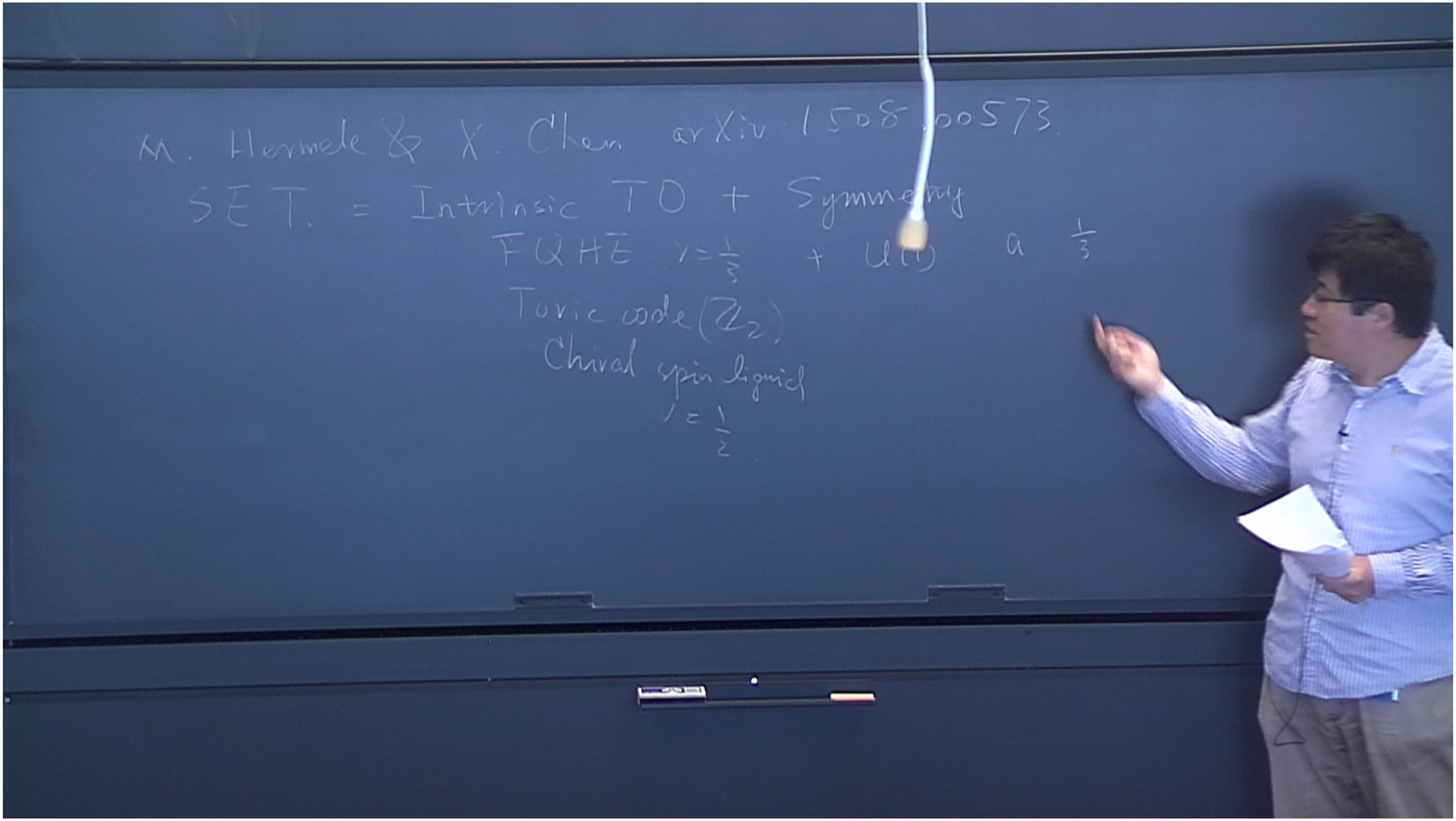
Title: Using anomaly tests to restrict crystal symmetry fractionalization in Z_2 and chiral spin liquids

Date: Aug 11, 2015 11:00 AM

URL: http://pirsa.org/15080116

Abstract: <p>Using the method of flux fusion anomaly test recently developed by M. Hermele and X. Chen (arXiv:1508.00573), we show that the possible ways of fractionalize crystal symmetry is greatly restricted if we assume the spin liquid has an SU(2) spin rotation symmetry and the spinon carries a half-integer spin. For a Z_2 spin liquid, under these assumptions the vison can only take the crystal symmetry fractionalization described by the Ising gauge theory. For a chiral spin liquid these assumptions imply that the spinon must also take fractionalized quantum numbers of crystal symmetries.</p>


M. Hermele & X. Chen arXiv:1508.00573


M. Hermele & X. Chen arXiv 1508.00573
S.E.T. = Intrinsic TO + Symmetry
FQHE ν = 1/3 + U(1) a 1/3
Toric code (Z2)
Chiral spin liquid
ν = 1/2

M. Hermele & X. Chen arXiv:1508.00573.

S.E.T. = Intrinsic TO + Symmetry

FQHE $\nu = \frac{1}{3}$ + U(1) a $\frac{1}{3}$ $a \times a \times a = 1$

Toric code ($\mathbb{Z}_2$)

Chiral spin liquid

$\nu = \frac{1}{2}$

M. Hermele & X. Chen arXiv 1508.00573.

S.E.T. = Intrinsic TO + Symmetry

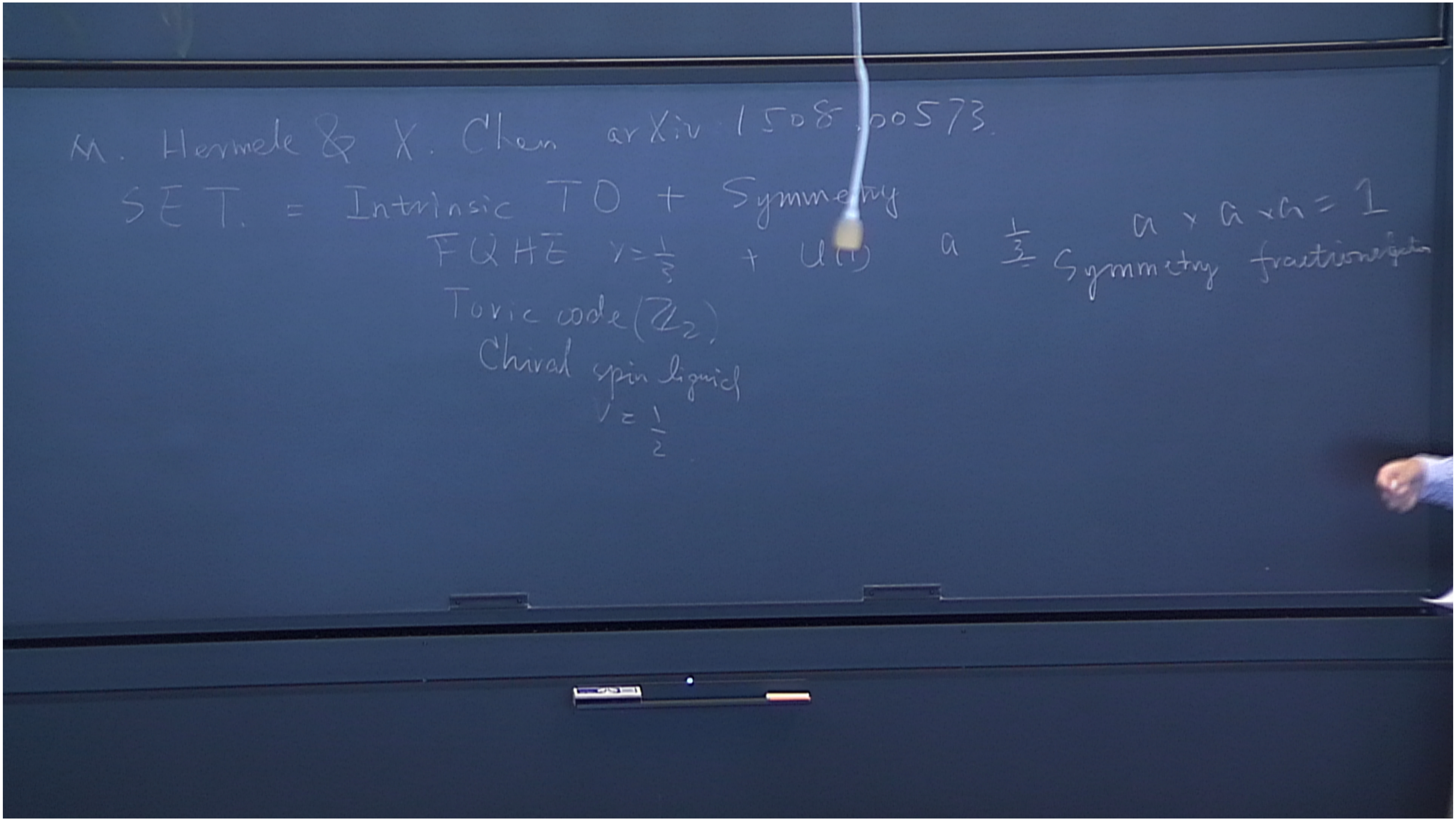
FQHE $\nu = \frac{1}{3}$ + $U(1)$ $\quad a \quad \frac{1}{3}$ Symmetry fractionalization

$a \times a \times a = 1$

Toric code ($\mathbb{Z}_2$)

Chiral spin liquid

$\nu = \frac{1}{2}$


M. Hermele & X. Chen arXiv 1508.00573.
S.E.T. = Intrinsic TO + Symmetry
FQHE ν=1/3 + U(1)
a 1/3
a × a × a = 1
Symmetry fractionalization
Toric code (Z2)
Chiral spin liquid
ν = 1/2

M. Hermele & X. Chen arXiv 1508.00573.

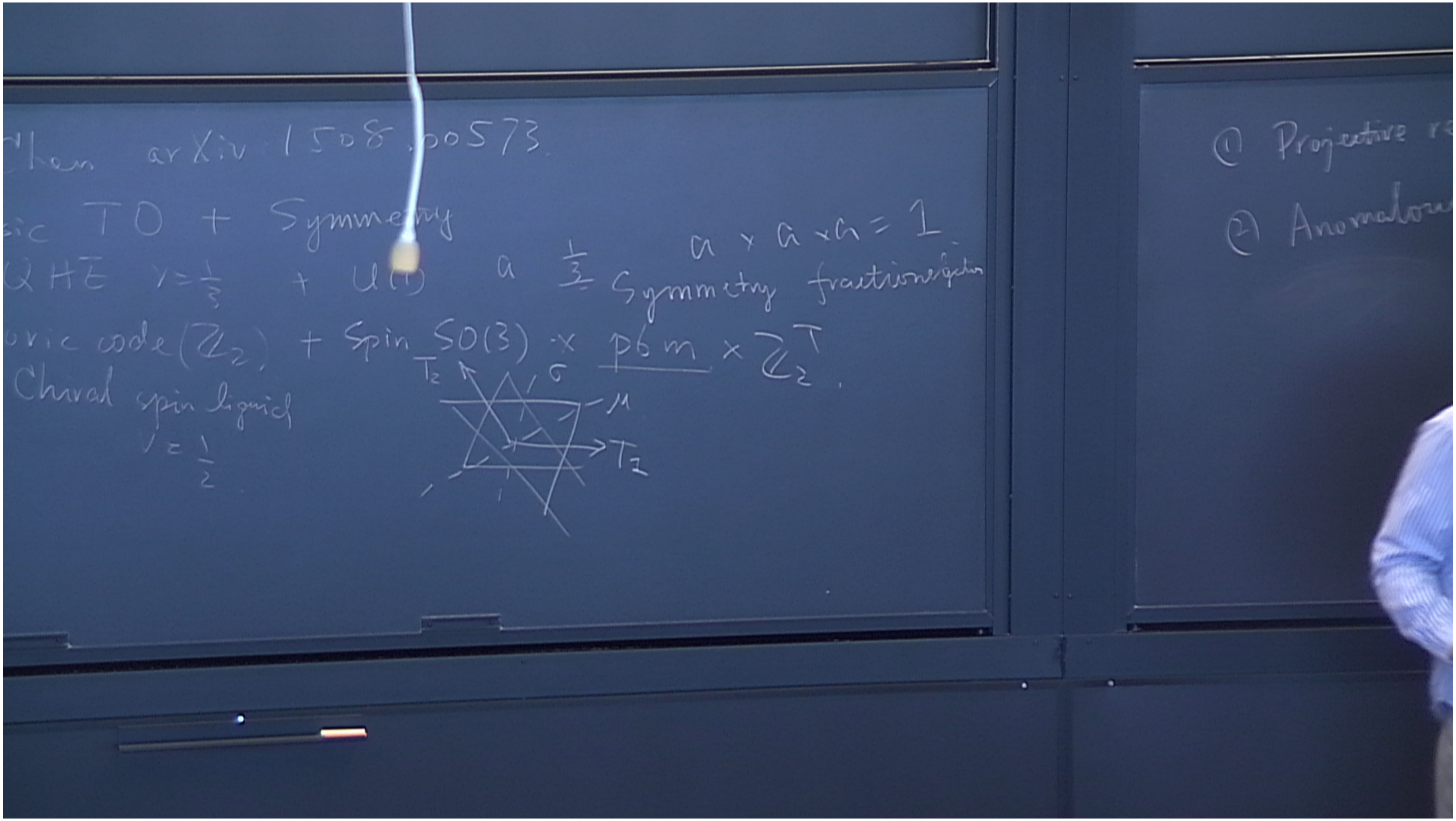
S.E.T. = Intrinsic TO + Symmetry

FQHE $\nu = \frac{1}{3}$ + $U(1)$ a $\frac{1}{3}$ $a \times a \times a = 1$ Symmetry fractionalization

Toric code ($\mathbb{Z}_2$)

Chiral spin liquid $\nu = \frac{1}{2}$

1. TO + SPT
2. $\boxed{\text{Sym. free}}$
3. $\mathbb{Z}_2$ $e \rightleftarrows m$


arXiv 1508.00573
TO + Symmetry
QHE ν = 1/3 + U(1)
a 1/3
a × a × a = 1
Symmetry fractionalization
code (Z2) + Spin SO(3) × p6m × Z2^T
Chiral spin liquid
ν = 1/2
T1
T2
σ
μ
① Projective
② Anomalous

① Projective rep. of $\underline{G}$

② Anomalous

$\underline{e}$, m, $e \times m = \varepsilon$

$e \times e = m \times m = \varepsilon \times \varepsilon = 1$.

$\underline{p6m \times \mathbb{Z}_2^T}$.

$\sigma^2 = +1 \longrightarrow \sigma_e^2 = \pm 1$.

$H^2(\underline{p6m \times \mathbb{Z}_2^T, \mathbb{Z}_2}) = \mathbb{Z}_2^7$

$T_1 T_2 = \pm T_2 T_1$

$\mu_1^2 = \pm 1$

$\sigma^2 = \pm 1$

$T^2 = \pm 1$.

$(\mu\sigma)^3 = I \;|\; \underline{I^2 = \pm 1}$.

$(\sigma T)^2 = \pm 1$

$(\mu T)^2 = \pm 1$

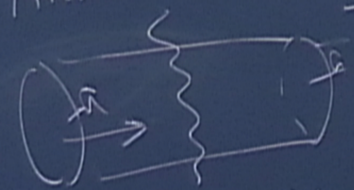
① Projective rep. of $\underline{G}$ $\quad$ $e, m, \quad e\times m = \varepsilon$

$e\times e = m\times m = \varepsilon\times\varepsilon = 1.$

② Anomalous $\underline{|a\rangle}$ $\quad a=1, e, m, \varepsilon.$ $\quad \underline{p6m\times \mathbb{Z}_2^T}.$

$\sigma^2 = +1 \longrightarrow \sigma_e^2 = \pm 1.$

$H^2(p6m\times \mathbb{Z}_2^T, \mathbb{Z}_2) = \mathbb{Z}_2^7$



$X^2 = \pm 1.$ a) X is antiunitary.

$|\psi\rangle = \sum_\alpha \lambda_\alpha |L_\alpha\rangle \otimes |\underline{R_\alpha}\rangle$

$X^2 |L_\alpha\rangle$ $\underline{\pm}$

$H^2(\mathbb{Z}_2^X, U(1)) = \mathbb{Z}_2.$

$T_1 T_2 = \pm T_2 T_1$

$M_1^2 = \pm 1$

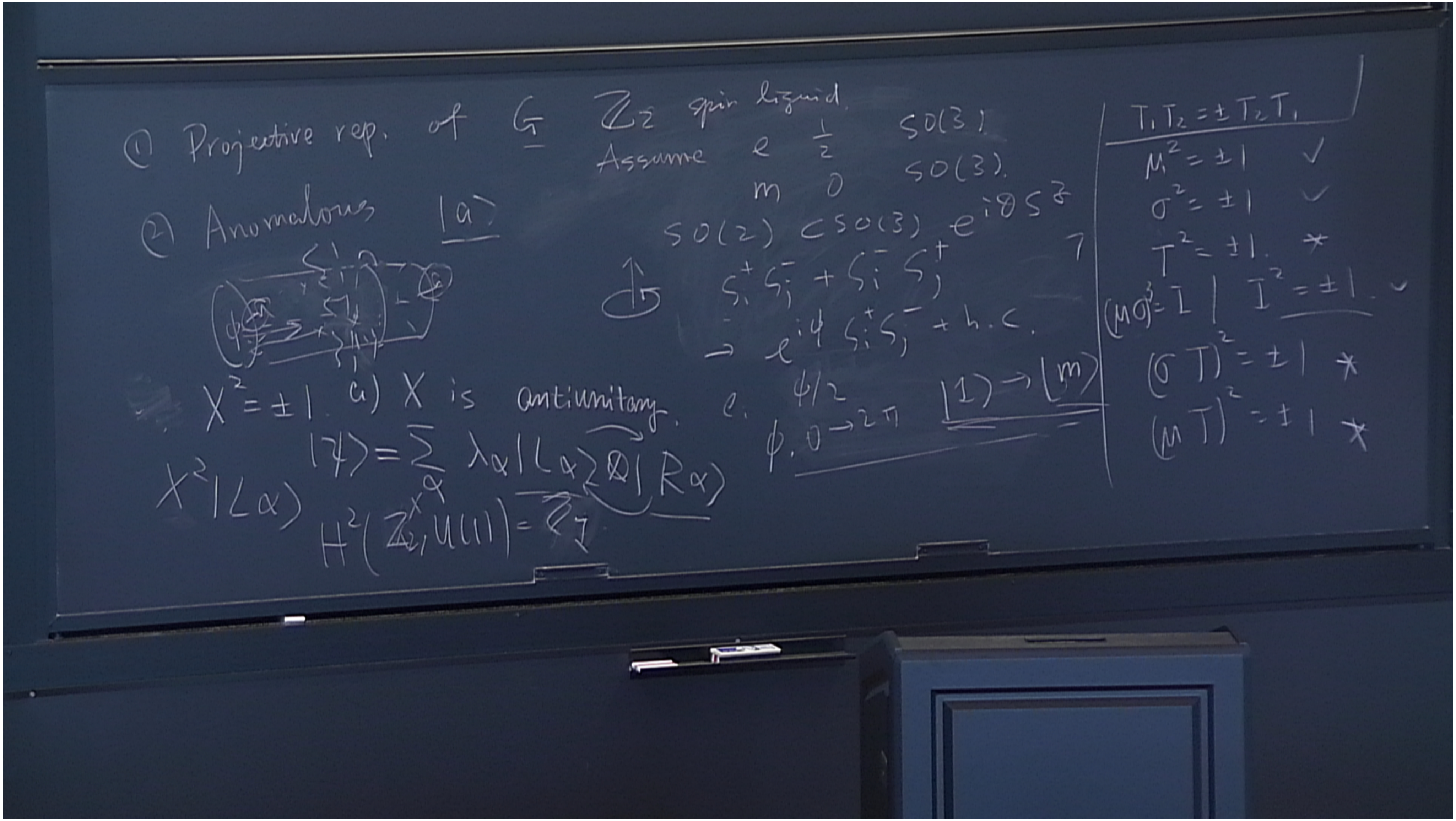
$\sigma^2 = \pm 1$

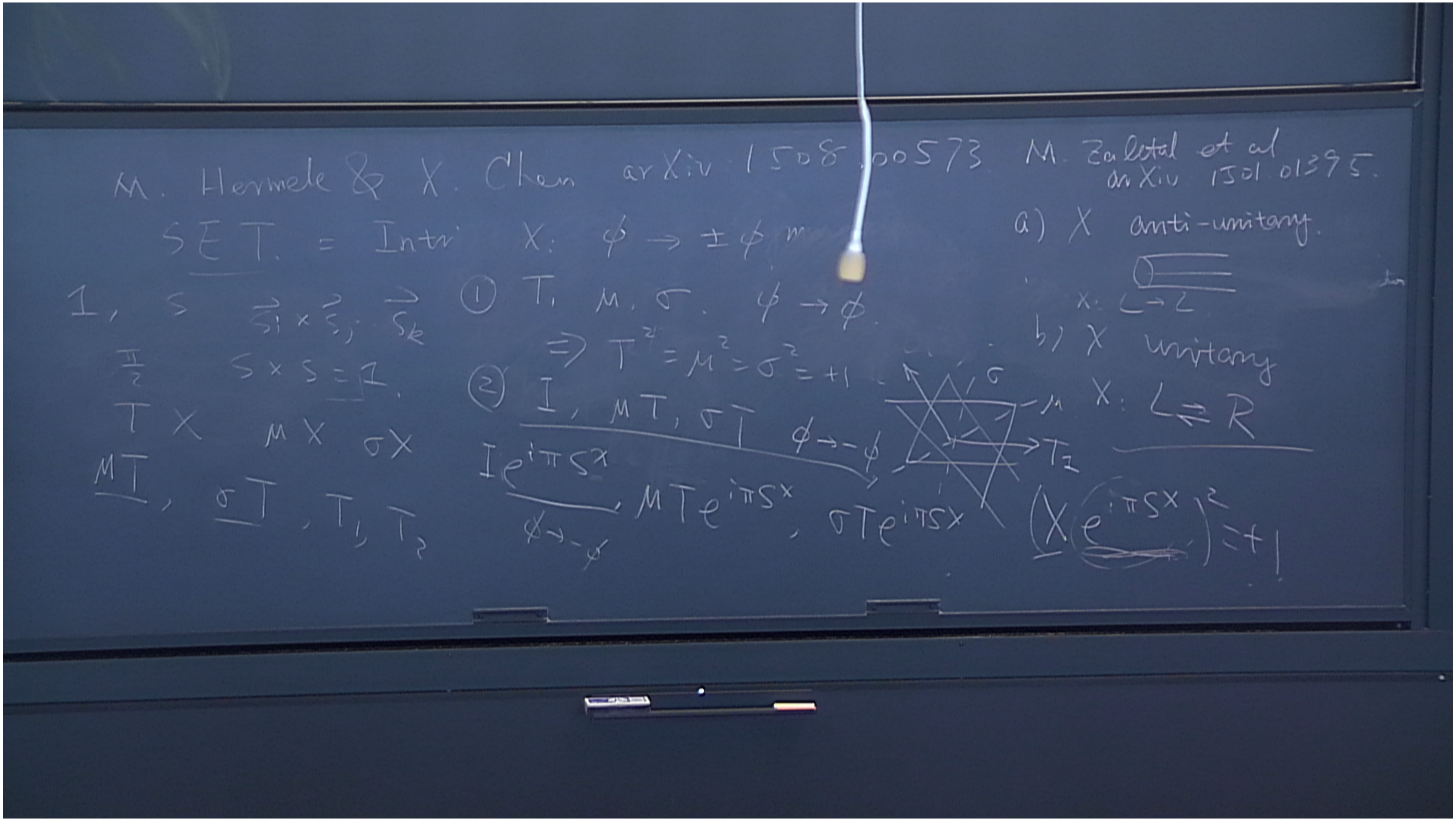
$T^2 = \pm 1.$

$(M\sigma)^3 = I \;|\; I^2 = \pm 1.$

$(\sigma T)^2 = \pm 1$

$(MT)^2 = \pm 1$


① Projective rep. of G
② Anomalous |a⟩
$X^2 = \pm 1$. a) X is antiunitary.
$|\psi\rangle = \sum_\alpha \lambda_\alpha |L_\alpha\rangle \otimes |R_\alpha\rangle$
$X^2 |L\alpha\rangle$
$H^2(\mathbb{Z}_2^X, U(1)) = \mathbb{Z}_2$
$\mathbb{Z}_2$ spin liquid.
Assume e $\frac{1}{2}$ SO(3)
m 0 SO(3).
$SO(2) \subset SO(3)$ $e^{i\theta S^z}$
$S_i^+ S_j^- + S_i^- S_j^+$
$\to e^{i\phi} S_i^+ S_j^- + h.c.$
e. $\phi/2$ $|1\rangle \to |m\rangle$
ϕ. $0 \to 2\pi$
$T_1 T_2 = \pm T_2 T_1$
$M_1^2 = \pm 1$ ✓
$\sigma^2 = \pm 1$ ✓
$T^2 = \pm 1$. *
$(M\sigma)^3 = I$ | $I^2 = \pm 1$ ✓
$(\sigma T)^2 = \pm 1$ *
$(M T)^2 = \pm 1$ *

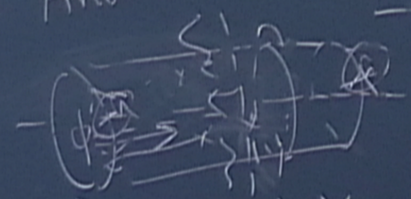

M. Hermele & X. Chen arXiv 1508.00573
M. Zaletel et al arXiv 1501.01395
SET = Intri X: φ → ±φ
a) X anti-unitary
X: L → L
b) X unitary
X: L ⇄ R
1, S, S_i × S_j · S_k
① T, M, σ. φ → φ
⇒ T² = M² = σ² = +1
S × S = 1
② I, MT, σT φ → −φ
T X M X σX
I e^{iπS^x}
MT, σT, T_1, T_2
MT e^{iπS^x}, σT e^{iπS^x}
(X e^{iπS^x})² = +1

① Projective rep. of G $\qquad$ $\mathbb{Z}_2$ spin liquid

Assume $\quad e \quad \frac{1}{2} \quad SO(3)$

$\qquad\qquad m \quad 0 \quad SO($

② Anomalous $\quad |a\rangle$



$SO(2) \subset SO(3) \quad e^{i}$

$S_i^+ S_j^- + S_i^- S_j^+$

$\to e^{i\phi} S_i^+ S_j^- + h.c.$

$e. \quad \phi/2 \qquad |1\rangle \xrightarrow{SO(3)}$

$\phi.\ 0 \to 2\pi \qquad \to |3\rangle$

$X^2 = \pm 1$. a) X is antiunitary.

$|\psi\rangle = \sum_\alpha \lambda_\alpha |L\alpha\rangle \otimes |R\alpha\rangle$

$X^2 |L\alpha\rangle$

$H^2(\mathbb{Z}_2^X, U(1)) = \mathbb{Z}_2$

$T_1 T_2 = \pm T_2 T_1$

$SO(2) \begin{cases} \cancel{M_1^2 = \pm 1} \\ \cancel{\sigma^2 = \pm 1} \\ \cancel{T^2 = \pm 1} \ * \end{cases}$

$(M\sigma)^3 = I \quad | \quad I^2 = \pm 1$. ✓

$SO(3) \begin{cases} (\sigma T)^2 = \pm 1 \ * \\ (M T)^2 = \pm 1 \ * \end{cases}$