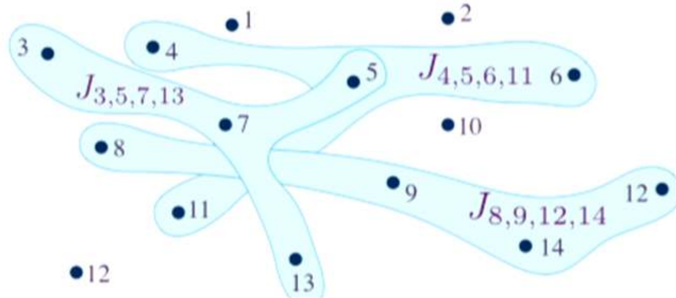
Eliminate $r_0$ between

$$\mathcal{Q} = \frac{r_0^{d-1}\sqrt{2d\left[(d-1)R^2 + (d+1)r_0^2\right]}}{\kappa^2 g_F}$$

$$\mathcal{E} = \frac{g_F r_0 \sqrt{2d\left[(d-1)R^2 + (d+1)r_0^2\right]}}{2\left[(d-1)^2R^2 + d(d+1)r_0^2\right]}$$

S. Sachdev, arXiv:1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

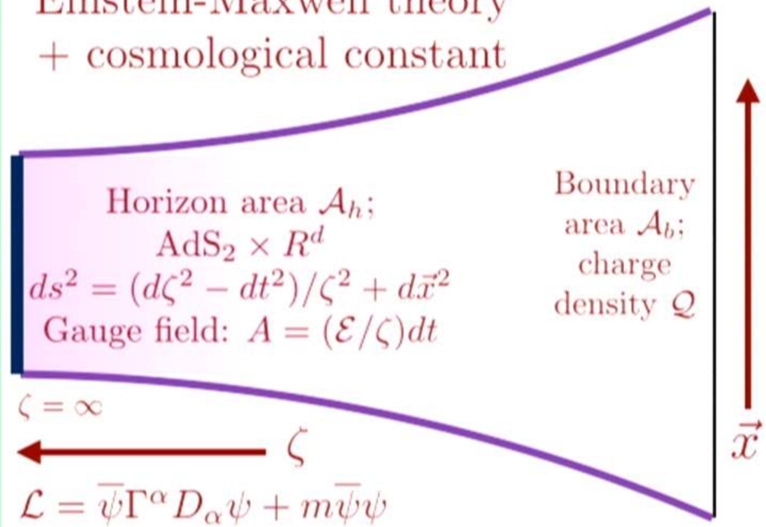
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant




$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

Local fermion density of states

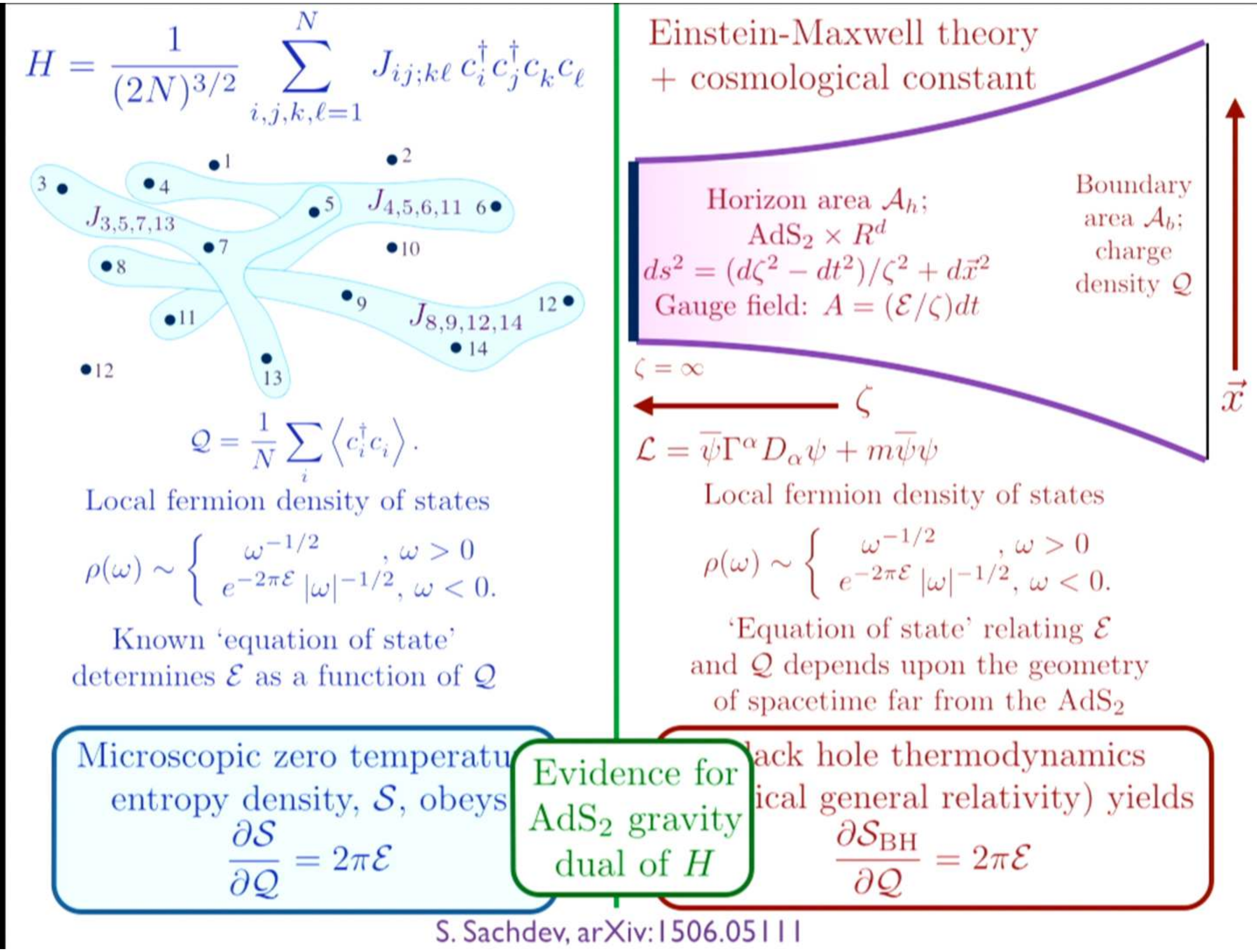
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the $AdS_2$

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511


$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$
$J_{3,5,7,13}$
$J_{4,5,6,11}$
$J_{8,9,12,14}$
$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle.$
Local fermion density of states
$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$
Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$
Einstein-Maxwell theory + cosmological constant
Horizon area $\mathcal{A}_h$;
$\mathrm{AdS}_2 \times R^d$
$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
Gauge field: $A = (\mathcal{E}/\zeta)dt$
Boundary area $\mathcal{A}_b$; charge density $\mathcal{Q}$
$\zeta = \infty$
$\zeta$
$\vec{x}$
$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$
Local fermion density of states
$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$
'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the $\mathrm{AdS}_2$
Microscopic zero temperatu
entropy density, $\mathcal{S}$, obeys
$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$
Evidence for $\mathrm{AdS}_2$ gravity dual of $H$
ack hole thermodynamics
ical general relativity) yields
$\frac{\partial \mathcal{S}_{\mathrm{BH}}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$
S. Sachdev, arXiv:1506.05111