Title: Entanglement renormalization for quantum fields

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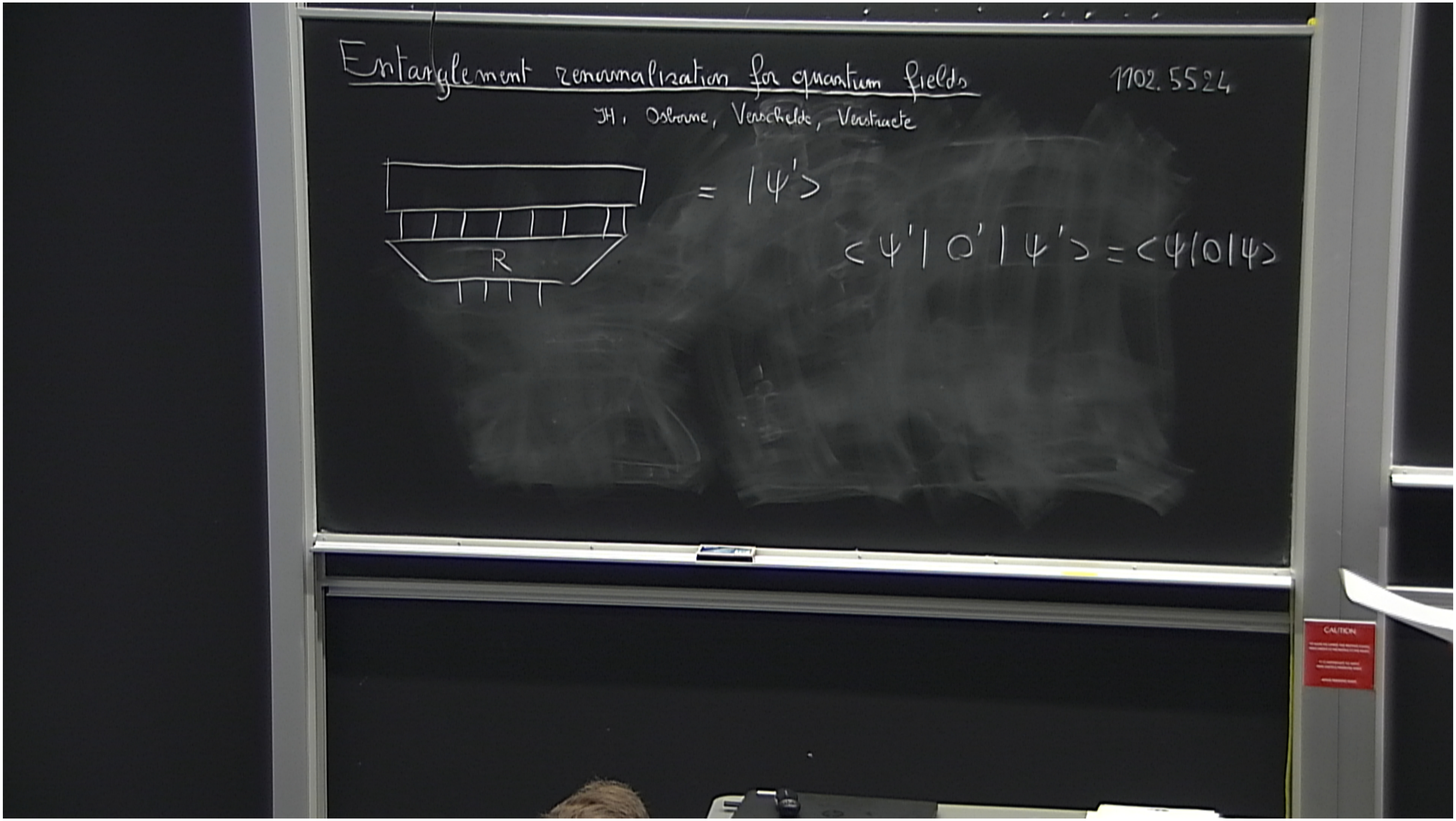
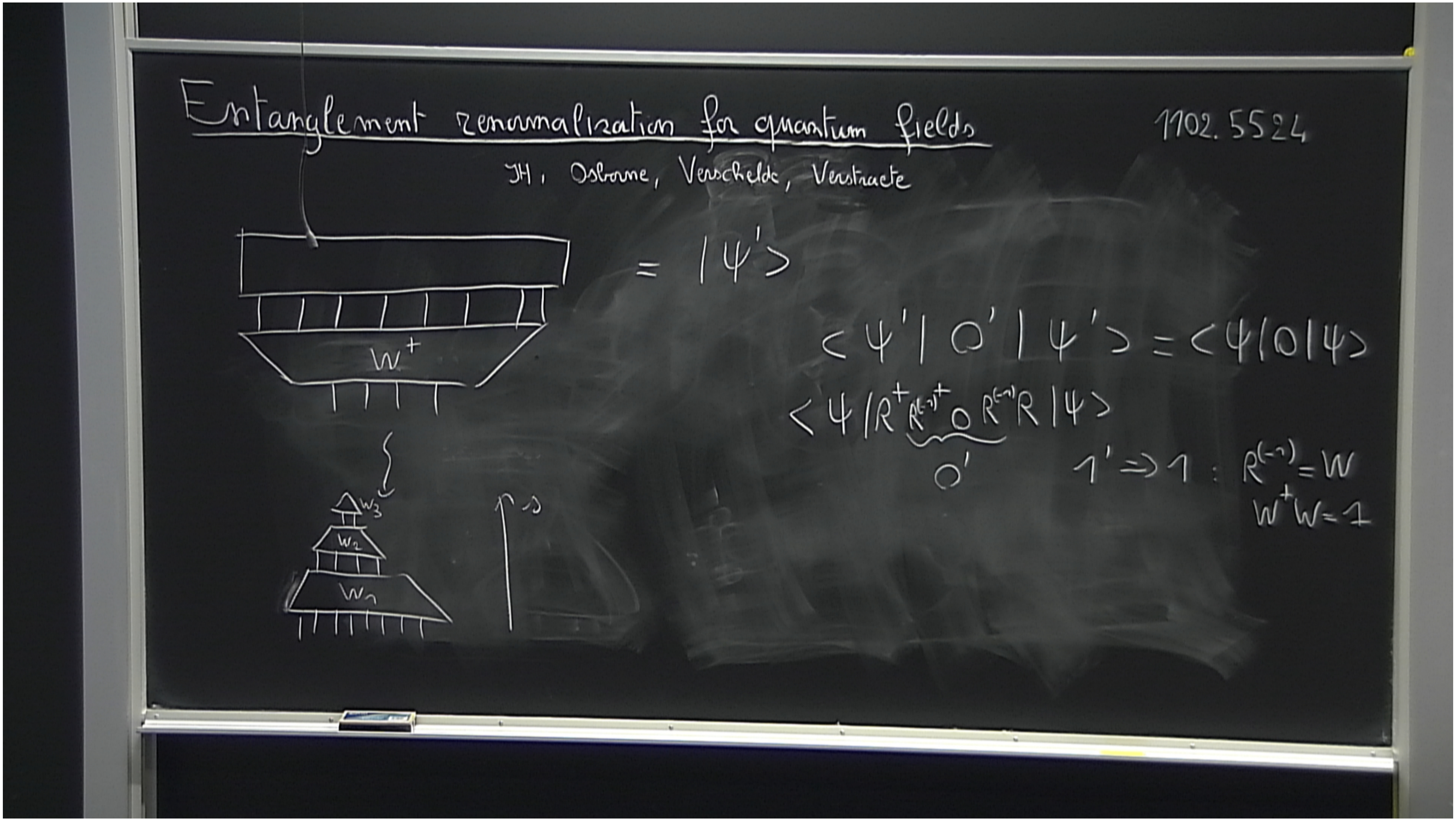
Abstract: The Multiscale Entanglement Renormalization Ansatz has proven to capture the ground state properties of strongly correlated quantum lattice systems, both in gapped regimes and at critical points, and realizes a lattice version of the holographic principle. In this talk, I will review a construction of entanglement renormalization that applies in the continuum (i.e. to quantum fields) and discuss several aspects such as the renormalization group equation and scaling exponents, illustrated using free field theories as example


Entanglement renormalization for quantum fields
1102.5524
JH, Osborne, Verschelde, Verstraete
= |ψ'>
R
<ψ'|O'|ψ'> = <ψ|O|ψ>


Entanglement renormalization for quantum fields
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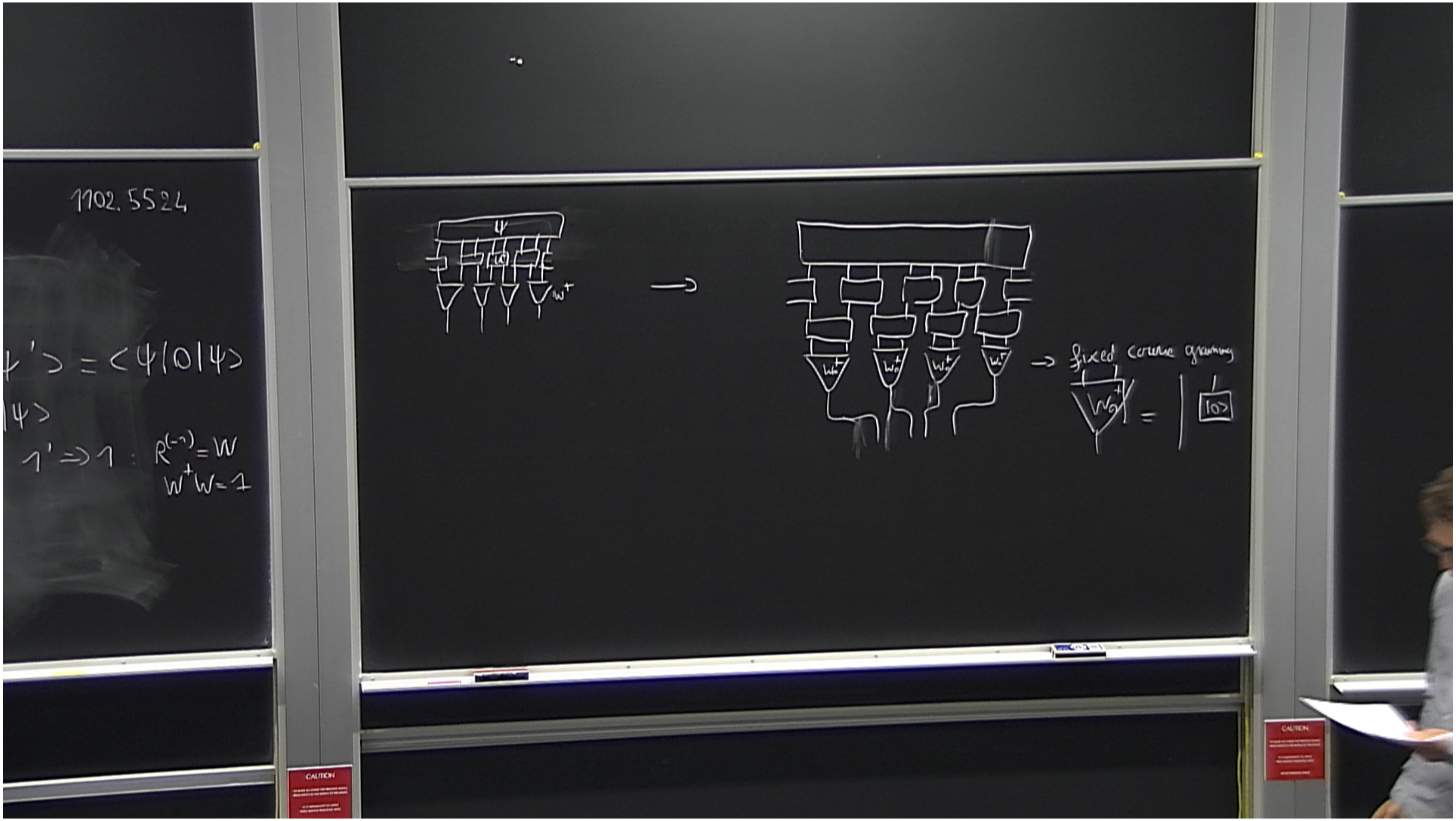
JH, Osborne, Verschelde, Verstraete
= |ψ'>
W+
W3
W2
W1
<ψ'| O' | ψ'> = <ψ|O|ψ>
<ψ|R+ R(-1)+ O R(-1) R |ψ>
O'
1' => 1 : R(-1) = W
W+W = 1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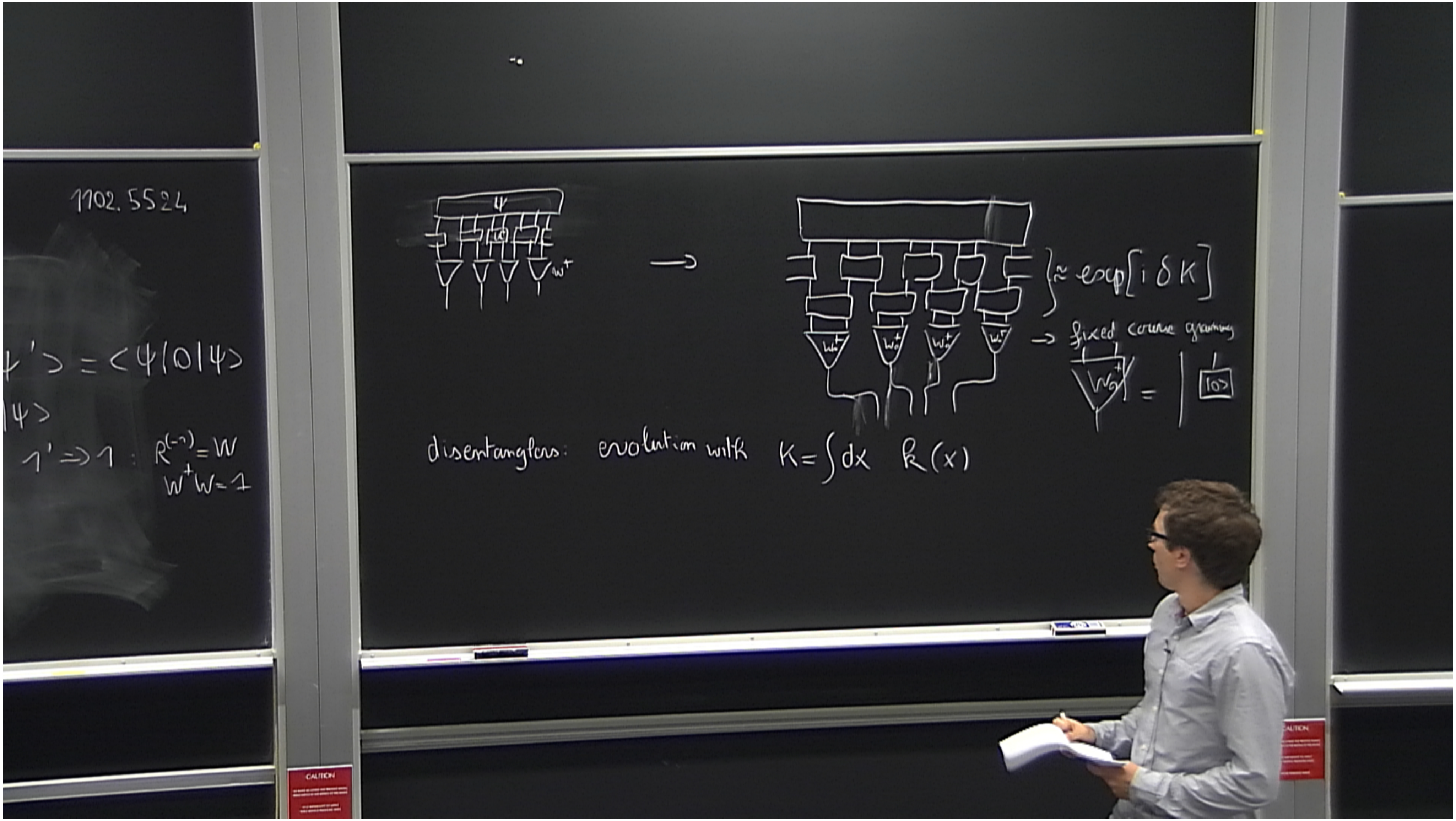
ψ
O
fixed coarse graining
|0>


1102.5524
disentanglers: evolution with K=∫dx k(x)
≈ exp[i δ K]
→ fixed coarse graining
|0⟩
W^†W=1


Entanglement renormalization for quantum fields
1102.5524

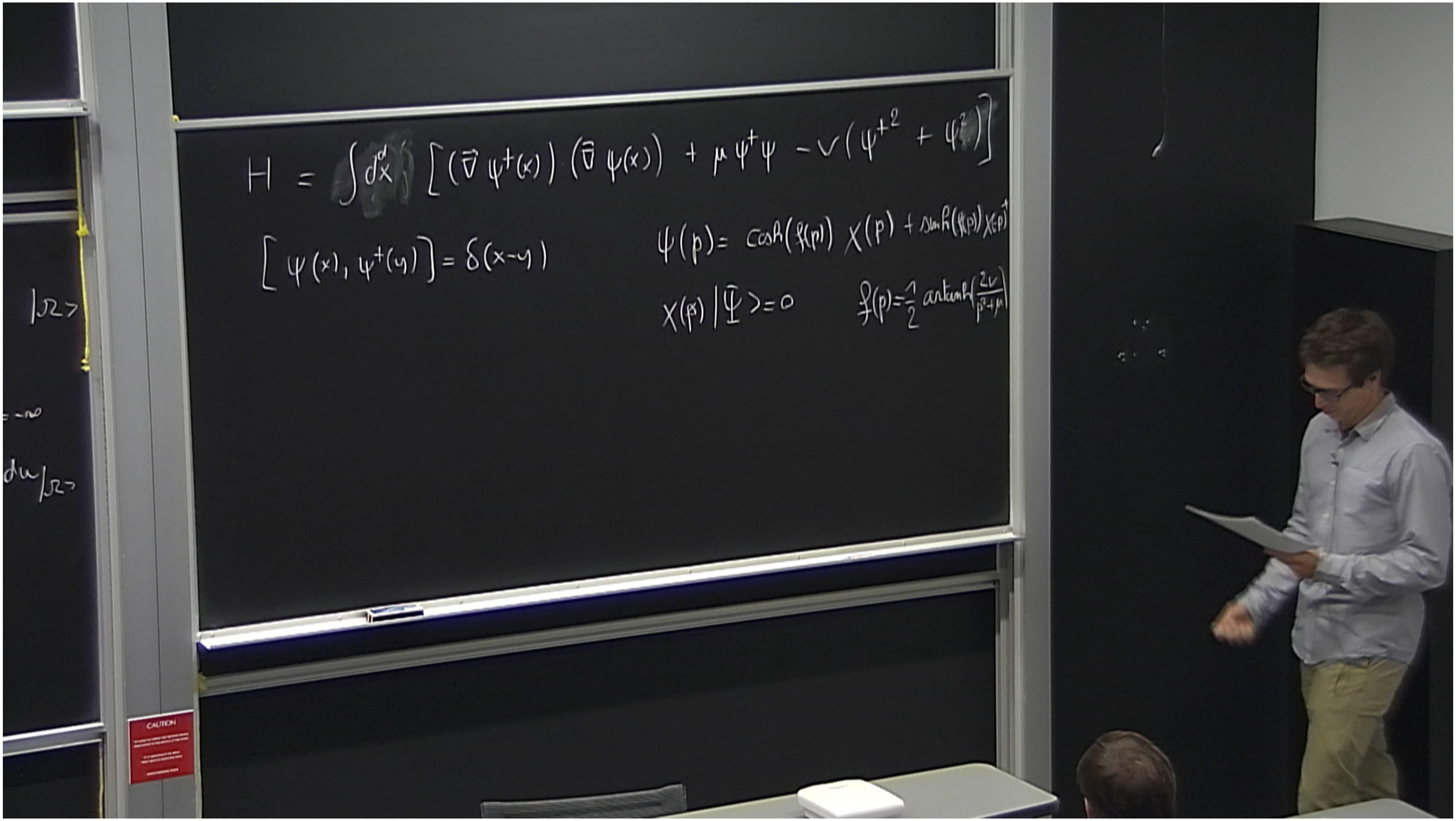
$$\langle \Psi | \hat{O} | \Psi \rangle = \langle \Omega | U(u, u_{IR})^\dagger \underbrace{U(u_{UV}, u)^\dagger \hat{O}\, U(u_{UV}, u)}_{\hat{O}_R(u)} \underbrace{U(u, u_{IR}) | \Omega \rangle}_{|\Psi(u)\rangle}$$

$$\frac{d}{du} O_R(u) = -i \left[K(u) + L,\ O_R(u) \right]$$

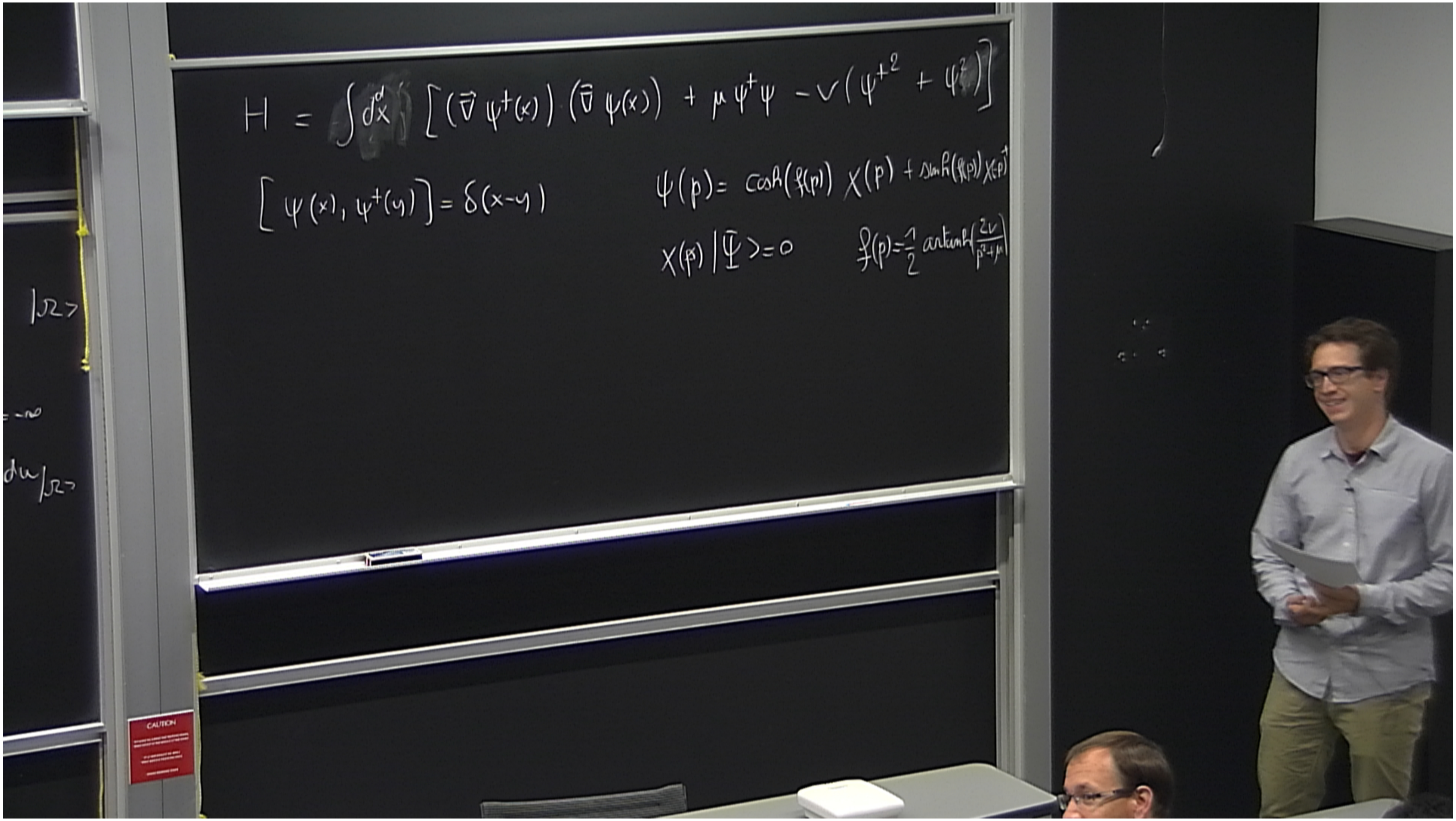
$$-i \left[K + L,\ O_R(u, x) \right] = \lambda\, O_R(u, x) + \vec{x} \cdot \vec{\nabla}\, O_R(e^u \vec{x})$$

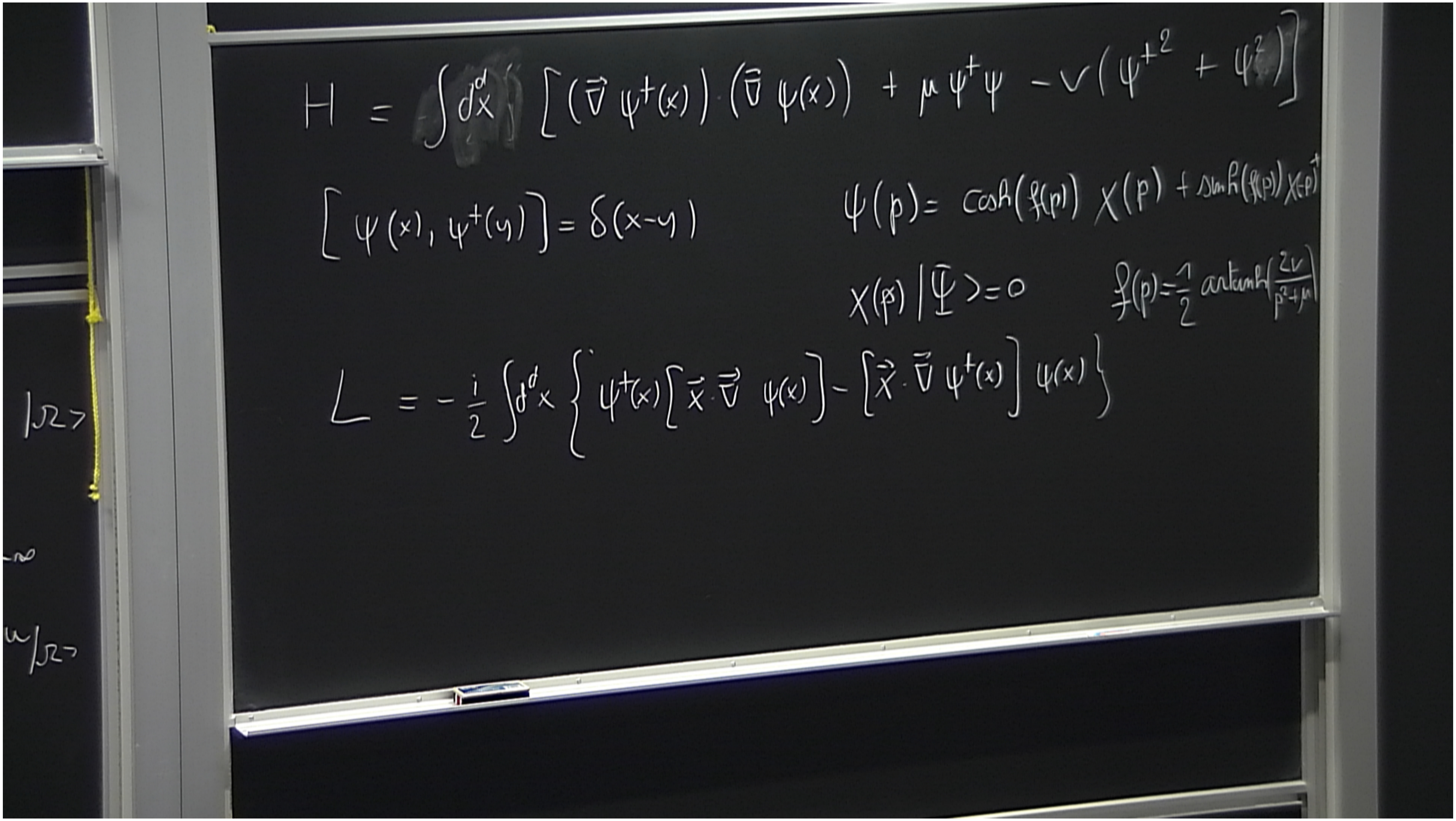
$$\langle \Psi | O_{\lambda_1}(\vec{x}_1)\, O_{\lambda_2}(\vec{x}_2) | \Psi \rangle = \langle \Psi(u) |\, e^{\lambda_1 u + \lambda_2 u}\, O_{\lambda_1}(e^u \vec{x}_1)\, O_{\lambda_2}(e^u \vec{x}_2) | \Psi(u) \rangle$$

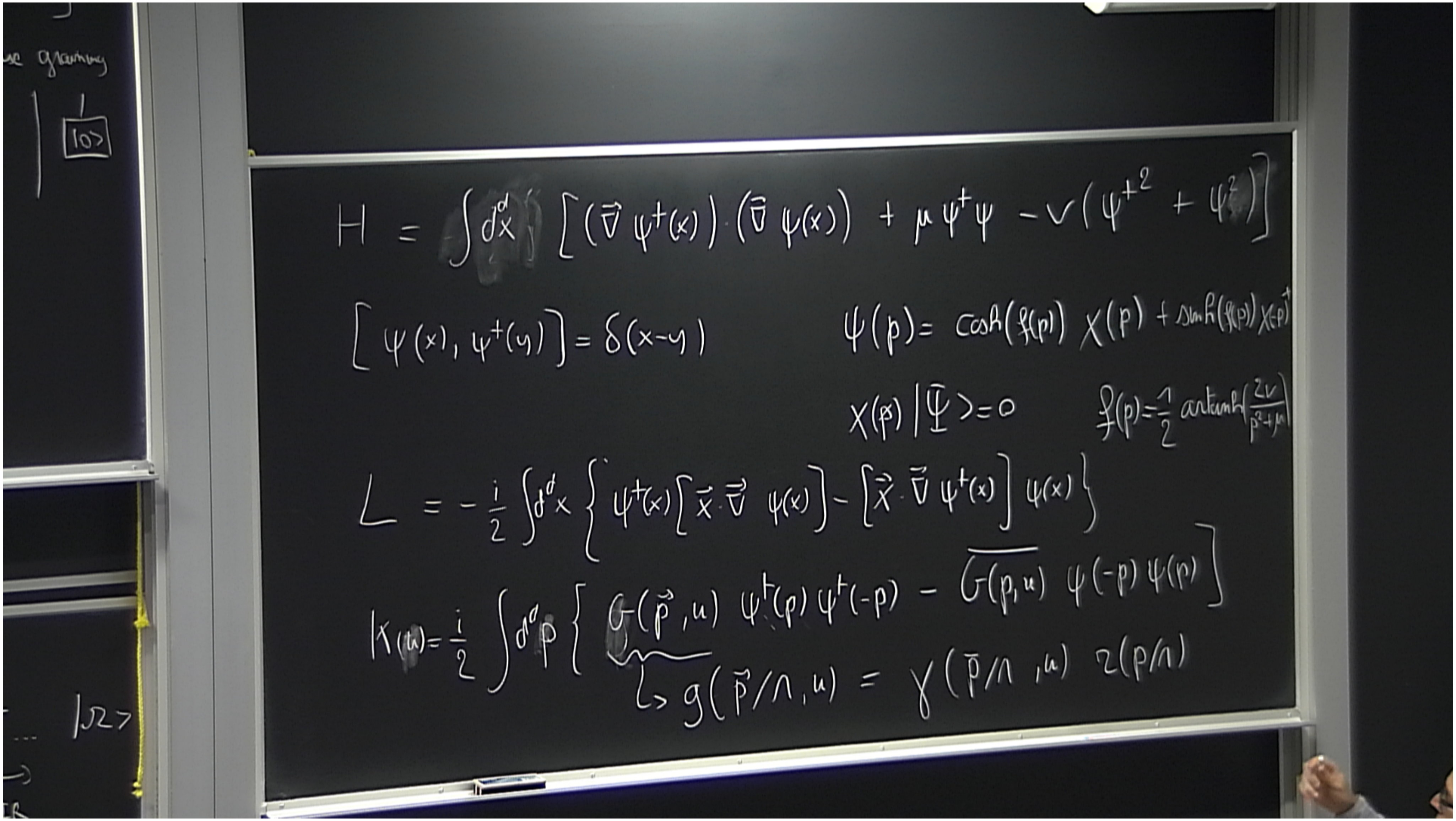
$$= \frac{\langle \Sigma \rangle}{|x_1 - x_2|^{\lambda_1 + \lambda_2}} \qquad u = -\log(\Lambda |x_1 - x_2|)$$

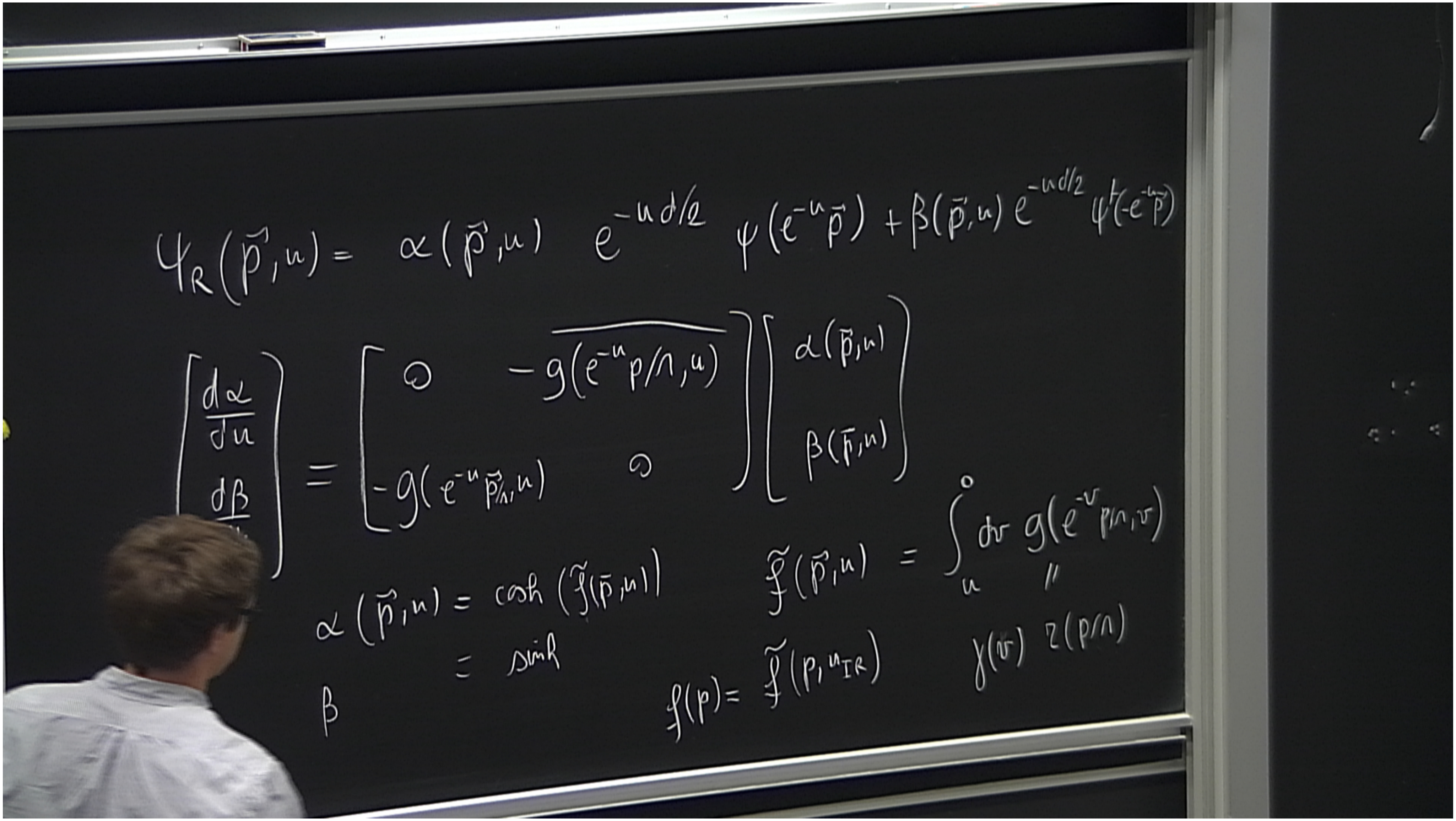


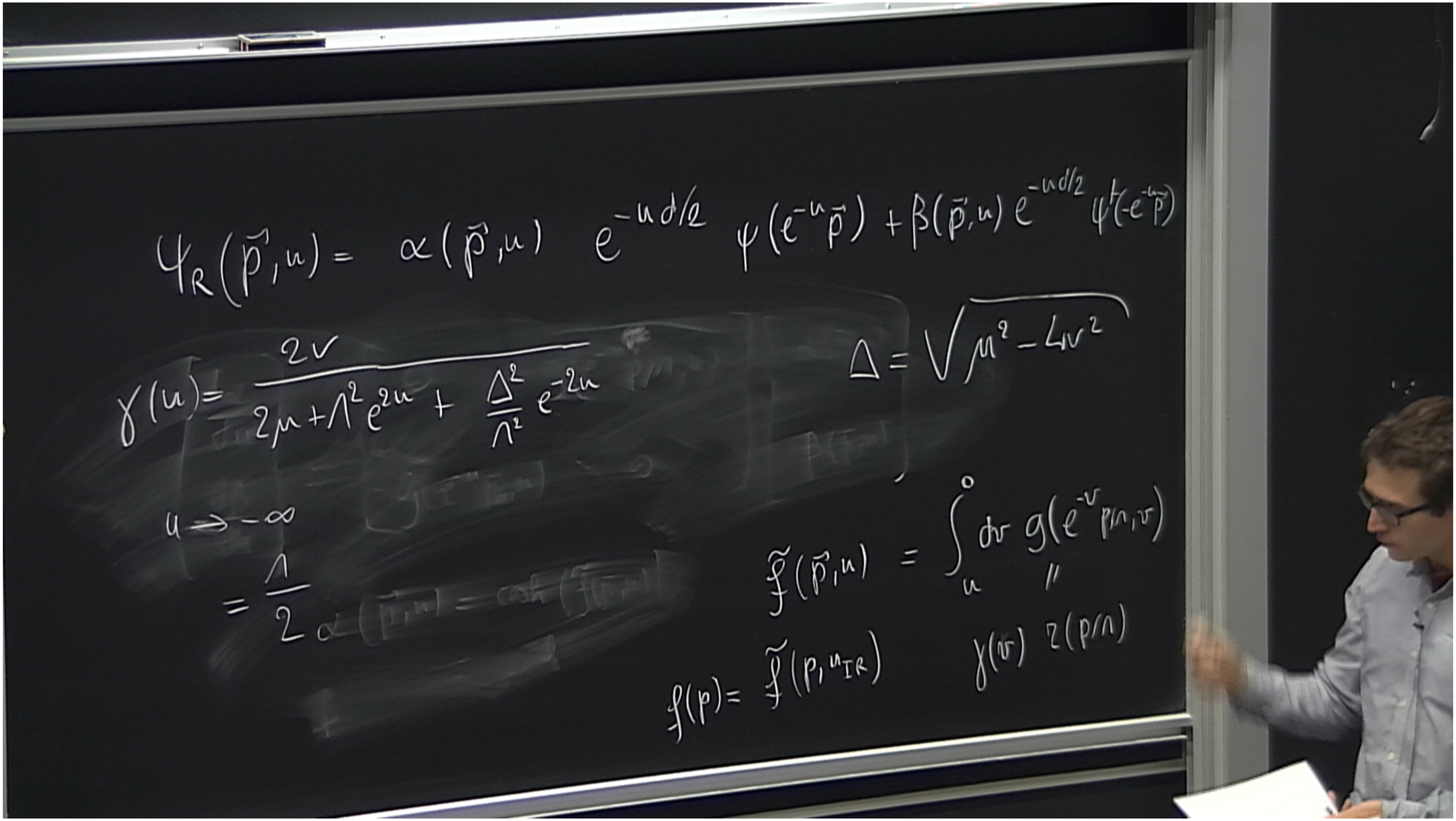
$H = \int d^d x \left[(\vec{\nabla} \psi^\dagger(x)) (\vec{\nabla} \psi(x)) + \mu \psi^\dagger \psi - \nu (\psi^{\dagger 2} + \psi^2) \right]$
$[\psi(x), \psi^\dagger(y)] = \delta(x-y)$
$\psi(p) = \cosh(f(p))\, \chi(p) + \sinh(f(p))\, \chi(-p)^\dagger$
$\chi(p) | \Psi \rangle = 0$
$f(p) = \frac{1}{2} \operatorname{arctanh}\left(\frac{2\nu}{p^2+\mu}\right)$
CAUTION

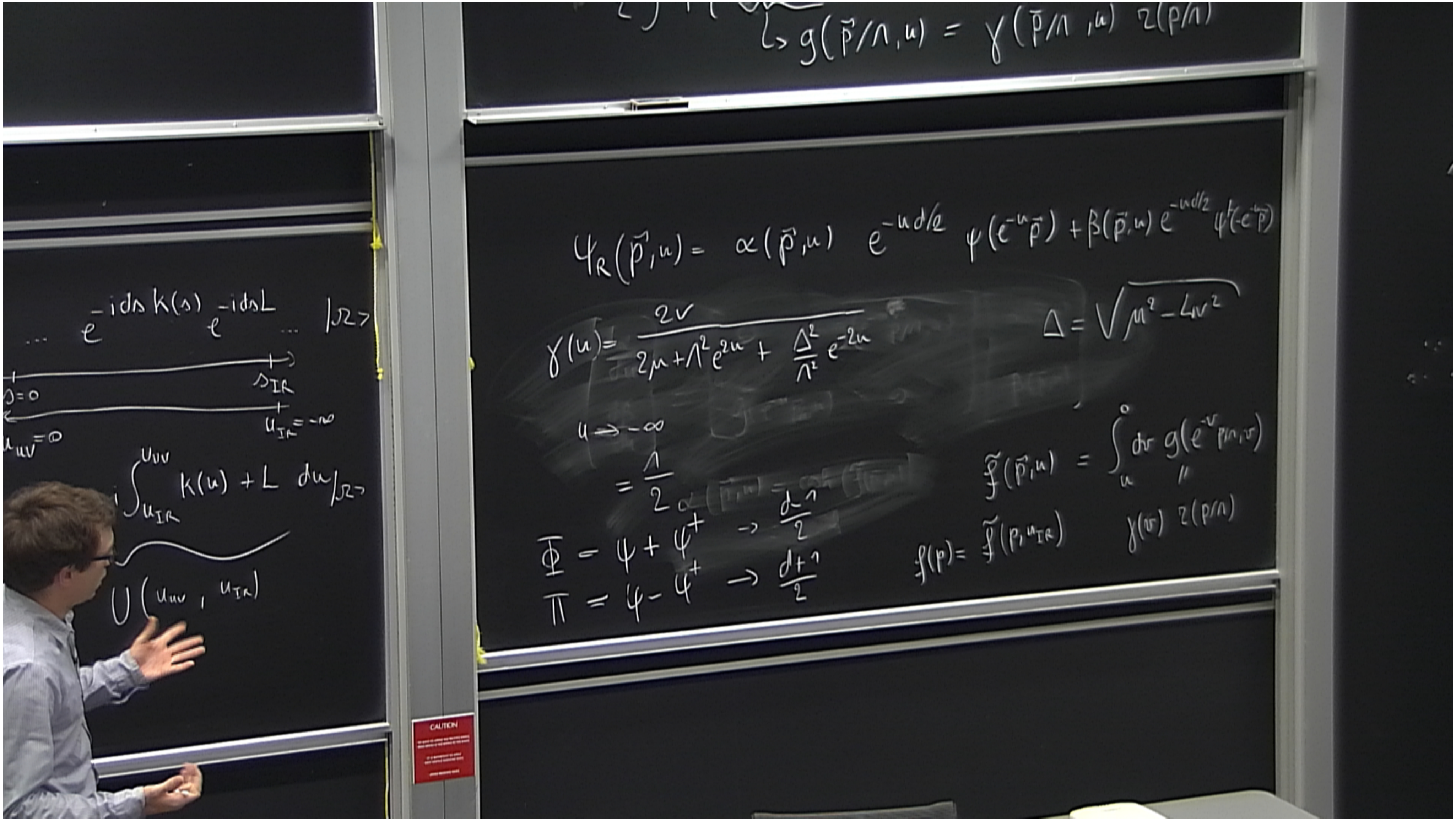

H = ∫d^d x [(∇ψ†(x)) (∇ψ(x)) + μψ†ψ − ν(ψ†² + ψ²)]
[ψ(x), ψ†(y)] = δ(x−y)
ψ(p) = cosh(f(p)) χ(p) + sinh(f(p)) χ(−p)†
χ(p) |Ψ> = 0
f(p) = −1/2 arctanh(2ν/(p²+μ))
|Ω>

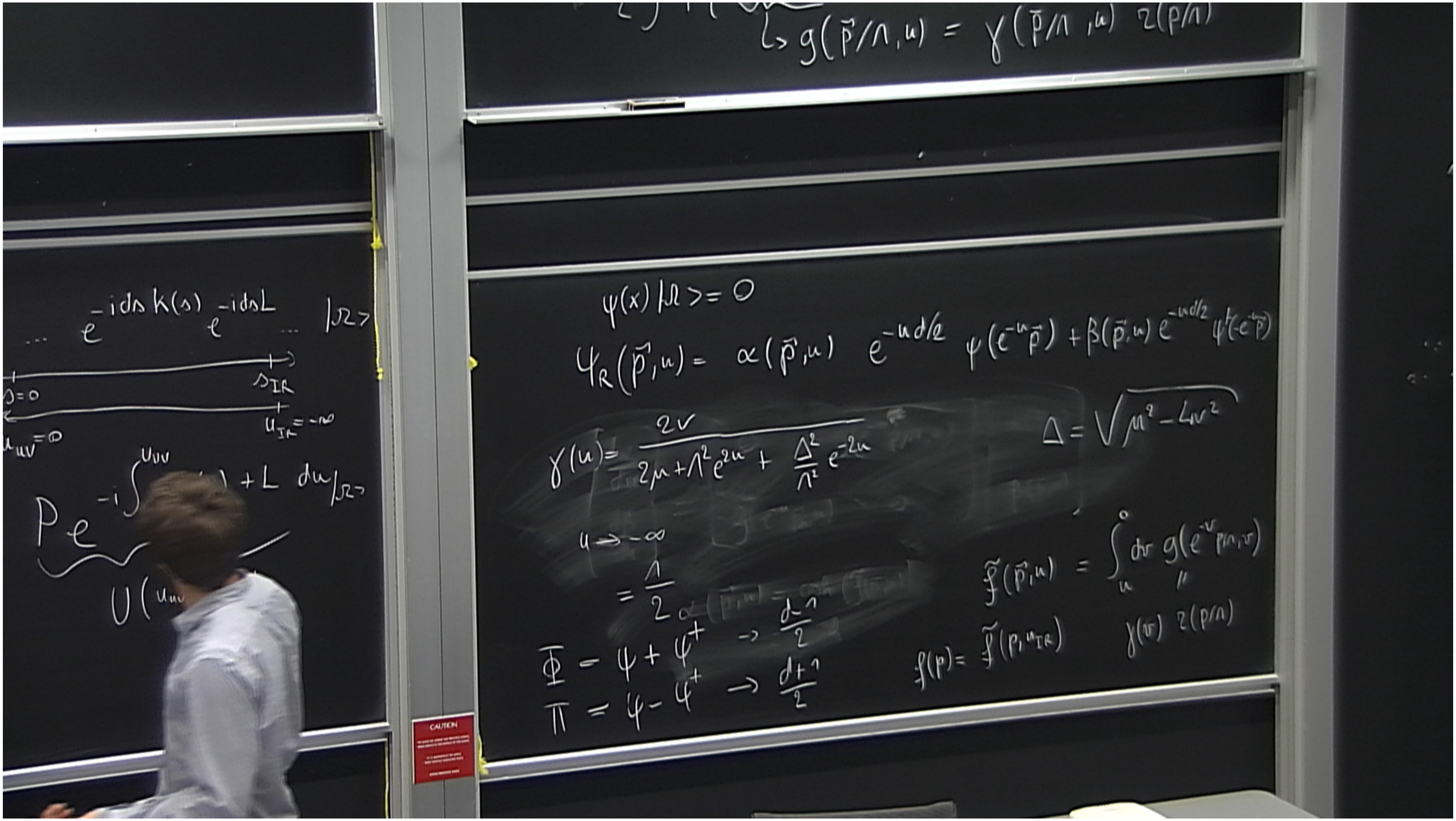

$H = \int d^d x \left[(\vec{\nabla} \psi^+(x)) \cdot (\vec{\nabla} \psi(x)) + \mu \psi^+ \psi - \nu (\psi^{+2} + \psi^2) \right]$
$[\psi(x), \psi^+(y)] = \delta(x-y)$
$\psi(p) = \cosh(f(p)) \chi(p) + \sinh(f(p)) \chi(-p)^+$
$\chi(p) | \Psi \rangle = 0$
$f(p) = \frac{1}{2} \operatorname{arctanh}\left(\frac{2\nu}{p^2+\mu}\right)$
$L = -\frac{i}{2} \int d^d x \left\{ \psi^+(x) [\vec{x} \cdot \vec{\nabla} \psi(x)] - [\vec{x} \cdot \vec{\nabla} \psi^+(x)] \psi(x) \right\}$
$|\Omega\rangle$
$\omega|\Omega\rangle$


$|0\rangle$
$H = \int d^d x \left[(\vec\nabla \psi^\dagger(x)) \cdot (\vec\nabla \psi(x)) + \mu \psi^\dagger \psi - \nu (\psi^{\dagger 2} + \psi^2) \right]$
$[\psi(x), \psi^\dagger(y)] = \delta(x-y)$
$\psi(p) = \cosh(f(p))\, \chi(p) + \sinh(f(p))\, \chi(-p)^\dagger$
$\chi(p) |\Psi\rangle = 0$
$f(p) = \frac{1}{2} \operatorname{arctanh}\left(\frac{2\nu}{p^2+\mu}\right)$
$L = -\frac{i}{2} \int d^d x \left\{ \psi^\dagger(x) [\vec x \cdot \vec\nabla \psi(x)] - [\vec x \cdot \vec\nabla \psi^\dagger(x)] \psi(x) \right\}$
$K(u) = \frac{i}{2} \int d^d p \left\{ G(\vec p, u)\, \psi^\dagger(p) \psi^\dagger(-p) - \overline{G(p,u)}\, \psi(-p) \psi(p) \right]$
$\hookrightarrow g(\vec p/\Lambda, u) = \gamma(\vec p/\Lambda, u)\, z(p/\Lambda)$
$|\Omega\rangle$


$\psi_R(\vec{p},u) = \alpha(\vec{p},u)\, e^{-ud/2}\, \psi(e^{-u}\vec{p}) + \beta(\vec{p},u)\, e^{-ud/2}\, \psi^\dagger(-e^{-u}\vec{p})$
$\begin{bmatrix} \frac{d\alpha}{du} \\ \frac{d\beta}{du} \end{bmatrix} = \begin{bmatrix} 0 & -\overline{g(e^{-u}p/\Lambda,u)} \\ -g(e^{-u}\vec{p}/\Lambda,u) & 0 \end{bmatrix} \begin{bmatrix} \alpha(\vec{p},u) \\ \beta(\vec{p},u) \end{bmatrix}$
$\alpha(\vec{p},u) = \cosh(\tilde{f}(\vec{p},u))$
$\beta = \sinh$
$\tilde{f}(\vec{p},u) = \int_u^0 dv\, g(e^{-v}p/\Lambda, v)$
$\gamma(v)\, z(p/\Lambda)$
$f(p) = \tilde{f}(p, u_{IR})$


ψR(p⃗,u) = α(p⃗,u) e^{-ud/2} ψ(e^{-u}p⃗) + β(p⃗,u) e^{-ud/2} ψ†(-e^{-u}p⃗)
γ(u) = 2ν / (2μ + Λ² e^{2u} + (Δ²/Λ²) e^{-2u})
Δ = √(μ² - 4ν²)
u → -∞
= Λ/2 α
f̃(p⃗,u) = ∫_u^0 dv g(e^{-v} p/Λ, v)
γ(v) z(p/Λ)
f(p) = f̃(p, u_IR)


↳ g(p⃗/Λ, u) = γ(p⃗/Λ, u) z(p/Λ)
ψ_R(p⃗, u) = α(p⃗, u) e^{-ud/2} ψ(e^{-u}p⃗) + β(p⃗, u) e^{-ud/2} ψ†(-e^{-u}p⃗)
γ(u) = 2v / (2μ + Λ² e^{2u} + (Δ²/Λ²) e^{-2u})
Δ = √(μ² − 4v²)
u → −∞
= Λ/2
Φ = ψ + ψ† → (d−1)/2
Π = ψ − ψ† → (d+1)/2
f̃(p⃗, u) = ∫_u^0 dv g(e^{-v} p/Λ, v)
γ(v) z(p/Λ)
f(p) = f̃(p, u_IR)
... e^{−idsK(s)} e^{−idsL} ... |Ω⟩
s = 0
s_IR
u_UV = 0
u_IR = −∞
i∫_{u_IR}^{u_UV} K(u) + L du |Ω⟩
U(u_UV, u_IR)
CAUTION


↳ g(p⃗/Λ, u) = γ(p⃗/Λ, u) z(p/Λ)
... e^{-idsK(s)} e^{-idsL} ... |Ω>
s=0
s_IR
u_IR = -∞
u_UV = 0
Pe^{-i∫^{u_UV} ... + L du} |Ω>
U(u_UV
ψ(x)|Ω> = 0
ψ_R(p⃗,u) = α(p⃗,u) e^{-ud/2} ψ(e^{-u}p⃗) + β(p⃗,u) e^{-ud/2} ψ†(-e^{-u}p⃗)
Δ = √(μ² - 4v²)
γ(u) = 2v / (2μ + Λ²e^{2u} + Δ²/Λ² e^{-2u})
u → -∞
= Λ/2
Φ = ψ + ψ† → (d-1)/2
Π = ψ - ψ† → (d+1)/2
f̃(p⃗,u) = ∫_u^0 dv g(e^{-v}p/Λ, v)
γ(v) z(p/Λ)
f(p) = f̃(p, u_IR)