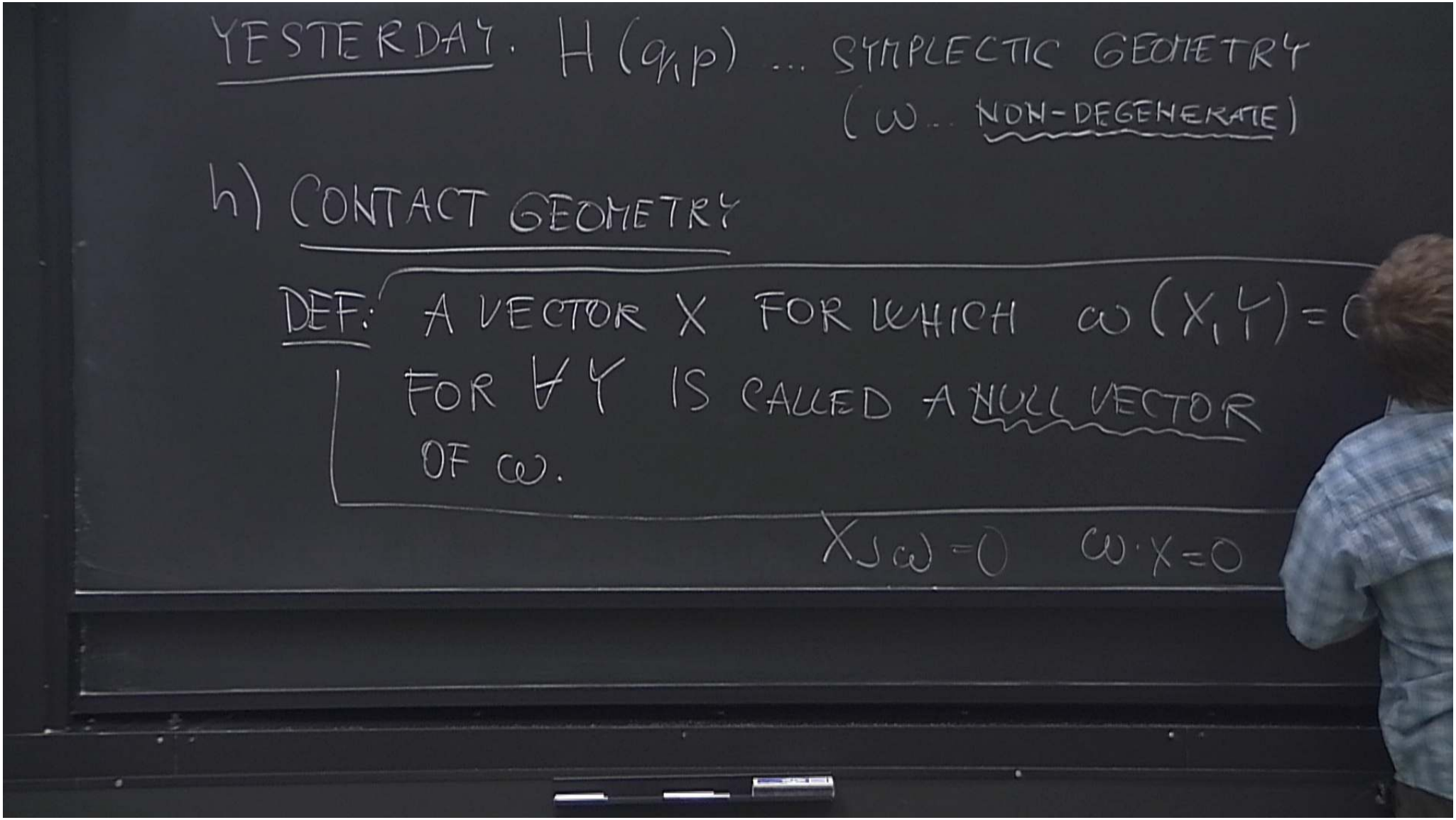
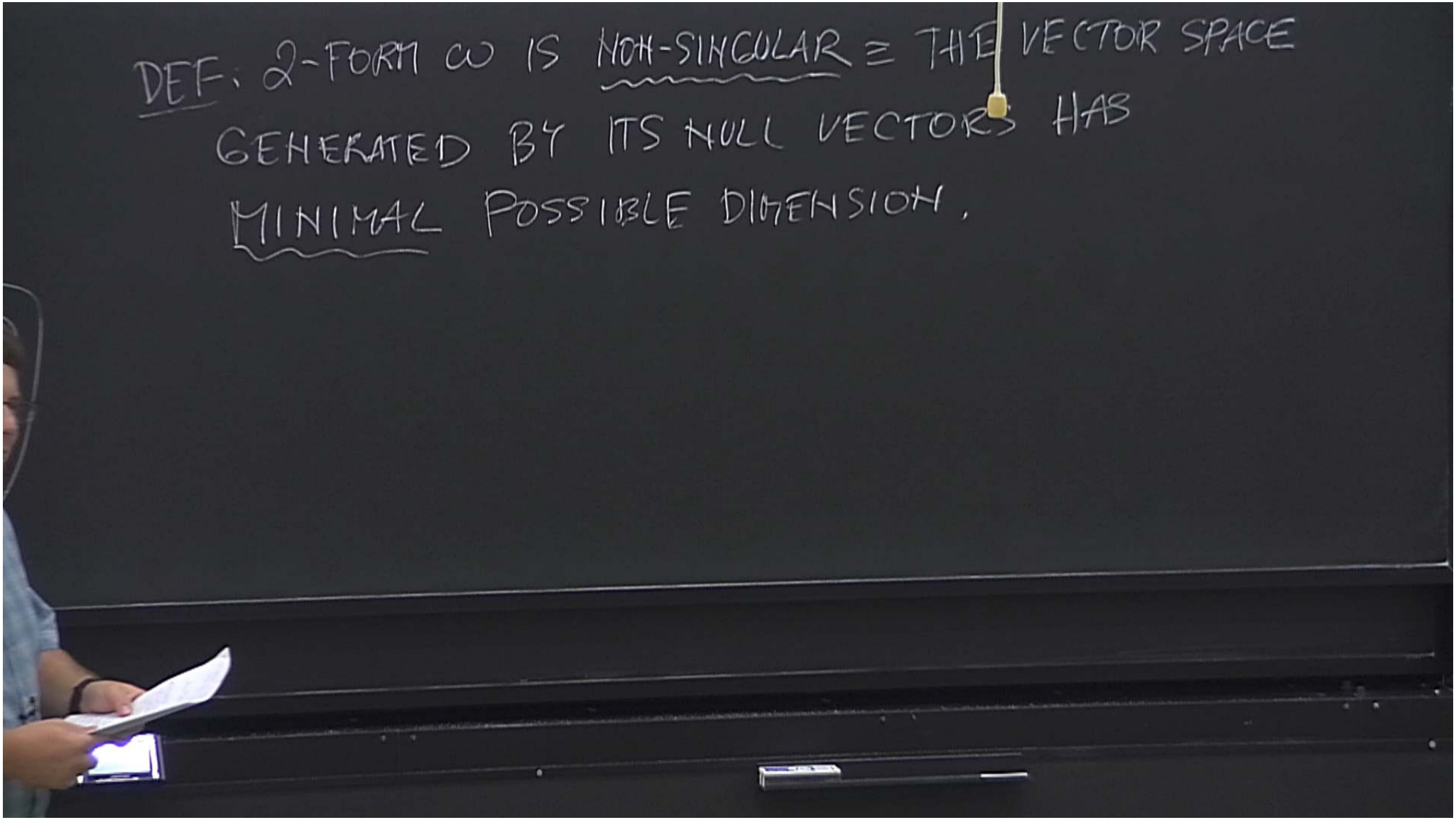
Title: Classical Mechanics - David Kubiznak

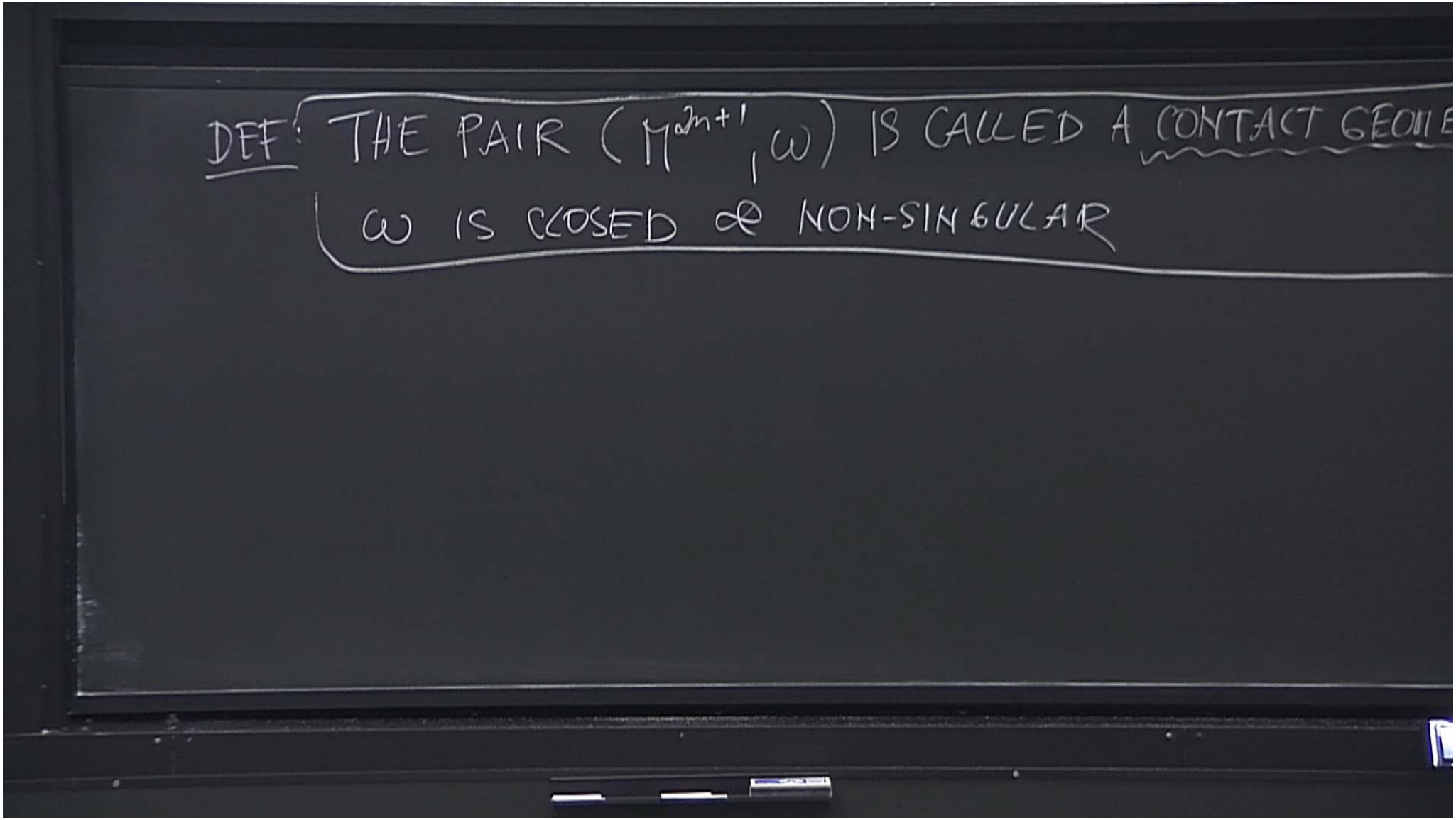
Date: Aug 27, 2015 09:00 AM

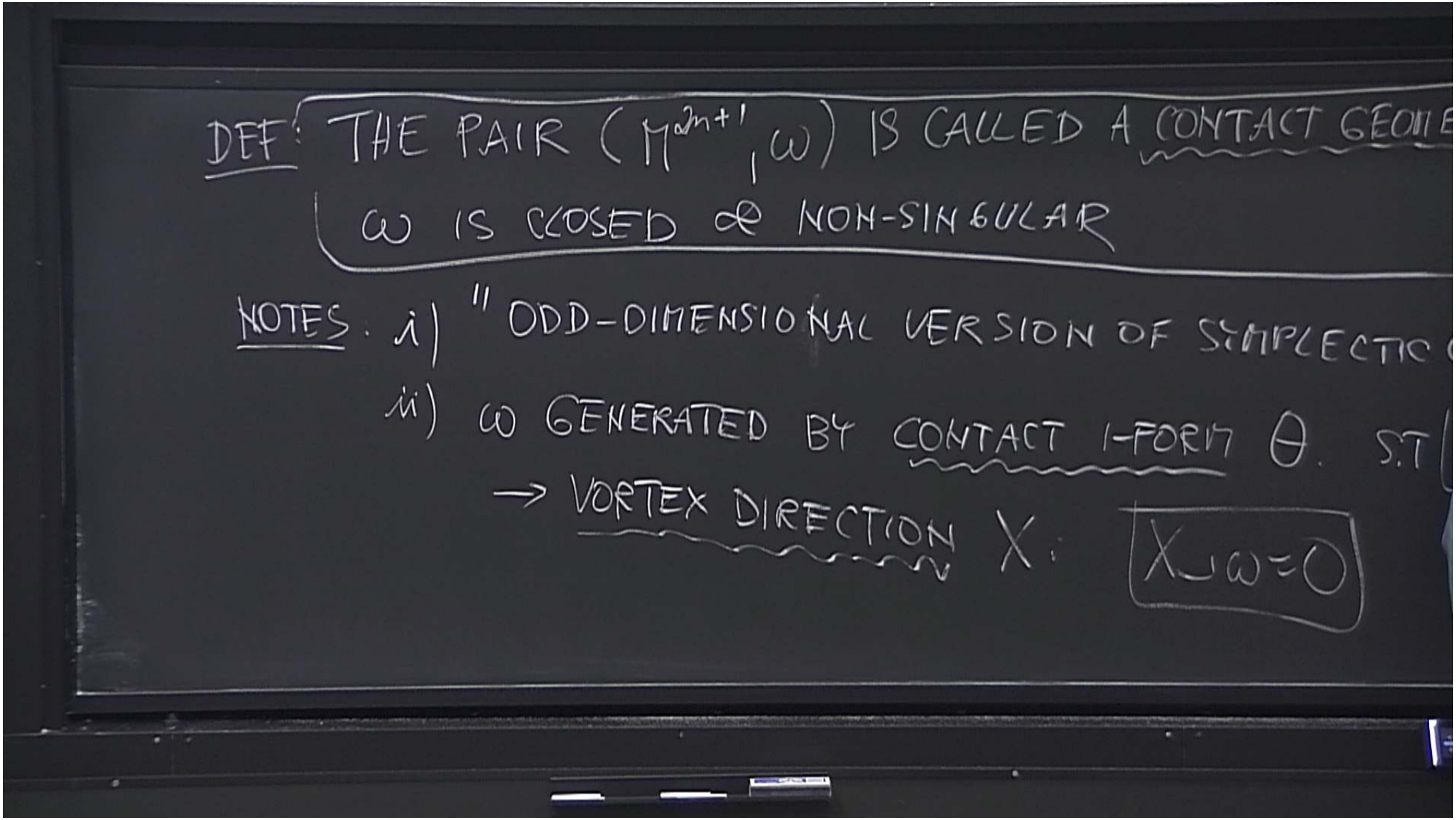
URL: http://pirsa.org/15080103

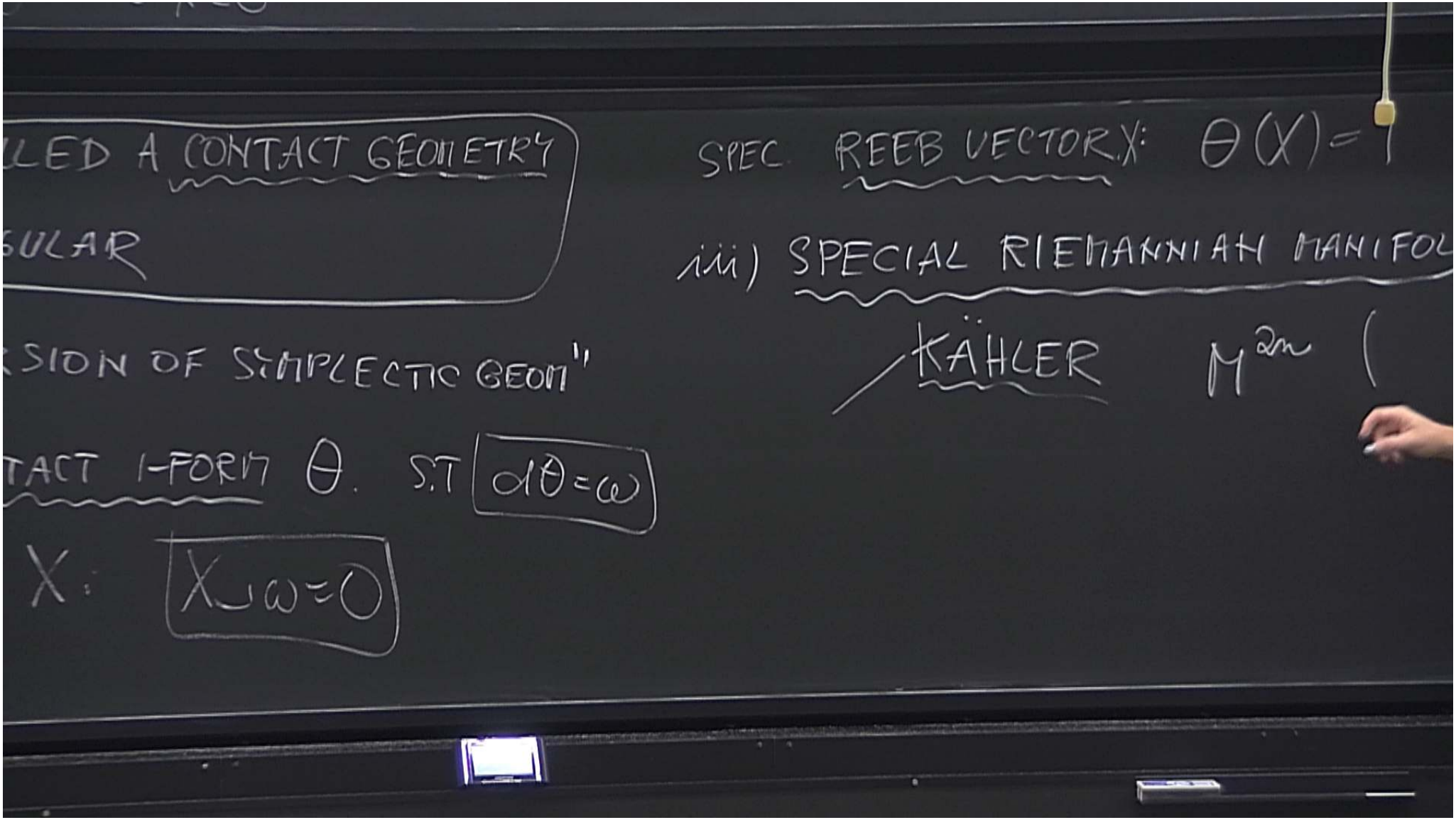
Abstract:

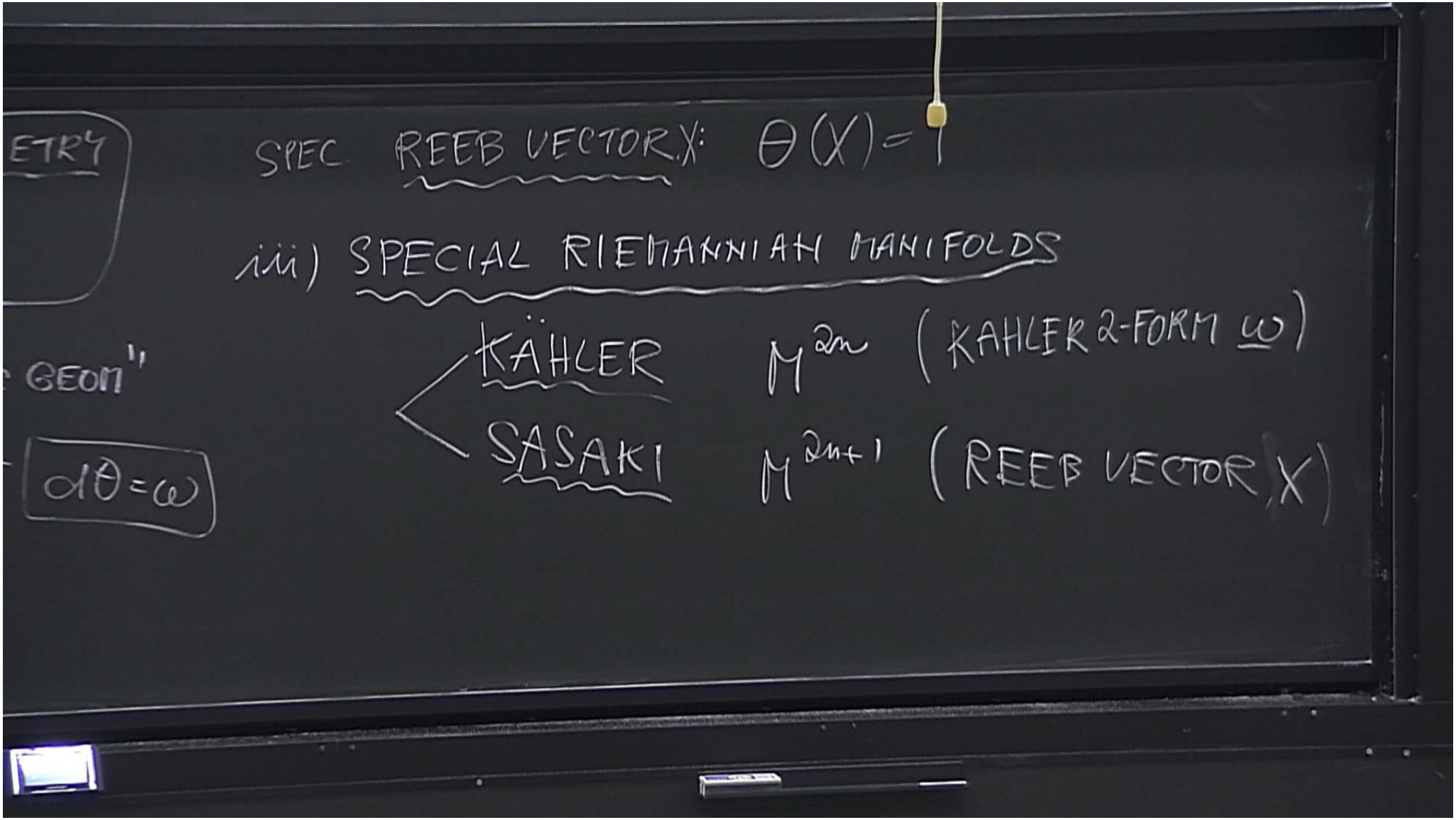

YESTERDAY. $H(q,p)$ ... SYMPLECTIC GEOMETRY
(ω ... NON-DEGENERATE)
h) CONTACT GEOMETRY
DEF: A VECTOR X FOR WHICH $\omega(X,Y)=0$
FOR $\forall Y$ IS CALLED A NULL VECTOR
OF ω.
$X \lrcorner \omega = 0$ $\omega \cdot X = 0$

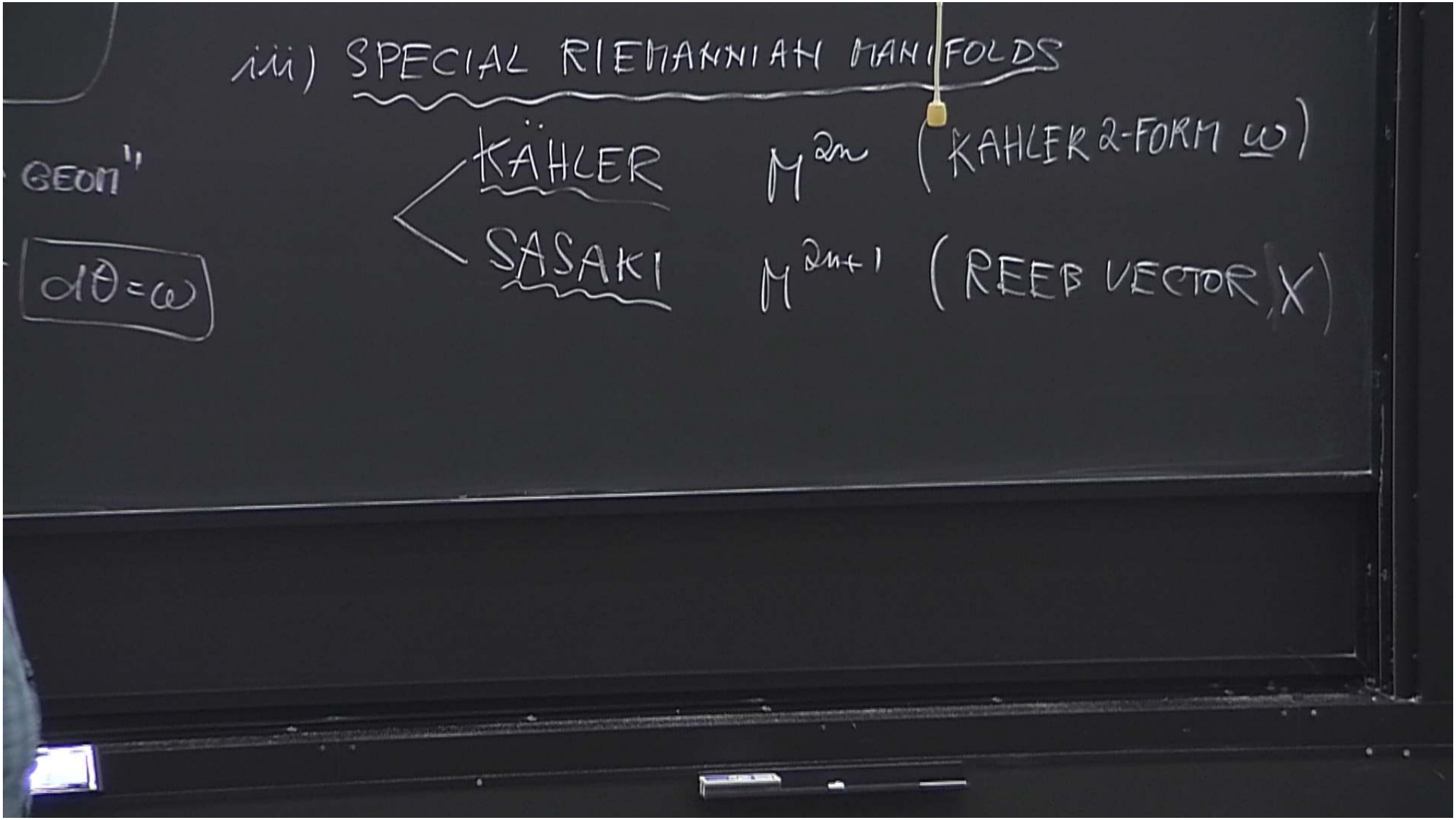

DEF: 2-FORM ω IS NON-SINGULAR ≡ THE VECTOR SPACE
GENERATED BY ITS NULL VECTORS HAS
MINIMAL POSSIBLE DIMENSION.


DEF: THE PAIR (M^{2n+1}, ω) IS CALLED A CONTACT GEOM
ω IS CLOSED & NON-SINGULAR


DEF: THE PAIR (M^{2n+1}, ω) IS CALLED A CONTACT GEOM
ω IS CLOSED & NON-SINGULAR
NOTES: i) " ODD-DIMENSIONAL VERSION OF SYMPLECTIC
ii) ω GENERATED BY CONTACT 1-FORM θ. S.T
→ VORTEX DIRECTION X: $X \lrcorner \omega = 0$


LED A CONTACT GEOMETRY
SULAR
SION OF SYMPLECTIC GEOM"
TACT 1-FORM θ. S.T dθ=ω
X: X⌟ω=0
SPEC. REEB VECTOR X: θ(X)=1
iii) SPECIAL RIEMANNIAN MANIFOL
KÄHLER M^2n (


ETRY
GEOM"
$d\theta = \omega$
SPEC. REEB VECTOR X: $\theta(X) = 1$
iii) SPECIAL RIEMANNIAN MANIFOLDS
KÄHLER M^{2n} (KÄHLER 2-FORM ω)
SASAKI M^{2n+1} (REEB VECTOR X)


iii) SPECIAL RIEMANNIAN MANIFOLDS
GEOM''
KÄHLER
SASAKI
M^{2n} (KAHLER 2-FORM ω)
M^{2n+1} (REEB VECTOR X)
$d\theta = \omega$

iv) $\boxed{\mathcal{L}_X \omega = 0}$

PROOF: $\mathcal{L}_X \omega = \underbrace{X \lrcorner d\omega}_{0} + d\left(\underbrace{X \lrcorner \omega}_{0}\right) = 0$

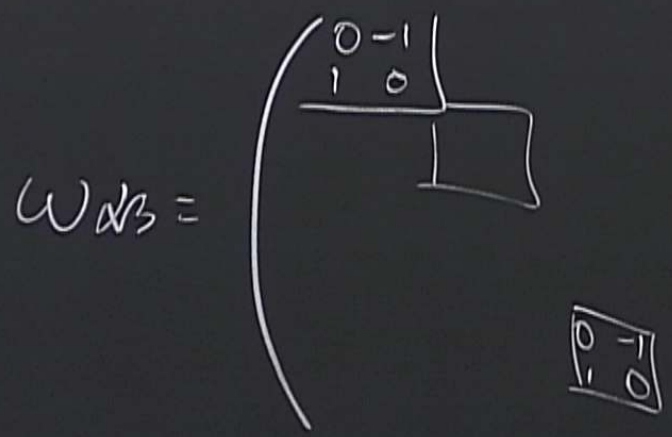
iv) $\boxed{\mathcal{L}_X \omega = 0}$

PROOF: $\mathcal{L}_X \omega = X \lrcorner \underbrace{d\omega}_{0} + d\left(\underbrace{X \lrcorner \omega}_{0}\right) = 0$

SPEC: TIME EVOLUTION $\equiv$ CANONICAL TRANSF ($\mathcal{L}_X \omega = 0$)

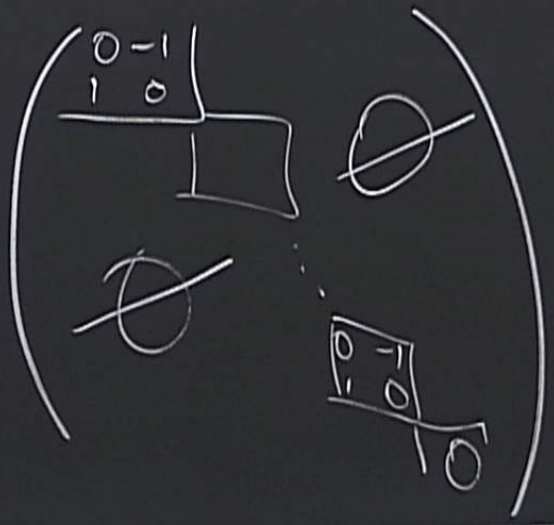
V) DARBOUX THEOREM SINCE ω CLOSED & NON-SING.

$\Rightarrow \exists$ CANONICAL COORDS (q^i, p_i, t) S.t

$\omega_{\mathbb{R}^3} =$ 

V) DARBOUX THEOREM SINCE ω CLOSED & NON-SING.

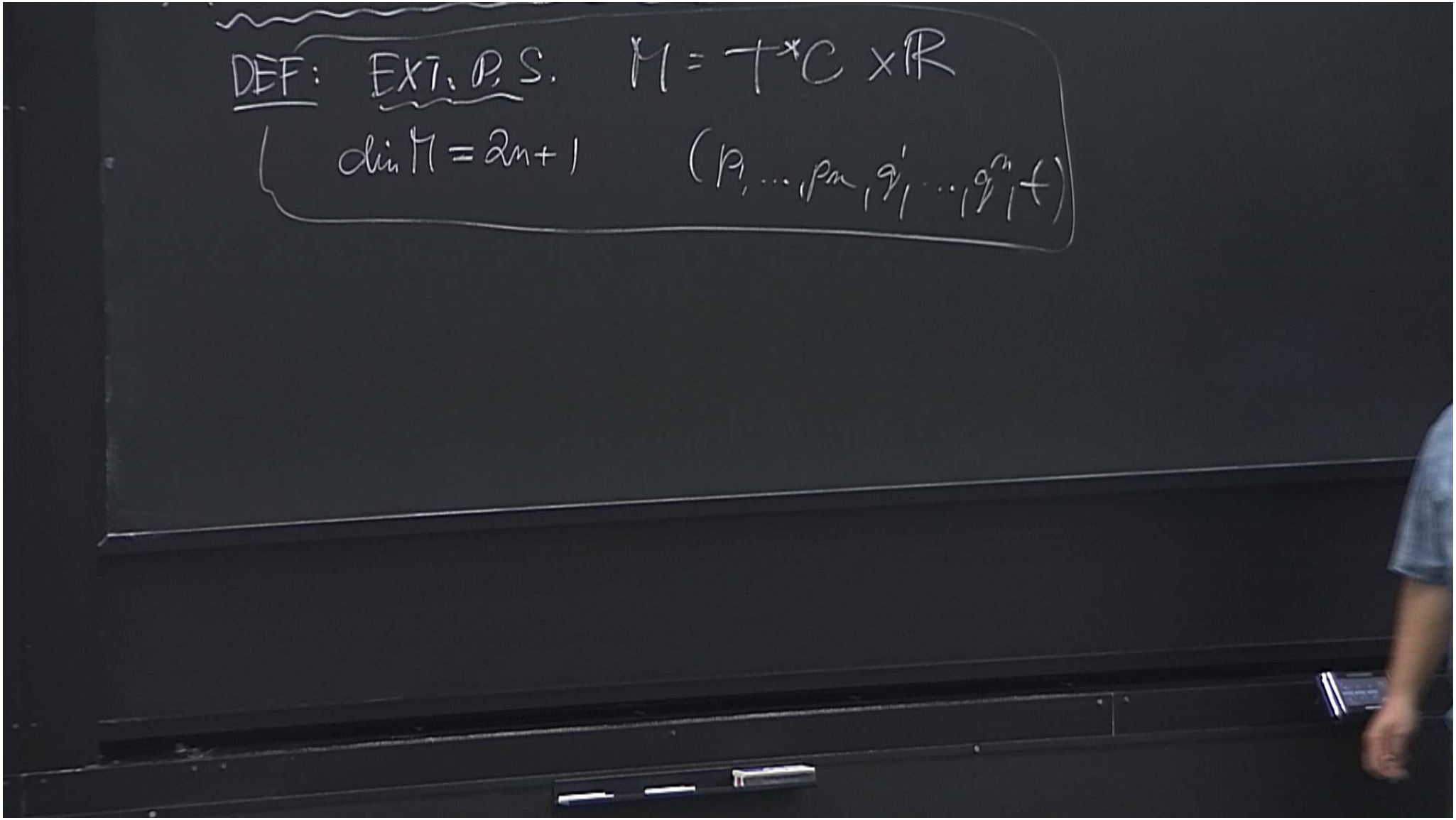
$\Rightarrow \exists$ CANONICAL COORDS (q^i, p_i, t) S.t

$\omega_{ab} =$ 

$\omega = \vec{dp_i} \wedge \vec{dq^i}$

$= dp_i \wedge dq^i$

$X = \frac{\partial}{\partial t}$


DEF: EXT. P.S. $M = T^*C \times \mathbb{R}$
$\dim M = 2n+1$ $(p_1, \dots, p_n, q^1, \dots, q^n, t)$

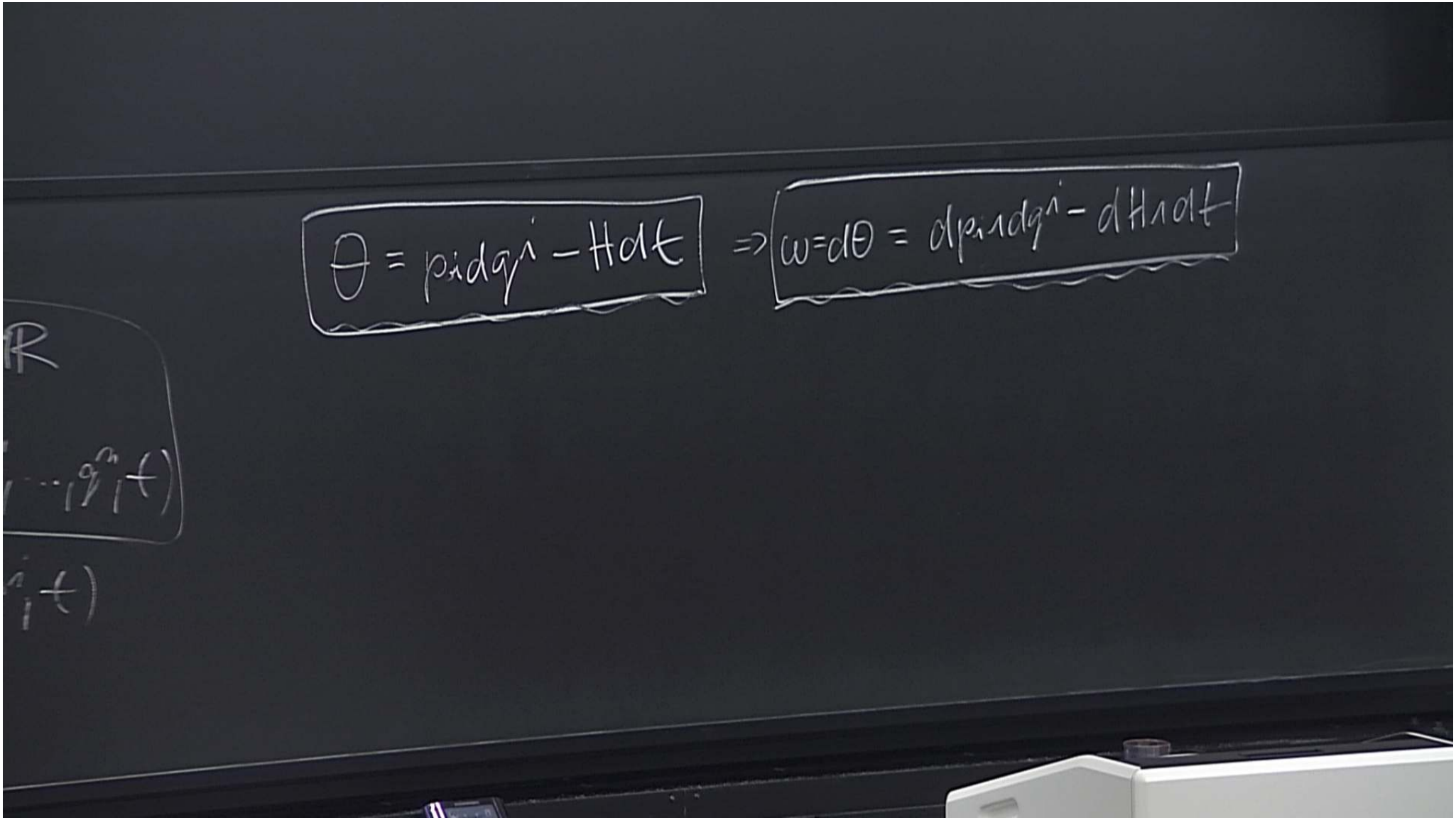
1) EXTENDED PHASE SPACE

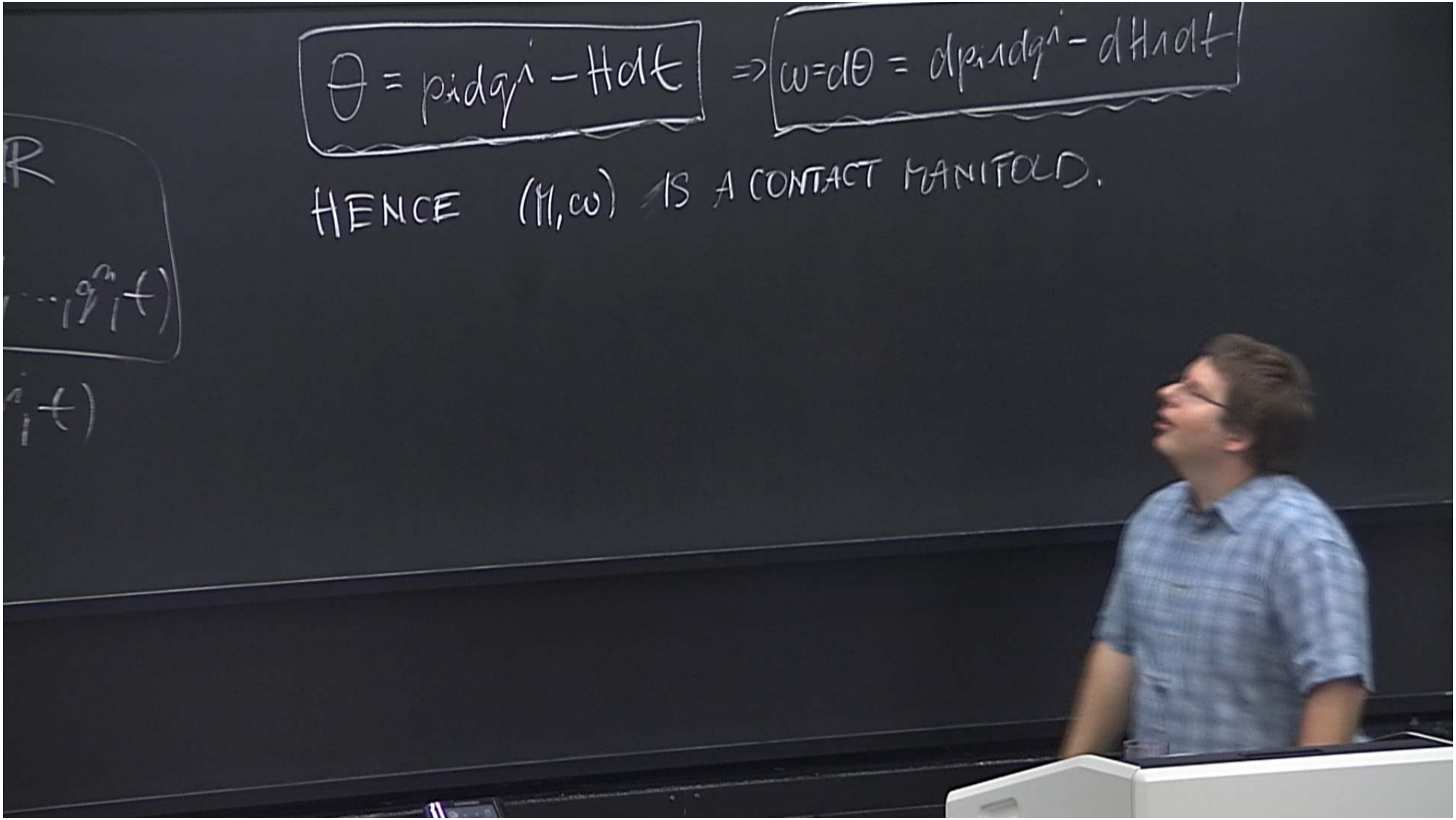
DEF: EXT. P.S. $M = T^*C \times \mathbb{R}$

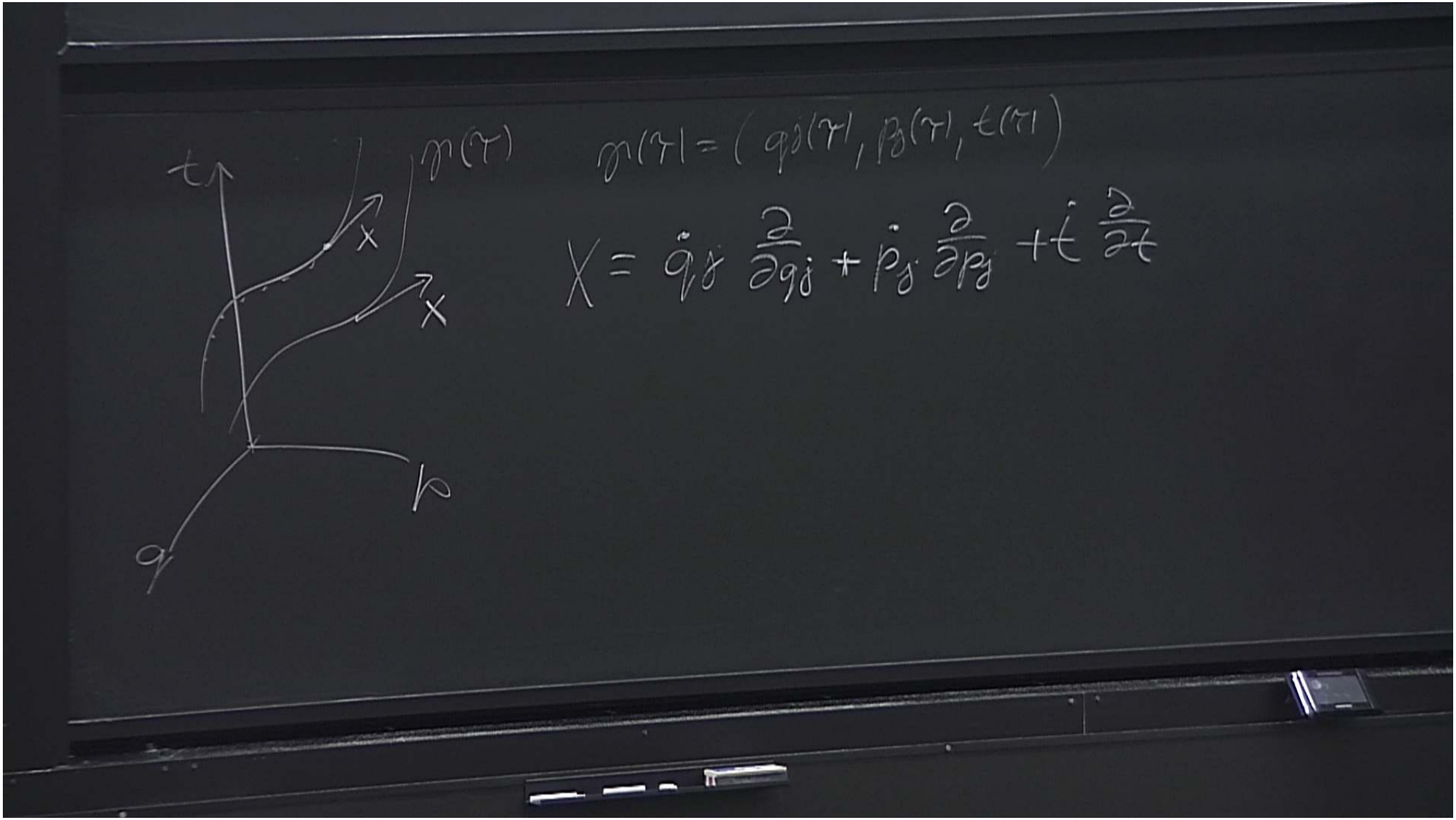
$\dim M = 2n+1$ $\quad (p_1, \dots, p_n, q^1, \dots, q^n, t)$

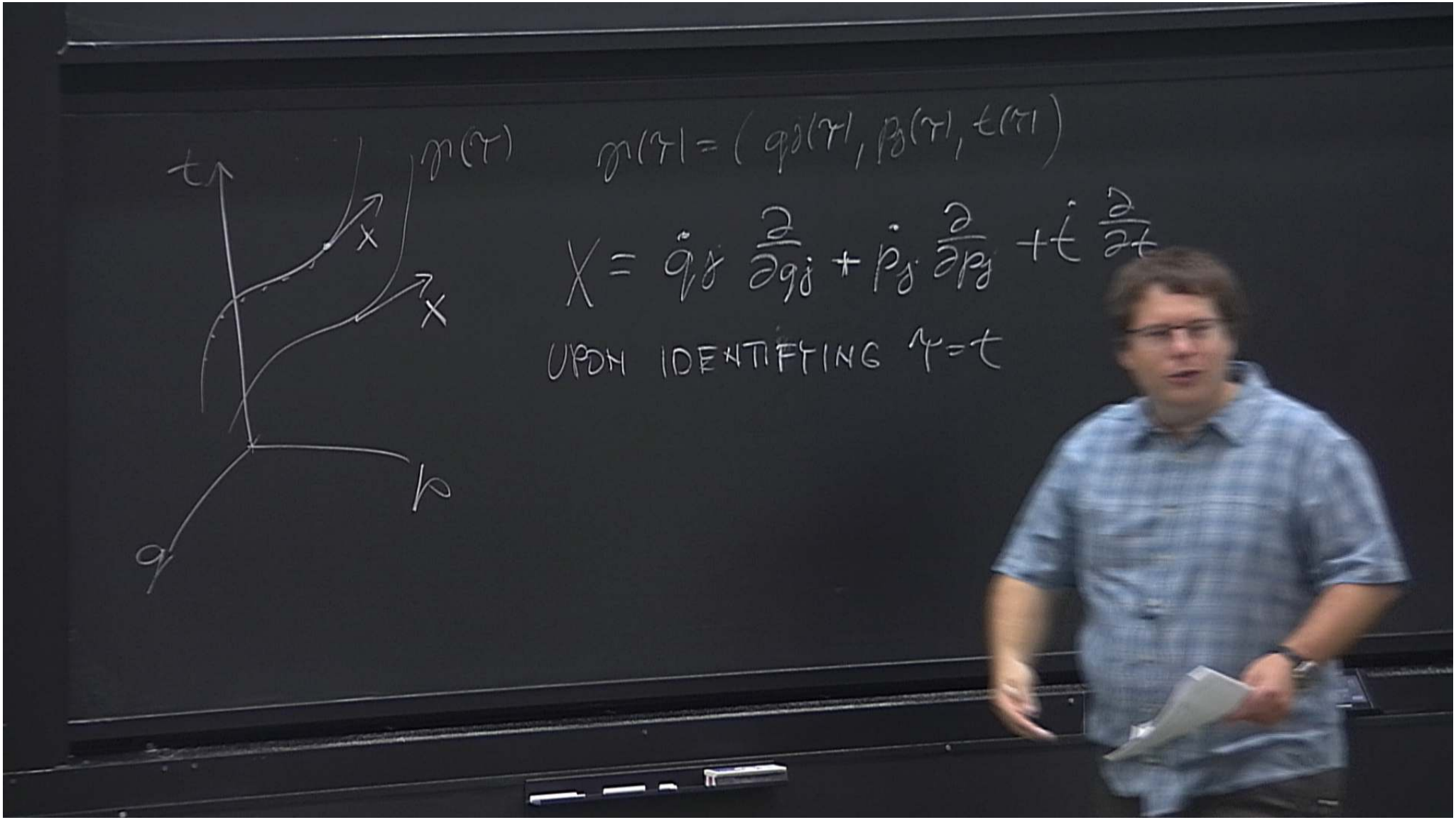
- GIVEN A HAMILTONIAN F. $H(p_i, q^i, t)$

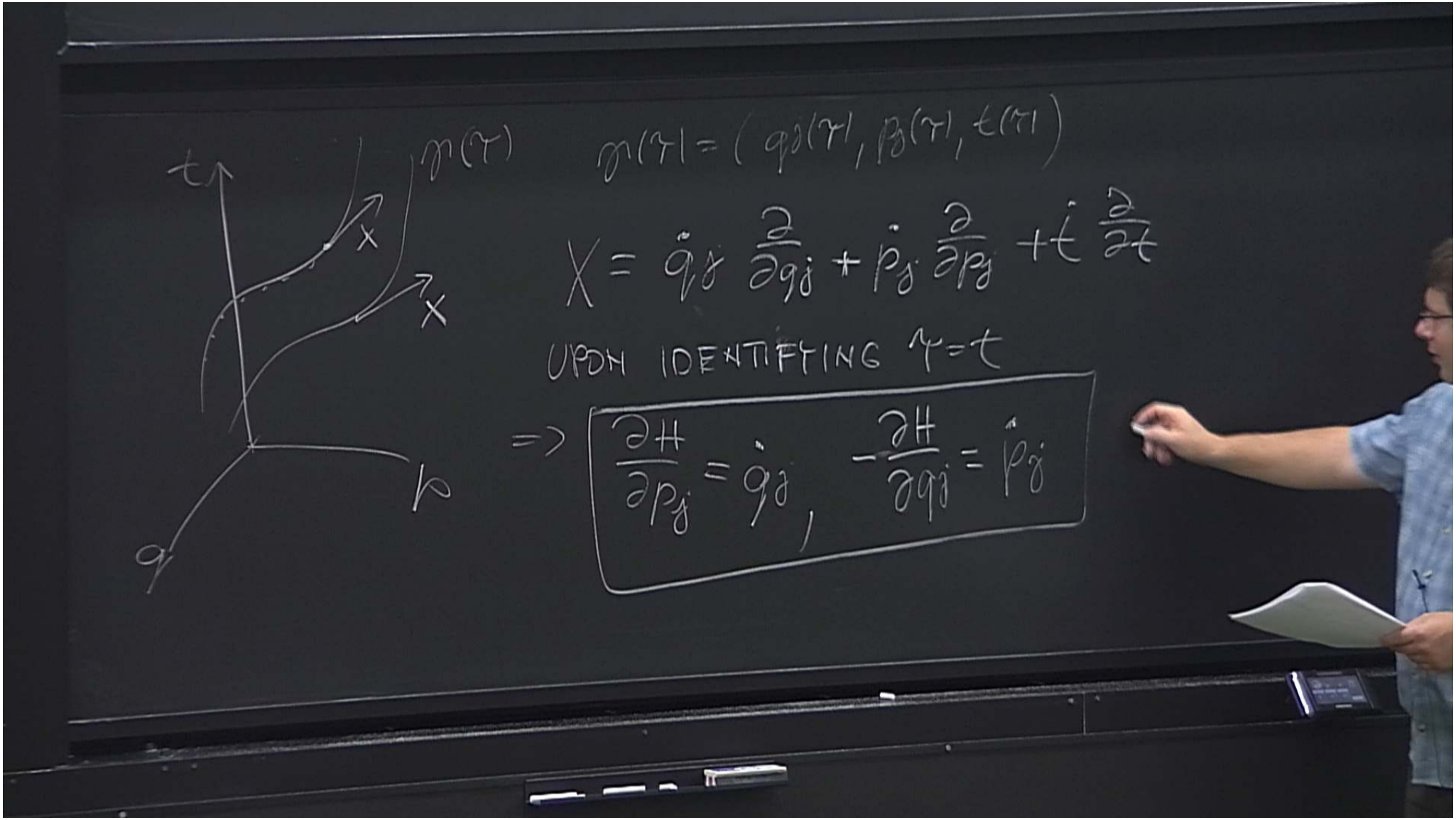
  M ADMITS CONTACT 1-FORM


$\theta = p_i dq^i - H dt$
$\Rightarrow \omega = d\theta = dp_i \wedge dq^i - dH \wedge dt$


$\theta = p_i dq^i - H dt$
$\Rightarrow \omega = d\theta = dp_i \wedge dq^i - dH \wedge dt$
HENCE (M, ω) IS A CONTACT MANIFOLD.


t
x
x
p
q
$\gamma(\tau)$
$\gamma(\tau) = (q_j(\tau), p_j(\tau), t(\tau))$
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$


t
x
x
p
q
$\gamma(\tau)$
$\gamma(\tau) = (q_j(\tau), p_j(\tau), t(\tau))$
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$
UPON IDENTIFYING $\tau = t$


t
x
x
p
q
$\gamma(\tau)$
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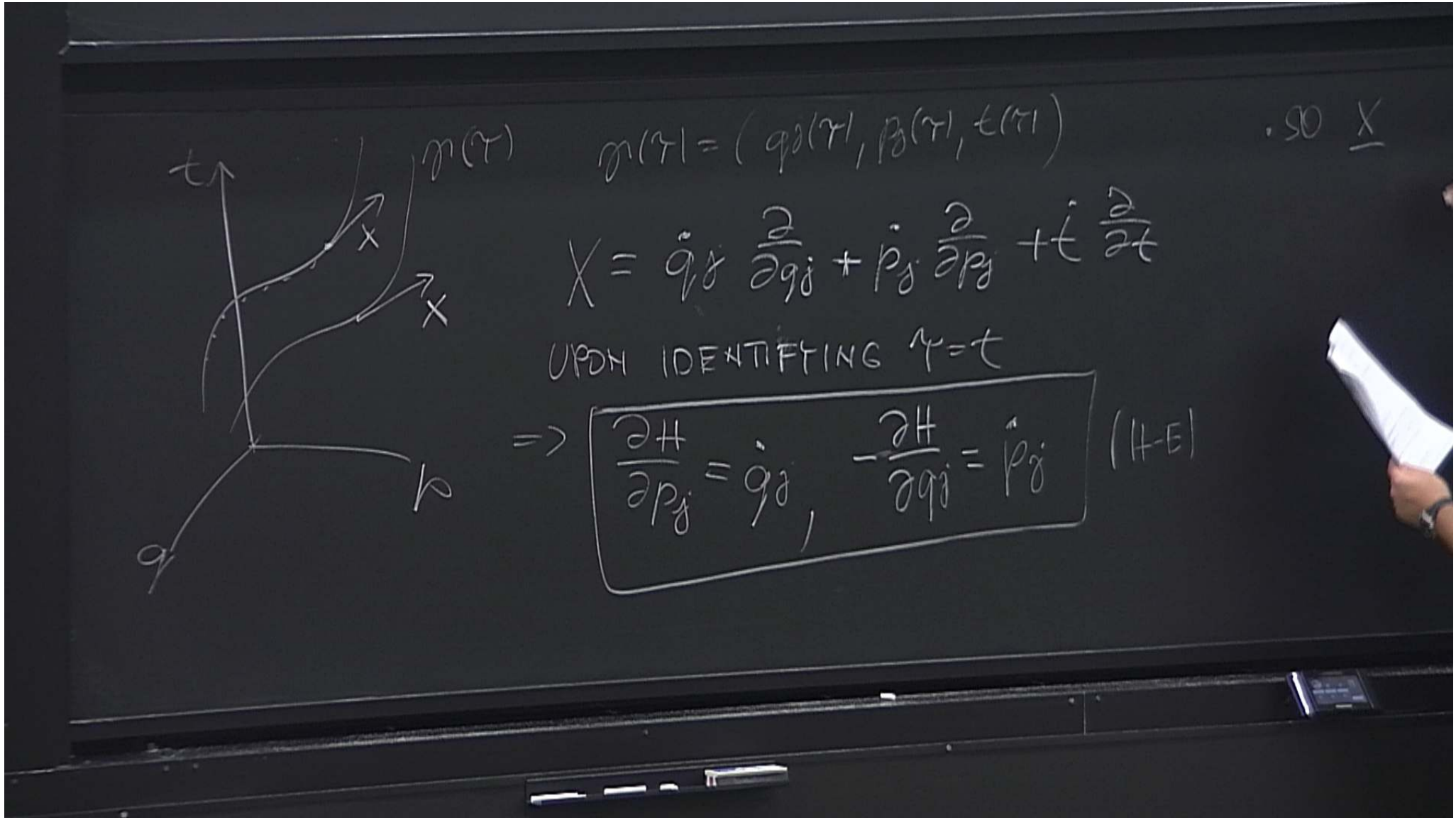
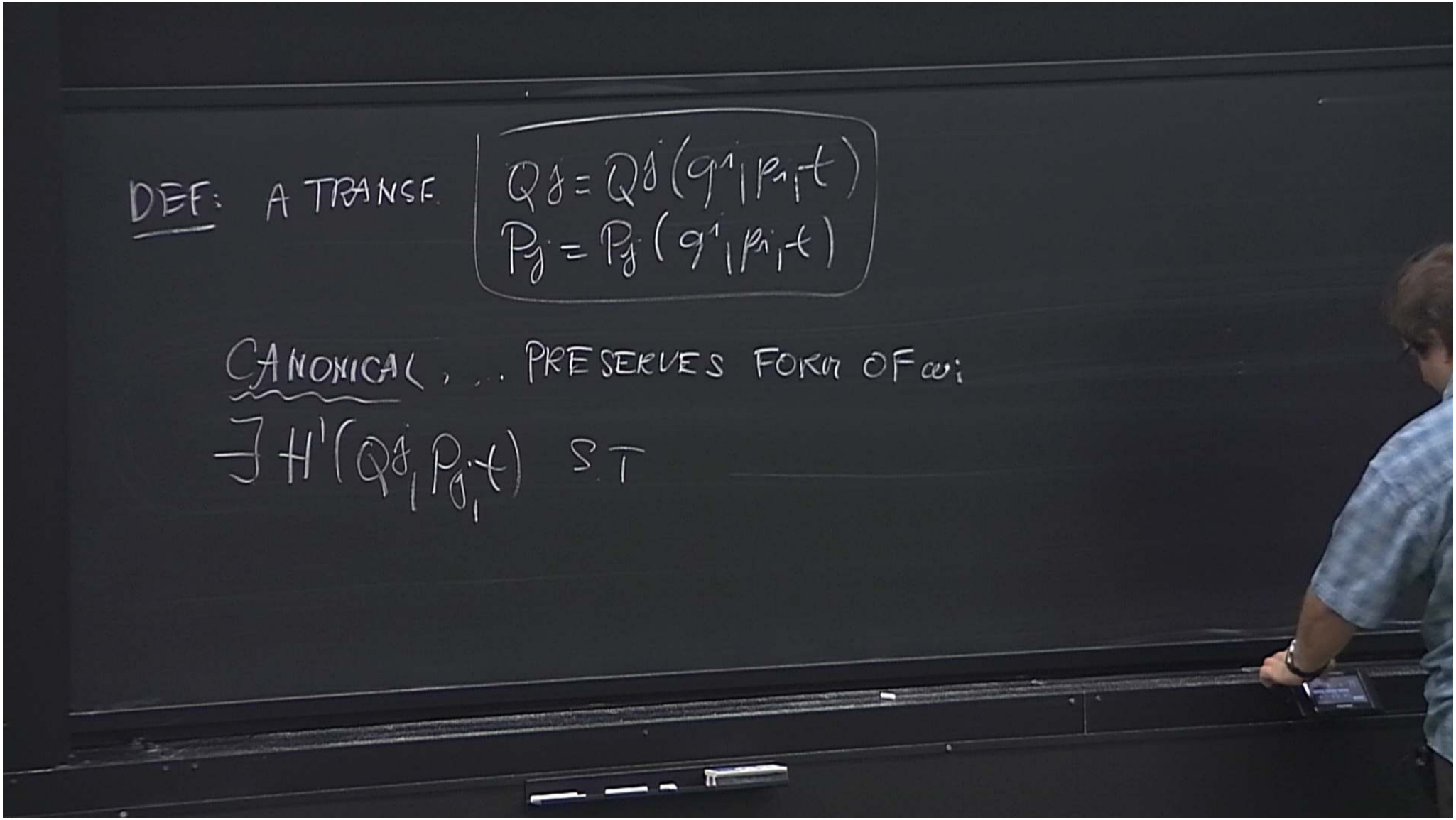
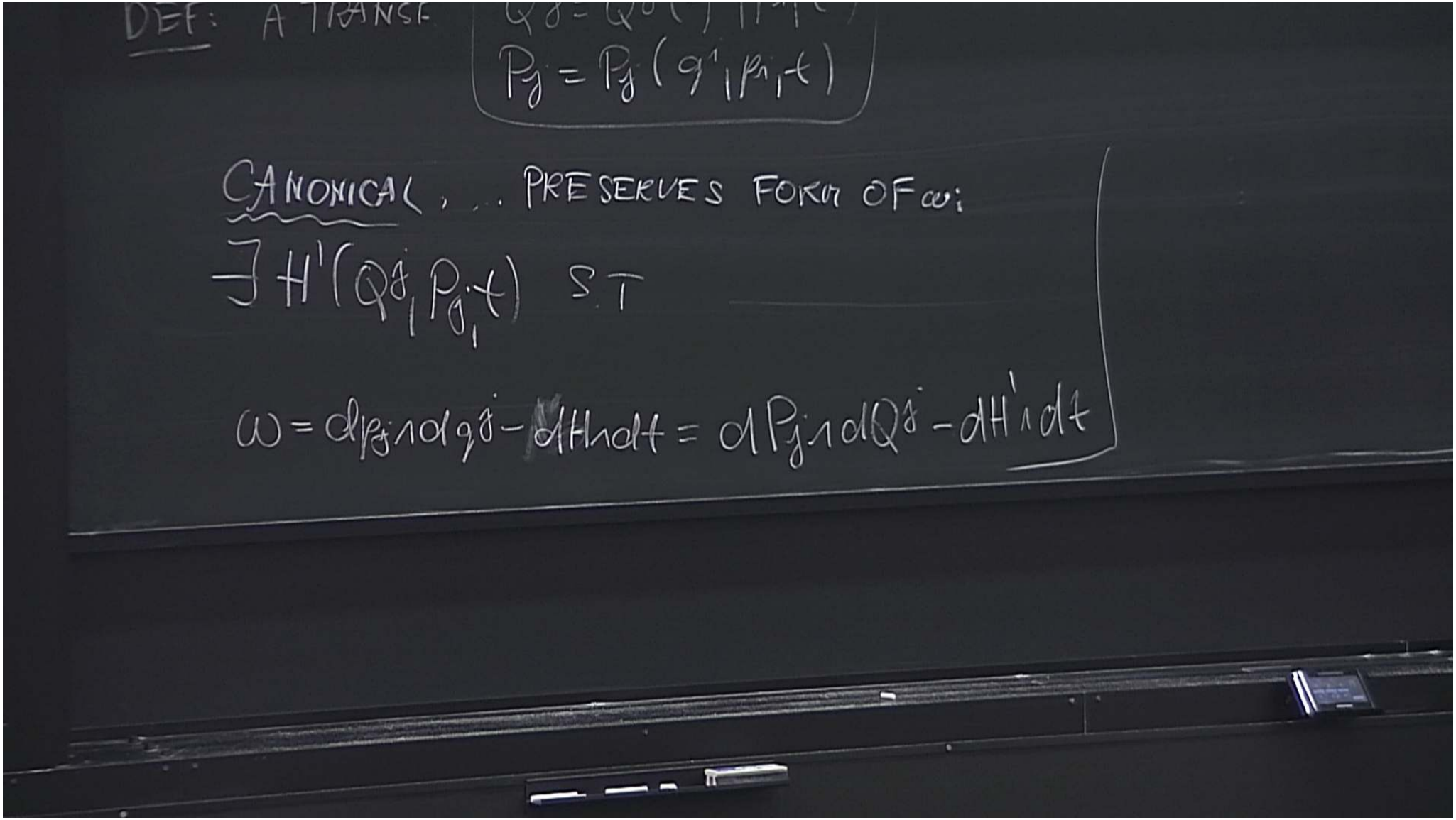
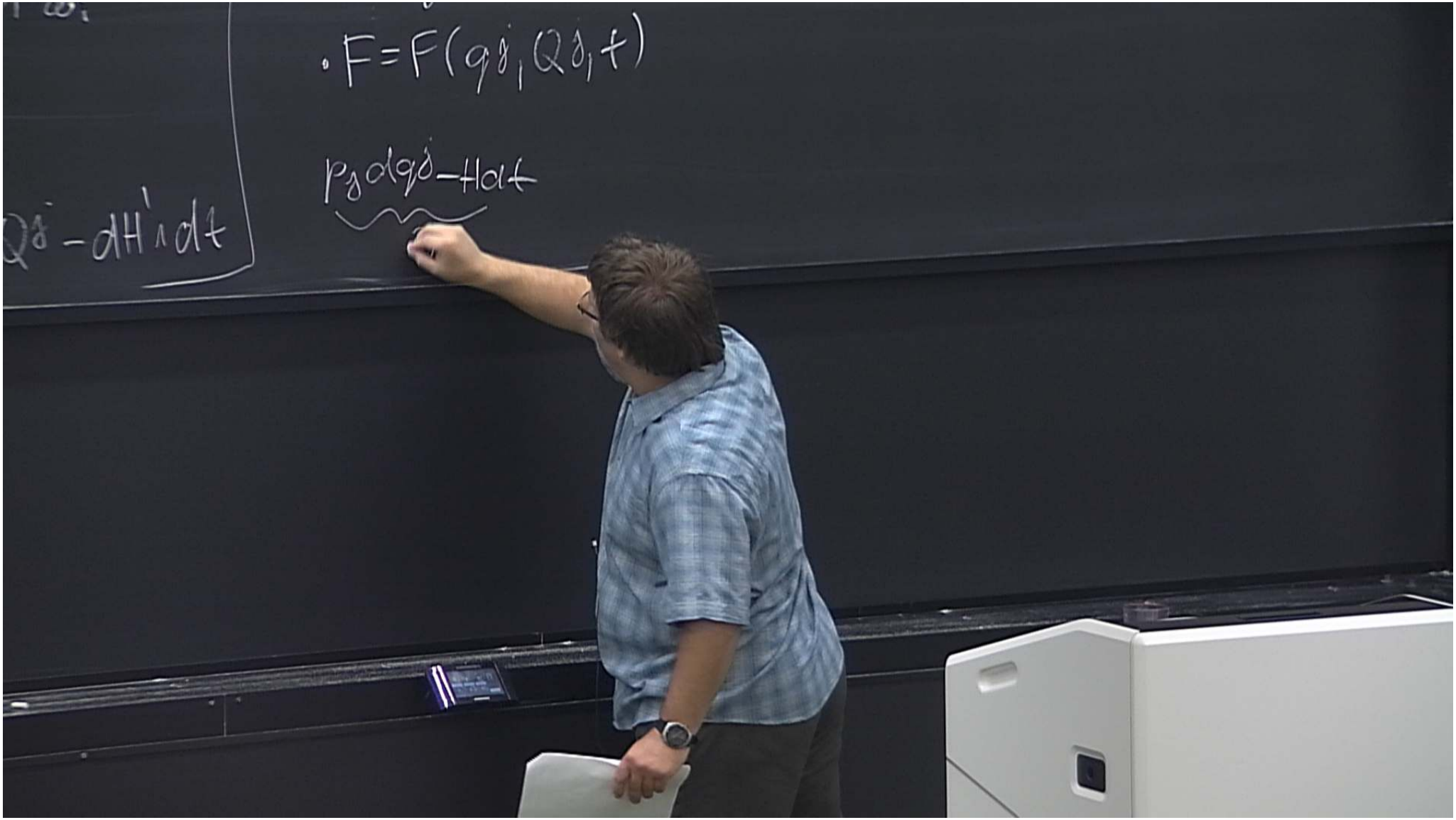
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$
UPON IDENTIFYING $\tau = t$
$\Rightarrow \boxed{\frac{\partial H}{\partial p_j} = \dot{q}_j, \quad -\frac{\partial H}{\partial q_j} = \dot{p}_j}$


t
x
x
p
q
$\gamma(\tau)$
$\gamma(\tau) = (q_j(\tau), p_j(\tau), t(\tau))$
SO X
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$
UPON IDENTIFYING $\tau = t$
$\Rightarrow \boxed{\frac{\partial H}{\partial p_j} = \dot{q}_j, \quad -\frac{\partial H}{\partial q_j} = \dot{p}_j}$ (H-E)


DEF: A TRANSF
$Q^j = Q^j(q^i, p_i, t)$
$P_j = P_j(q^i, p_i, t)$
CANONICAL, ... PRESERVES FORM OF ω:
$\exists H'(Q^j, P_j, t)$ S.T


DEF: A TRANSF
$P_j = P_j(q^i, P_i, t)$
CANONICAL, ... PRESERVES FORM OF ω:
$\exists H'(Q^i, P_i, t)$ S.T.
$\omega = dP_j \wedge dq^i - dH \wedge dt = dP_j \wedge dQ^i - dH' \wedge dt$


F=F(qs, Qs, t)
Ps dqs − Hdt
Qs − dH'/dt

$\tilde{\theta} = P_j dQ^j - H' dt \qquad \omega = d\tilde{\theta}$

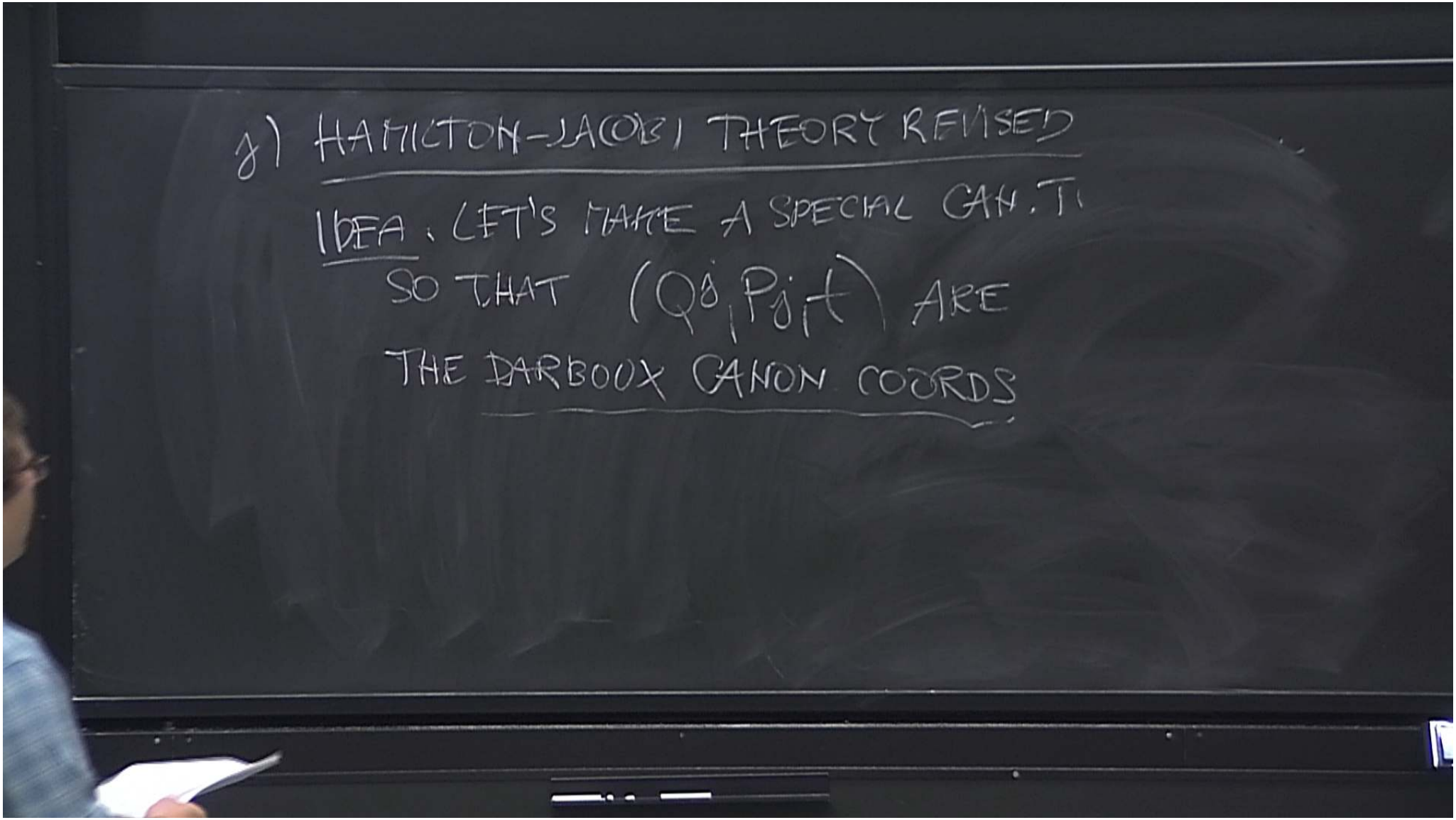
$d(F)$ .. GEN. F.

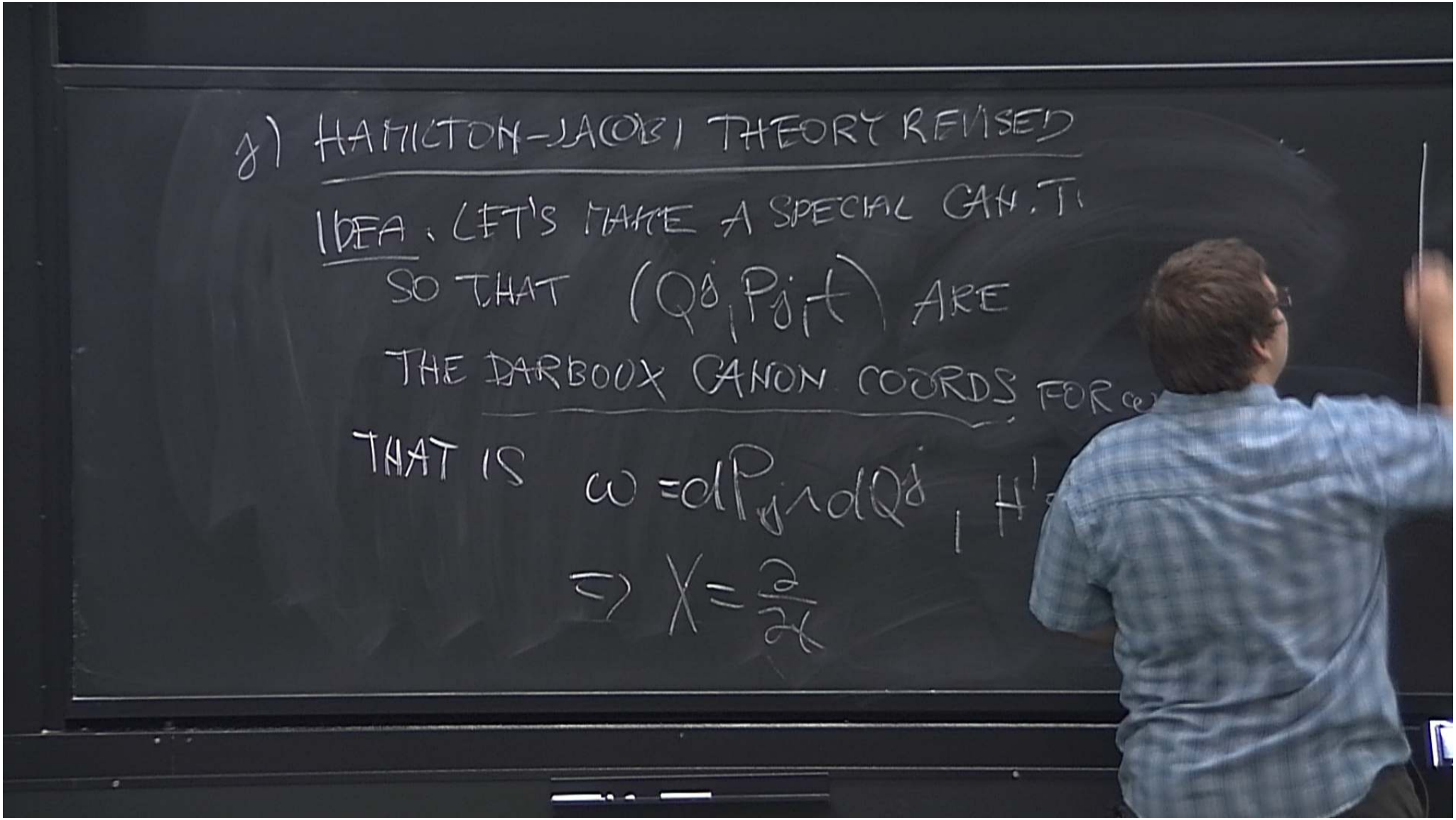
$\cdot F = F(q^j, Q^j, t)$

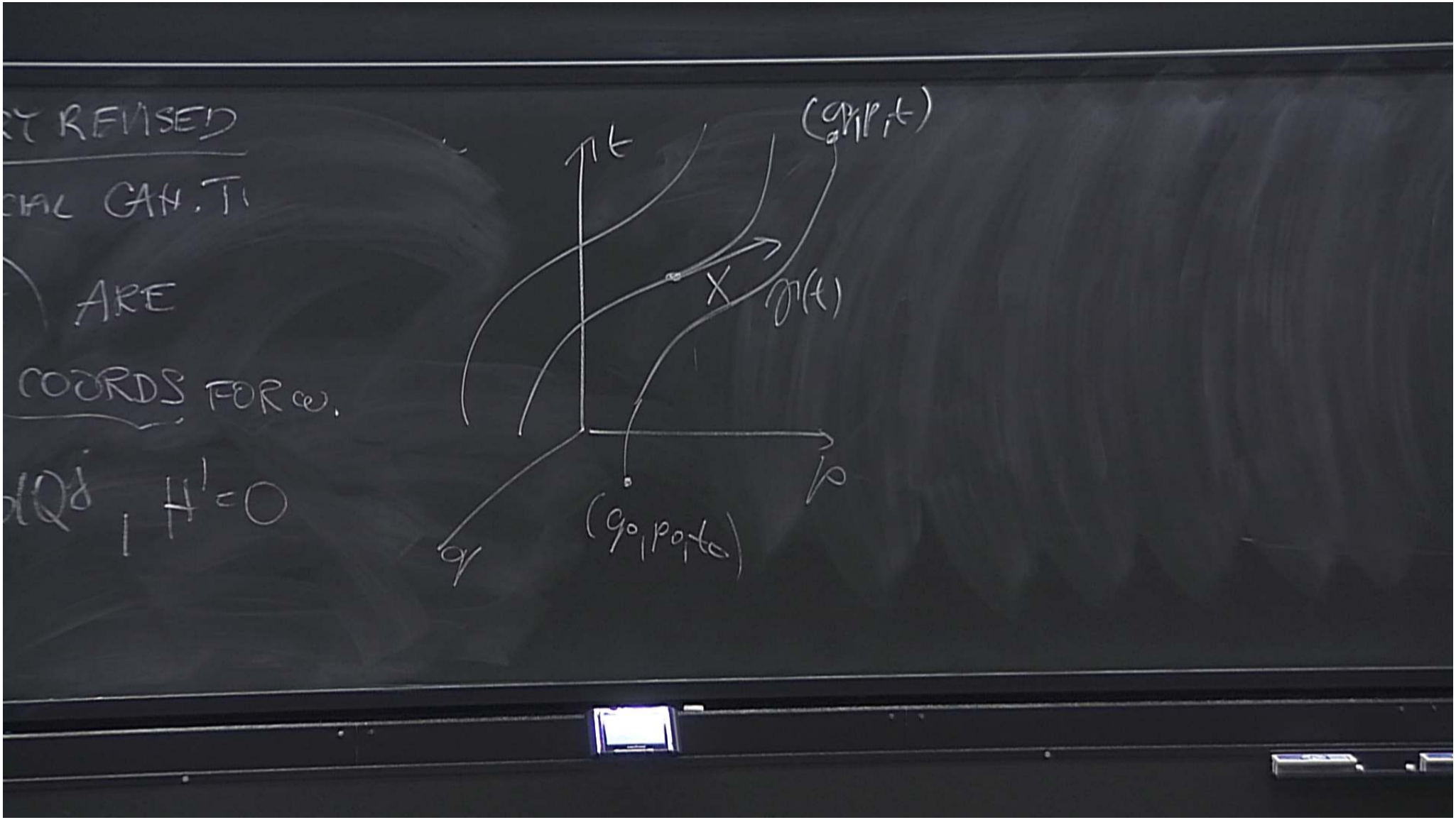
$$\underbrace{P_j dq^j - H dt}_{\theta} - \underbrace{P_j dQ^j + H' dt}_{-\tilde{\theta}} = \underbrace{\frac{\partial F}{\partial q^j} dq^j + \frac{\partial F}{\partial Q^j} dQ^j}_{dF} + \frac{\partial F}{\partial t} dt$$

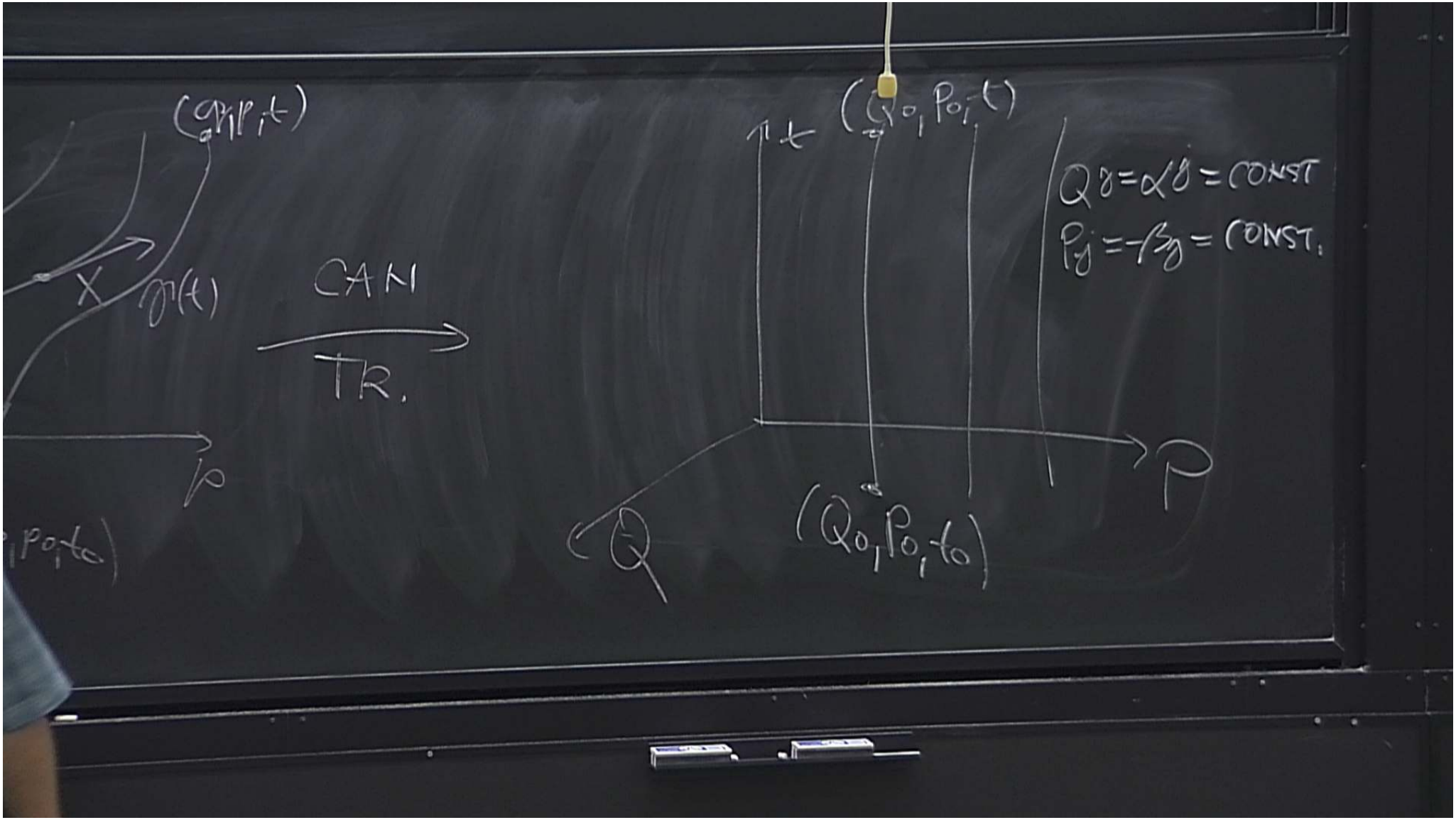
$F \omega_i$

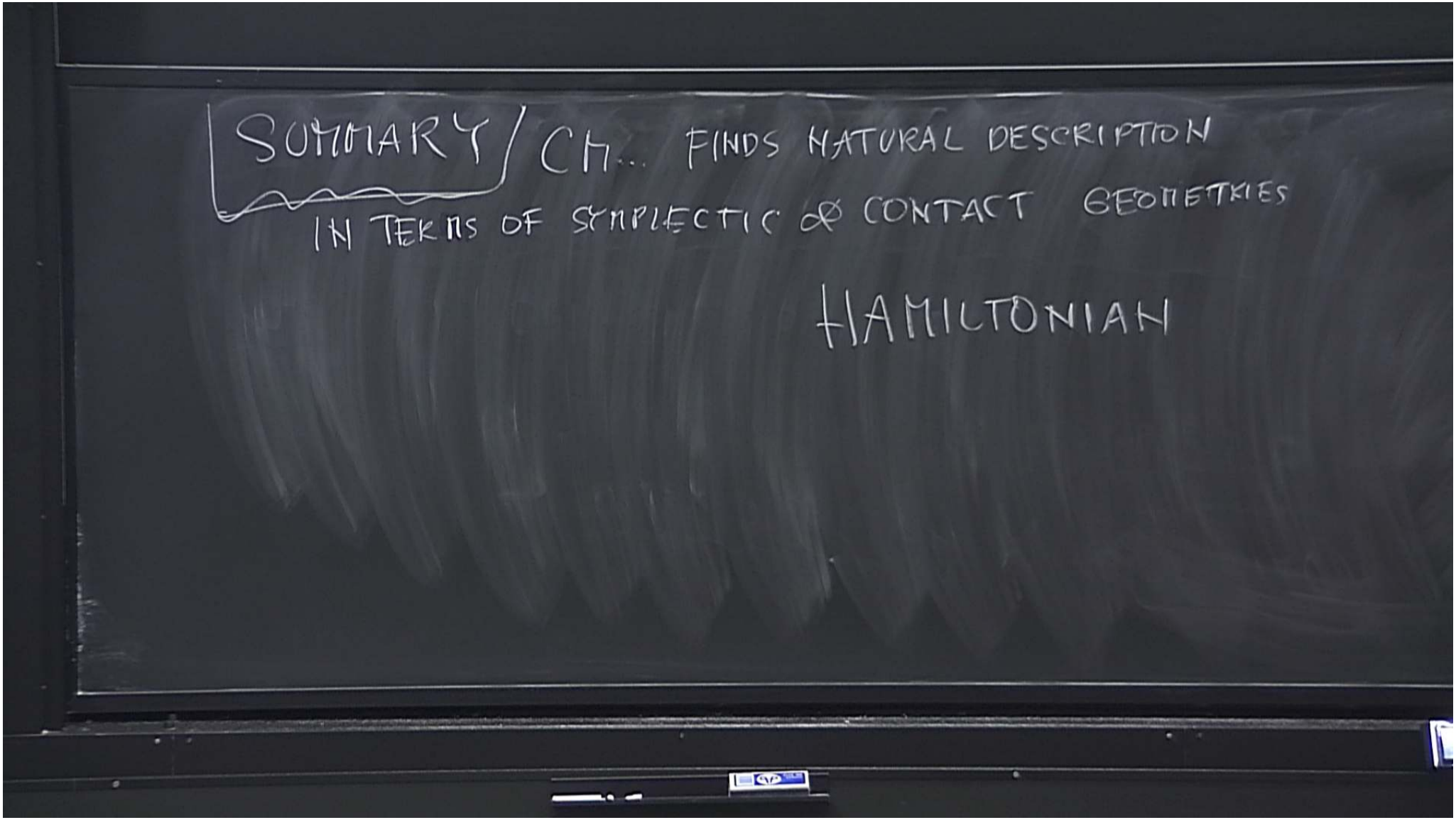
$Q^j - dH' \wedge dt$

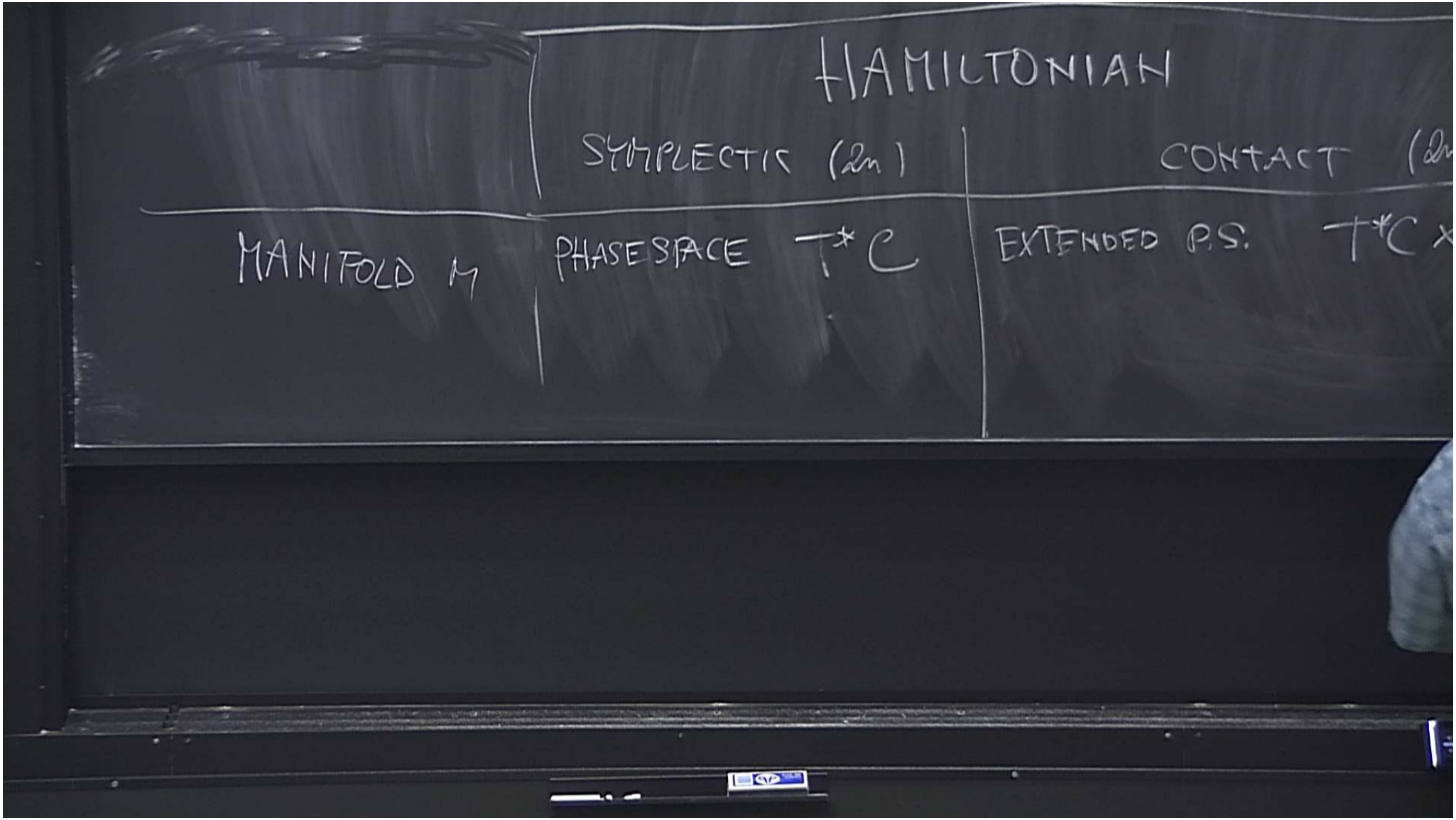

7) HAMILTON-JACOBI THEORY REVISED
IDEA. LET'S MAKE A SPECIAL CAN. T
SO THAT (Qi, Pj, t) ARE
THE DARBOUX CANON COORDS

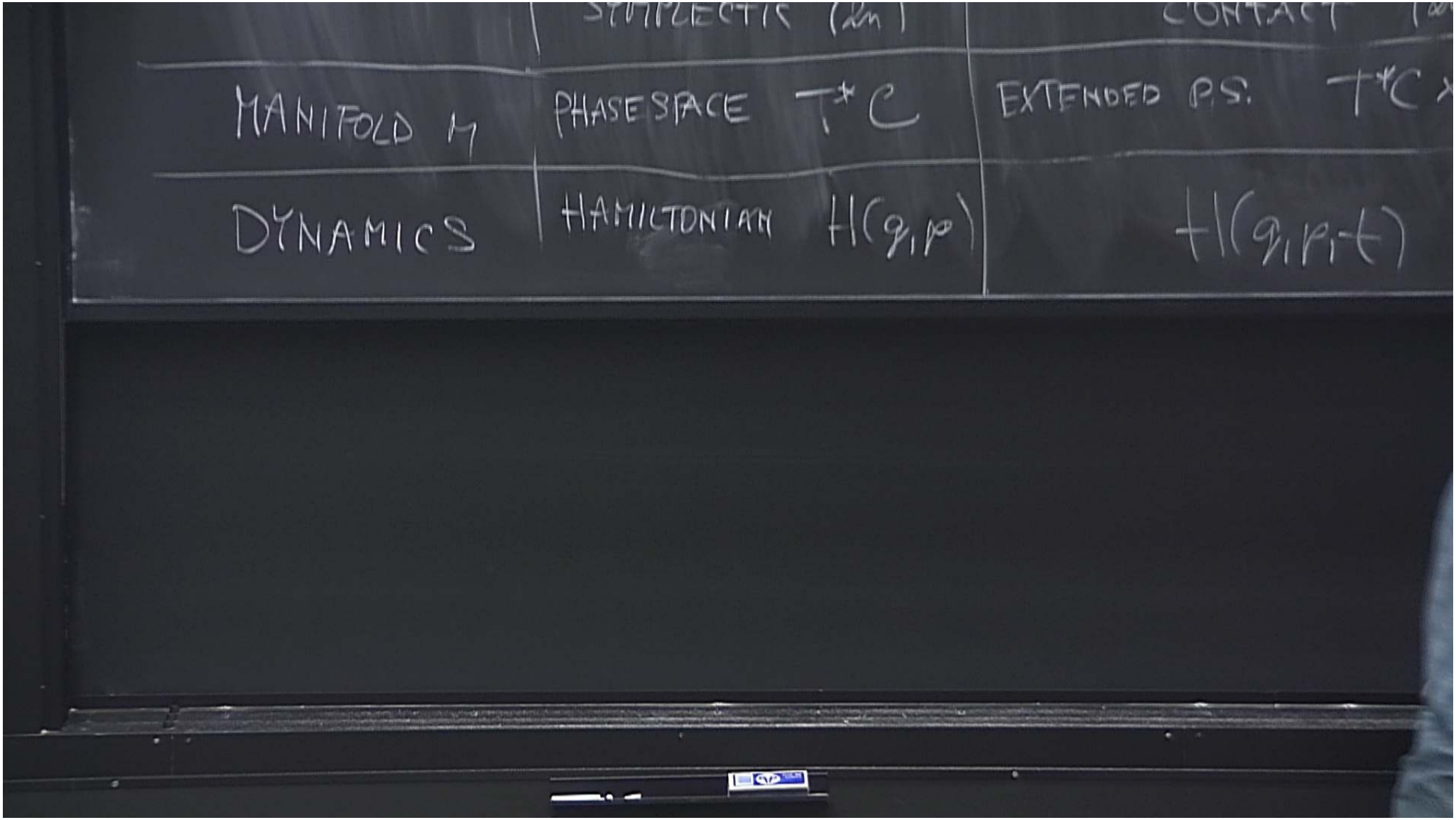

7) HAMILTON-JACOBI THEORY REVISED
IDEA: LET'S MAKE A SPECIAL CAN. T
SO THAT (Q^i, P_j, t) ARE
THE DARBOUX CANON COORDS FOR
THAT IS $\omega = dP_j \wedge dQ^j$, H'
$\Rightarrow \chi = \frac{\partial}{\partial t}$

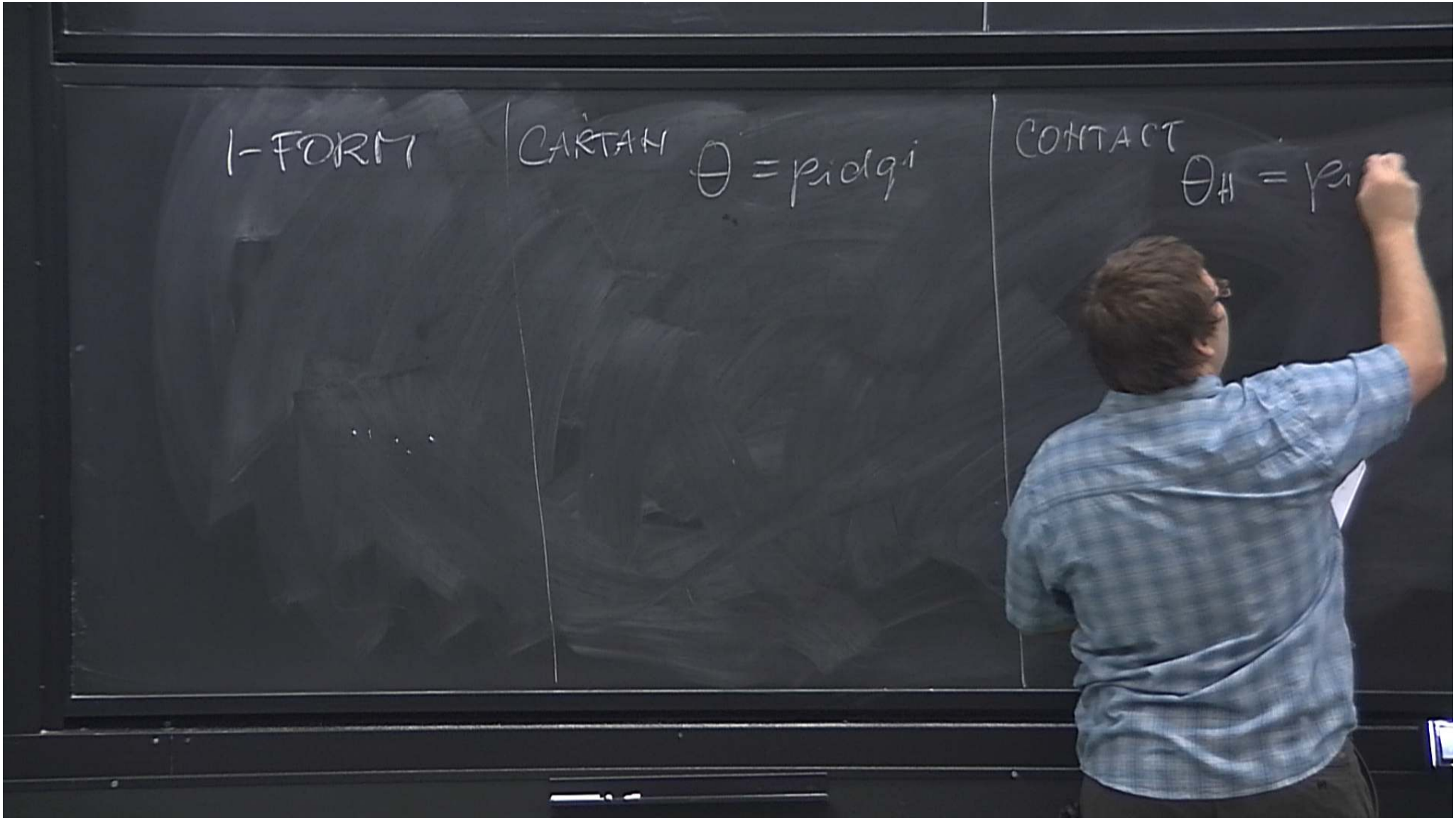

REVISED
ARE
COORDS FOR ∞.
Π^t
(q_a, p_a, t)
X
$\pi(t)$
p
q
(q_0, p_0, t_0)

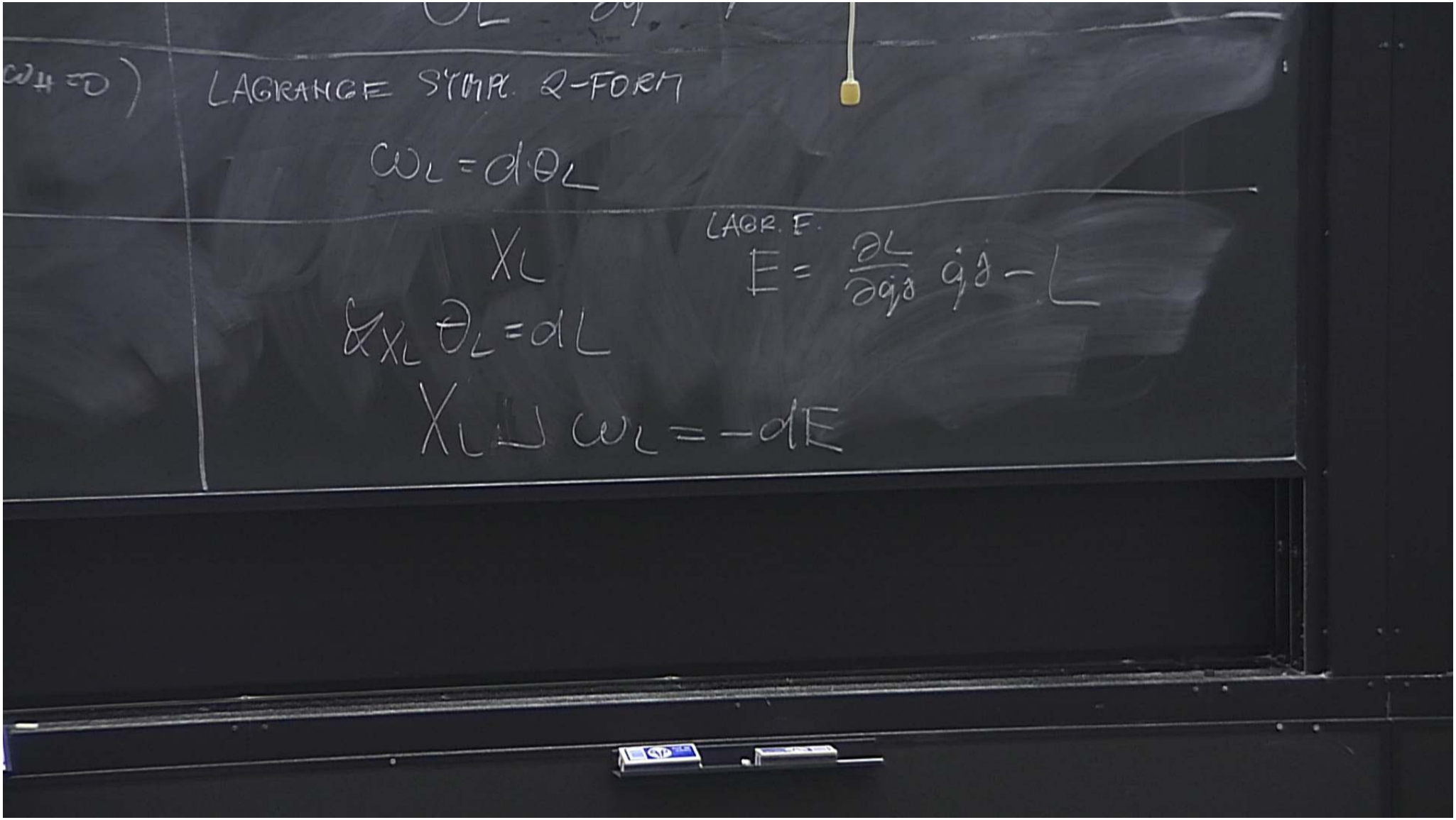

(Q0, P0, t)
X
CAN
TR.
P
Q
(Q0, P0, to)
P
Qd = αD = CONST
Pd = βS = CONST.

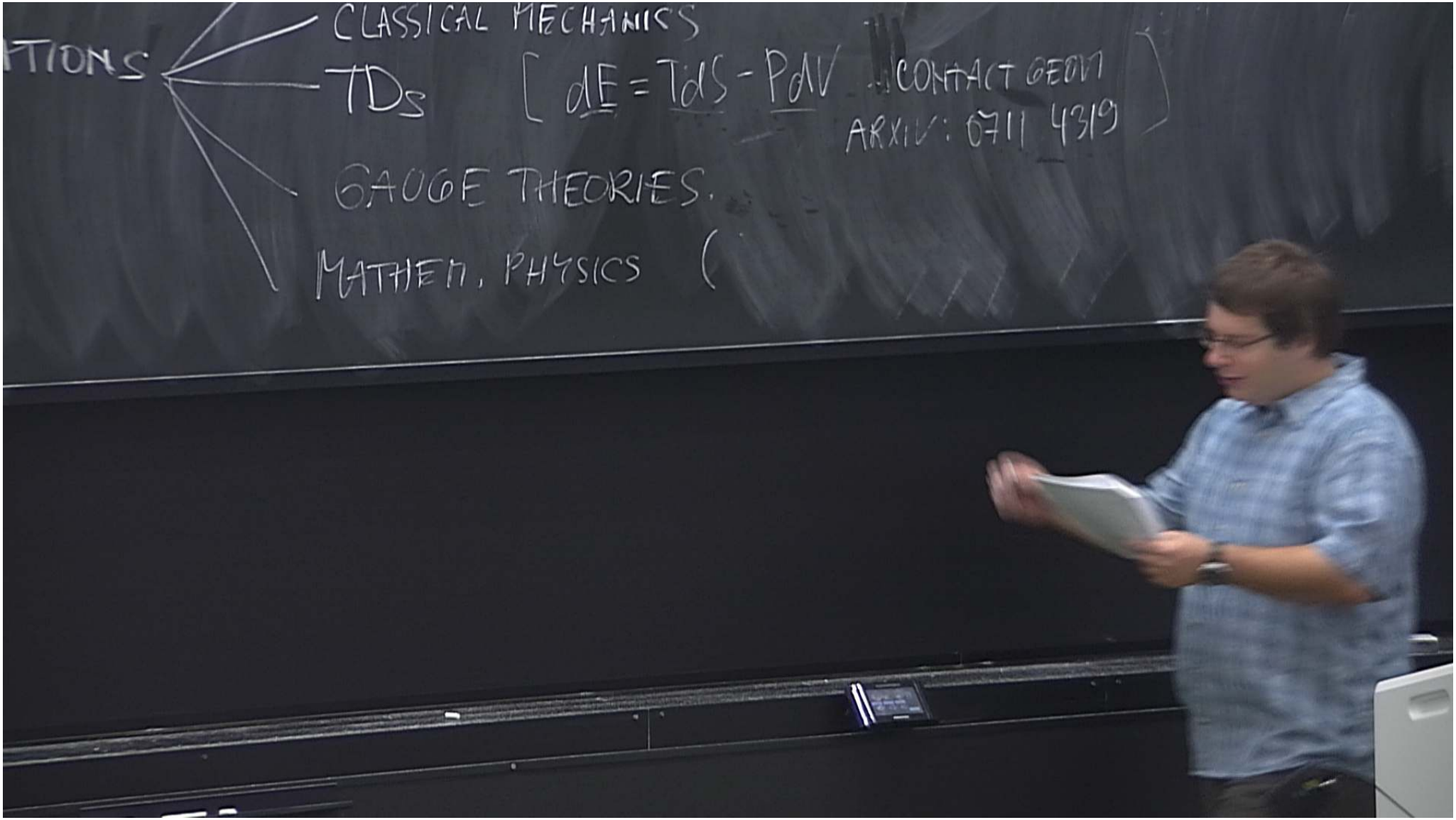

SUMMARY / CH.. FINDS NATURAL DESCRIPTION
IN TERMS OF SYMPLECTIC & CONTACT GEOMETRIES
HAMILTONIAN


HAMILTONIAN
SYMPLECTIC (2n)
CONTACT (2n
MANIFOLD M
PHASE SPACE T*C
EXTENDED P.S. T*C ×


SYMPLECTIC (2n)
CONTACT
MANIFOLD M
PHASE SPACE T^*C
EXTENDED P.S. $T^*C \times$
DYNAMICS
HAMILTONIAN $H(q,p)$
$H(q,p,t)$


1-FORM
CARTAN
$\theta = p_i dq_i$
CONTACT
$\theta_H = p_i$


$\omega_L = 0$)
LAGRANGE SYMPL. 2-FORM
$\omega_L = d\theta_L$
X_L
$\iota_{X_L}\theta_L = dL$
$X_L \lrcorner \omega_L = -dE$
LAGR. F.
$E = \frac{\partial L}{\partial \dot{q}^i}\dot{q}^i - L$


TIONS
CLASSICAL MECHANICS
TDs
[dE = TdS - PdV
CONTACT GEOM
ARXIV: 0711 4319
GAUGE THEORIES.
MATHEM. PHYSICS (

$$p_\delta = \frac{\partial F}{\partial q_\delta}, \quad -P_\delta = \frac{\partial F}{\partial Q_\delta}, \quad H' - H = \frac{\partial F}{\partial t} \qquad (C)$$

QFT: $\frac{\partial}{\partial}$ $q(t)$

GR: IN CM: CENTRAL OBJECT $\underset{\sim}{\omega}$

NON-DEG, ANTISYMM. $\omega_{\alpha\beta}$, $d\omega = 0$

IN GR: CENTRAL OBJECT g

NON-DEG., SYMMETRIC $g_{\alpha\beta}$, $\nabla g = 0$