Title: PSI 2015/2016 Classical Mechanics 3 - David Kubiznak

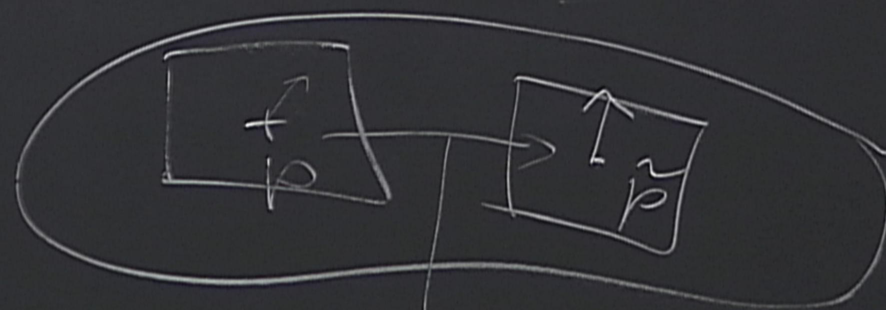
Date: Aug 20, 2015 09:00 AM

URL: http://pirsa.org/15080098

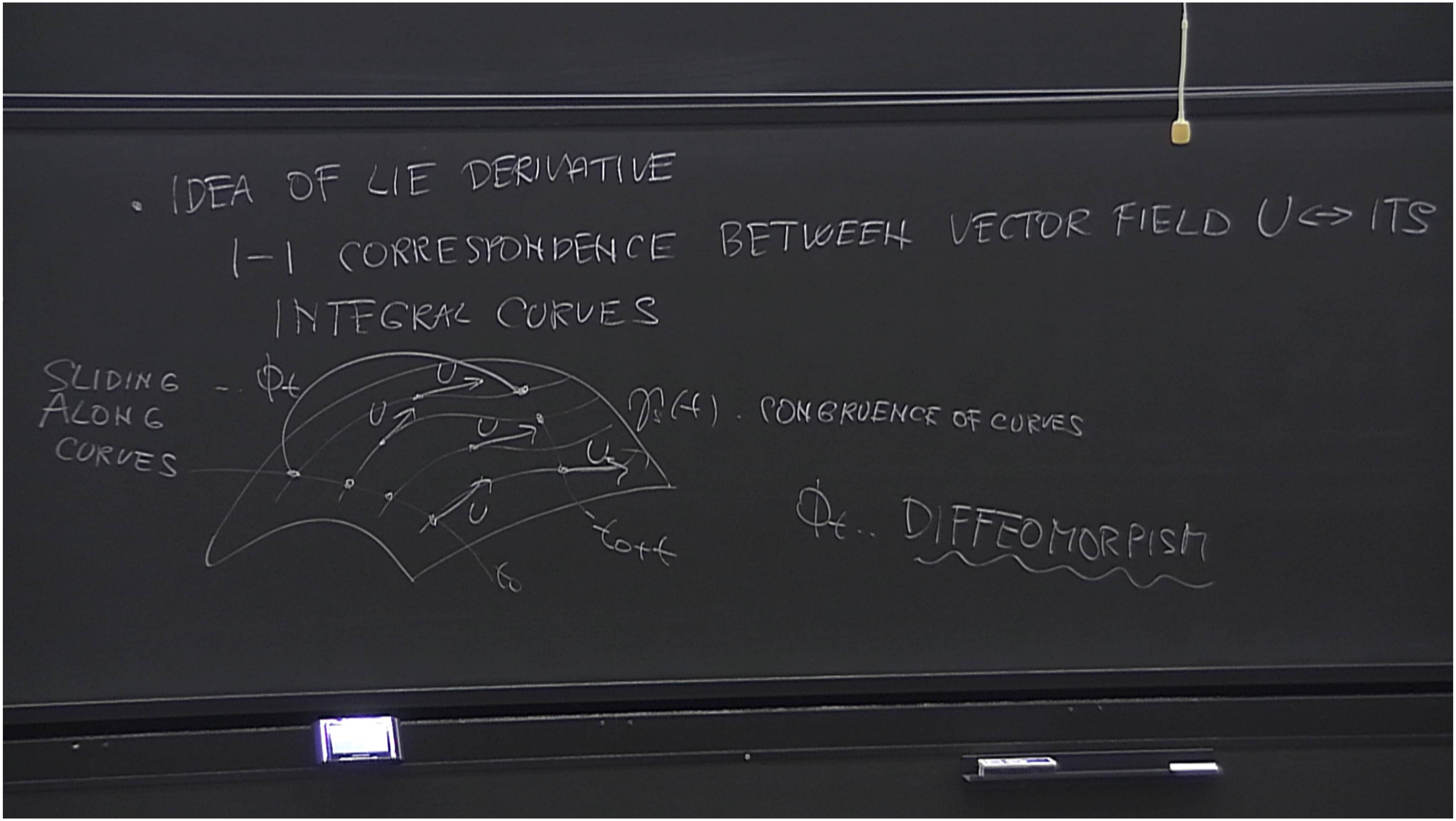
Abstract:

d) LIE DERIVATIVE

- TO DIFFERENTIATE VECTORS



HOW TO IDENTIFY ?



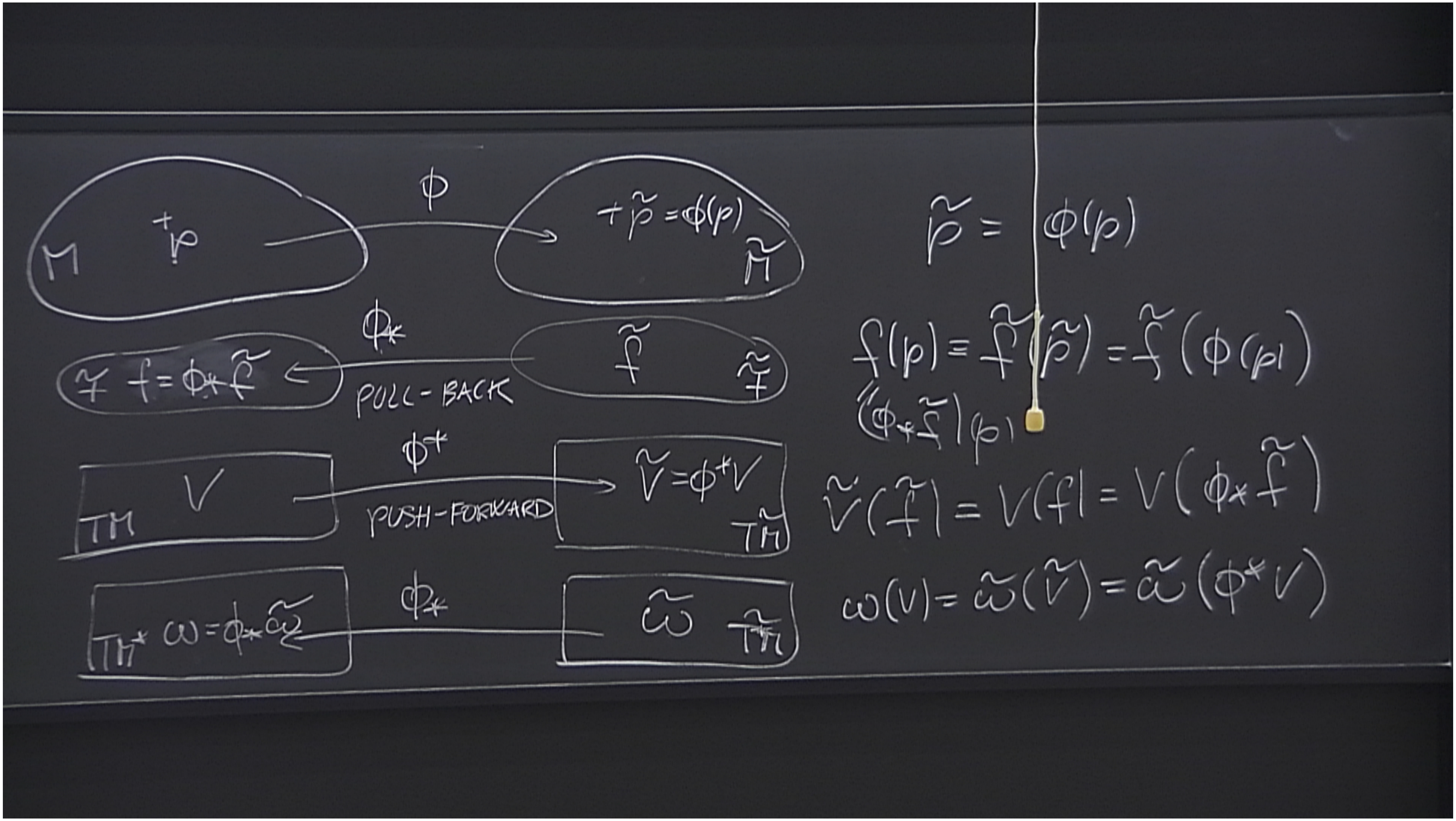
• IDEA OF LIE DERIVATIVE
1–1 CORRESPONDENCE BETWEEN VECTOR FIELD U ⟷ ITS
INTEGRAL CURVES
SLIDING ALONG CURVES .. ϕ_t
$\gamma_s(t)$ .. CONGRUENCE OF CURVES
U
t_0
t_0+t
ϕ_t .. DIFFEOMORPISM

SIDE REMARK: MAPS BETWEEN MANIFOLDS.

LET M & $\tilde{M}$ MANIFOLDS & $\phi : M \to \tilde{M}$ SMOOTH

THEN WE FIND THE FOLLOWING

INDUCED MAPS:


ϕ
M
$+p$
$+\tilde{p} = \phi(p)$
$\tilde{M}$
ϕ_*
$f = \phi_* \tilde{f}$
$\tilde{f}$
PULL-BACK
ϕ^*
TM
V
$\tilde{V} = \phi^* V$
$T\tilde{M}$
PUSH-FORWARD
ϕ_*
TM^*
$\omega = \phi_* \tilde{\omega}$
$\tilde{\omega}$
$T\tilde{M}^*$
$\tilde{p} = \phi(p)$
$f(p) = \tilde{f}(\tilde{p}) = \tilde{f}(\phi(p))$
$(\phi_* \tilde{f})(p)$
$\tilde{V}(\tilde{f}) = V(f) = V(\phi_* \tilde{f})$
$\omega(V) = \tilde{\omega}(\tilde{V}) = \tilde{\omega}(\phi^* V)$

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THEN WE FIND THE FOLLOWING

INDUCED MAPS:

FOR A GIVEN TYPE OF OBJECT

THIS IS ONLY 1-WAY

M

TM

TM^*

TH.

DEF: LET Φ_t BE A 1-PARAMETRIC GROUP OF DIFFEOMORPHISMS GENERATED BY VECTOR FIELD U. THEN THE LIE DERIVATIVE $\mathcal{L}_U$ W.R.T. U IS DEFINED:

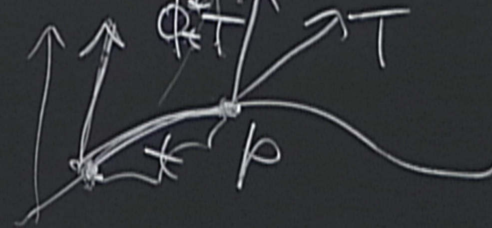
$$\mathcal{L}_U T\big|_p = \lim_{t \to 0} \frac{T\big|_p - \Phi_t^* T\big|_p}{t}$$

MORPHISM. $\phi : M \to N$, 1-1, ONTO, ϕ^{-1} IS SMOOTH.

CAN USE ϕ^{-1} TO DEFINE ABOVE MAPS

BOTH WAYS.

ϕ^* : TENSORS ON $M \to$ TENSORS ON $\hat{M}$.



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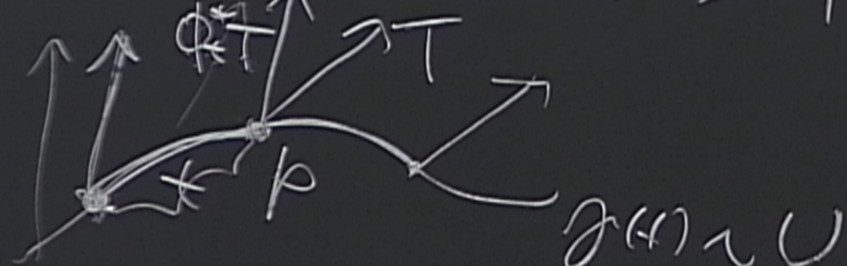
. $\Phi: M \to N$, 1-1, ONTO, ϕ^{-1} IS SMOOTH.

ϕ^{-1} TO DEFINE ABOVE MAPS

S.

Φ^*: TENSORS ON M → TENSORS ON $\hat{M}$.



DEF

O

B

L

~

IS

"WHAT WAS THERE − WHAT I TRANSPORTED THERE"

EXAMPLE: $\mathcal{L}_v f = \lim_{t \to 0} \frac{f(t_0) - \tilde{f}(t_0)}{t} = \left| \tilde{f}(t_0) = f(t_0 - t) \right|$

$$= \frac{df}{dt} = v^\mu \frac{\partial f}{\partial x^\mu} = v(f)$$

PROPERTIES:

i) $\mathcal{L}_U$ MAPS (r,s) TENSORS TO (r,s) TENSORS.

ii) $\mathcal{L}_U$ LINEAR & PRESERVES CONTRACTION

iii) LEIBNITZ. $\mathcal{L}_U(T\otimes S) = (\mathcal{L}_U T)\otimes S + T\otimes(\mathcal{L}_U S)$

iv) $\mathcal{L}_U f = U f = U^\mu \frac{\partial f}{\partial x^\mu}$

$\mathcal{L}_U V = [U, V] = UV - VU \ldots$ LIE BRACKET

$\mathcal{L}_U T^{\alpha\ldots}{}_{\beta\ldots} = U^\gamma \frac{\partial}{\partial x^\gamma} T^{\alpha\ldots}{}_{\beta\ldots} - T^{\gamma\ldots}{}_{\beta\ldots} \frac{\partial}{\partial x^\gamma} U^{\alpha}\ldots \; + \ldots$

e) DIFFERENTIAL FORMS

DEF: A DIFFERENT. p-FORM $\underline{\omega}$ IS A TOT. ANTISYMMETRIC TENSOR OF TYPE $(0,p)$.

$$\omega_{\alpha_1 \dots \alpha_p} = \omega_{[\alpha_1 \dots \alpha_p]} = \frac{1}{p!} \sum_{\substack{\text{PERMS} \\ \pi}} \text{SIGN}(\pi)\, \omega_{\alpha_{\pi(1)} \dots \alpha_{\pi(p)}}$$

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- ANTISYM. UNDER EXCHANGE OF ANY 2 INDICES
- Λ^p_x .. VECTOR SPACE OF p-FORMS

$$\dim \Lambda^p_x = \binom{n}{p}$$

$\ldots d\pi(p)$

• WEDGE PRODUCT. $\wedge : \Lambda^p \times \Lambda^q \to \Lambda^{p+q}$

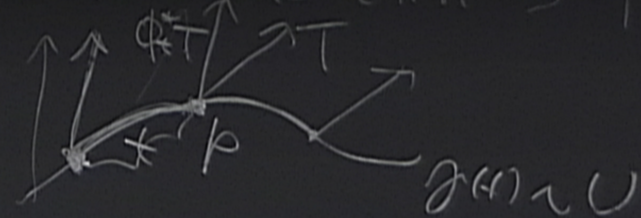
$$(\omega \wedge \nu)_{\alpha_1 \dots \alpha_{p+q}} = \frac{(p+q)!}{p!\,q!}\, \omega_{[\alpha_1 \dots \alpha_p}\, \nu_{\alpha_{p+1} \dots \alpha_{p+q}]}$$

• $\omega \wedge \nu = (-)^{pq}\, \nu \wedge \omega$

• SINCE dx^α IS A COORDINATE BASIS OF 1-FORMS GENERAL p-FORM CAN BE WRITTEN

$$\omega = \frac{1}{p!}\, \omega_{\alpha_1 \dots \alpha_p}\, \overbrace{dx^{\alpha_1} \wedge \dots \wedge dx^{\alpha_p}}^{\text{BASIS OF } p\text{-FORMS}}$$

TENSORS ON M.



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BASIS OF p-FORMS

$$\omega = \frac{1}{p!}\,\omega_{\alpha_1\dots\alpha_p}\, dx^{\alpha_1}\wedge\dots\wedge dx^{\alpha_p}$$

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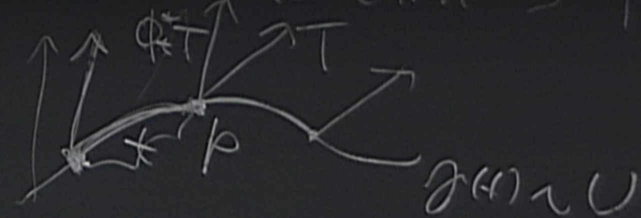
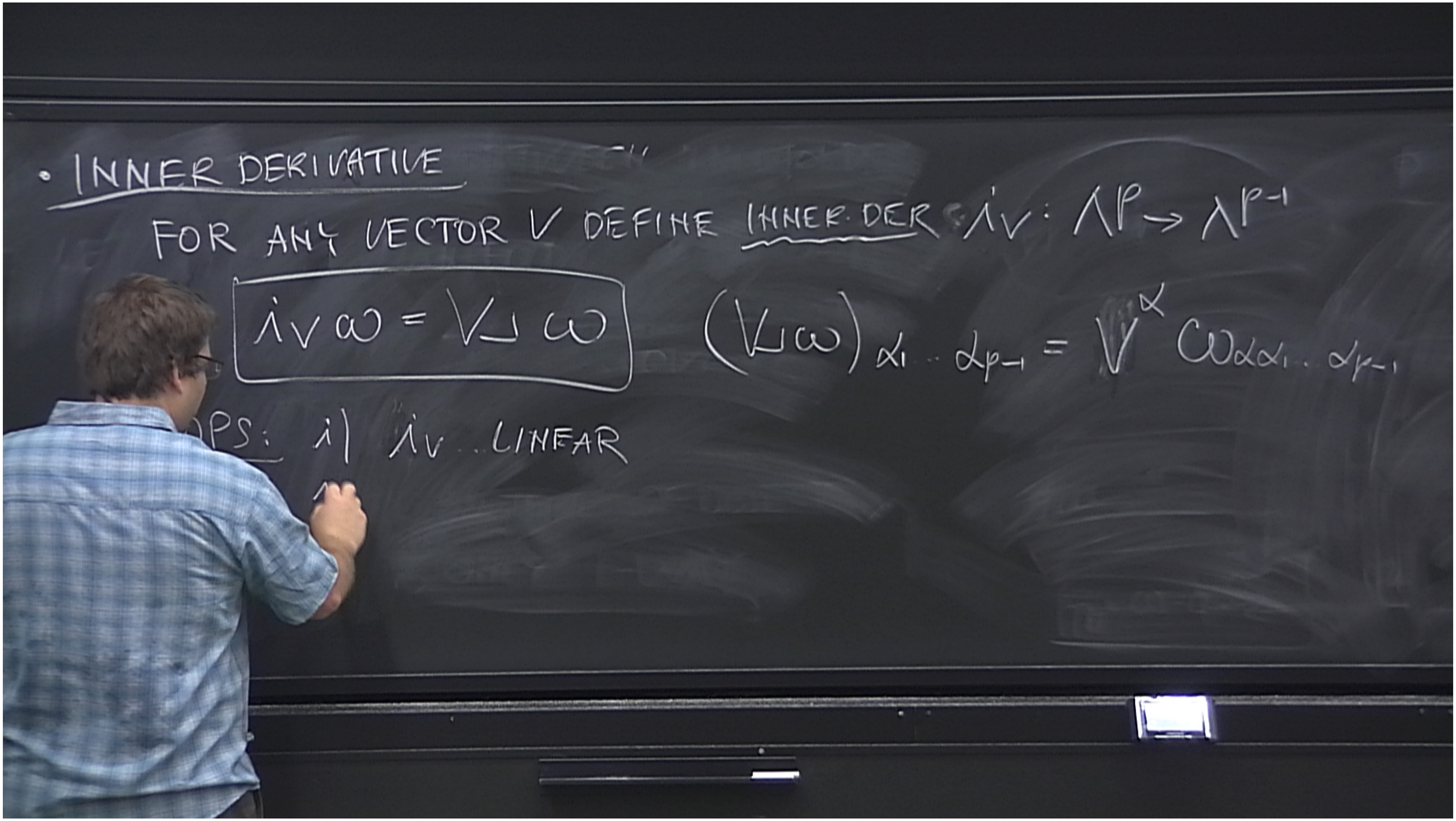
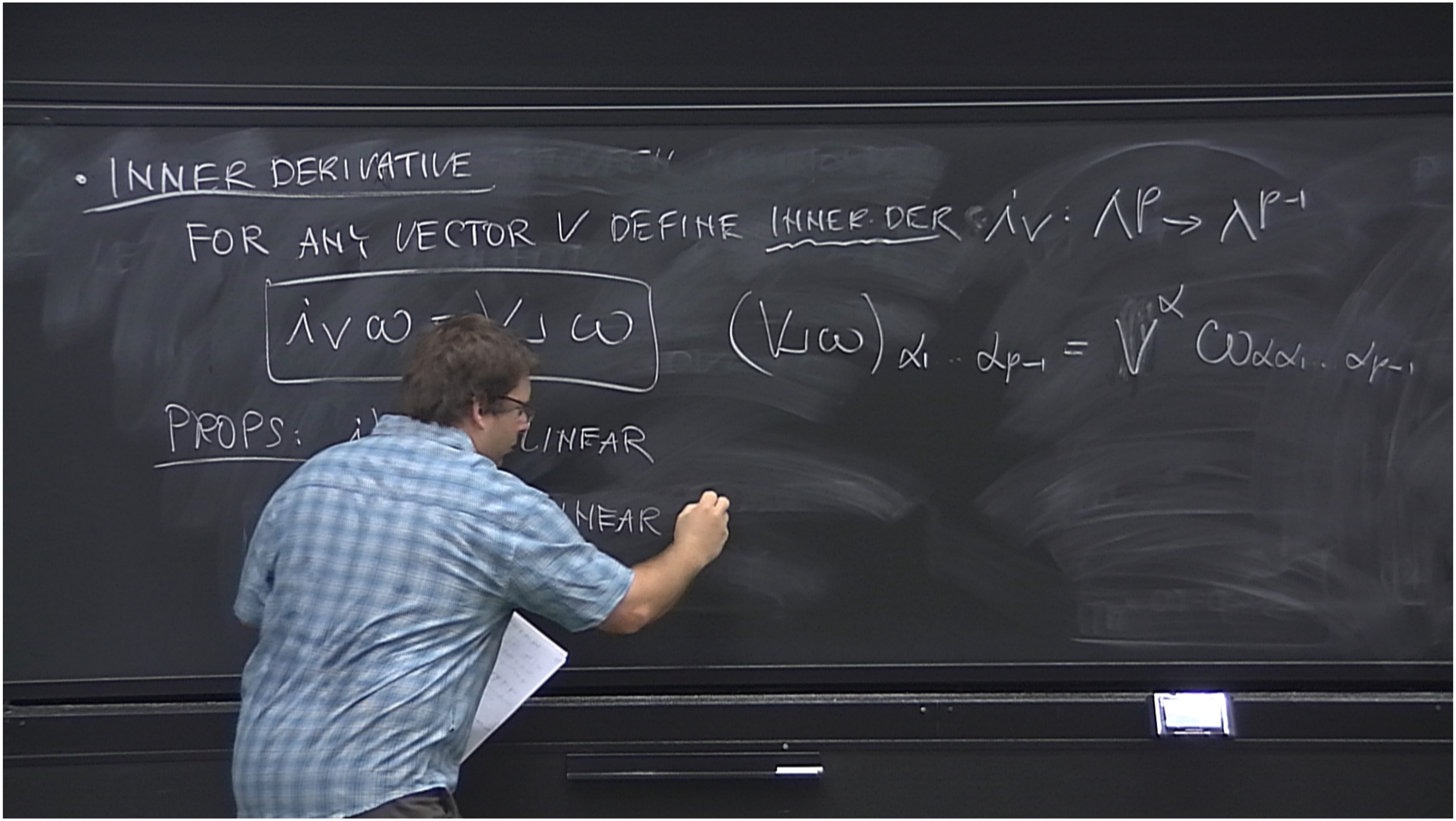



• INNER DERIVATIVE
FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
$i_V \omega = V \lrcorner \omega$
$(V \lrcorner \omega)_{\alpha_1 \ldots \alpha_{p-1}} = V^\alpha \omega_{\alpha \alpha_1 \ldots \alpha_{p-1}}$
PROPS: i) i_V ... LINEAR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PROPS:
LINEAR
NEAR

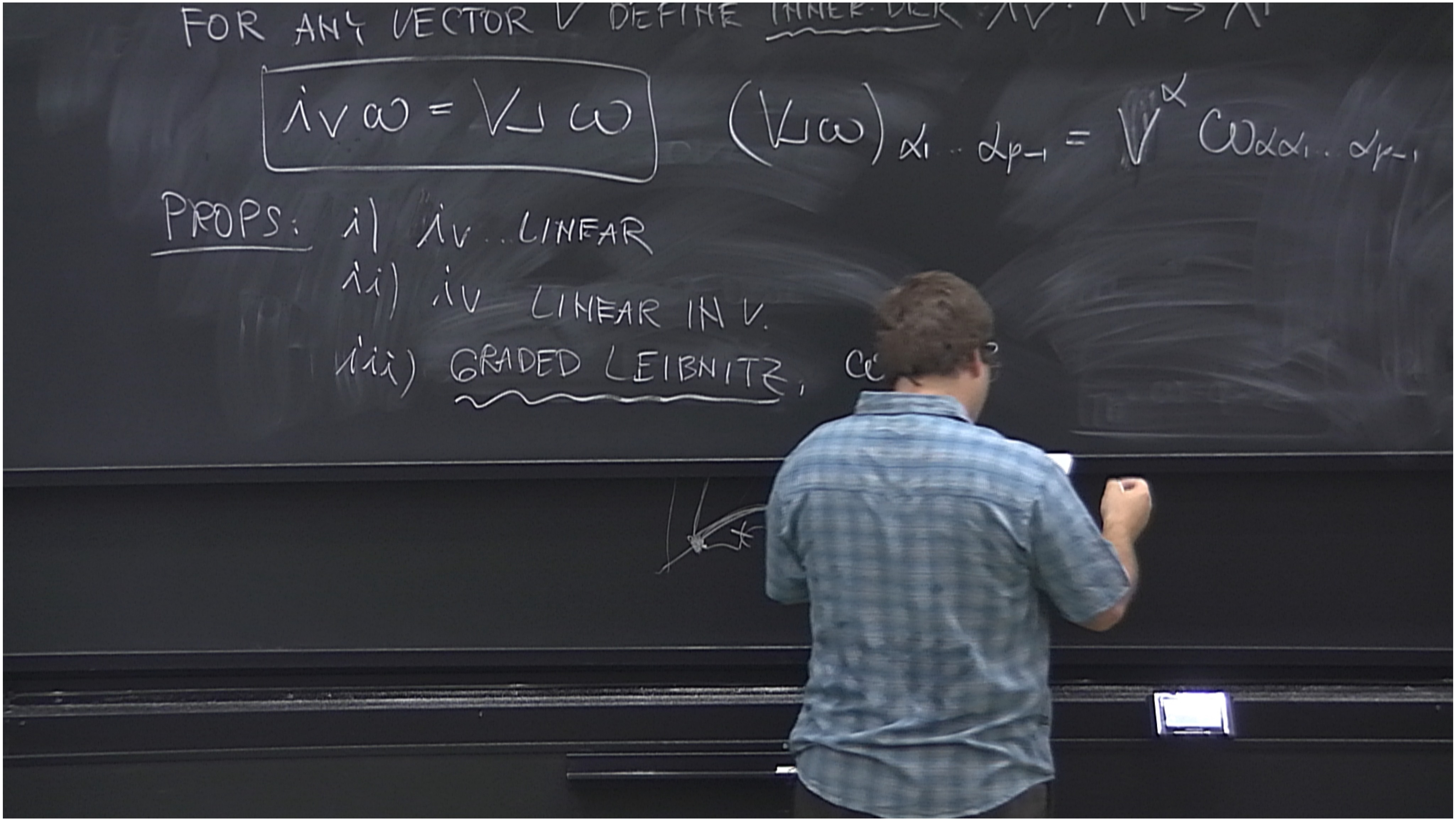
• INNER DERIVATIVE

FOR ANY VECTOR V DEFINE INNER DER. $i_V : \Lambda^p \to \Lambda^{p-1}$

$$\boxed{i_V \omega = V \lrcorner \omega} \qquad (V \lrcorner \omega)_{\alpha_1 \ldots \alpha_{p-1}} = V^\alpha \omega_{\alpha \alpha_1 \ldots \alpha_{p-1}}$$

PROPS: i) i_V ... LINEAR

ii) i_V LINEAR IN V.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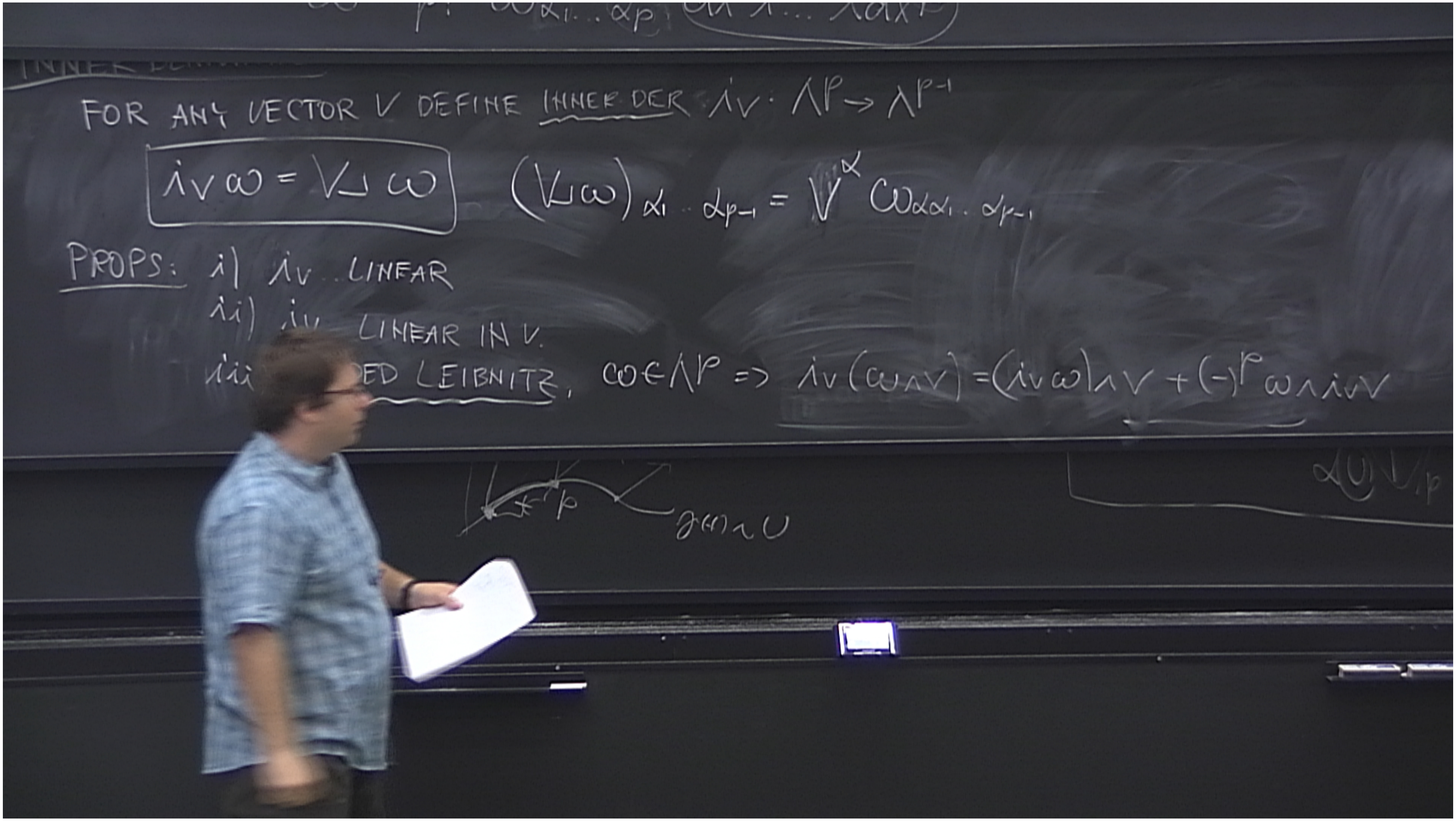
PROPS: i) i_V ... LINEAR
ii) i_V LINEAR IN V.
iii) GRADED LEIBNITZ,


FOR ANY VECTOR V DEFINE INNER DER $i_V : \Lambda^p \to \Lambda^{p-1}$
$i_V \omega = V \lrcorner \omega$
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PROPS: i) i_V .. LINEAR
ii) i_V .. LINEAR IN V.
LEIBNITZ, $\omega \in \Lambda^p \Rightarrow i_V(\omega \wedge \nu) = (i_V \omega) \wedge \nu + (-)^p \omega \wedge i_V \nu$

FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$

$$\boxed{i_V \omega = V \lrcorner\, \omega} \qquad (V \lrcorner\, \omega)_{\alpha_1 \ldots \alpha_{p-1}} = V^{\alpha}\, \omega_{\alpha \alpha_1 \ldots \alpha_{p-1}}$$

iv) $i_V i_W$

PROPS: i) i_V ... LINEAR

ii) i_V LINEAR IN V.

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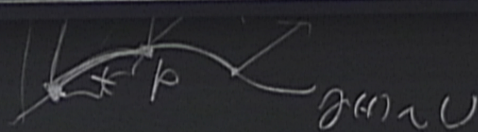
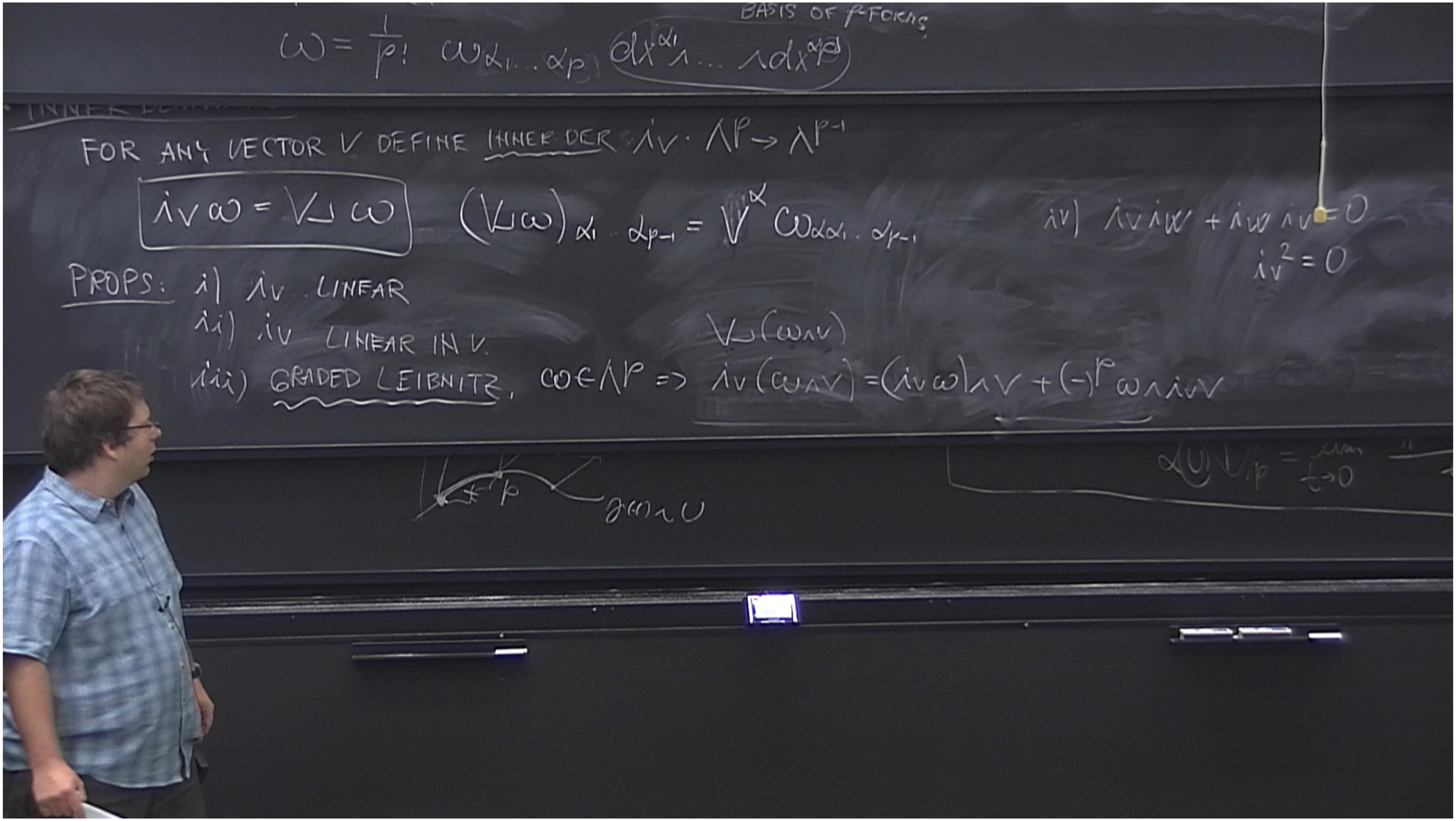
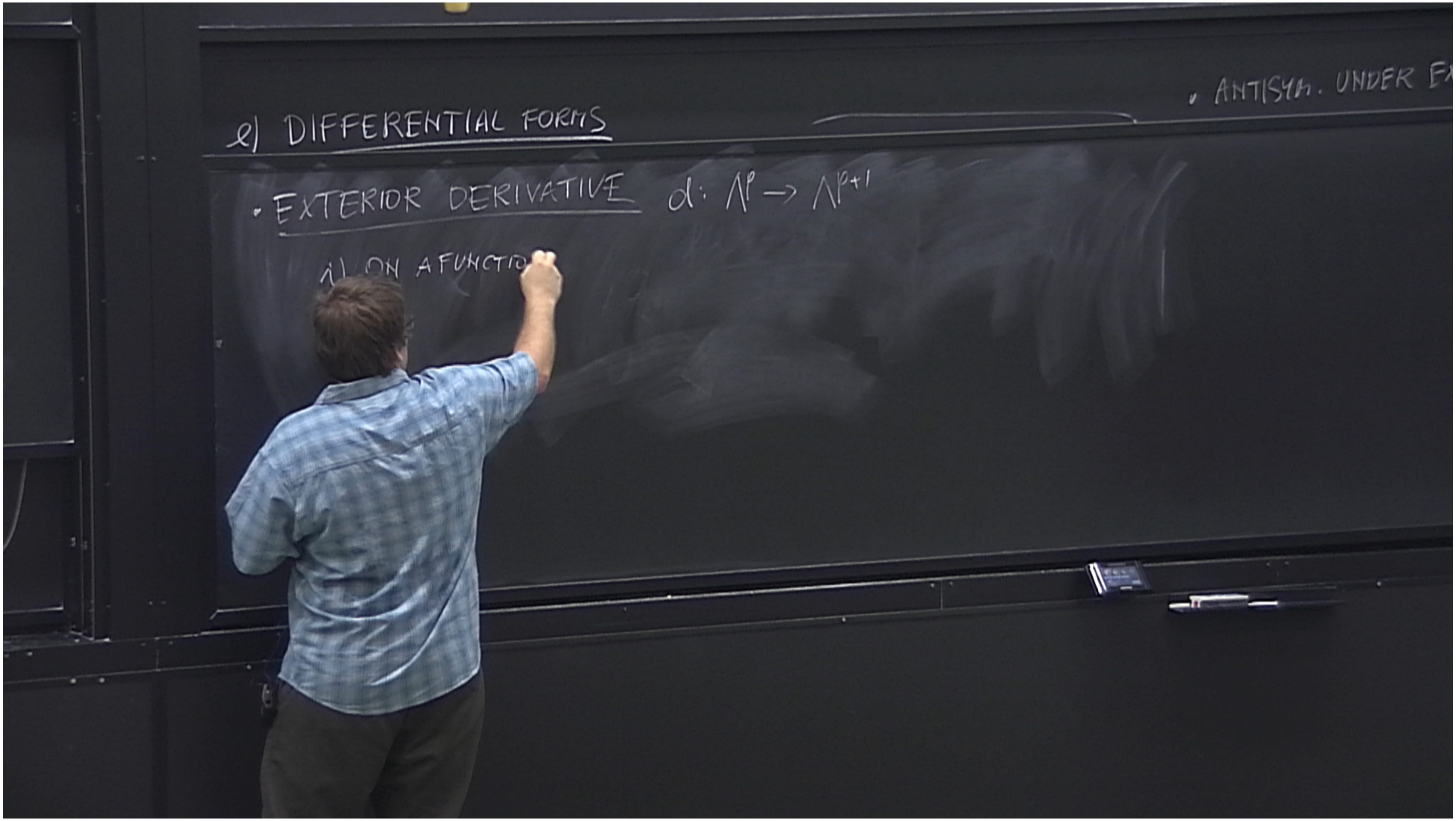



BASIS OF p-FORMS
$\omega = \frac{1}{p!} \omega_{\alpha_1 \ldots \alpha_p} dx^{\alpha_1} \wedge \ldots \wedge dx^{\alpha_p}$
FOR ANY VECTOR V DEFINE INNER DER $i_V : \Lambda^p \to \Lambda^{p-1}$
$i_V \omega = V \lrcorner \omega$
$(V \lrcorner \omega)_{\alpha_1 \ldots \alpha_{p-1}} = V^\alpha \omega_{\alpha \alpha_1 \ldots \alpha_{p-1}}$
iv) $i_V i_W + i_W i_V = 0$
$i_V^2 = 0$
PROPS: i) i_V LINEAR
ii) i_V LINEAR IN V.
$V \lrcorner (\omega \wedge v)$
iii) GRADED LEIBNITZ, $\omega \in \Lambda^p \Rightarrow i_V(\omega \wedge v) = (i_V \omega) \wedge v + (-)^p \omega \wedge i_V v$

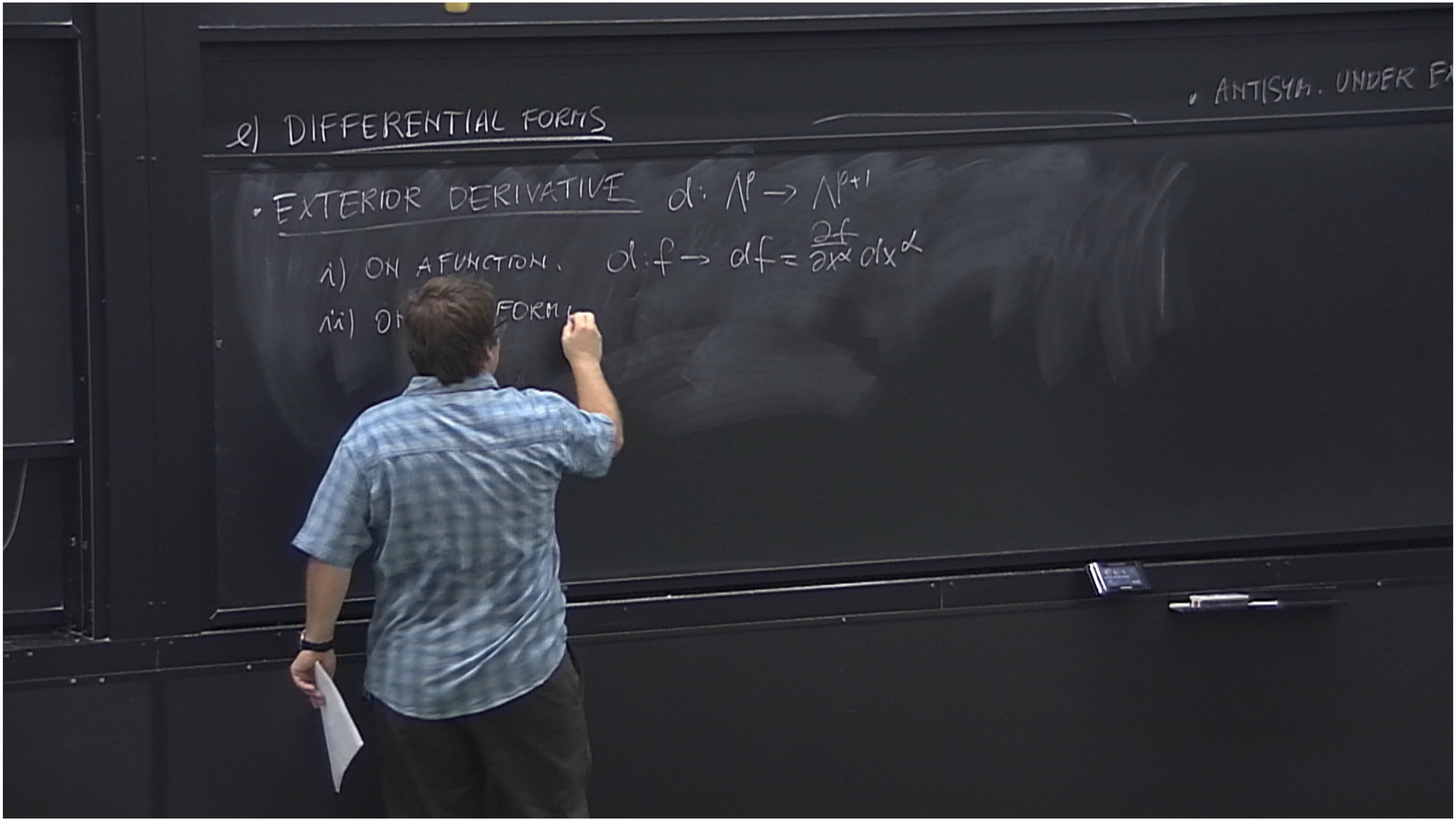

e) DIFFERENTIAL FORMS
• EXTERIOR DERIVATIVE d: Λp → Λp+1
• ANTISYM. UNDER

e) DIFFERENTIAL FORMS

• ANTISYM. UNDER EX

• EXTERIOR DERIVATIVE $d: \Lambda^p \to \Lambda^{p+1}$

i) ON A FUNCTION. $d: f \to df = \frac{\partial f}{\partial x^\alpha} dx^\alpha$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e) DIFFERENTIAL FORMS
• EXTERIOR DERIVATIVE d: Λ^p → Λ^{p+1}
i) ON A FUNCTION. d: f → df = ∂f/∂x^α dx^α
ii) ON FORM.

• ANTISYM. UNDER E

e) DIFFERENTIAL FORMS

• EXTERIOR DERIVATIVE $d : \Lambda^p \to \Lambda^{p+1}$

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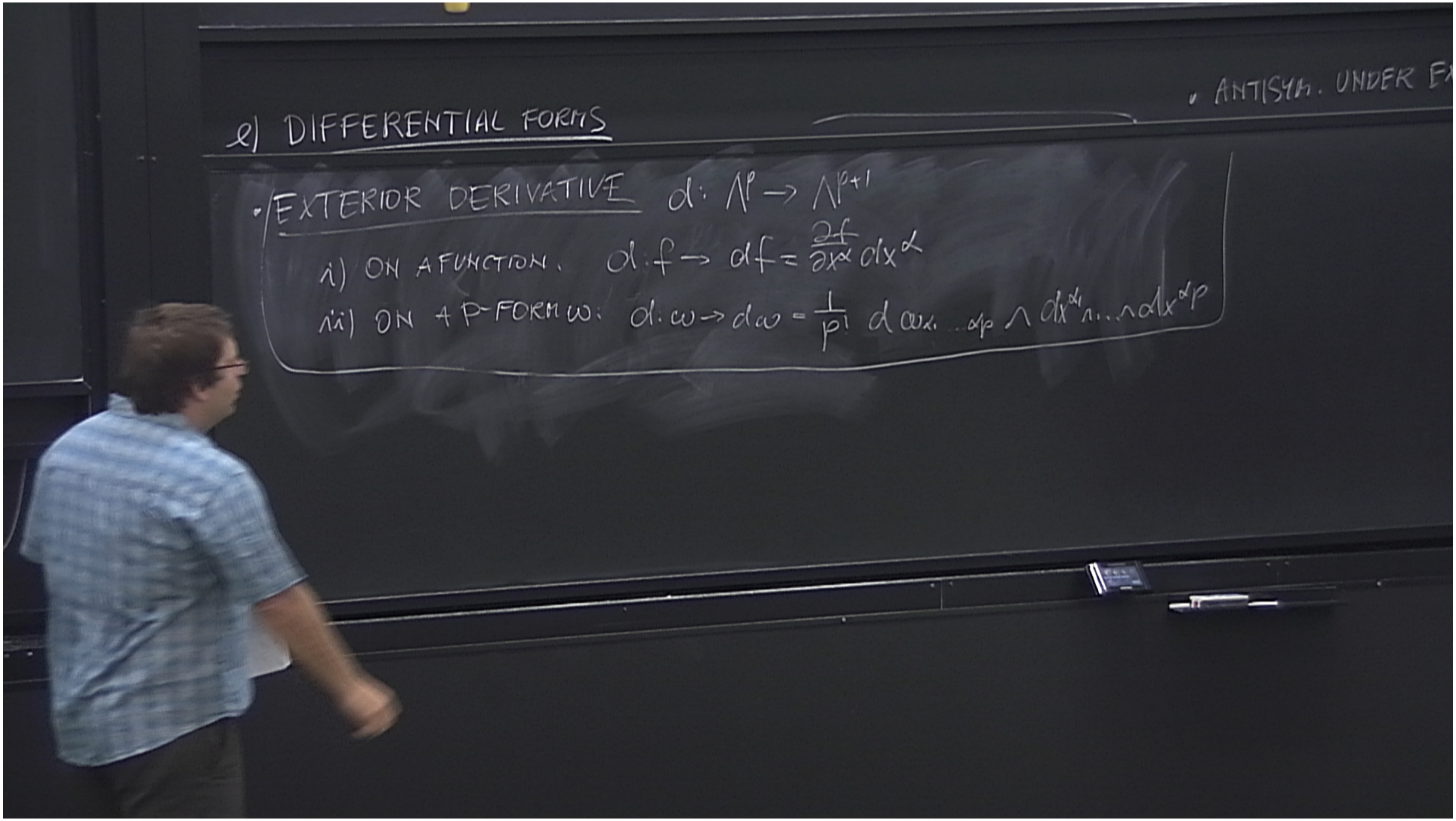
ii) ON A P-FORM ω: $d : \omega \to d\omega =$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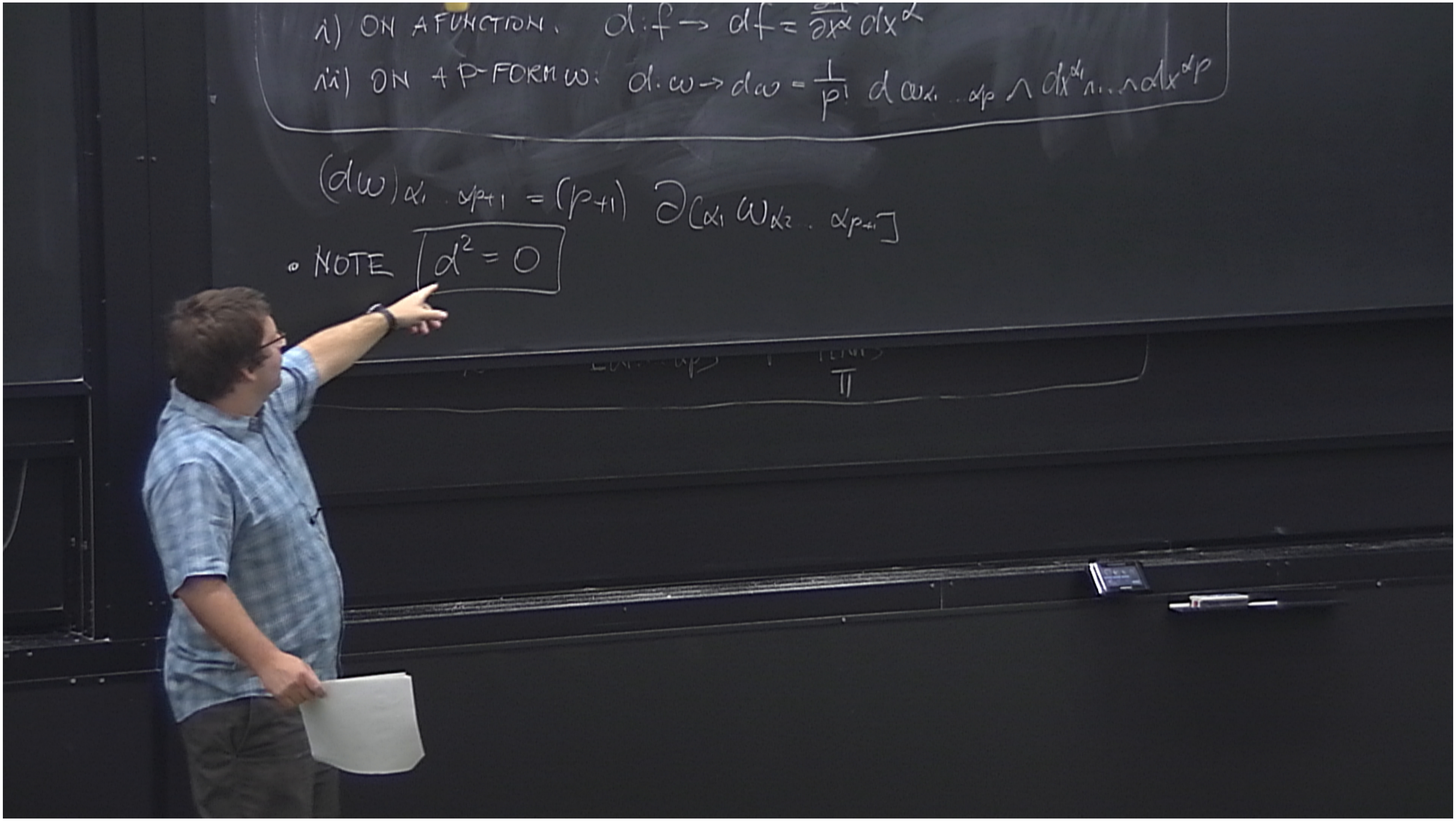
- NOTE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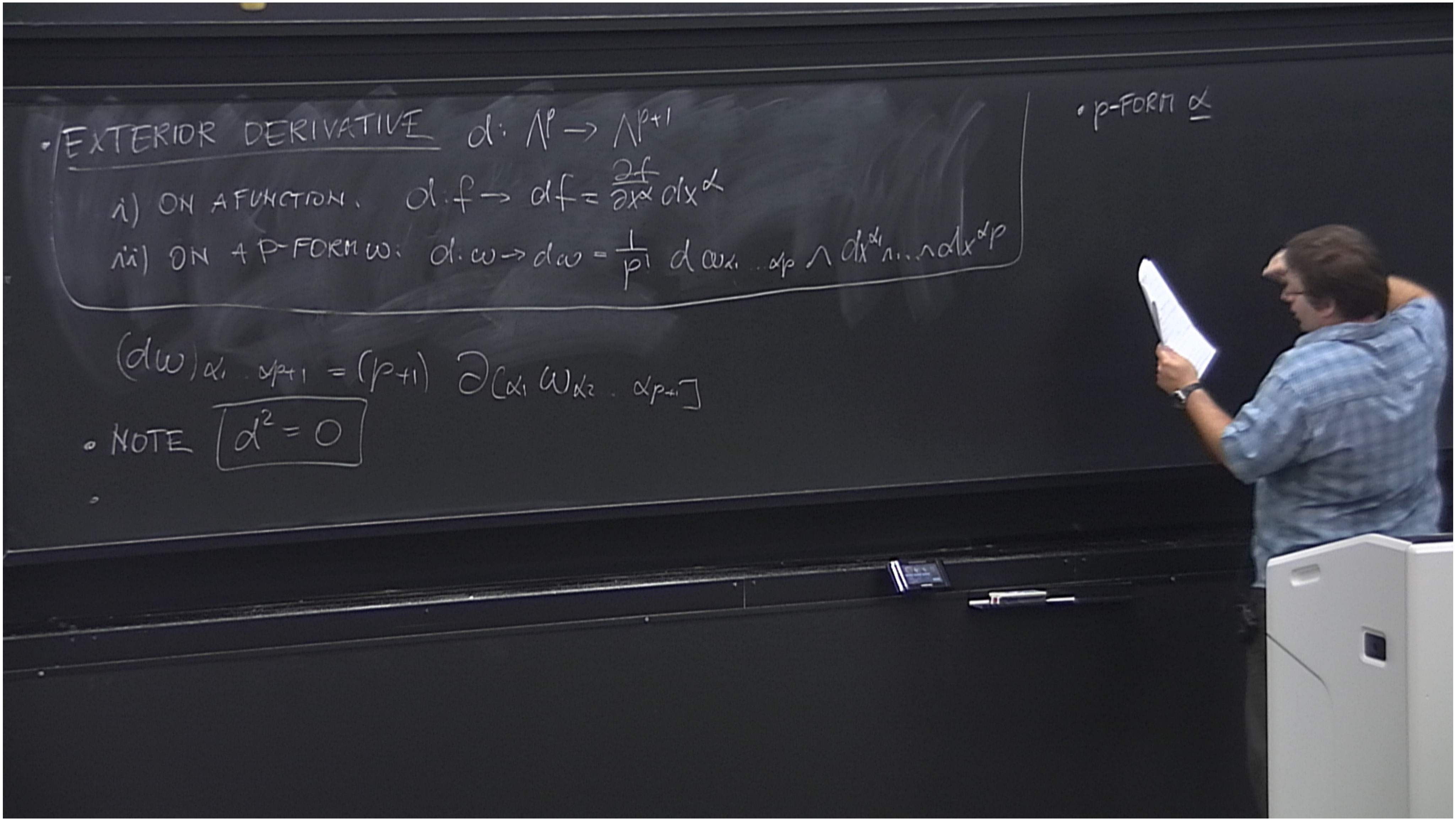
• NOTE $\boxed{d^2 = 0}$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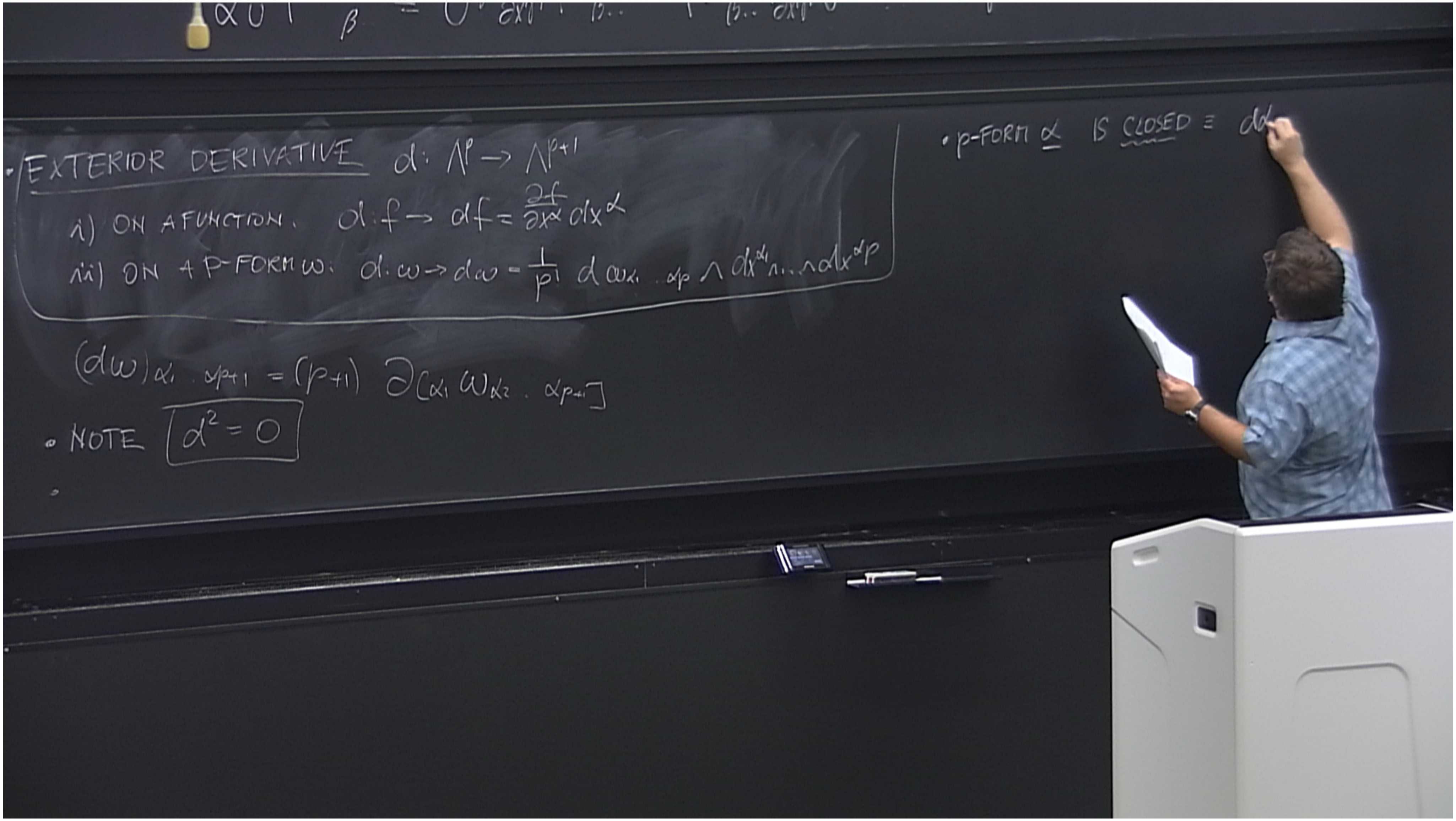
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p-FORM α IS CLOSED $\equiv$ $d\alpha$

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• NOTE $\boxed{d^2 = 0}$

• p-FORM α IS CLOSED $\equiv$ $d\alpha = 0$

EXACT $\equiv$ $\alpha = d\beta$

• **EXTERIOR DERIVATIVE** $d: \Lambda^p \to \Lambda^{p+1}$

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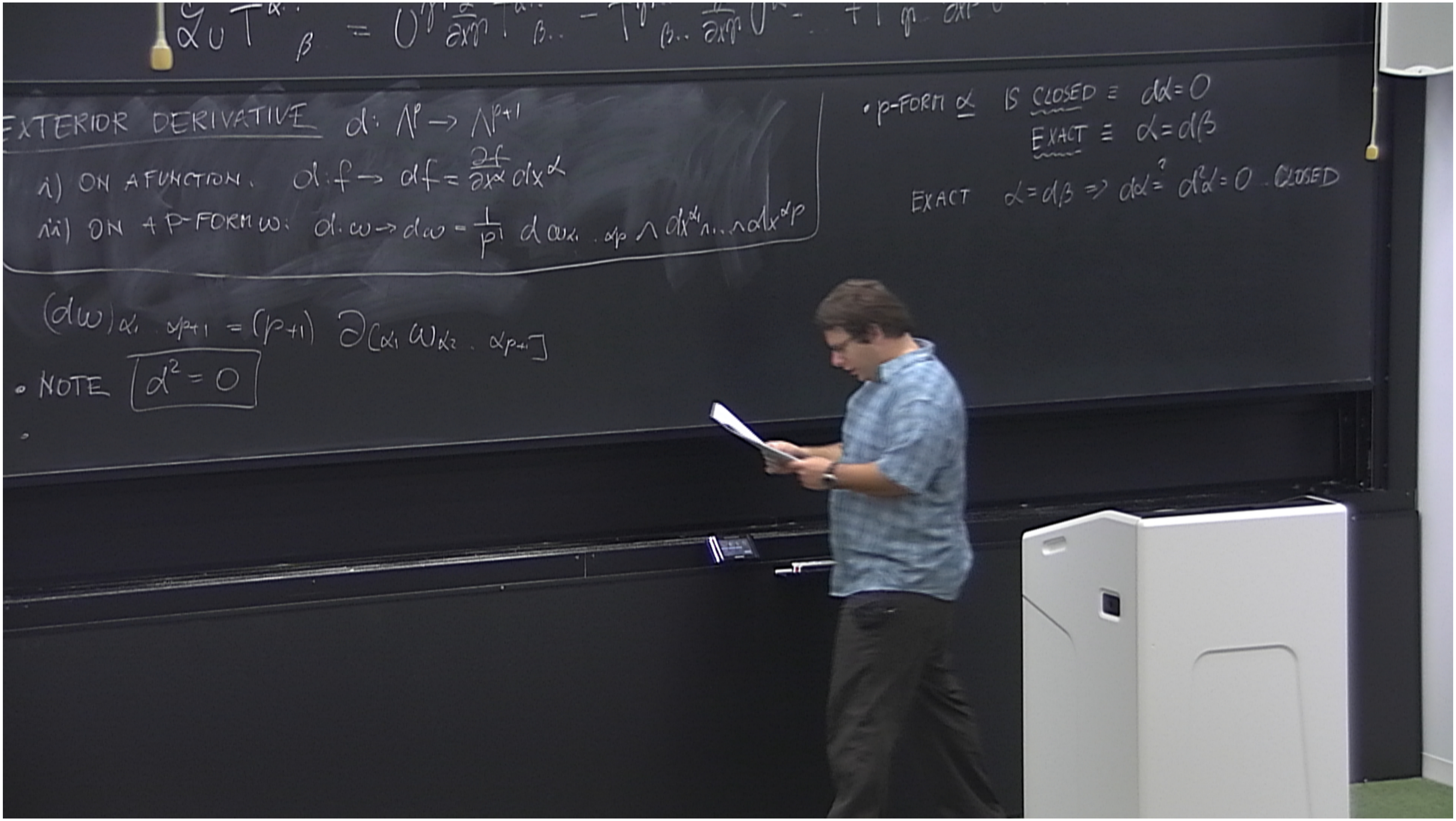
EXACT $\alpha = d\beta \Rightarrow d\alpha \overset{?}{=} d^2\alpha = 0$ .. CLOS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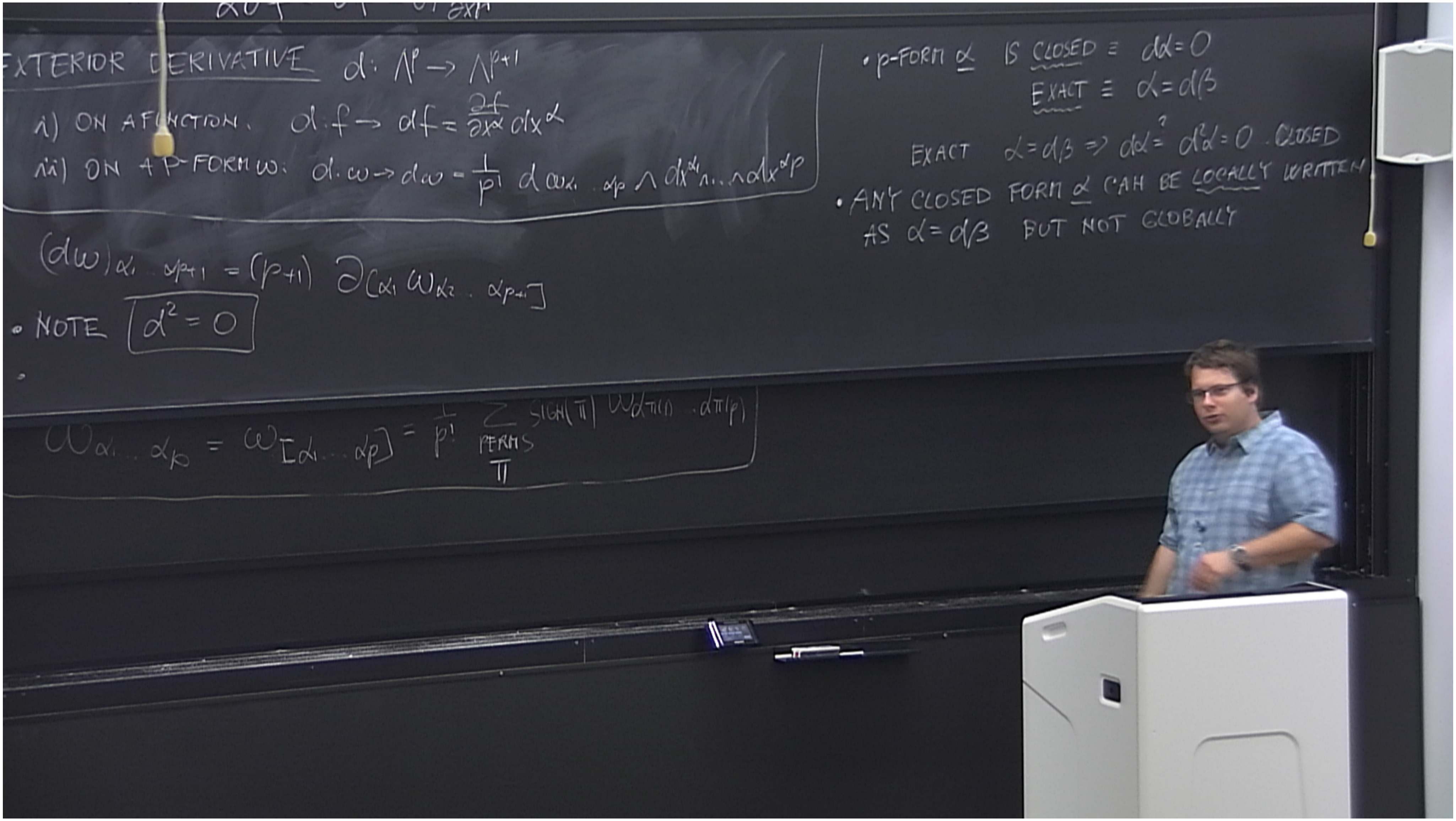
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• p-FORM α IS CLOSED $\equiv d\alpha = 0$
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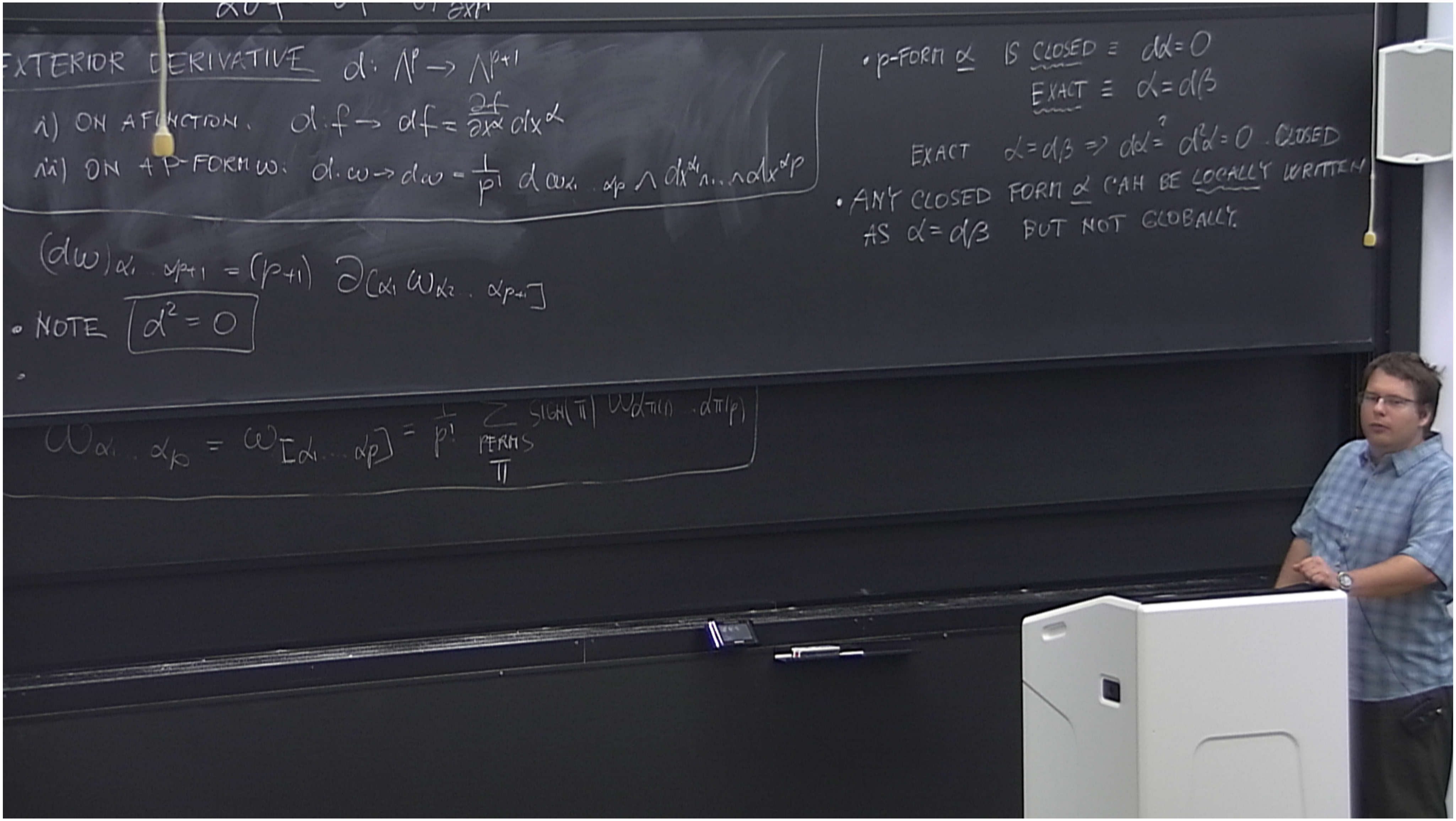
EXACT $\alpha = d\beta \Rightarrow d\alpha \overset{?}{=} d^2\alpha = 0$. CLOSED
• ANY CLOSED FORM α CAN BE LOCALLY WRITTEN AS $\alpha = d\beta$ BUT NOT GLOBALLY



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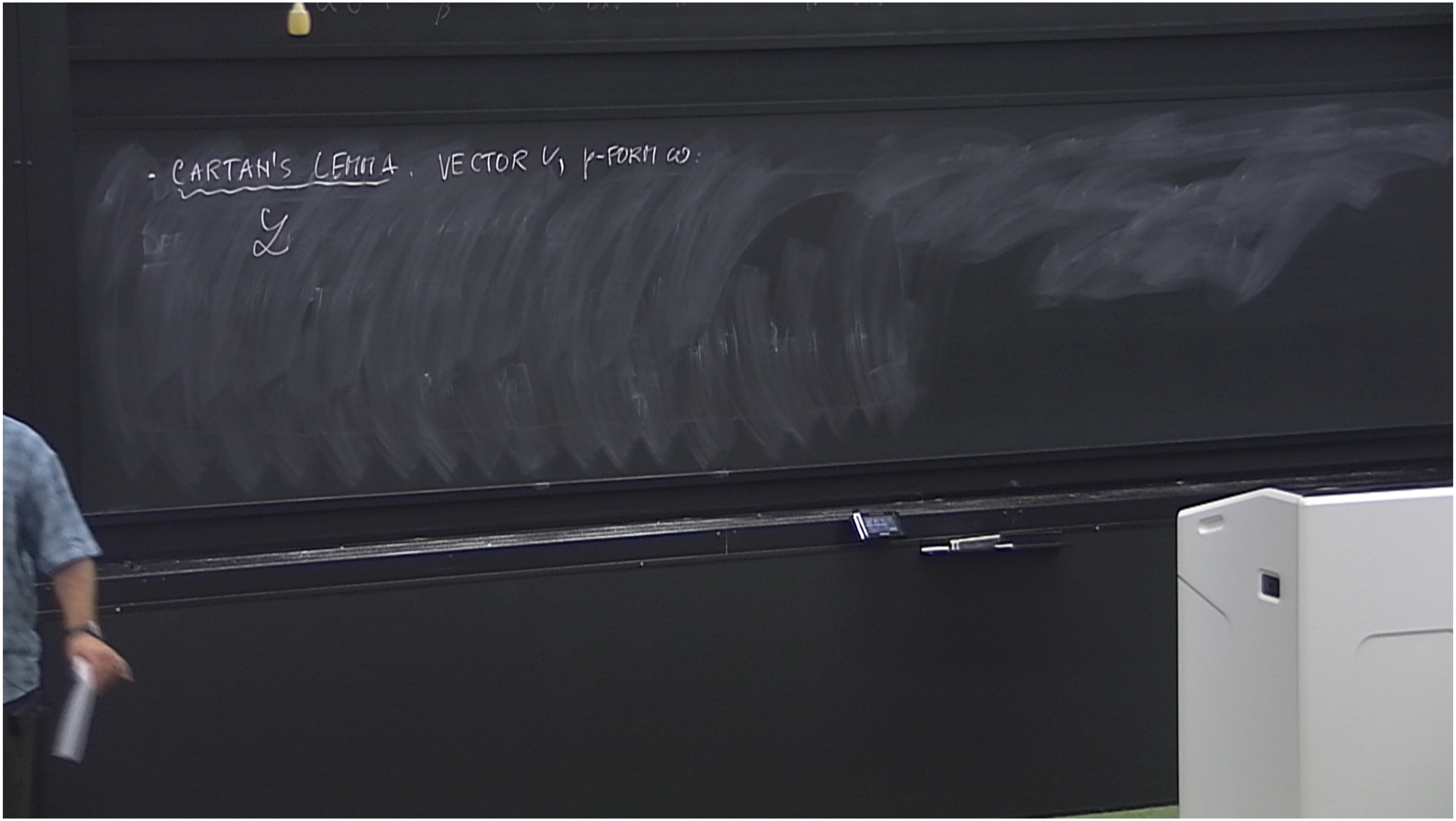
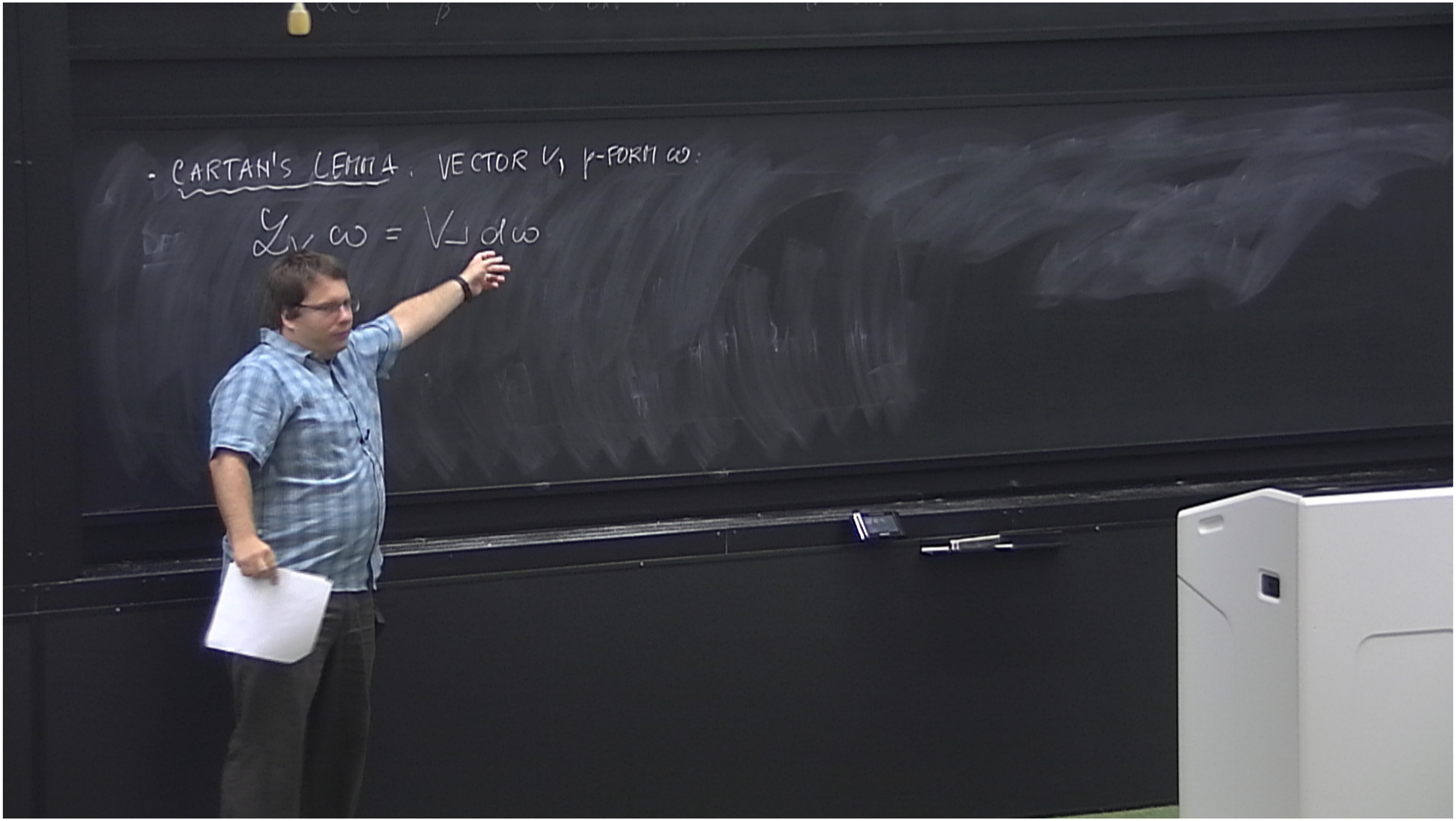
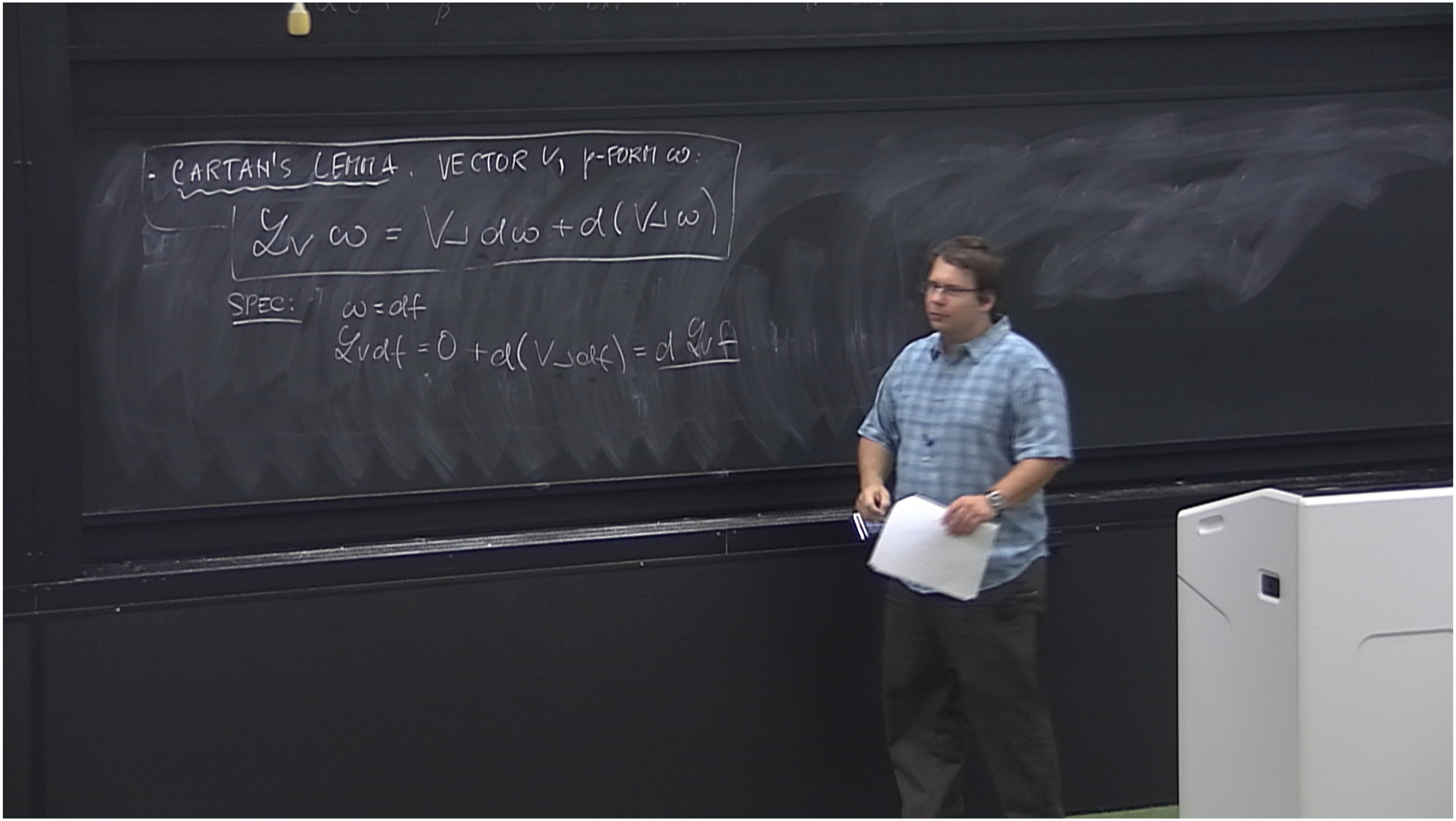
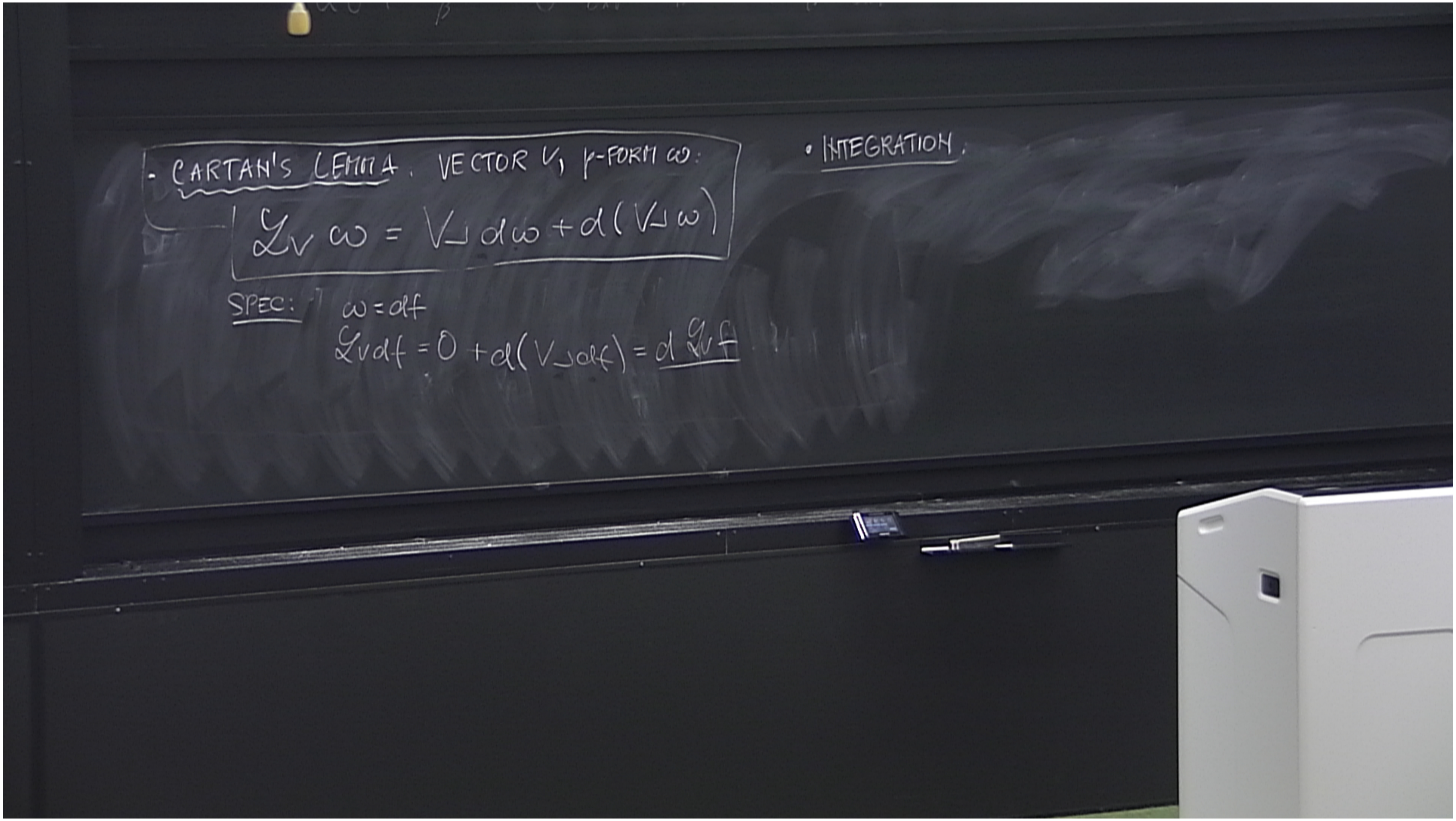
EXACT $\alpha = d\beta \Rightarrow d\alpha \overset{?}{=} d^2\alpha = 0$. CLOSED
• ANY CLOSED FORM α CAN BE LOCALLY WRITTEN AS $\alpha = d\beta$ BUT NOT GLOBALLY.


• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}$


• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega$



CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
SPEC:
$\omega = df$
$\mathcal{L}_V df = 0 + d(V \lrcorner df) = d\,\mathcal{L}_V f$



• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
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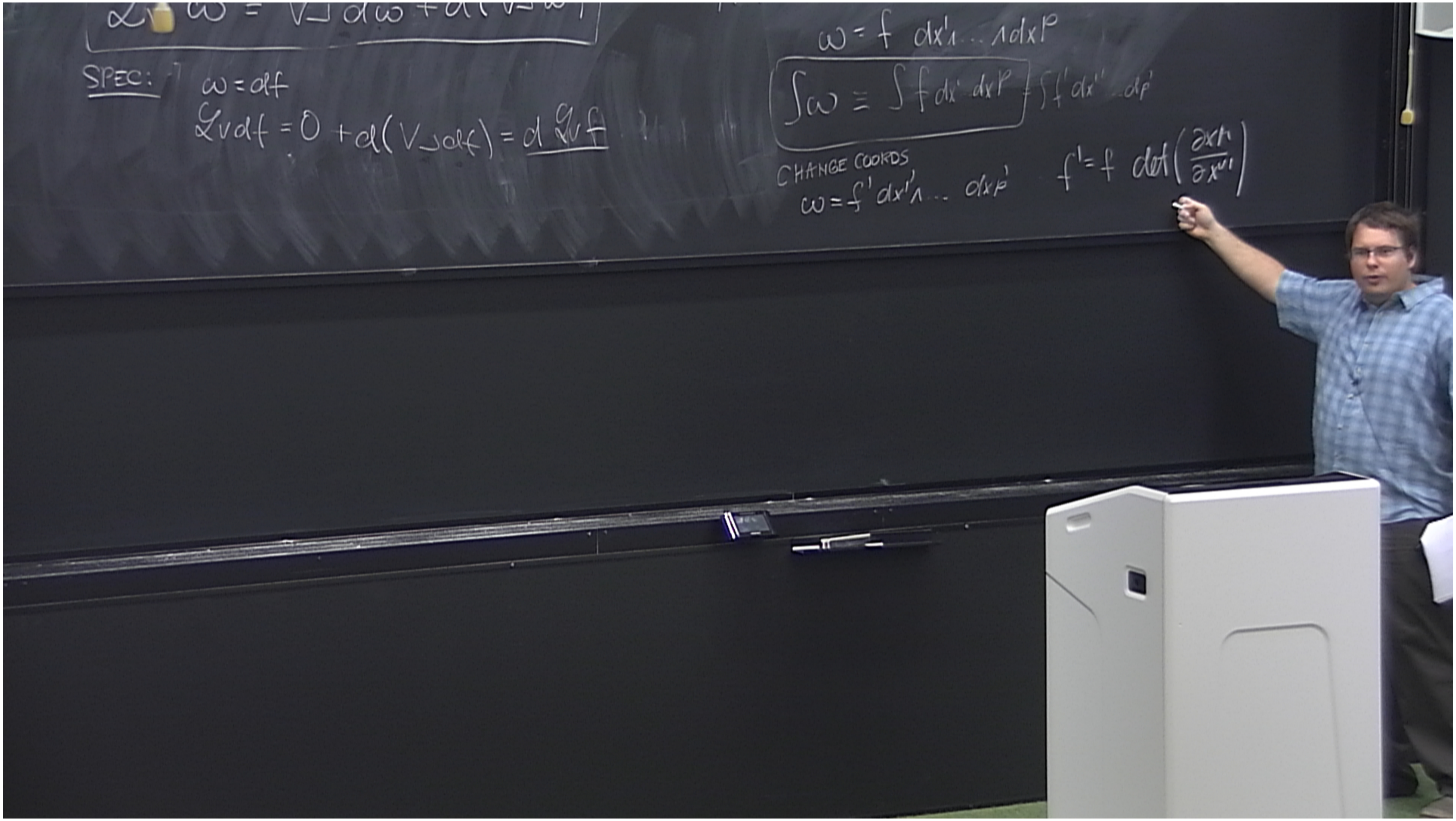
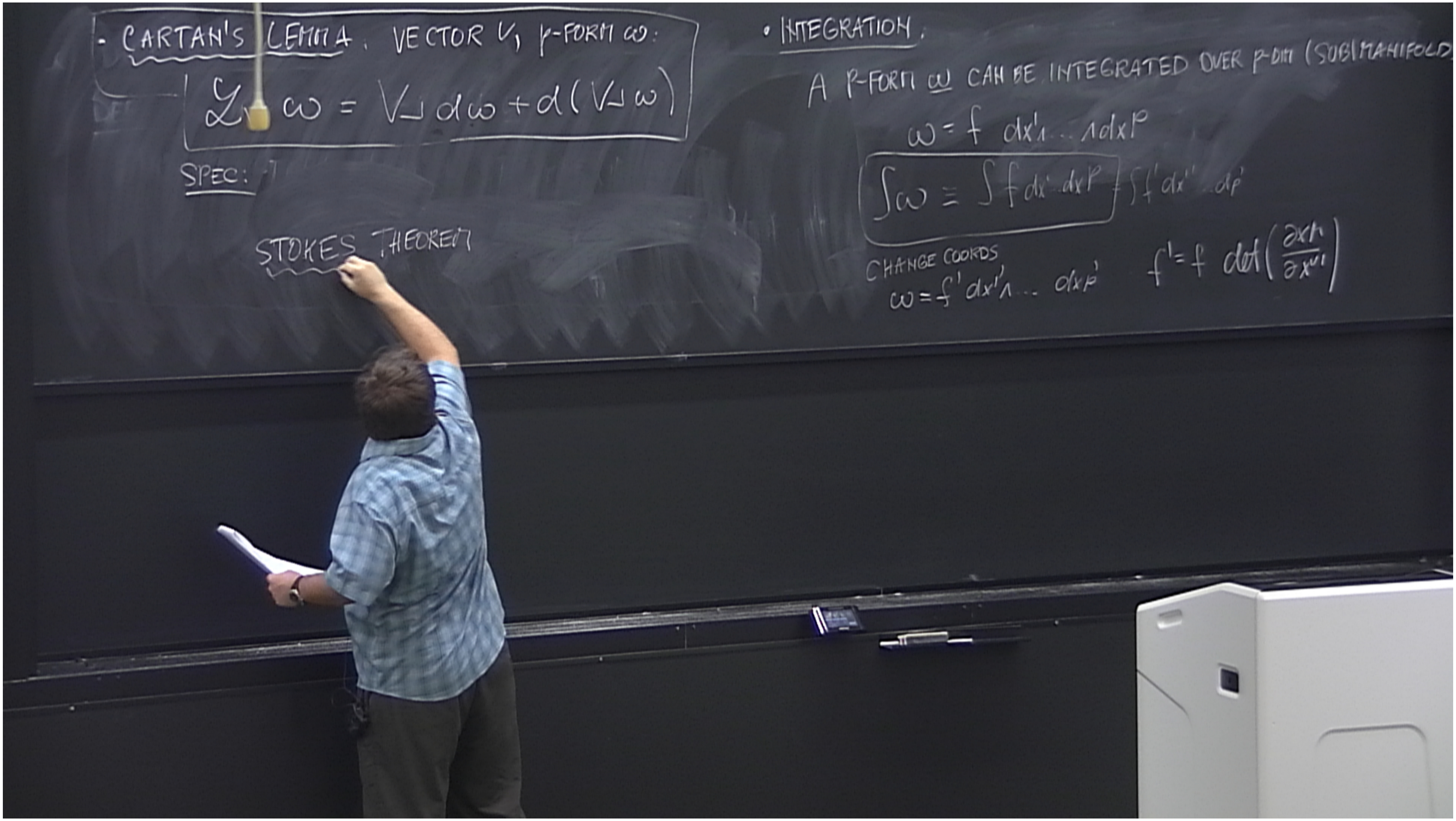
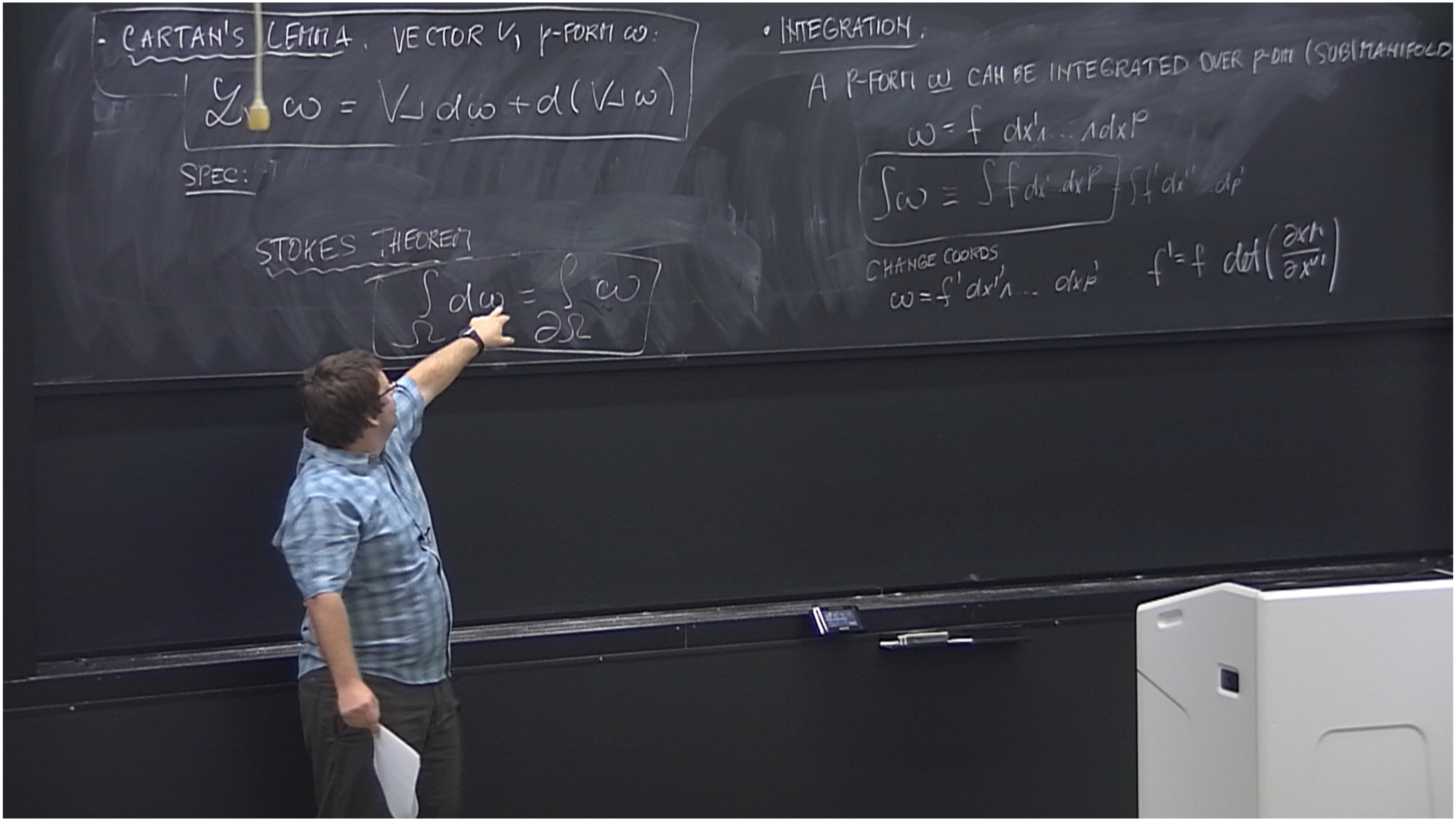
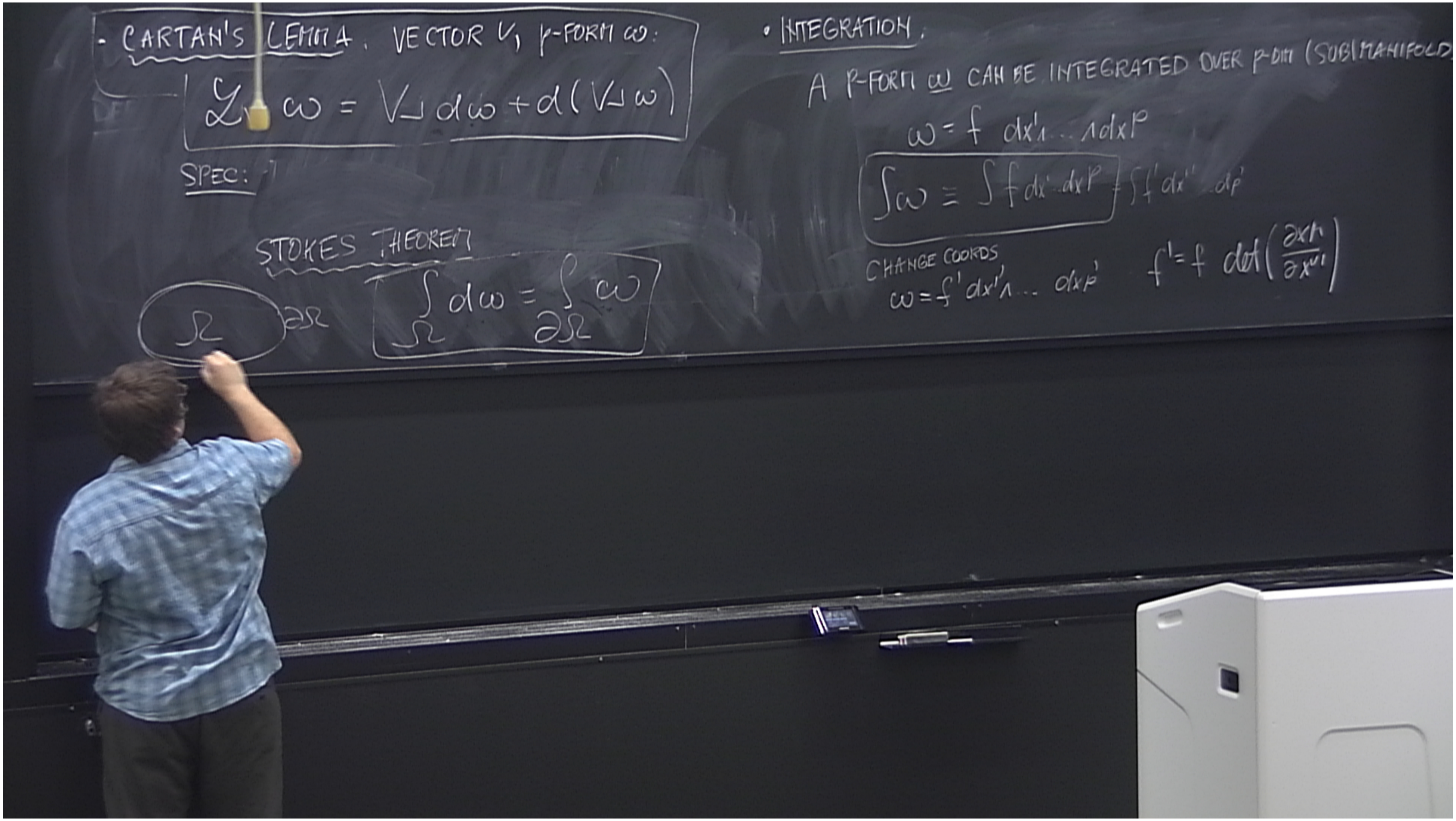
$\mathcal{L}_V df = 0 + d(V \lrcorner df) = d\mathcal{L}_V f$
• INTEGRATION.


SPEC:
ω = df
𝓛v df = 0 + d(V⌟df) = d 𝓛v f
ω = f dx'∧…∧dx^p
∫ω = ∫ f dx' dx^p
CHANGE COORDS
ω = f' dx''∧… dx^p'
f' = f det(∂x^μ/∂x'^ν)



• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
ℒ_V ω = V⌟dω + d(V⌟ω)
SPEC:
STOKES THEOREM
• INTEGRATION.
A p-FORM ω CAN BE INTEGRATED OVER p-DIM (SUB)MANIFOLD
ω = f dx¹∧…∧dx^p
∫ω ≡ ∫ f dx¹…dx^p = ∫ f′ dx′¹…dx′^p
CHANGE COORDS
ω = f′ dx′¹∧… dx′^p
f′ = f det(∂x^μ/∂x′^ν)


• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
SPEC:
STOKES THEOREM
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$
• INTEGRATION.
A p-FORM ω CAN BE INTEGRATED OVER p-DIM (SUB)MANIFOLDS
$\omega = f\, dx^1 \wedge \dots \wedge dx^p$
$\int \omega \equiv \int f\, dx^1 \dots dx^p = \int f' dx'^1 \dots dx'^p$
CHANGE COORDS
$\omega = f' dx'^1 \wedge \dots dx'^p$
$f' = f \det\left(\frac{\partial x^\mu}{\partial x'^\nu}\right)$



CARTAN'S LEMMA . VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
SPEC:
STOKES THEOREM
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$
Ω
∂Ω
INTEGRATION.
A p-FORM ω CAN BE INTEGRATED OVER p-DIM (SUB)MANIFOLD
$\omega = f\, dx^1 \wedge \ldots \wedge dx^p$
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CHANGE COORDS
$\omega = f' dx'^1 \wedge \ldots dx'^p$
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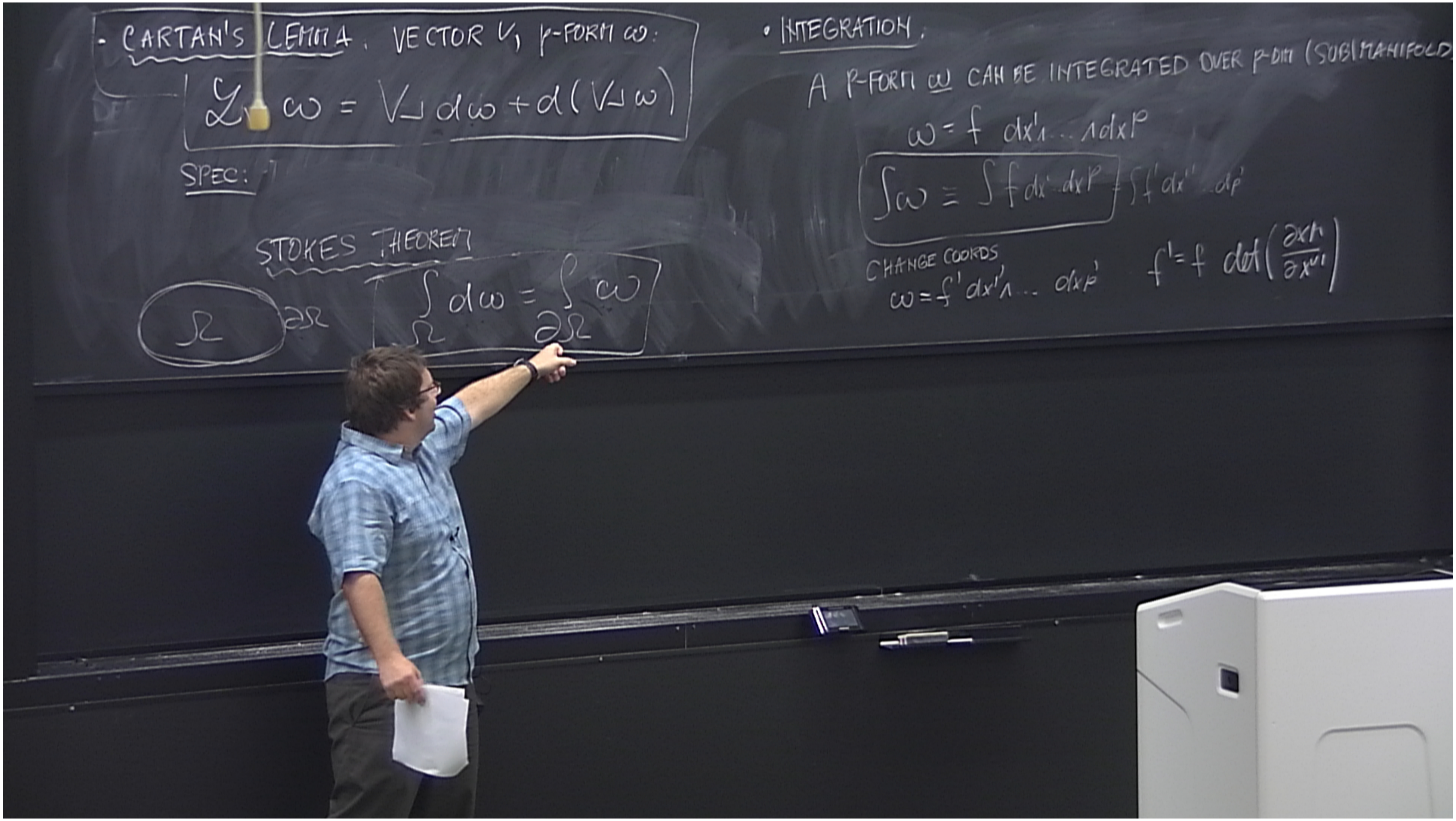

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$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
SPEC:
STOKES THEOREM
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$
Ω
∂Ω
• INTEGRATION .
A p-FORM ω CAN BE INTEGRATED OVER p-DIM (SUB)MANIFOLD
$\omega = f \, dx^1 \wedge \ldots \wedge dx^p$
$\int \omega \equiv \int f \, dx^1 \ldots dx^p = \int f' dx'^1 \ldots dx'^p$
CHANGE COORDS
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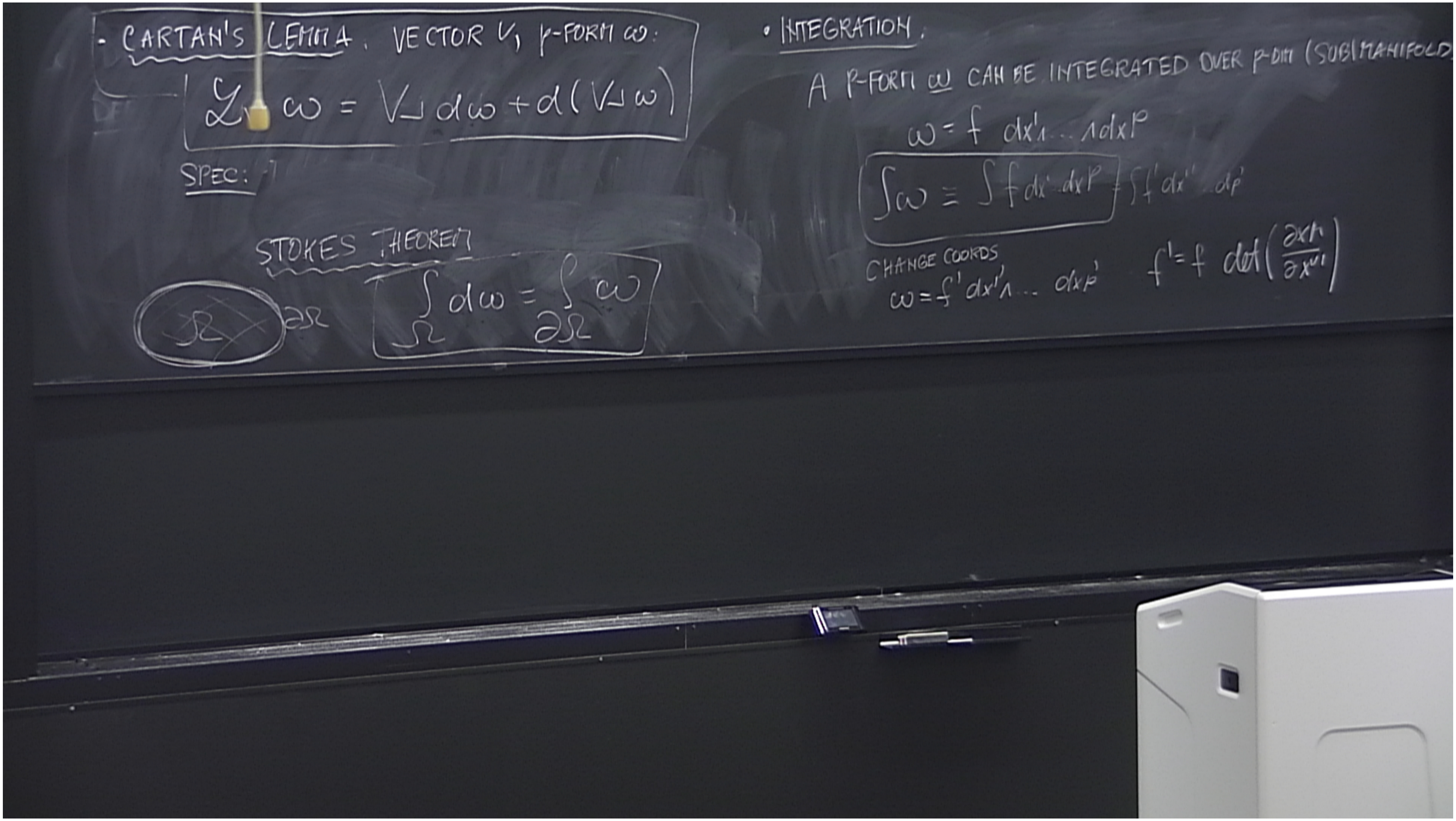
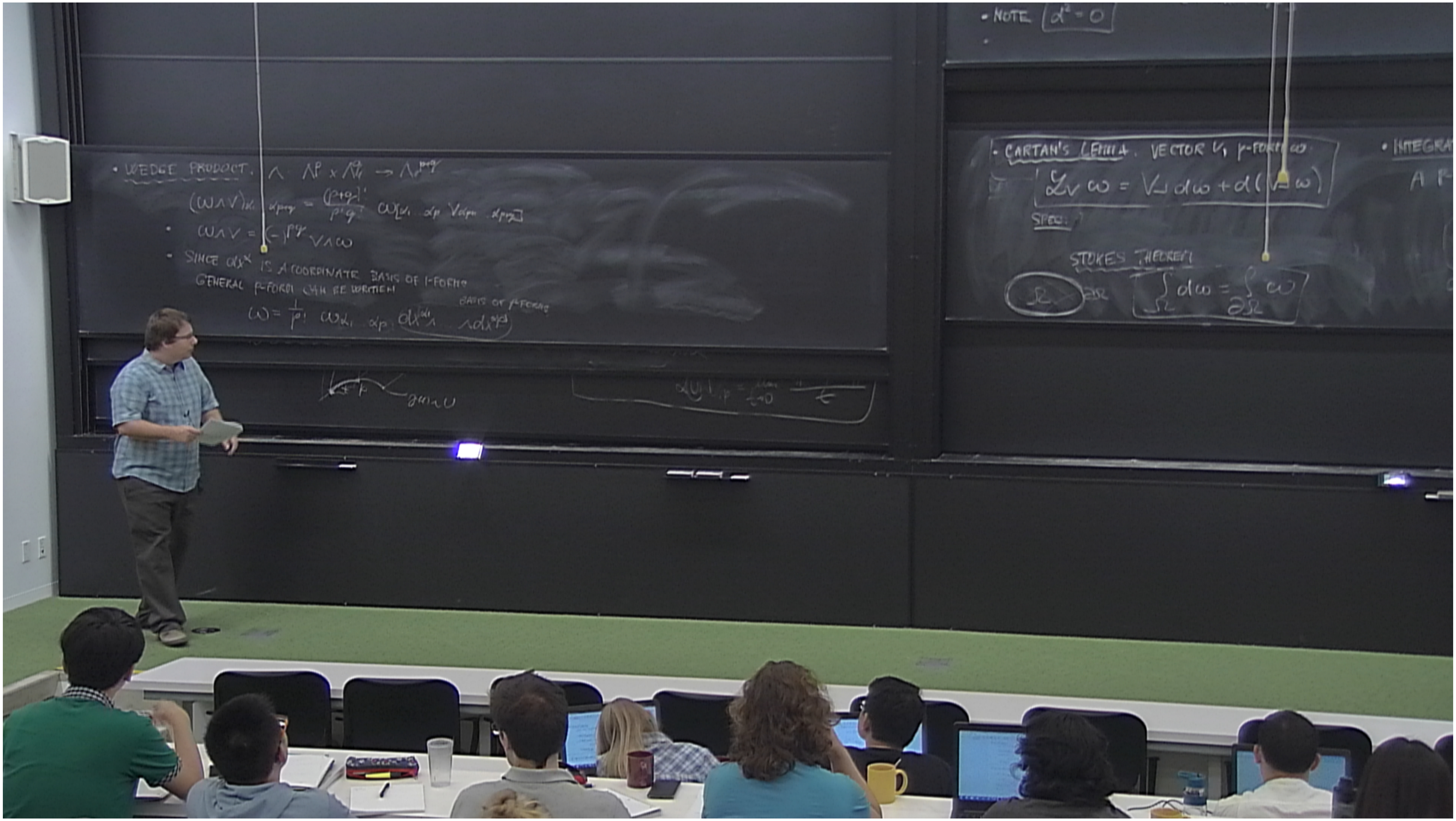
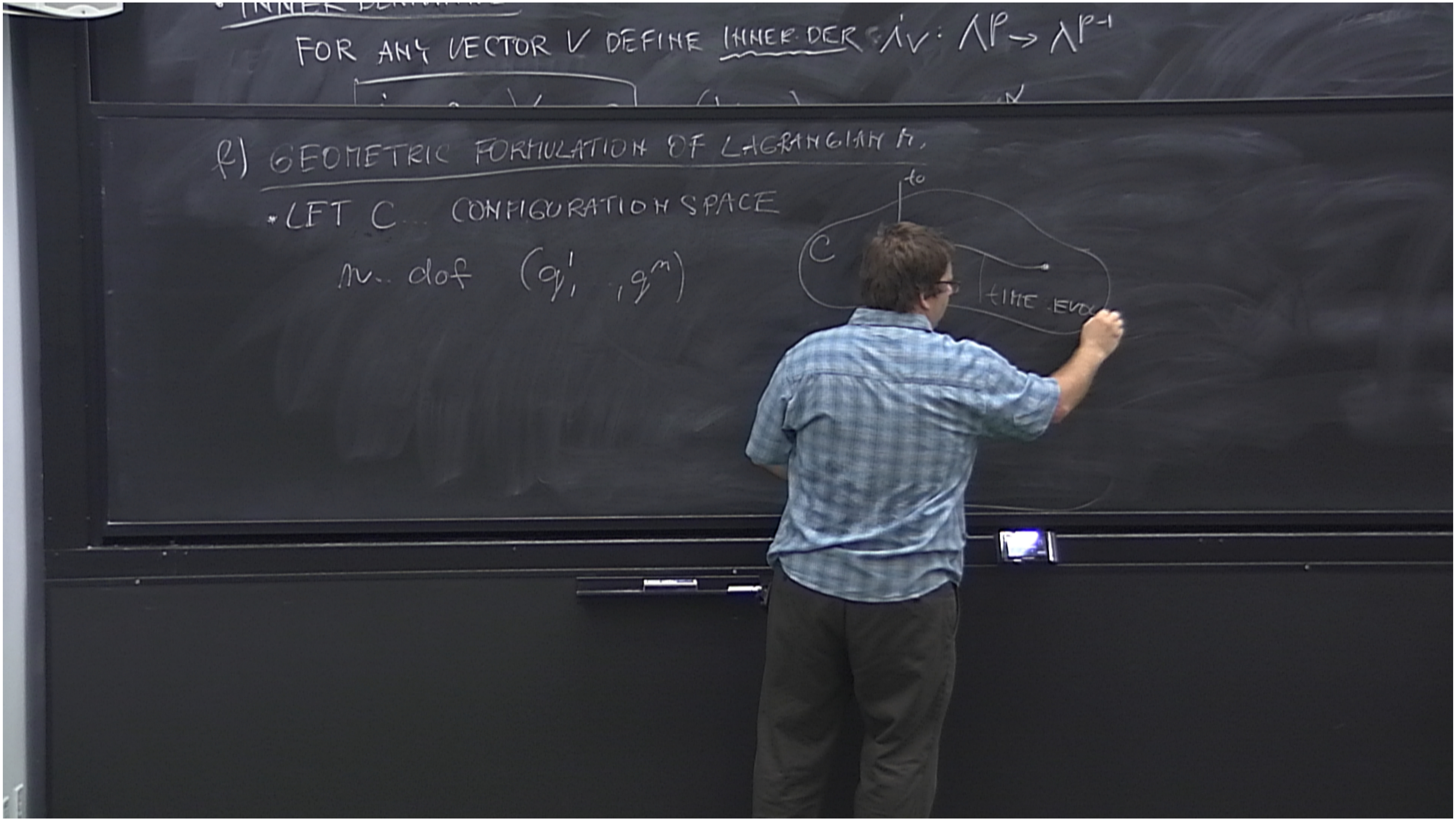
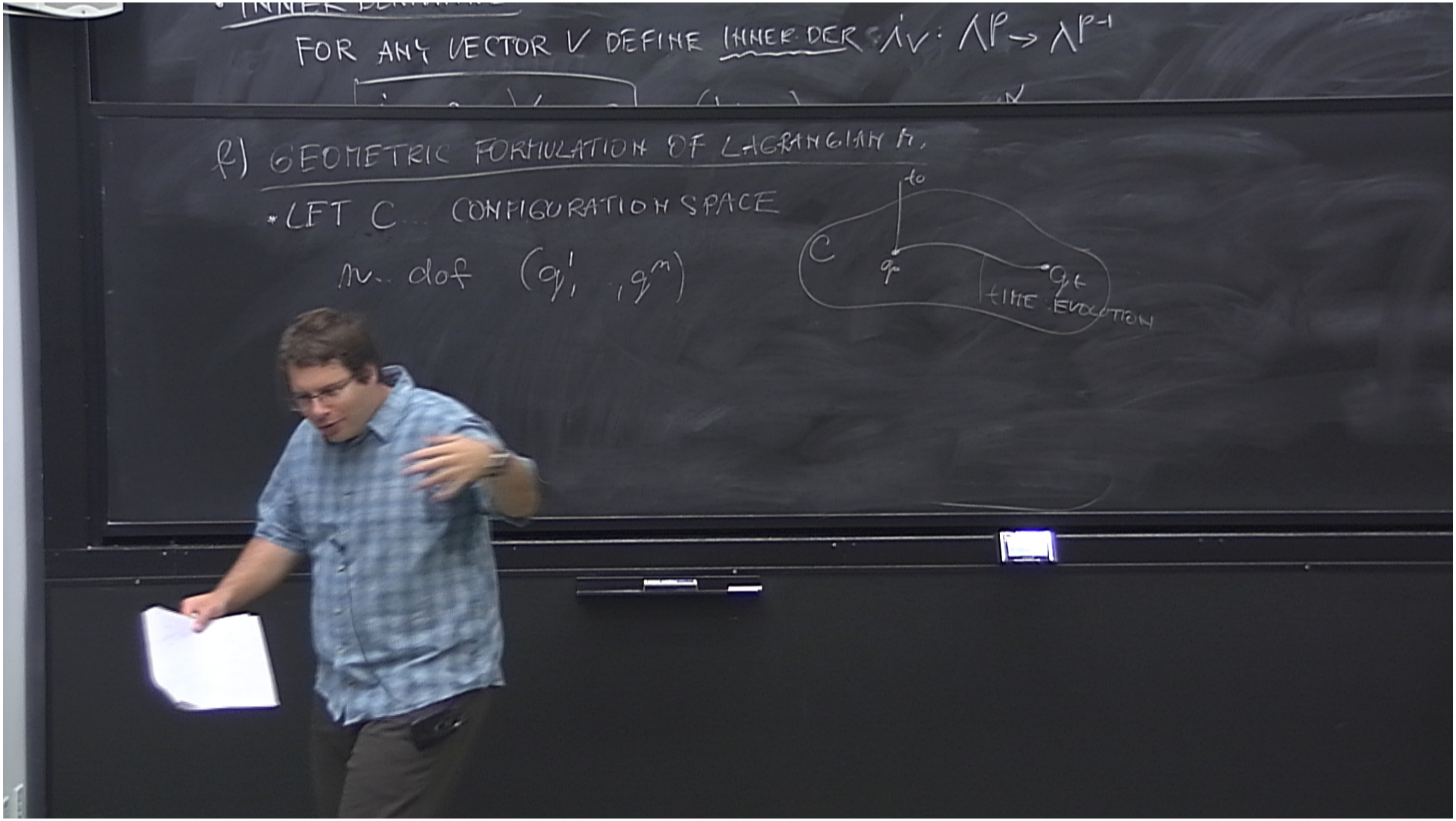
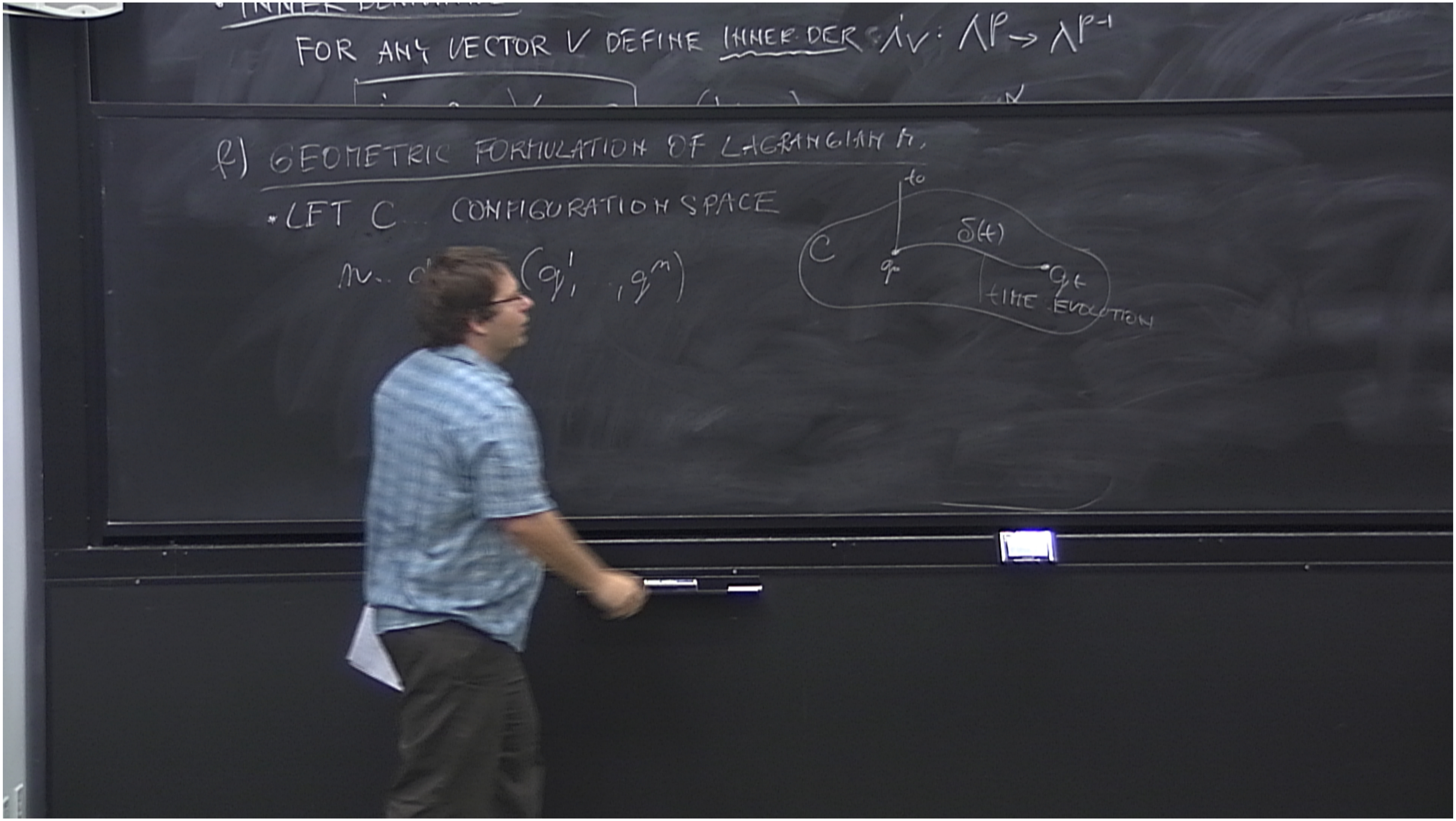
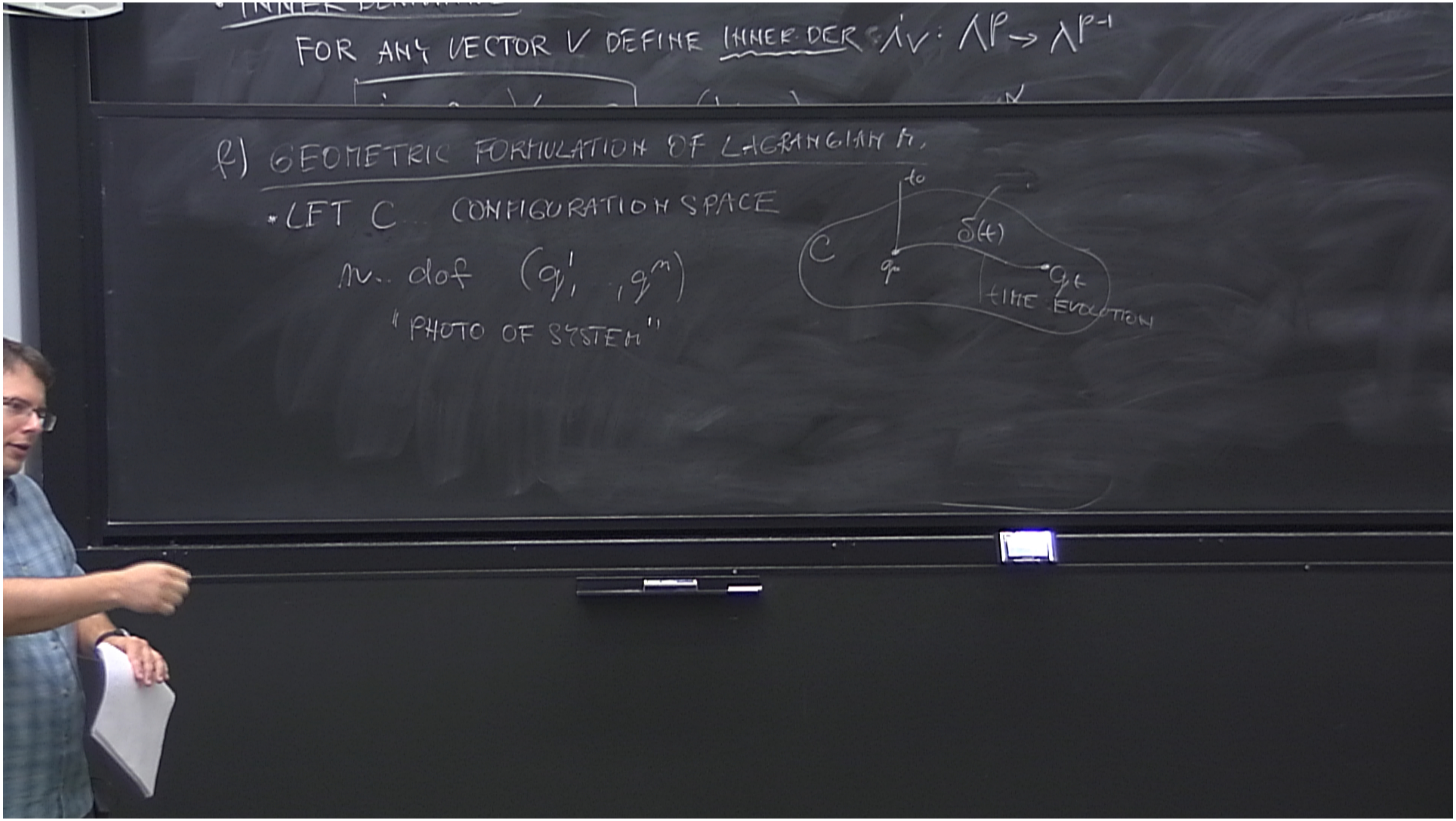

• CARTAN'S LEMMA. VECTOR V, p-FORM ω:
$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
SPEC:
STOKES THEOREM
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$
Ω $\partial\Omega$
• INTEGRATION.
A p-FORM ω CAN BE INTEGRATED OVER p-DIM (SUB)MANIFOLDS
$\omega = f\, dx^1 \wedge \dots \wedge dx^p$
$\int \omega \equiv \int f\, dx^1 \dots dx^p = \int f'\, dx'^1 \dots dx'^p$
CHANGE COORDS
$\omega = f'\, dx'^1 \wedge \dots dx'^p$
$f' = f \det\left(\frac{\partial x^\mu}{\partial x'^\nu}\right)$

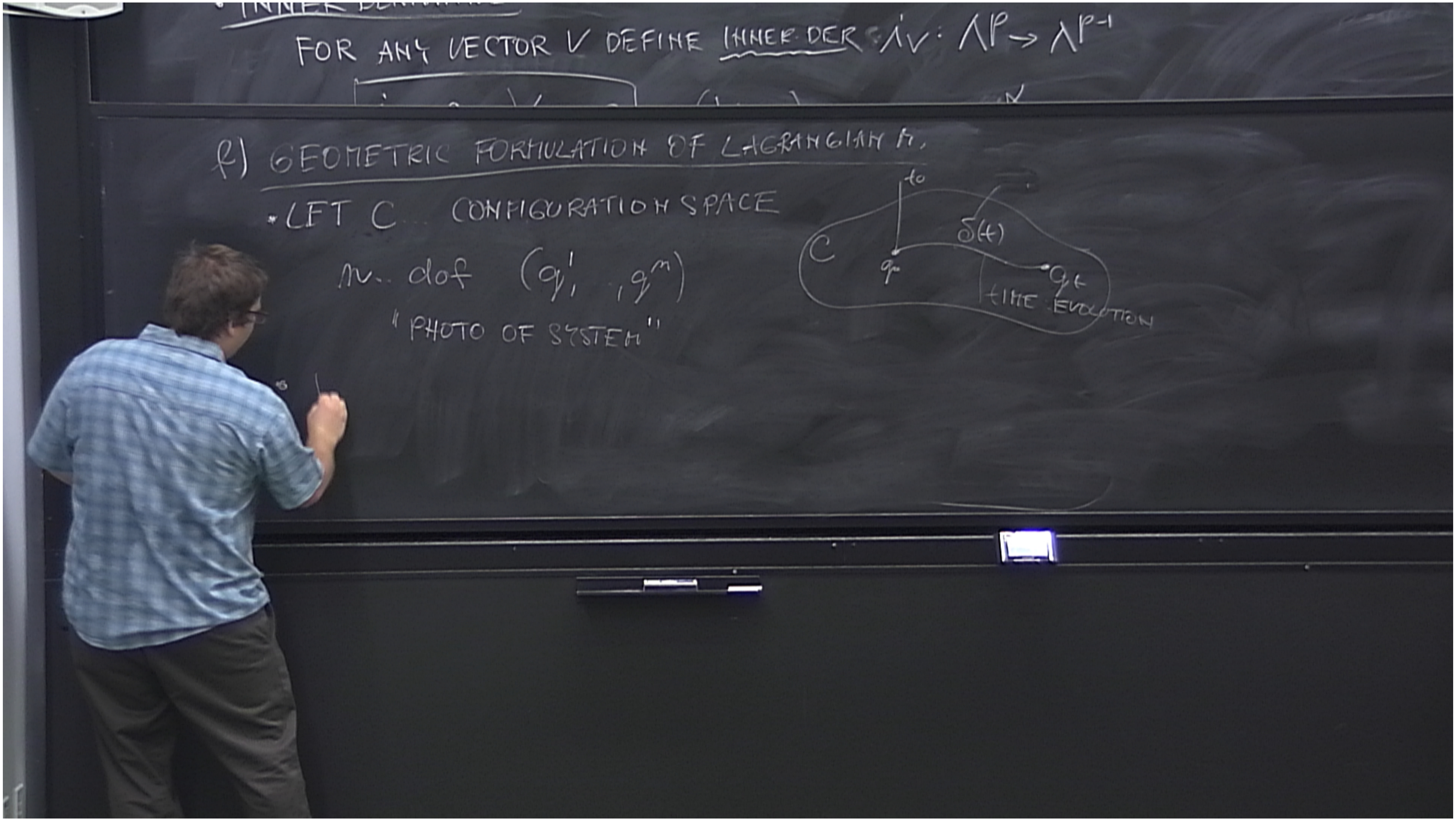

• WEDGE PRODUCT.
• SINCE dx IS A COORDINATE BASIS OF 1-FORMS
GENERAL p-FORM CAN BE WRITTEN
BASIS OF p-FORMS
• NOTE
• CARTAN'S LEMMA. VECTOR V, p-FORM ω
SPEC:
STOKES THEOREM


FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C ... CONFIGURATION SPACE
n .. dof $(q^1, \dots, q^n)$
C
t_0
time evol

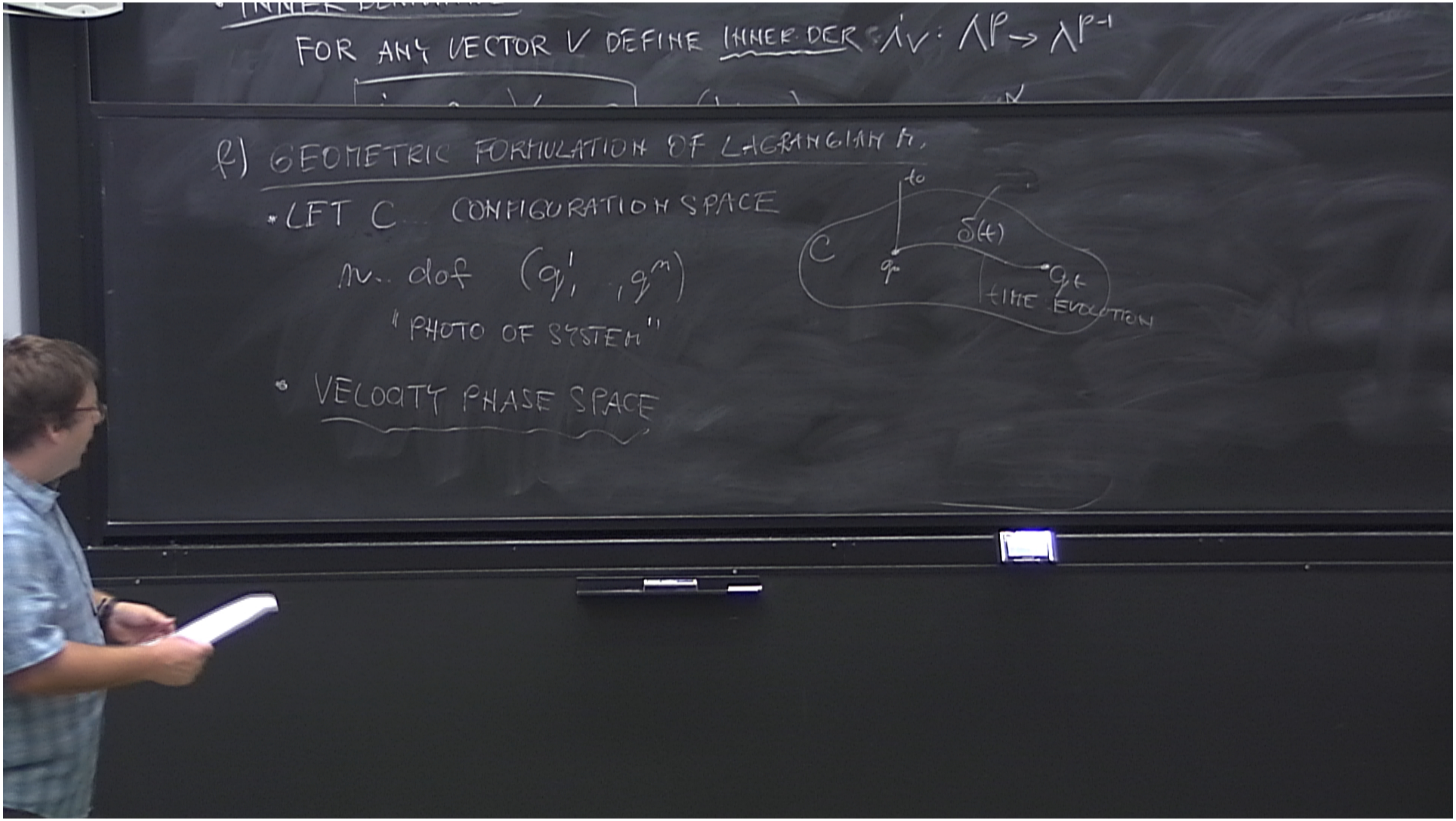

FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C ... CONFIGURATION SPACE
n.. dof $(q^1, \ldots, q^n)$
C
t_0
q_0
q_t
TIME EVOLUTION

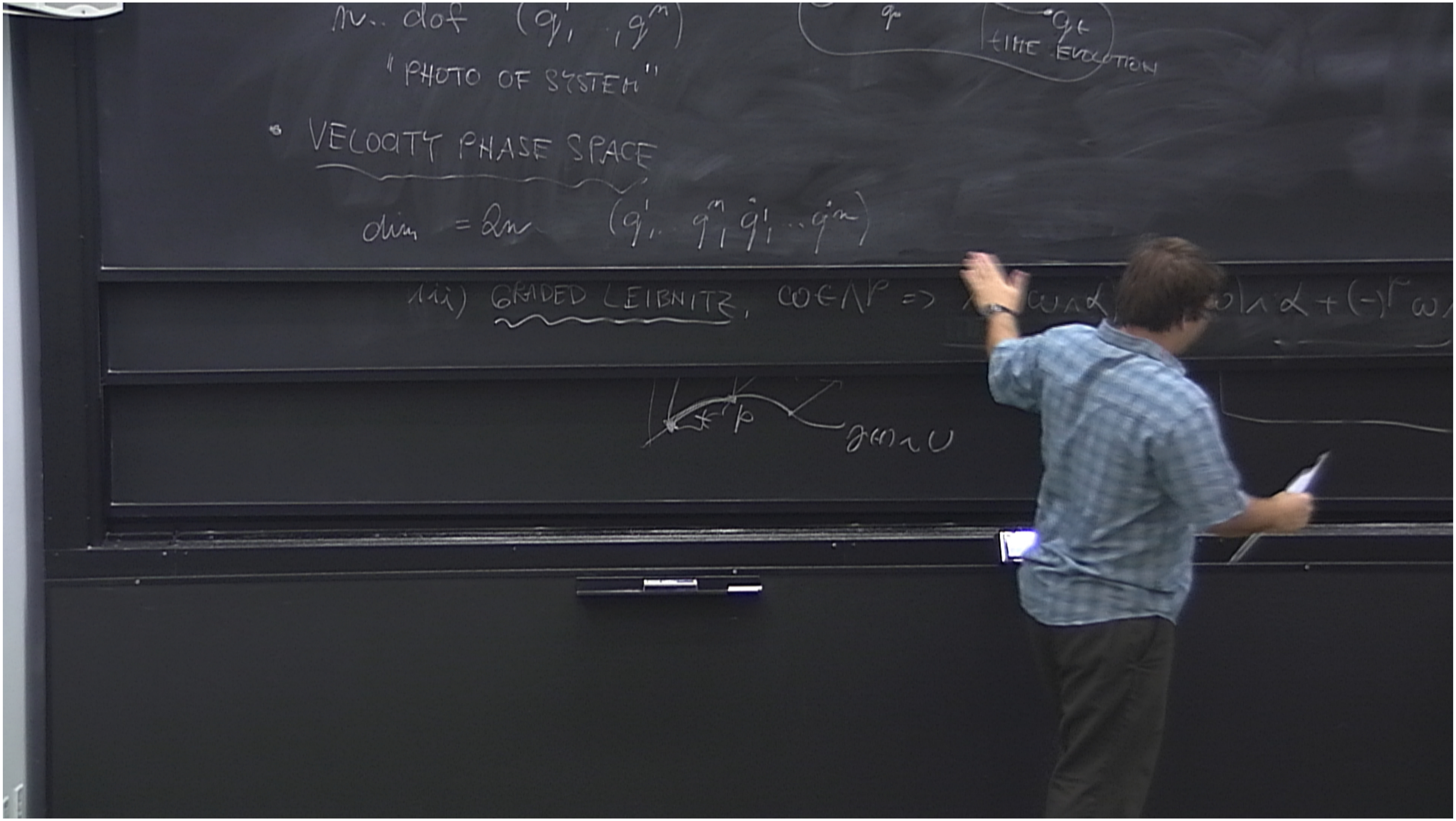

FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C ... CONFIGURATION SPACE
$(q^1, \ldots, q^n)$
t_0
C
$\gamma(t)$
q_0
q_t
TIME EVOLUTION


FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C .. CONFIGURATION SPACE
n.. dof $(q^1, \ldots, q^n)$
"PHOTO OF SYSTEM"
t_0
C
$\delta(t)$
q_0
q_t
time EVOLUTION

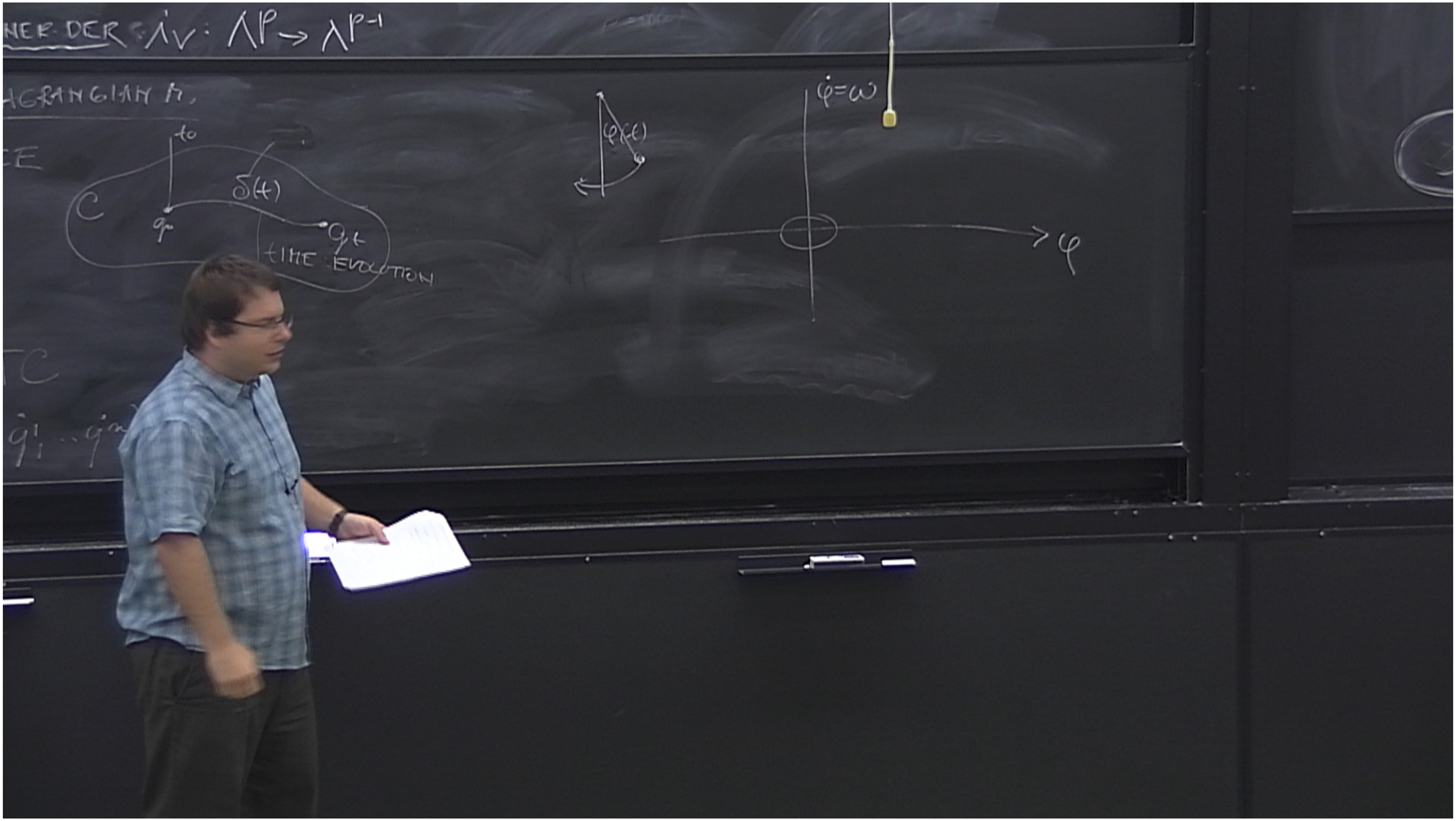


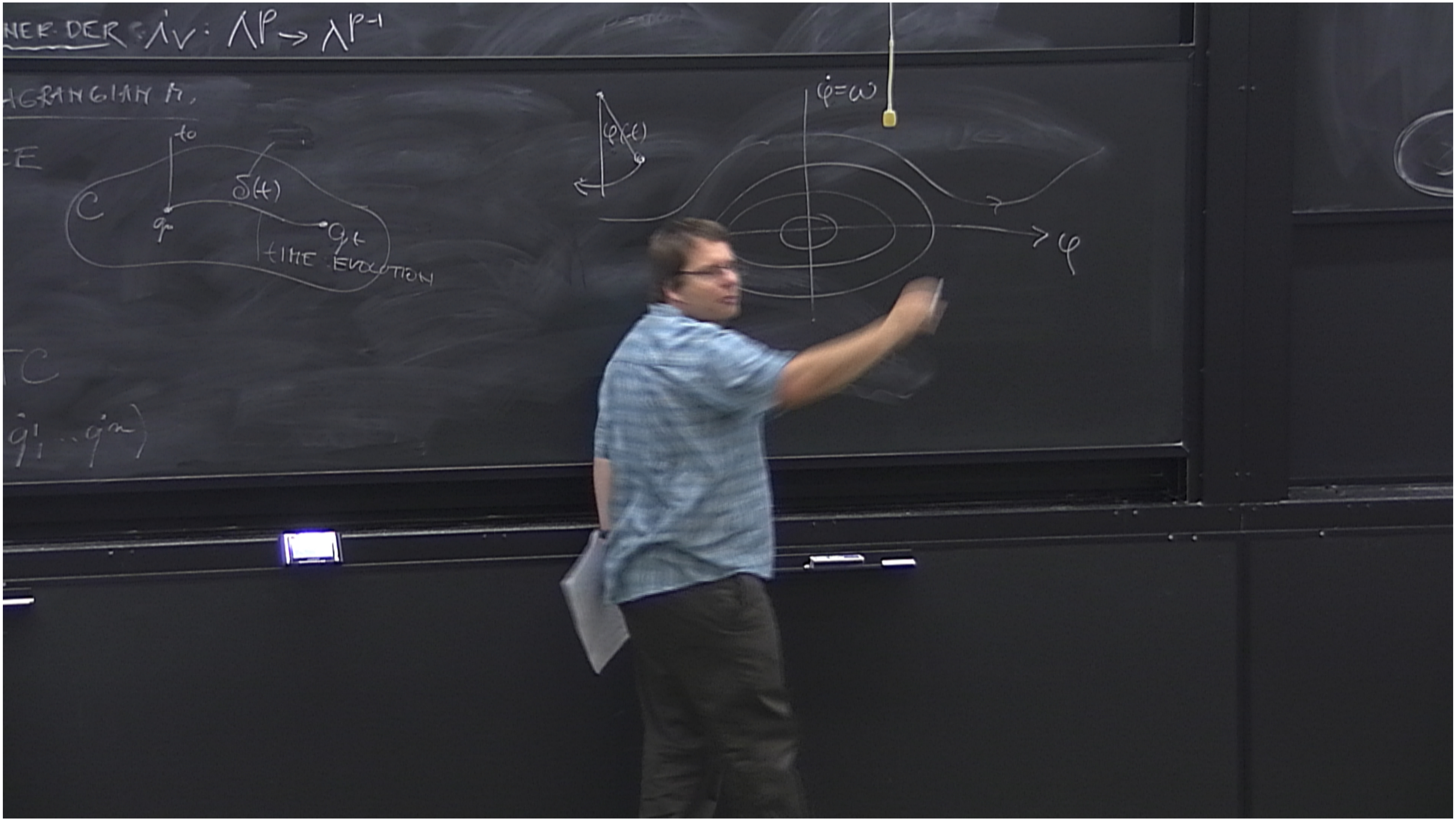
FOR ANY VECTOR V DEFINE INNER DER: $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C ... CONFIGURATION SPACE
n .. dof $(q^1, \ldots, q^n)$
"PHOTO OF SYSTEM"
C
t_0
q_0
$\gamma(t)$
q_t
TIME EVOLUTION


FOR ANY VECTOR V DEFINE INNER DER. $i_V : \Lambda^p \to \Lambda^{p-1}$
f) GEOMETRIC FORMULATION OF LAGRANGIAN M.
• LET C … CONFIGURATION SPACE
n.. dof $(q^1, \ldots, q^n)$
"PHOTO OF SYSTEM"
• VELOCITY PHASE SPACE
t_0
$\delta(t)$
C
q_0
q_t
TIME EVOLUTION

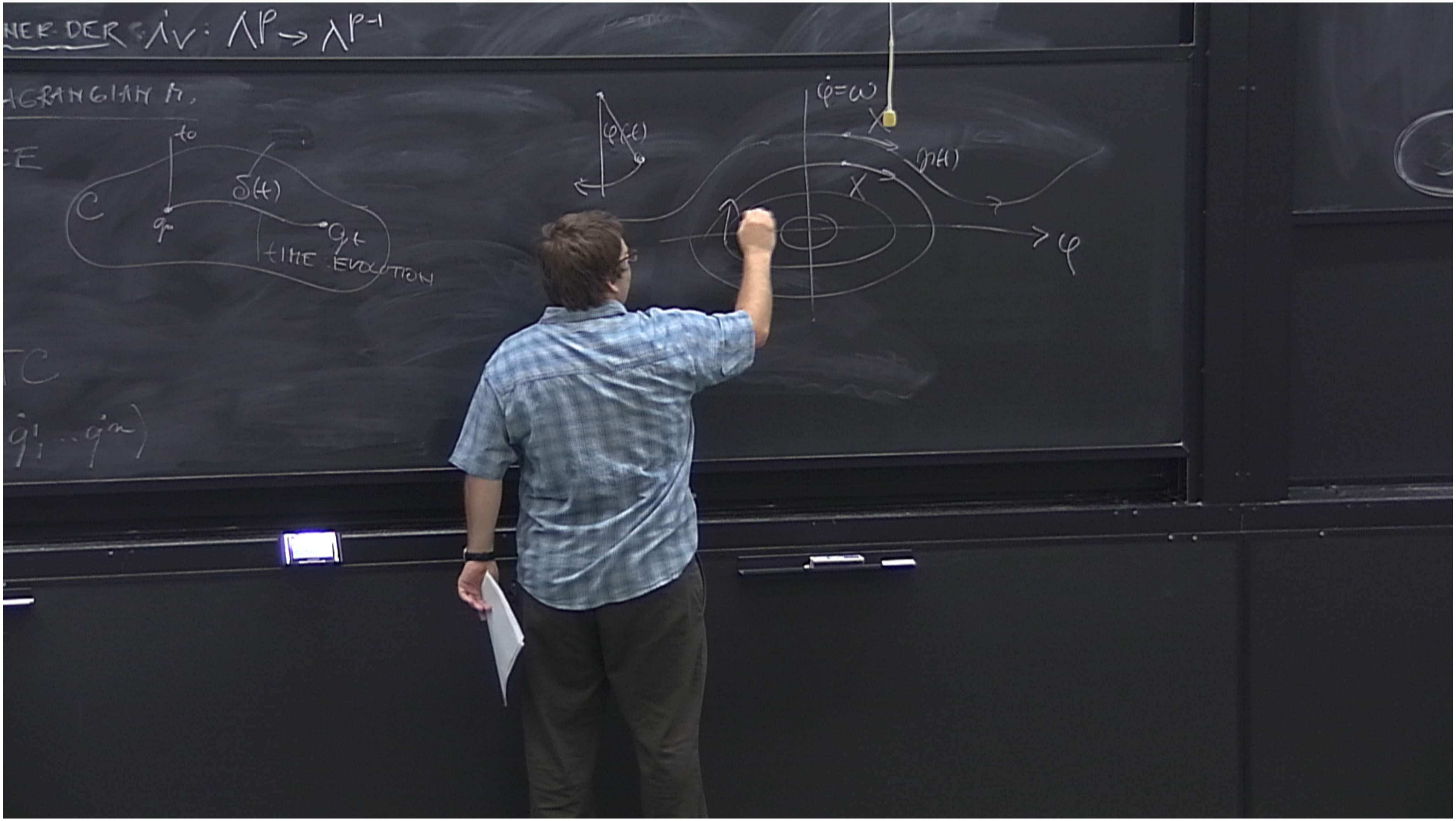


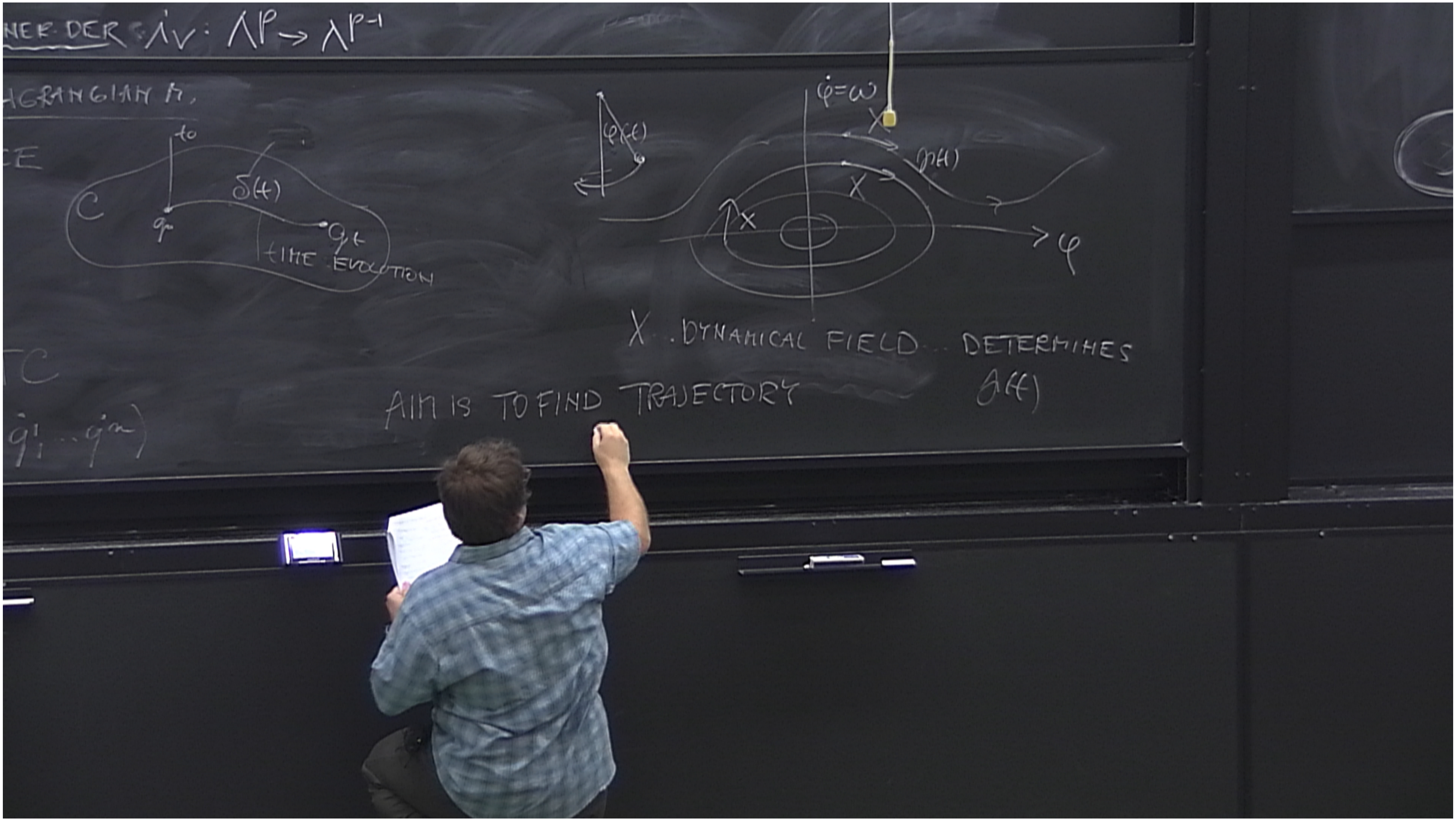
"PHOTO OF SYSTEM"
VELOCITY PHASE SPACE
TIME EVOLUTION
iii) GRADED LEIBNITZ





$\dot\varphi = \omega$
$\varphi(t)$
C
$\delta(t)$
TIME EVOLUTION


TIME EVOLUTION


TIME EVOLUTION
X .. DYNAMICAL FIELD .. DETERMINES
AIM IS TO FIND TRAJECTORY