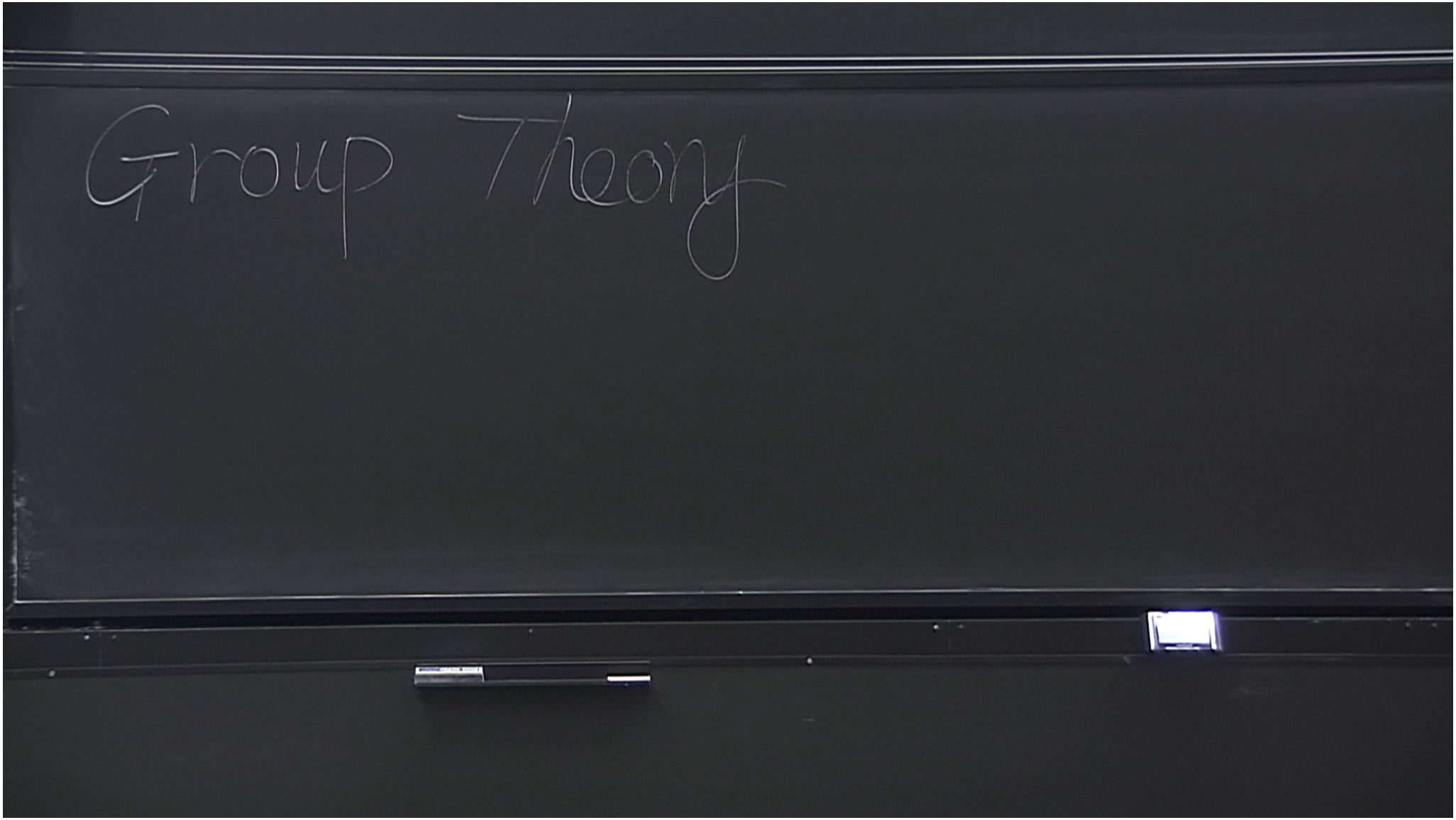
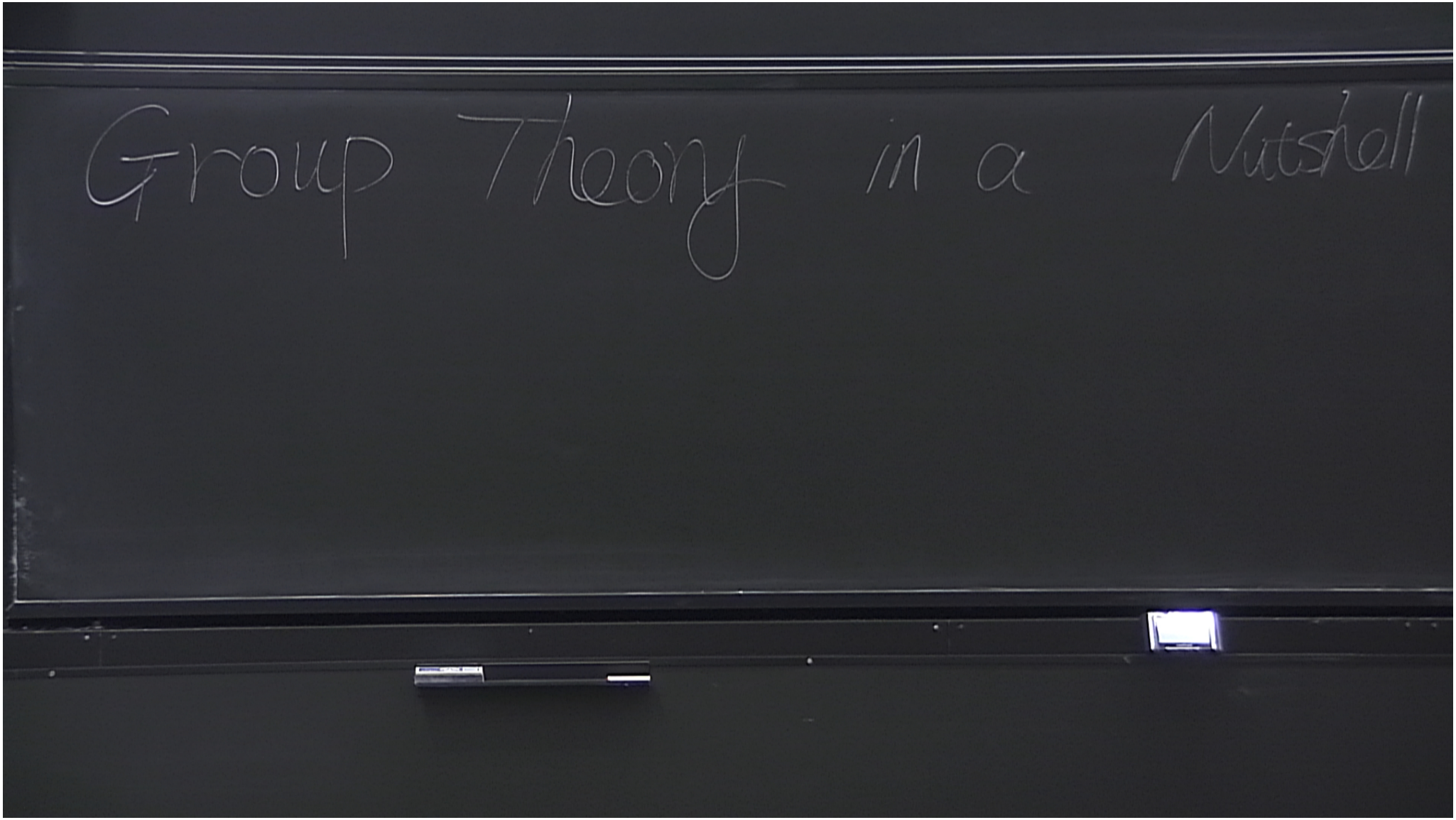
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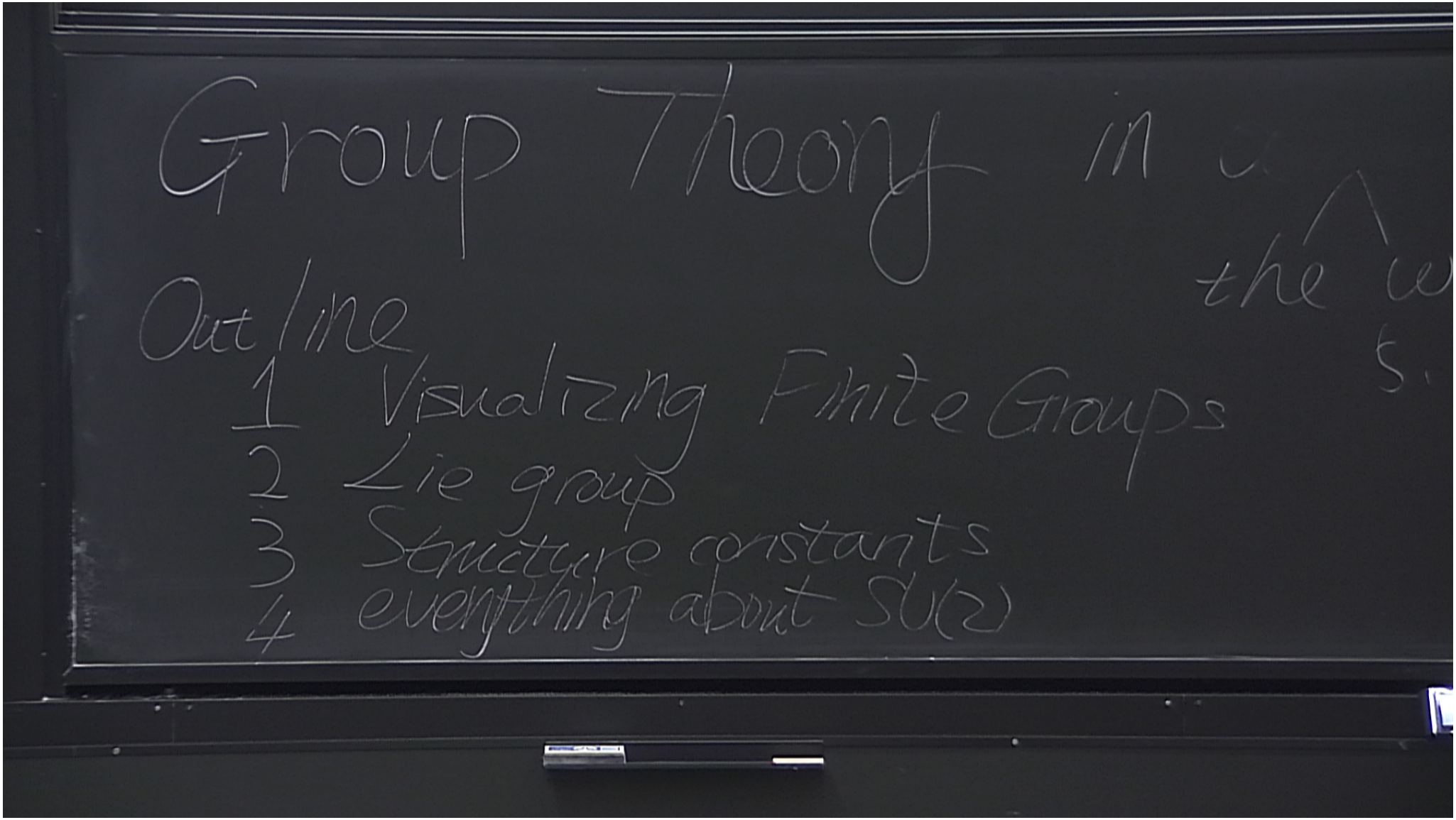
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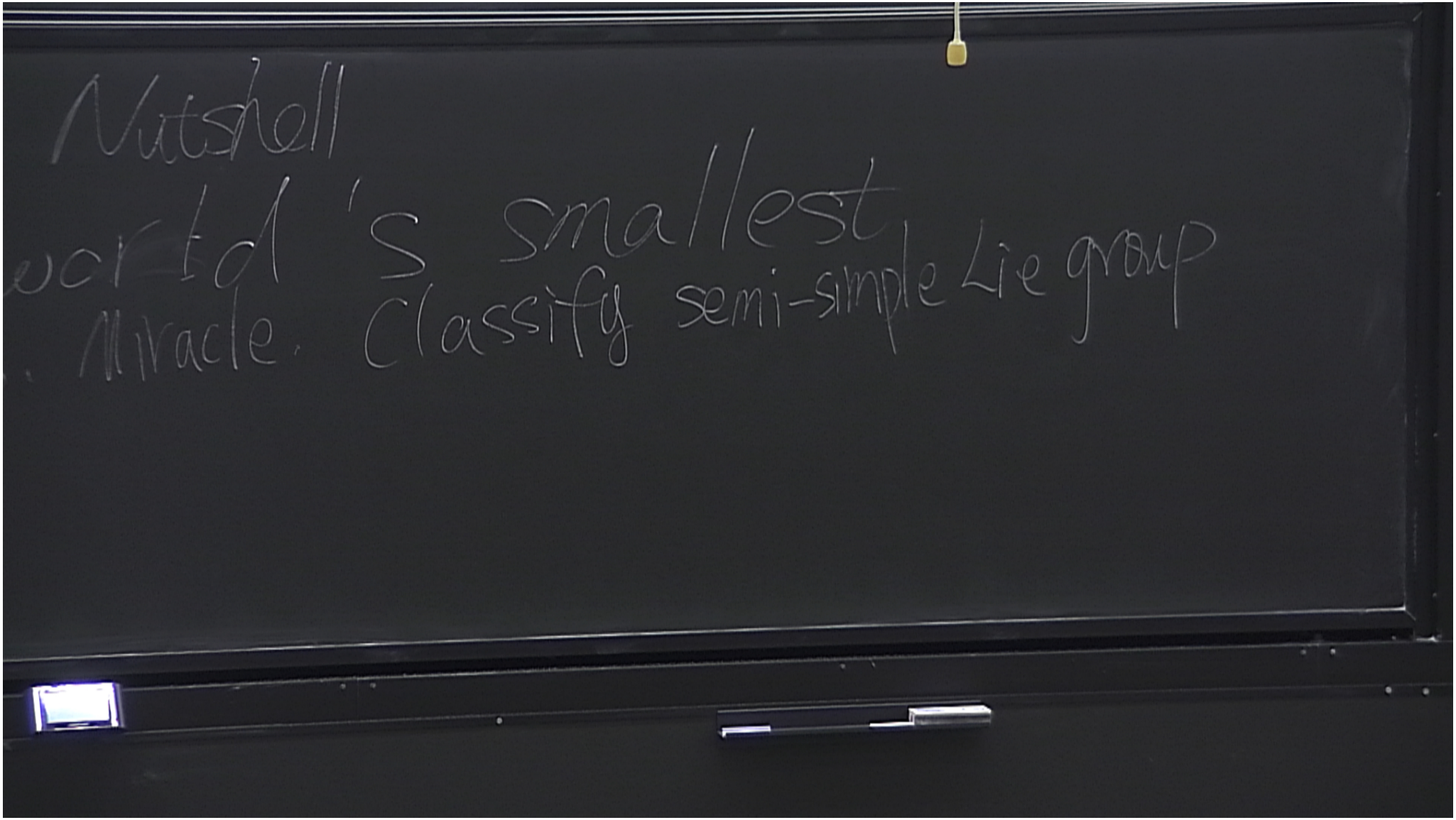
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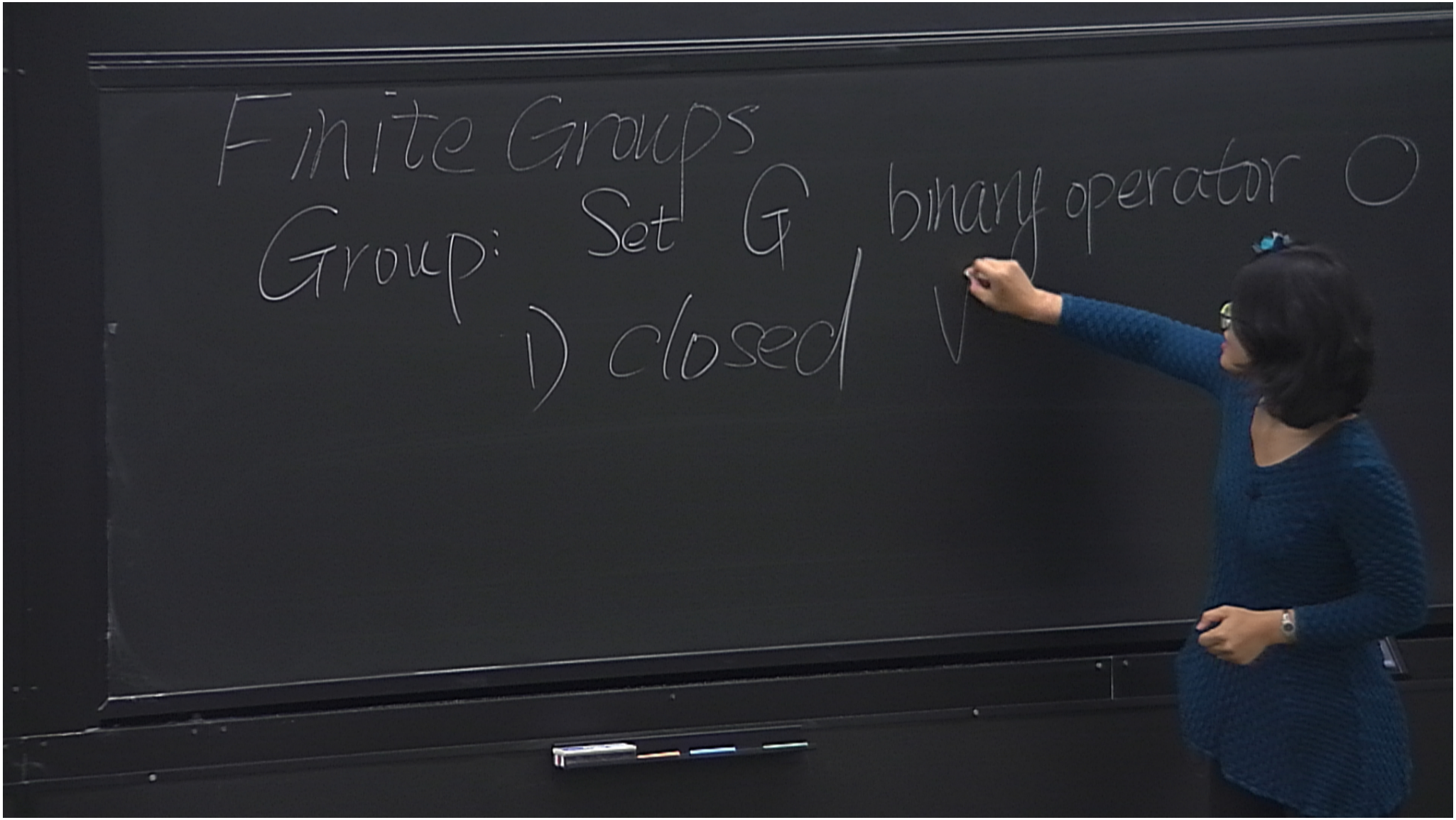
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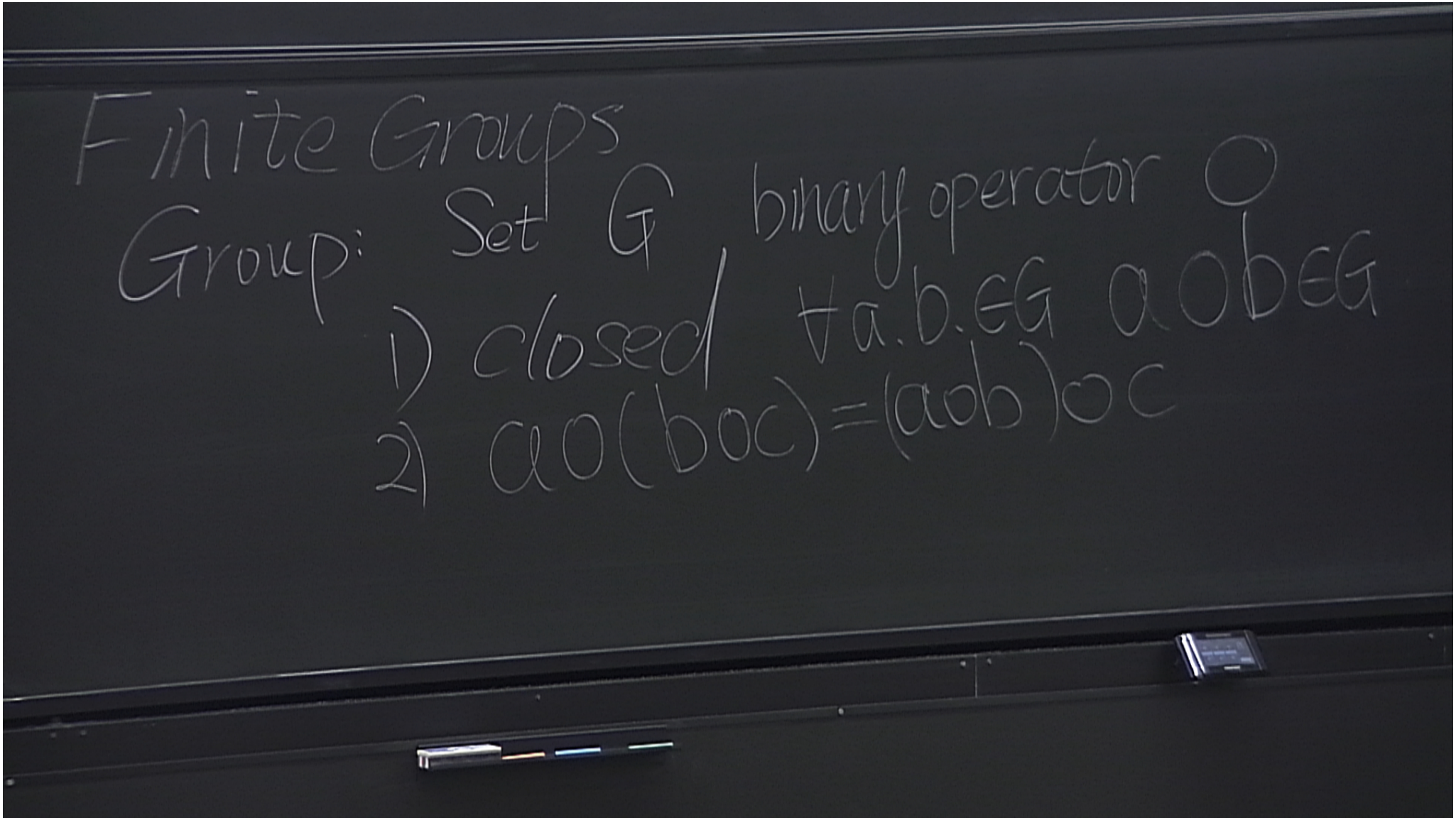

Group Theory


Group Theory in a Nutshell


Group Theory in
the
Outline
1 Visualizing Finite Groups
2 Lie group
3 Structure constants
4 everything about SU(2)

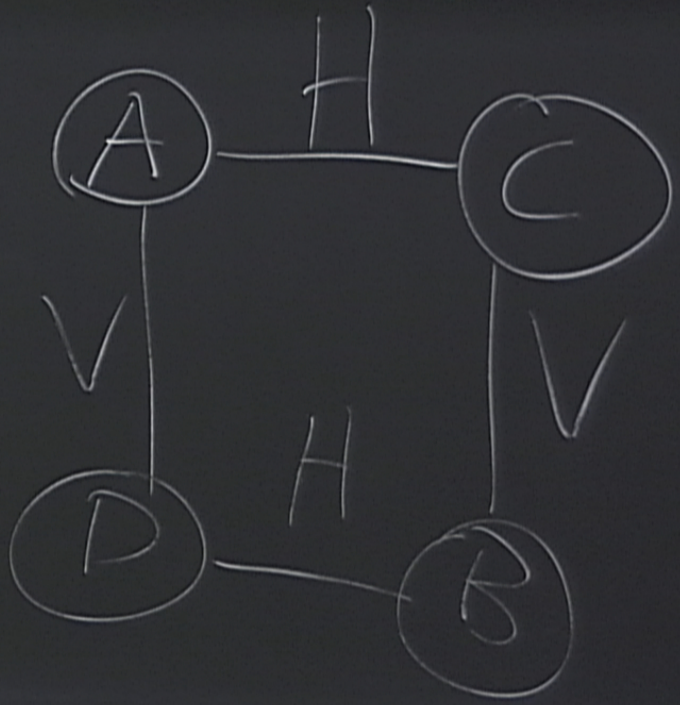

Nutshell
world's smallest
Miracle. Classify semi-simple Lie group

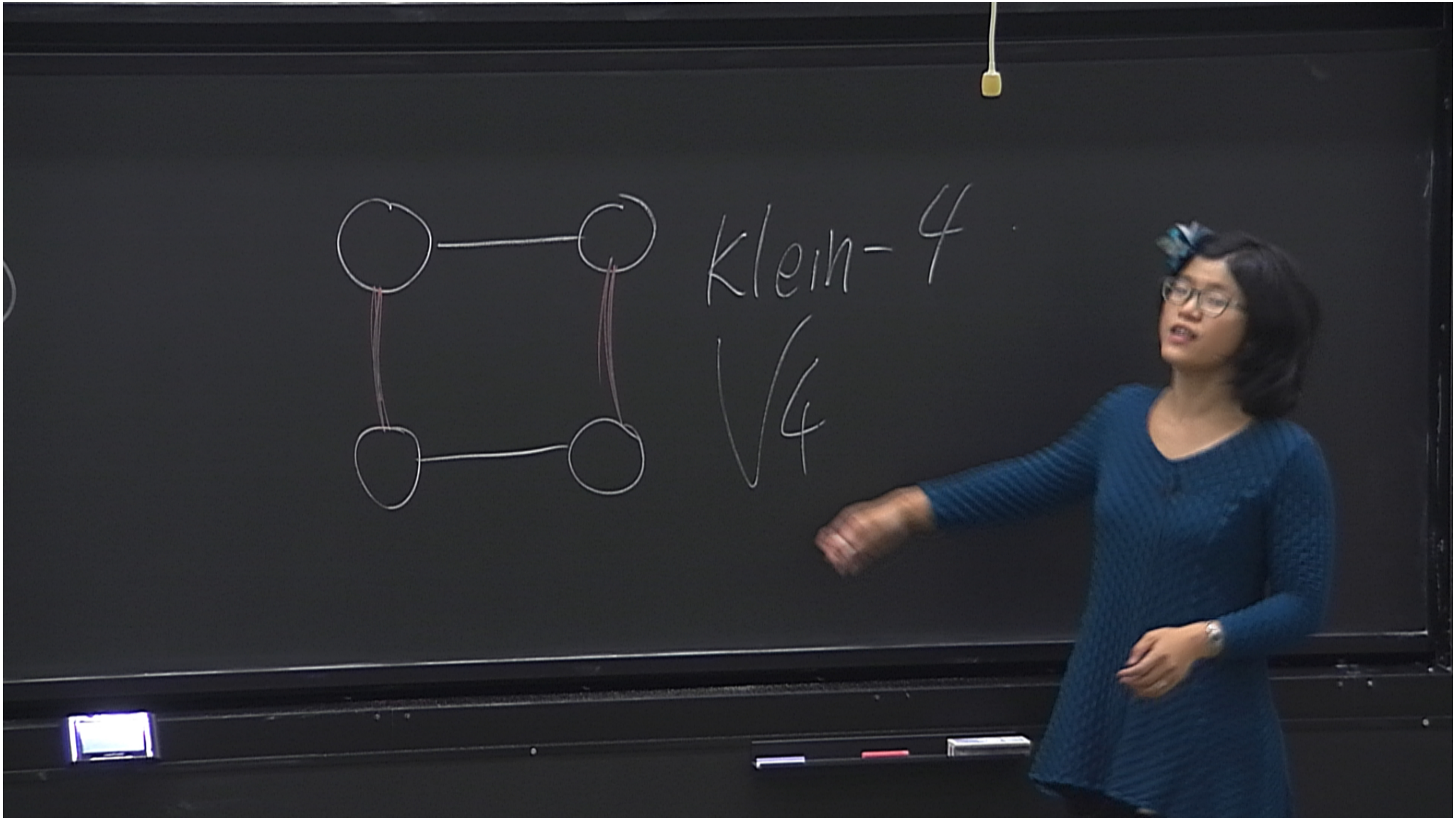

Finite Groups
Group: Set G, binary operator ○
1) closed

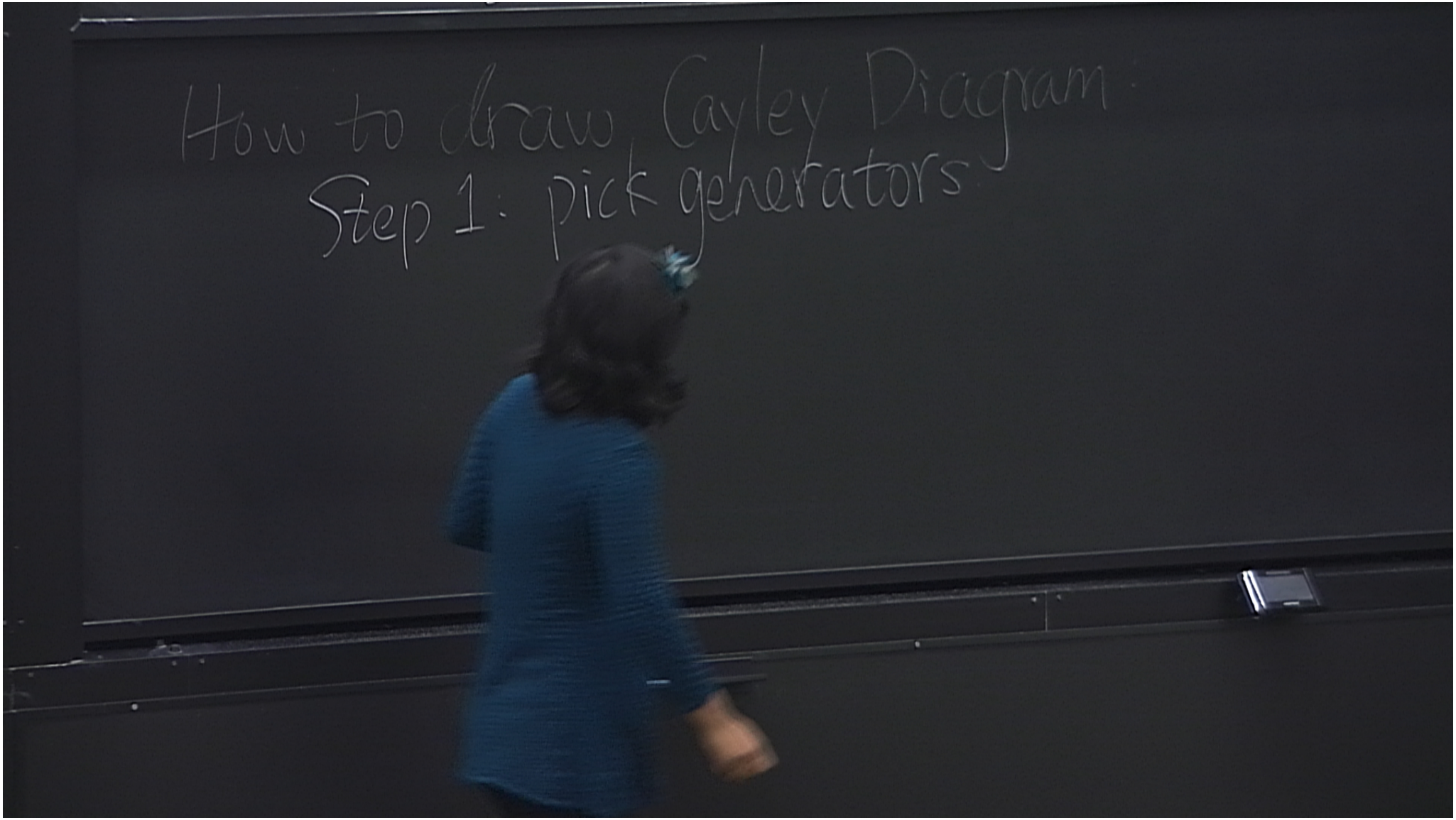

Finite Groups
Group: Set G binary operator ○
1) closed ∀a.b.∈G a○b∈G
2) a○(b○c)=(a○b)○c

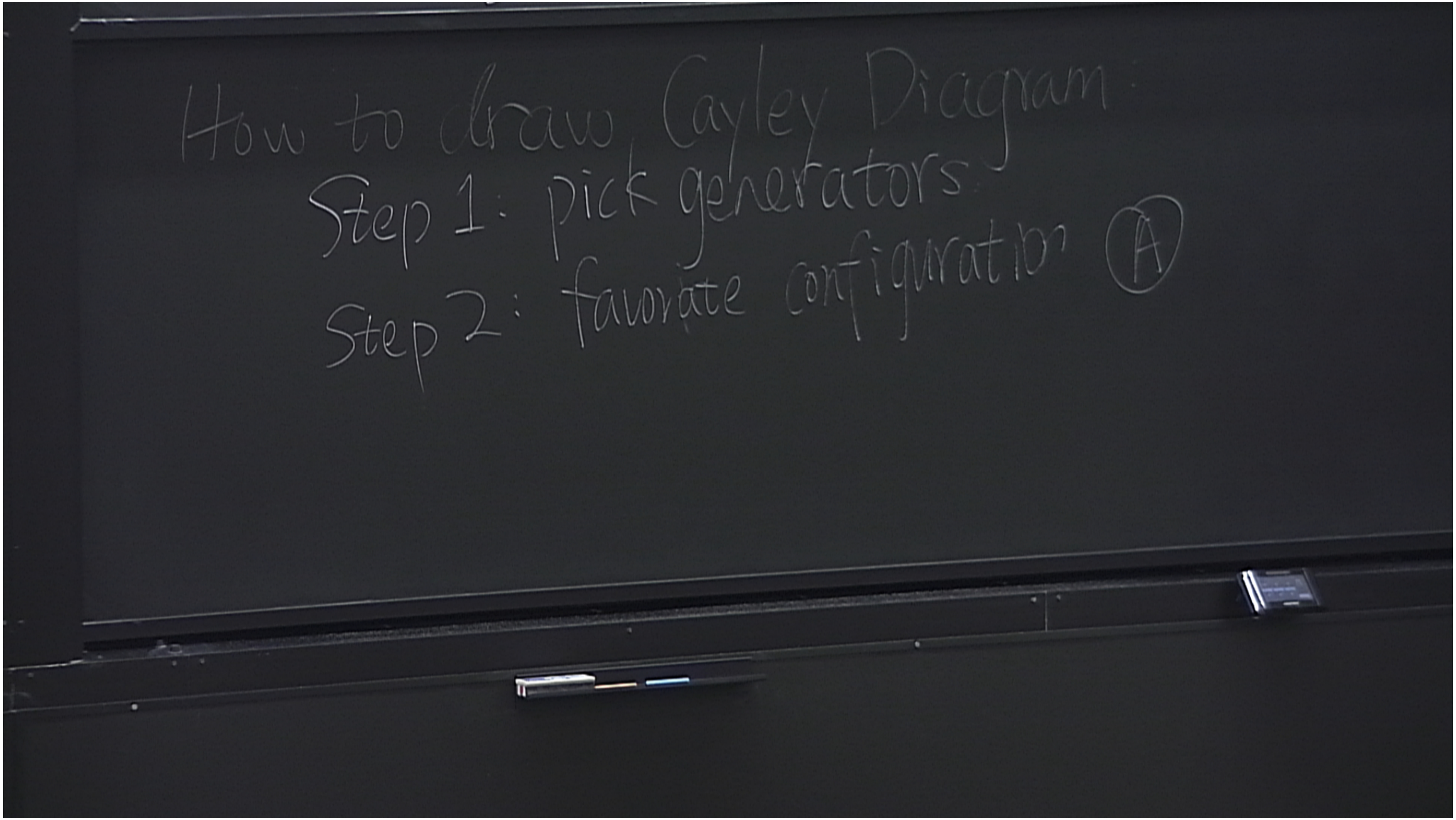
Cayley diagram.

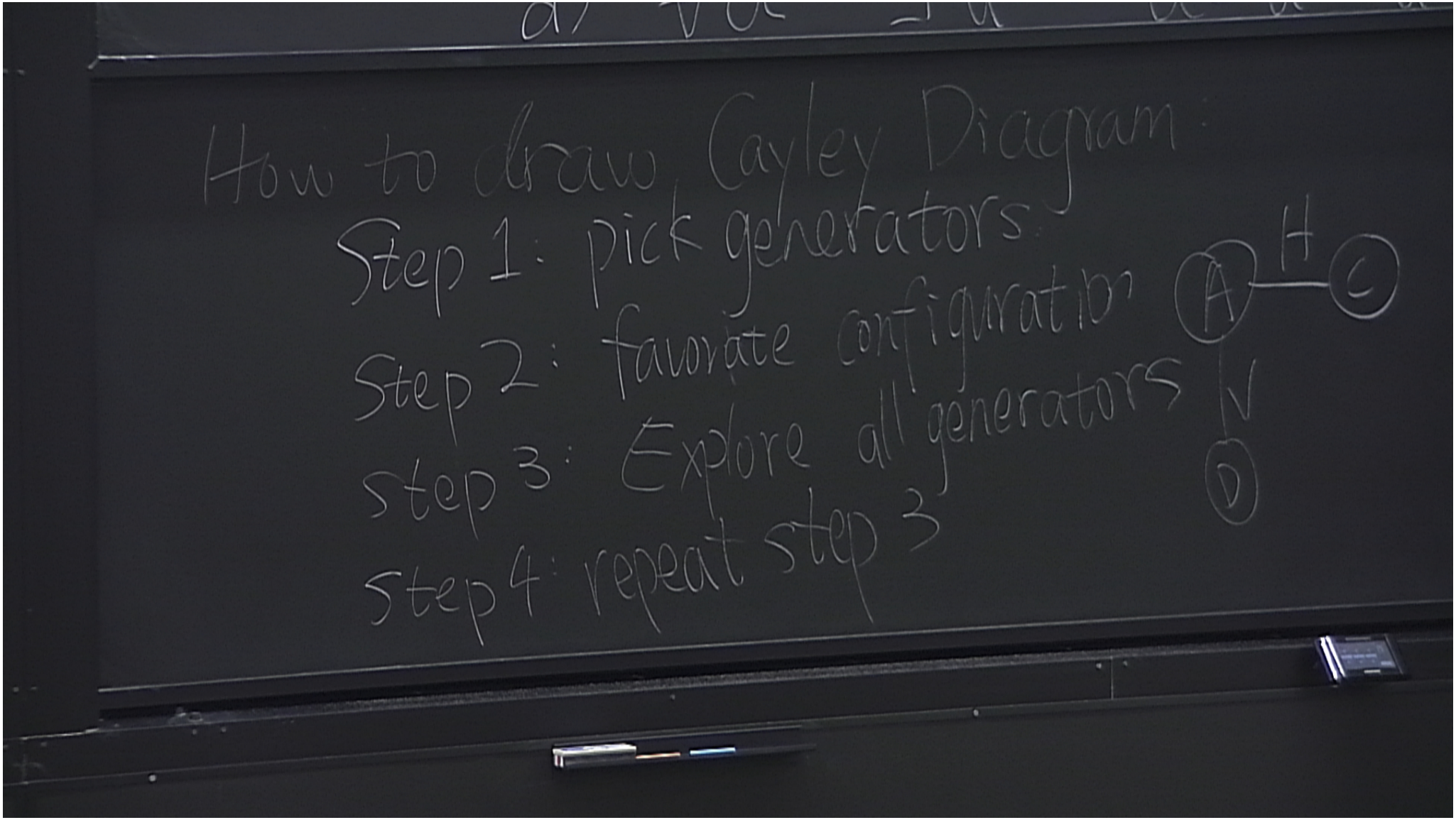
Mattress group




Klein-4
V4


How to draw Cayley Diagram:
Step 1: pick generators


How to draw Cayley Diagram:
Step 1: pick generators
Step 2: favorate configuration
A


How to draw Cayley Diagram:
Step 1: pick generators
Step 2: favorate configuration
step 3: Explore all generators
step 4: repeat step 3
A
H
C
V
D

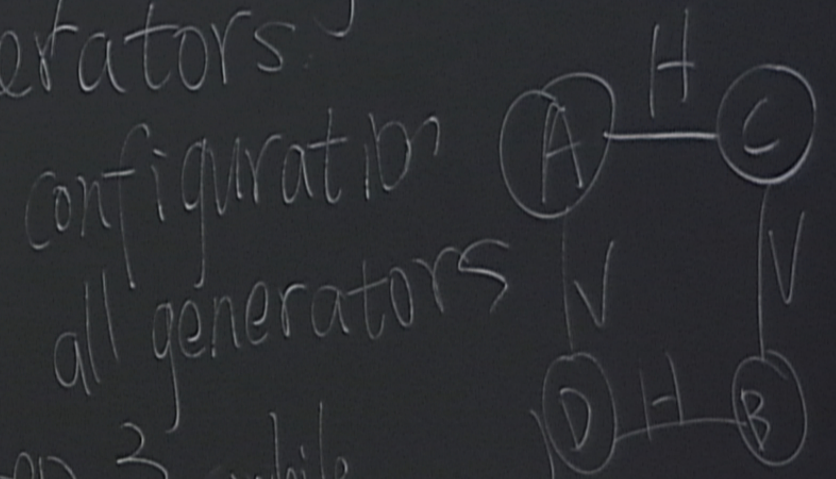
How to draw Cayley Diagram:

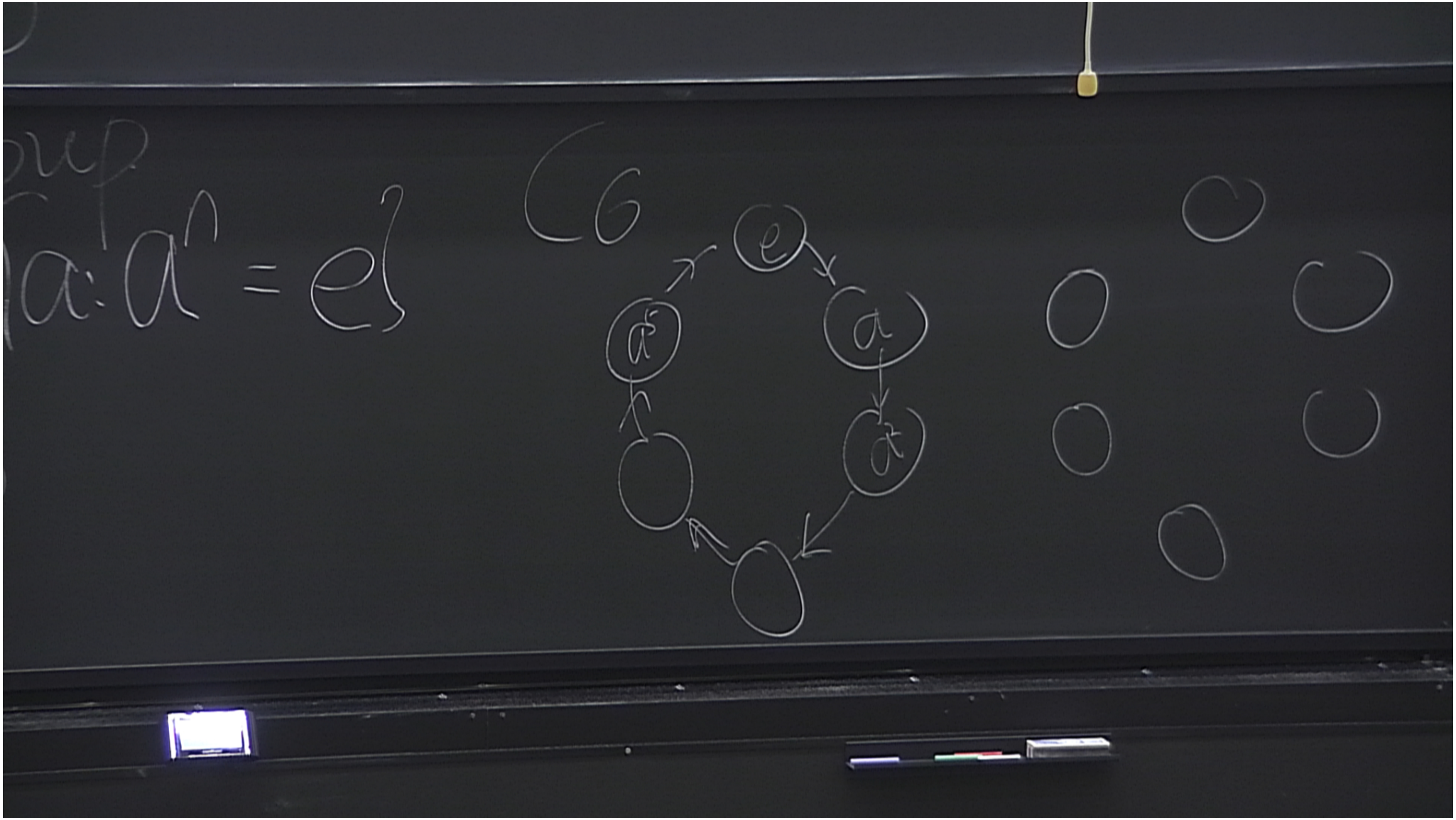
Step 1: pick generators

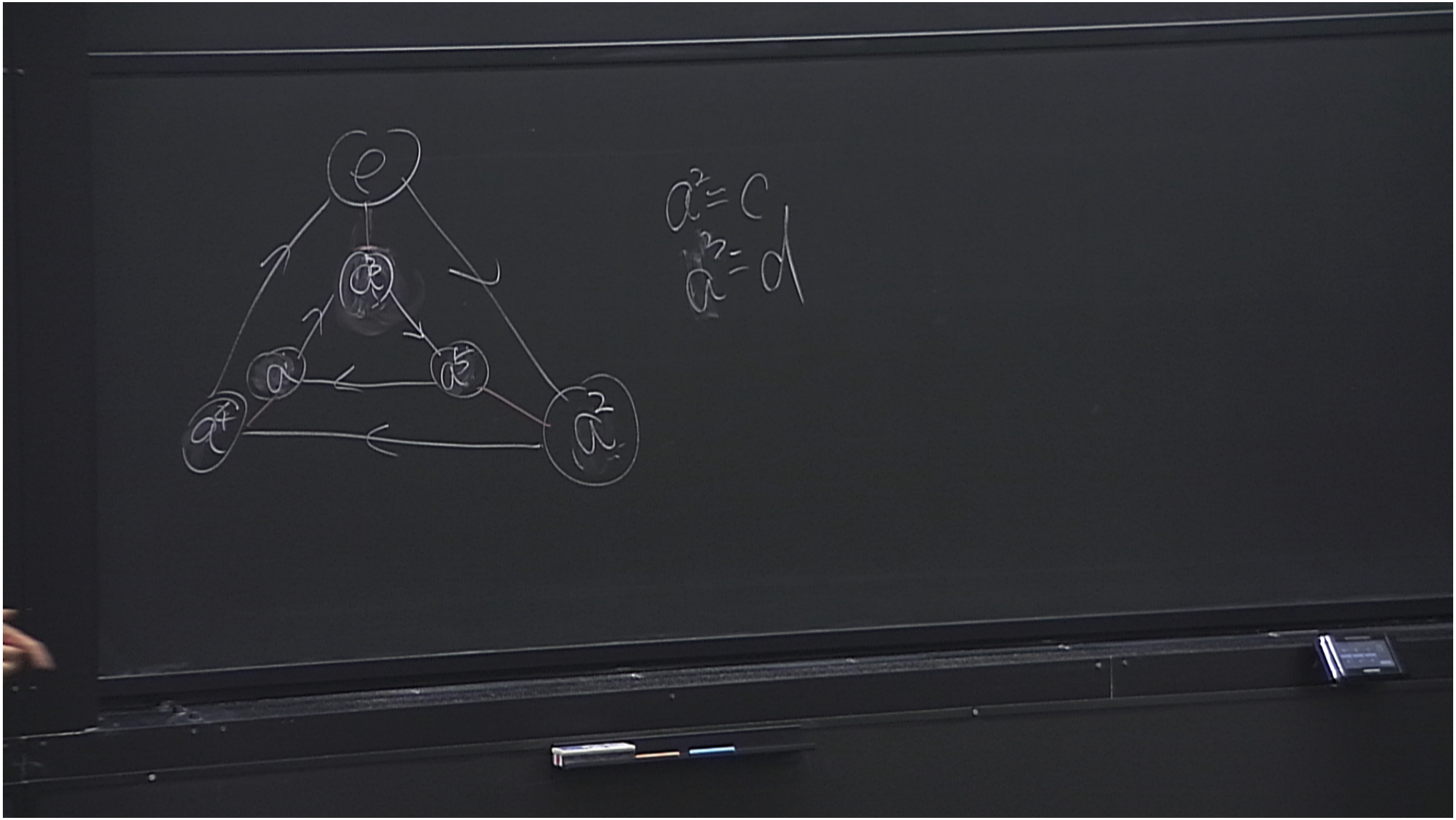
Step 2: favorite configuration

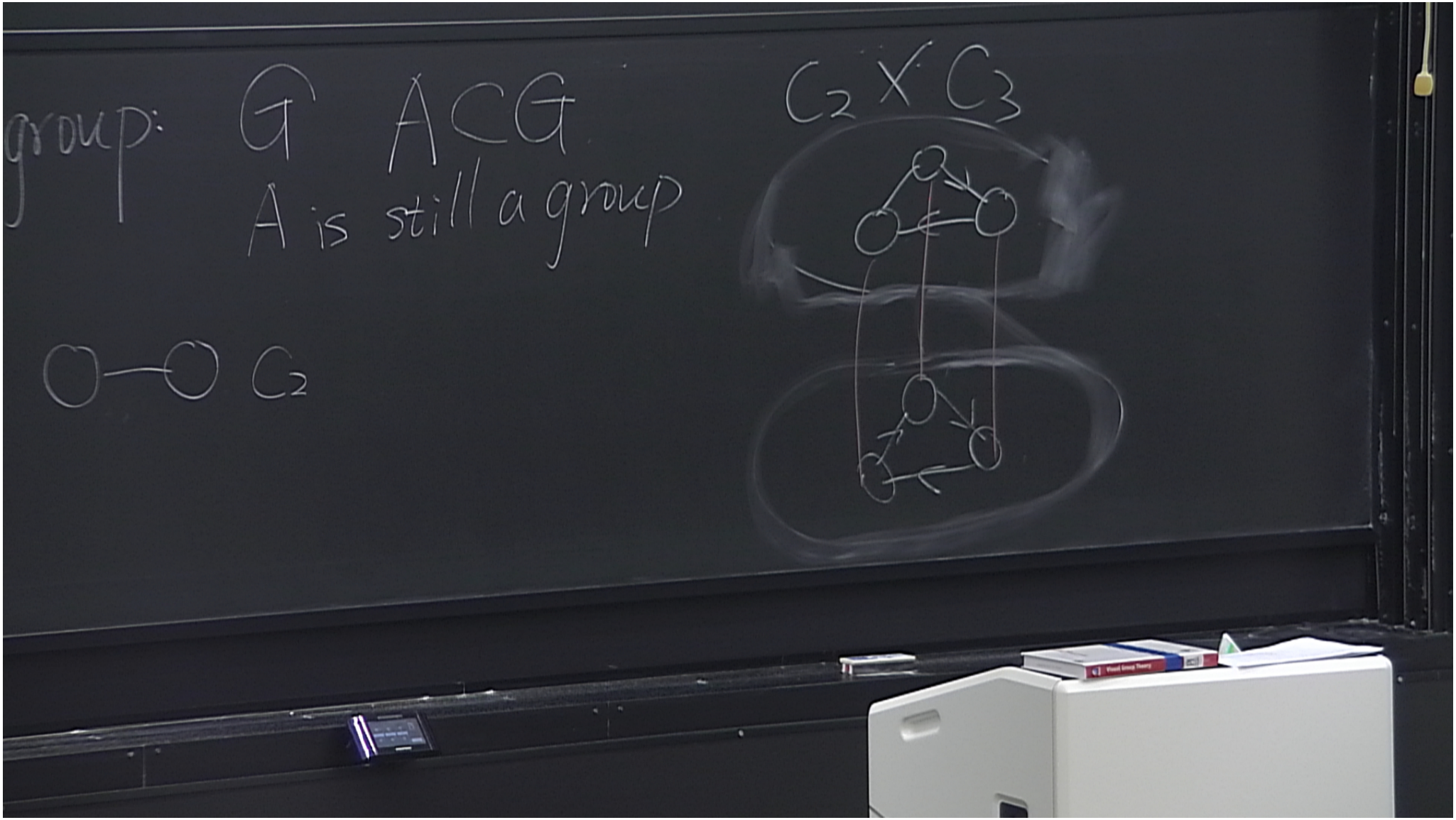
step 3: Explore all generators

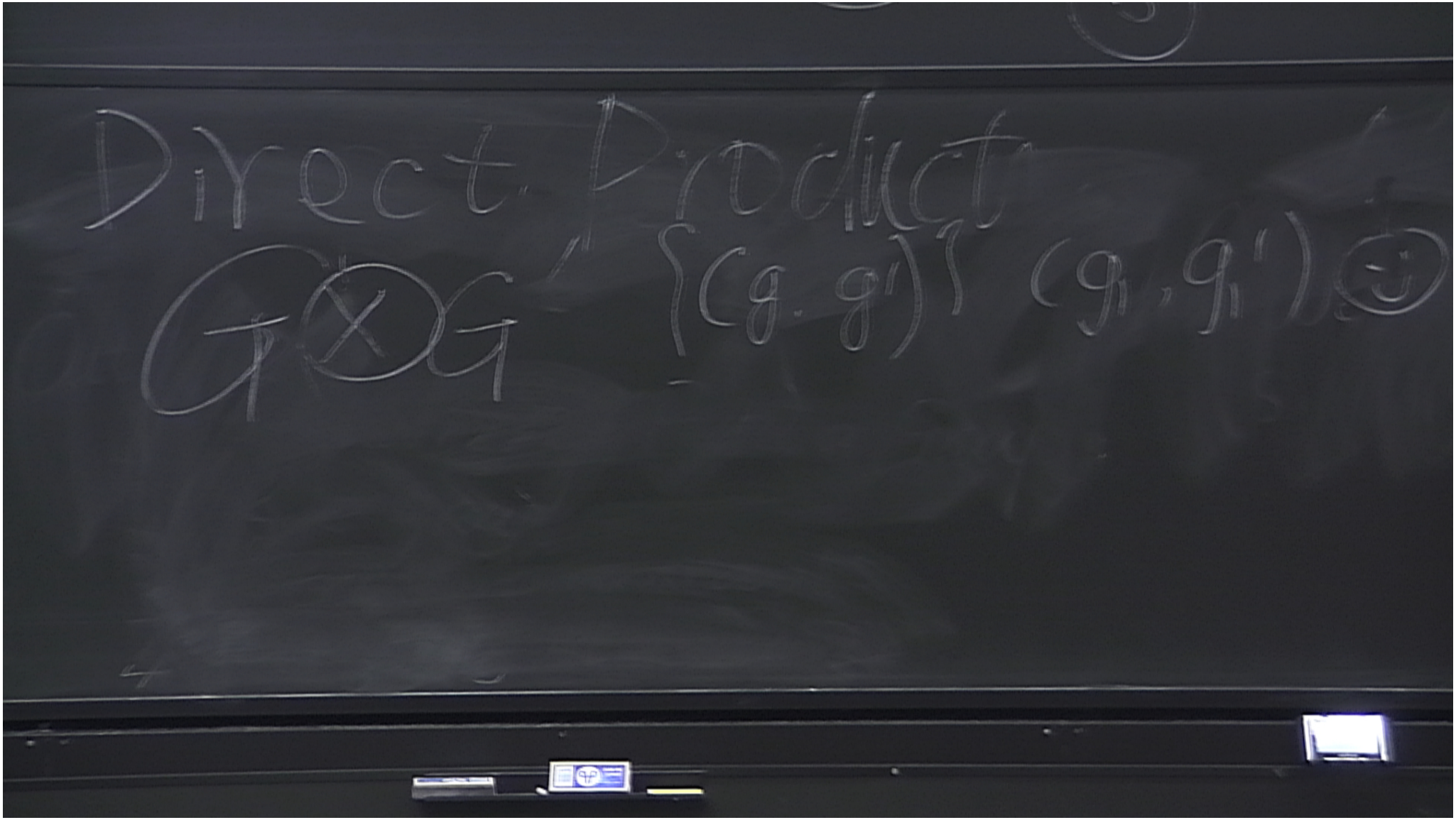
Do Step 4: repeat step 3, while

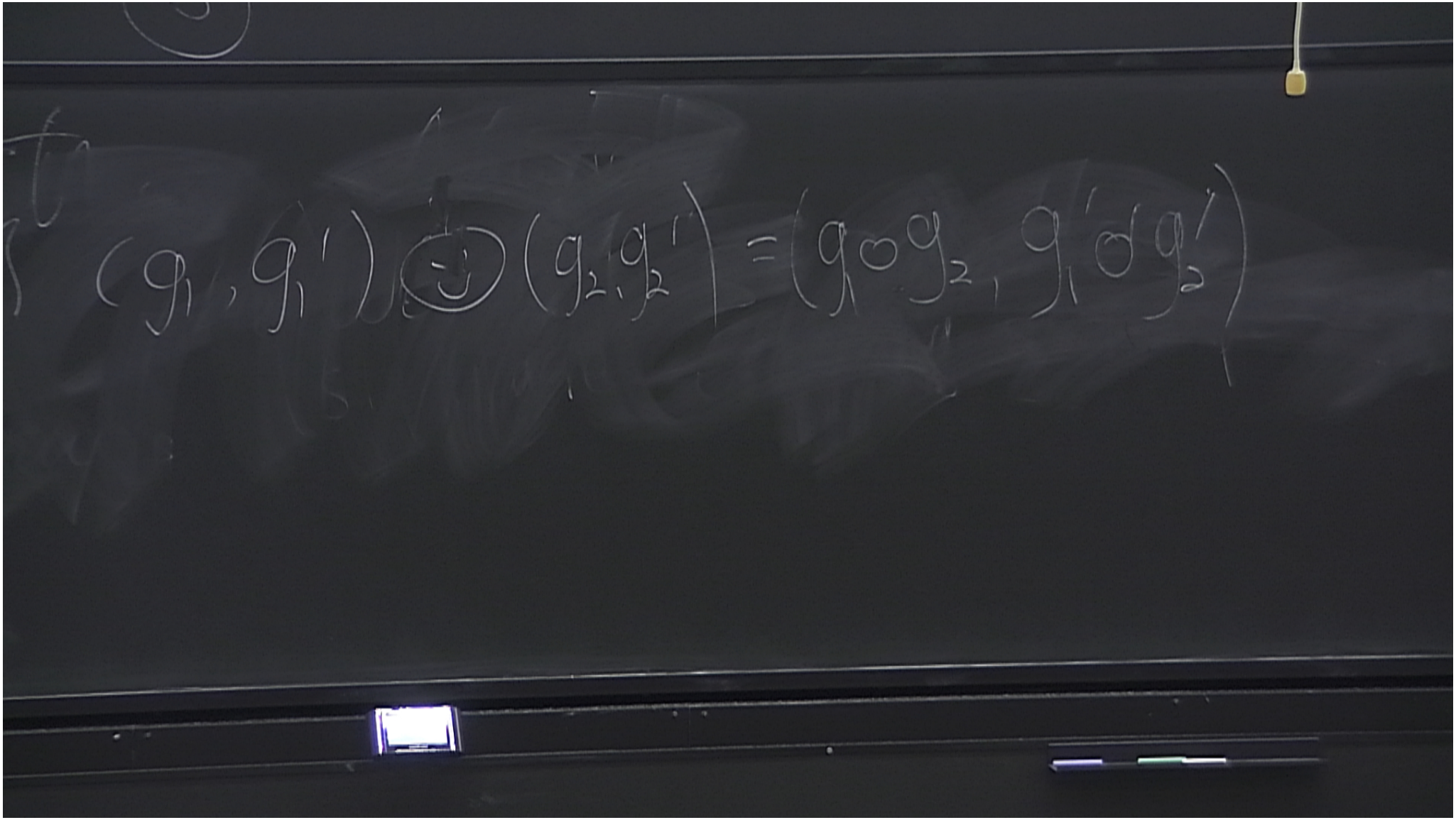


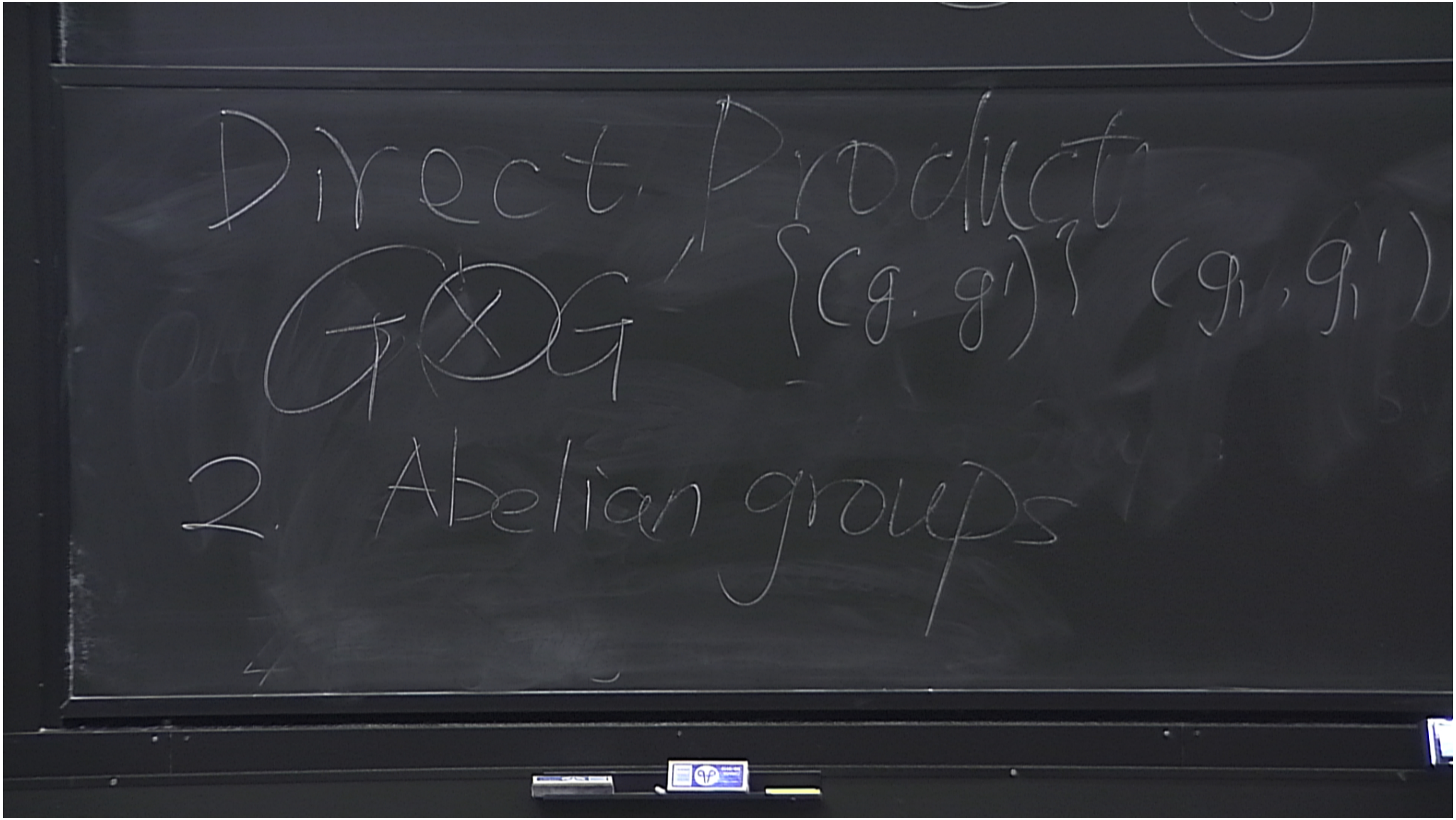

a: a^n = e
G
e
a
a^2
a^3

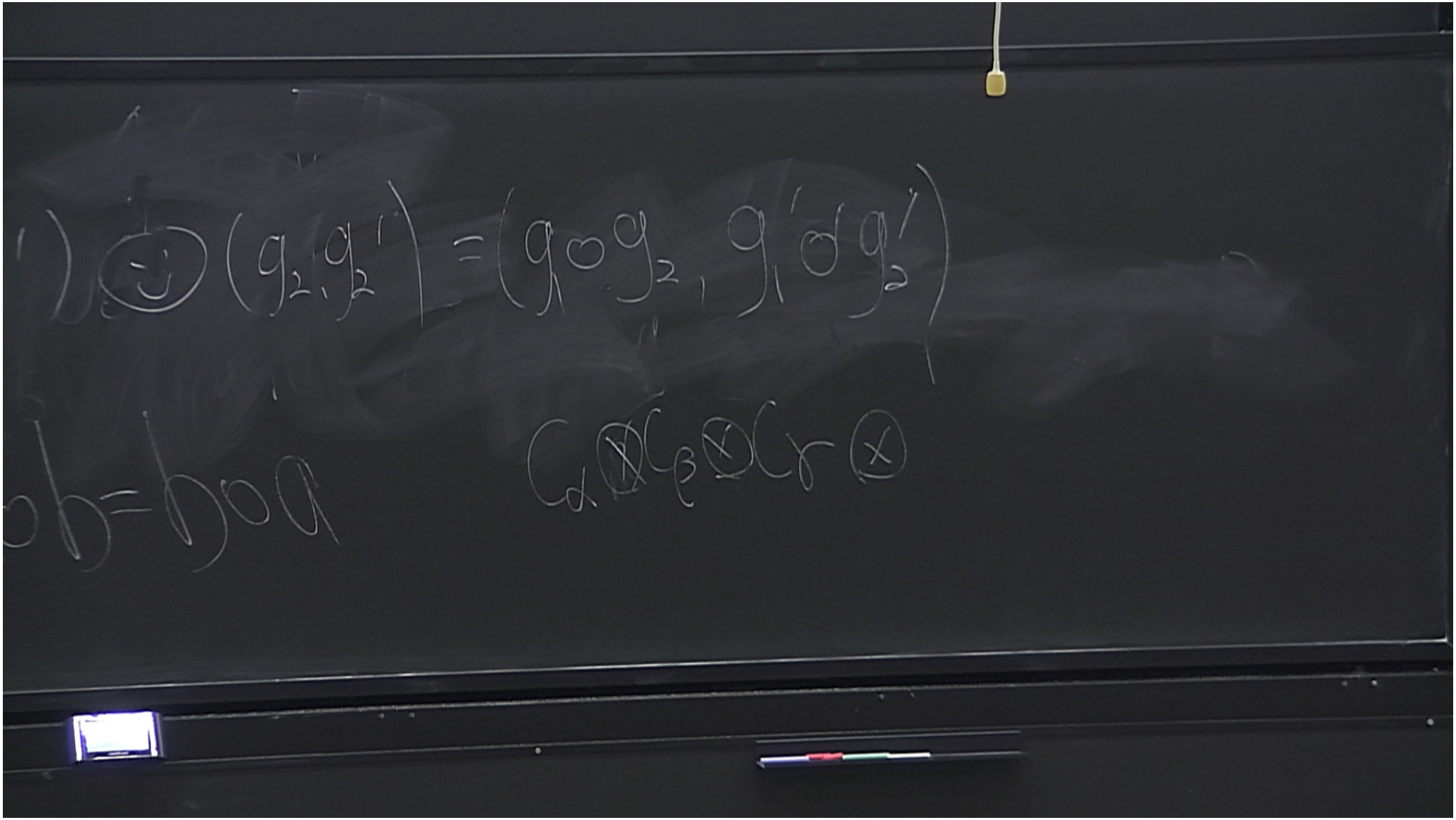

e
a
a²
a³
a⁴
a⁵
a² = c
a³ = d

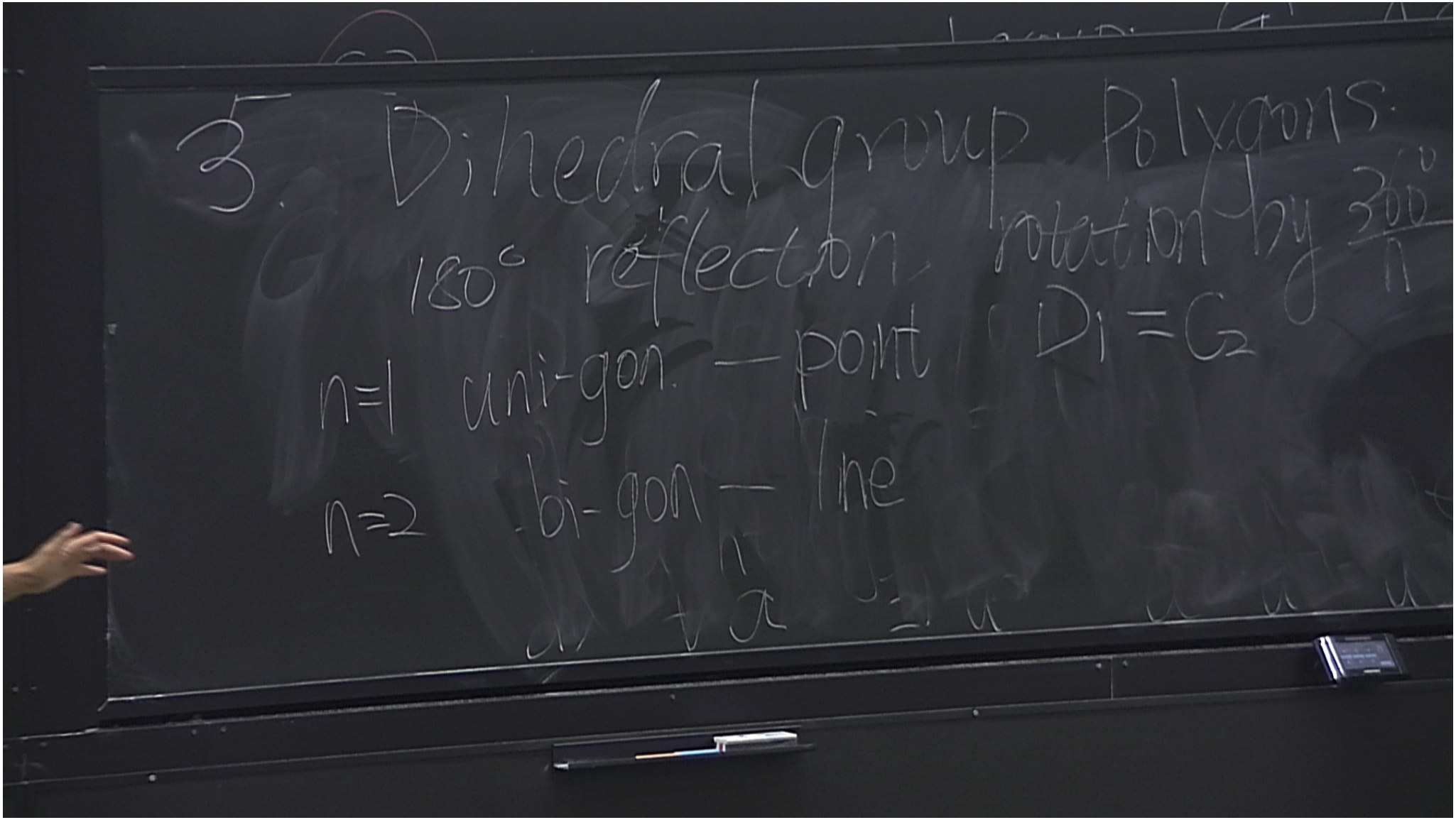

group: G ACG
A is still a group
C2
C2 X C3

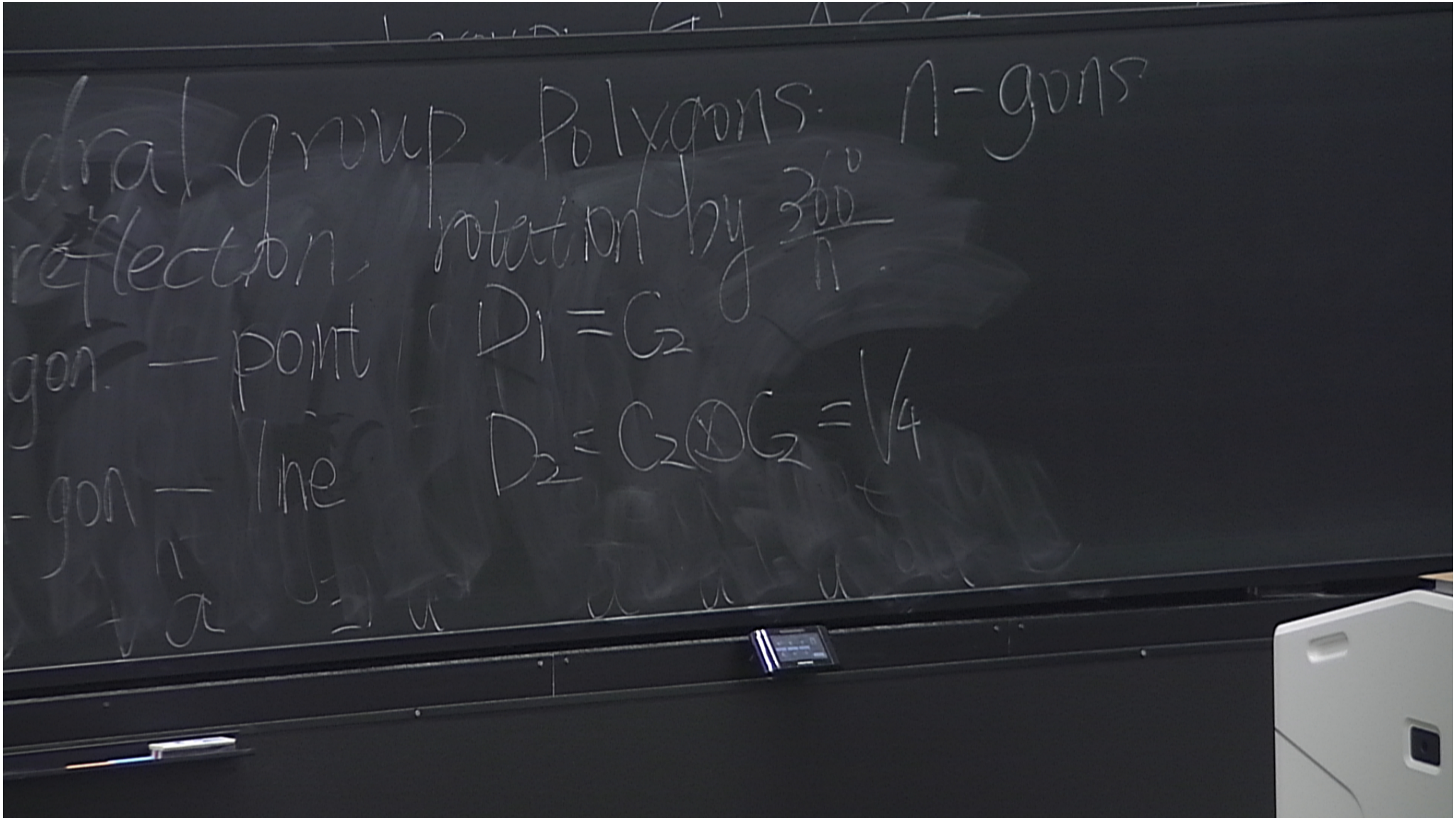

Direct Product
G × G'
(g, g')(g, g')

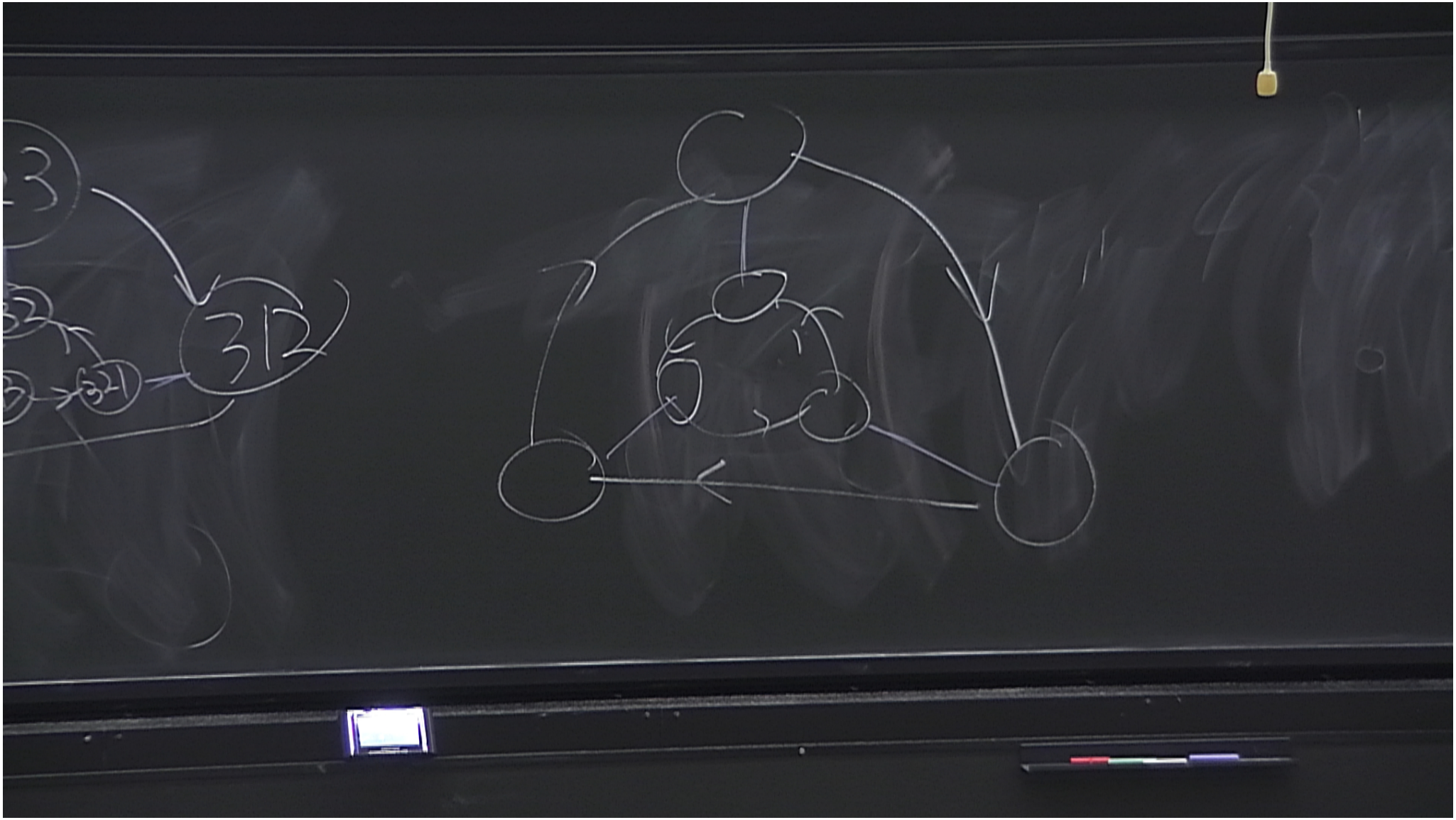

$(g_1, g_1') \oplus (g_2, g_2') = (g_1 \circ g_2, g_1' \circ g_2')$

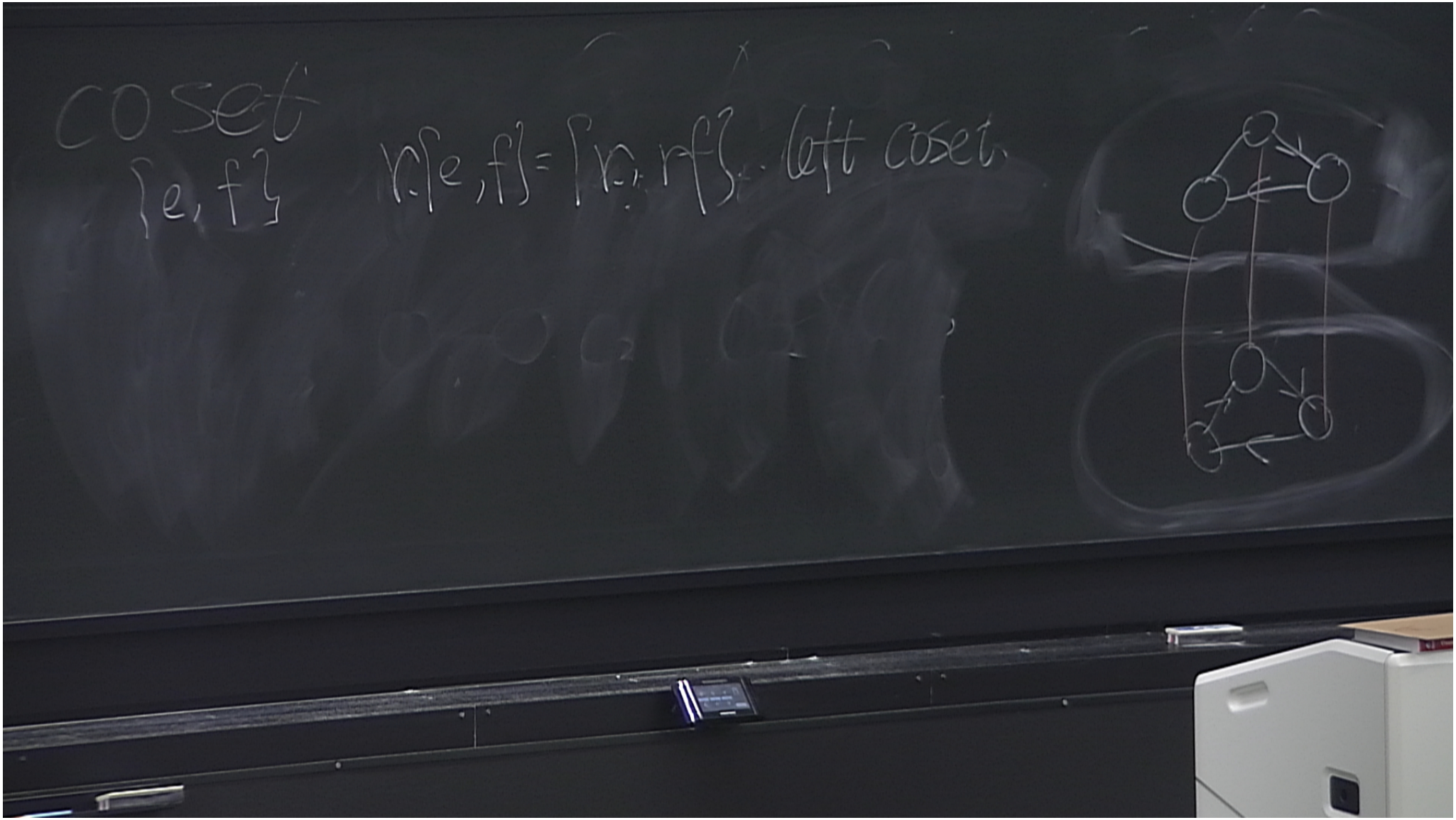

Direct Product
G ⊕ G
(g, g')
(g, g')
2. Abelian groups

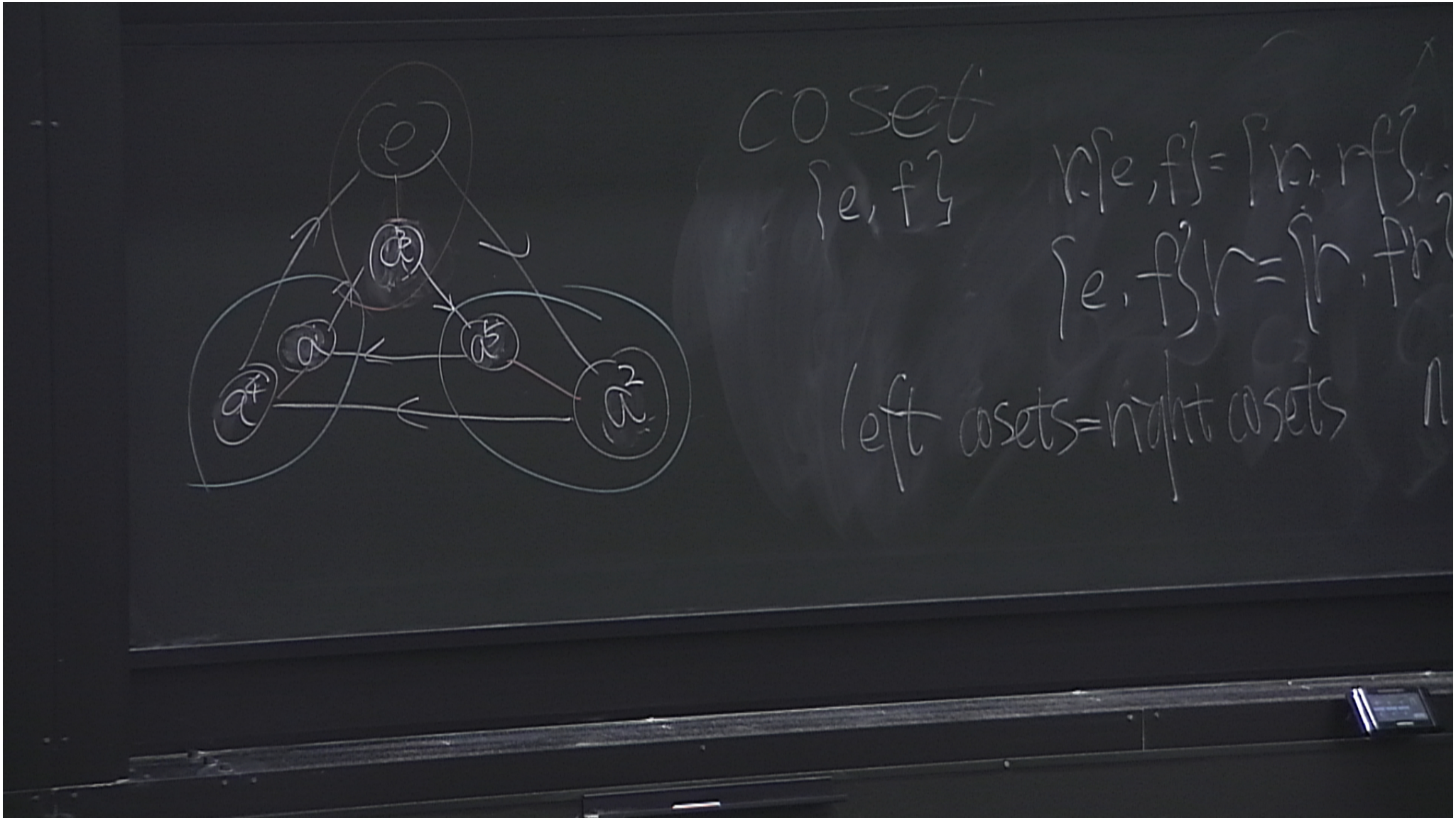

$(g_1, g_1') \circ (g_2, g_2') = (g_1 \circ g_2, g_1' \circ g_2')$
$C_\alpha \otimes C_\beta \otimes C_\gamma \otimes$

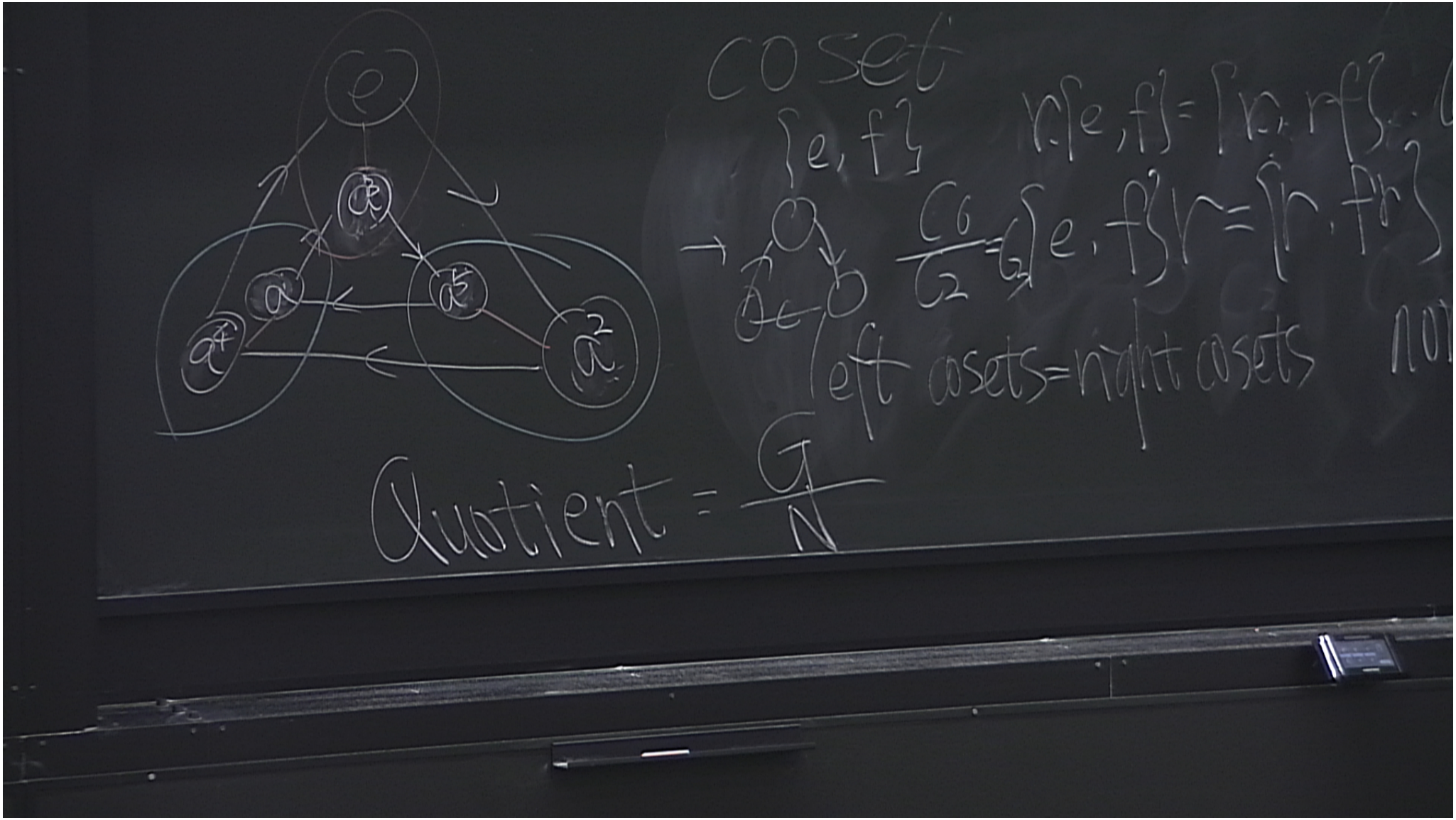

3. Dihedral group Polygons:
180° reflection rotation by 360°/n
n=1 uni-gon — point D1 = G2
n=2 bi-gon — line

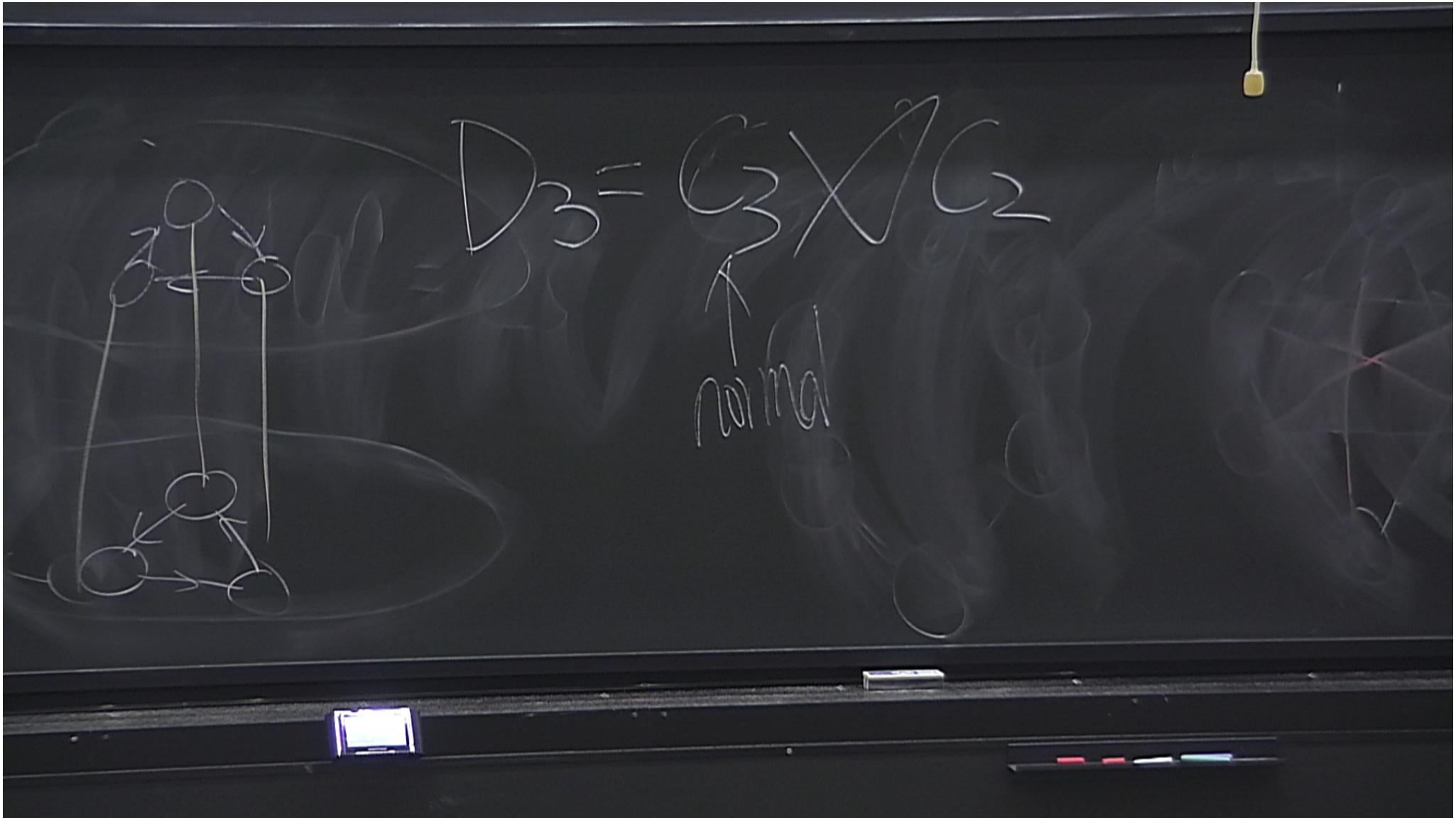

dral group Polygons: n-gons
reflection rotation by 360°/n
gon — point D1 = G2
-gon — line D2 = G2 ⊗ G2 = V4

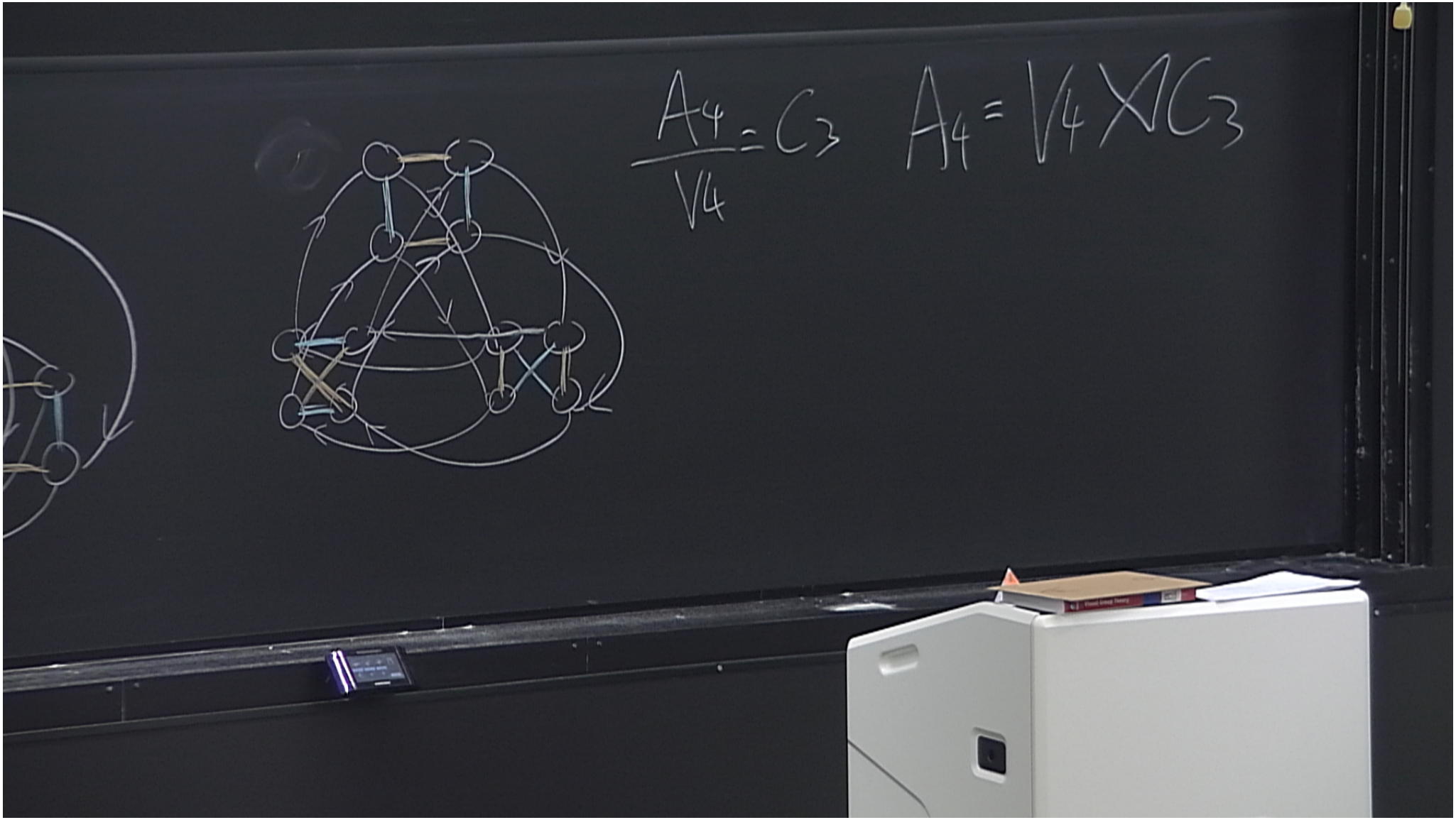

312
321


coset
{e, f}
n{e, f} = {n, nf}. left coset.


e
a
a⁴
a³
a²
a⁵
coset
{e, f}
left cosets = right cosets


coset
{e, f}
Quotient = G/N
left cosets = right cosets


$D_3 = C_3 \rtimes C_2$
normal


$\frac{A_4}{V_4} = C_3$
$A_4 = V_4 \times C_3$