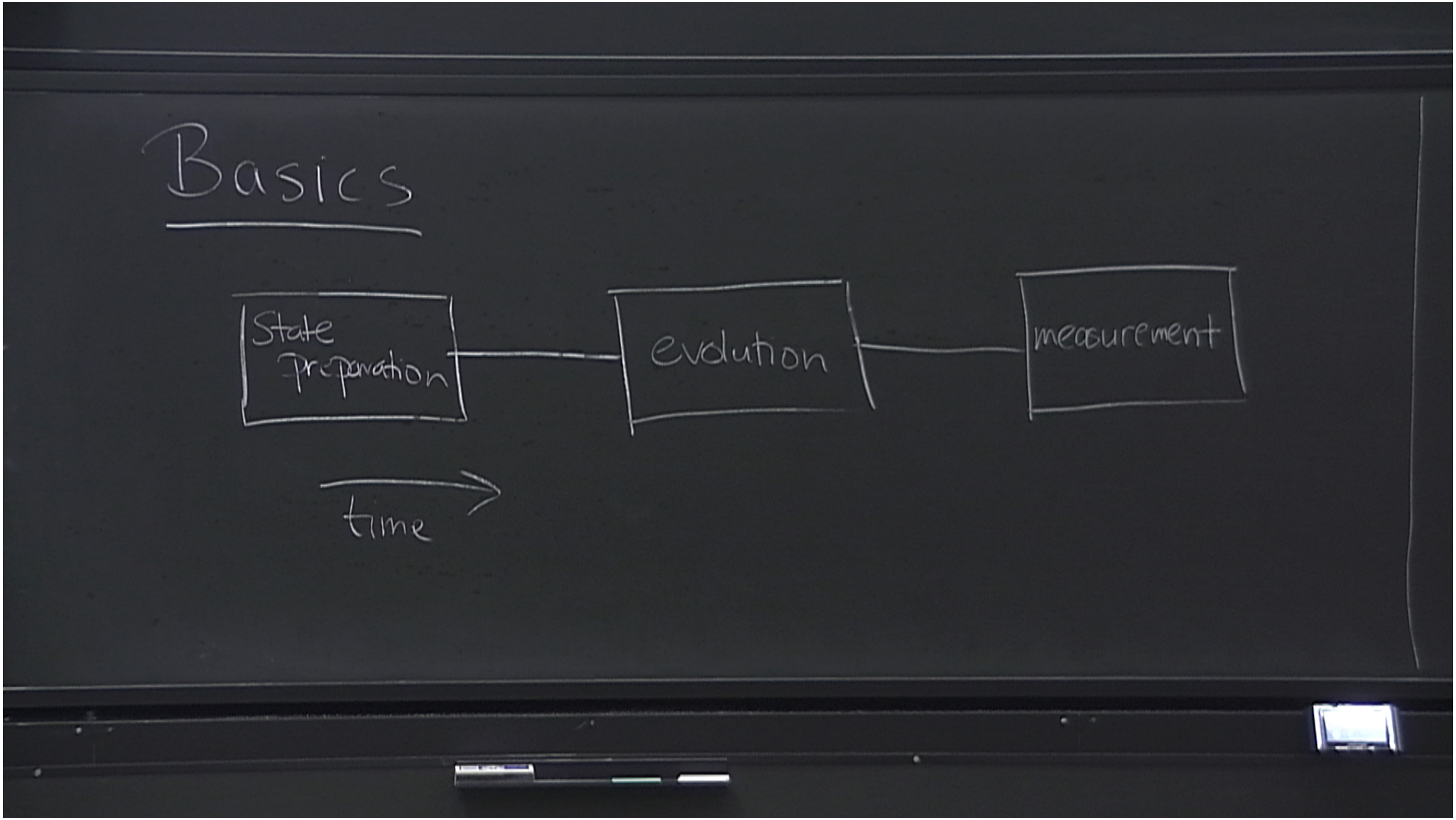
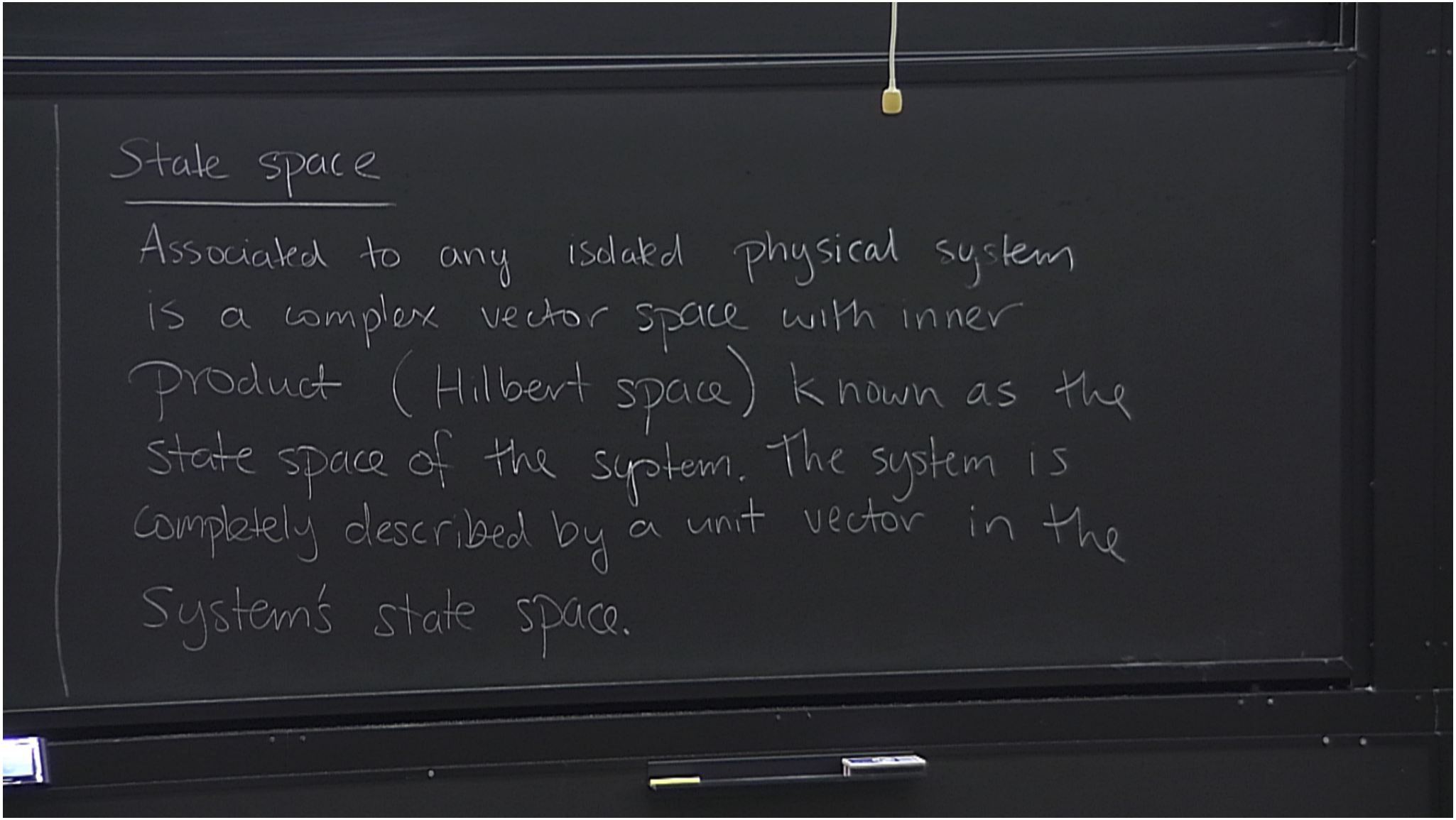
Title: Quantum Mechanics

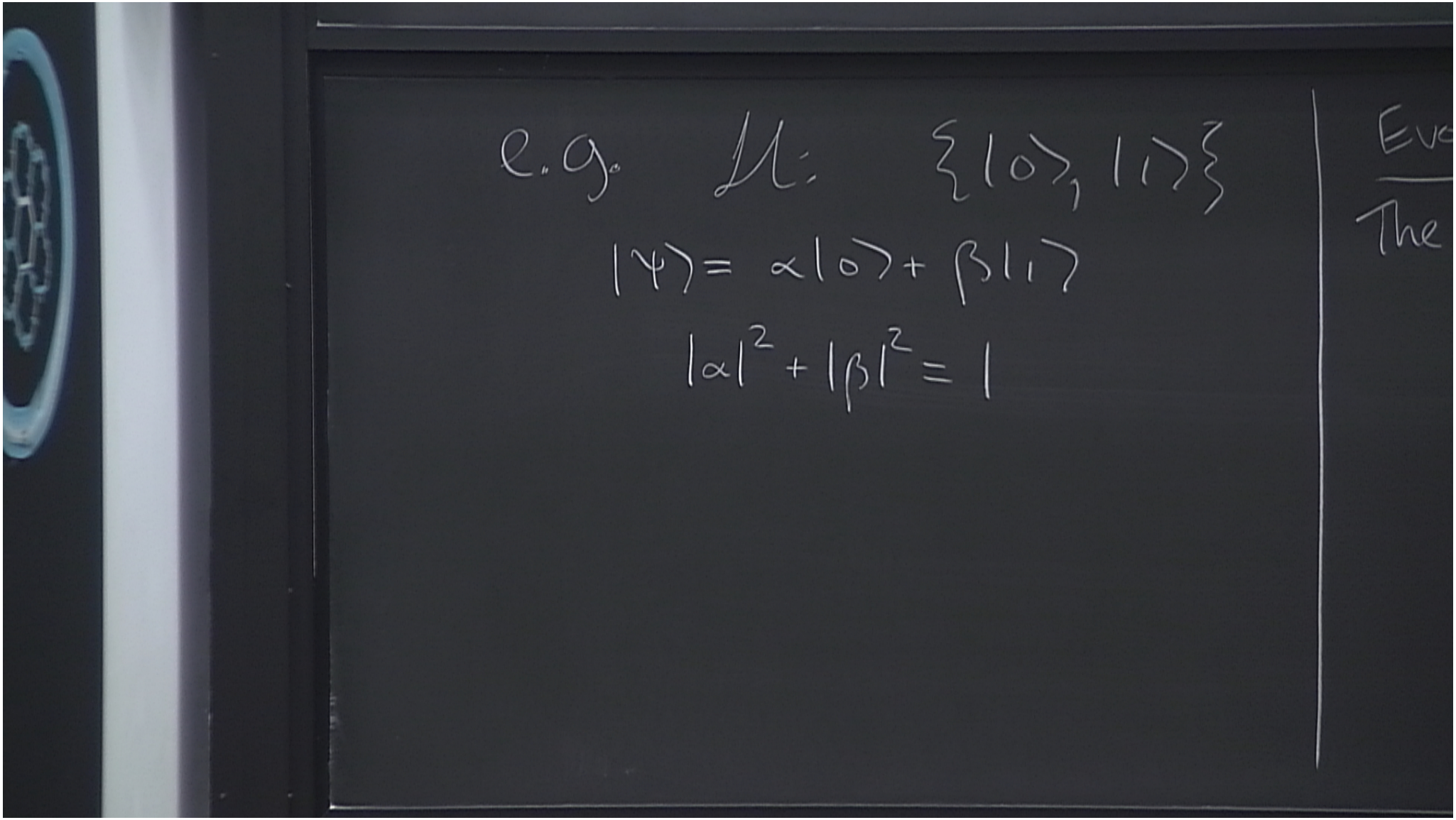
Date: Aug 14, 2015 10:30 AM

URL: http://pirsa.org/15080095

Abstract:


Basics
State preparation
evolution
measurement
time


State space
Associated to any isolated physical system is a complex vector space with inner product (Hilbert space) known as the state space of the system. The system is completely described by a unit vector in the system's state space.


e.g. $\mathcal{H}$: $\{|0\rangle, |1\rangle\}$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$|\alpha|^2 + |\beta|^2 = 1$
Eve
The

Measurement

A projective meas. is described by an observable M, a Hermit
of the system being observed. The observable has a spectral
wher P_m is the projector onto the eigenspace of M with e

e.g. $\{P_m\} = \{|0\rangle\langle 0|, |1\rangle\langle 1|\}$

$\{P_m\} = \{|+\rangle\langle +|, |-\rangle\langle -|\}$ where $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$

transformation. That is, the state $|\psi\rangle$ at time

by a unitary operator which depends on t,

e.g. $U = |0\rangle\langle 0| + e^{i\phi}|1\rangle\langle 1|$

$U = |0\rangle\langle 1| - |1\rangle\langle 0|$

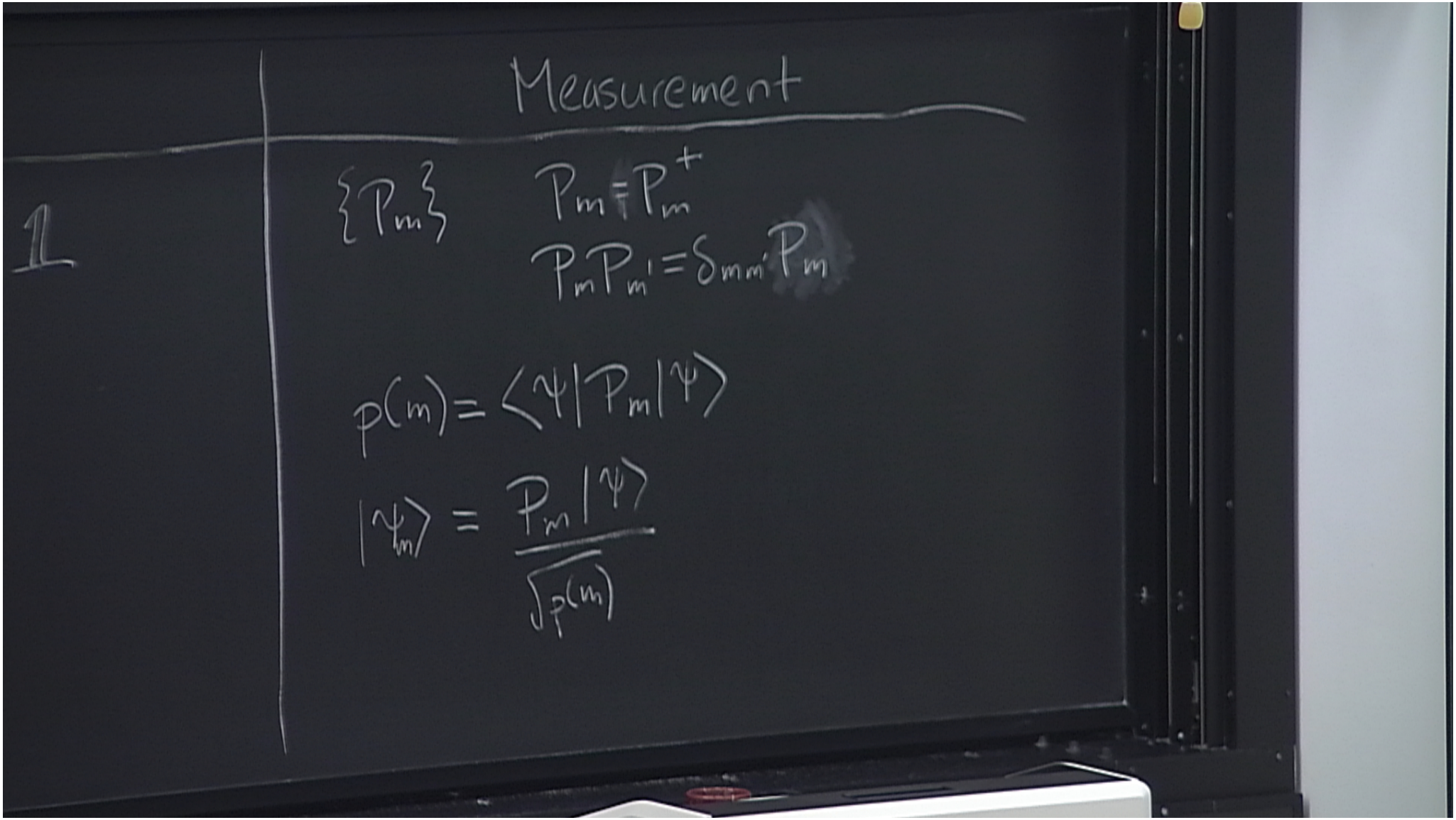
[0 1]

ed by an observable M, a Hermitian opera

$U = |0\rangle\langle 1| - |1\rangle\langle 0|$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

observable M, a Hermitian operator on H
ervable has a spectral decomposition M
eigenspace of M with eigenvalues m.


Measurement
1
$\{P_m\}$
$P_m = P_m^\dagger$
$P_m P_{m'} = \delta_{mm'} P_m$
$p(m) = \langle\psi| P_m |\psi\rangle$
$|\psi_m\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}}$

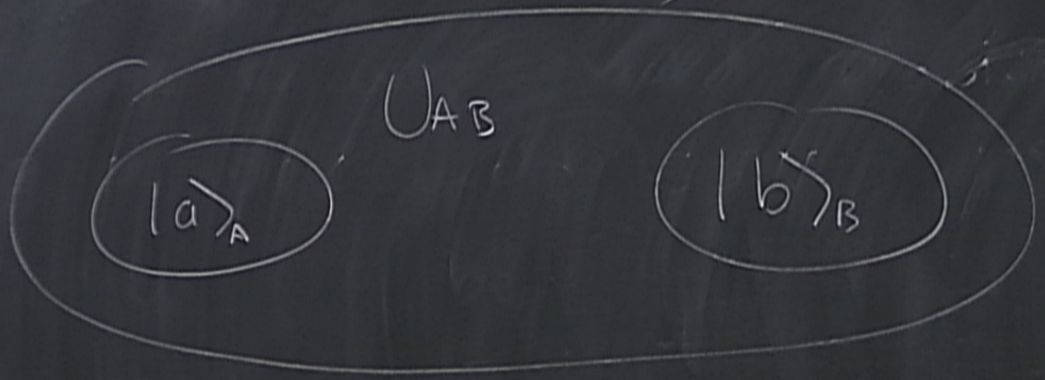
Composite System

The state space of a composite system is the the tensor product of the component physical systems.

e.g. $\mathcal{H}_A$: $\{|0\rangle_A, |1\rangle_A\}$; $\mathcal{H}_B$ $\{|0\rangle_B, |1\rangle_B\}$

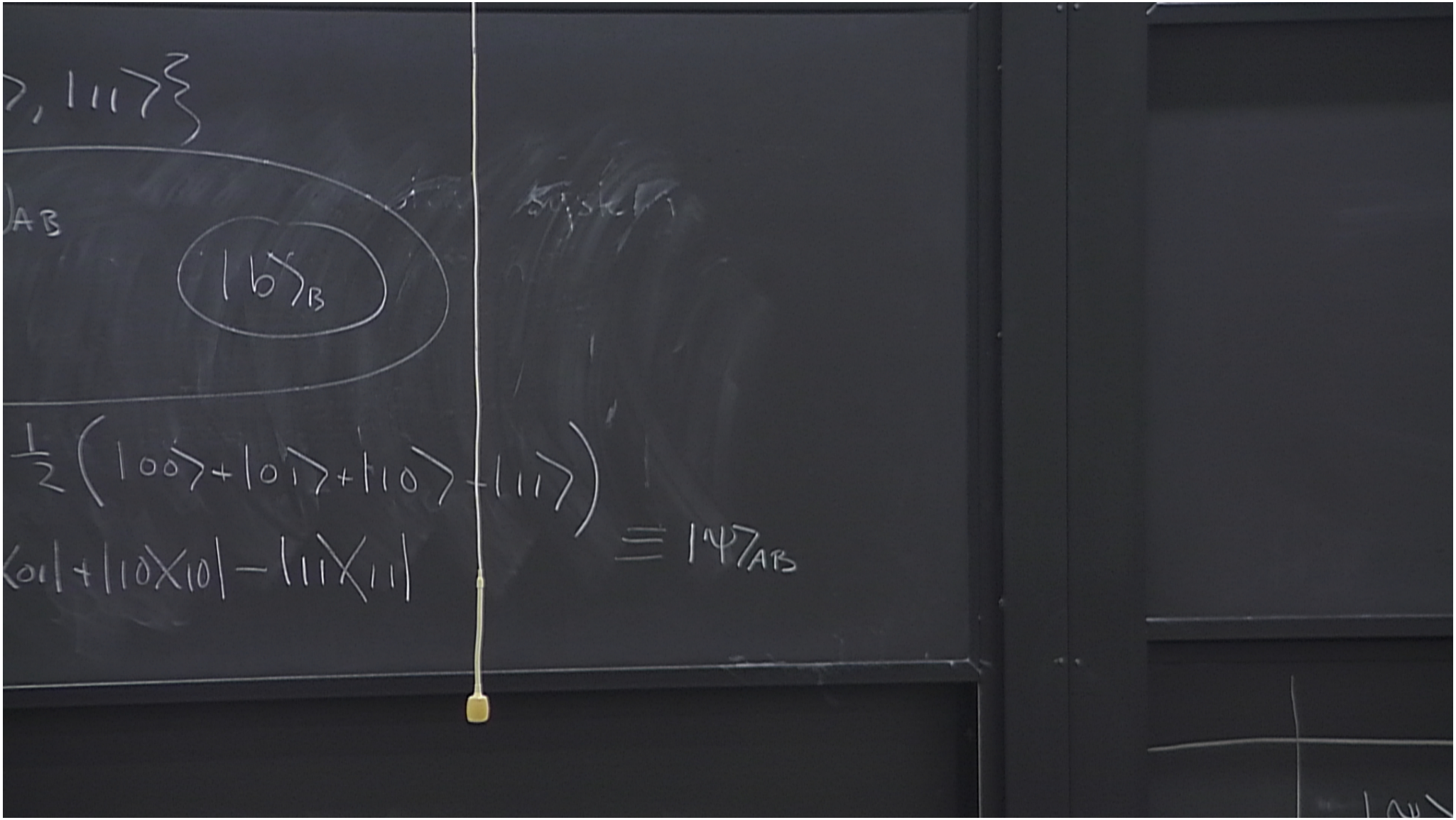
$\mathcal{H}_{AB}$ $\{|0\rangle_A|0\rangle_B, |0\rangle_A|1\rangle_B, |1\rangle|0\rangle, |1\rangle|1\rangle\}$

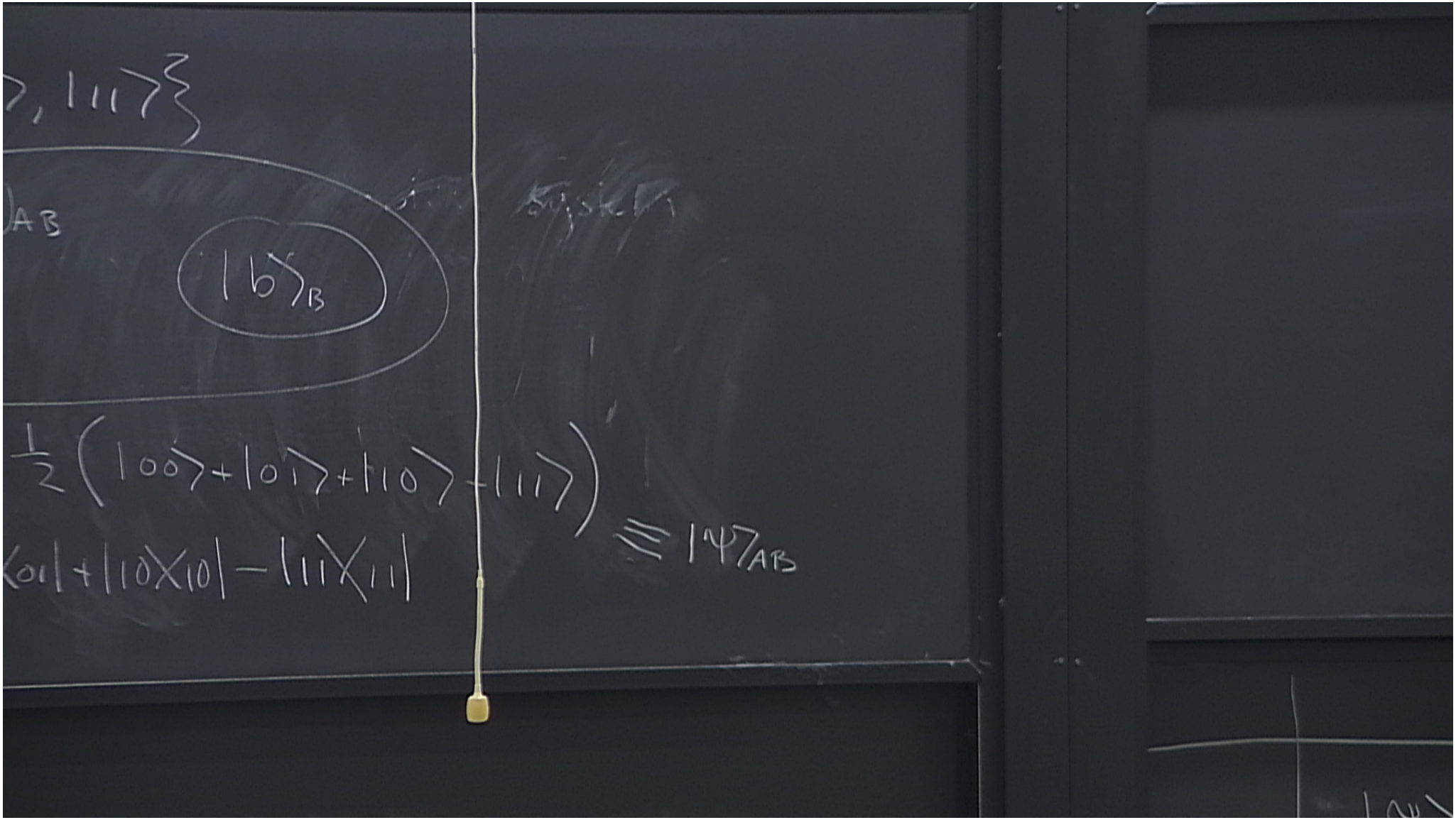
$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$



e.g. $|+\rangle_A|+\rangle_B = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

$U_{AB} = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$


$|0\rangle_B$
$\frac{1}{2}(|00\rangle+|01\rangle+|10\rangle-|11\rangle)$
$\equiv |\Psi\rangle_{AB}$


$|11\rangle\}$
$|b\rangle_B$
$\frac{1}{2}(|00\rangle+|01\rangle+|10\rangle-|11\rangle) \equiv |\Psi\rangle_{AB}$
$|01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$

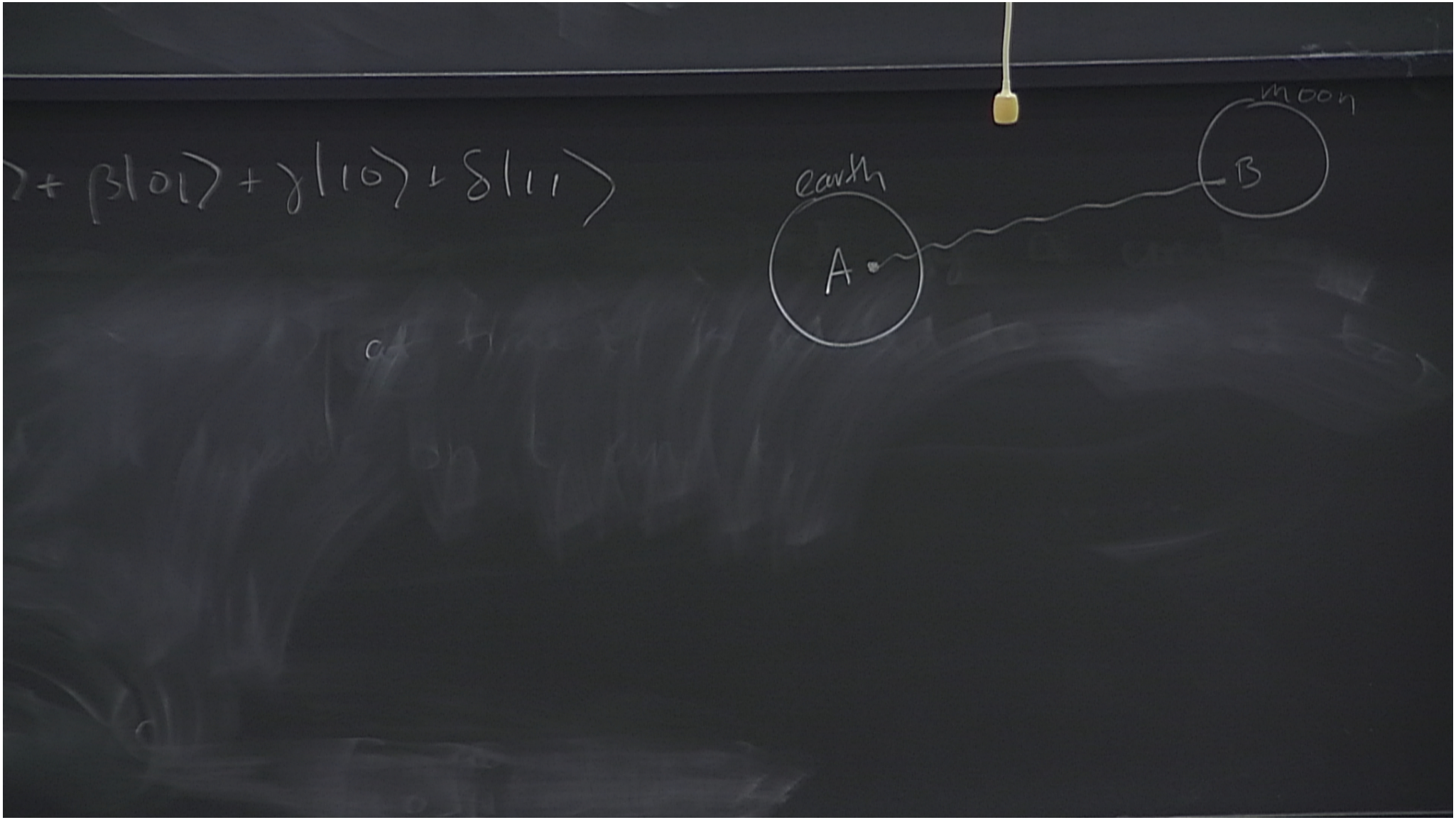
$$|\psi'\rangle_{AB} = U_{AB}\,|\psi\rangle_{AB}$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|+\rangle|0\rangle + |-\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_A|+\rangle_B + |1\rangle_A|-\rangle_B\right)$$

$$\neq |\phi_a\rangle_A\,|\phi_b\rangle_B$$

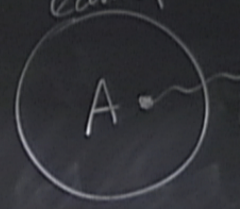

+ β|01⟩ + γ|10⟩ + δ|11⟩
earth
A
B
moon

$$|\Psi_G\rangle_{AB} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$\hat{\rho}_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi|_{AB}\right]$$



$$|\psi_G\rangle_{AB} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

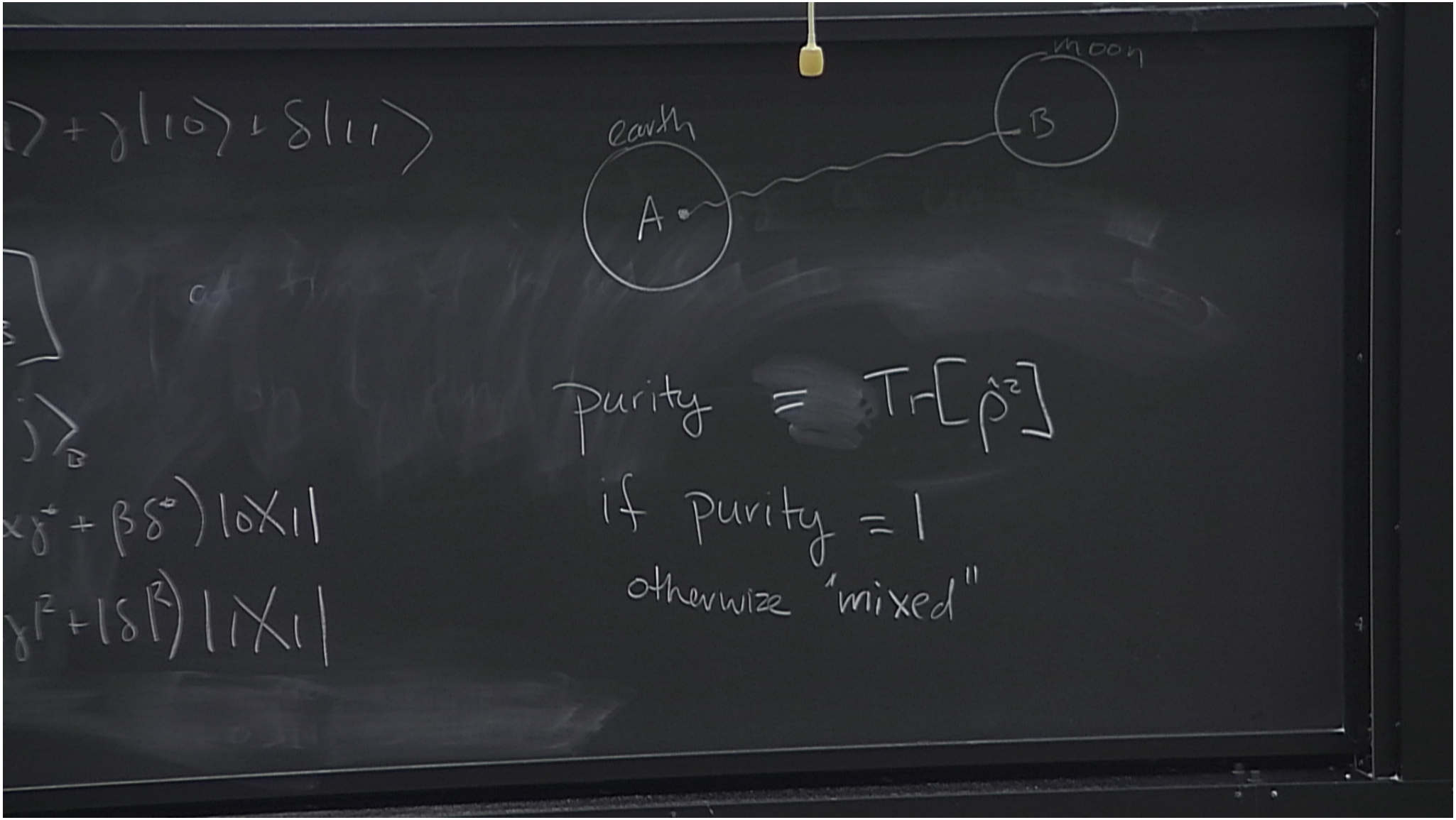


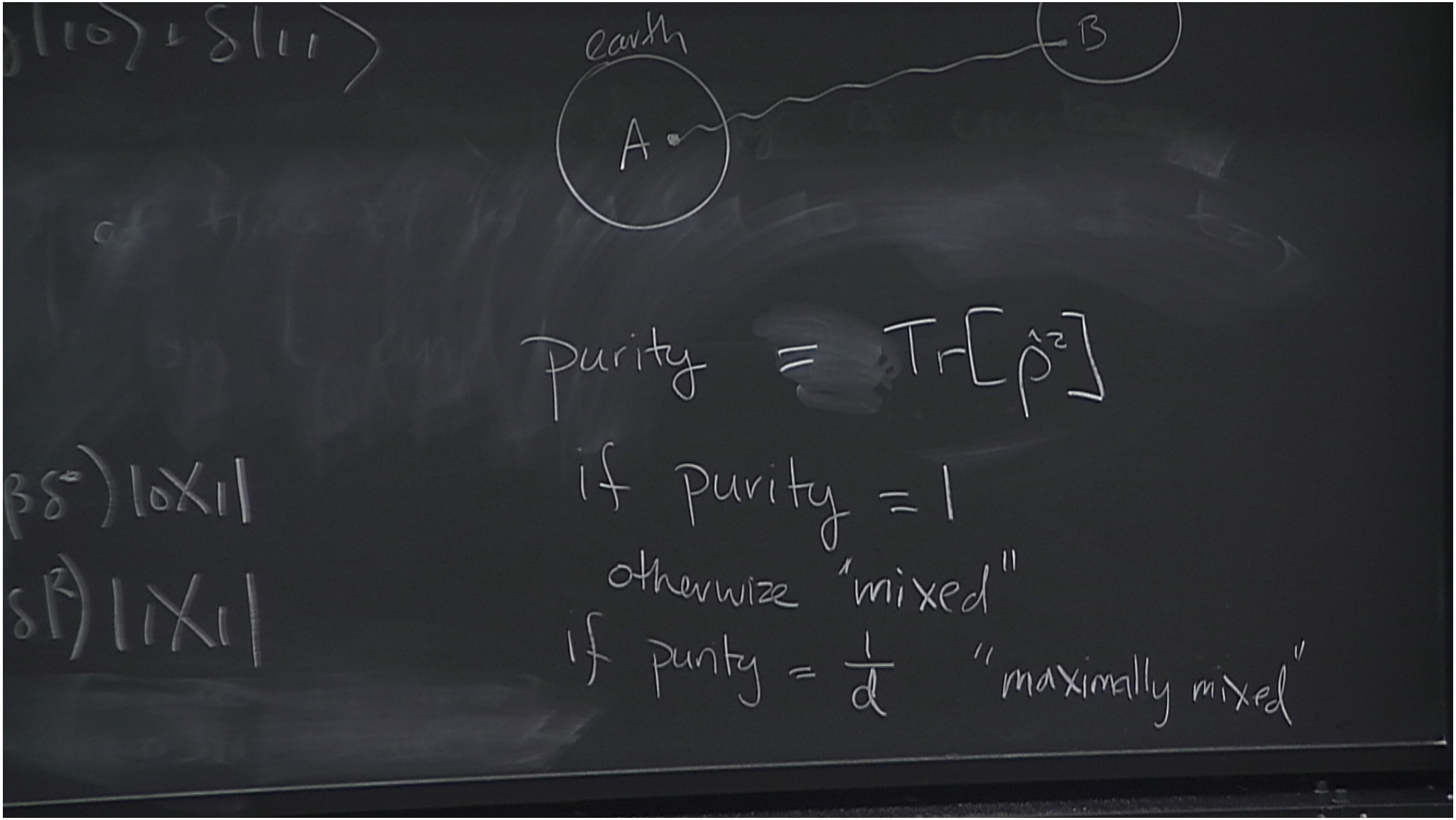
$$\hat{\rho}_A = \mathrm{Tr}_B\left[|\psi\rangle_{AB}\langle\psi|_{AB}\right]$$

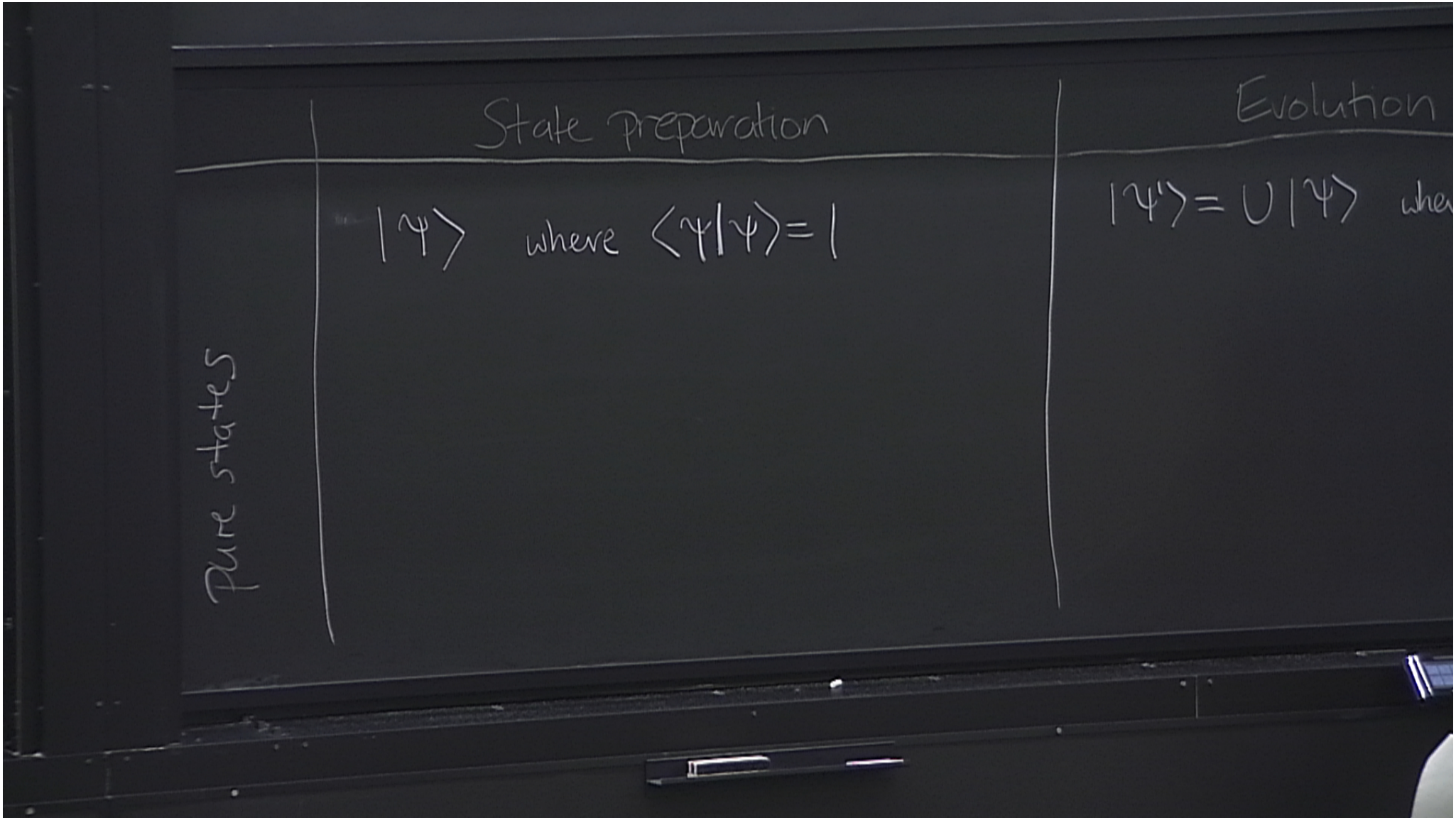
$$= \sum_j {}_B\langle j|\psi\rangle_{AB}\langle\psi|_{AB}|j\rangle_B$$

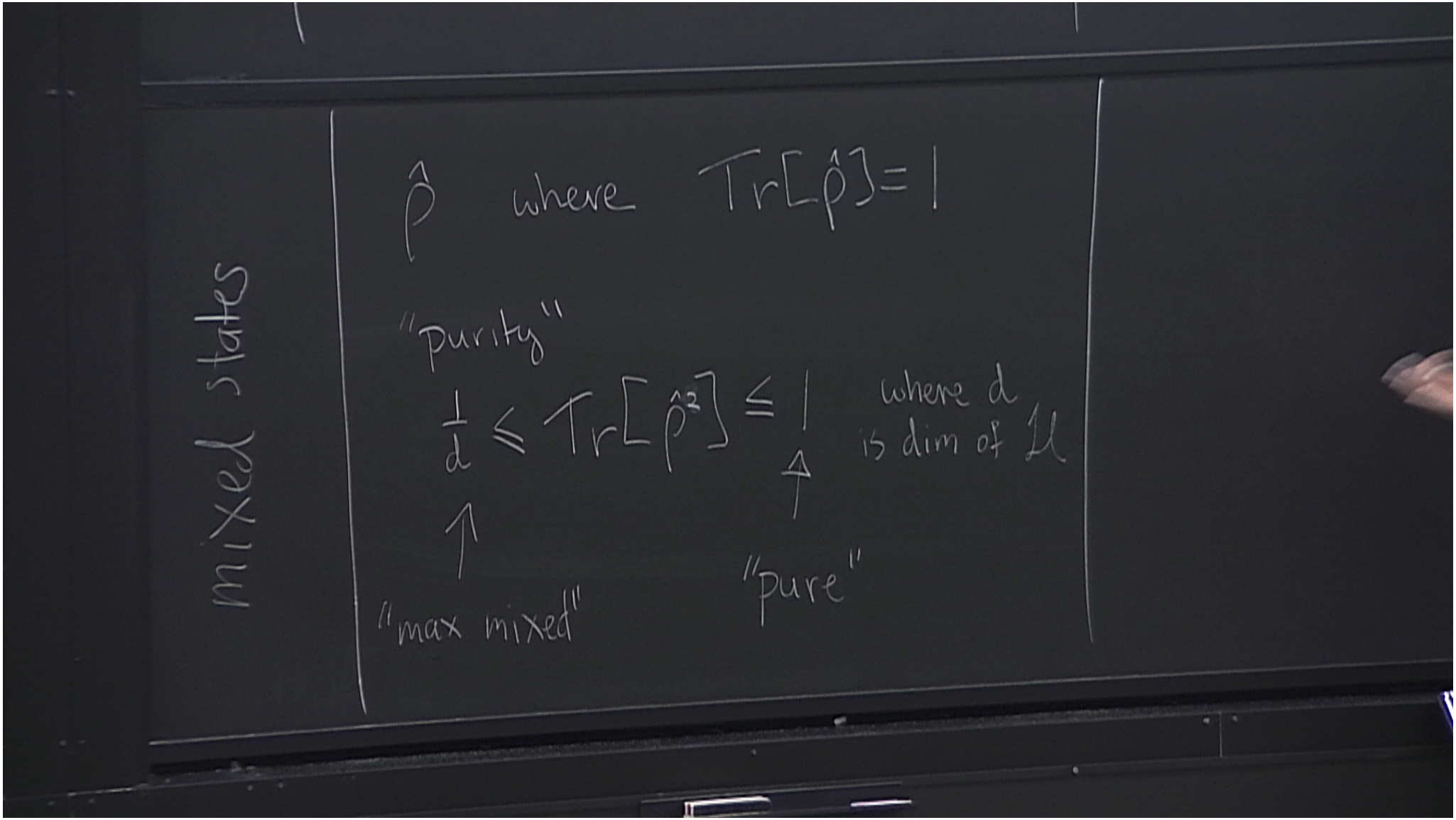
$$= \left(|\alpha|^2 + |\beta|^2\right)|0\rangle\langle 0| + \left(\alpha\gamma^* + \beta\delta^*\right)|0\rangle\langle 1|$$

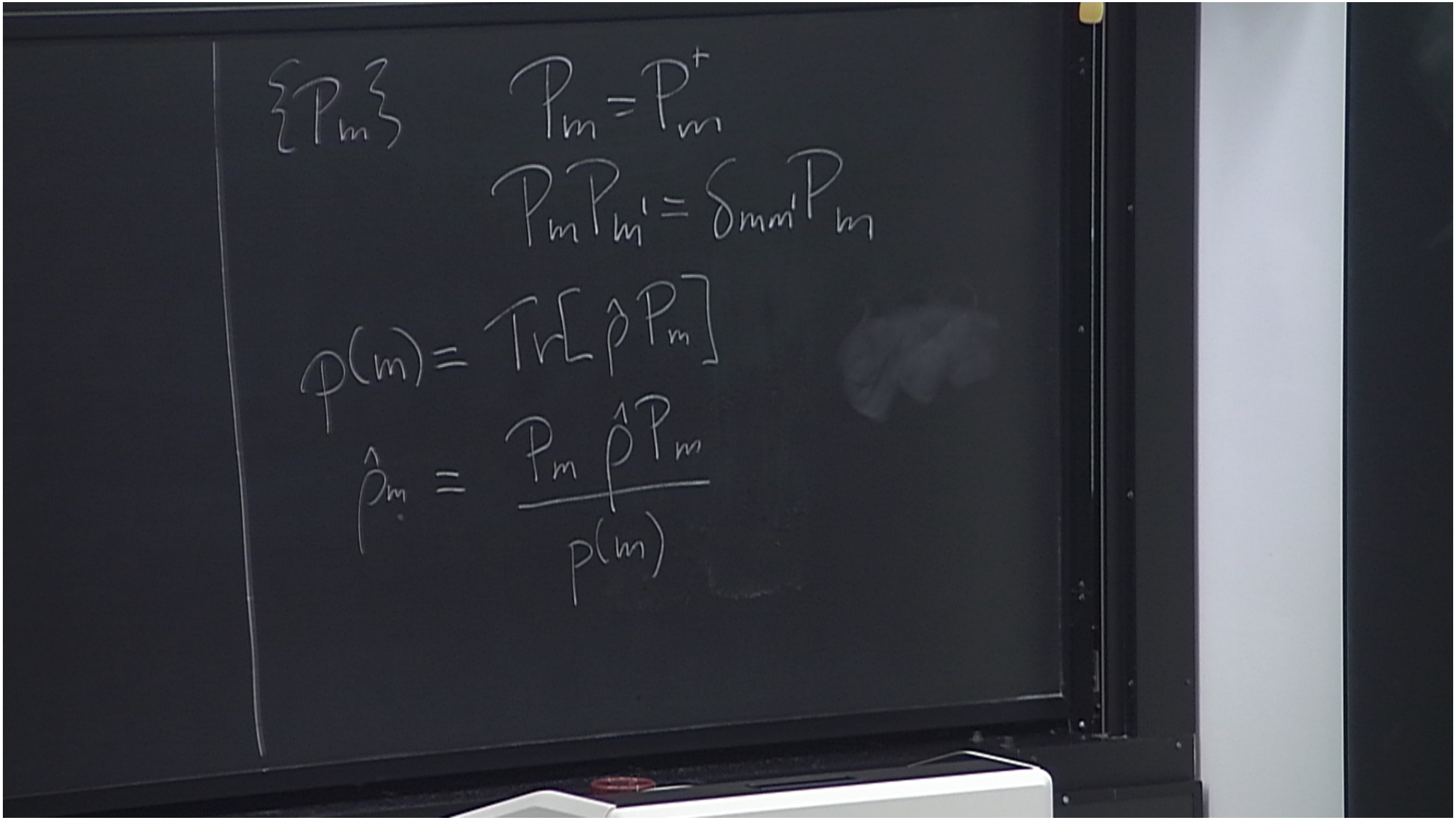
$$+ \left(\gamma\alpha^* + \delta\beta^*\right)|1\rangle\langle 0| + \left(|\gamma|^2 + |\delta|^2\right)|1\rangle\langle 1|$$


earth
A
moon
B
Purity = Tr[ρ̂²]
if Purity = 1
otherwise "mixed"


earth
A
B
Purity = Tr[ρ̂²]
if Purity = 1
otherwise "mixed"
if purity = 1/d "maximally mixed"


State preparation
Evolution
Pure states
$|\psi\rangle$ where $\langle\psi|\psi\rangle = 1$
$|\psi'\rangle = U|\psi\rangle$ whe


mixed states
$\hat{\rho}$ where $Tr[\hat{\rho}] = 1$
"purity"
$\frac{1}{d} \leq Tr[\hat{\rho}^2] \leq 1$
where d is dim of $\mathcal{H}$
"max mixed"
"pure"


$\{P_m\}$
$P_m = P_m^+$
$P_m P_{m'} = \delta_{mm'} P_m$
$p(m) = \mathrm{Tr}[\hat{\rho} P_m]$
$\hat{\rho}_m = \frac{P_m \hat{\rho} P_m}{p(m)}$

Joint evolution

$$\rho_A' = \mathrm{Tr}_B\left[U_{AB} \left(|a\rangle_A |b\rangle_B \right) \left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \right]$$

$$= \sum_k {}_B\langle k| \, U_{AB} \left(|a\rangle_A |b\rangle_B \right) \left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \, |k\rangle_B$$

$) U_{AB}^{\dagger}]$

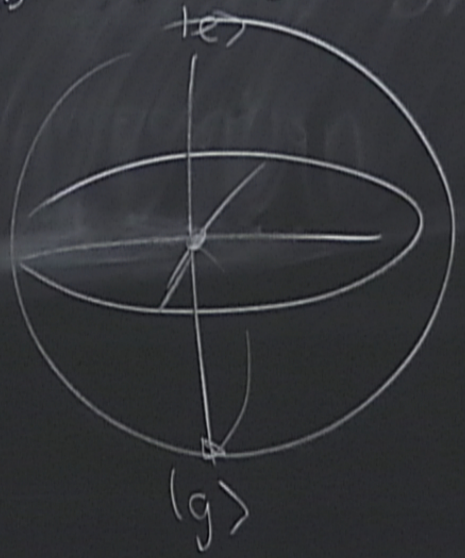
define $A_k = \langle k|_B U_{AB} |b\rangle_B$

$U_{AB}^{\dagger} |k\rangle_B$

$\hat{\rho} = \frac{|e\rangle\langle e| + |g\rangle\langle g|}{2}$

define $A_k = \langle k|_B U_{AB} |b\rangle_B$

$\hat{\rho} = \frac{|e\rangle\langle e| + |g\rangle\langle g|}{2}$



Joint evolution

$$\rho_A' = \mathrm{Tr}_B\left[U_{AB} \left(|a\rangle_A |b\rangle_B \right)\left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \right]$$

$$= \sum_k {}_B\langle k| \, U_{AB} \left(|a\rangle_A |b\rangle_B \right)\left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \, |k\rangle_B$$

$$= \sum_k A_k |a\rangle\langle a|_A A_k^\dagger$$

"Krauss operators"

bove

$$\rho' = \sum_k A_k \hat{\rho} A_k^\dagger$$

where $\sum_k A_k^\dagger A_k = \mathbb{1}$

Joint evolution

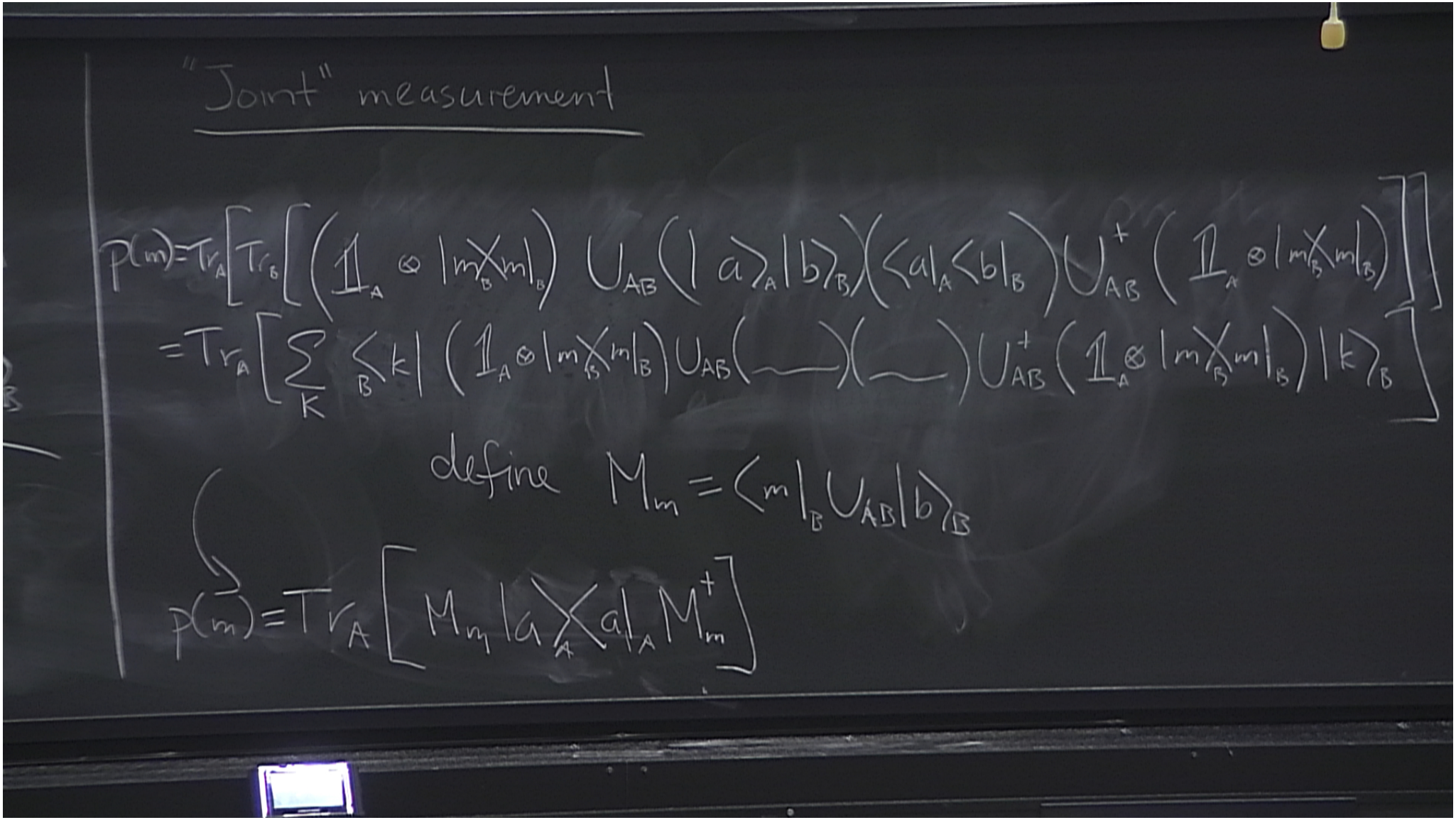
$$\rho_A' = \mathrm{Tr}_B\left[U_{AB} \left(|a\rangle_A |b\rangle_B \right)\left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \right]$$

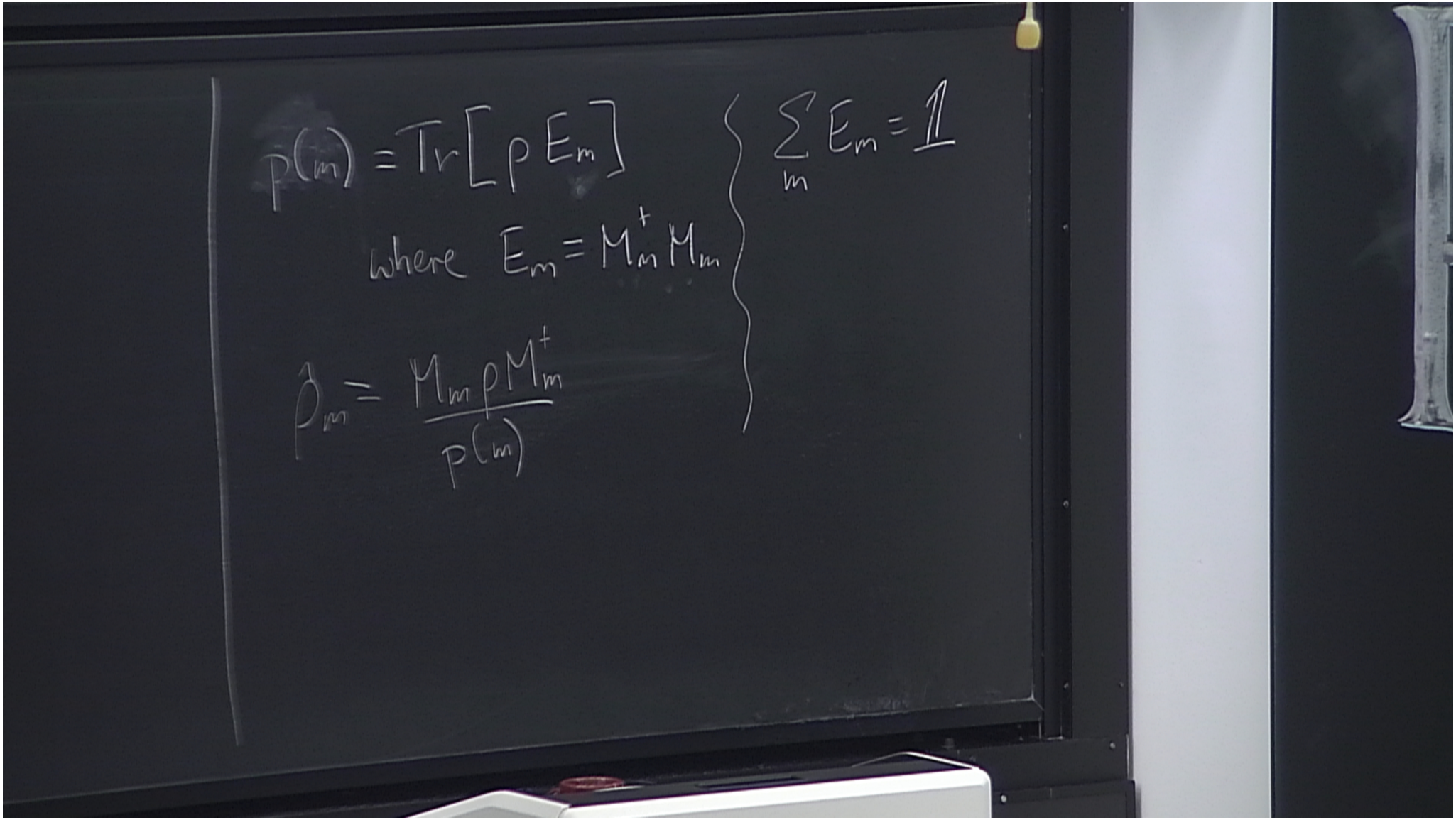
$$= \sum_k {}_B\langle k| \, U_{AB} \left(|a\rangle_A |b\rangle_B \right)\left(\langle a|_A \langle b|_B \right) U_{AB}^\dagger \, |k\rangle_B$$

$$= \sum_k A_k |a\rangle\langle a|_A A_k^\dagger$$

"Krauss operators"

$$A_k = {}_B\langle k| U_{AB} |b\rangle_B$$


"Joint" measurement
$p(m) = Tr_A\left[Tr_B\left[\left(\mathbb{1}_A \otimes |m\rangle\langle m|_B\right) U_{AB} \left(|a\rangle_A|b\rangle_B\right)\left(\langle a|_A\langle b|_B\right) U_{AB}^{\dagger} \left(\mathbb{1}_A \otimes |m\rangle\langle m|_B\right)\right]\right]$
$= Tr_A\left[\sum_k {}_B\langle k| \left(\mathbb{1}_A \otimes |m\rangle\langle m|_B\right) U_{AB} (\quad)(\quad) U_{AB}^{\dagger} \left(\mathbb{1}_A \otimes |m\rangle\langle m|_B\right) |k\rangle_B\right]$
define $M_m = \langle m|_B U_{AB} |b\rangle_B$
$p(m) = Tr_A\left[M_m |a\rangle\langle a|_A M_m^{\dagger}\right]$


$p(m) = \mathrm{Tr}[\rho E_m]$
where $E_m = M_m^\dagger M_m$
$\rho_m = \frac{M_m \rho M_m^\dagger}{p(m)}$
$\sum_m E_m = \mathbb{1}$