Title: Using anomaly tests to restrict crystal symmetry fractionalization in Z_2 and chiral spin liquids

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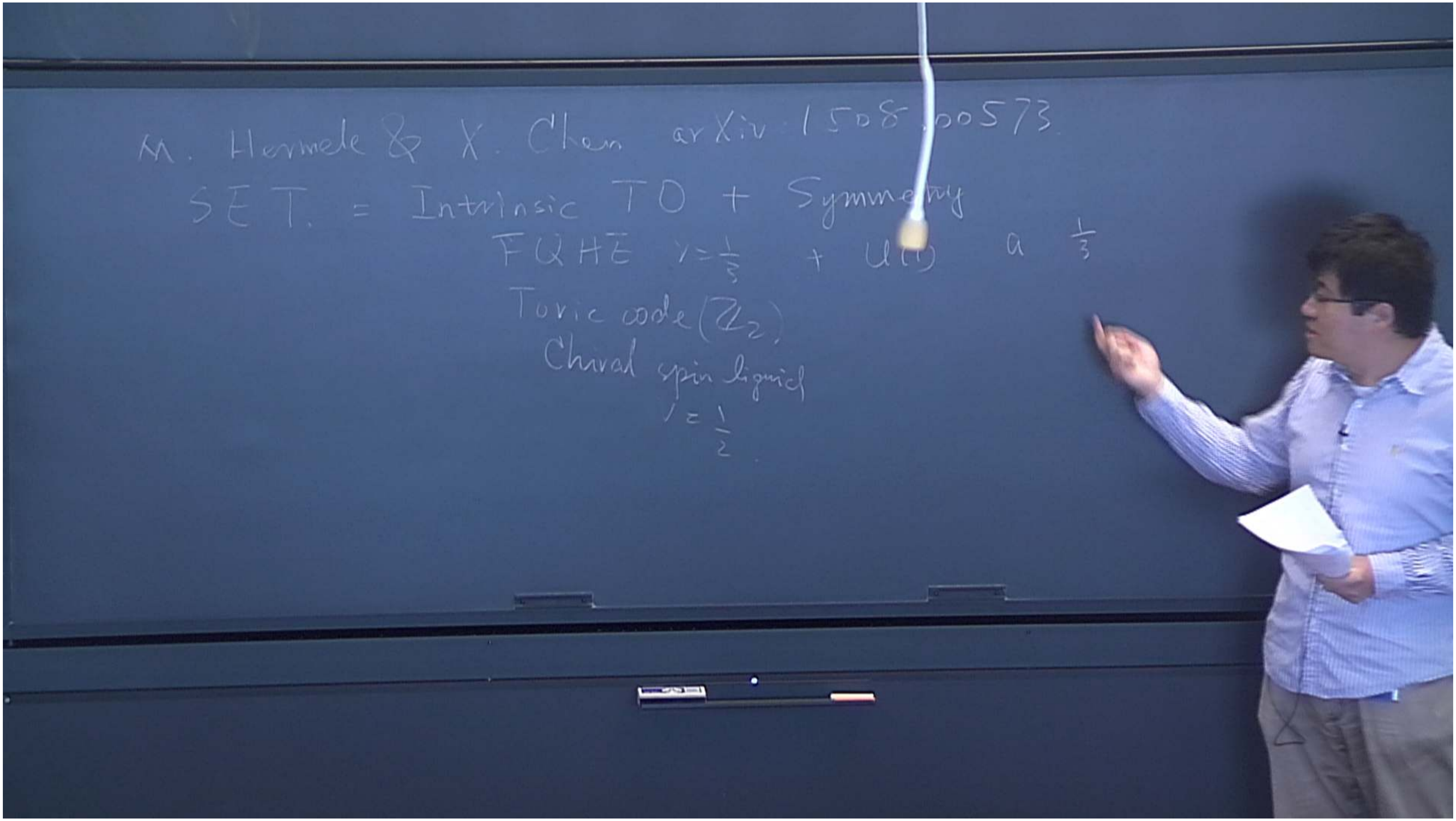
Abstract: <p>Using the method of flux fusion anomaly test recently developed by M. Hermele and X. Chen (arXiv:1508.00573), we show that the possible ways of fractionalize crystal symmetry is greatly restricted if we assume the spin liquid has an SU(2) spin rotation symmetry and the spinon carries a half-integer spin. For a Z_2 spin liquid, under these assumptions the vison can only take the crystal symmetry fractionalization described by the Ising gauge theory. For a chiral spin liquid these assumptions imply that the spinon must also take fractionalized quantum numbers of crystal symmetries.</p>


M. Hermele & X. Chen arXiv:1508.00573


M. Hermele & X. Chen arXiv 1508.00573
S.E.T. = Intrinsic TO + Symmetry
FQHE $\nu = \frac{1}{3}$ + U(1) a $\frac{1}{3}$
Toric code ($\mathbb{Z}_2$)
Chiral spin liquid
$\nu = \frac{1}{2}$

M. Hermele & X. Chen arXiv 1508.00573.

S.E.T. = Intrinsic TO + Symmetry

FQHE $\nu = \frac{1}{3}$ + $U(1)$ a $\frac{1}{3}$ $a \times a \times a = 1$

Toric code ($\mathbb{Z}_2$)

Chiral spin liquid

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Symmetry fractionalization

Toric code ($\mathbb{Z}_2$)

Chiral spin liquid

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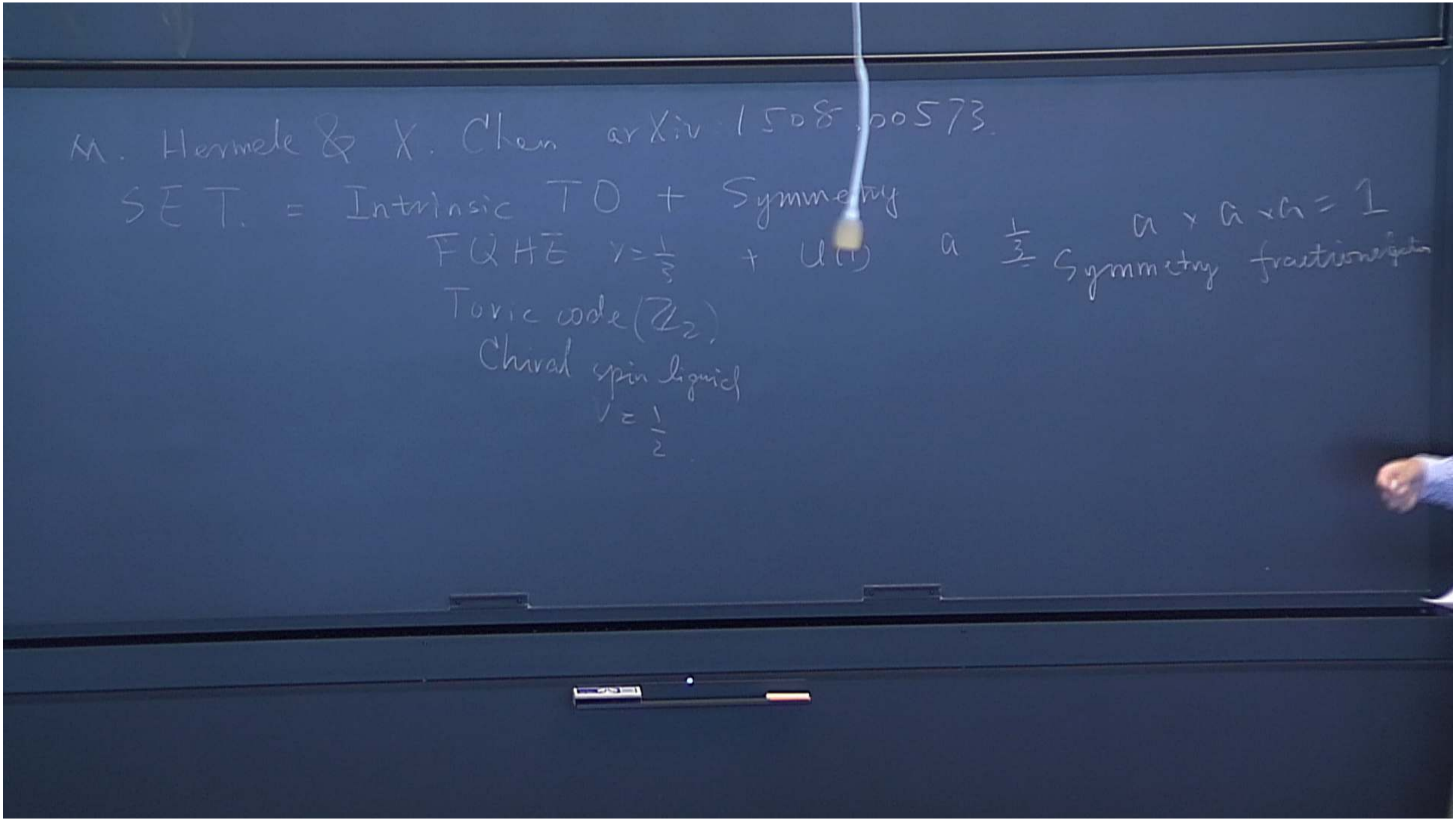

M. Hermele & X. Chen arXiv 1508.00573.
SET = Intrinsic TO + Symmetry
FQHE $\nu = \frac{1}{3}$ + U(1) a $\frac{1}{3}$ a × a × a = 1
Symmetry fractionalization
Toric code ($\mathbb{Z}_2$)
Chiral spin liquid
$\nu = \frac{1}{2}$

M. Hermele & X. Chen arXiv 1508.00573.

S.E.T. = Intrinsic TO + Symmetry

FQHE $\nu = \frac{1}{3}$ + $U(1)$

a $\frac{1}{3}$ Symmetry fraction[alization]

$a \times a \times a = 1$

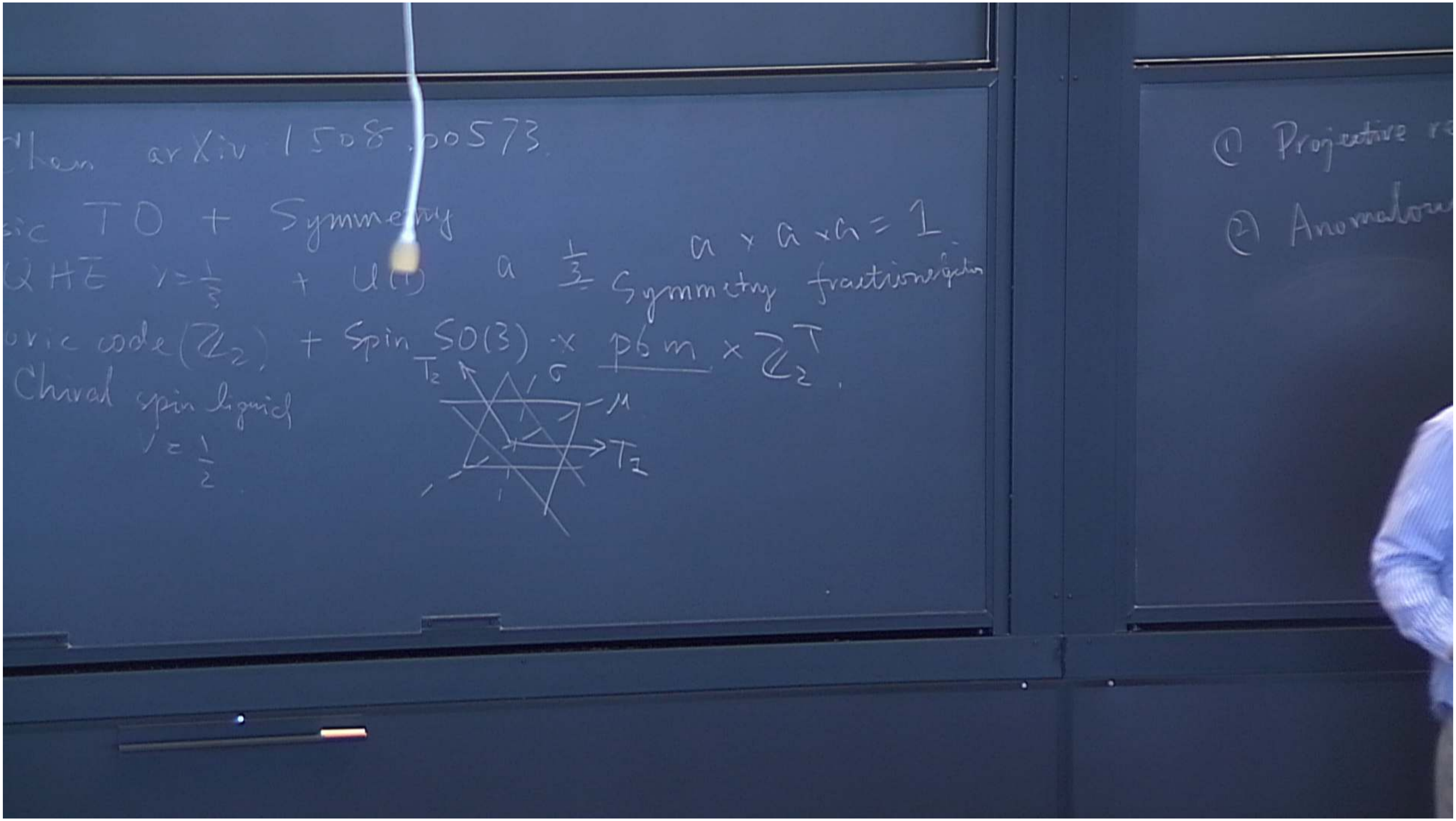
Toric code ($\mathbb{Z}_2$)

Chiral spin liquid $\nu = \frac{1}{2}$.

① TO + SPT

$\boxed{② \text{ Sym. free}}$

③ $\mathbb{Z}_2$ $e \rightleftarrows m$



Chen arXiv 1508.00573
TO + Symmetry
QHE ν = 1/3 + U(1)
a 1/3
a × a × a = 1
Symmetry fractionalization
code(Z2) + Spin SO(3) × p6m × Z2^T
Chiral spin liquid
ν = 1/2
T2
σ
μ
T1
① Projective
② Anomalou

① Projective rep. of $\underline{G}$

② Anomalous

$\underline{e}$, m, $e \times m = \varepsilon$

$e \times e = m \times m = \varepsilon \times \varepsilon = 1$.

$\underline{p6m \times \mathbb{Z}_2^T}$.

$\sigma^2 = +1 \longrightarrow \sigma_e^2 = \pm 1$.

$H^2(\underline{p6m \times \mathbb{Z}_2^T}, \mathbb{Z}_2) = \mathbb{Z}_2^7$

$T_1 T_2 = \pm T_2 T_1$

$\mu_1^2 = \pm 1$

$\sigma^2 = \pm 1$

$T^2 = \pm 1$.

$(\mu\sigma)^3 = I$ | $\underline{I^2 = \pm 1}$.

$(\sigma T)^2 = \pm 1$

$(\mu T)^2 = \pm 1$

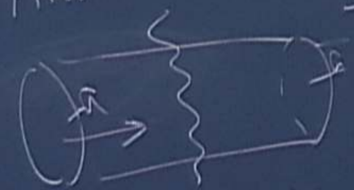
① Projective rep. of $\underline{G}$ $\quad$ $\underline{e}$, m, $\quad e \times m = \varepsilon$

$e \times e = m \times m = \varepsilon \times \varepsilon = 1$.

② Anomalous $\underline{|a\rangle}$ $\quad G = 1, e, m, \varepsilon$. $\quad \underline{pbm \times \mathbb{Z}_2^T}$.

$\sigma^2 = +1 \longrightarrow \sigma_e^2 = \pm 1$.

$H^2(pbm \times \mathbb{Z}_2^T, \mathbb{Z}_2) = \mathbb{Z}_2^7$



$X^2 = \pm 1$. a) X is antiunitary.

$|\psi\rangle = \sum_\alpha \lambda_\alpha |L_\alpha\rangle \otimes |\underline{R_\alpha}\rangle$

$\underline{X}^2 |L\alpha\rangle$

$H^2(\mathbb{Z}_2^{\times T}, U(1)) = \mathbb{Z}_2$.

$\underline{T_1 T_2 = \pm T_2 T_1}$

$M_1^2 = \pm 1$

$\sigma^2 = \pm 1$

$T^2 = \pm 1$.

$(M\sigma)^3 = I$ | $\underline{I^2 = \pm 1}$.

$(\sigma T)^2 = \pm 1$

$(M T)^2 = \pm 1$

① Projective rep. of G

② Anomalous $|a\rangle$

$X^2 = \pm 1$. (a) X is antiunitary.

$|\psi\rangle = \sum_\alpha \lambda_\alpha |L\alpha\rangle \otimes |R\alpha\rangle$

$X^2 |L\alpha\rangle$

$H^2(Z_2^X, U(1)) = Z_2$

Z_2 spin liquid

Assume e $\frac{1}{2}$ $SO(3)$

m 0 $SO(3)$.

$SO(2) \subset SO(3)$ $e^{i\theta S^z}$

$S_i^+ S_j^- + S_i^- S_j^+$

$\rightarrow e^{i\phi} S_i^+ S_j^- + h.c.$

e: $\phi/2$

$\phi: 0 \rightarrow 2\pi$ $|1\rangle \rightarrow |m\rangle$

$T_1 T_2 = \pm T_2 T_1$

$M_1^2 = \pm 1$ ✓

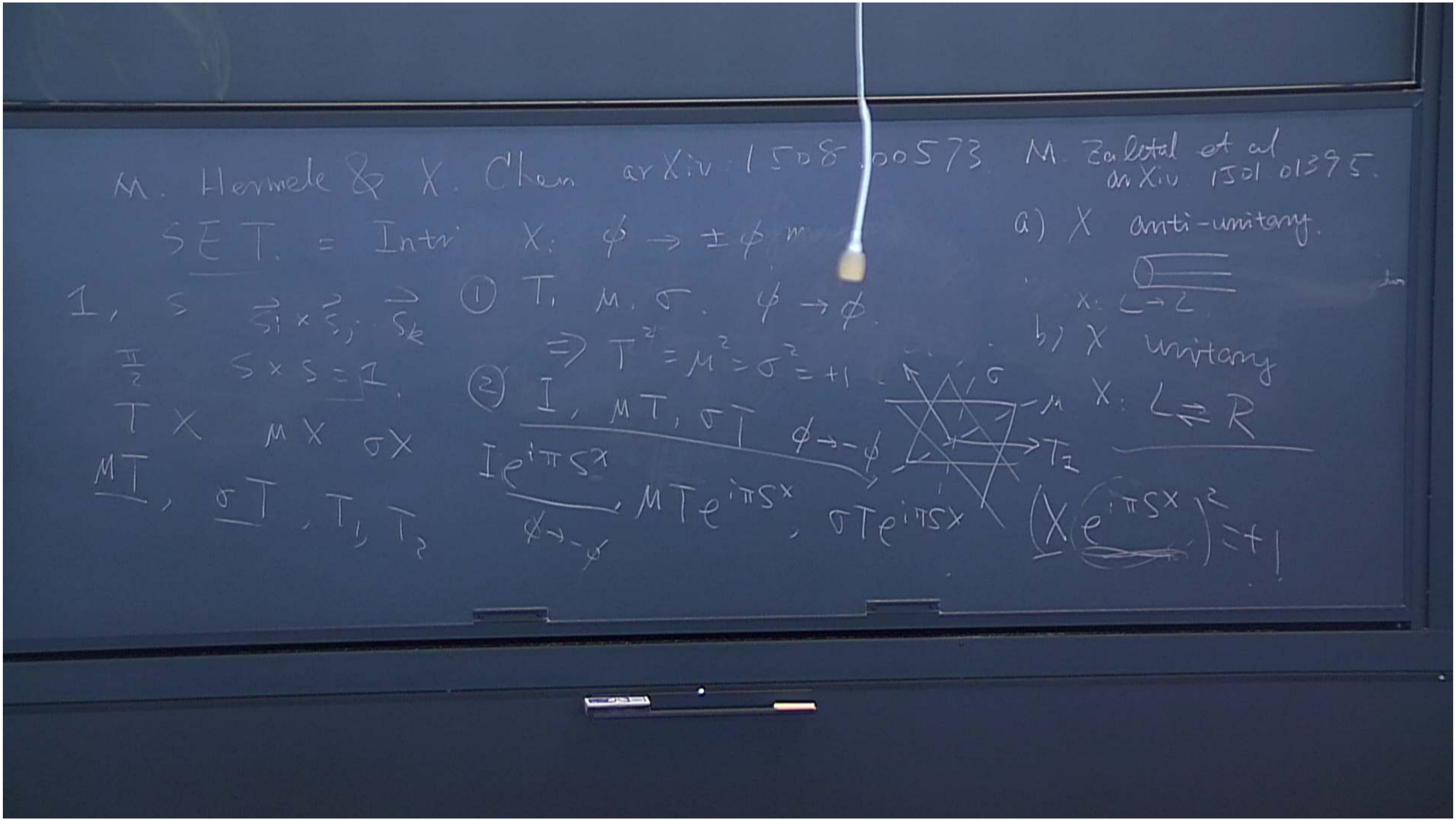
$\sigma^2 = \pm 1$ ✓

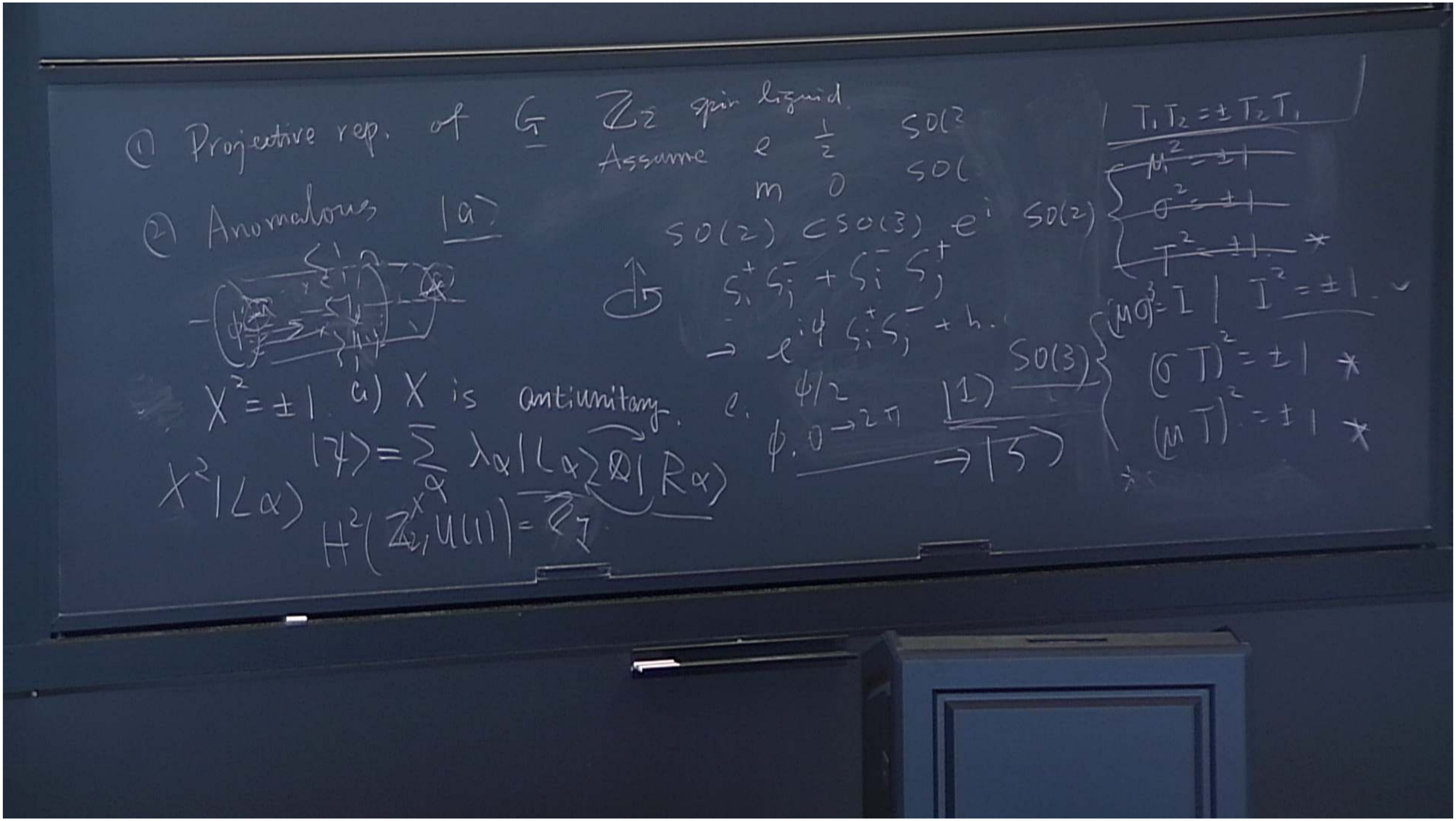
$T^2 = \pm 1$. *

$(M\sigma)^3 = I$ | $I^2 = \pm 1$. ✓

$(\sigma T)^2 = \pm 1$ *

$(MT)^2 = \pm 1$ *


M. Hermele & X. Chen arXiv 1508.00573
M. Zaletel et al arXiv 1501.01395.
SET = Intr X: $\phi \to \pm\phi$
1, S $\vec{S}_i \times \vec{S}_j \cdot \vec{S}_k$
① T, M, σ. $\phi \to \phi$.
$\frac{\pi}{2}$ $S \times S = 1$.
$\Rightarrow T^2 = M^2 = \sigma^2 = +1$
② I, MT, σT $\phi \to -\phi$
TX MX σX
$\underline{MT}$, $\underline{\sigma T}$, T_1, T_2
$I e^{i\pi S^x}$ $\phi \to -\phi$, $MT e^{i\pi S^x}$, $\sigma T e^{i\pi S^x}$
σ M T_2
a) X anti-unitary.
X: $L \to L$
b) X unitary
X: $L \rightleftarrows R$
$(X e^{i\pi S^x})^2 = +1$


① Projective rep. of G
② Anomalous |a⟩
Z2 spin liquid
Assume e 1/2 SO(3
m 0 SO(
SO(2) ⊂ SO(3) e^i SO(2)
S_i^+ S_j^- + S_i^- S_j^+
→ e^{iφ} S_i^+ S_j^- + h.c.
X^2 = ±1 a) X is antiunitary. e: φ/2
|ψ⟩ = Σ_α λ_α |Lα⟩ ⊗ |Rα⟩
X^2 |Lα⟩
H^2(Z_2^X, U(1)) = Z_2
φ: 0 → 2π
|1⟩ SO(3)
→ |3⟩
T_1 T_2 = ± T_2 T_1
M_1^2 = ±1
σ^2 = ±1
T^2 = ±1 *
(Mσ)^3 = I | I^2 = ±1
(σT)^2 = ±1 *
(MT)^2 = ±1 *