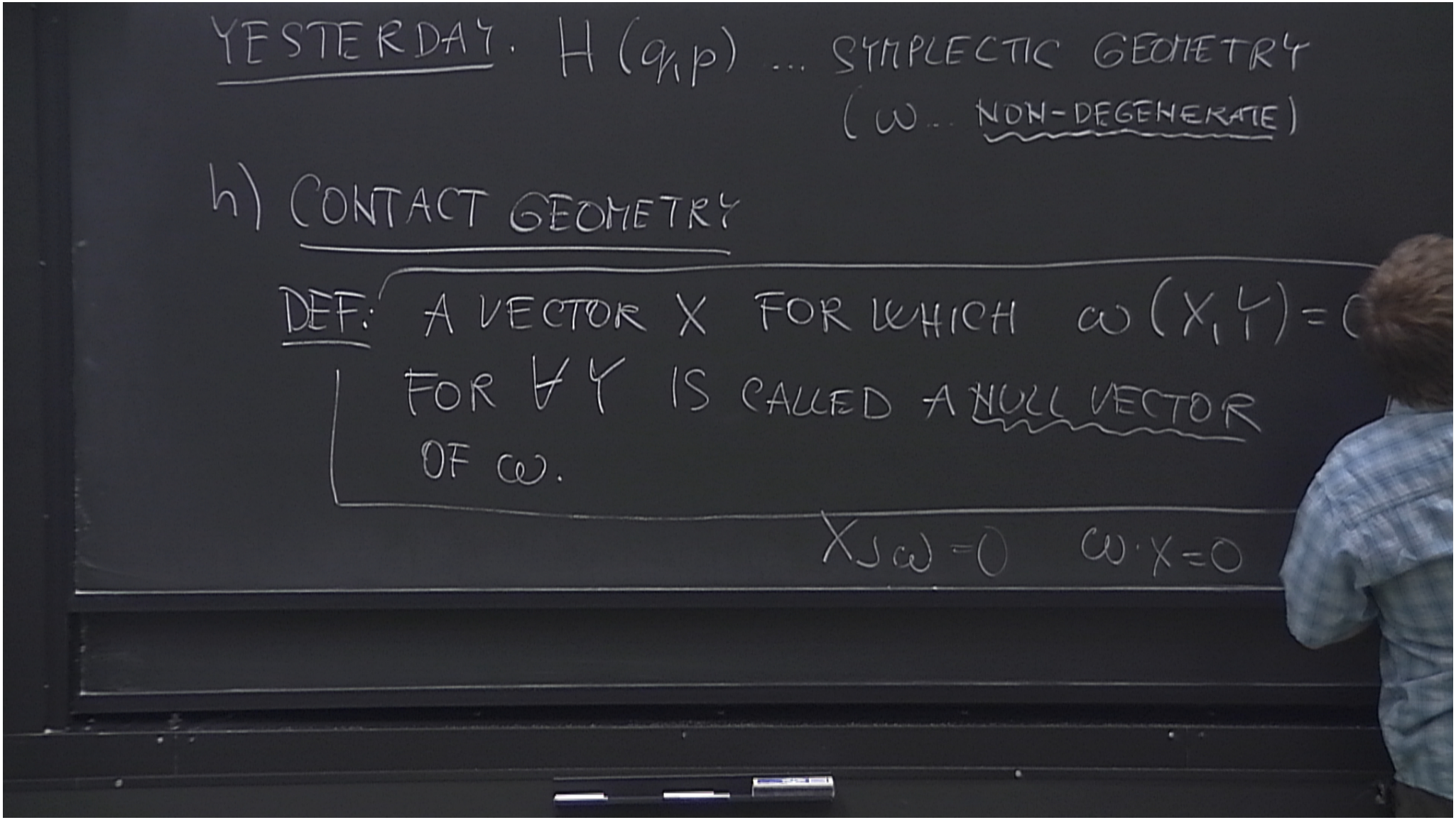
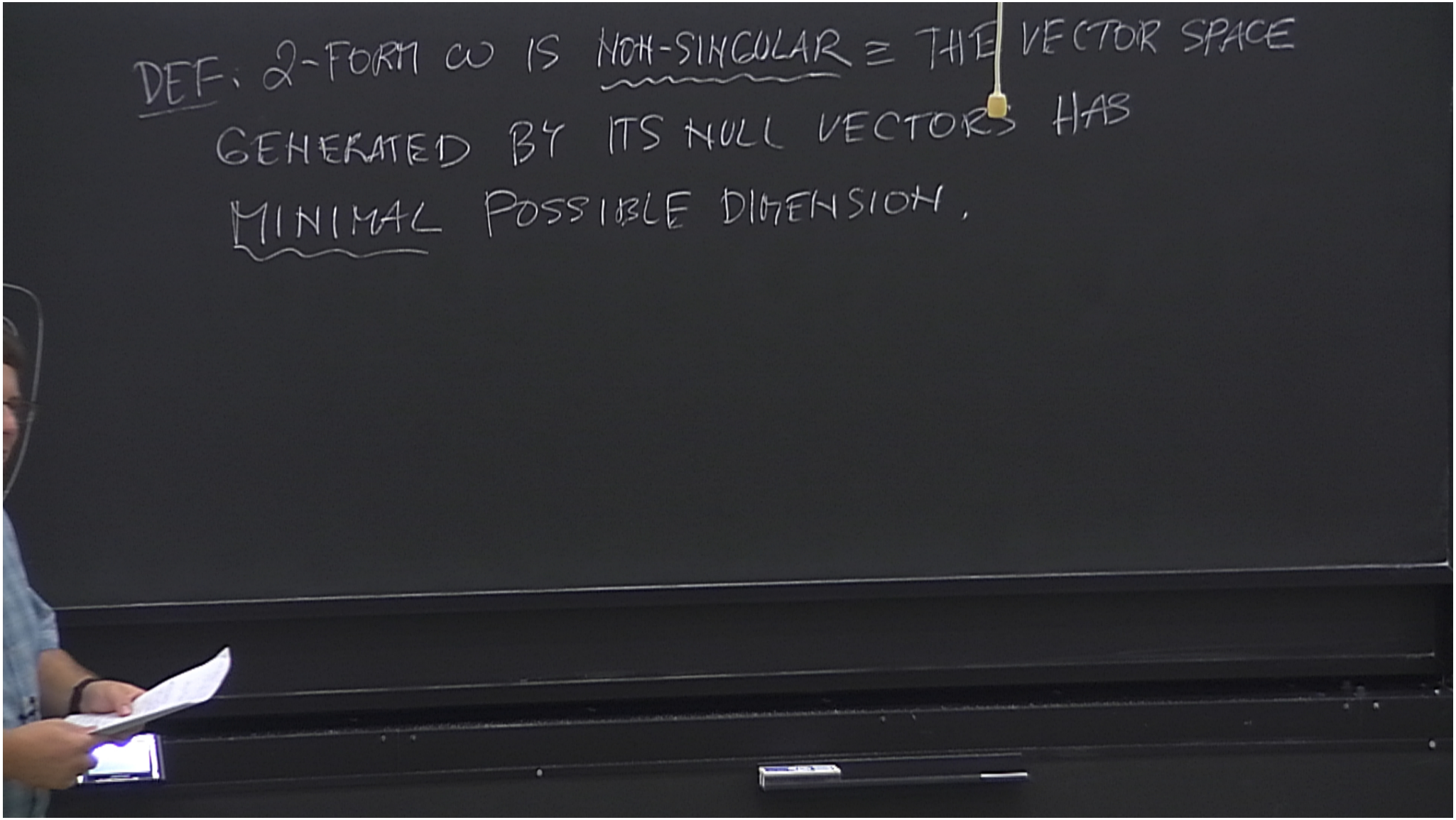
Title: Classical Mechanics - David Kubiznak

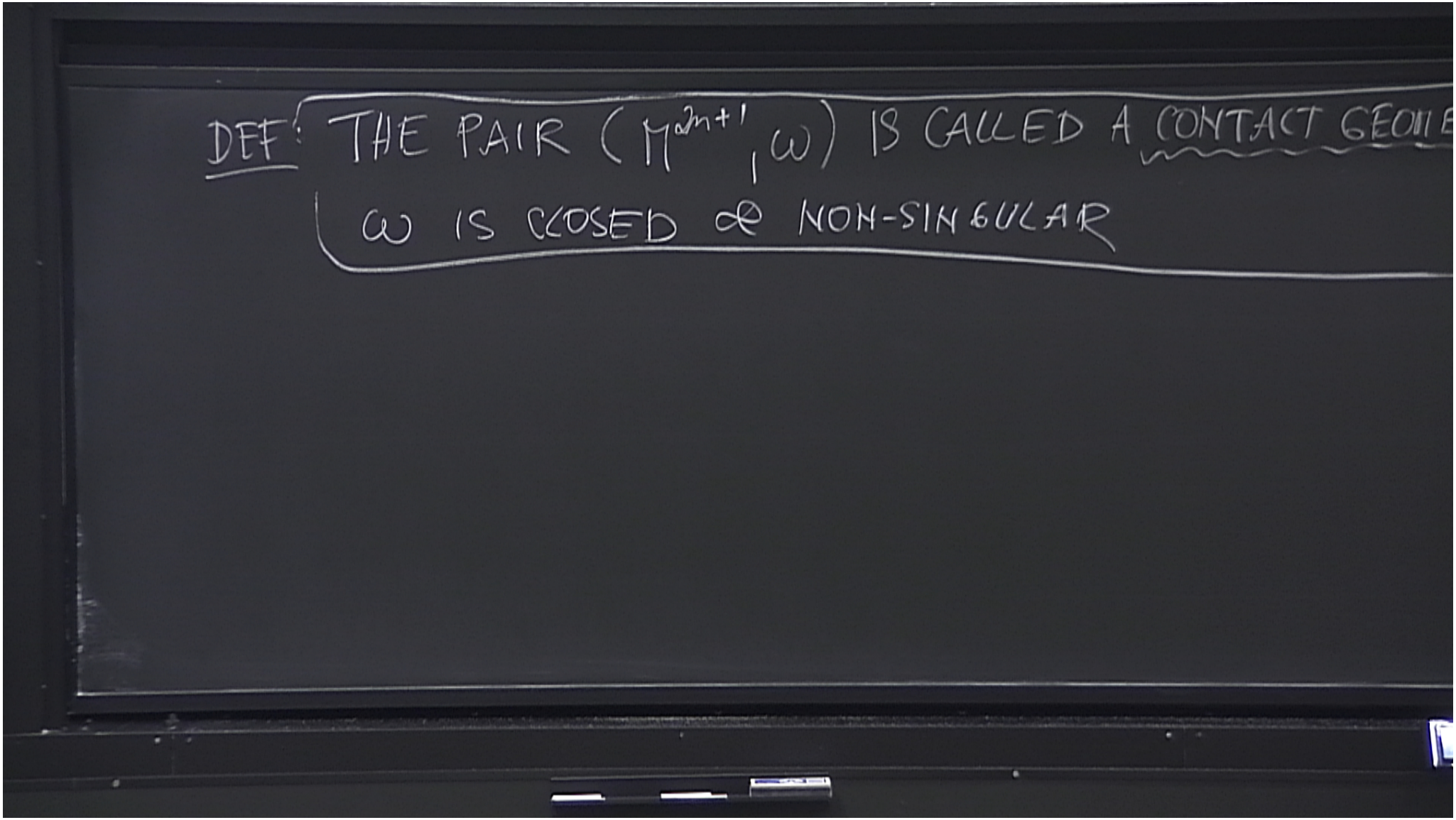
Date: Aug 27, 2015 09:00 AM

URL: http://pirsa.org/15080103

Abstract:


YESTERDAY. $H(q,p)$ ... SYMPLECTIC GEOMETRY
(ω ... NON-DEGENERATE)
h) CONTACT GEOMETRY
DEF: A VECTOR X FOR WHICH $\omega(X,Y)=0$
FOR $\forall Y$ IS CALLED A NULL VECTOR
OF ω.
$X \lrcorner \omega = 0$ $\omega \cdot X = 0$


DEF: 2-FORM ω IS NON-SINGULAR ≡ THE VECTOR SPACE
GENERATED BY ITS NULL VECTORS HAS
MINIMAL POSSIBLE DIMENSION.


DEF: THE PAIR (M^{2n+1}, ω) IS CALLED A CONTACT GEOM
ω IS CLOSED & NON-SINGULAR

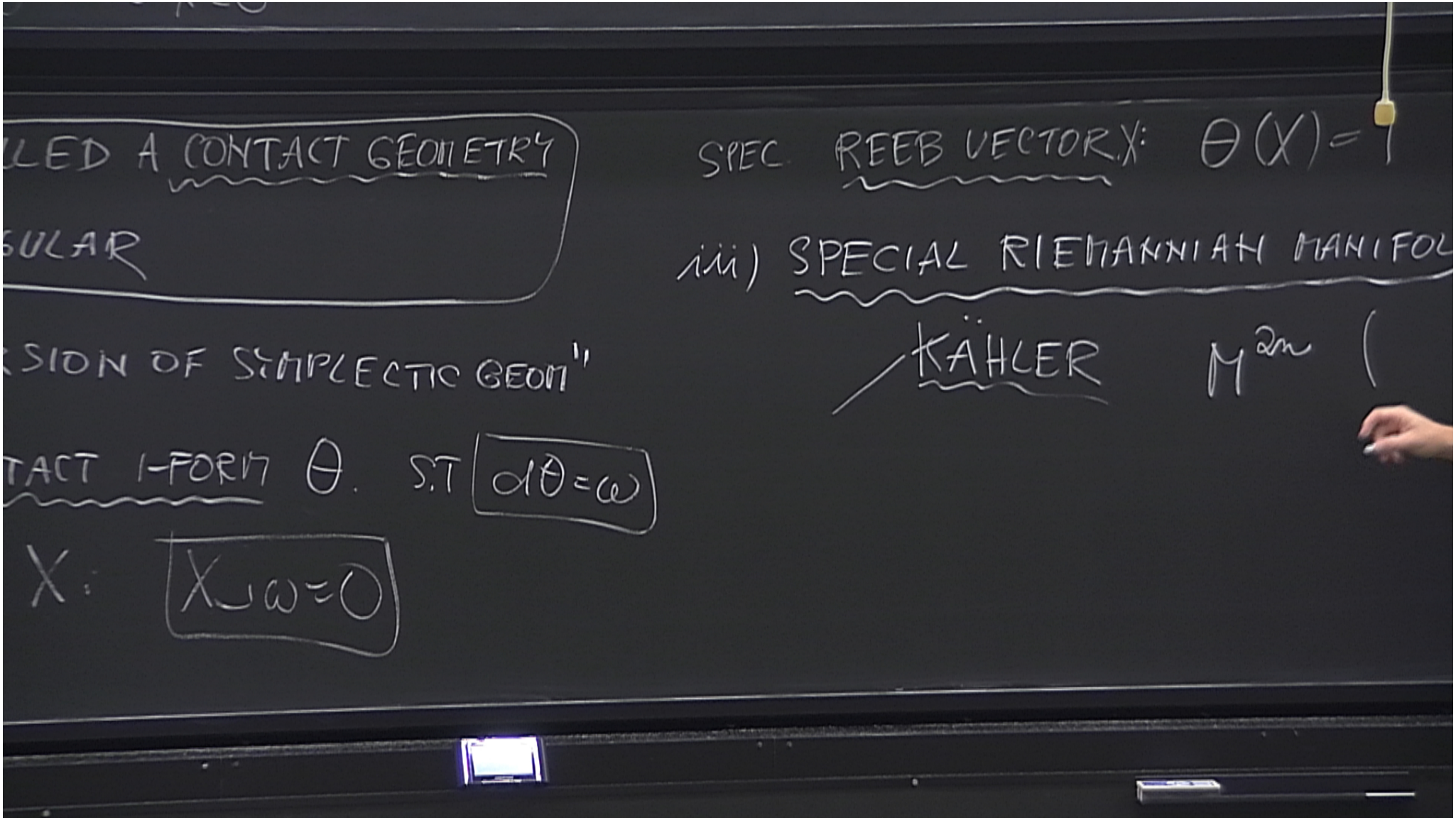
DEF: THE PAIR (M^{2n+1}, ω) IS CALLED A CONTACT GEOME

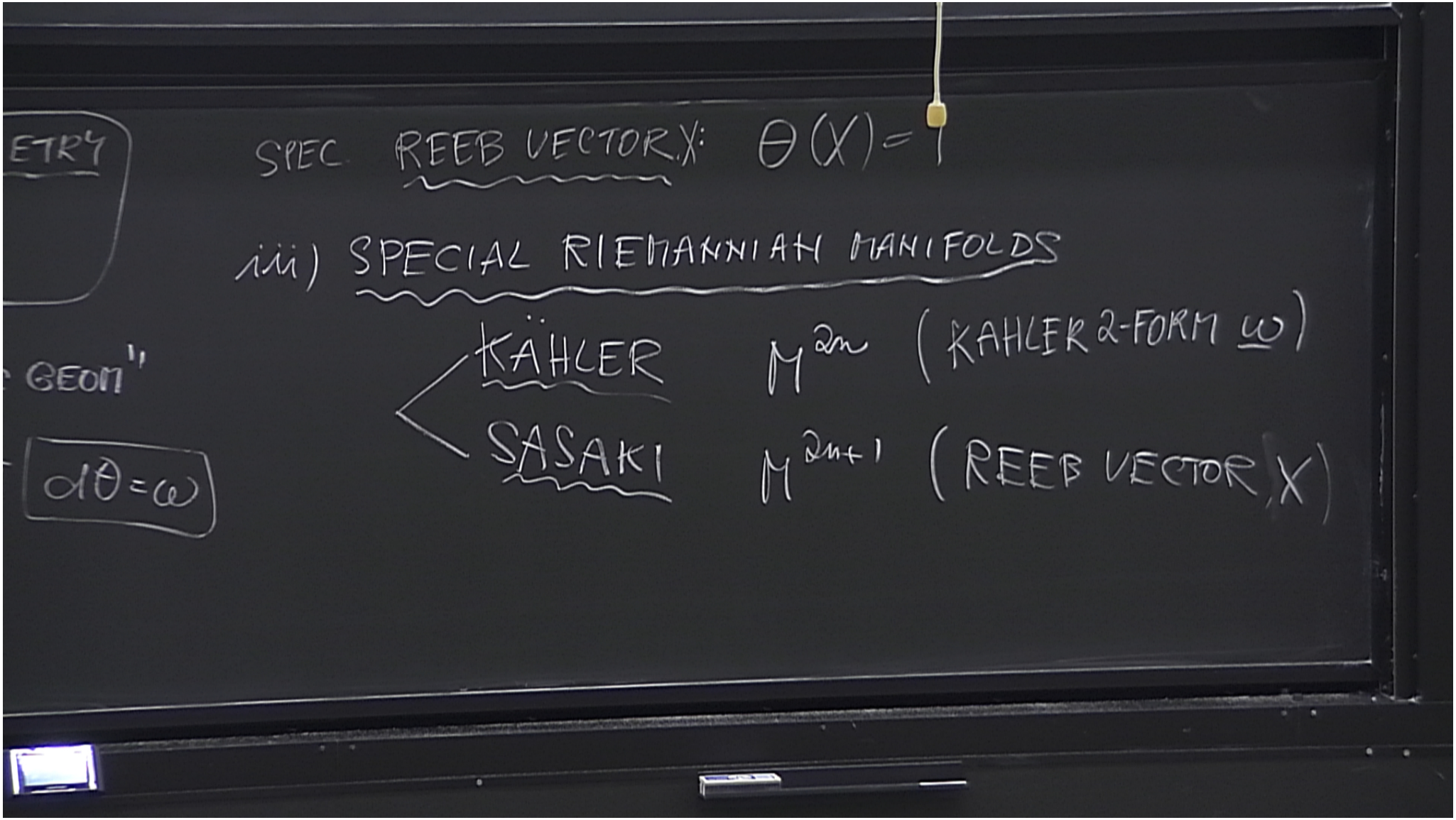
ω IS CLOSED & NON-SINGULAR

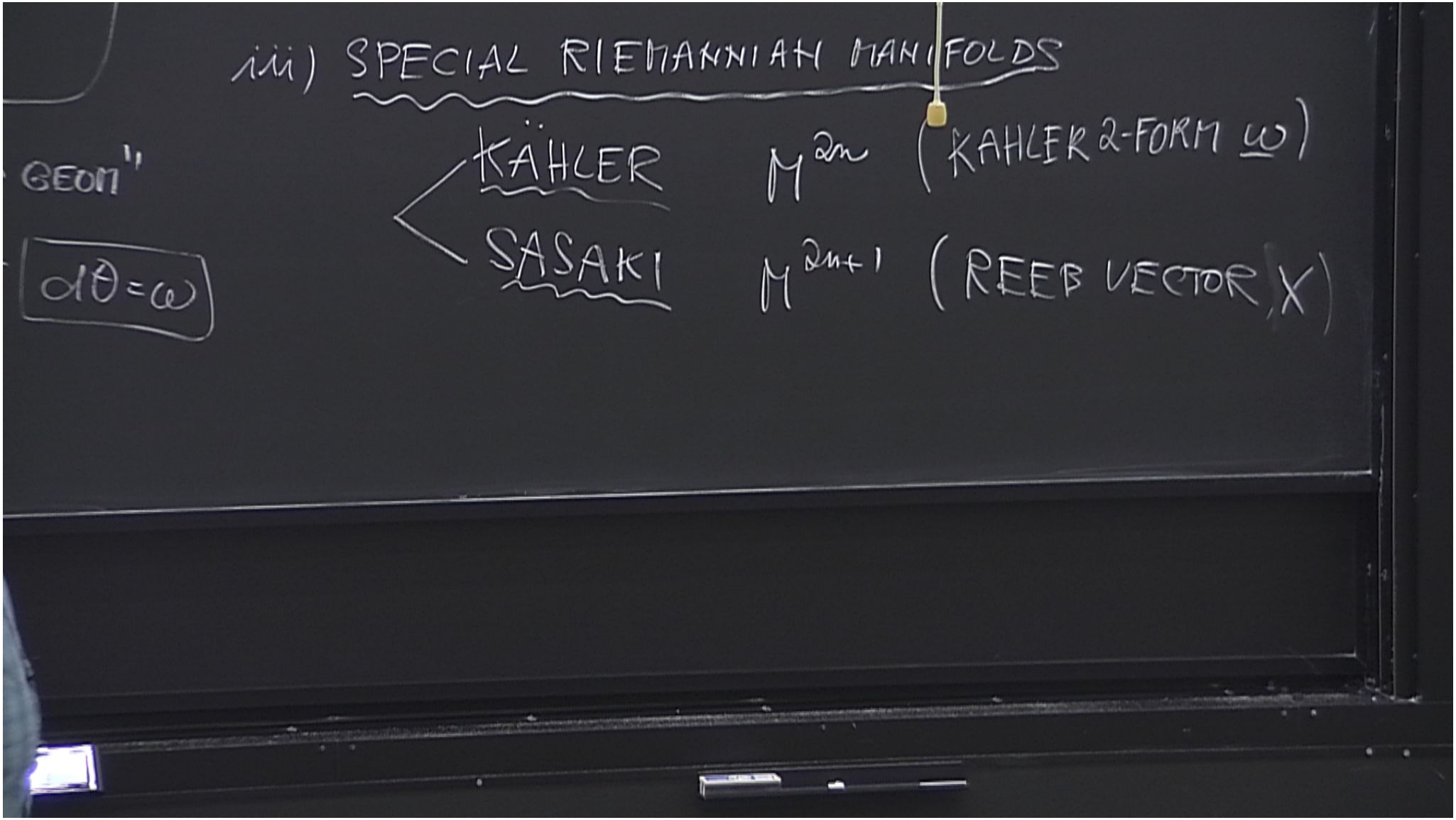
NOTES: i) " ODD-DIMENSIONAL VERSION OF SYMPLECTIC

ii) ω GENERATED BY CONTACT 1-FORM θ. S.T

→ VORTEX DIRECTION X: $\boxed{X \lrcorner \omega = 0}$


LED A CONTACT GEOMETRY
SULAR
SION OF SYMPLECTIC GEOM"
TACT 1-FORM θ. S.T dθ=ω
X: X⌟ω=0
SPEC. REEB VECTOR X: θ(X)=1
iii) SPECIAL RIEMANNIAN MANIFOL
KÄHLER M^2n (


ETRY
GEOM"
$d\theta = \omega$
SPEC. REEB VECTOR X: $\theta(X) = 1$
iii) SPECIAL RIEMANNIAN MANIFOLDS
KÄHLER M^{2n} (KÄHLER 2-FORM ω)
SASAKI M^{2n+1} (REEB VECTOR X)


iii) SPECIAL RIEMANNIAN MANIFOLDS
GEOM''
KÄHLER
SASAKI
M^{2n} (KAHLER 2-FORM ω)
M^{2n+1} (REEB VECTOR X)
$d\theta = \omega$

iv) $\boxed{\mathcal{L}_X \omega = 0}$

PROOF: $\mathcal{L}_X \omega = X \lrcorner \underbrace{d\omega}_{0} + d\left(\underbrace{X \lrcorner \omega}_{0}\right) = 0$

iv) $\boxed{\mathcal{L}_X \omega = 0}$

v)

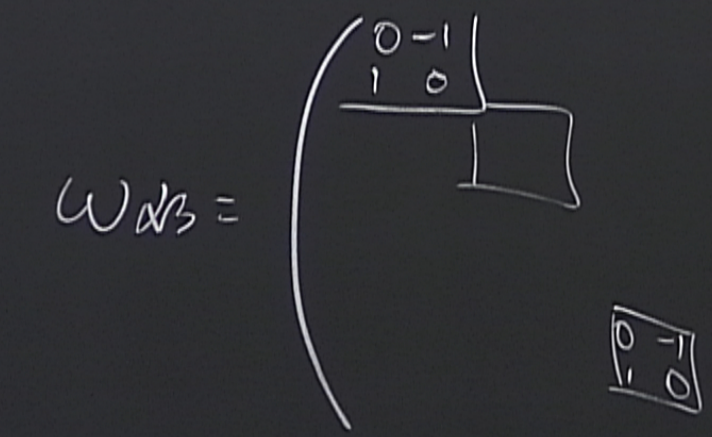
PROOF: $\mathcal{L}_X \omega = X \lrcorner \underbrace{d\omega}_{0} + d\left(\underbrace{X \lrcorner \omega}_{\emptyset}\right) = \emptyset$

SPEC: TIME EVOLUTION $\equiv$ CANONICAL TRANSF ($\mathcal{L}_X \omega = 0$)

V) DARBOUX THEOREM SINCE ω CLOSED & NON-SING.

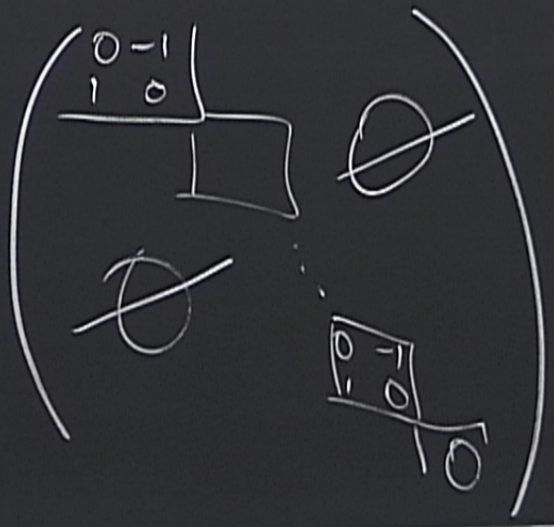
$\Rightarrow \exists$ CANONICAL COORDS (q^i, p_i, t) S.t

$\omega_{\mathbb{R}^3} =$



V) DARBOUX THEOREM SINCE ω CLOSED & NON-SING.

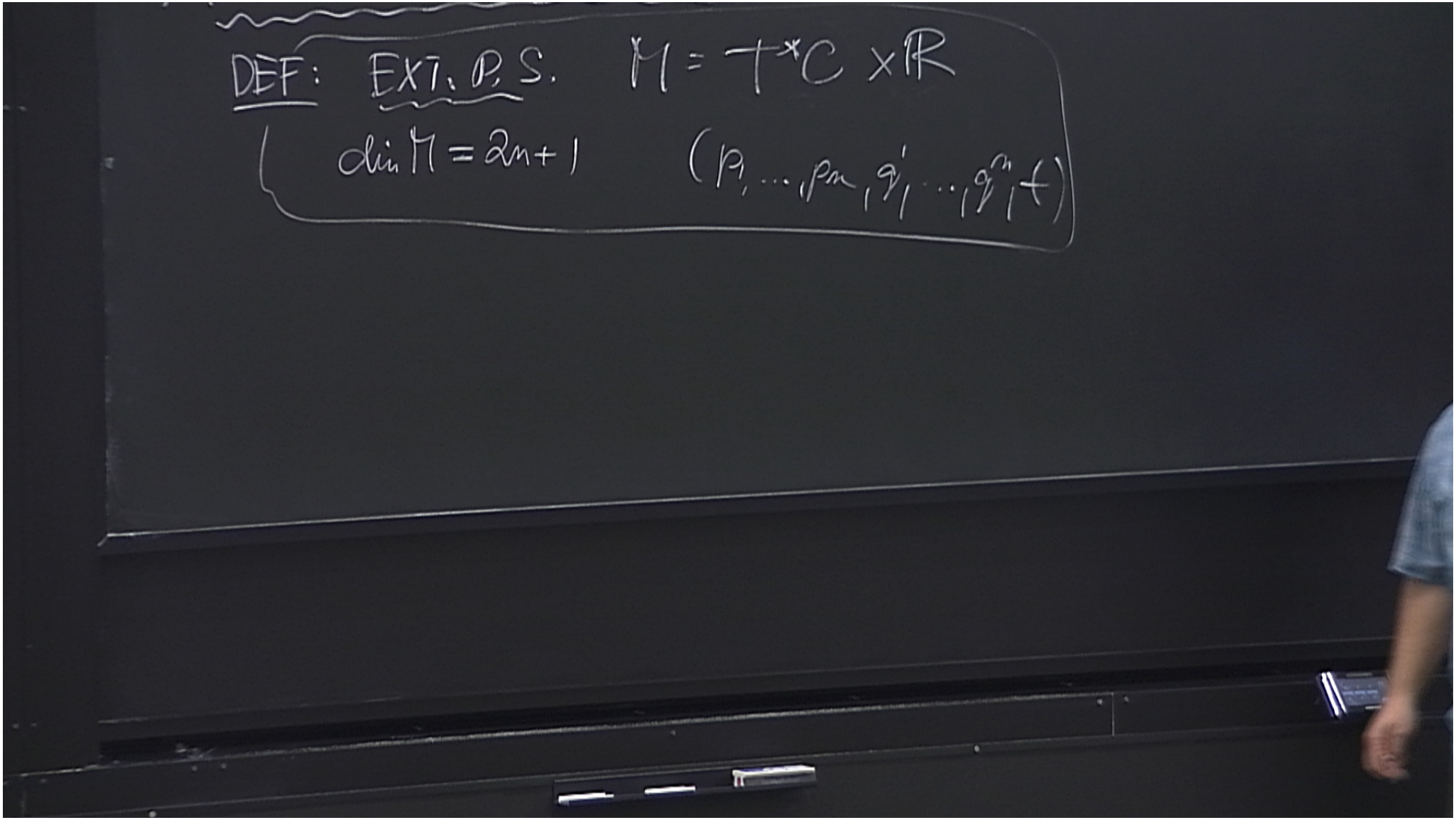
$\Rightarrow \exists$ CANONICAL COORDS (q^i, p_i, t) S.t

$\omega_{ab} =$ 

$\omega = \vec{dp_i} \wedge \vec{dq^i}$

$= dp_i \wedge dq^i$

$X = \frac{\partial}{\partial t}$


DEF: EXT. P.S. $M = T^*C \times \mathbb{R}$
$\dim M = 2n+1$ $(p_1, \dots, p_n, q^1, \dots, q^n, t)$

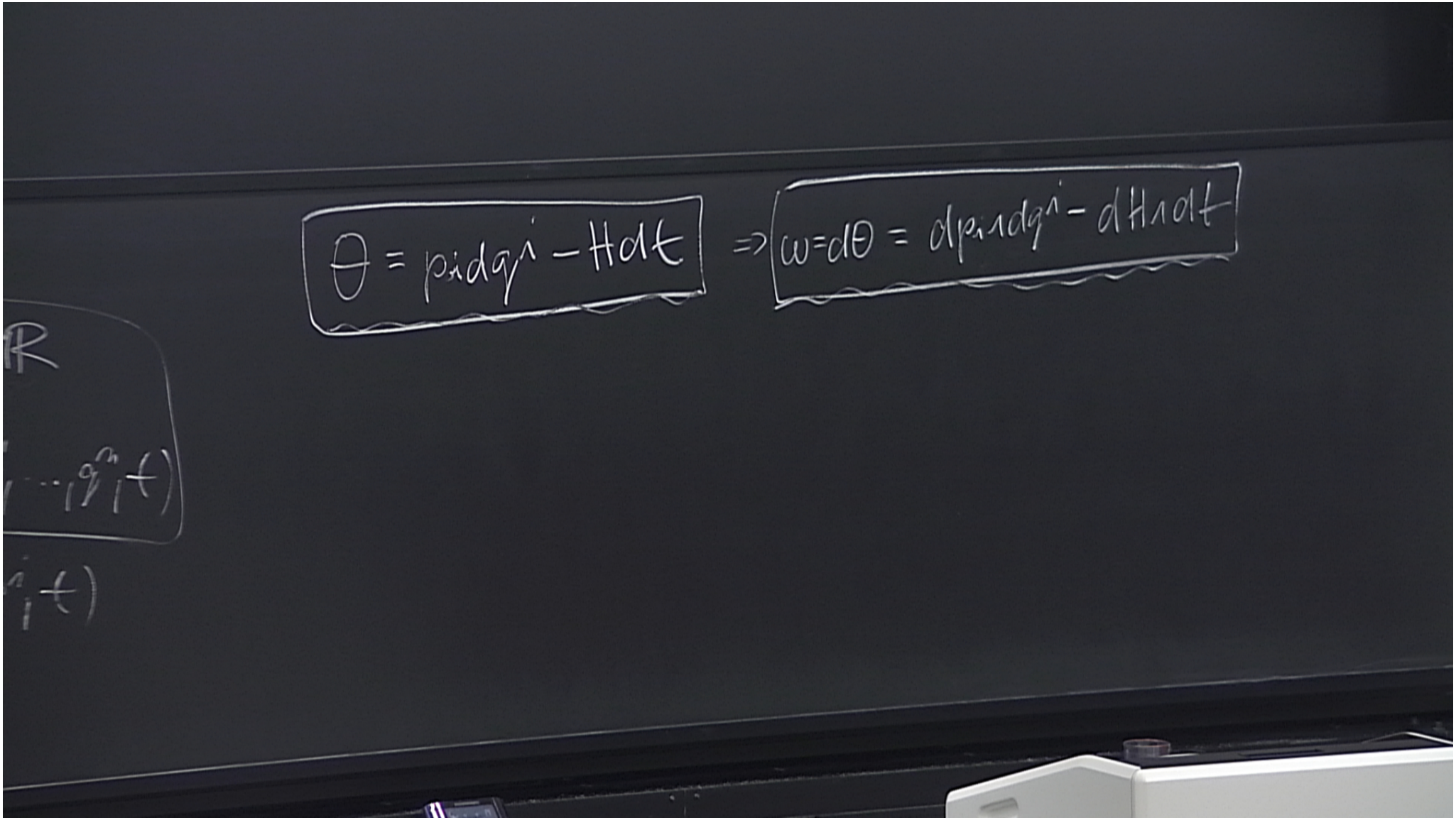
1) EXTENDED PHASE SPACE

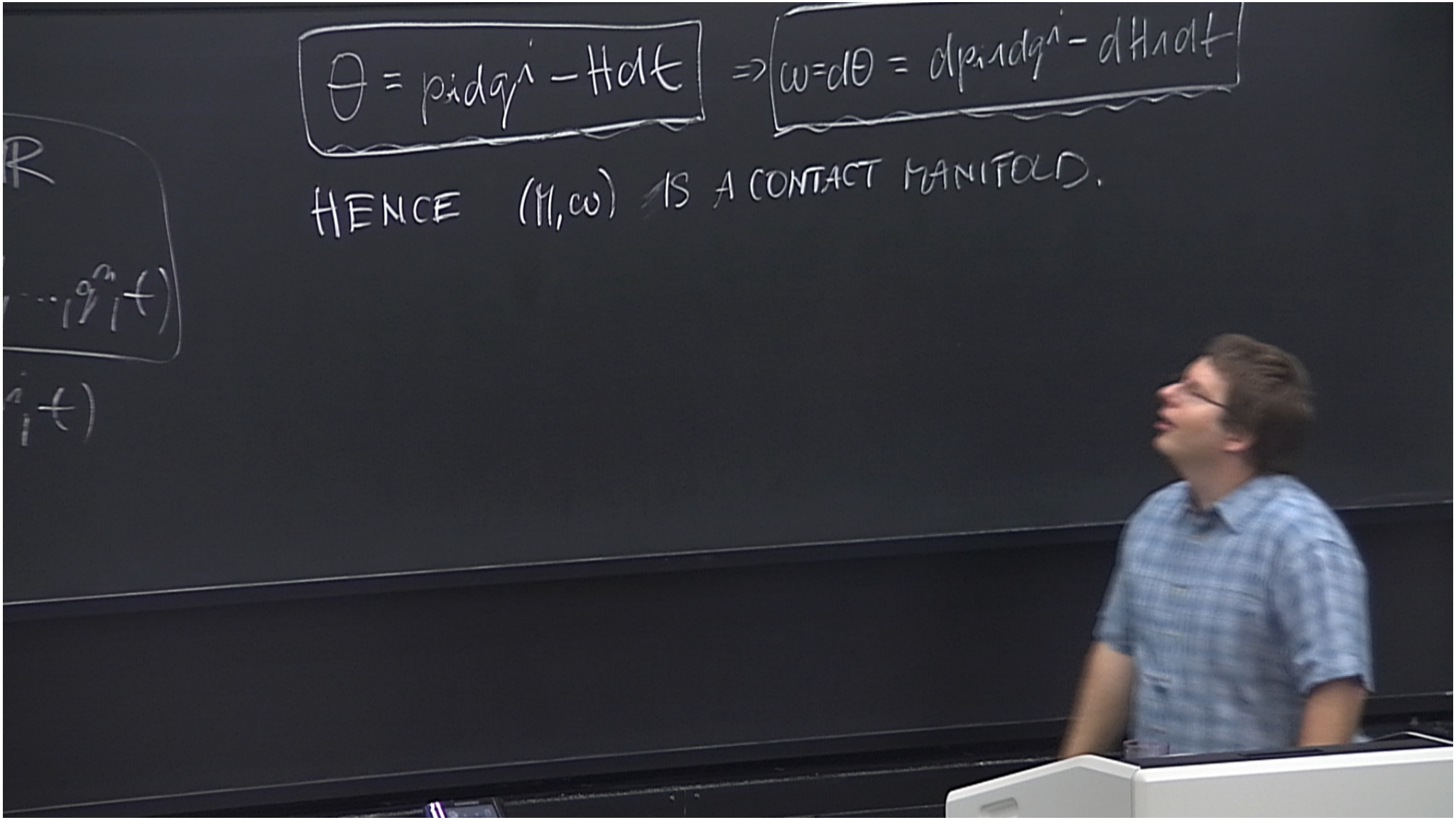
DEF: EXT. P.S. $M = T^*C \times \mathbb{R}$

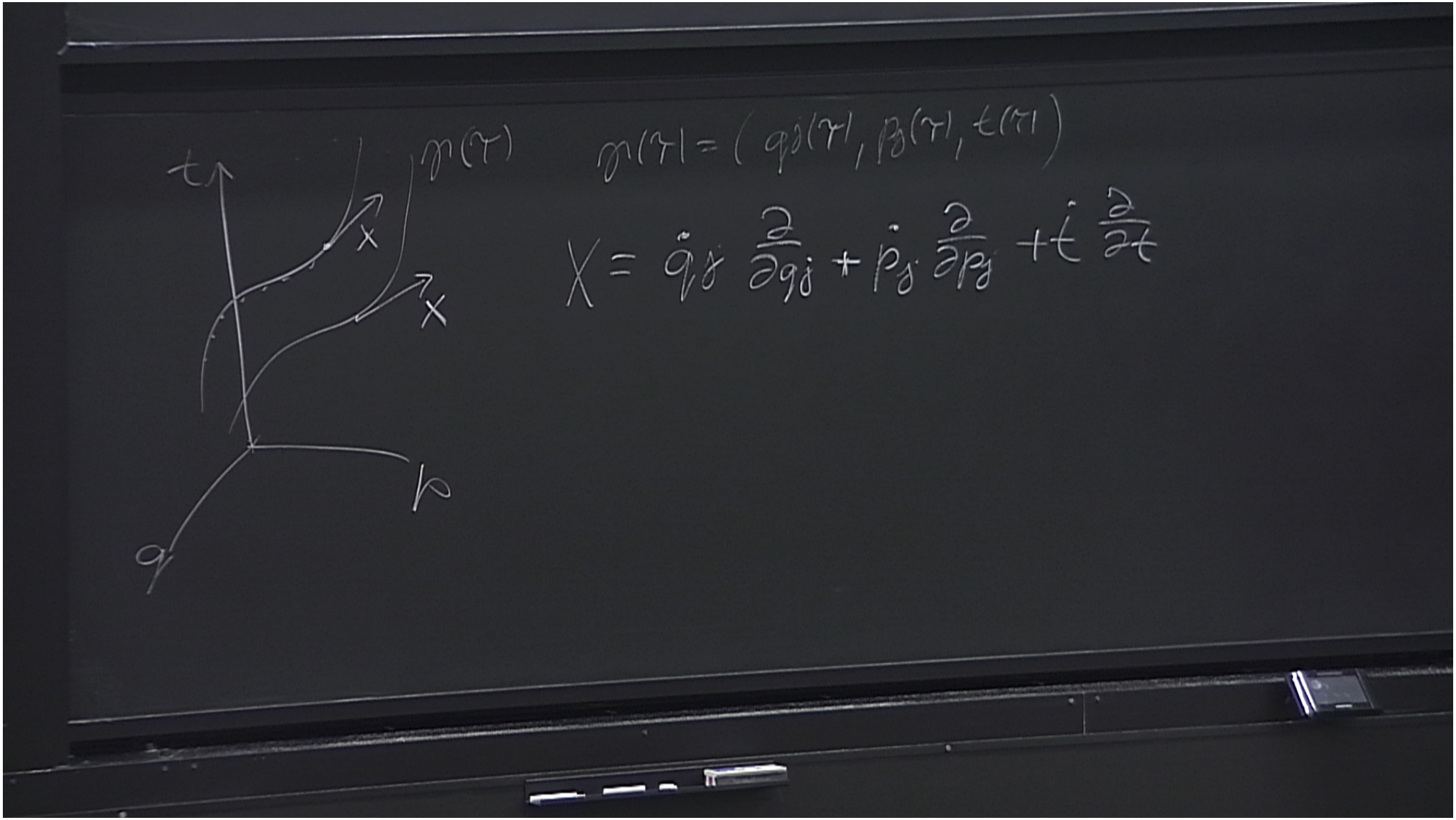
$\dim M = 2n+1 \qquad (p_1, \dots, p_n, q^1, \dots, q^n, t)$

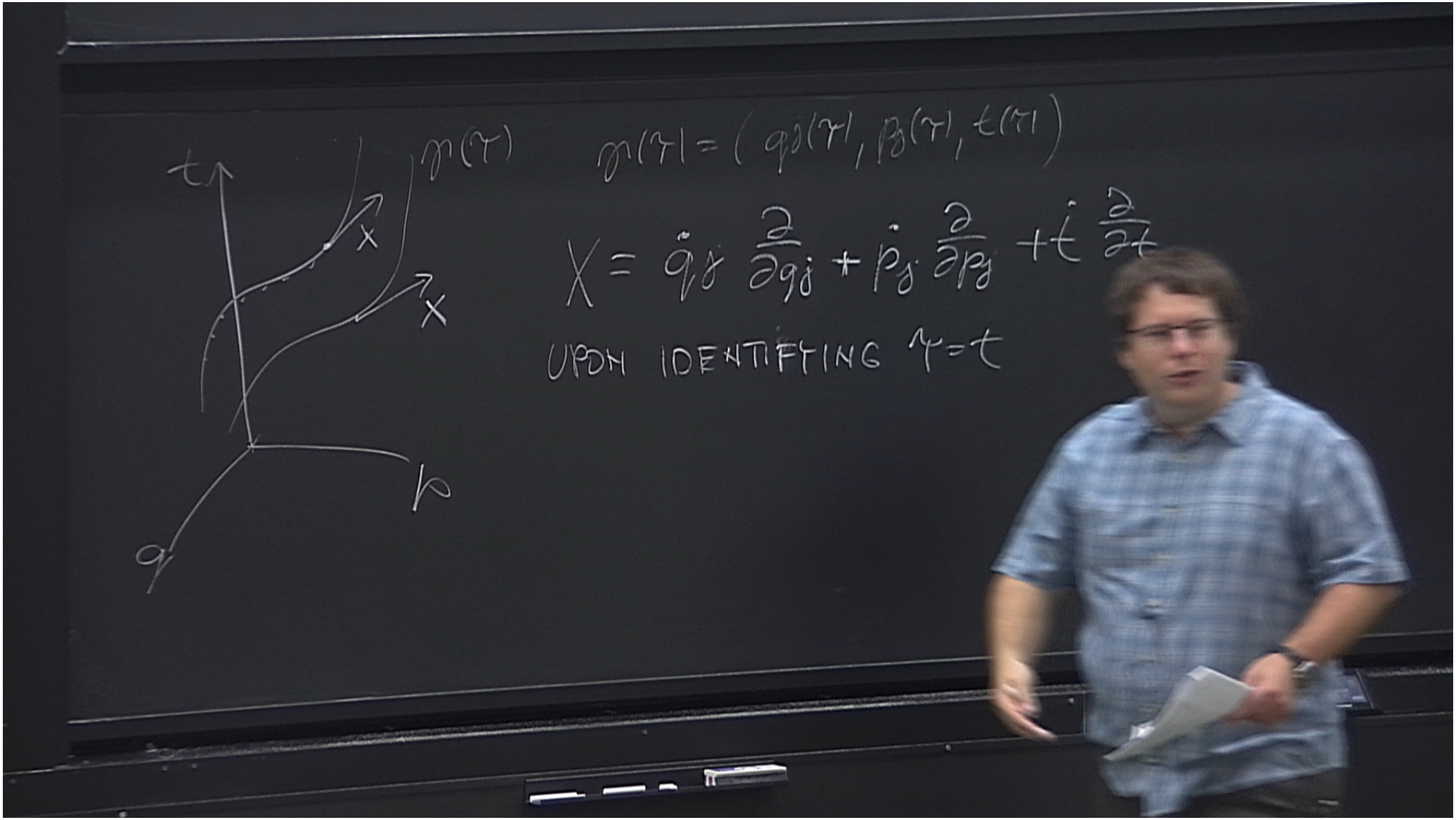
- GIVEN A HAMILTONIAN F. $H(p_i, q^i, t)$

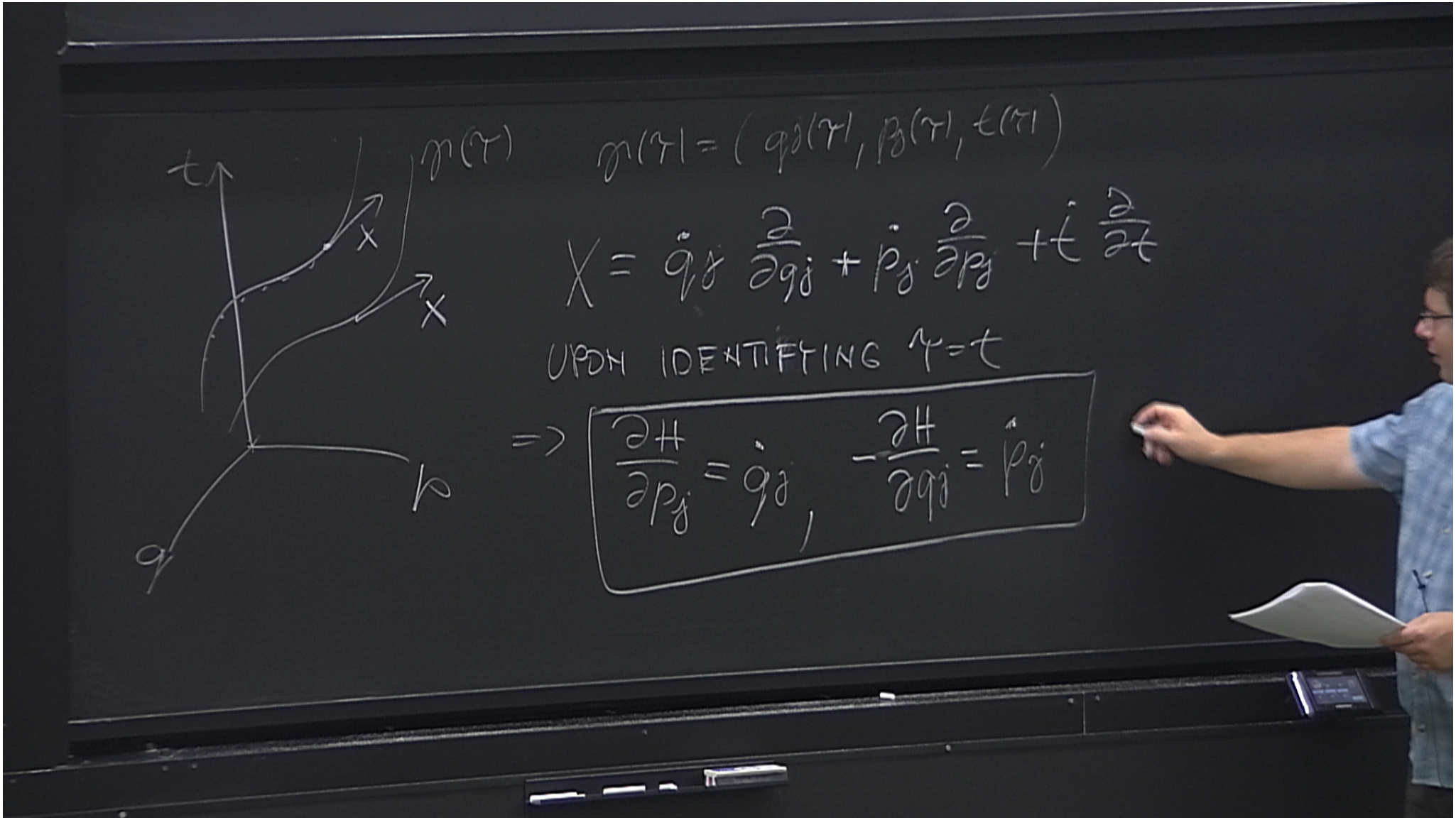
  M ADMITS CONTACT 1-FORM

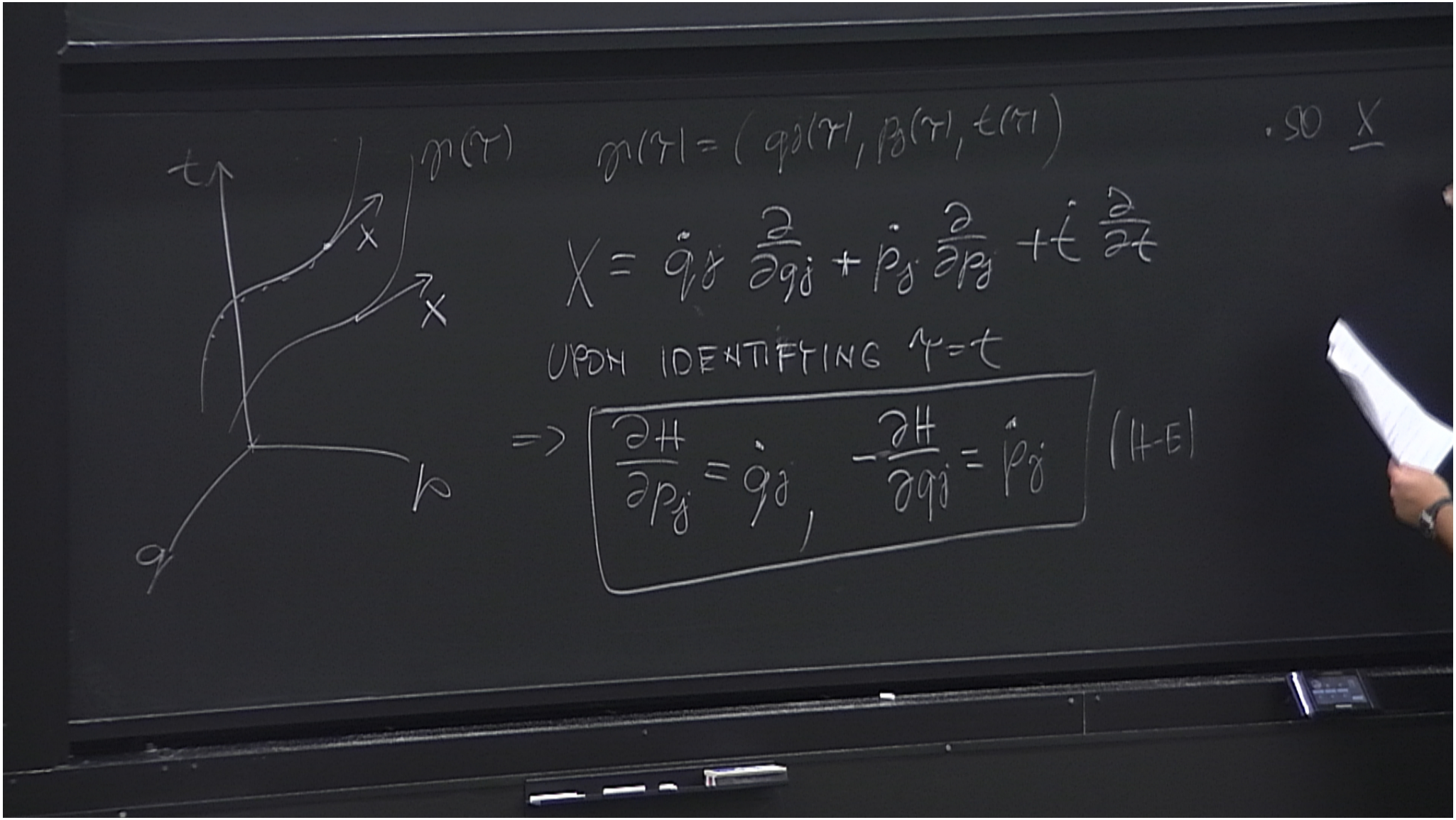

$\theta = p_i dq^i - H dt$
$\Rightarrow \omega = d\theta = dp_i \wedge dq^i - dH \wedge dt$

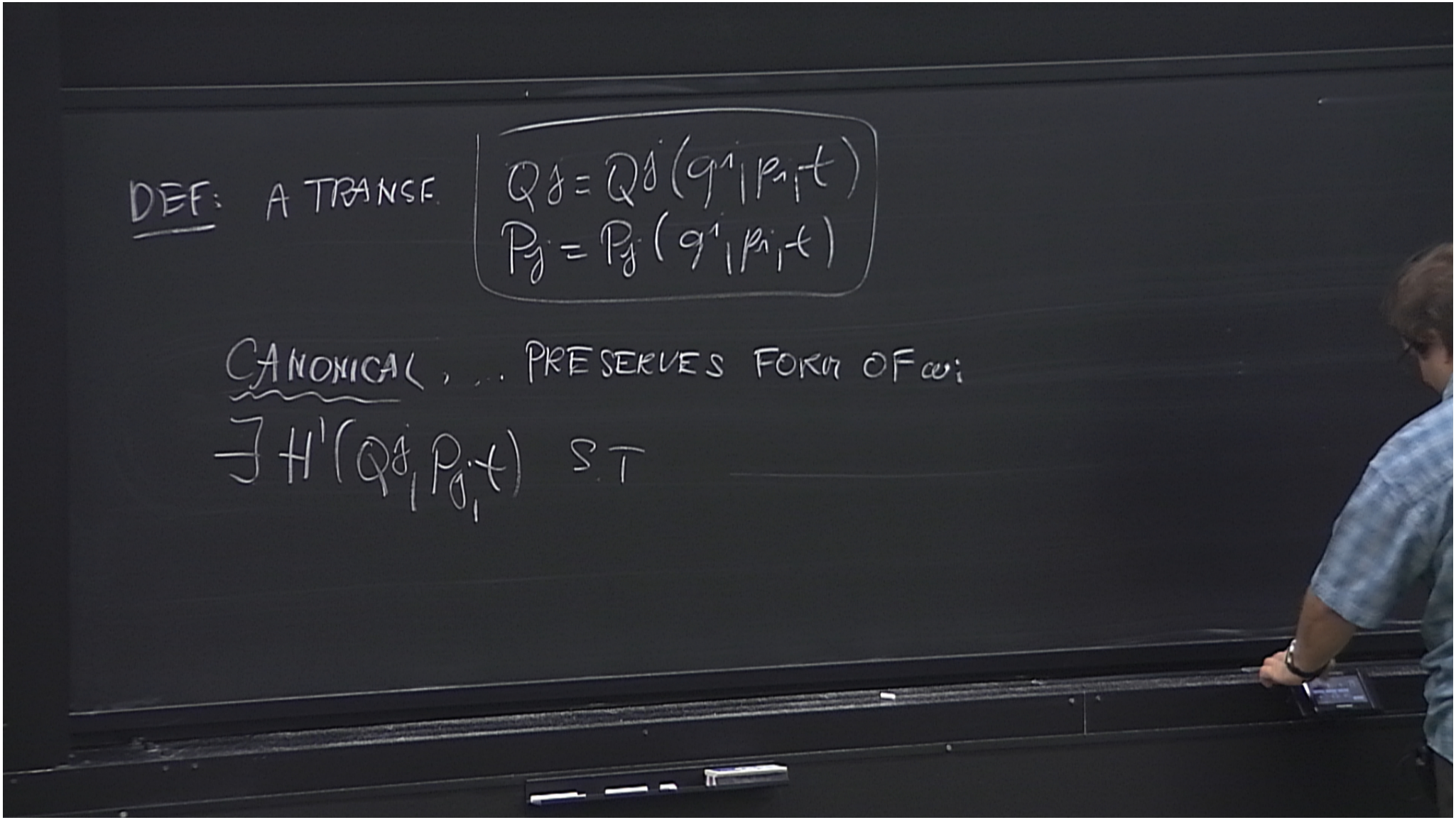

$\theta = p_i dq^i - H dt$
$\Rightarrow \omega = d\theta = dp_i \wedge dq^i - dH \wedge dt$
HENCE (M, ω) IS A CONTACT MANIFOLD.

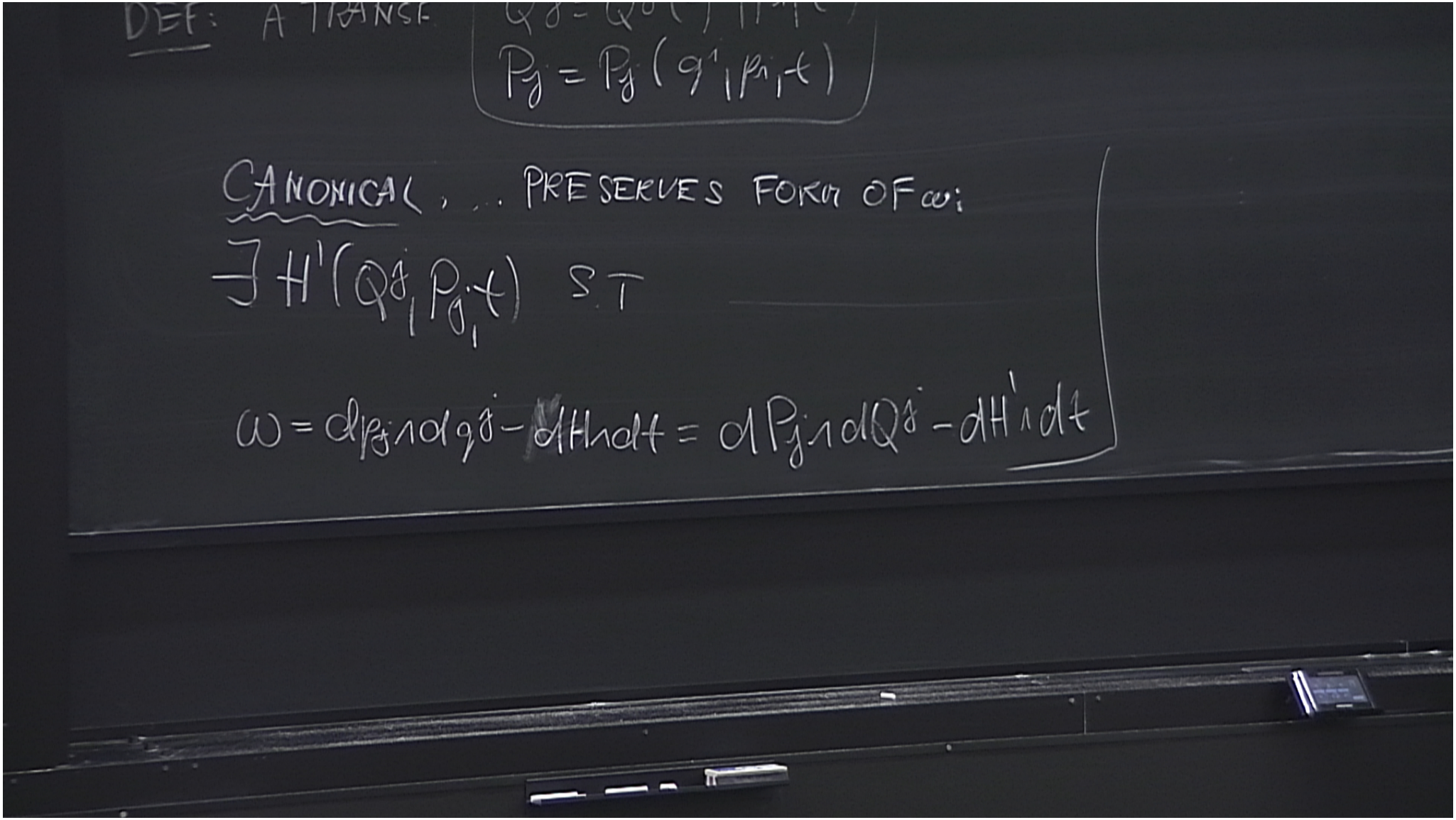

t
x
x
p
q
$\gamma(\tau)$
$\gamma(\tau) = (q_j(\tau), p_j(\tau), t(\tau))$
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$

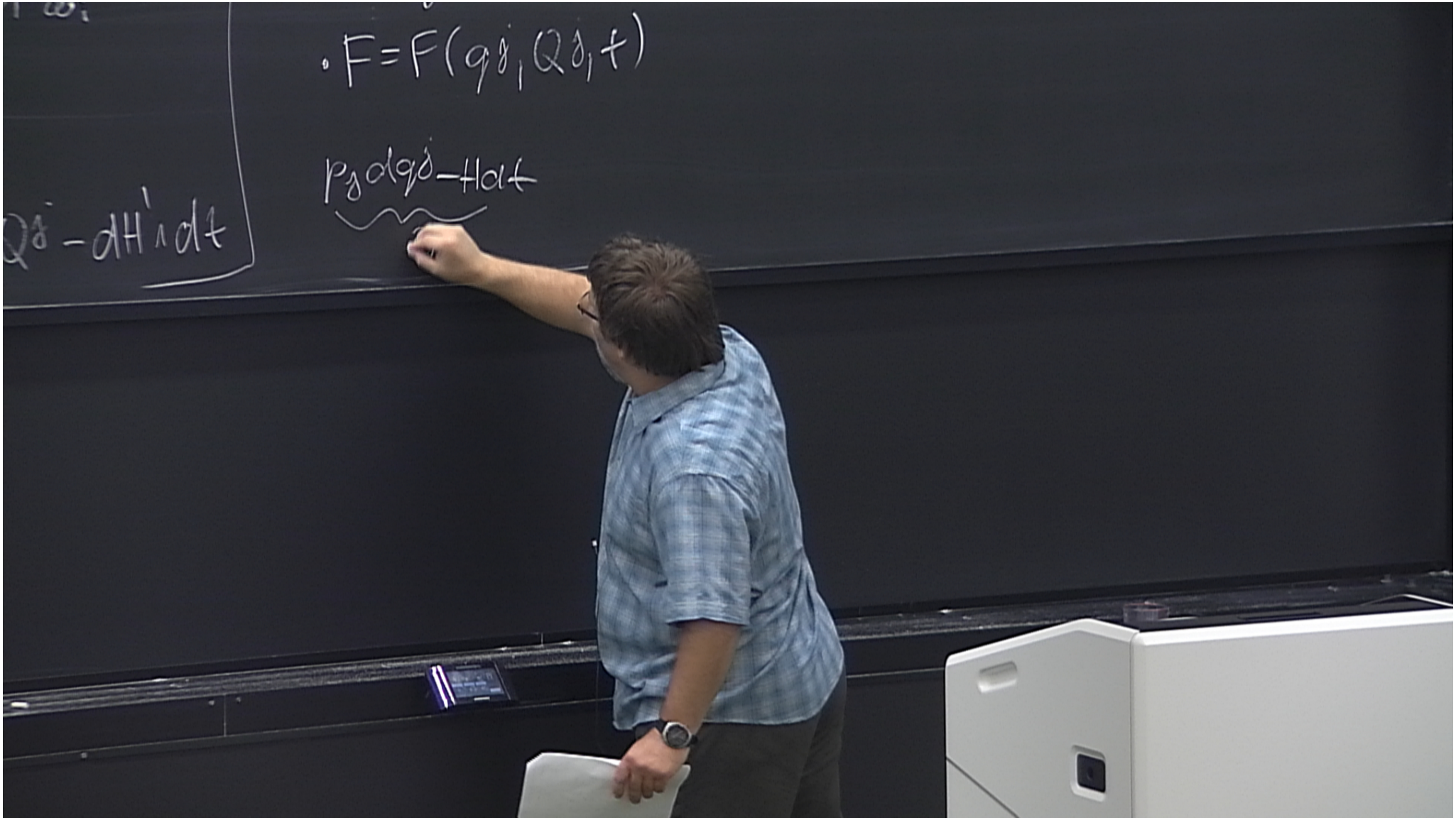

t
x
x
p
q
$\gamma(\tau)$
$\gamma(\tau) = (q_j(\tau), p_j(\tau), t(\tau))$
$X = \dot{q}_j \frac{\partial}{\partial q_j} + \dot{p}_j \frac{\partial}{\partial p_j} + \dot{t} \frac{\partial}{\partial t}$
UPON IDENTIFYING $\tau = t$

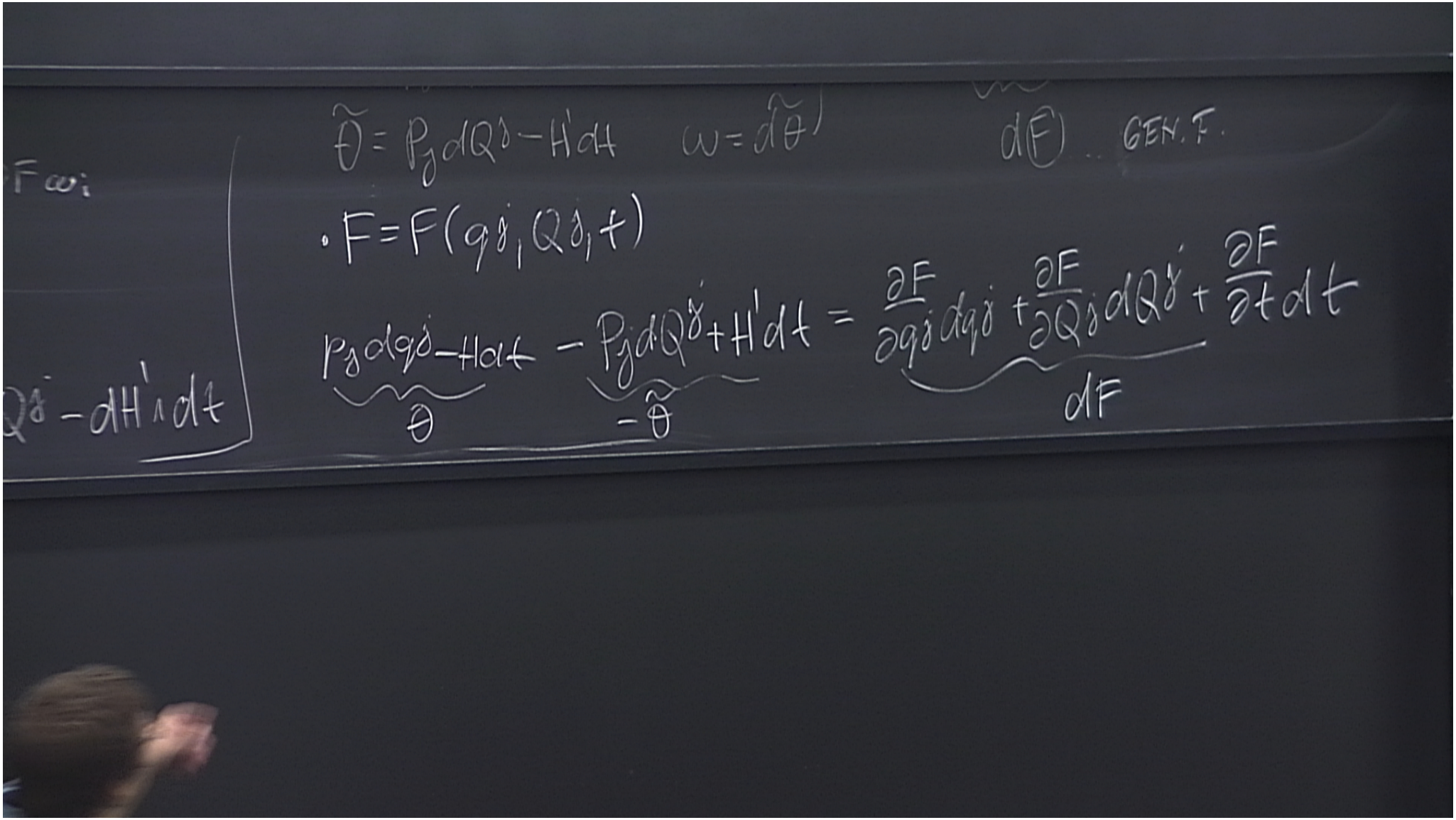

t
x(τ)
x
x
p
q
x(τ) = (q_j(τ), p_j(τ), t(τ))
X = q̇_j ∂/∂q_j + ṗ_j ∂/∂p_j + ṫ ∂/∂t
UPON IDENTIFYING τ = t
=> ∂H/∂p_j = q̇_j , −∂H/∂q_j = ṗ_j

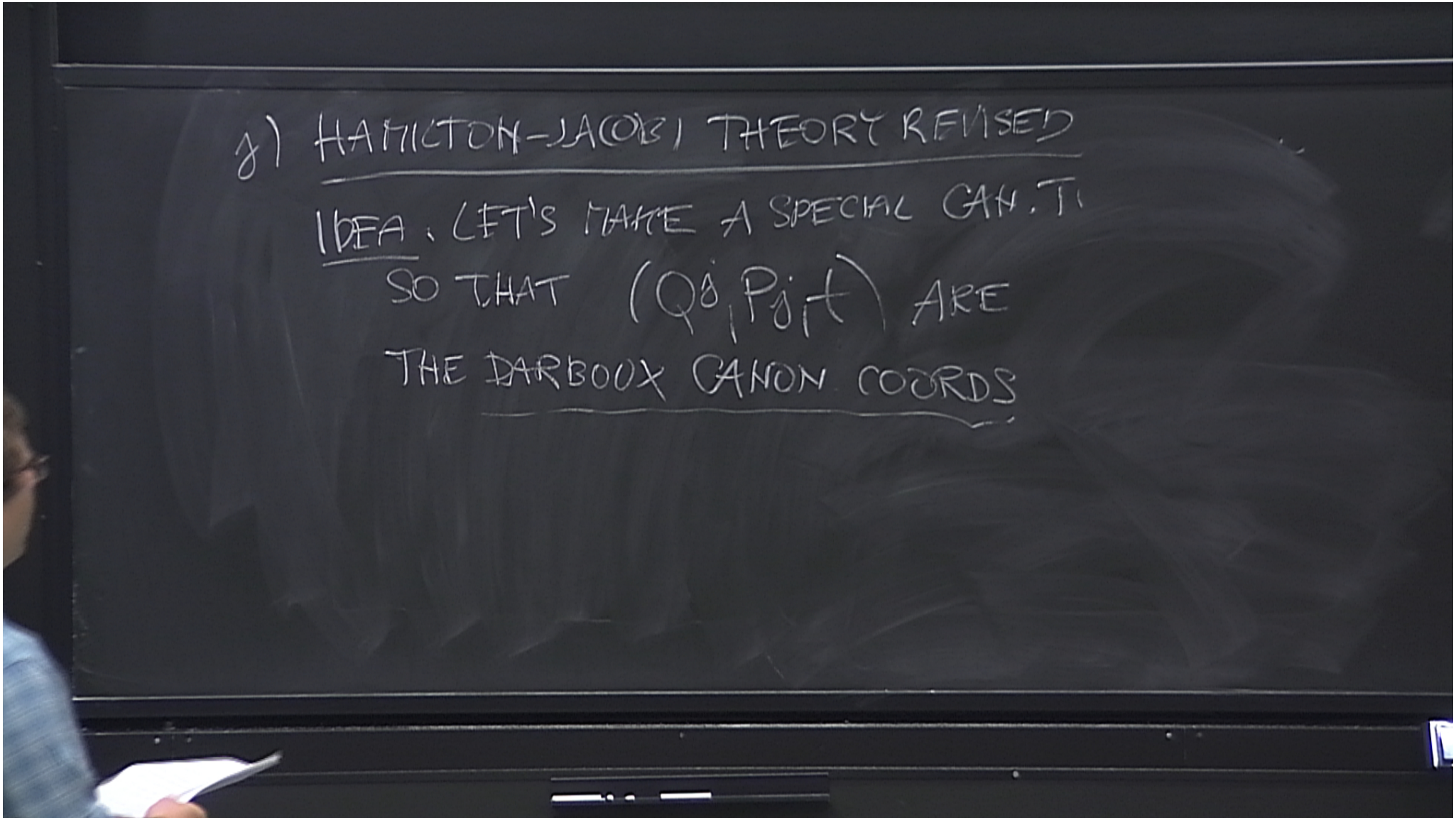

t
x(τ)
x
x
p
q
x(τ) = (qj(τ), pj(τ), t(τ))
.so X
X = q̇j ∂/∂qj + ṗj ∂/∂pj + ṫ ∂/∂t
UPON IDENTIFYING τ = t
=> ∂H/∂pj = q̇j , −∂H/∂qj = ṗj
(H-E)

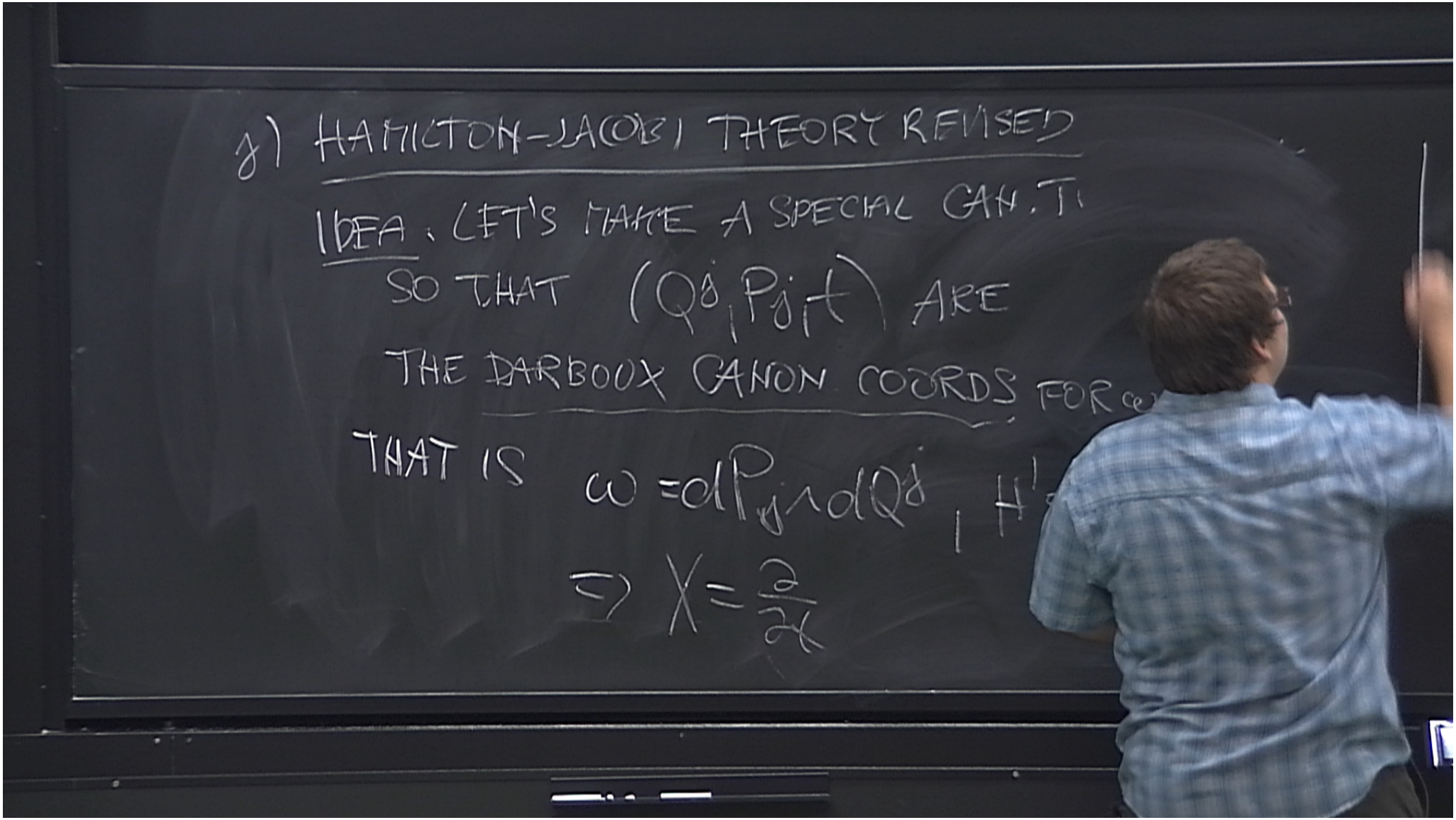

DEF: A TRANSF
$Q^j = Q^j(q^i, p_i, t)$
$P_j = P_j(q^i, p_i, t)$
CANONICAL, ... PRESERVES FORM OF ω:
$\exists H'(Q^j, P_j, t)$ S.T

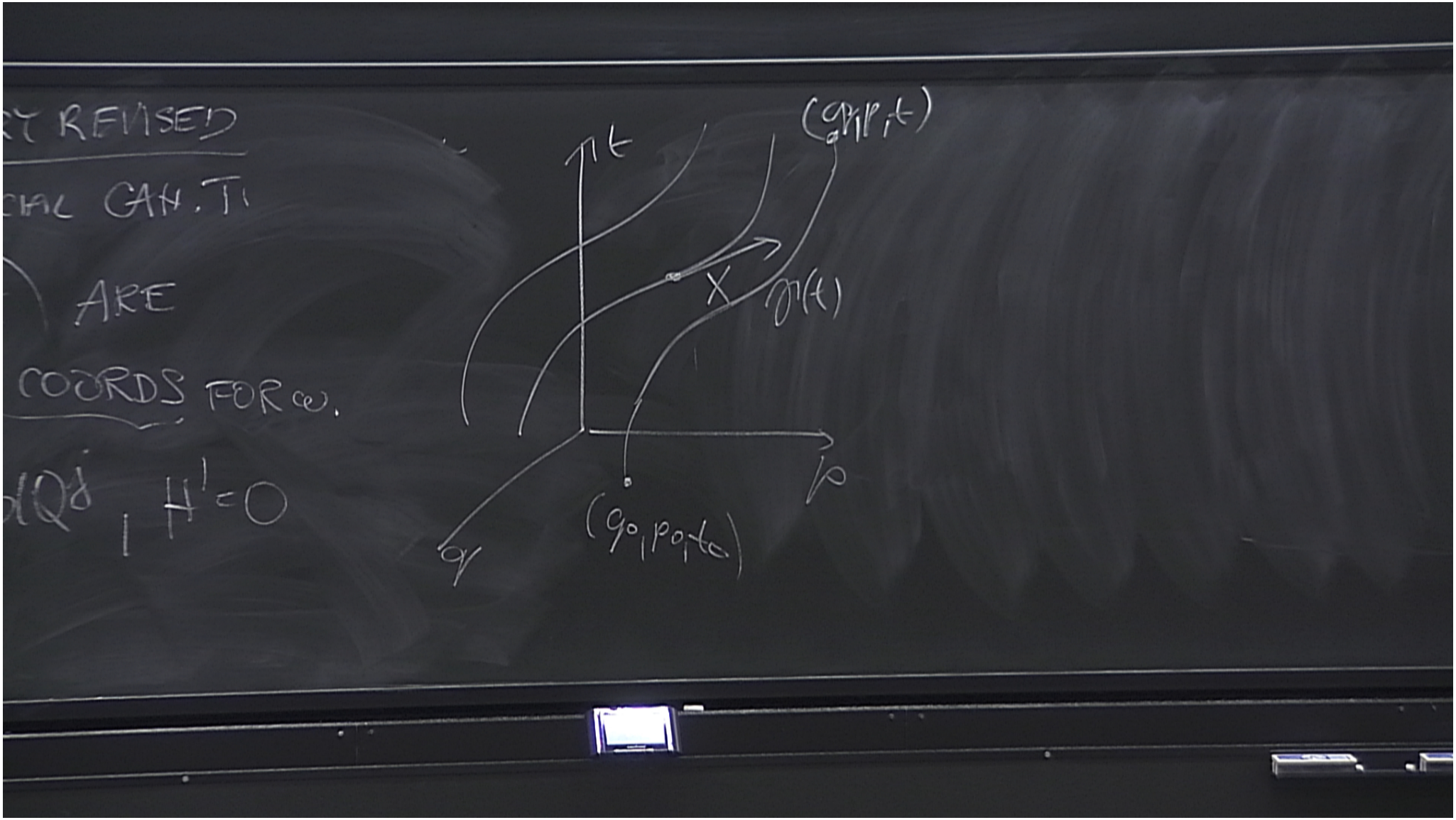

DEF: A TRANSF
$P_j = P_j(q^i, P_i, t)$
CANONICAL, ... PRESERVES FORM OF ω:
$\exists H'(Q^i, P_i, t)$ S.T
$\omega = dP_j \wedge dq^i - dH \wedge dt = dP_j \wedge dQ^i - dH' \wedge dt$

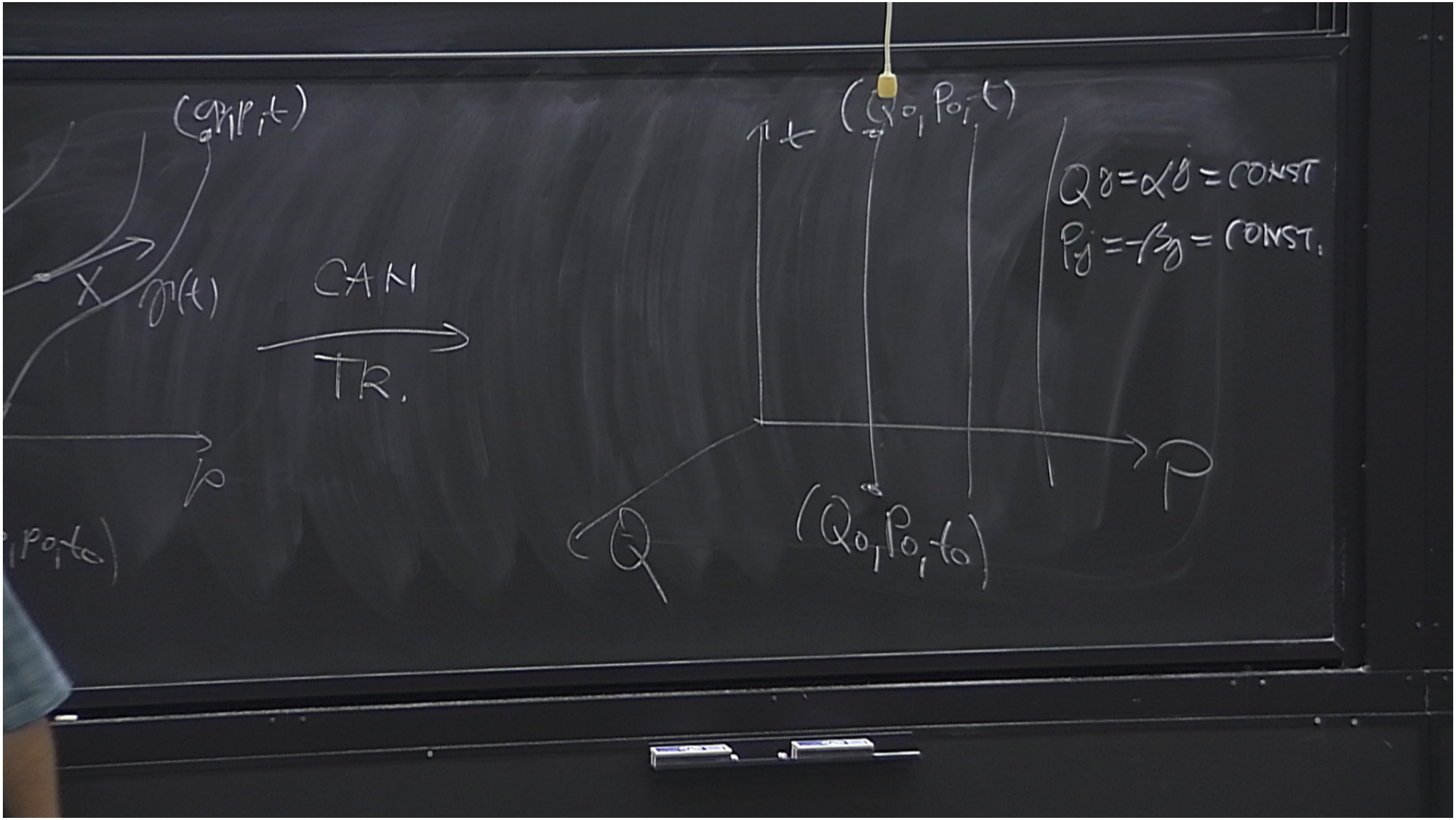

F=F(qi,Qi,t)
Qi - dH'/dt

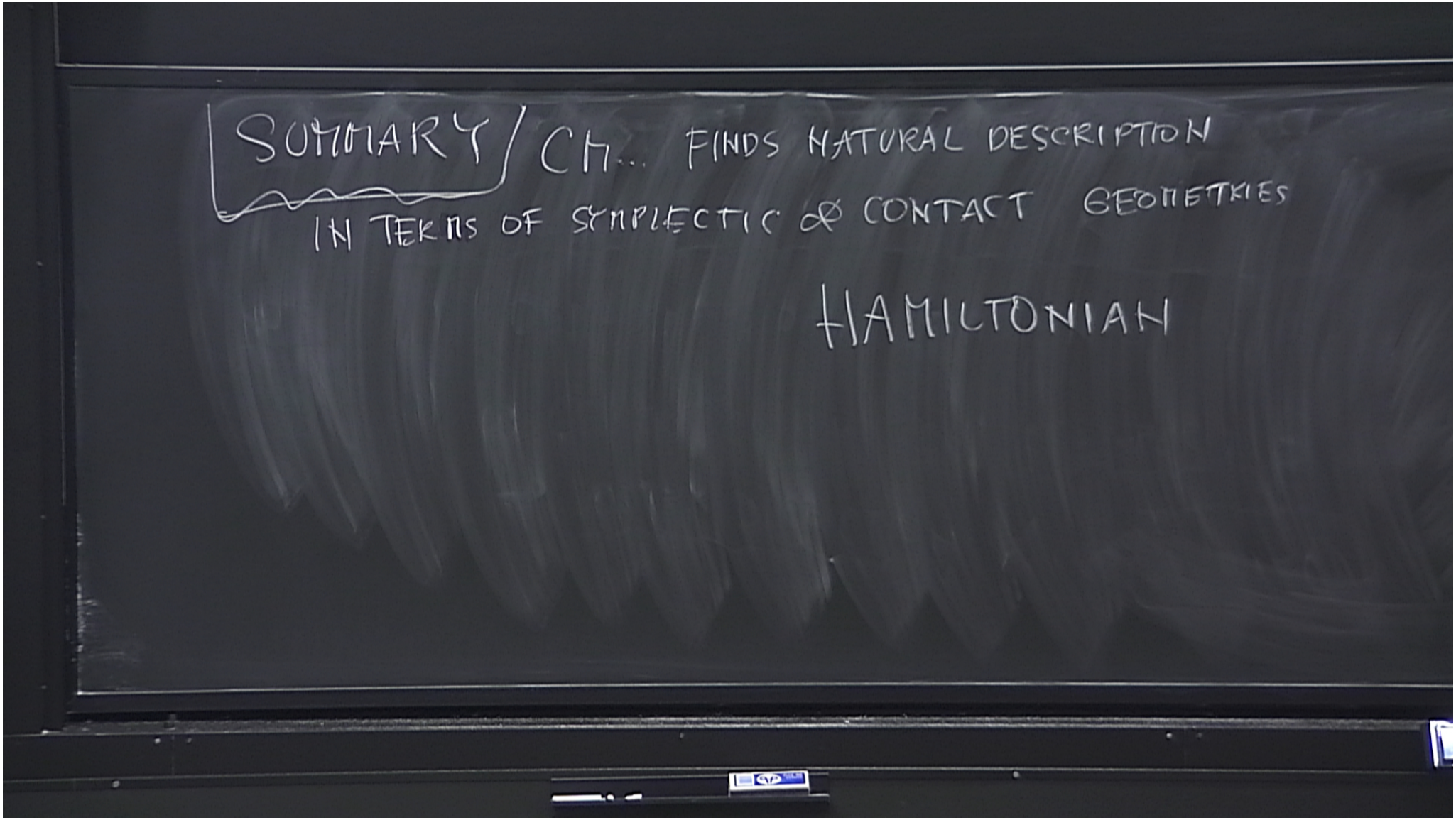

$\tilde{\theta} = P_j dQ^j - H' dt \quad \omega = d\tilde{\theta}$
$d(F)$ .. GEN. F.
$\cdot F = F(q^j, Q^j, t)$
$P_j dq^j - H dt - P_j dQ^j + H' dt = \frac{\partial F}{\partial q^j} dq^j + \frac{\partial F}{\partial Q^j} dQ^j + \frac{\partial F}{\partial t} dt$
θ
$-\tilde{\theta}$
dF
$Q^j - dH' \wedge dt$

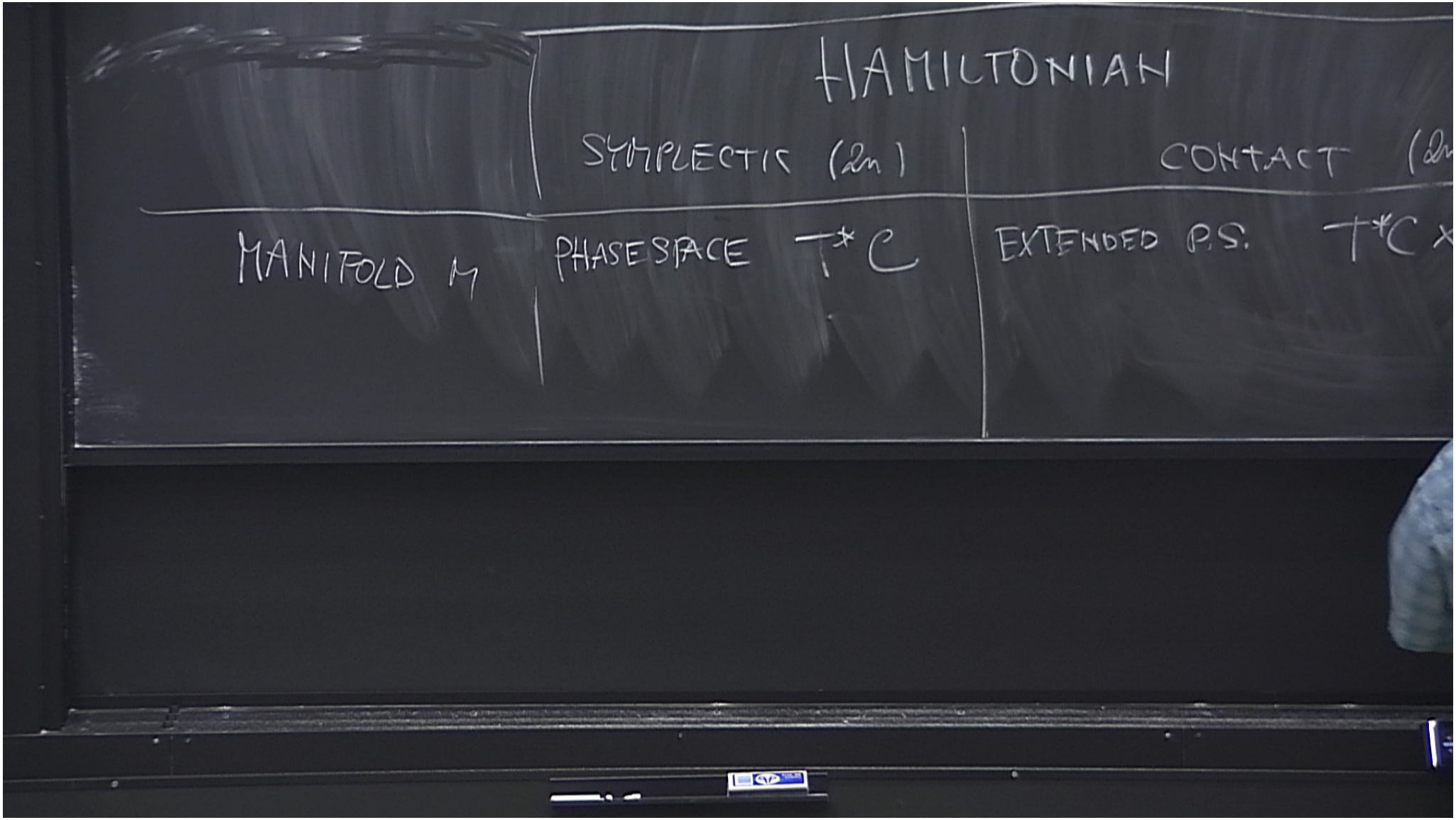

7) HAMILTON-JACOBI THEORY REVISED
IDEA: LET'S MAKE A SPECIAL CAN. T
SO THAT (Qi, Pj, t) ARE
THE DARBOUX CANON COORDS

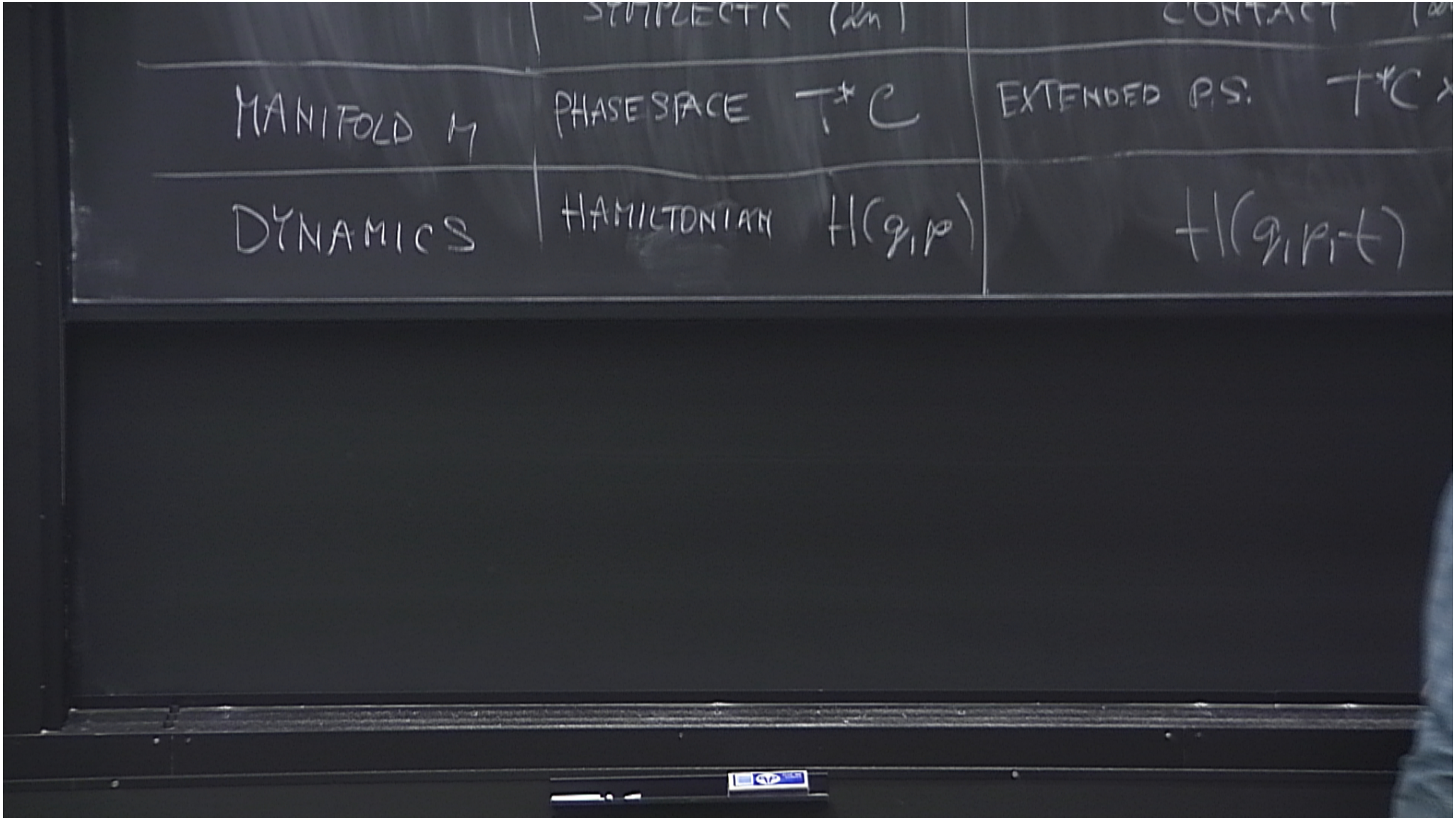

7) HAMILTON-JACOBI THEORY REVISED
IDEA: LET'S MAKE A SPECIAL CAN. T
SO THAT (Q^i, P_j, t) ARE
THE DARBOUX CANON COORDS FOR
THAT IS $\omega = dP_j \wedge dQ^j$, H'
$\Rightarrow \chi = \frac{2}{2t}$

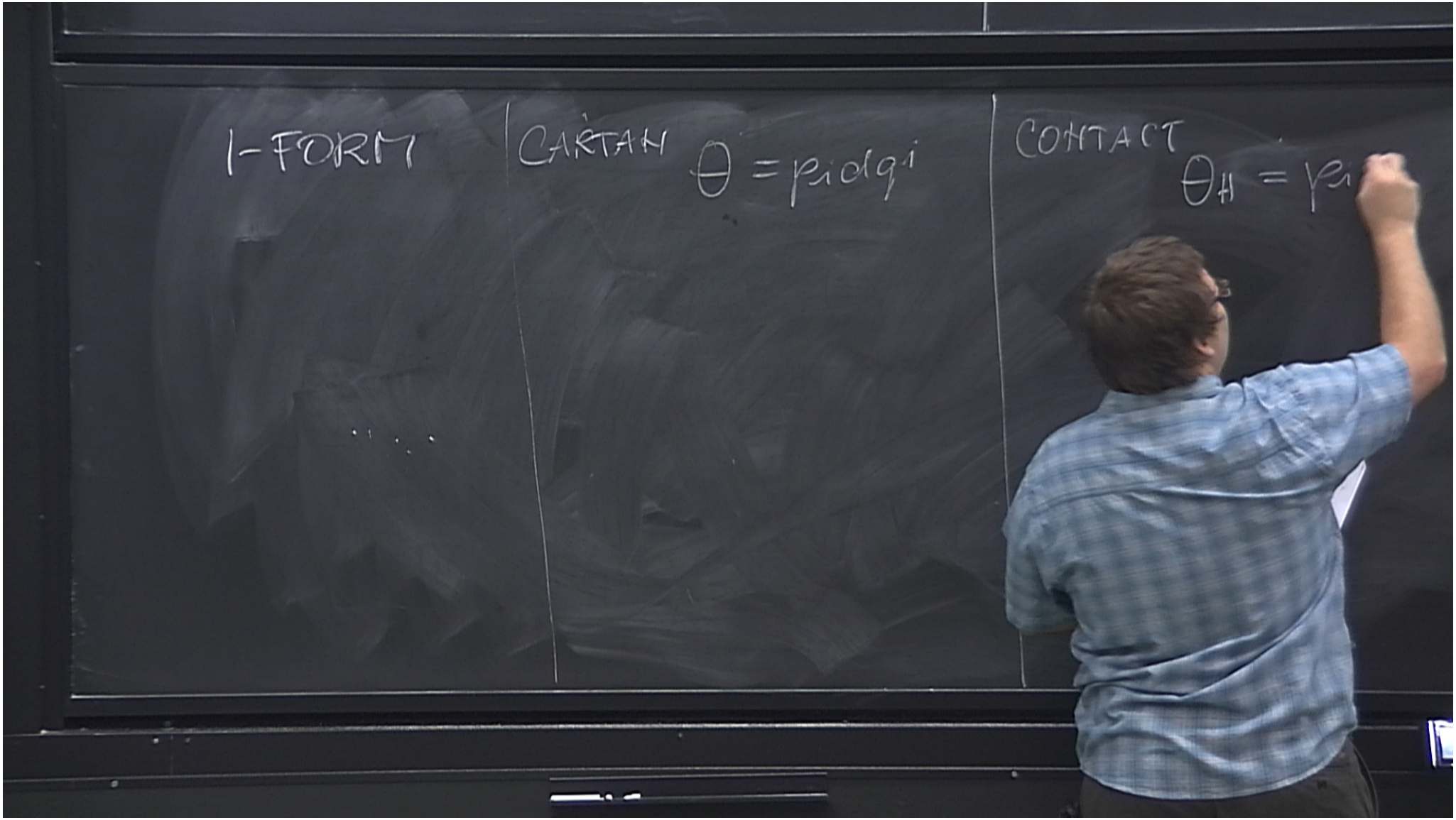

REVISED
CAN. TI
ARE
COORDS FOR ∞.
H'=0
π t
(q*, p*, t)
X
π(t)
p
q
(q°, p°, t°)

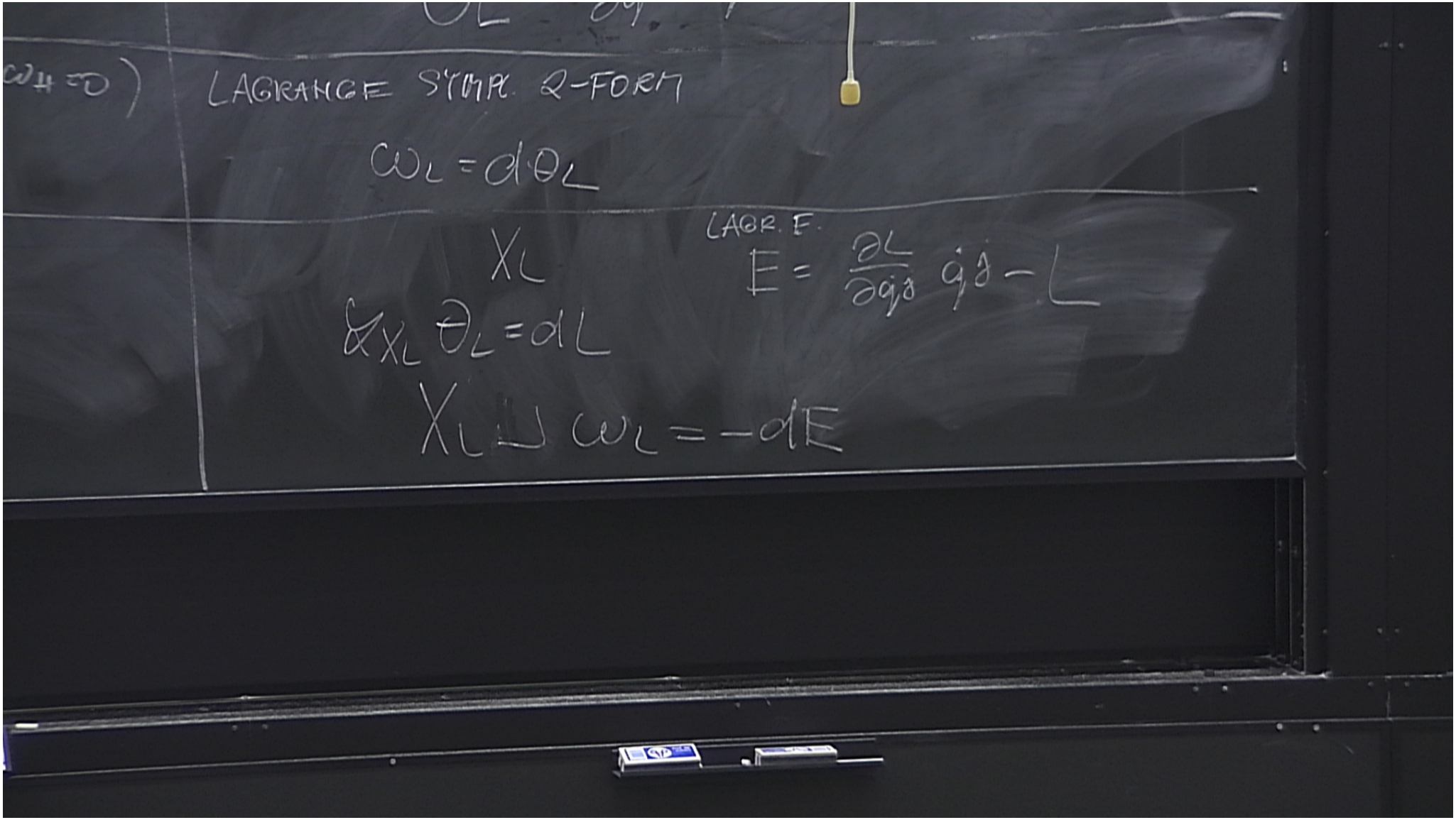

(Q0, P0, t)
X
CAN
TR.
P
Q
(Q0, P0, to)
P
Qj = αj = CONST.
Pj = βj = CONST.

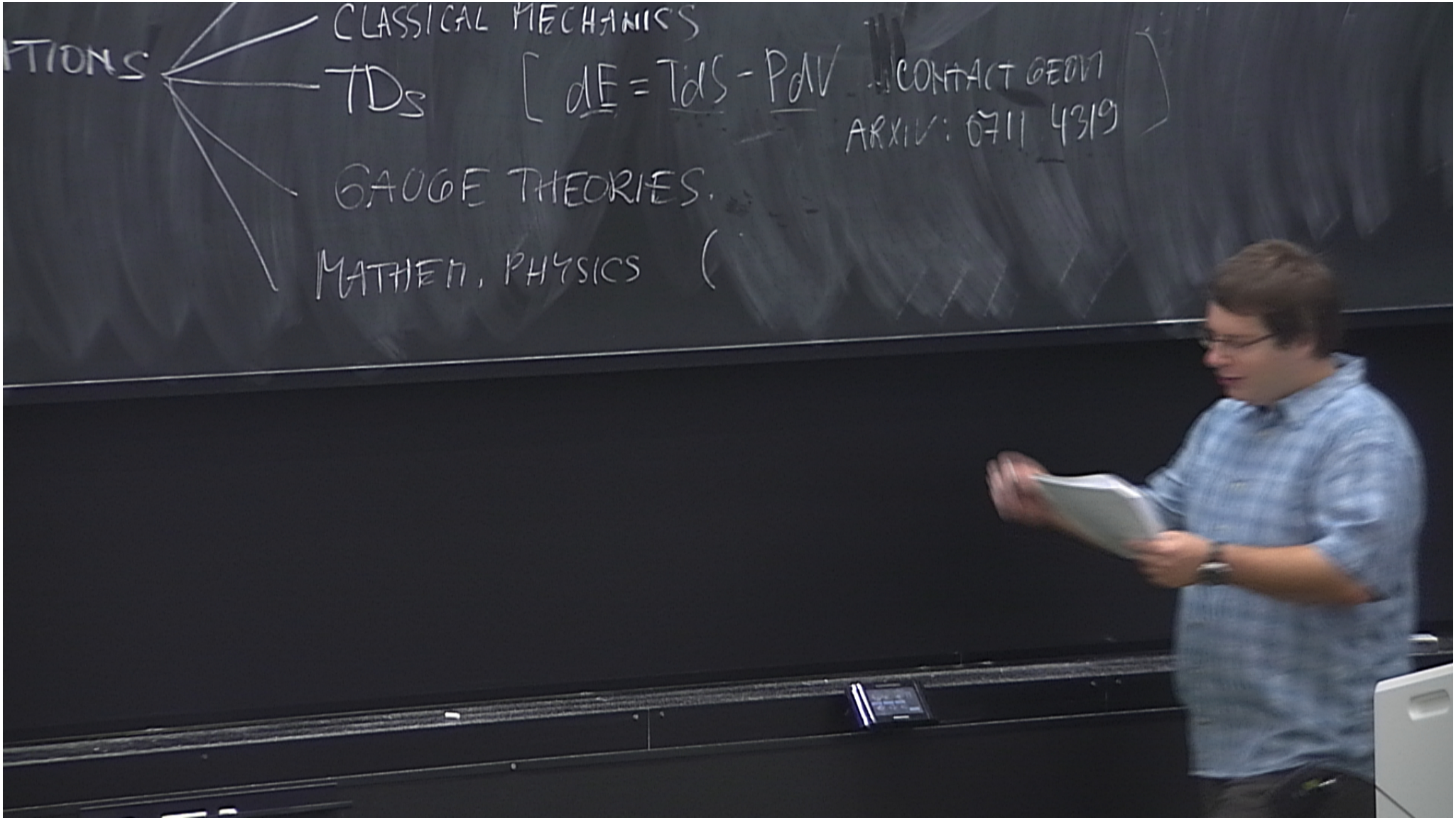

SUMMARY / CH.. FINDS NATURAL DESCRIPTION
IN TERMS OF SYMPLECTIC & CONTACT GEOMETRIES
HAMILTONIAN


HAMILTONIAN
SYMPLECTIC (2n)
CONTACT (2n
MANIFOLD M
PHASE SPACE T*C
EXTENDED P.S. T*C ×


SYMPLECTIC (2n)
CONTACT
MANIFOLD M
PHASE SPACE T^*C
EXTENDED P.S. $T^*C \times$
DYNAMICS
HAMILTONIAN $H(q,p)$
$H(q,p,t)$


1-FORM
CARTAN
$\theta = p_i dq_i$
CONTACT
$\theta_H = p_i$


$\omega_L = 0$)
LAGRANGE SYMPL. 2-FORM
$\omega_L = d\theta_L$
X_L
$\mathcal{L}_{X_L}\theta_L = dL$
$X_L \lrcorner \omega_L = -dE$
LAGR. F.
$E = \frac{\partial L}{\partial \dot{q}^i}\dot{q}^i - L$


TIONS
CLASSICAL MECHANICS
TDs
[dE = TdS - PdV ... CONTACT GEOM
ARXIV: 0711 4319]
GAUGE THEORIES.
MATHEM. PHYSICS (

$$p_i = \frac{\partial F}{\partial q_i}, \quad -P_i = \frac{\partial F}{\partial Q_i}, \quad H' - H = \frac{\partial F}{\partial t} \qquad (C)$$

QFT: $\frac{\partial}{\partial}$ $q(t)$

GR: IN CM: CENTRAL OBJECT ω

NON-DEG, ANTISYMM. $\omega_{\alpha\beta}$, $d\omega = 0$

IN GR: CENTRAL OBJECT g

NON-DEG., SYMMETRIC $g_{\alpha\beta}$, $\nabla g = 0$