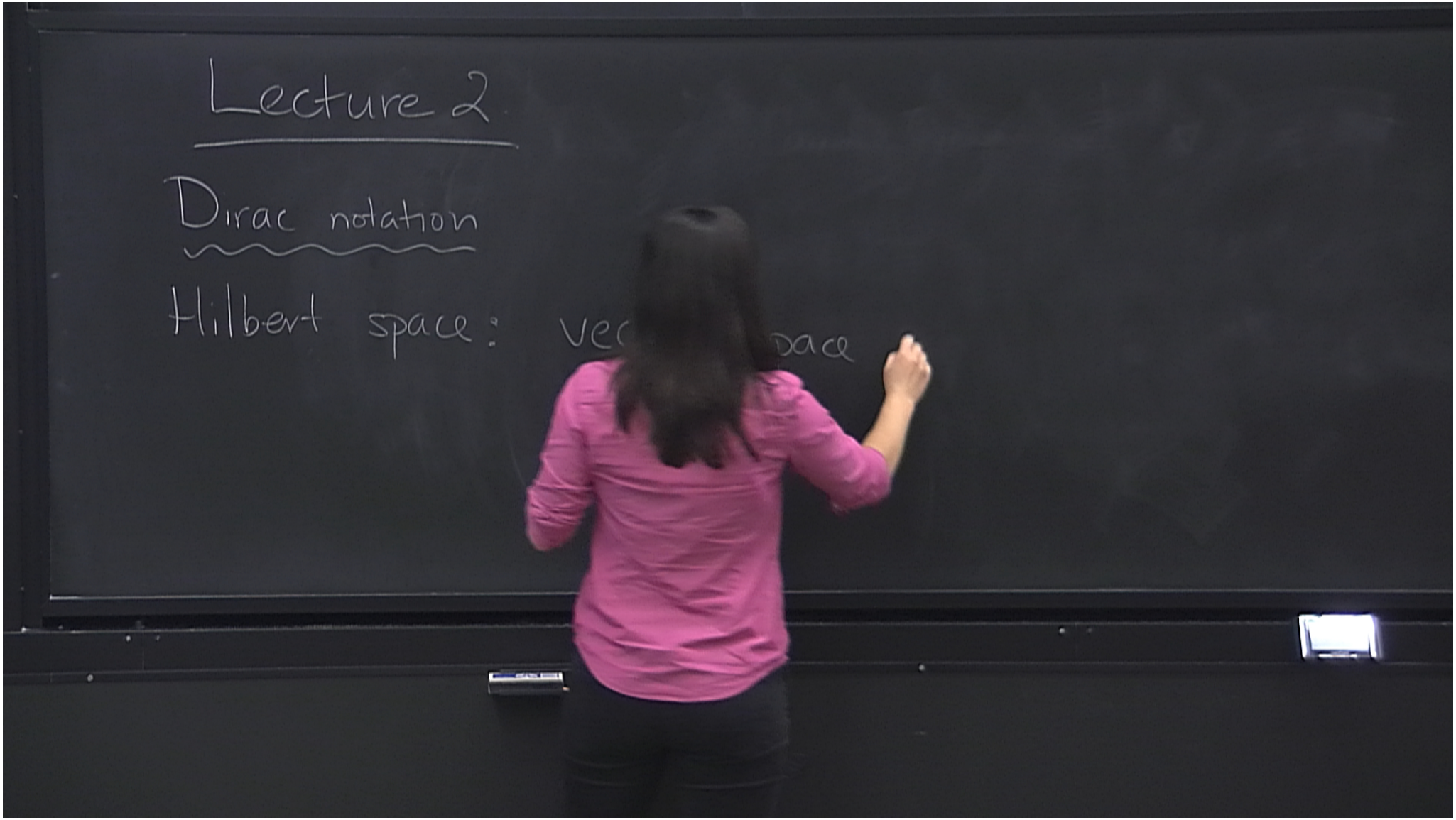
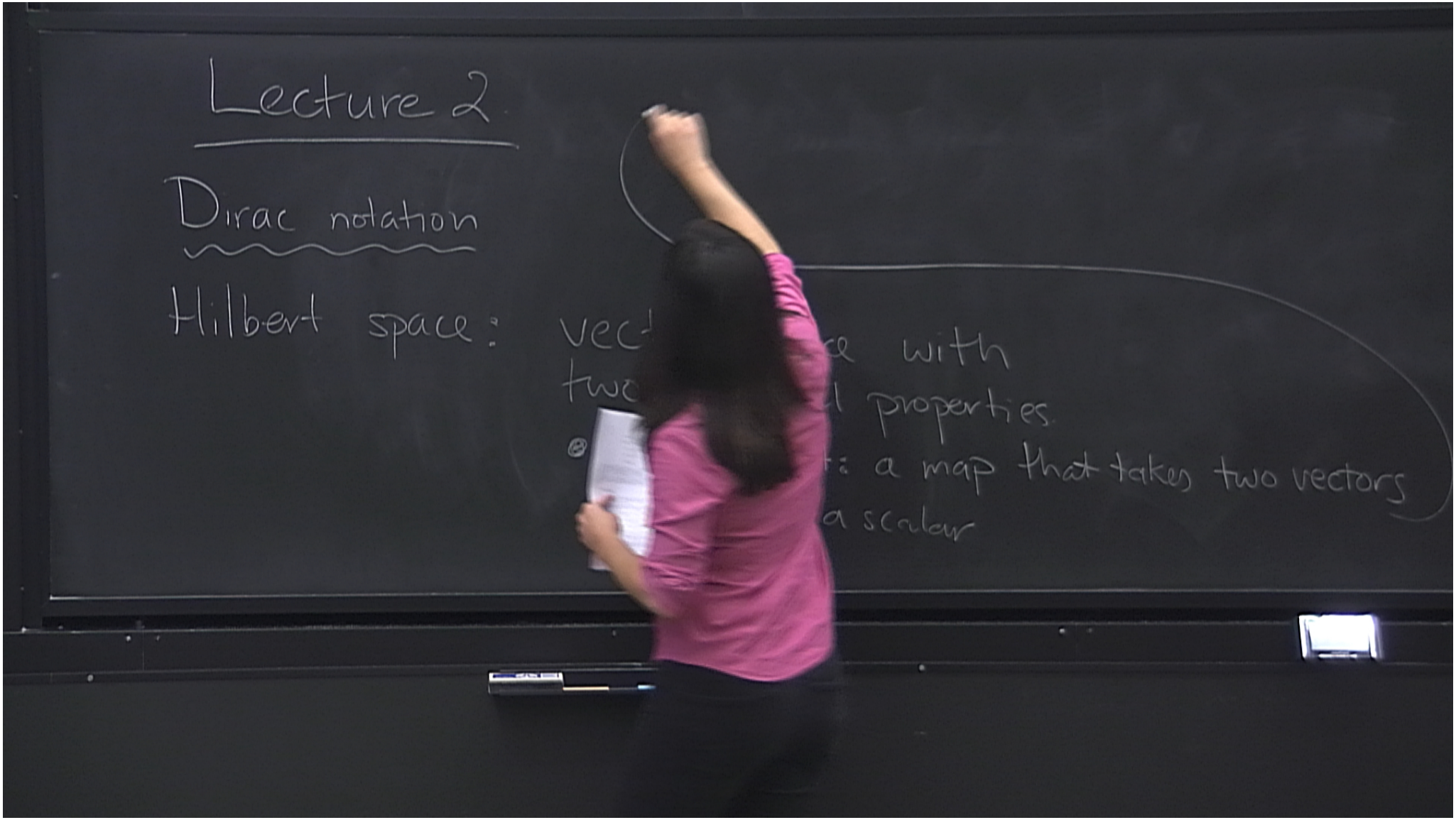
Title: Quantum Mechanics

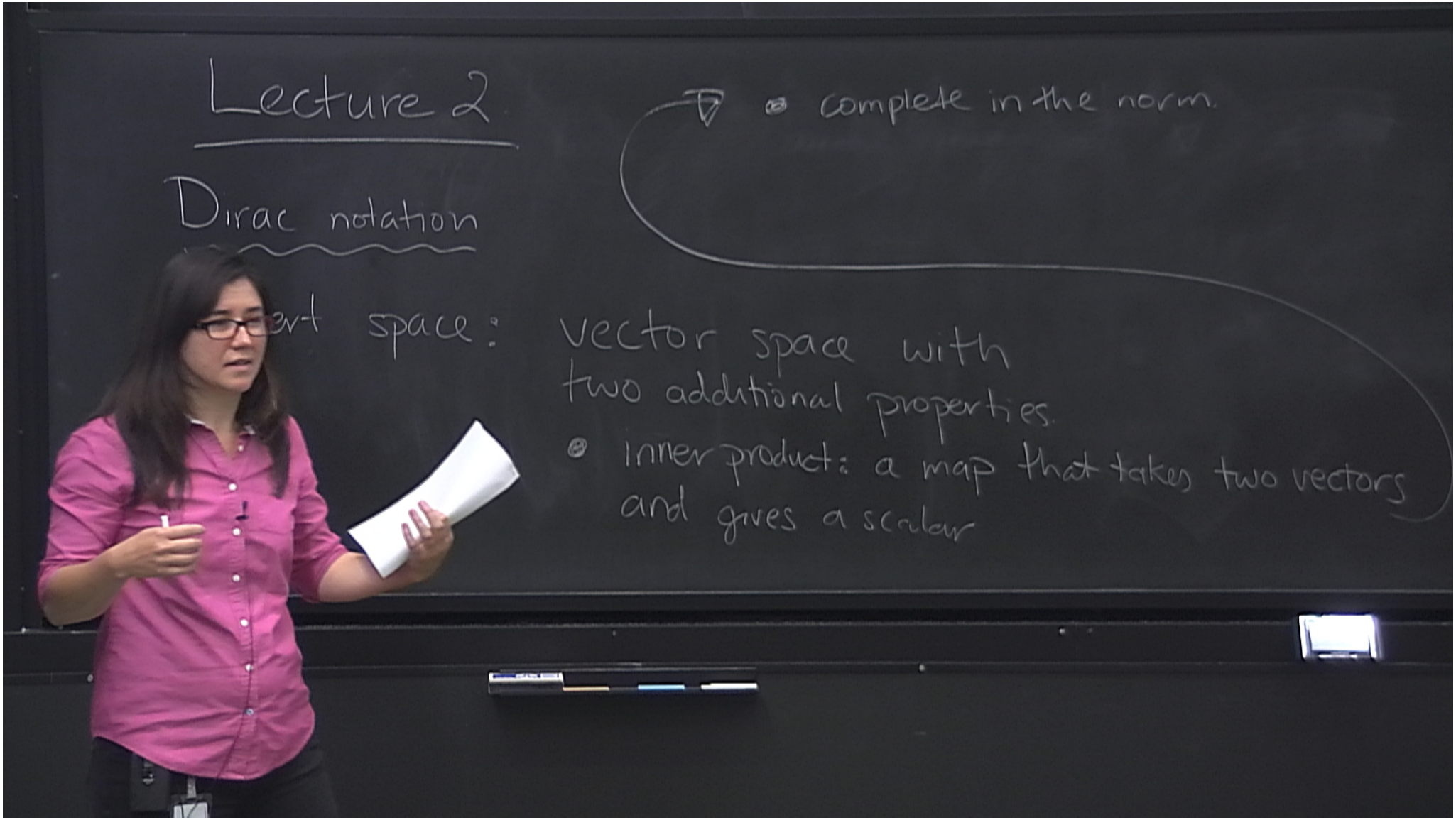
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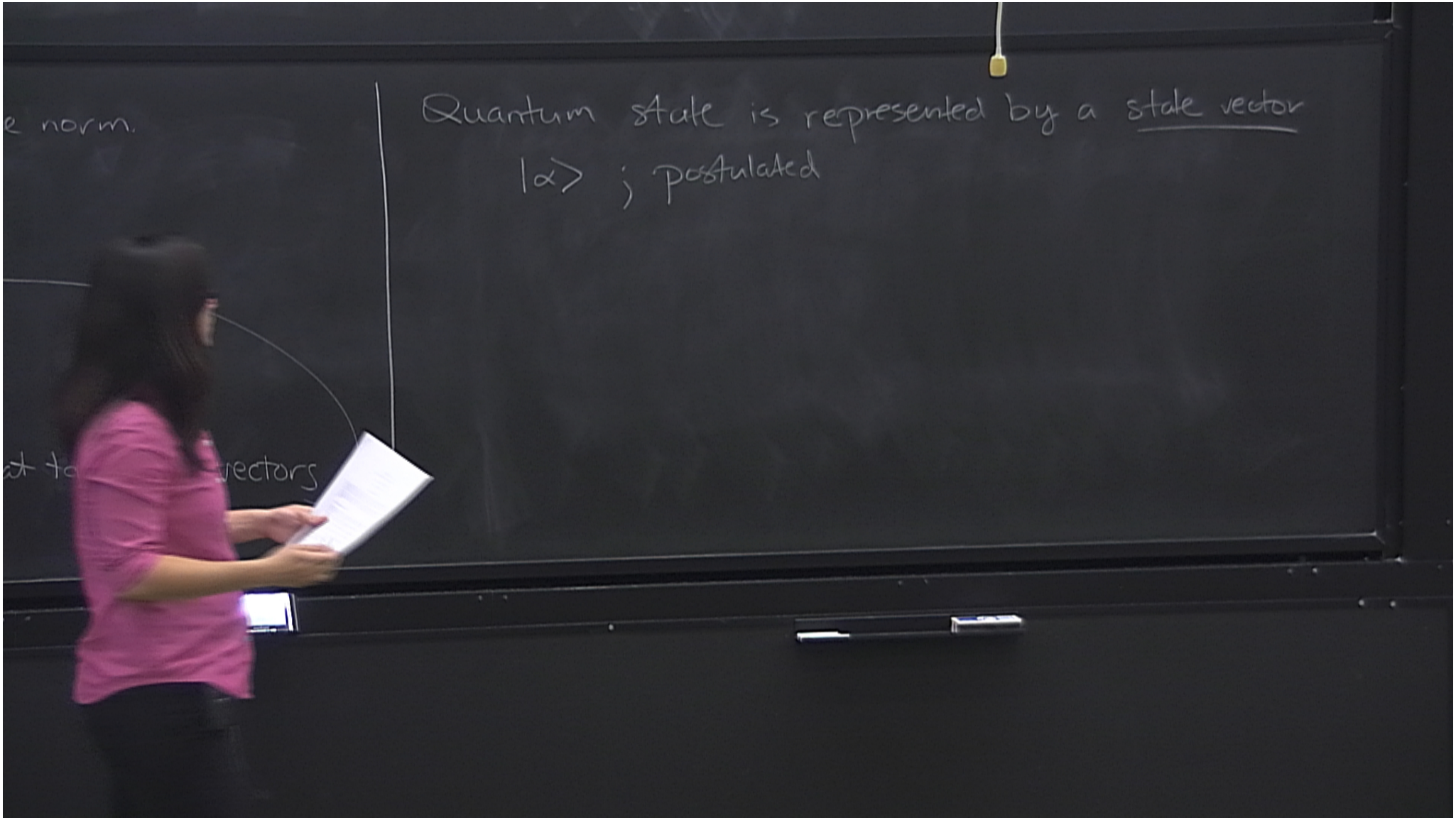
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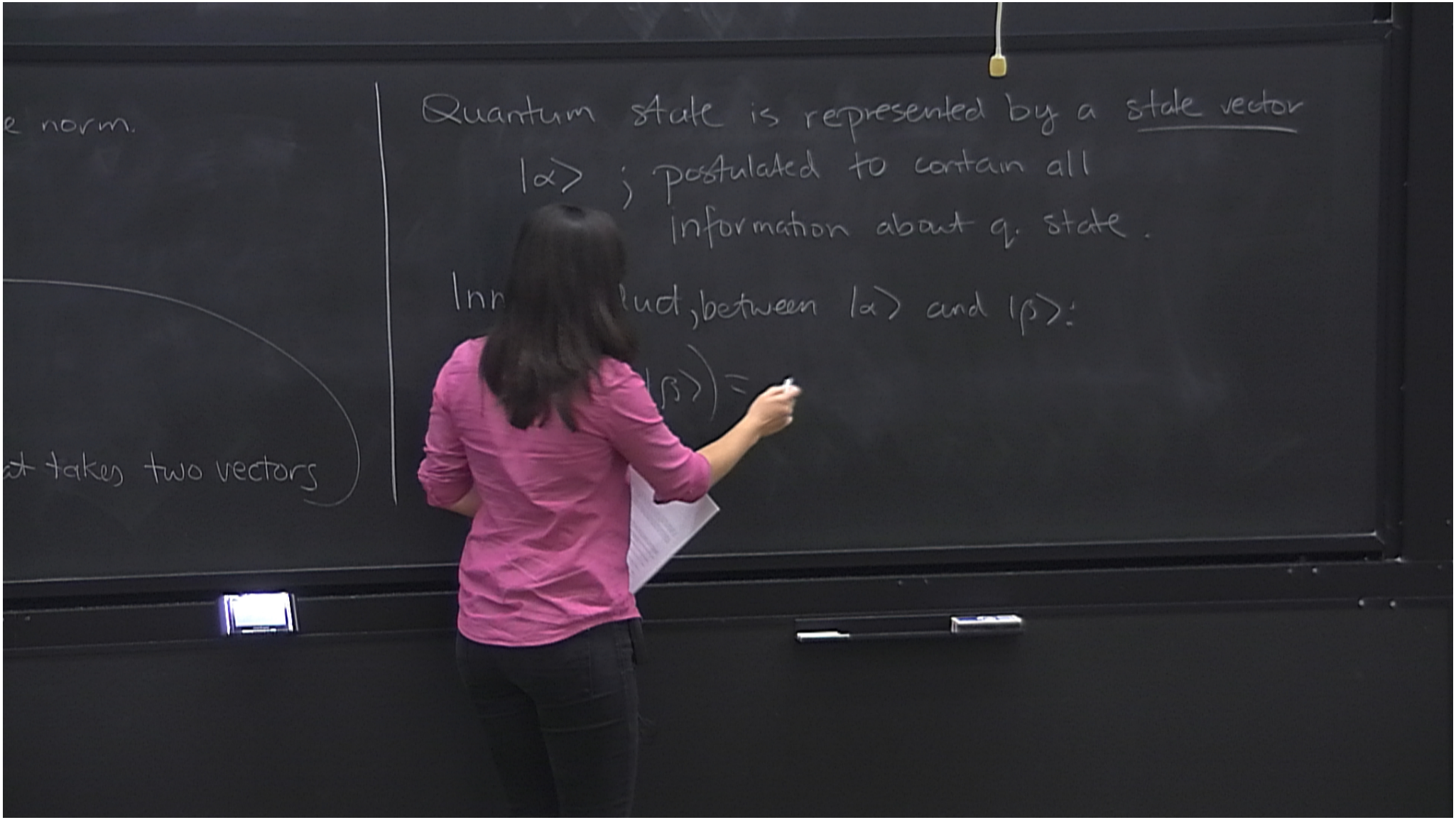
Abstract:

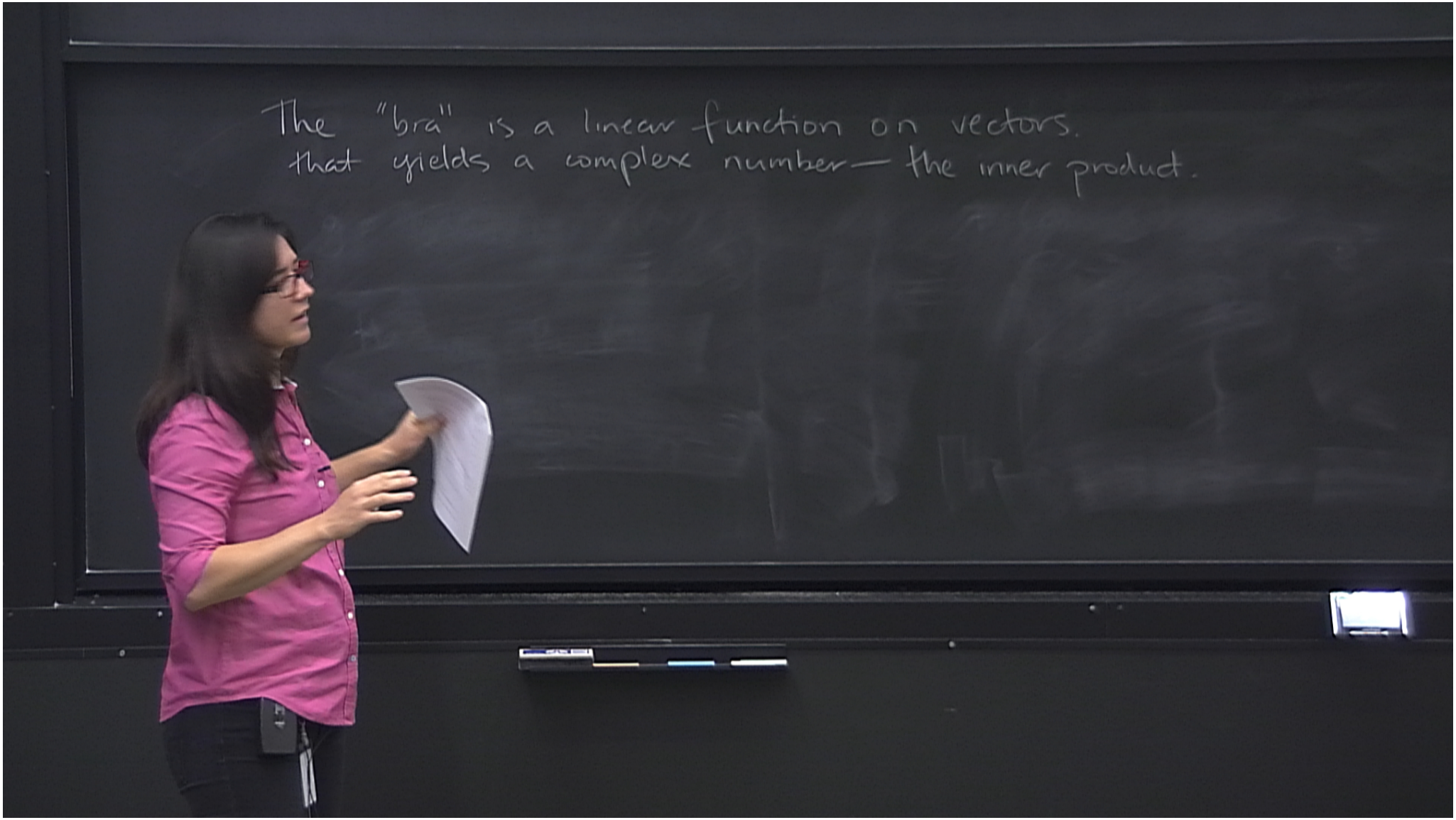

Lecture 2
Dirac notation
Hilbert space:

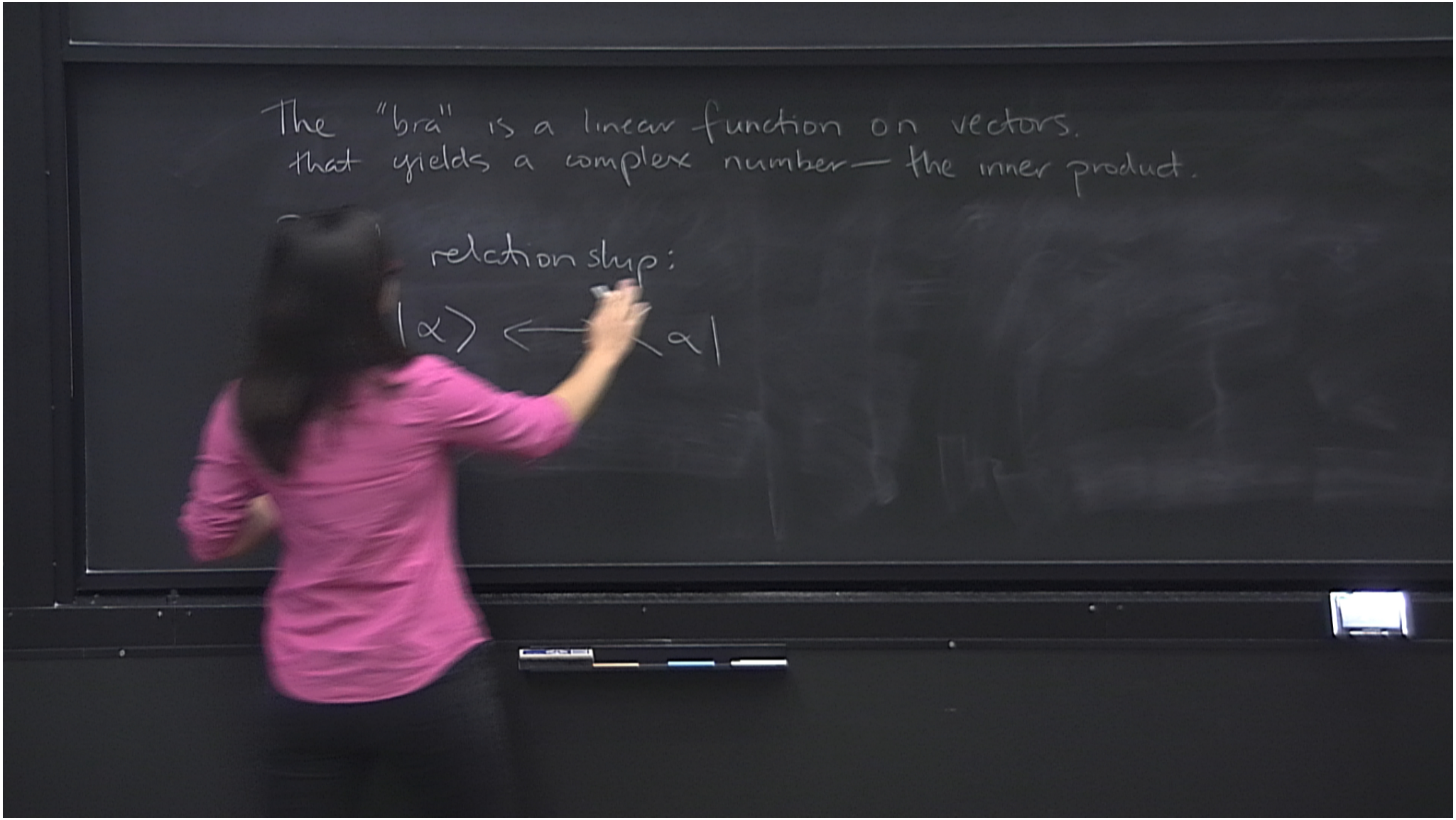

Lecture 2
Dirac notation
Hilbert space:
with
properties
: a map that takes two vectors
a scalar

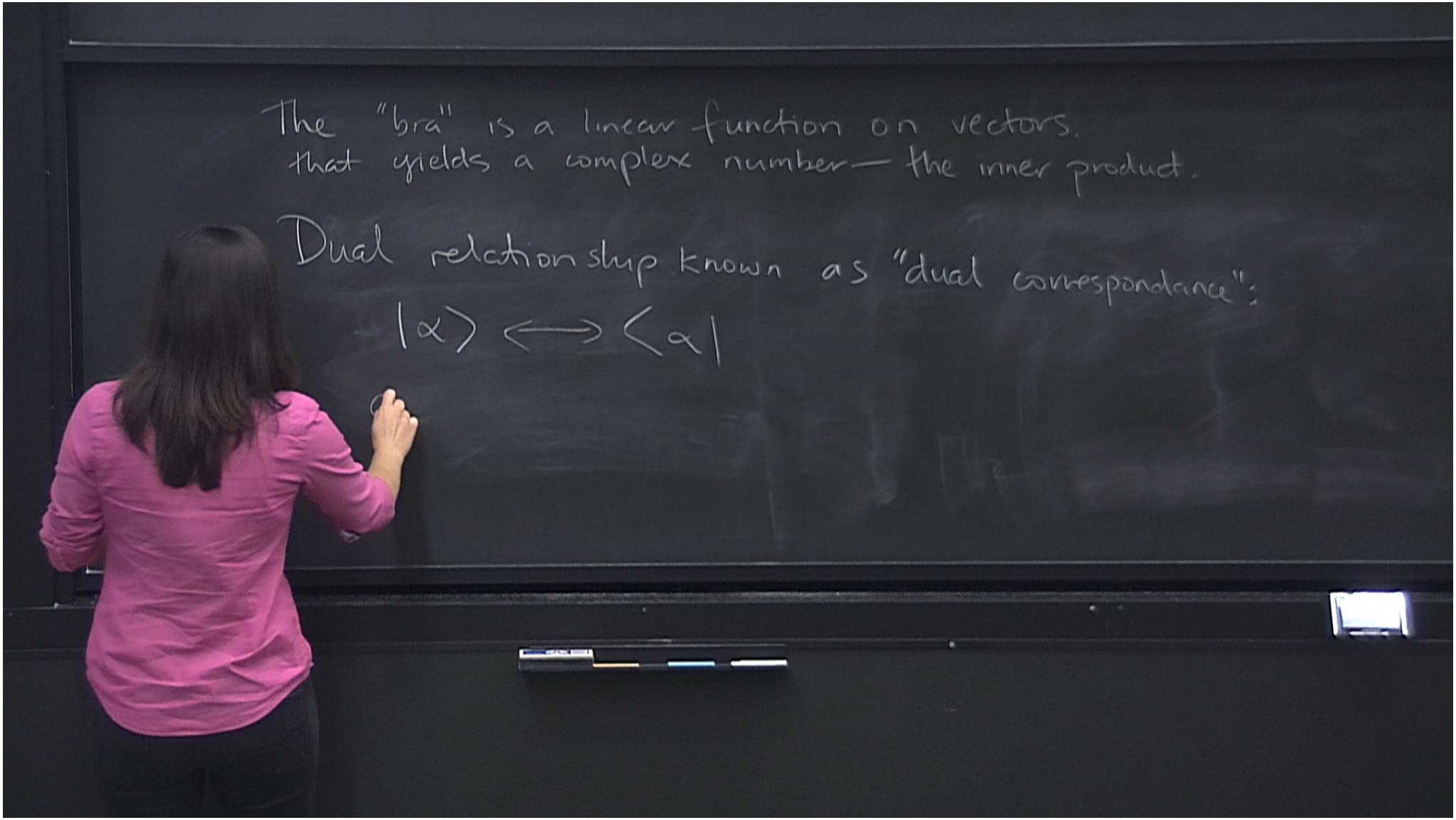

Lecture 2
Dirac notation
complete in the norm.
vector space with
two additional properties.
inner product: a map that takes two vectors
and gives a scalar

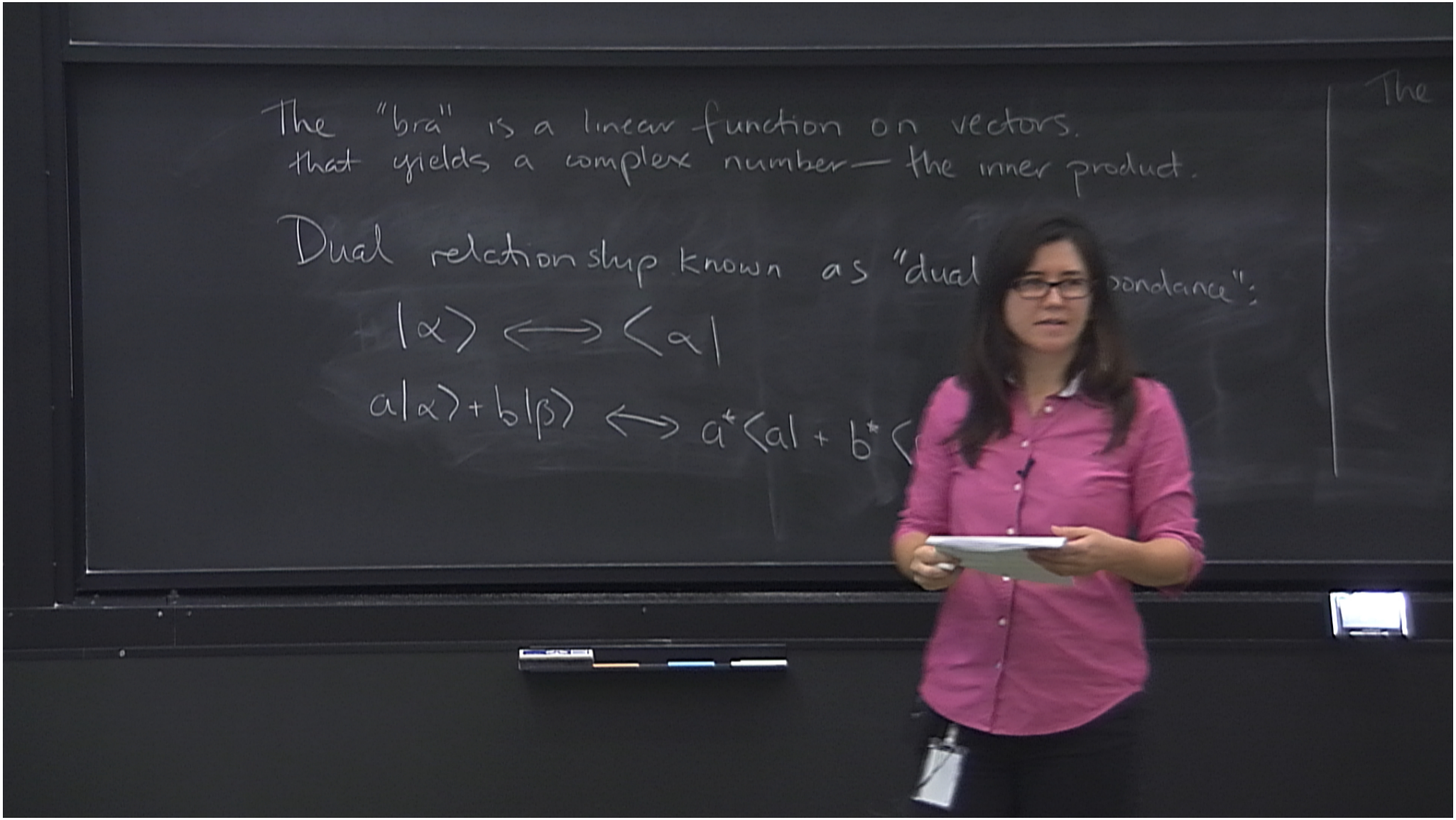

e norm.
Quantum state is represented by a state vector
|α> ; postulated
vectors

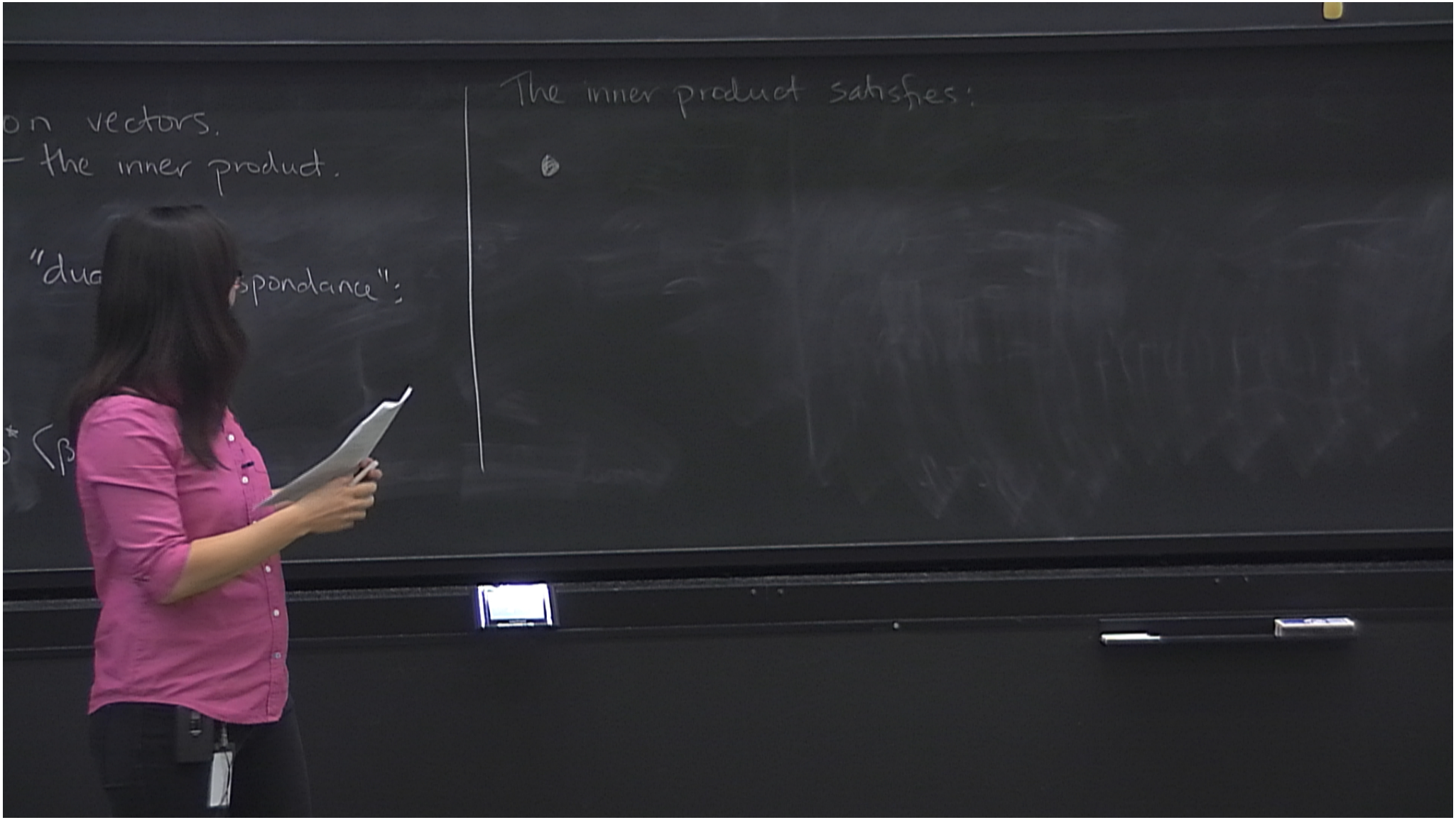

e norm.
t takes two vectors
Quantum state is represented by a state vector
|α> ; postulated to contain all
information about q. state.
Inn uct, between |α> and |β>:


The "bra" is a linear function on vectors.
that yields a complex number — the inner product.


The "bra" is a linear function on vectors.
that yields a complex number — the inner product.
relationship:
|α⟩ ⟷ ⟨α|


The "bra" is a linear function on vectors.
that yields a complex number — the inner product.
Dual relationship known as "dual correspondance":
$|\alpha\rangle \longleftrightarrow \langle\alpha|$


The "bra" is a linear function on vectors.
that yields a complex number — the inner product.
Dual relationship known as "dual
$|\alpha\rangle \longleftrightarrow \langle\alpha|$
$a|\alpha\rangle + b|\beta\rangle \longleftrightarrow a^*\langle\alpha| + b^*$
The


on vectors.
– the inner product.
"dua spondance":
The inner product satisfies:

roduct.

spondance":

The inner product satisfies:

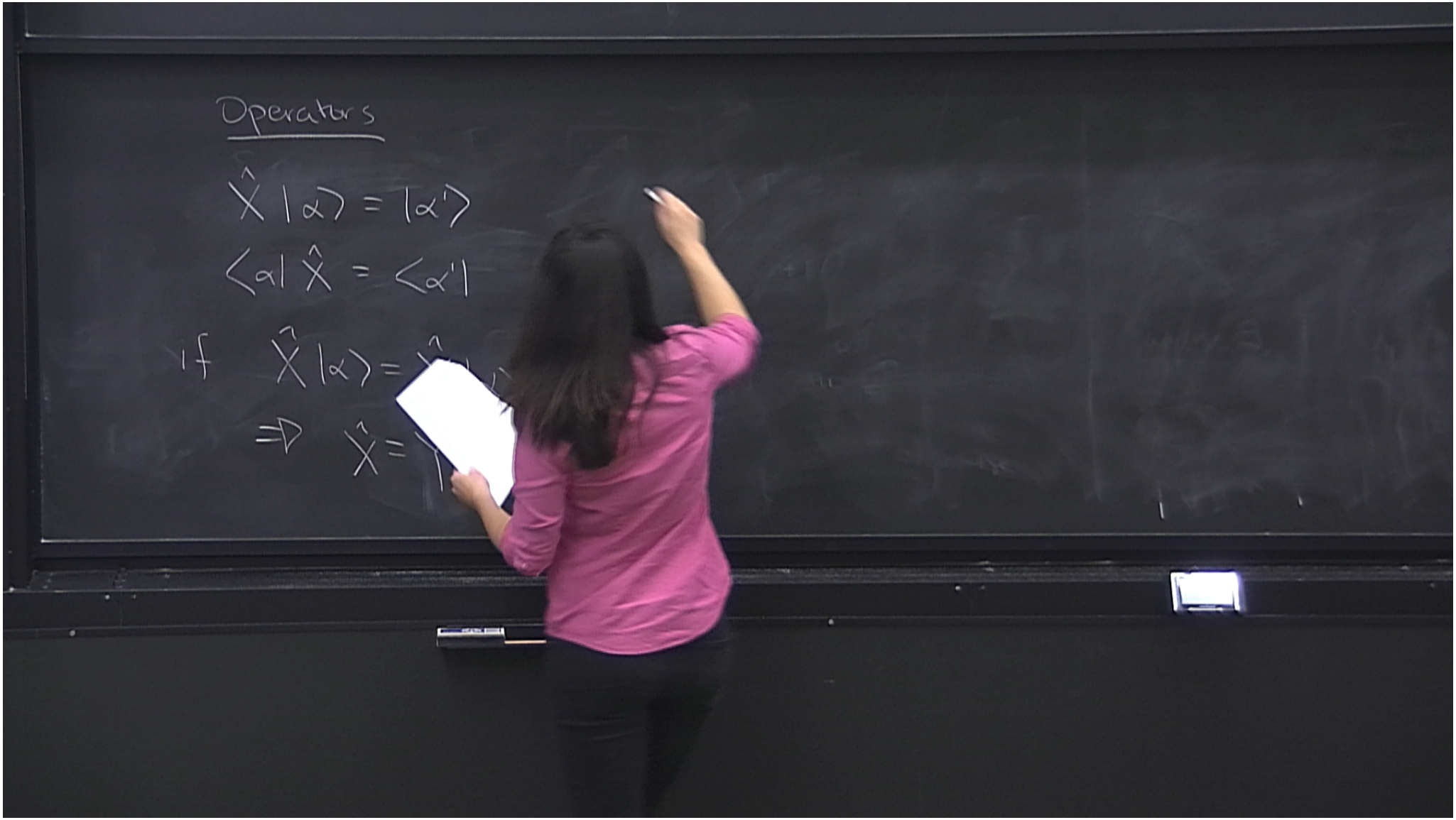
- $\langle \beta | \alpha \rangle = \langle \alpha | \beta \rangle^*$
- $\langle \beta | (a|\alpha_1\rangle + b|\alpha_2\rangle) = a\langle \beta|\alpha_1\rangle + b\langle \beta|\alpha_2\rangle$

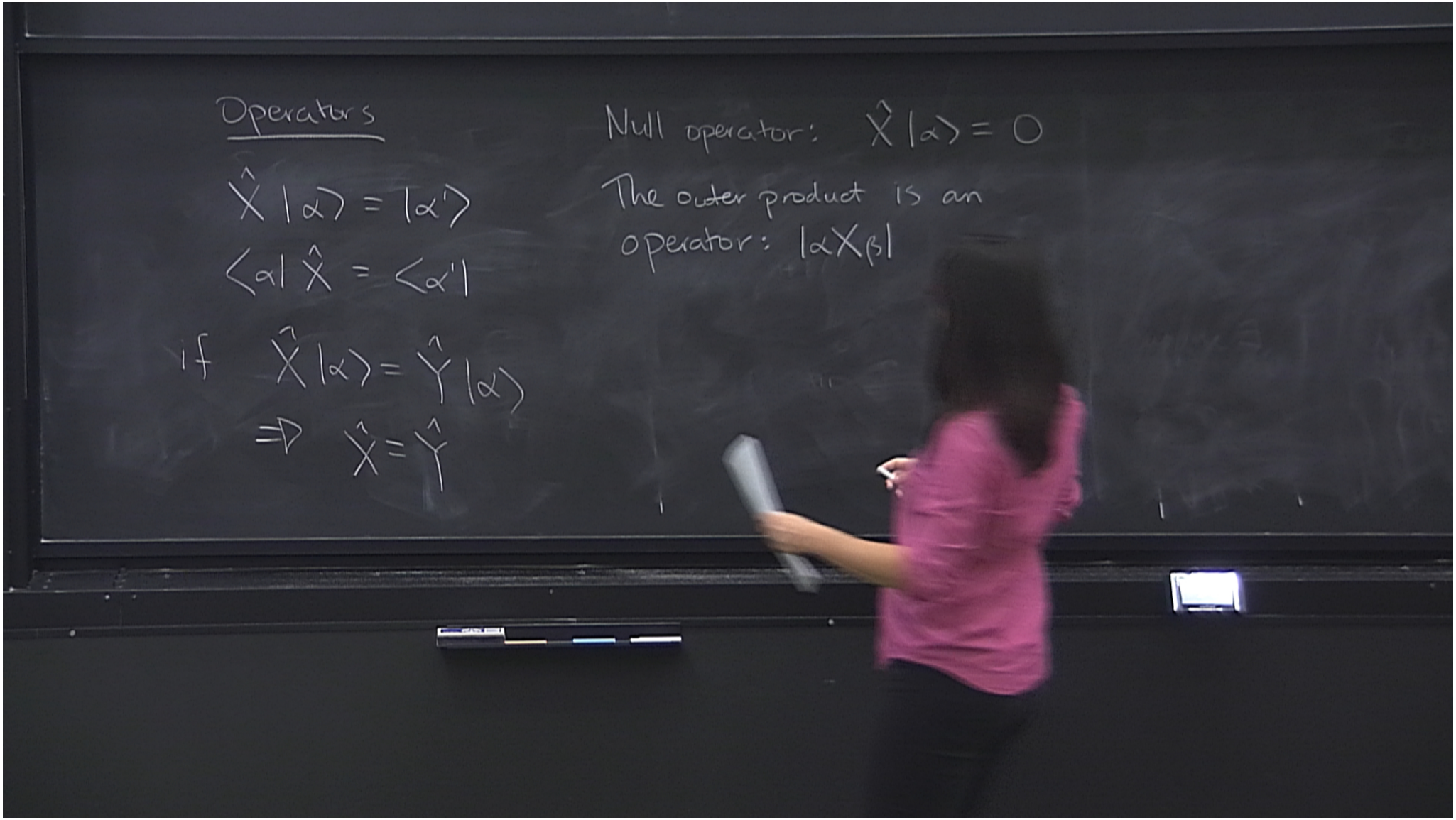
roduct.

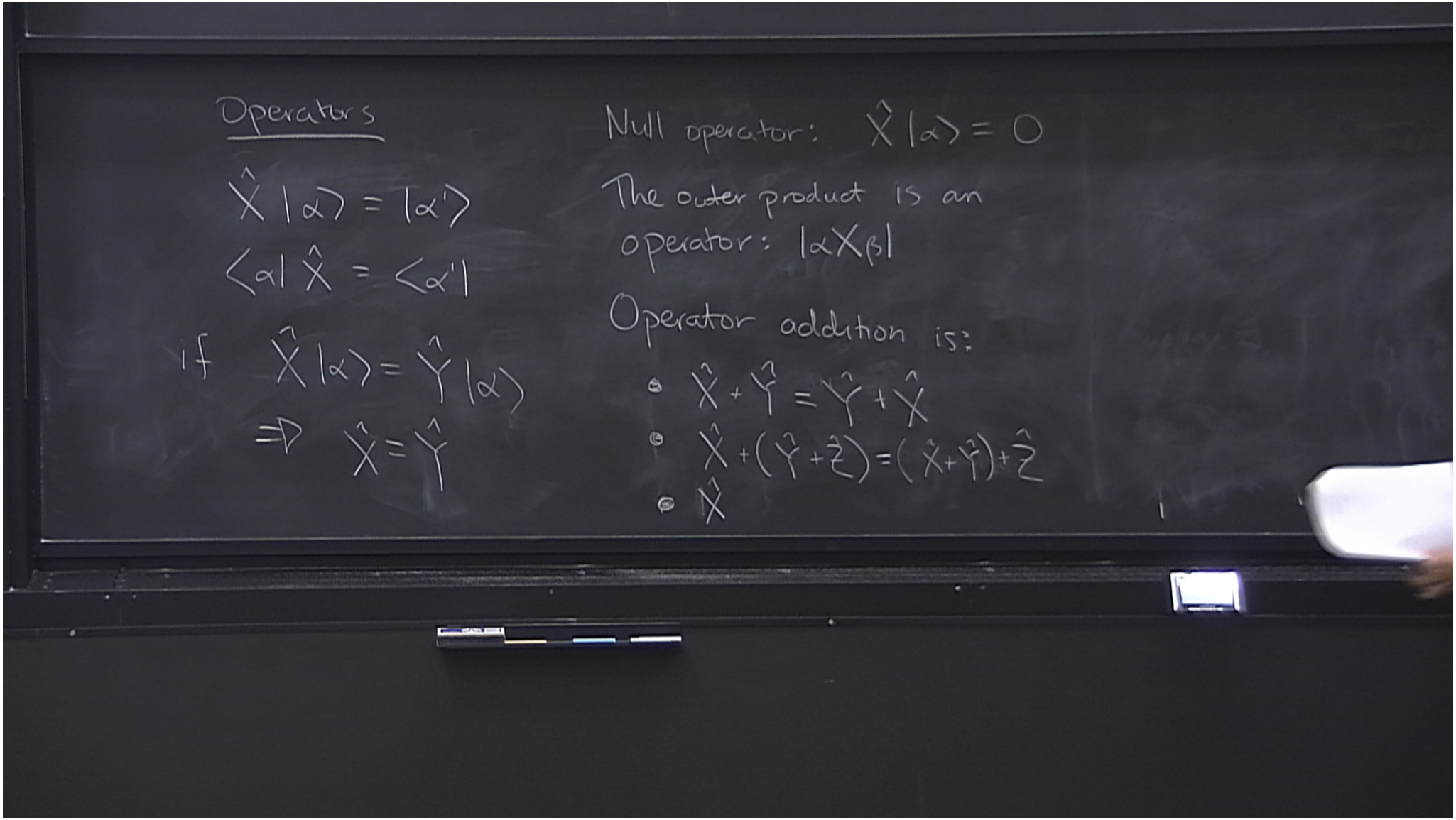
spondance":

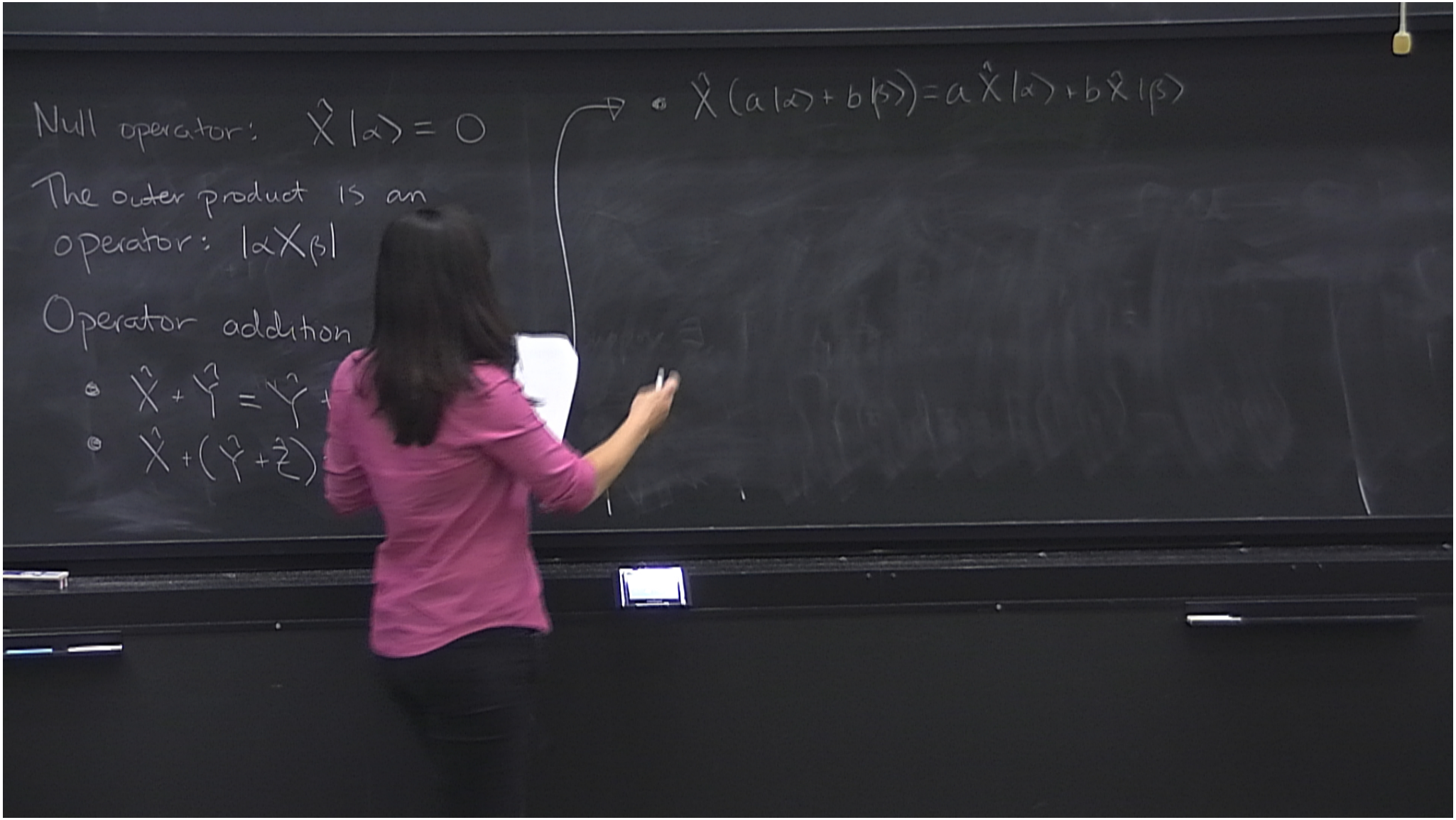
The inner product satisfies:

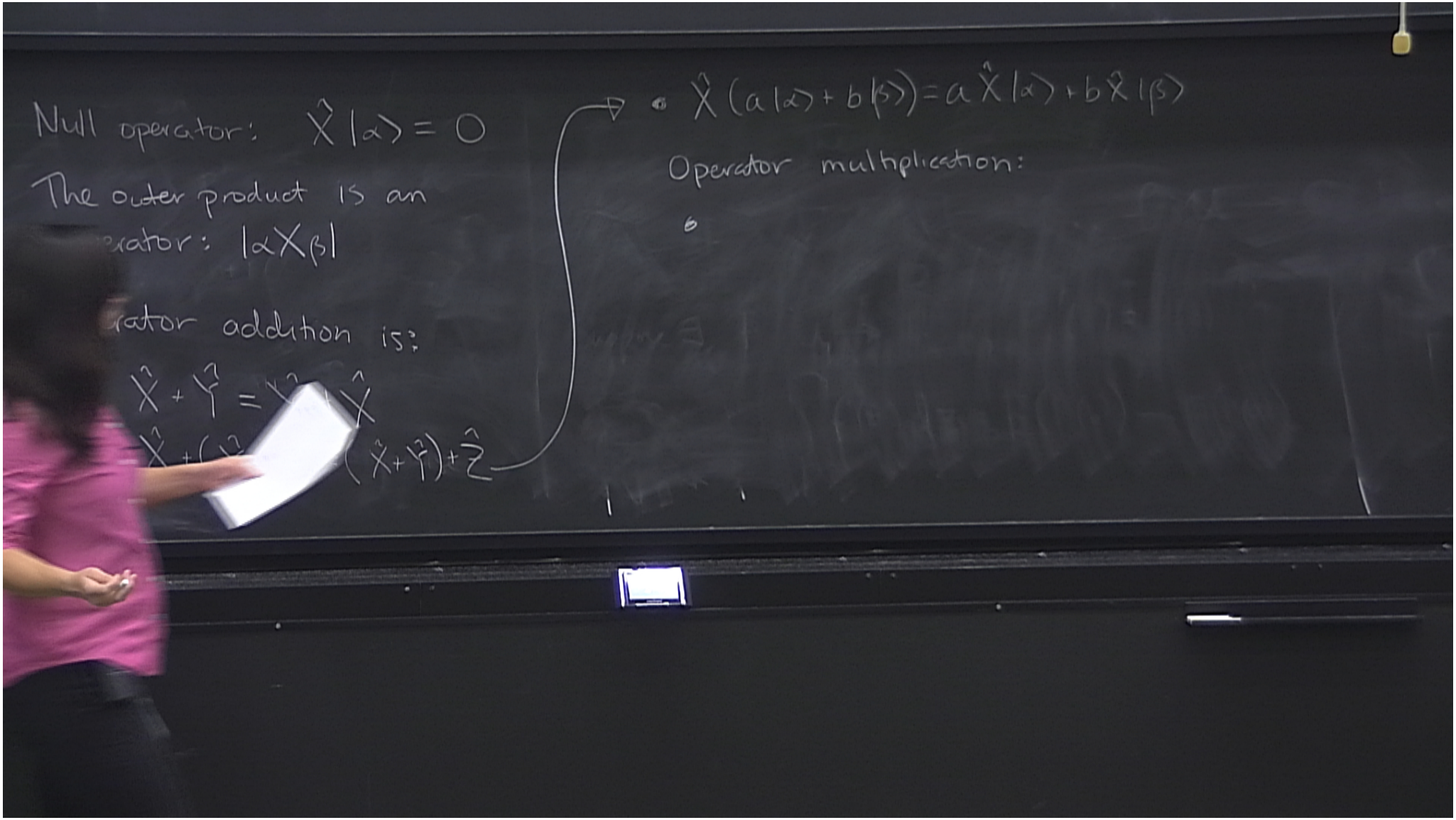
1. $\langle \beta | \alpha \rangle = \langle \alpha | \beta \rangle^*$
2. $\langle \beta | (a|\alpha_1\rangle + b|\alpha_2\rangle) = a\langle \beta|\alpha_1\rangle + b\langle \beta|\alpha_2\rangle$
3. $\langle \alpha | \alpha \rangle \geq 0$

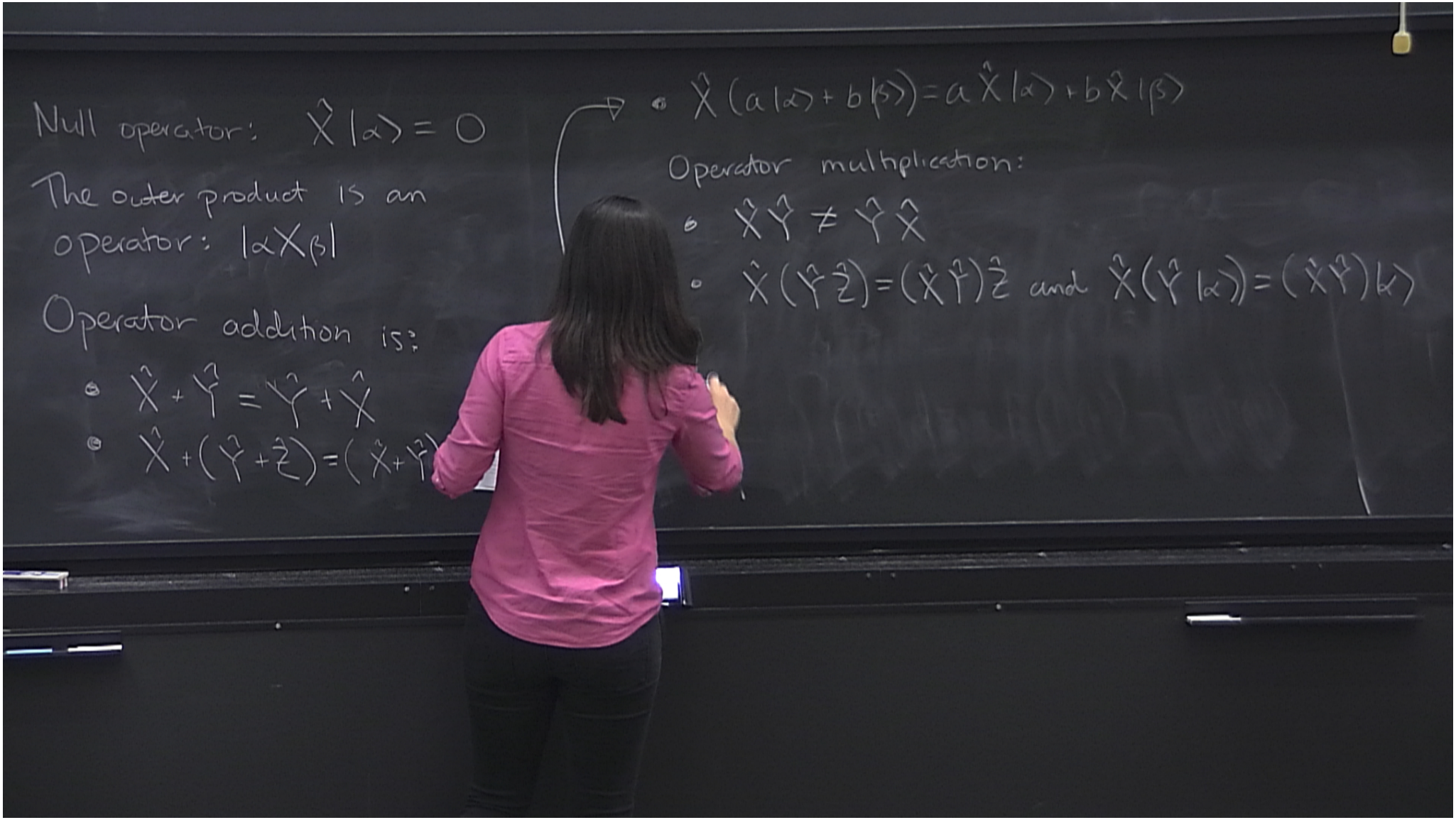

Operators
$\hat{X}|\alpha\rangle = |\alpha\rangle$
$\langle\alpha|\hat{X} = \langle\alpha'|$
if
$\Rightarrow$

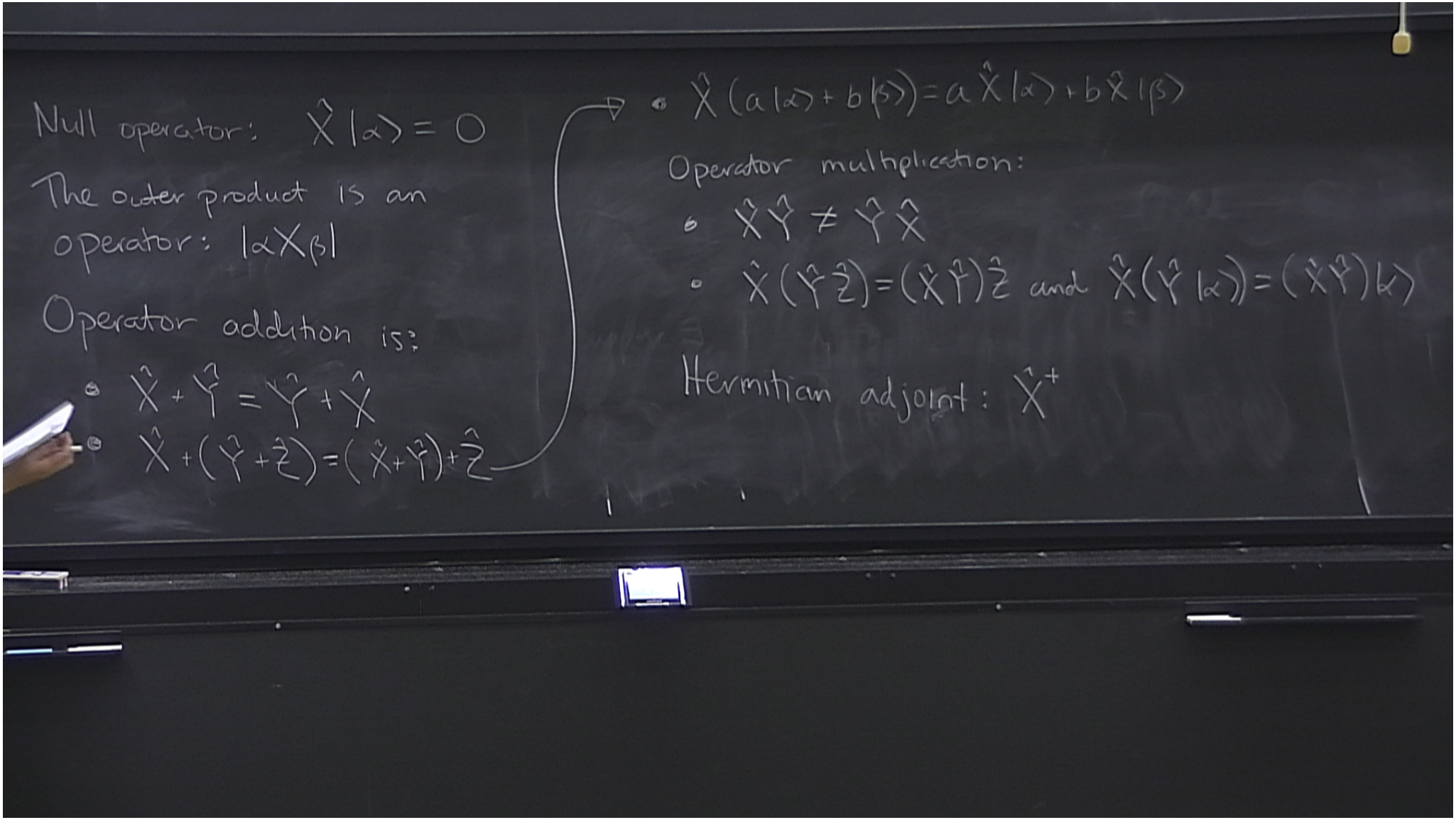

Operators
$\hat{X}|\alpha\rangle = |\alpha\rangle$
$\langle\alpha|\hat{X} = \langle\alpha'|$
if $\hat{X}|\alpha\rangle = \hat{Y}|\alpha\rangle$
$\Rightarrow \hat{X} = \hat{Y}$
Null operator: $\hat{X}|\alpha\rangle = 0$
The outer product is an operator: $|\alpha X \beta|$

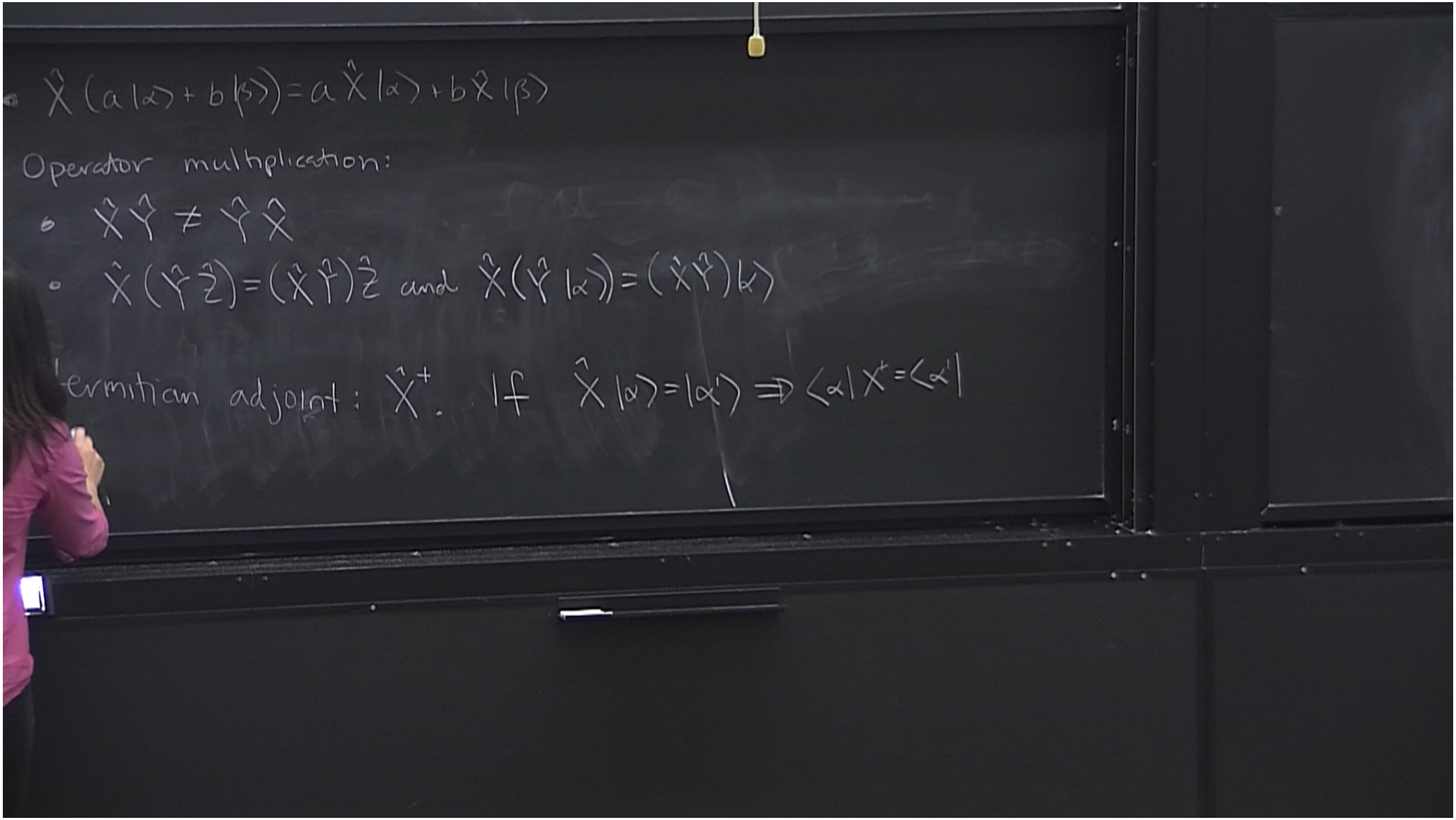

Operators
$\hat{X}|\alpha\rangle = |\alpha'\rangle$
$\langle\alpha|\hat{X} = \langle\alpha'|$
if $\hat{X}|\alpha\rangle = \hat{Y}|\alpha\rangle$
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The outer product is an operator: $|\alpha\rangle\langle\beta|$
Operator addition is:
$\hat{X} + \hat{Y} = \hat{Y} + \hat{X}$
$\hat{X} + (\hat{Y} + \hat{Z}) = (\hat{X} + \hat{Y}) + \hat{Z}$
$\hat{X}$

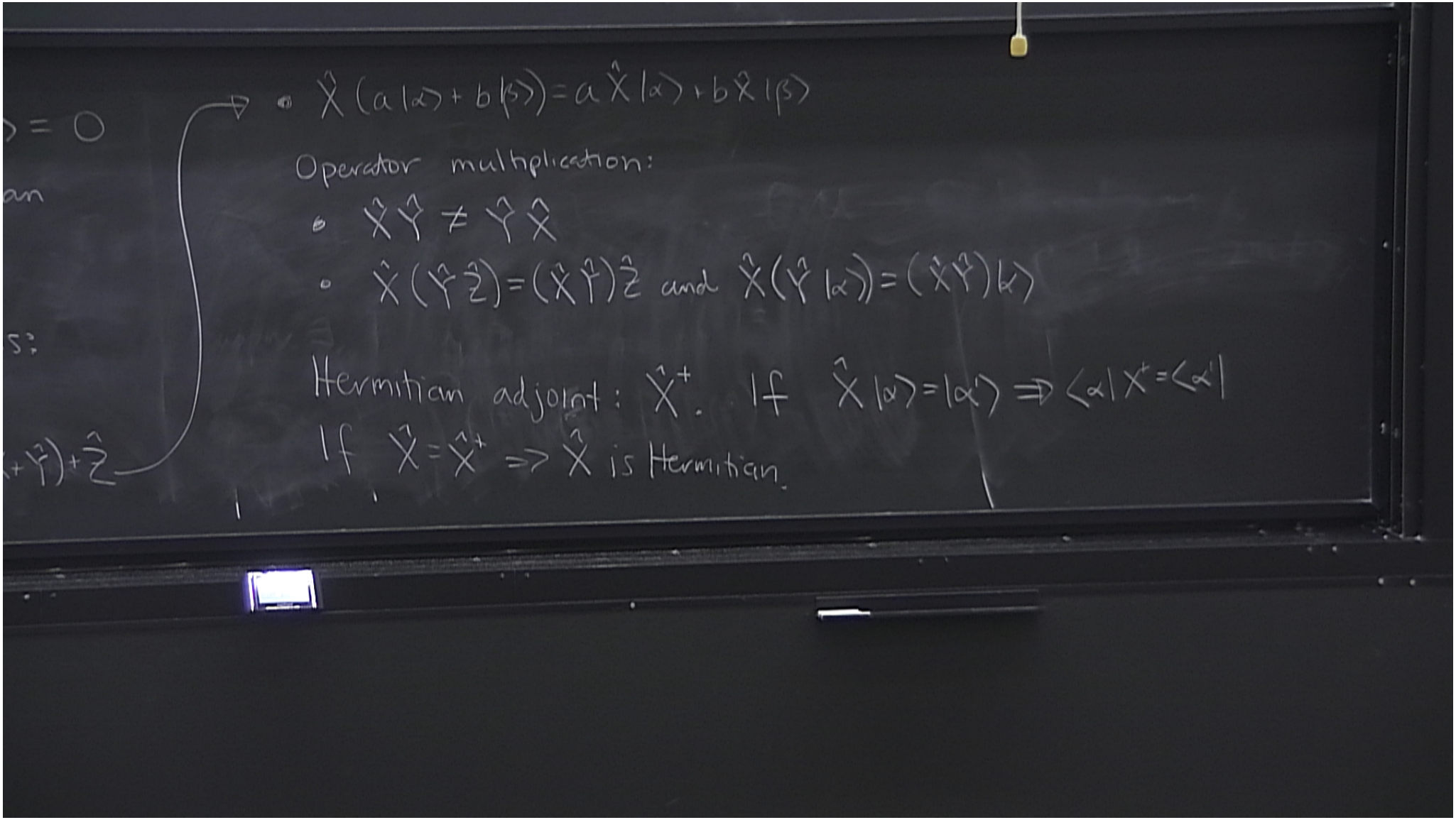

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Operator addition
$\hat{X} + \hat{Y} = \hat{Y} +$
$\hat{X} + (\hat{Y} + \hat{Z})$
$\hat{X}(a|\alpha\rangle + b|\beta\rangle) = a\hat{X}|\alpha\rangle + b\hat{X}|\beta\rangle$

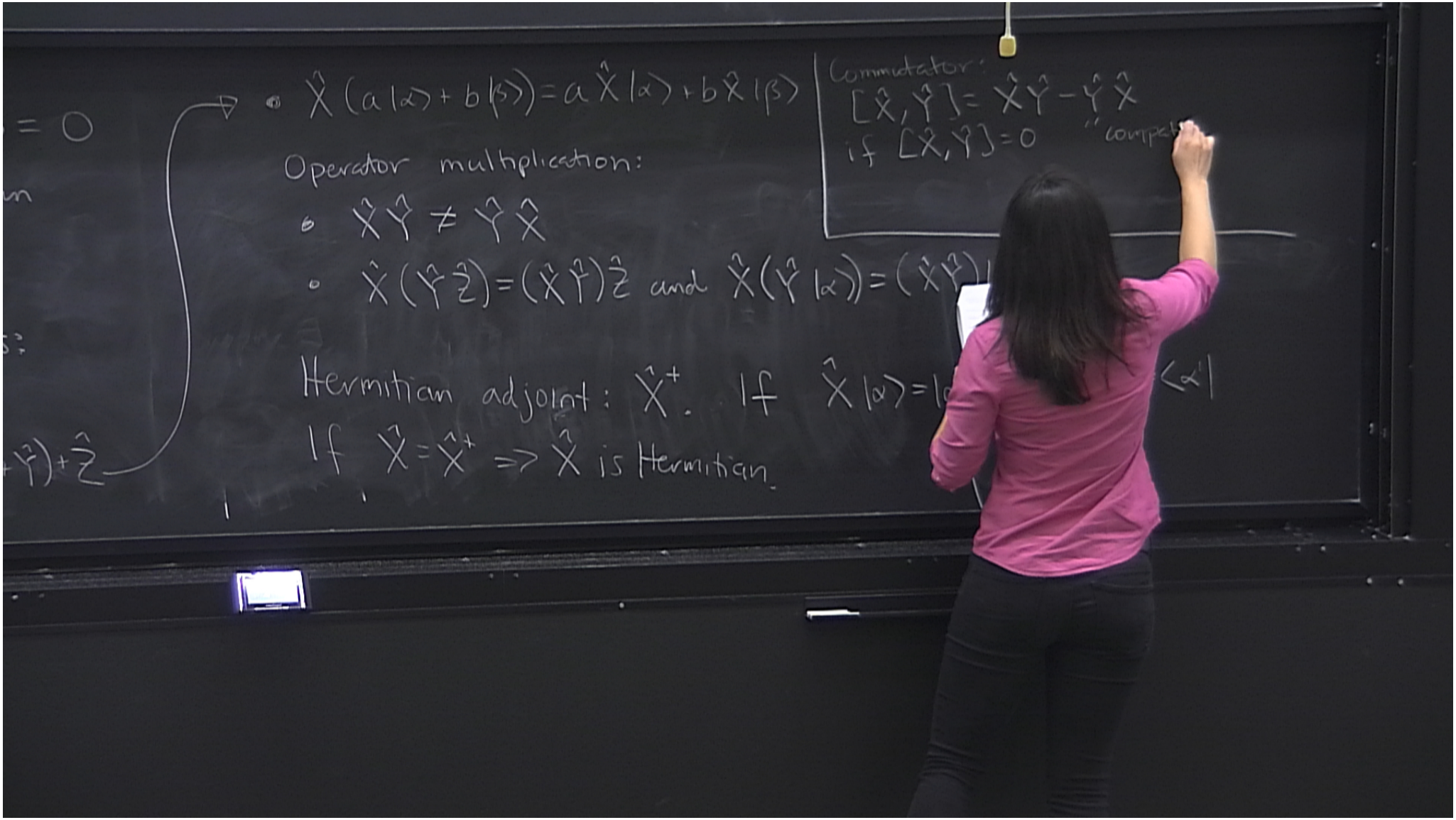

Null operator: $\hat{X}|\alpha\rangle = 0$
The outer product is an
rator: $|\alpha X \beta|$
rator addition is:
$\hat{X} + \hat{Y} = \hat{X}$
$\hat{X} + (\hat{X}+\hat{Y}) + \hat{Z}$
$\hat{X}(a|\alpha\rangle + b|\beta\rangle) = a\hat{X}|\alpha\rangle + b\hat{X}|\beta\rangle$
Operator multiplication:

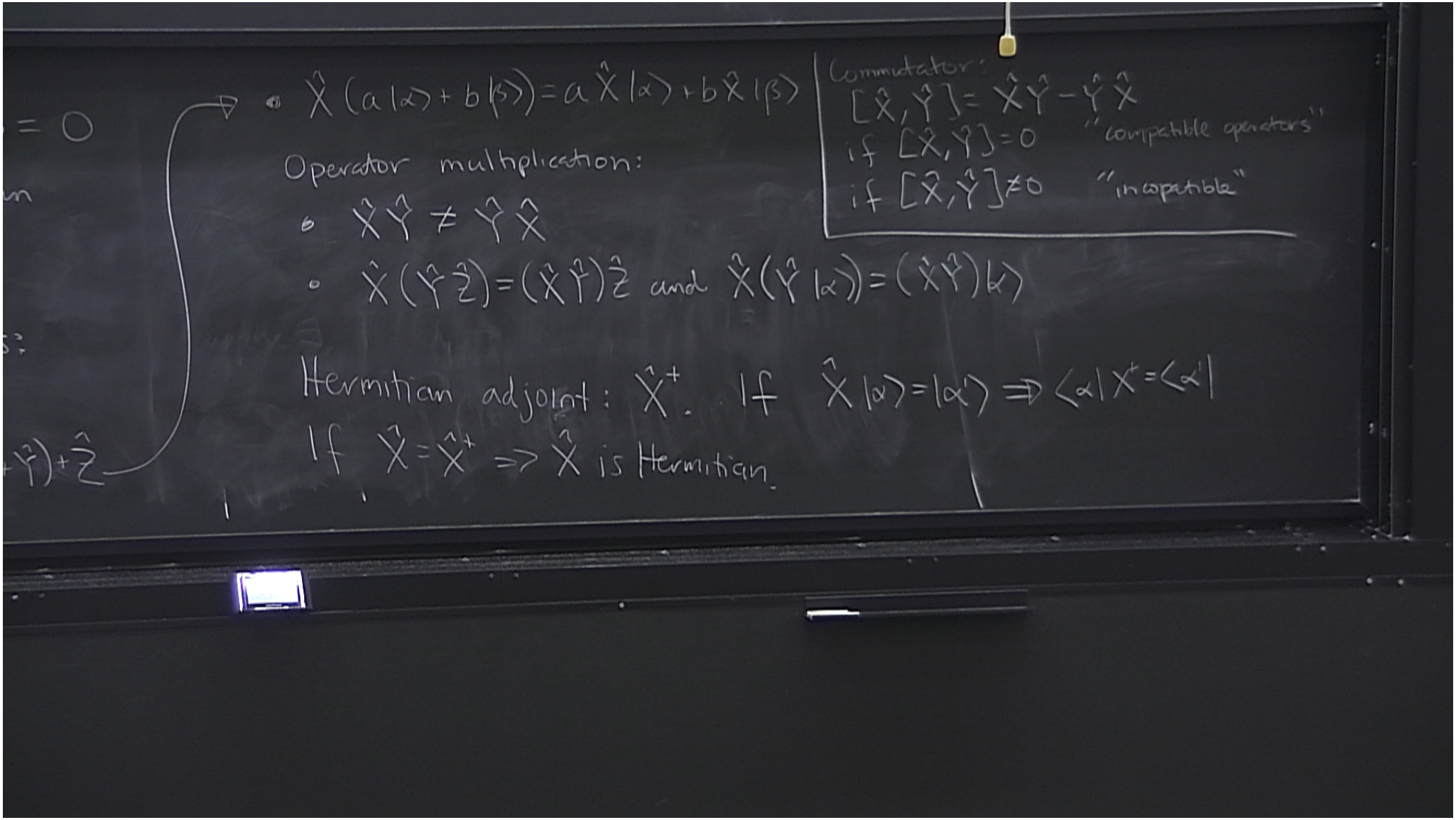

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Operator multiplication:
$\hat{X}\hat{Y} \neq \hat{Y}\hat{X}$
$\hat{X}(\hat{Y}\hat{Z}) = (\hat{X}\hat{Y})\hat{Z}$ and $\hat{X}(\hat{Y}|\alpha\rangle) = (\hat{X}\hat{Y})|\alpha\rangle$


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Hermitian adjoint: $\hat{X}^\dagger$

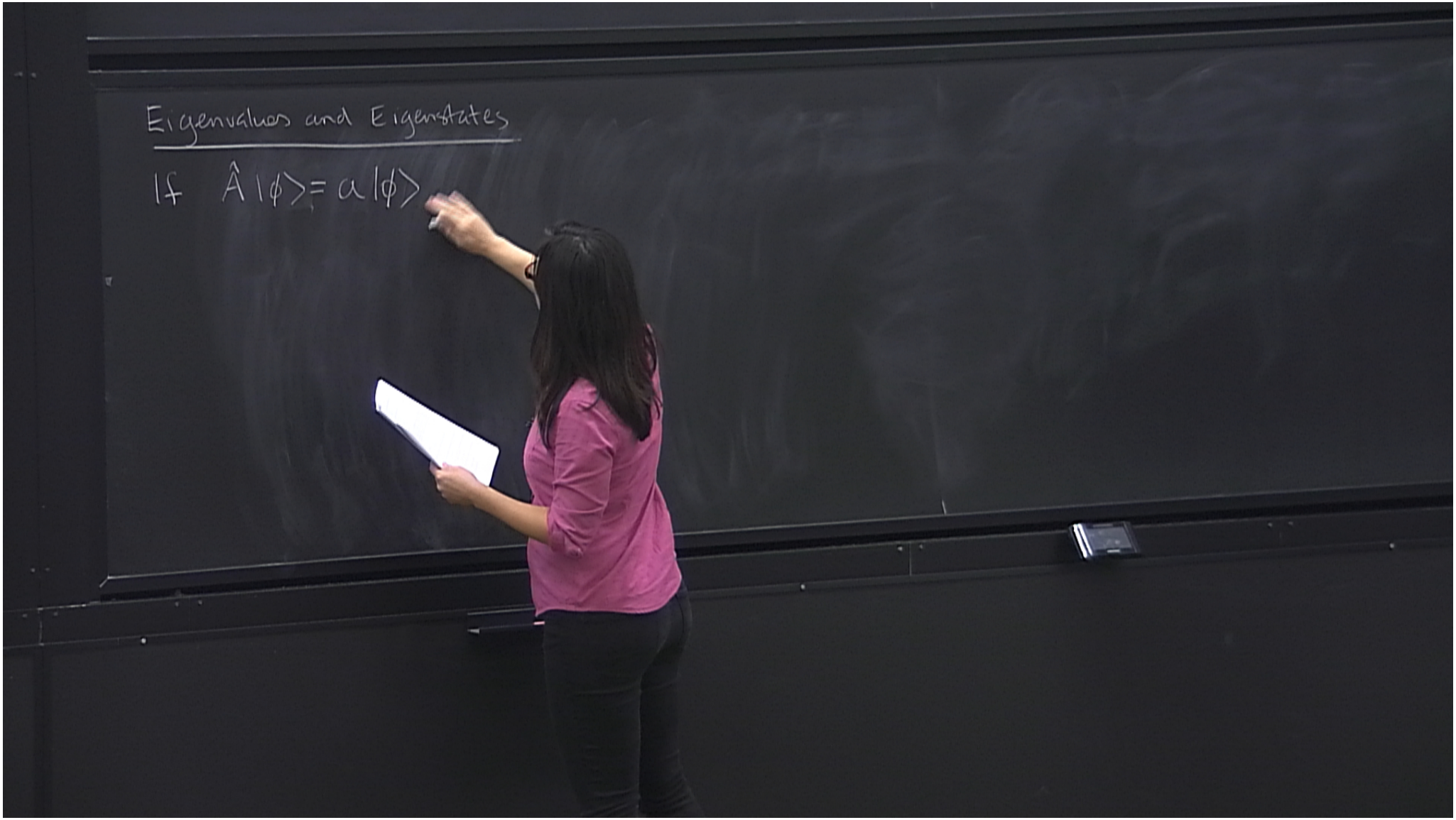

$\hat{X}(a|\alpha\rangle + b|\beta\rangle) = a\hat{X}|\alpha\rangle + b\hat{X}|\beta\rangle$
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Hermitian adjoint: $\hat{X}^\dagger$. If $\hat{X}|\alpha\rangle = |\alpha'\rangle \Rightarrow \langle\alpha|\hat{X}^\dagger = \langle\alpha'|$

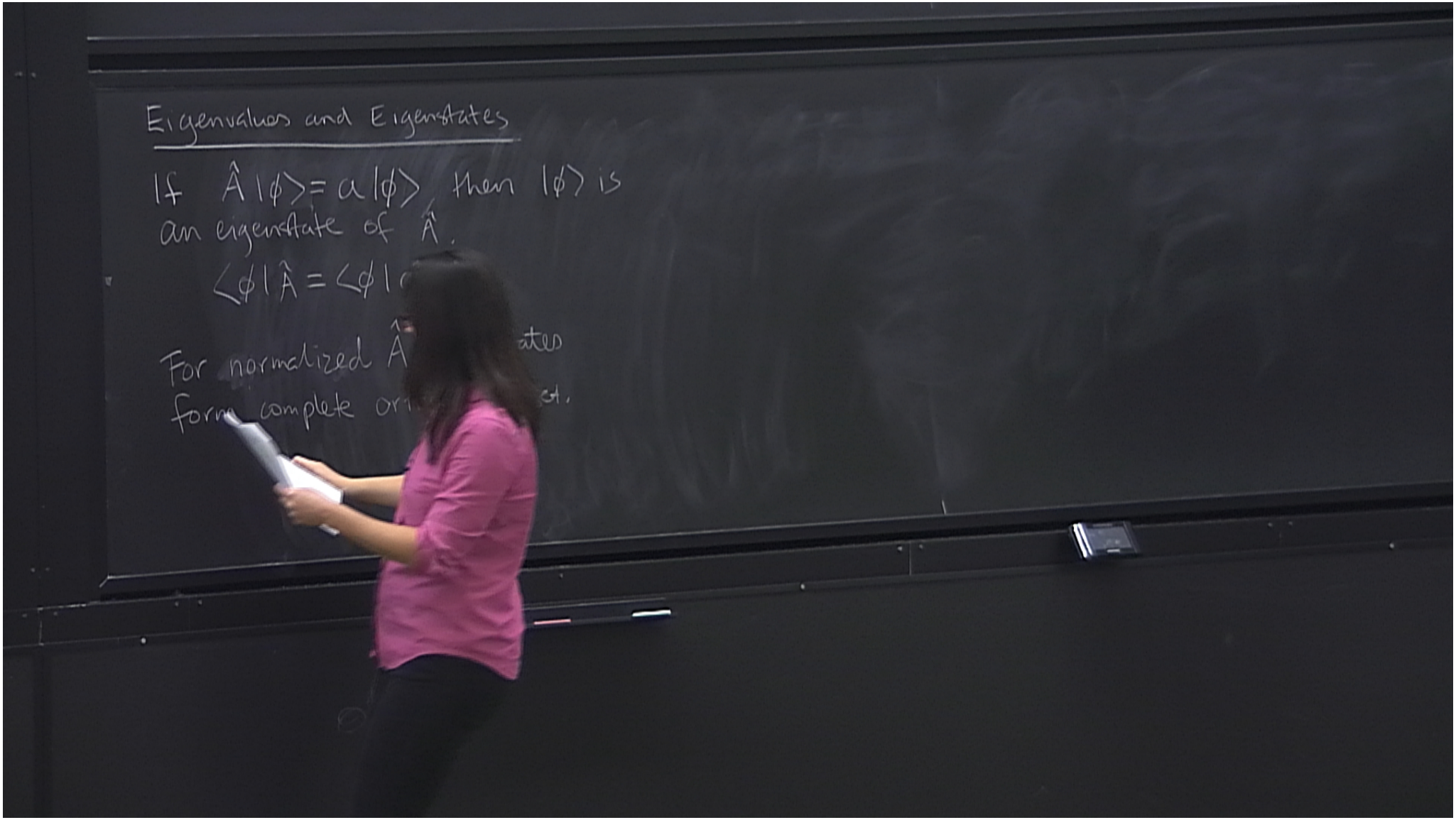

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Hermitian adjoint: $\hat{X}^\dagger$. If $\hat{X}|\alpha\rangle = |\alpha'\rangle \Rightarrow \langle\alpha|X^\dagger = \langle\alpha'|$
If $\hat{X} = \hat{X}^\dagger \Rightarrow \hat{X}$ is Hermitian.

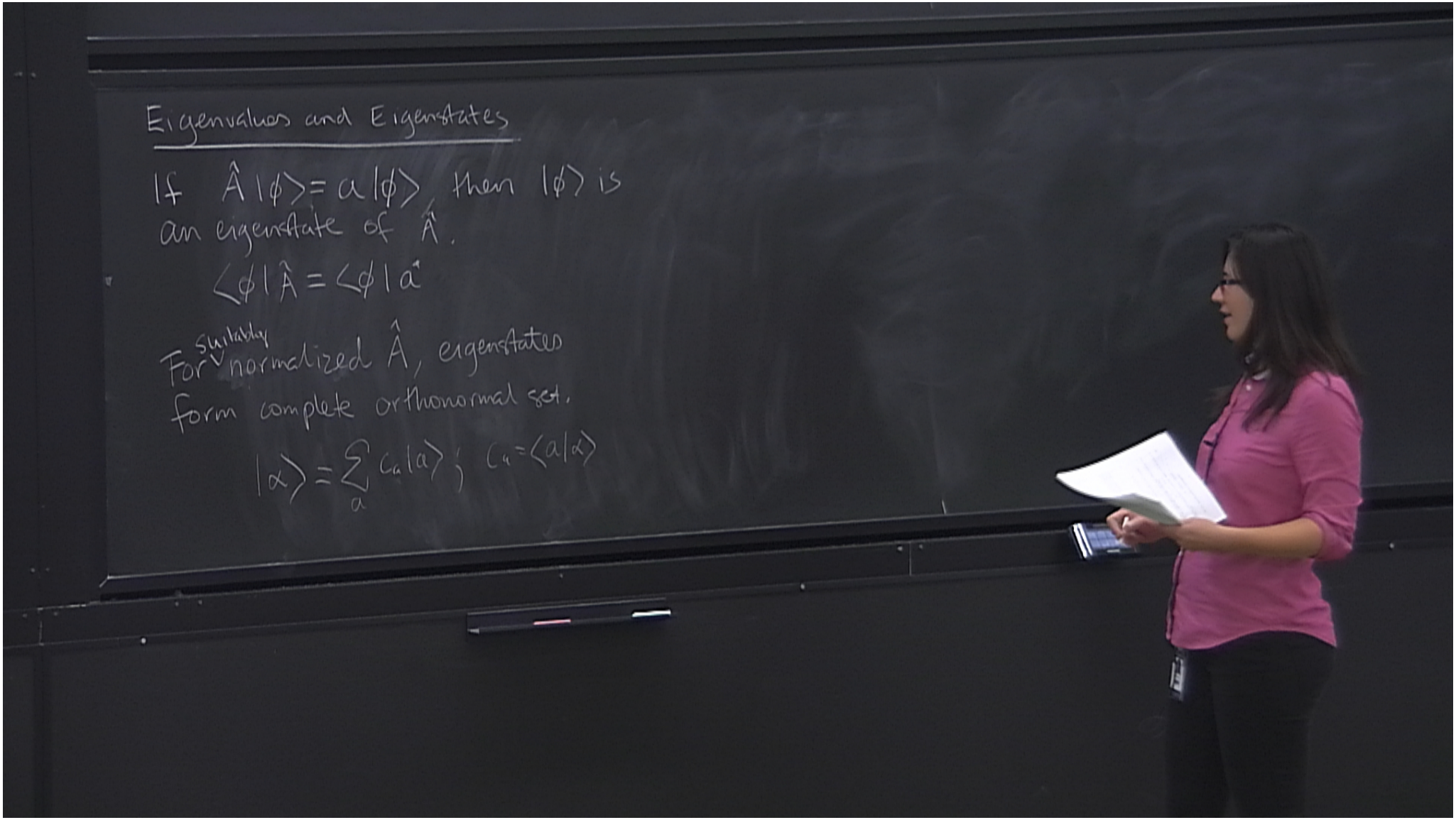

Commutator:
if
Operator multiplication:
Hermitian adjoint:
If is Hermitian.

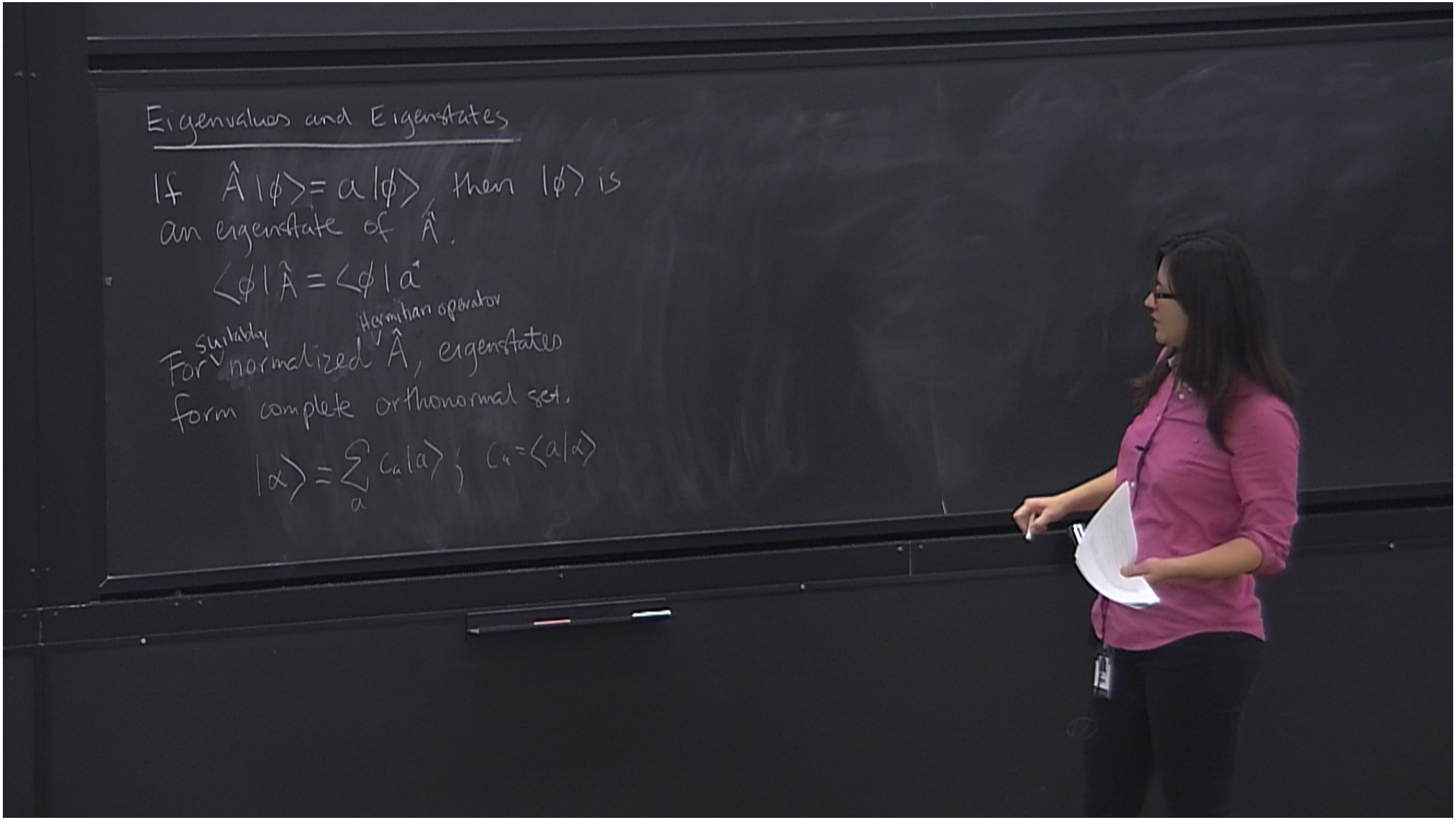

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Operator multiplication:
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If $\hat{X} = \hat{X}^\dagger \Rightarrow \hat{X}$ is Hermitian.
Commutator:
$[\hat{X}, \hat{Y}] = \hat{X}\hat{Y} - \hat{Y}\hat{X}$
if $[\hat{X}, \hat{Y}] = 0$ "compatible operators"
if $[\hat{X}, \hat{Y}] \neq 0$ "incompatible"

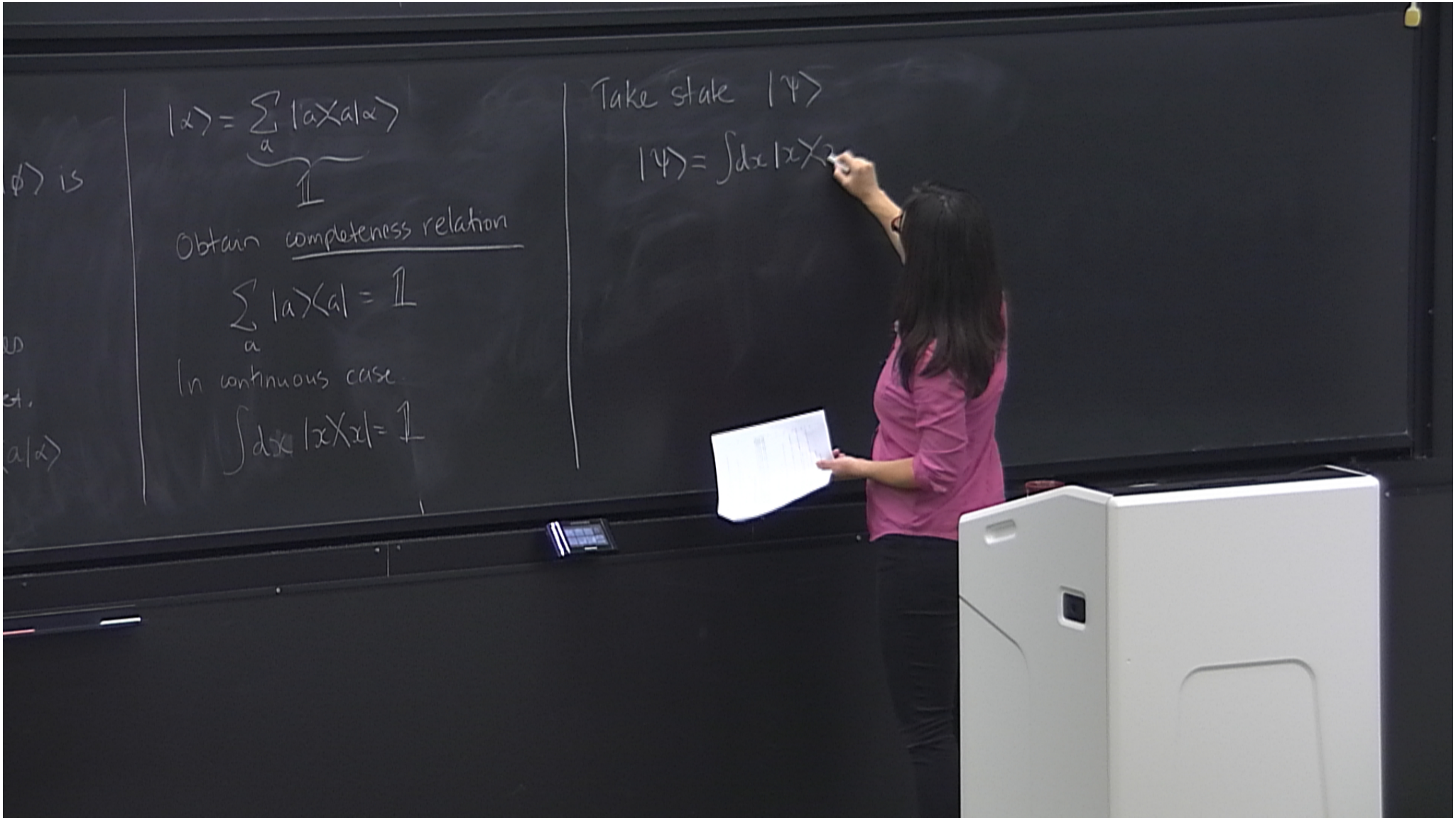

Eigenvalues and Eigenstates

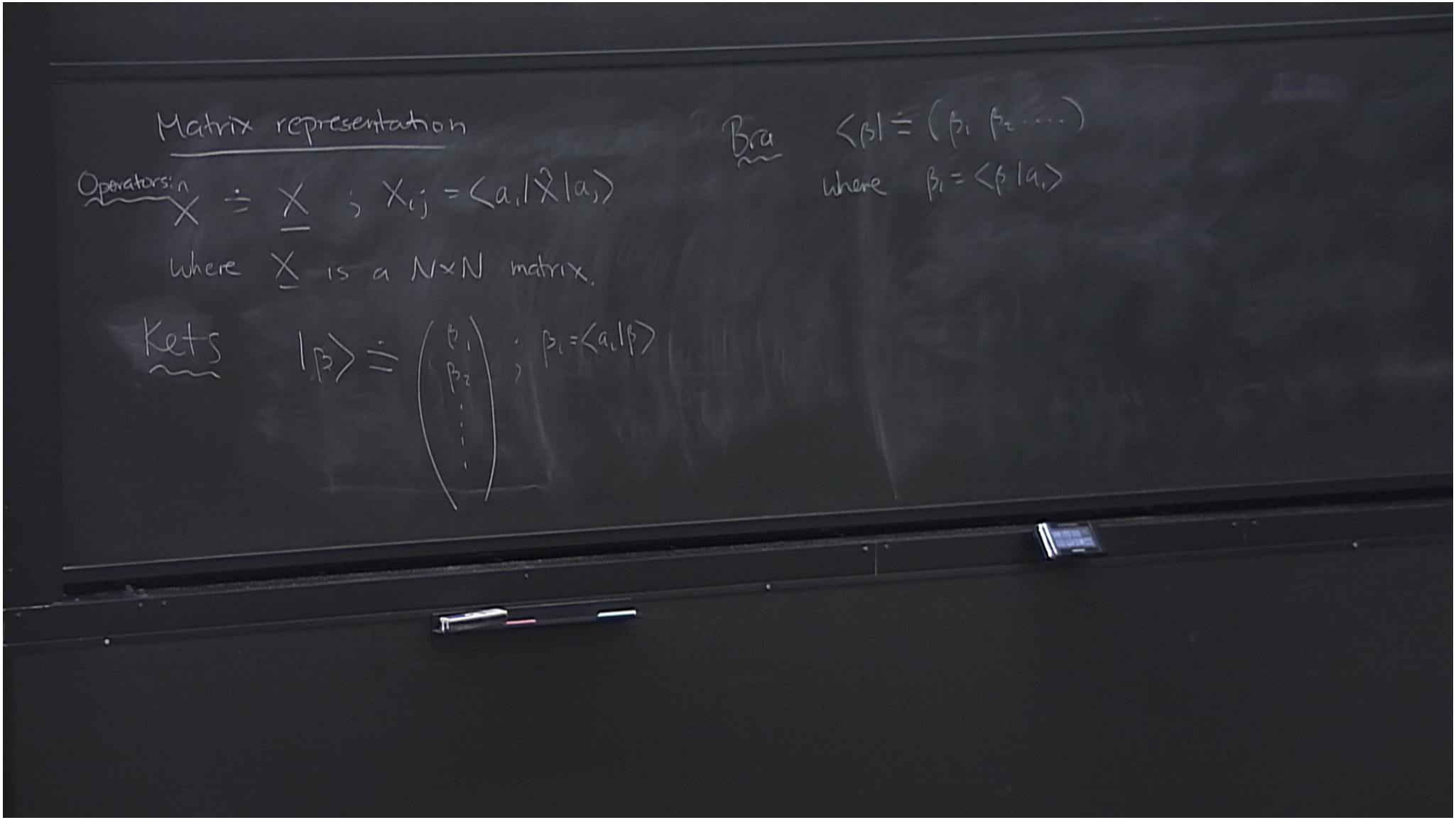

Eigenvalues and Eigenstates
If $\hat{A}|\phi\rangle = a|\phi\rangle$

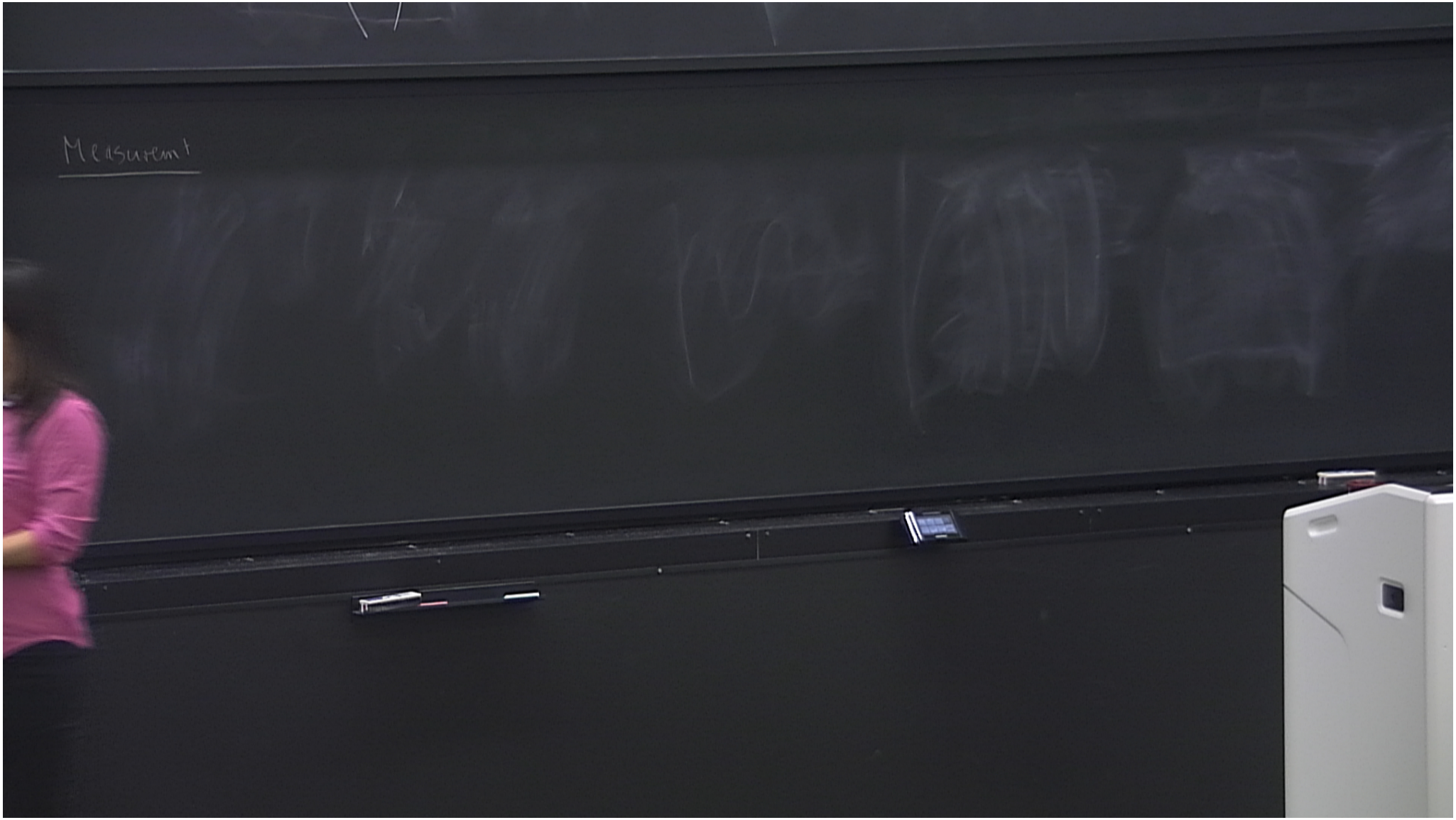

Eigenvalues and Eigenstates
If $\hat{A}|\phi\rangle = a|\phi\rangle$ then $|\phi\rangle$ is an eigenstate of $\hat{A}$.
$\langle\phi|\hat{A} = \langle\phi|a$
For normalized $\hat{A}$ ates
form complete or

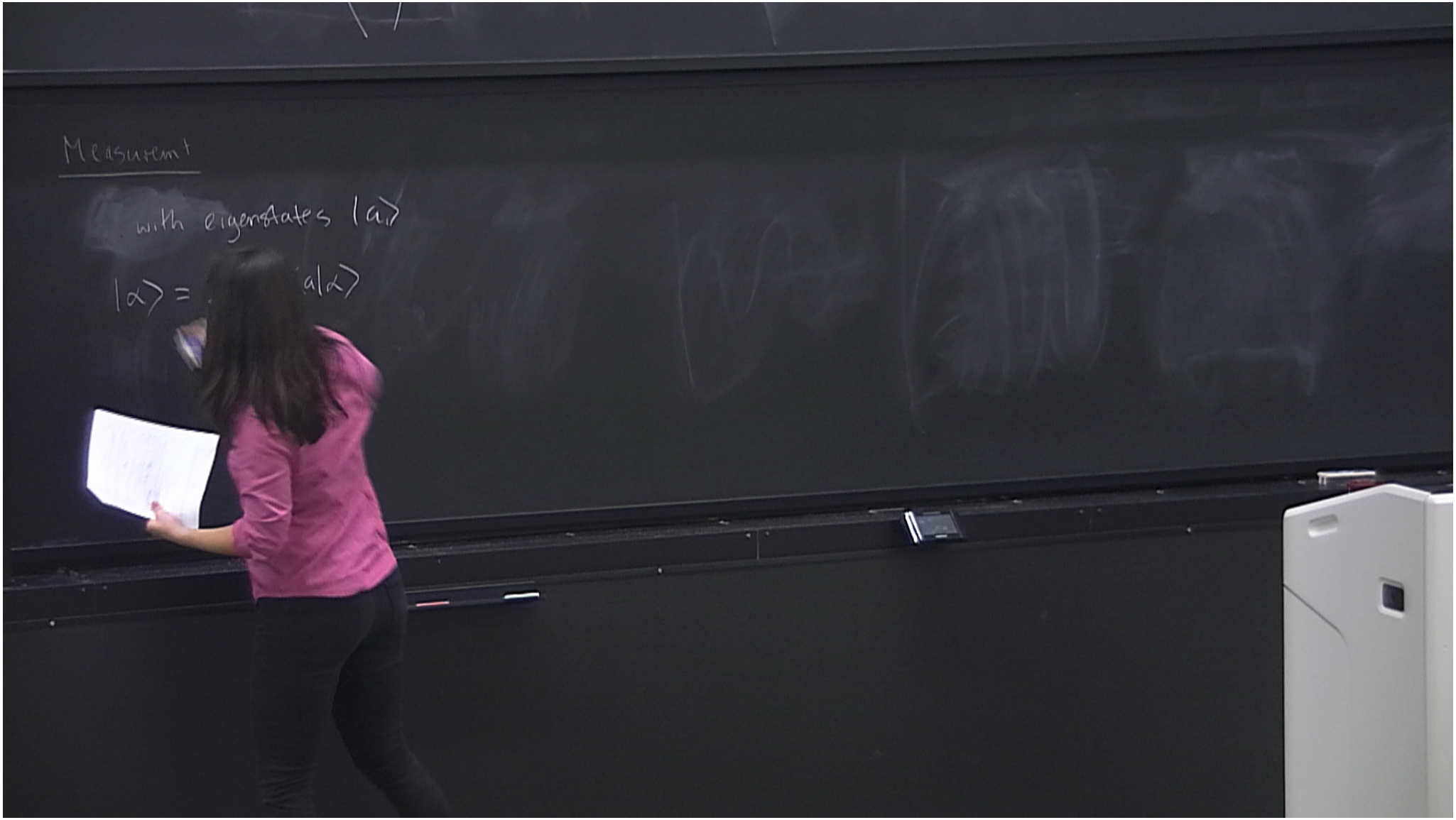

Eigenvalues and Eigenstates
If $\hat{A}|\phi\rangle = a|\phi\rangle$ then $|\phi\rangle$ is an eigenstate of $\hat{A}$.
$\langle\phi|\hat{A} = \langle\phi|a$
For suitably normalized $\hat{A}$, eigenstates form complete orthonormal set.
$|\alpha\rangle = \sum_a c_a|a\rangle$; $c_a = \langle a|\alpha\rangle$

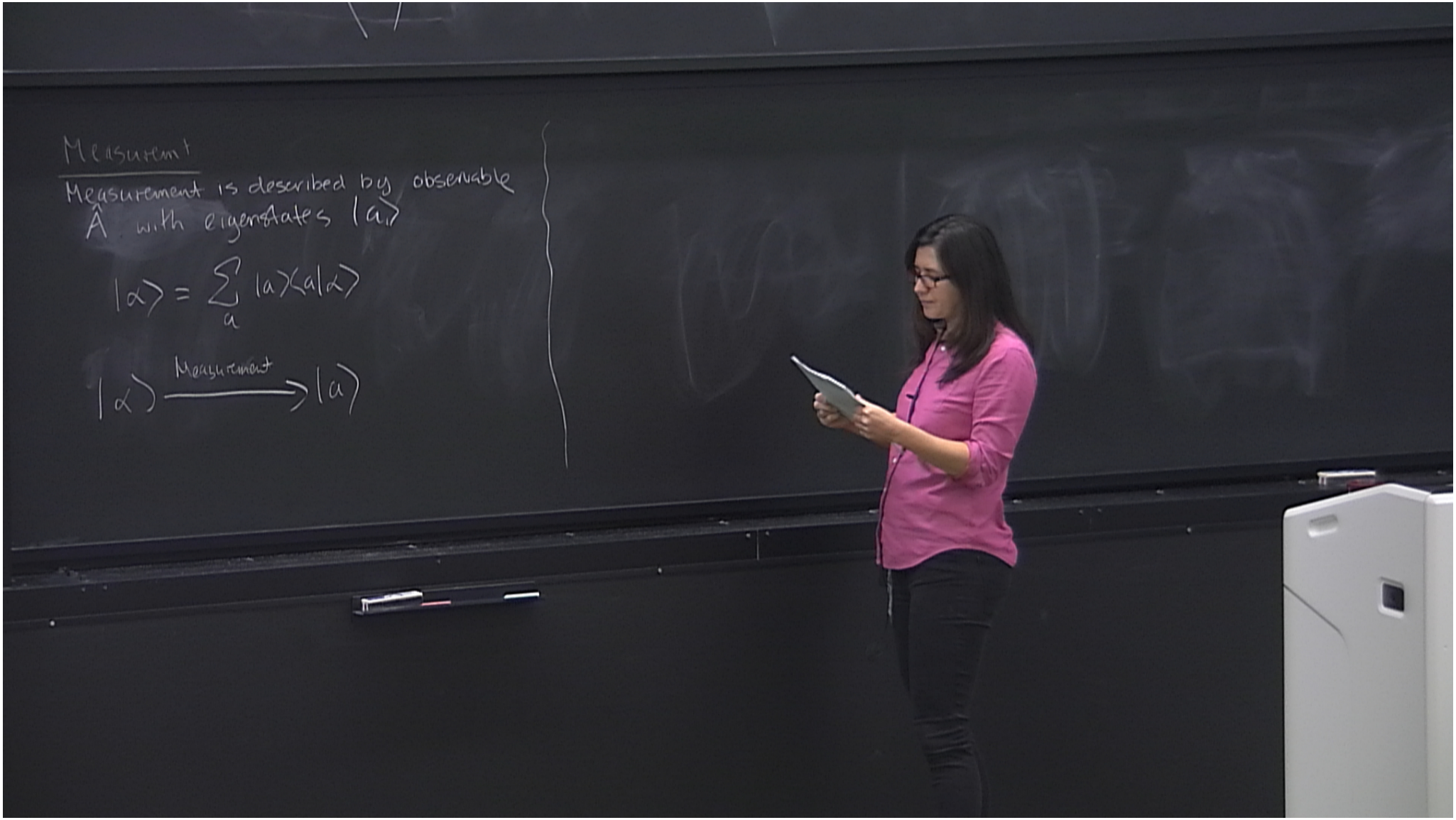

Eigenvalues and Eigenstates
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For suitably normalized Hermitian operator $\hat{A}$, eigenstates form complete orthonormal set.
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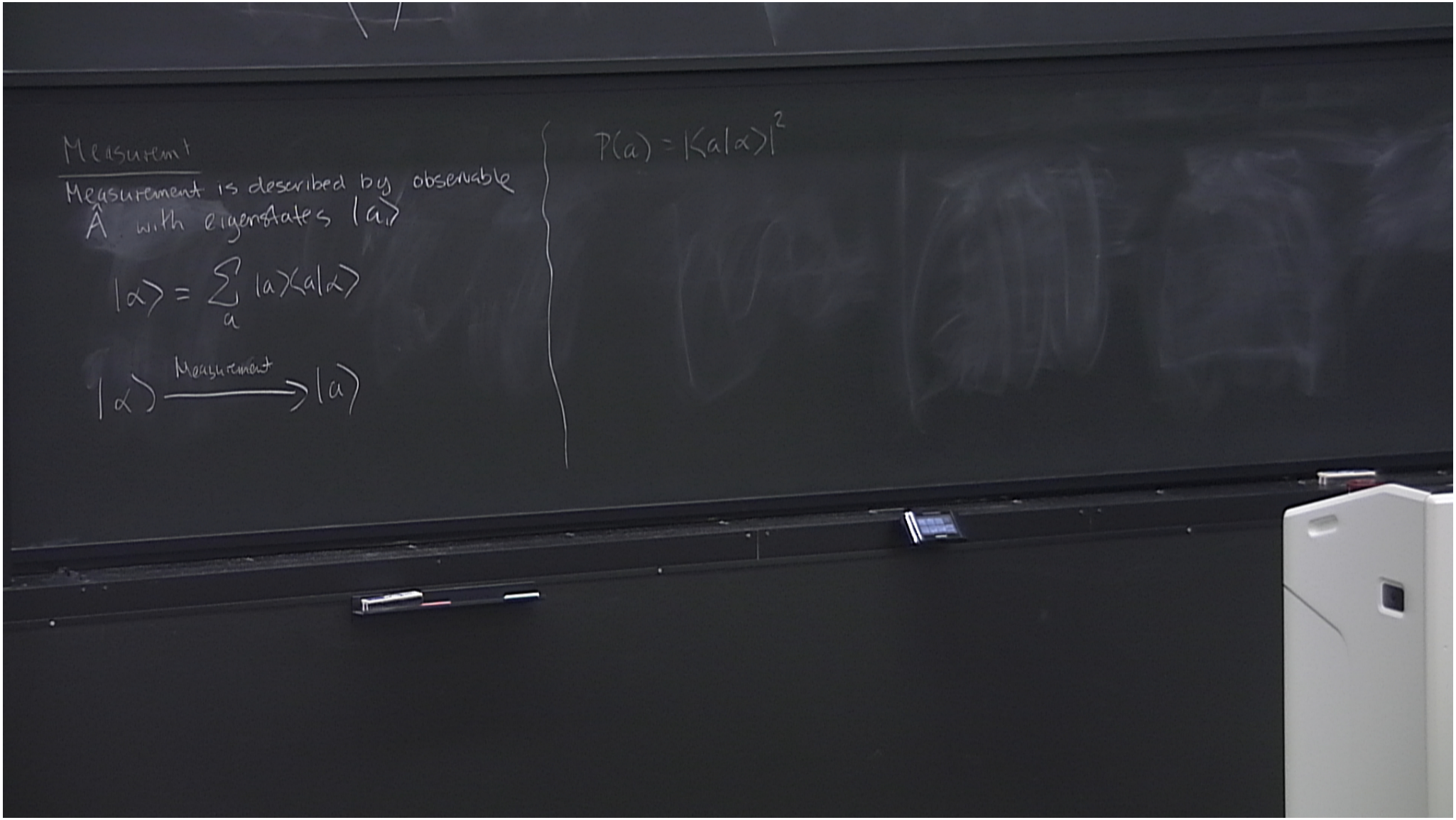

$|\alpha\rangle = \sum_a |a\rangle\langle a|\alpha\rangle$
$\mathbb{1}$
Obtain completeness relation
$\sum_a |a\rangle\langle a| = \mathbb{1}$
In continuous case
$\int dx\, |x\rangle\langle x| = \mathbb{1}$
Take state $|\psi\rangle$
$|\psi\rangle = \int dx\, |x\rangle\langle$

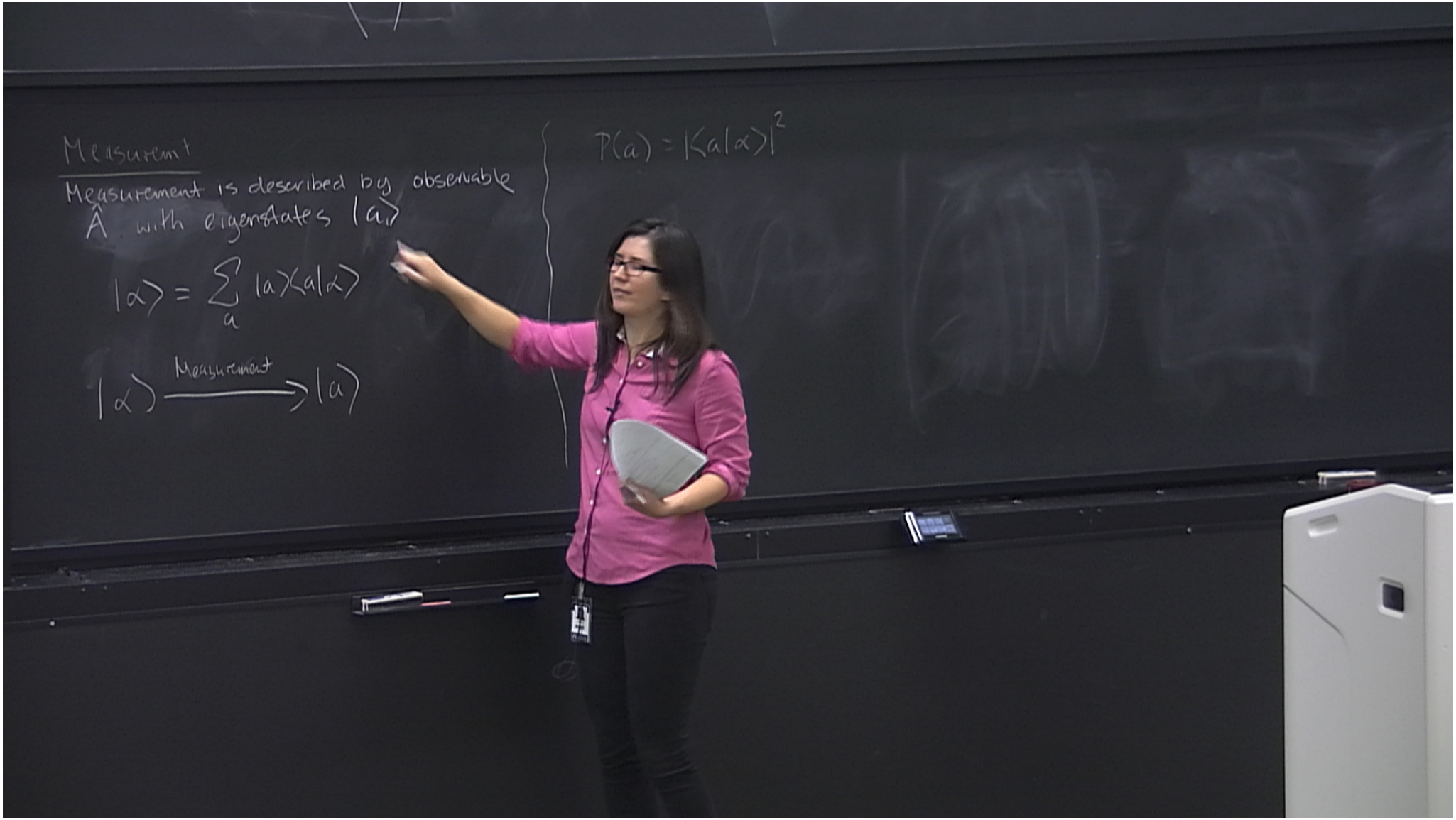

Matrix representation
Operators: $\hat{X} \doteq \underline{X}$; $X_{ij} = \langle a_i | \hat{X} | a_j \rangle$
where $\underline{X}$ is a $N \times N$ matrix.
Kets $|\beta\rangle \doteq \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \end{pmatrix}$; $\beta_i = \langle a_i | \beta \rangle$
Bra $\langle \beta | \doteq (\beta_1^* \ \beta_2^* \cdots)$
where $\beta_i^* = \langle \beta | a_i \rangle$

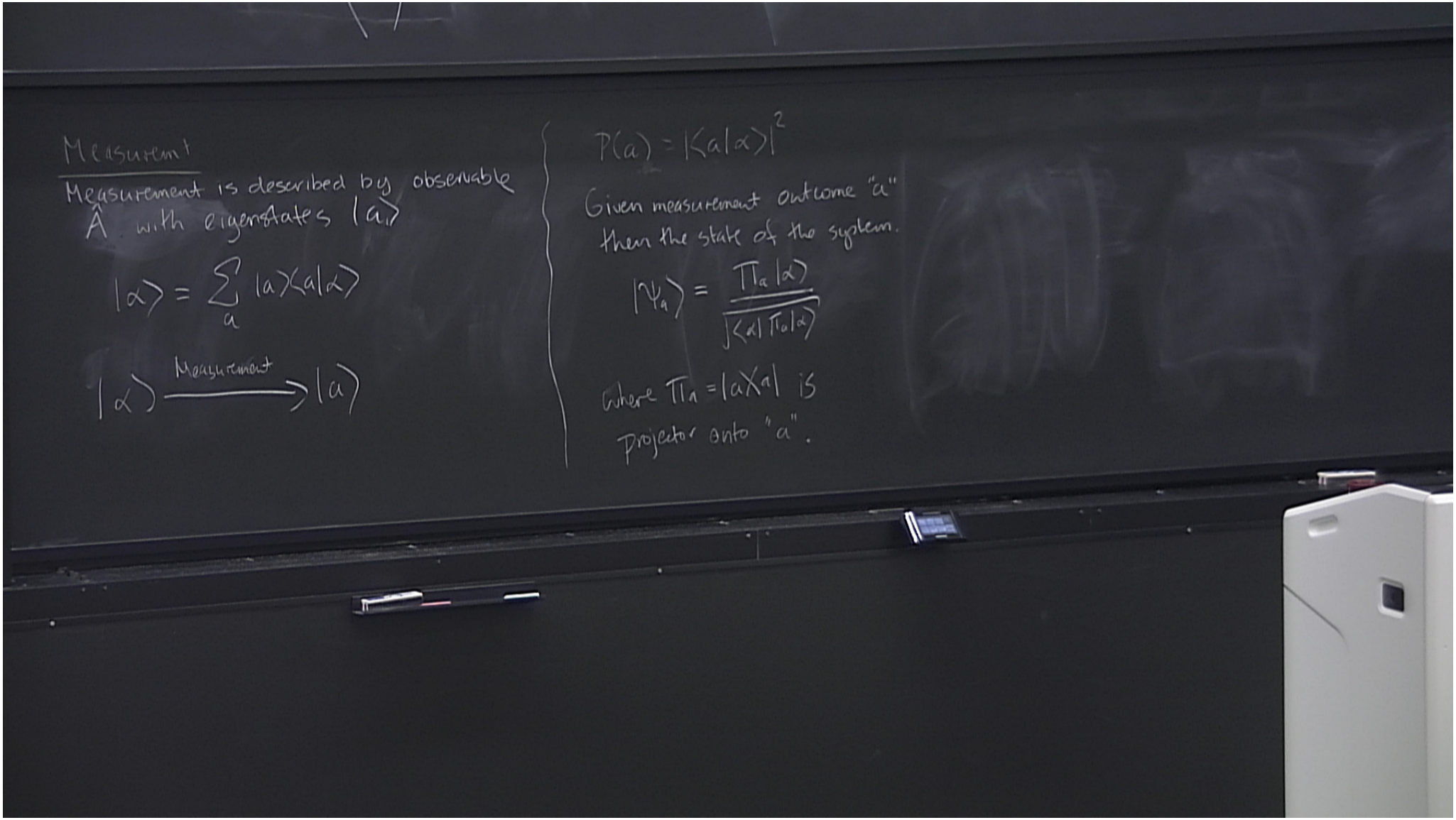

Measurement

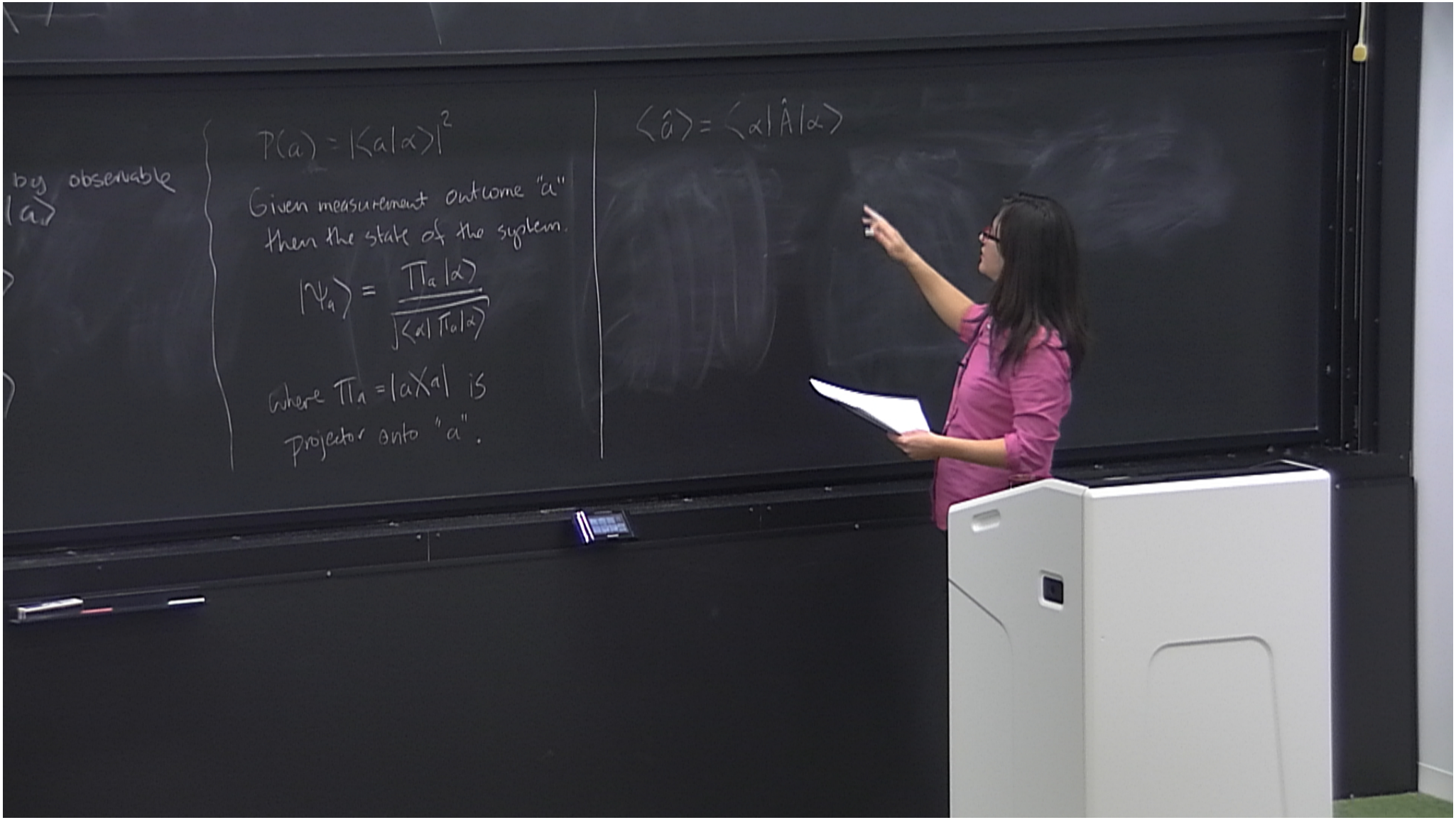

Measurement
with eigenstates |a⟩
|α⟩ = a|α⟩

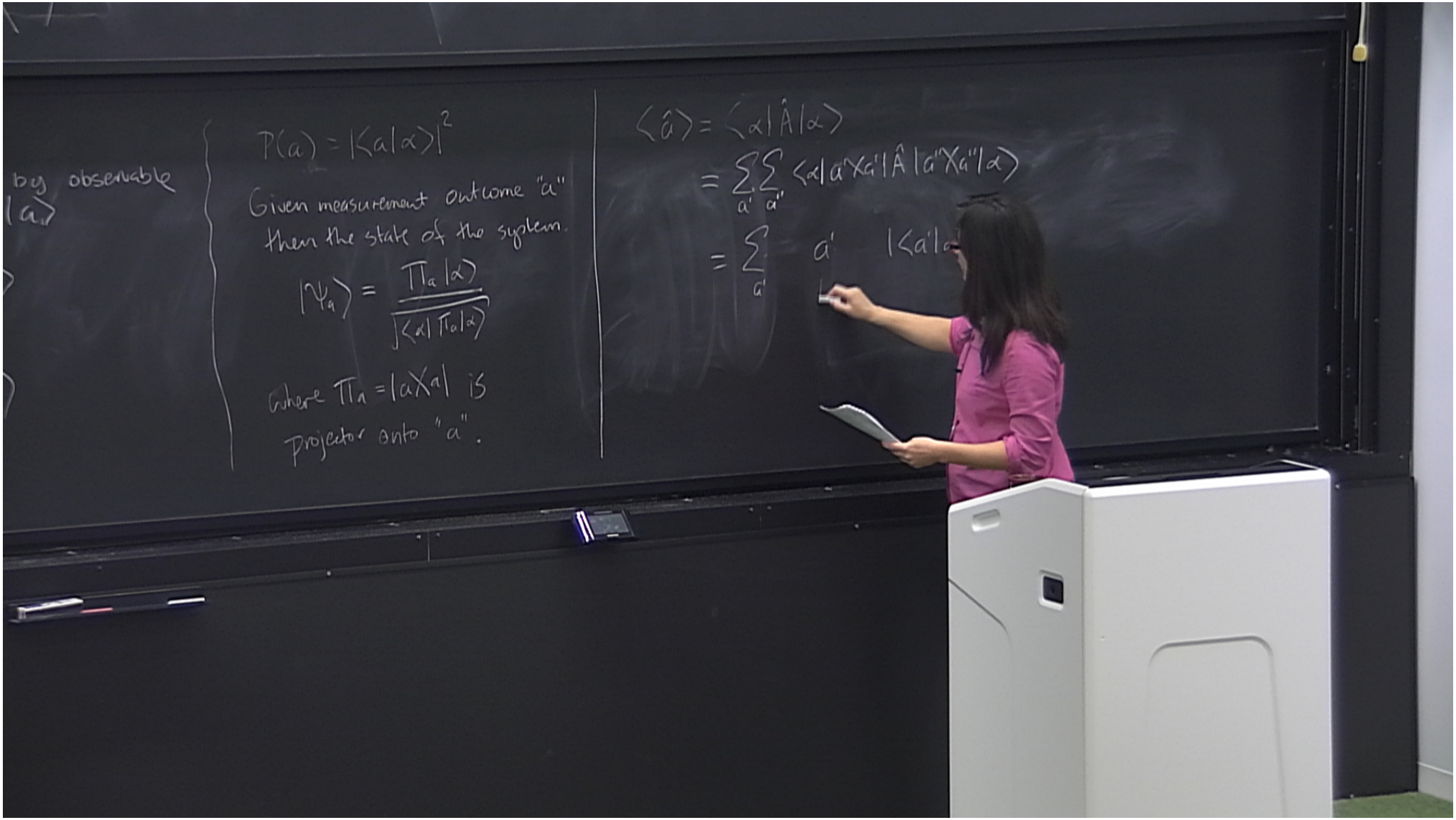

Measurement
Measurement is described by observable $\hat{A}$ with eigenstates $|a\rangle$
$|\alpha\rangle = \sum_a |a\rangle\langle a|\alpha\rangle$
$|\alpha\rangle \xrightarrow{\text{Measurement}} |a\rangle$

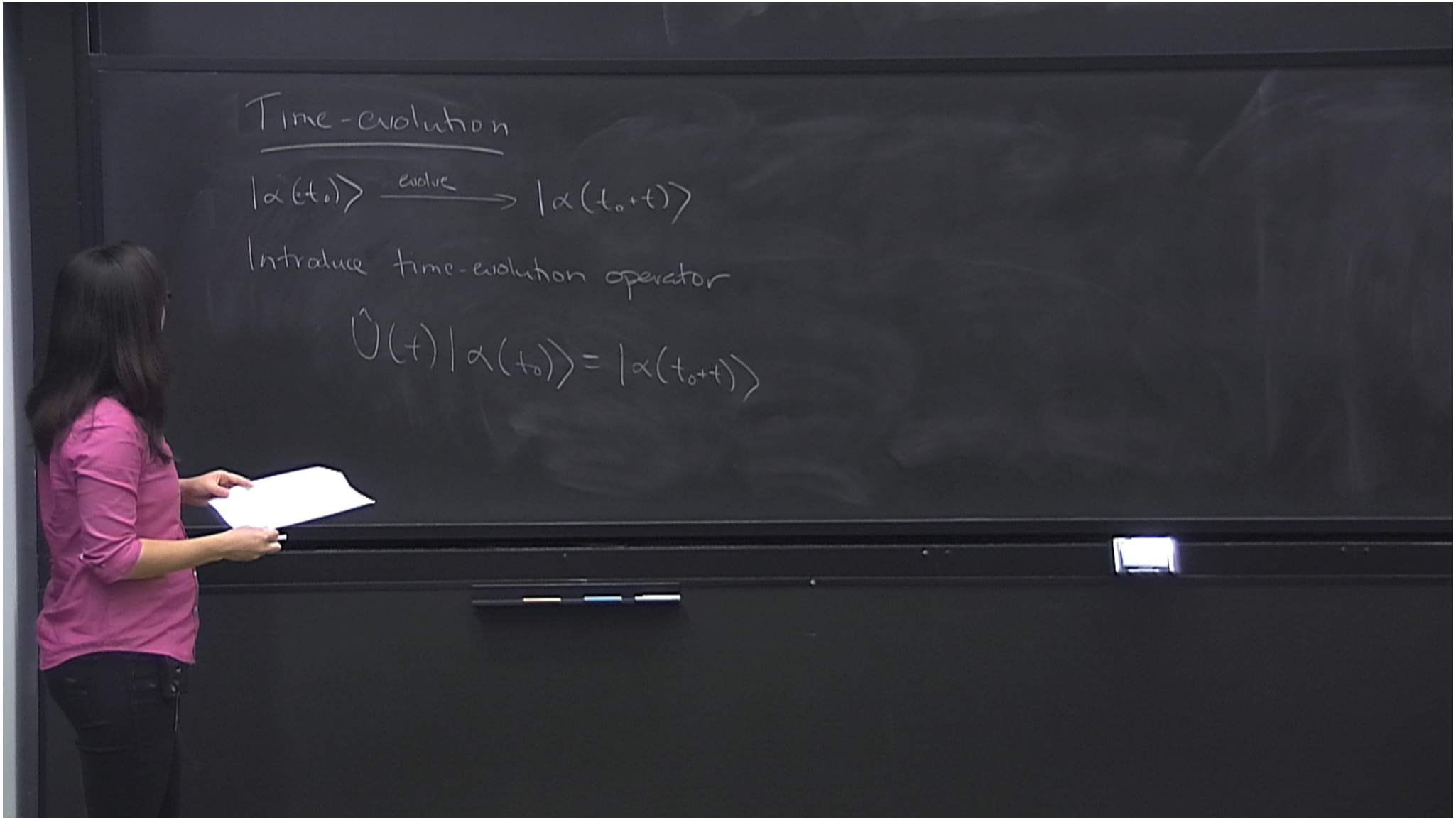

Measurement
Measurement is described by observable $\hat{A}$ with eigenstates $|a\rangle$
$|\alpha\rangle = \sum_a |a\rangle\langle a|\alpha\rangle$
$|\alpha\rangle \xrightarrow{\text{Measurement}} |a\rangle$
$P(a) = |\langle a|\alpha\rangle|^2$

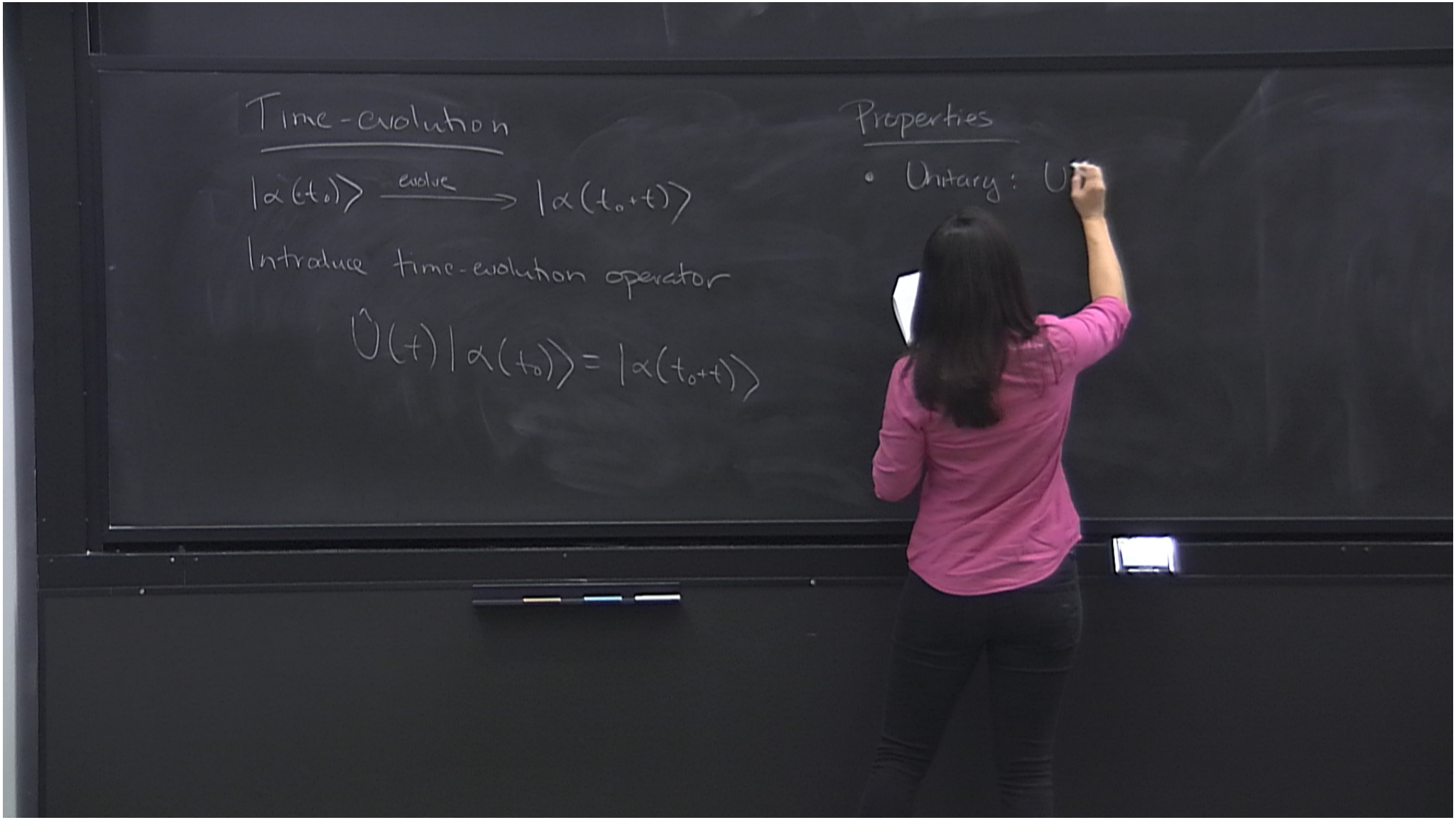

Measurement
Measurement is described by observable
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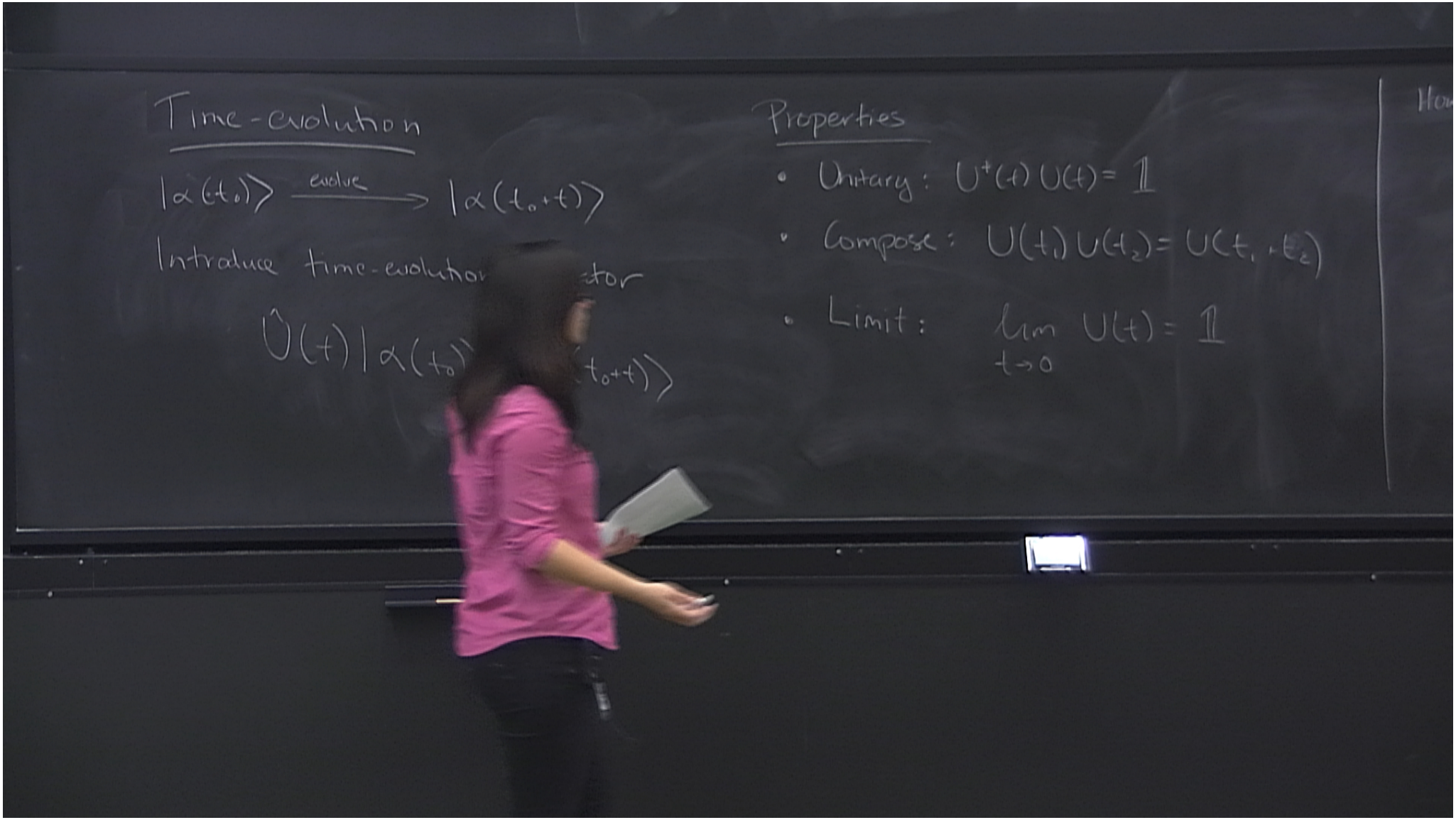

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Given measurement outcome "a" then the state of the system.
$|\psi_a\rangle = \frac{\Pi_a|\alpha\rangle}{\sqrt{\langle\alpha|\Pi_a|\alpha\rangle}}$
where $\Pi_a = |a\rangle\langle a|$ is Projector onto "a".

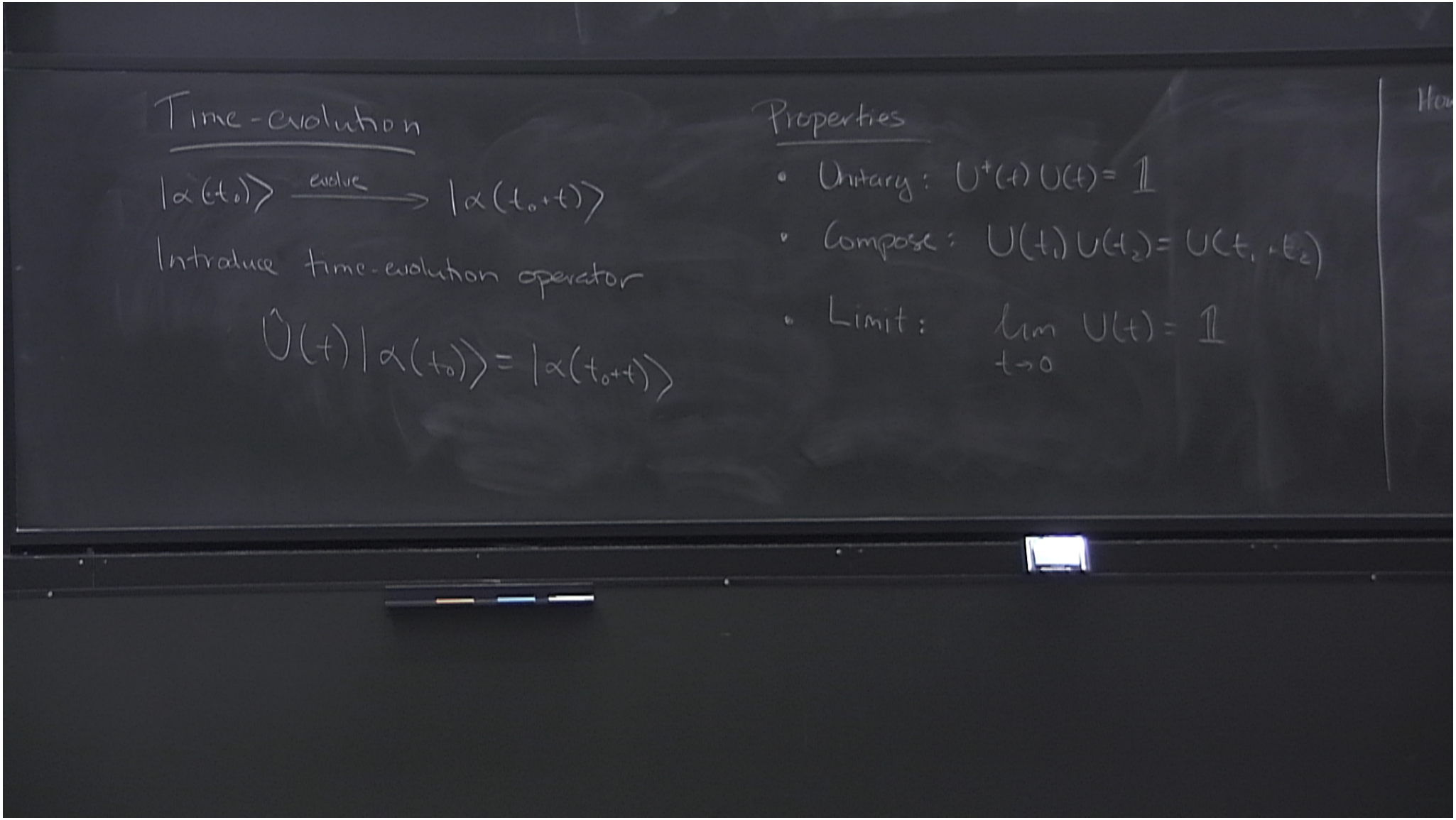

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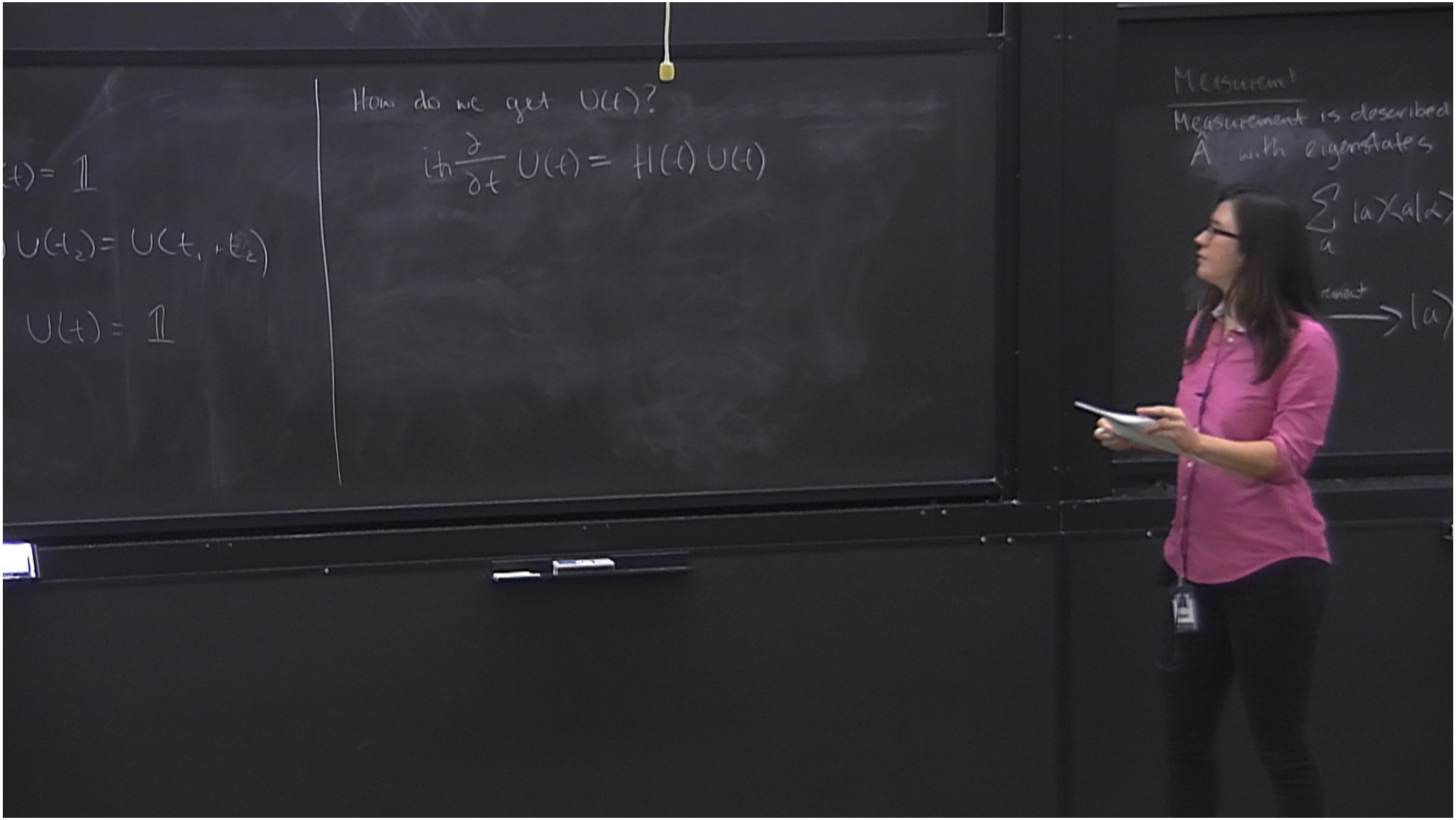

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$= \sum_{a'}$ a' $|\langle a'|$

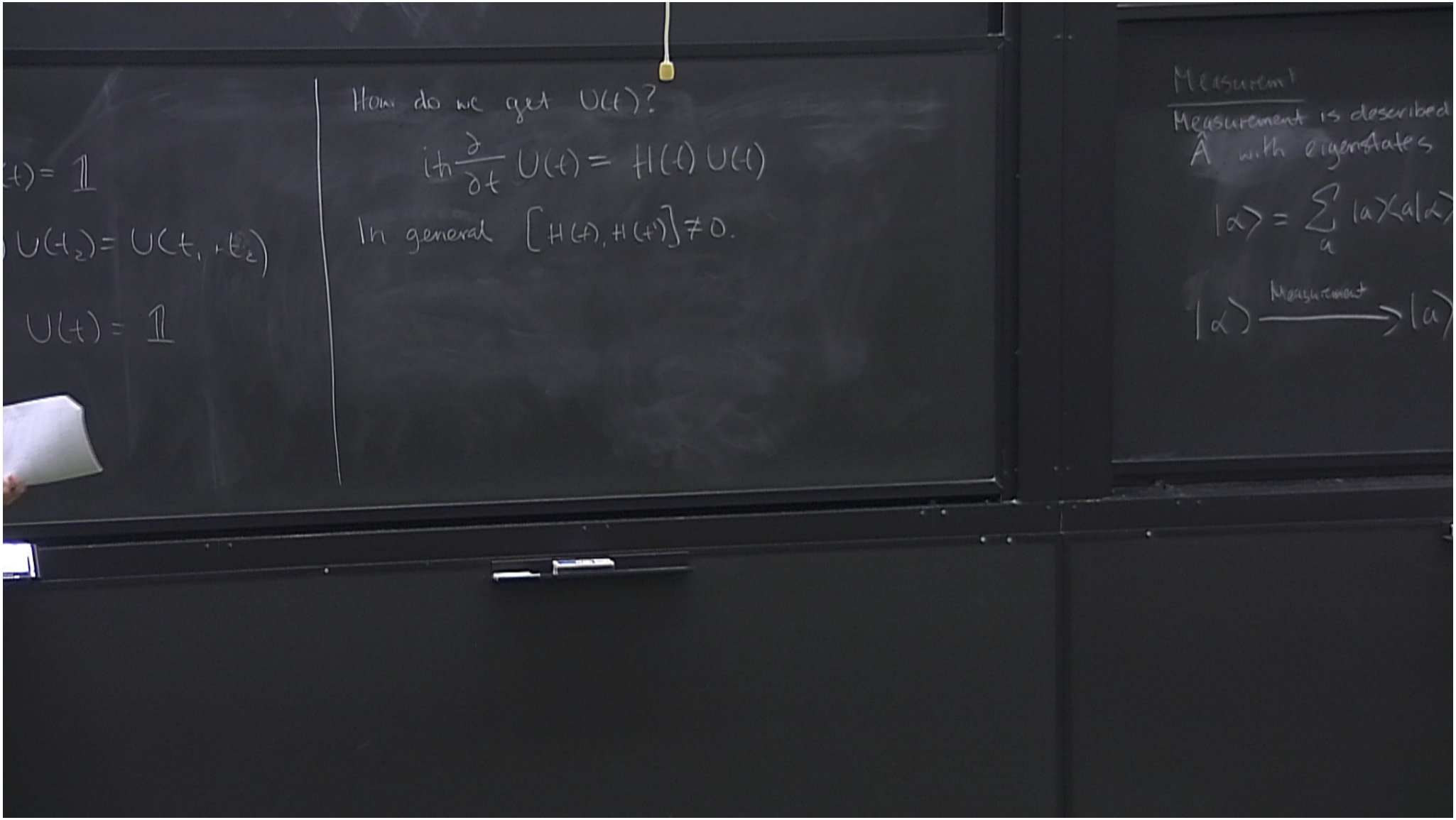

Time-evolution
$|\alpha(t_0)\rangle$ evolve $|\alpha(t_0+t)\rangle$
Introduce time-evolution operator
$\hat{U}(t)|\alpha(t_0)\rangle = |\alpha(t_0+t)\rangle$

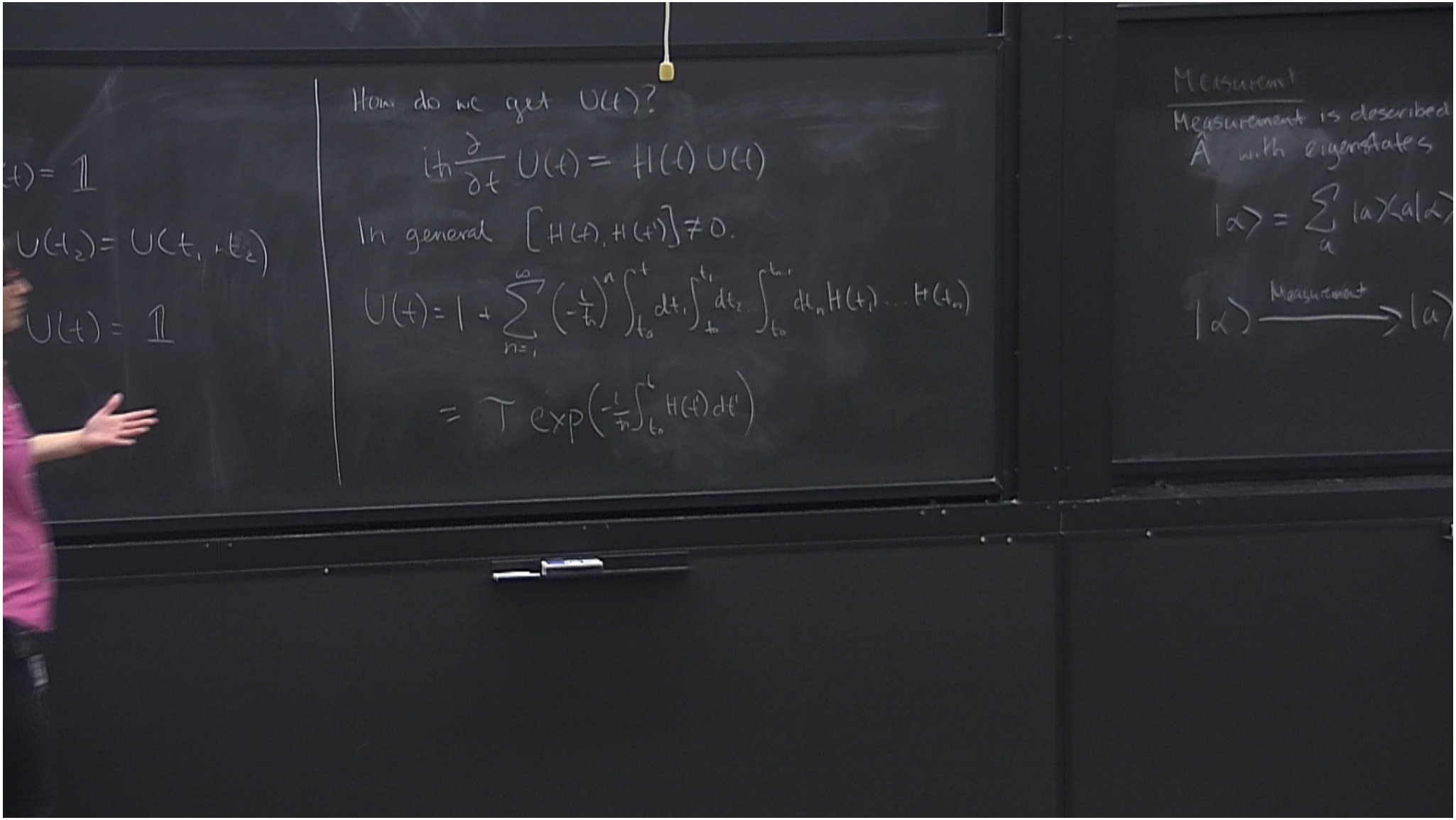

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$\hat{U}(t)|\alpha(t_0)$
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• Compose: $U(t_1)\,U(t_2) = U(t_1+t_2)$
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How do we get U(t)?
$i\hbar \frac{\partial}{\partial t} U(t) = H(t) U(t)$
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In general $[H(t), H(t')] \neq 0$.
Measurement
Measurement is described
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How do we get U(t)?
$i\hbar \frac{\partial}{\partial t} U(t) = H(t)\,U(t)$
In general $[H(t), H(t')] \neq 0$.
$U(t) = 1 + \sum_{n=1}^{\infty} \left(\frac{-i}{\hbar}\right)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n H(t_1) \cdots H(t_n)$
$= T \exp\left(\frac{-i}{\hbar} \int_{t_0}^{t} H(t')\,dt'\right)$
Measurement
Measurement is described
$\hat{A}$ with eigenstates
$|\alpha\rangle = \sum_a |a\rangle\langle a|\alpha\rangle$
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Sometimes life is simpler: $[H(t), H(t')] = 0$

$$U(t) = \exp\left(-\frac{i}{\hbar}\int_{t_0}^{t} H(t)dt\right)$$

Sometimes it's even simpler: $\hat{H}$ is not time dependent

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Time-independent perturbation the



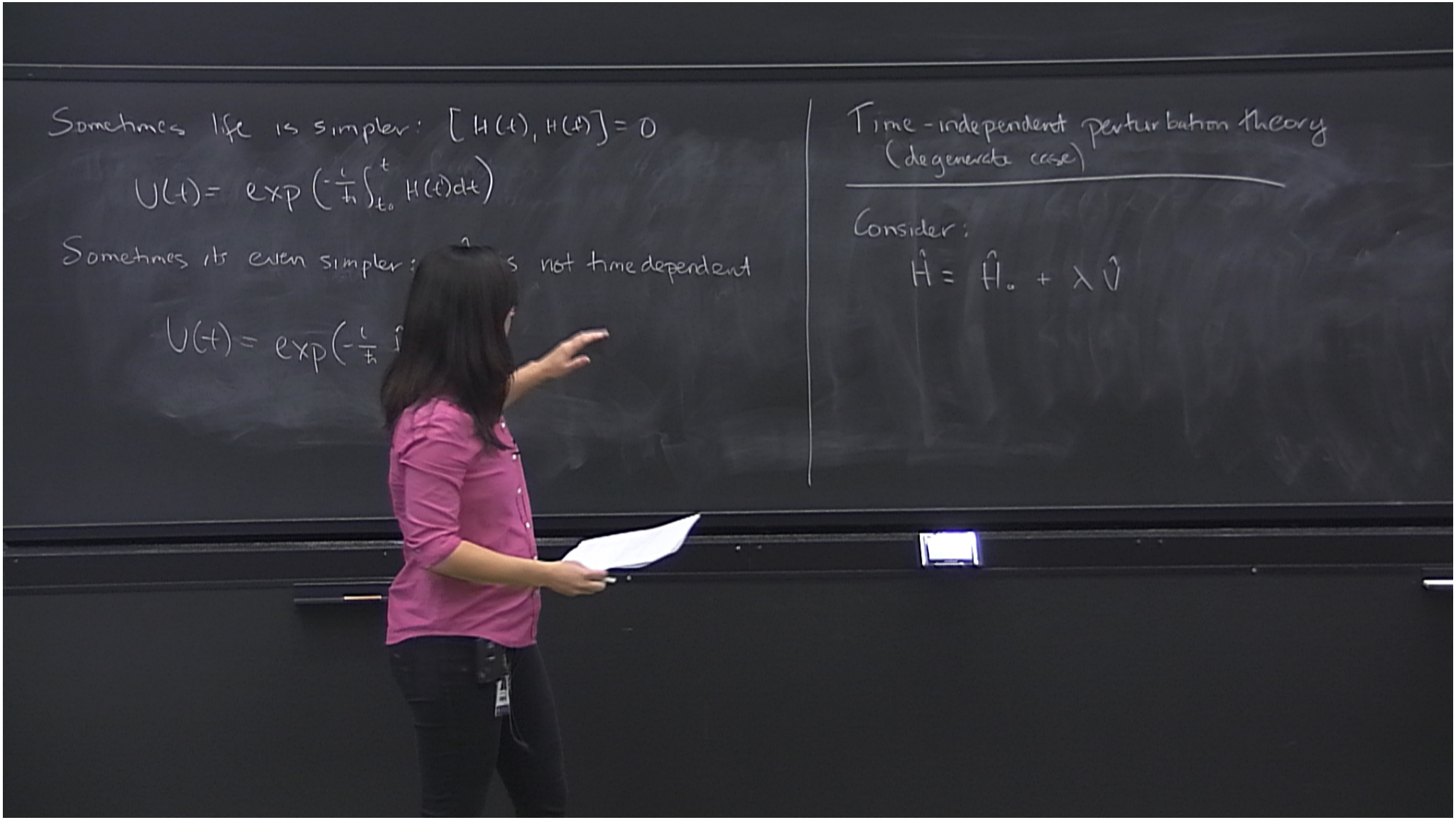
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Time-independent perturbation theory
(degenerate case)


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Consider:
$\hat{H} = \hat{H}_0 + \lambda \hat{V}$

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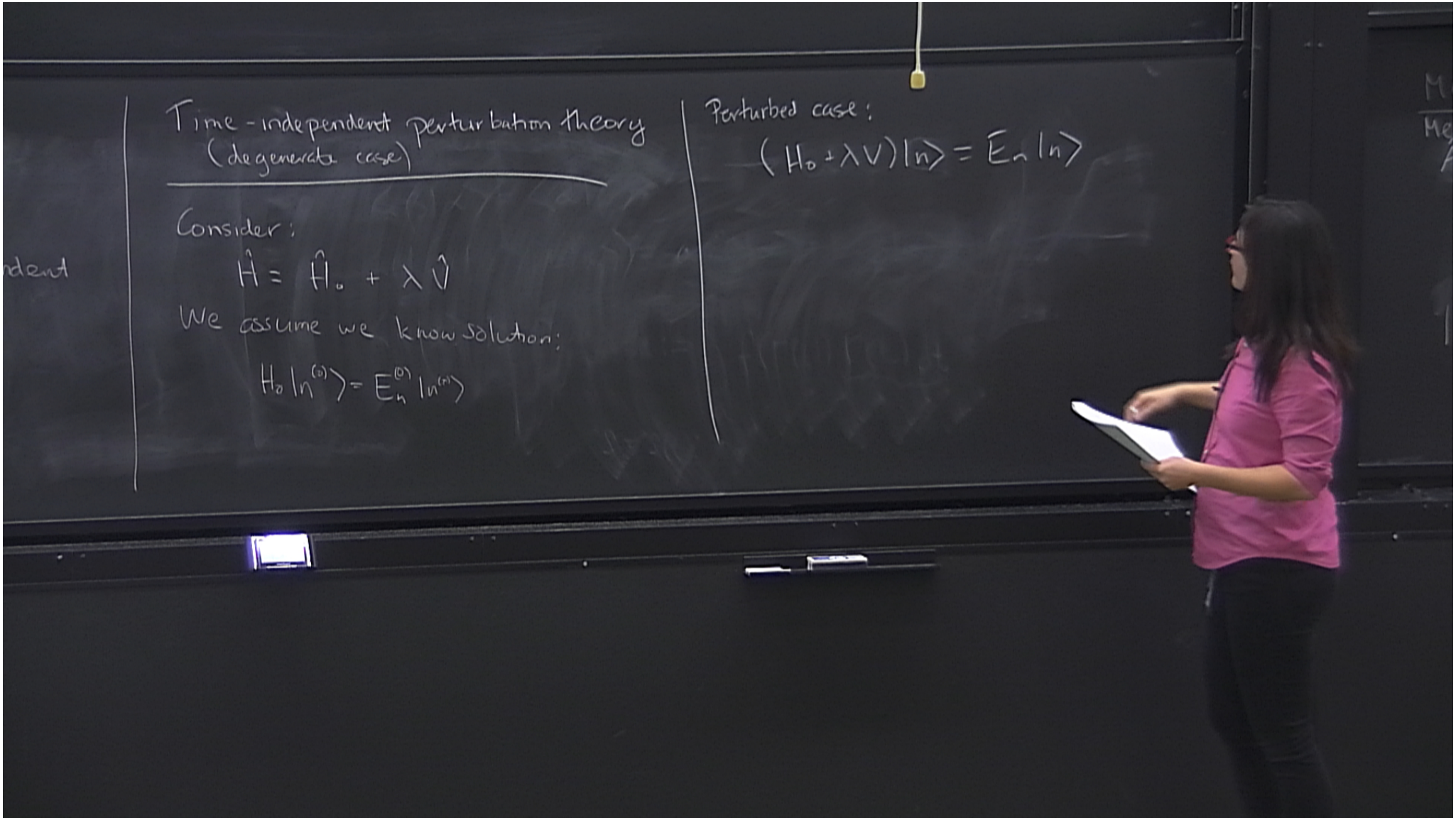
Time-independent perturbation theory (degenerate case)

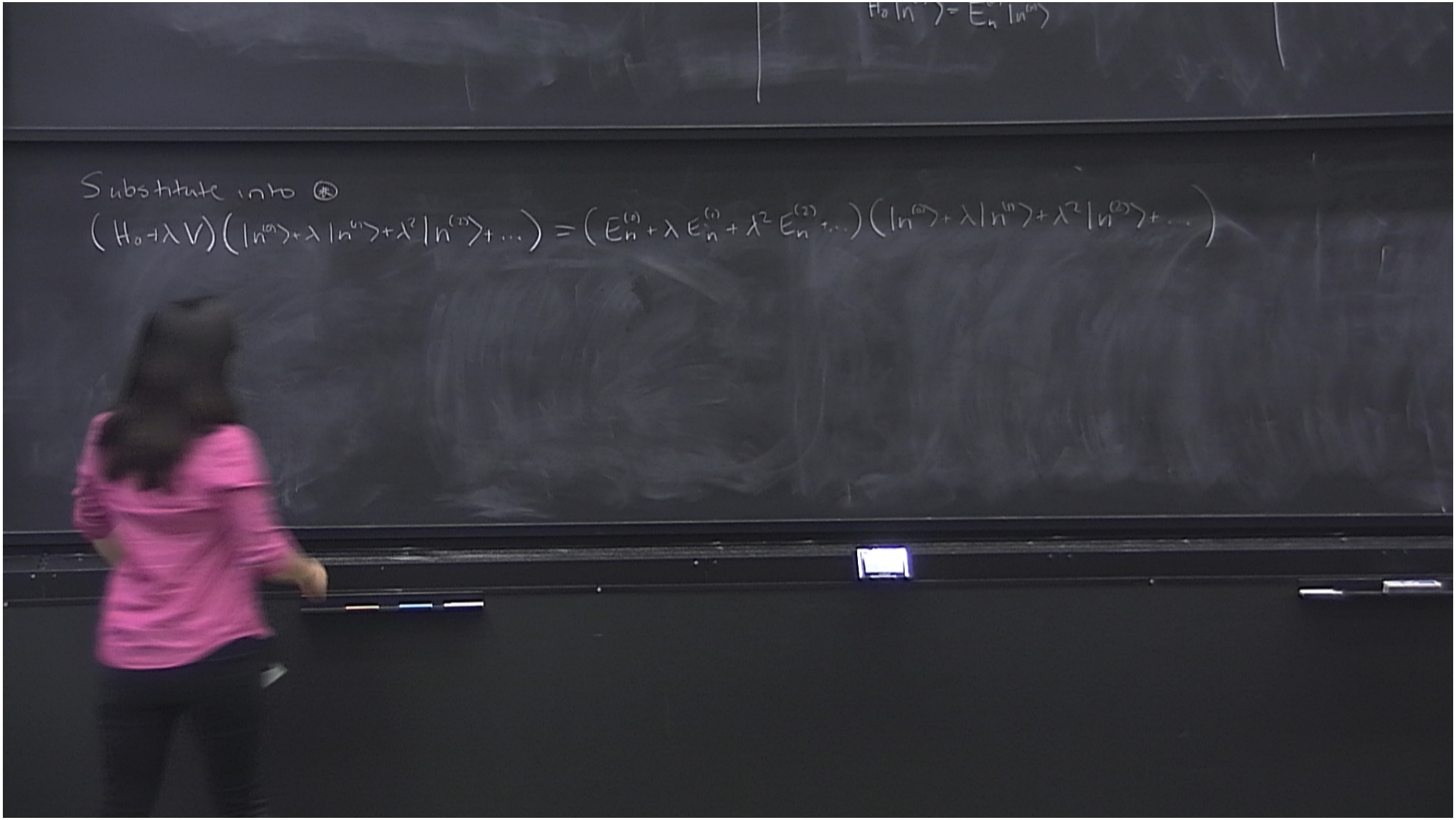
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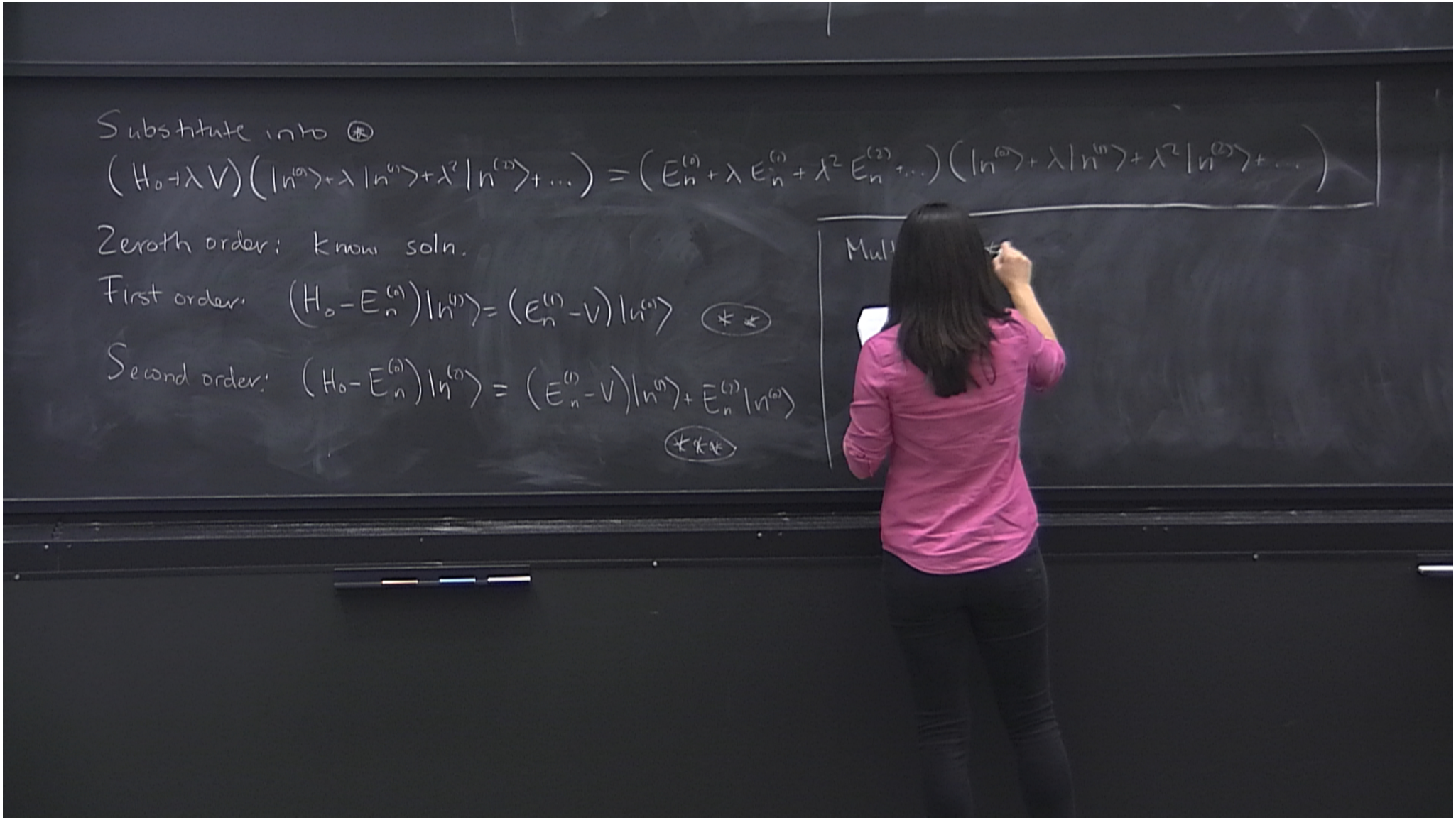
$$\hat{H} = \hat{H}_0 + \lambda\hat{V}$$

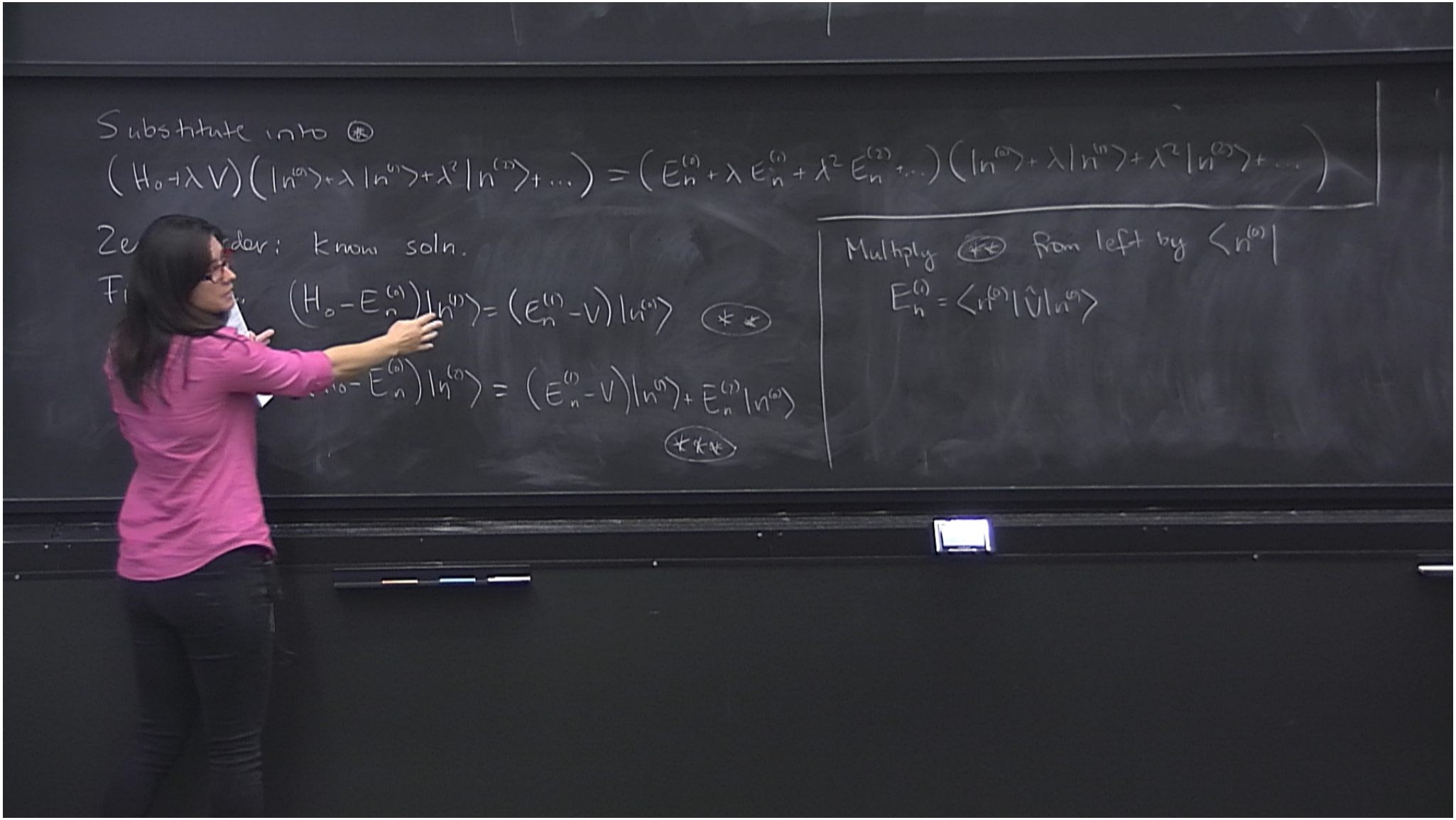
We assume we know solution:

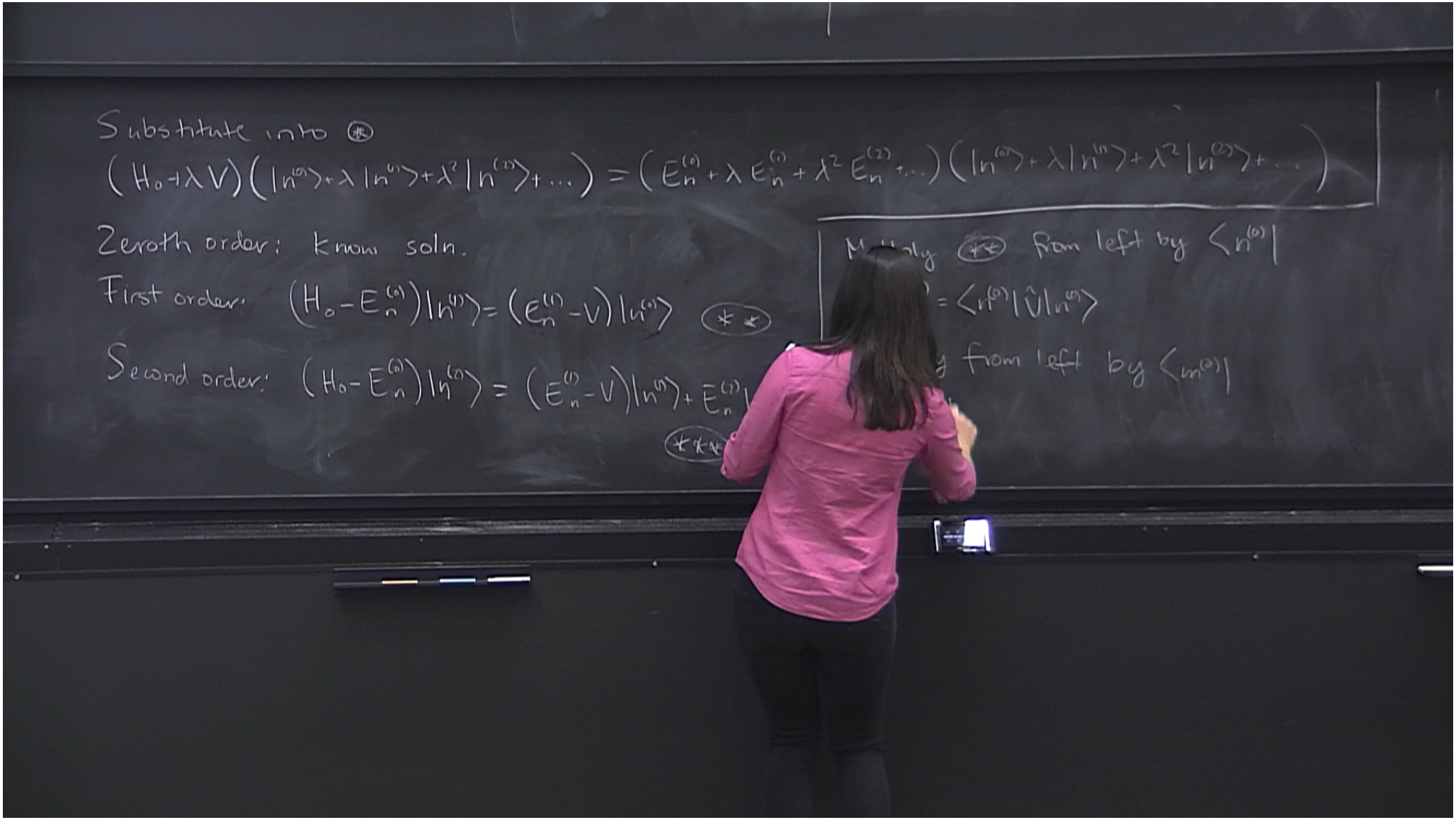
$$H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle$$


Time-independent perturbation theory
(degenerate case)
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We assume we know solution:
$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$
Perturbed case:
$(H_0 + \lambda V)|n\rangle = E_n |n\rangle$


Substitute into ⊛
$(H_0+\lambda V)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots)=(E_n^{(0)}+\lambda E_n^{(1)}+\lambda^2 E_n^{(2)}+\dots)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots)$


Substitute into (*)
$(H_0+\lambda V)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots) = (E_n^{(0)}+\lambda E_n^{(1)}+\lambda^2 E_n^{(2)}+\dots)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots)$
Zeroth order: known soln.
First order: $(H_0-E_n^{(0)})|n^{(1)}\rangle=(E_n^{(1)}-V)|n^{(0)}\rangle$ (**)
Second order: $(H_0-E_n^{(0)})|n^{(2)}\rangle=(E_n^{(1)}-V)|n^{(1)}\rangle+E_n^{(2)}|n^{(0)}\rangle$ (***)


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(***)
Multiply (**) from left by $\langle n^{(0)}|$
$E_n^{(1)} = \langle n^{(0)}|\hat{V}|n^{(0)}\rangle$


Substitute into (*)
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from left by $\langle n^{(0)}|$
$=\langle n^{(0)}|\hat{V}|n^{(0)}\rangle$
from left by $\langle m^{(0)}|$

Substitute into (*)

$$(H_0+\lambda V)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots)=(E_n^{(0)}+\lambda E_n^{(1)}+\lambda^2 E_n^{(2)}+\dots)(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\dots)$$

Zeroth order: known soln.

First order: $(H_0-E_n^{(0)})|n^{(1)}\rangle=(E_n^{(1)}-V)|n^{(0)}\rangle$ (**)

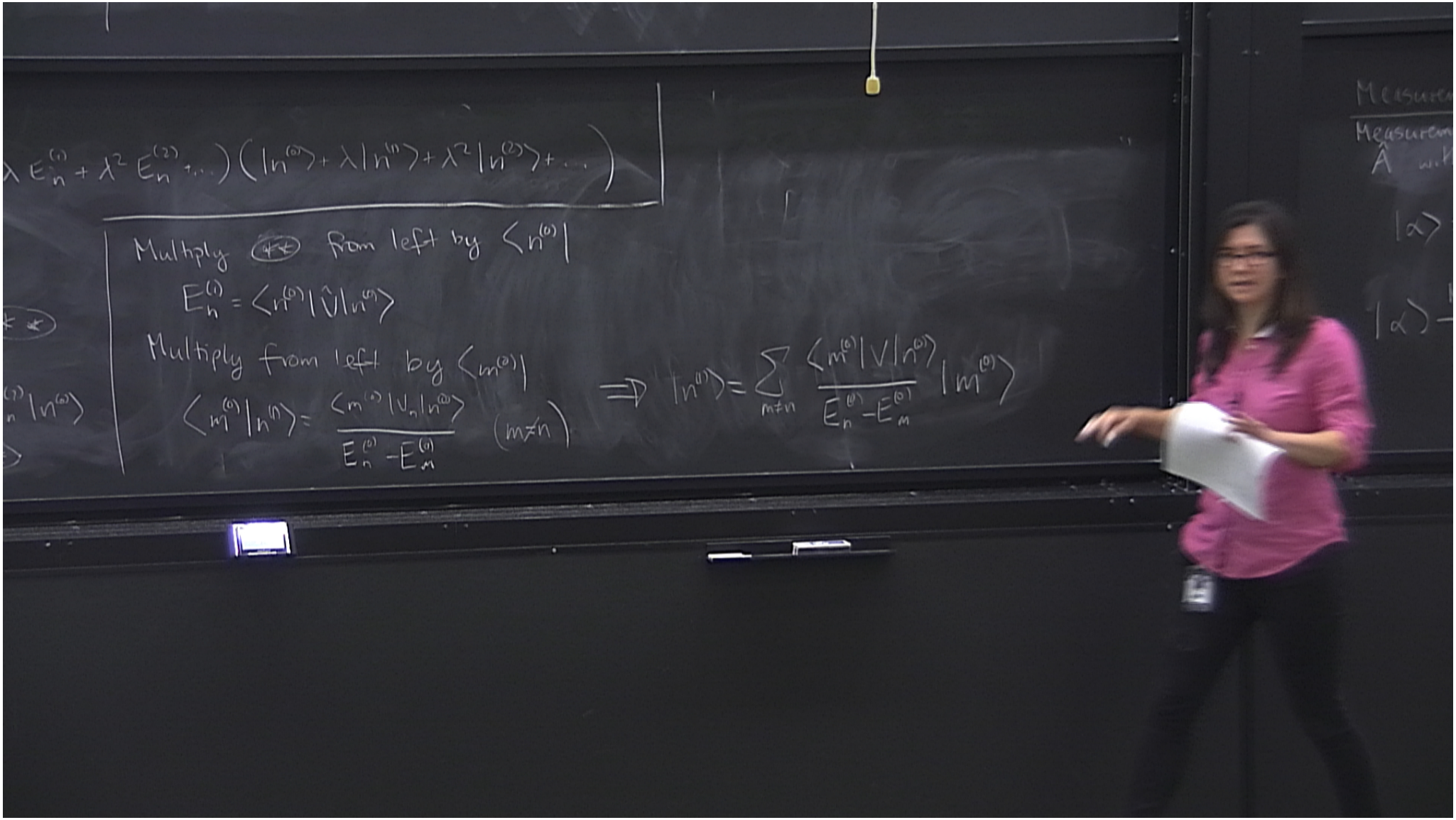
Second order: $(H_0-E_n^{(0)})|n^{(2)}\rangle=(E_n^{(1)}-V)|n^{(1)}\rangle+E_n^{(2)}|n^{(0)}\rangle$ (***)

Multiply (**) from left by $\langle n^{(0)}|$

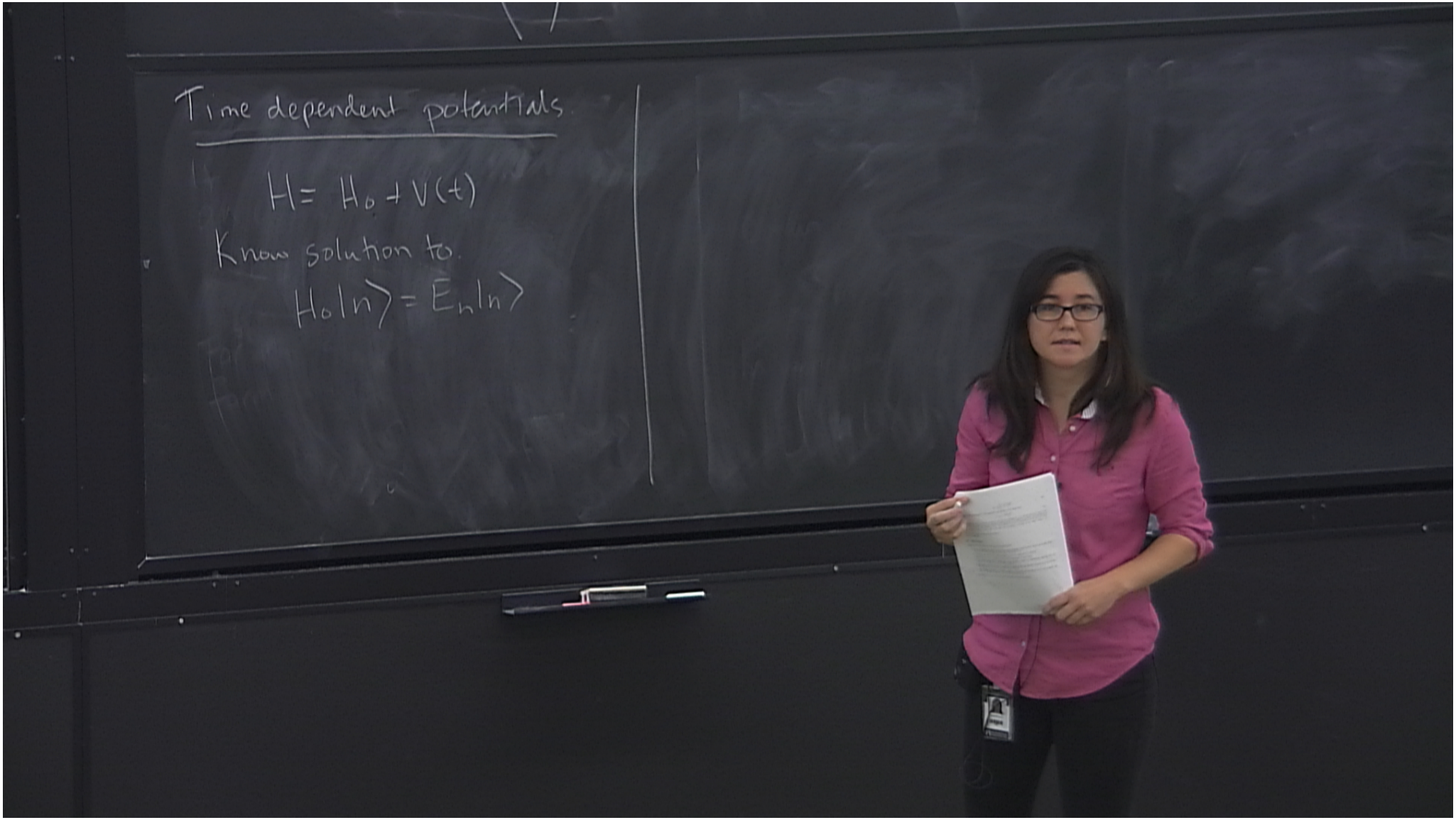
$$E_n^{(1)}=\langle n^{(0)}|\hat{V}|n^{(0)}\rangle$$

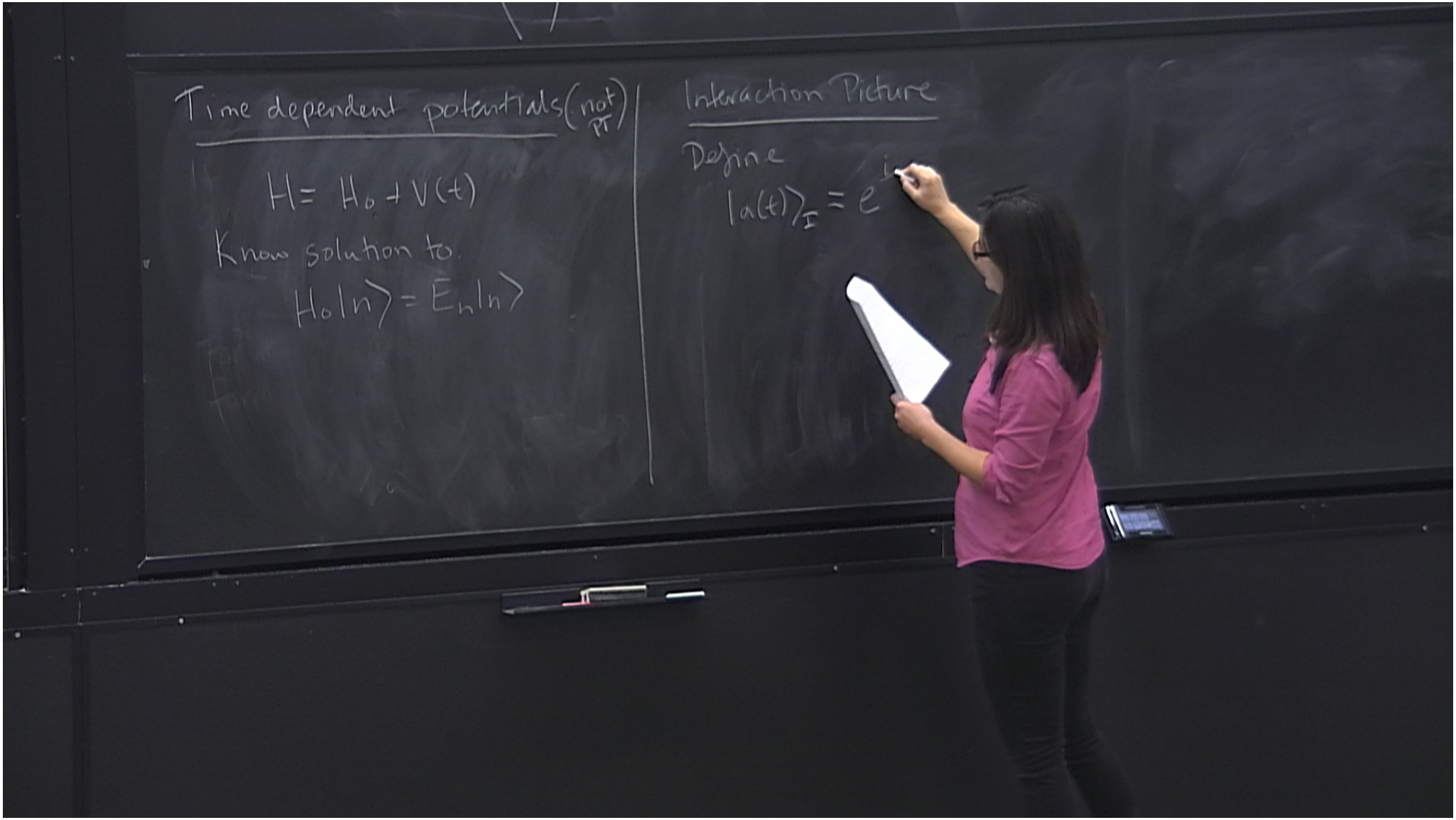
Multiply from left by $\langle m^{(0)}|$

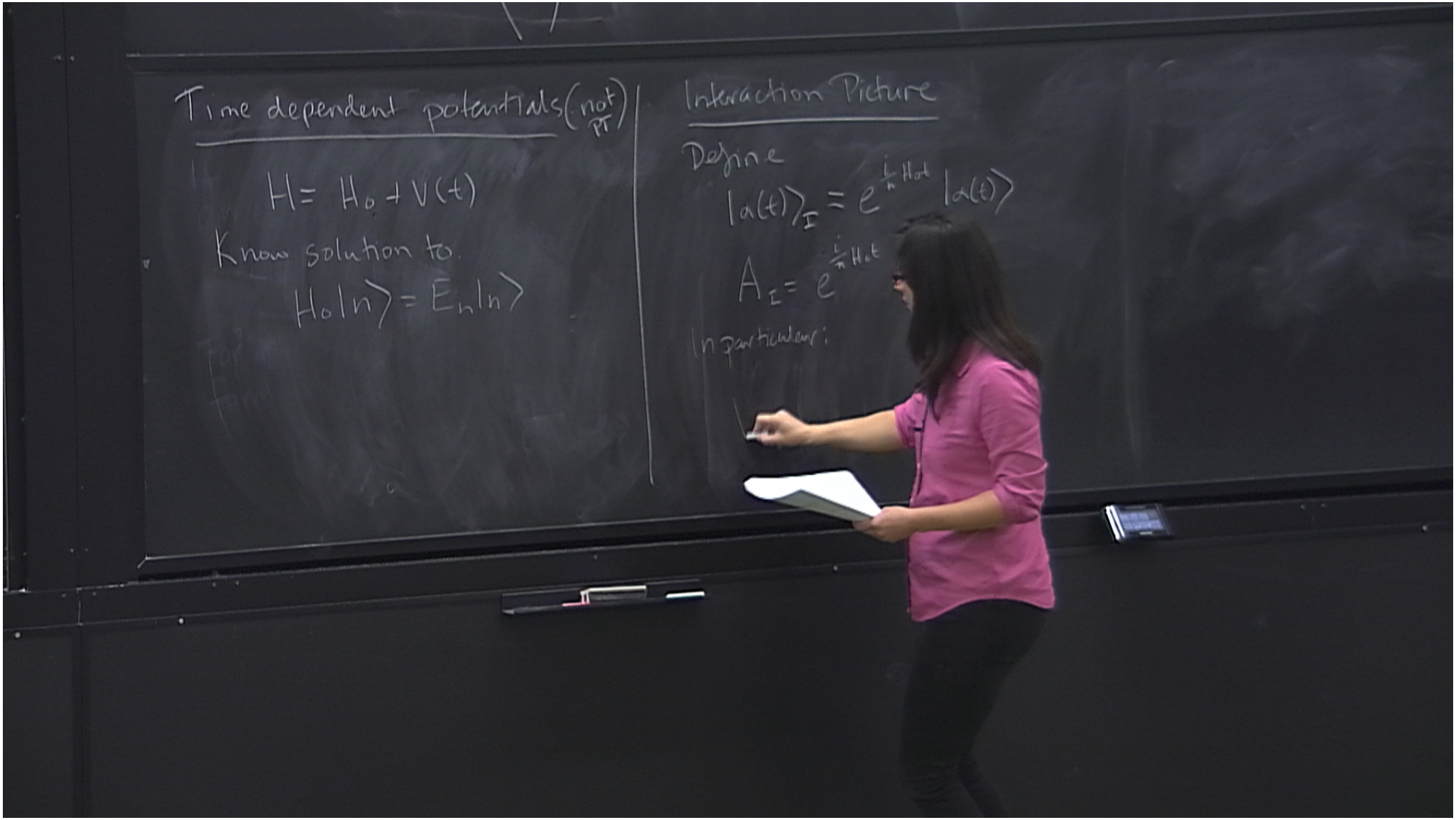
$$\langle m^{(0)}|n^{(1)}\rangle=\frac{\langle m^{(0)}|V_n|n^{(0)}\rangle}{E_n^{(0)}-E_m^{(0)}}\quad(m\neq n)\quad\Rightarrow\quad|n^{(1)}\rangle=$$

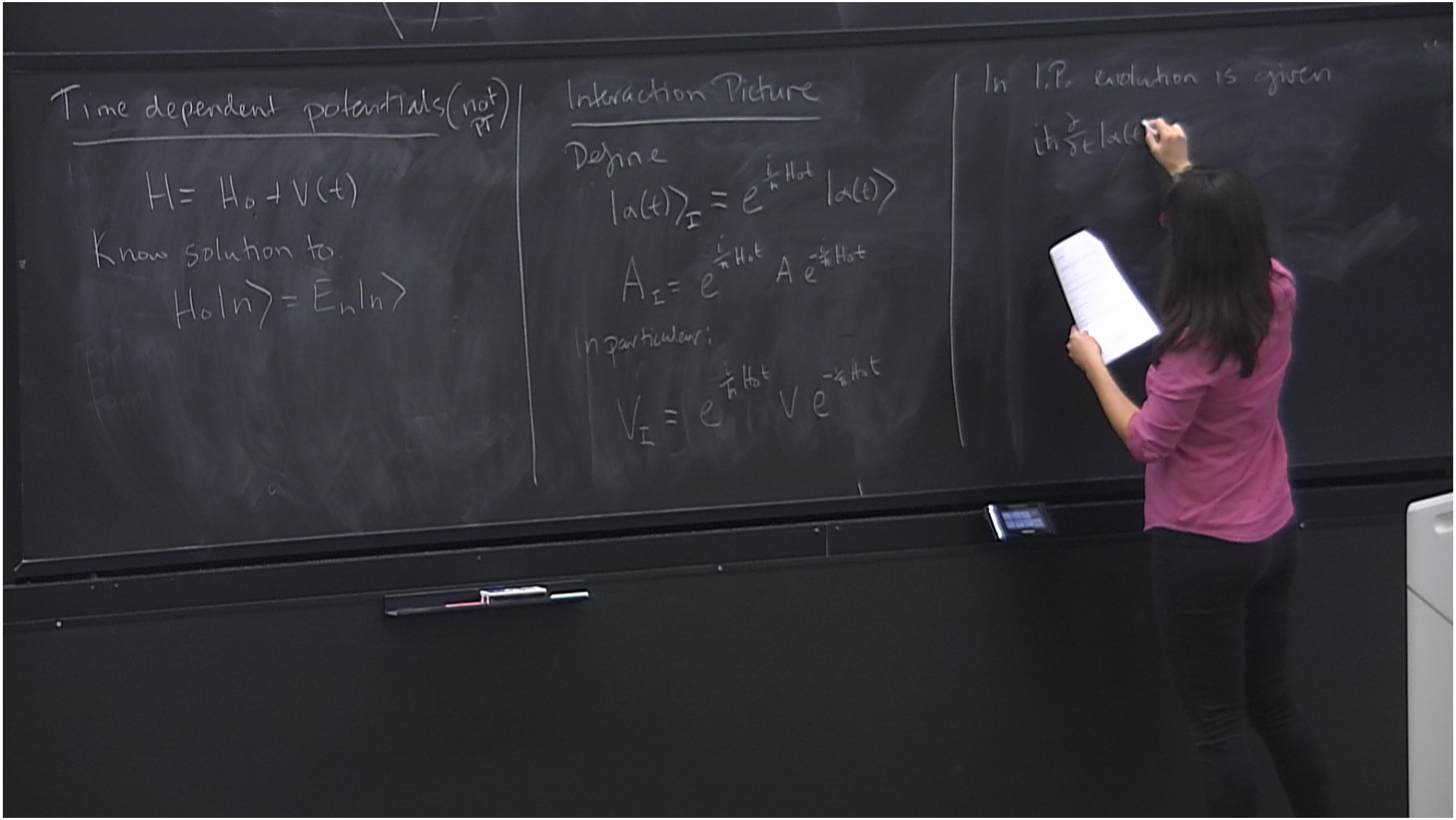

$\lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots)(|n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \lambda^2|n^{(2)}\rangle + \ldots)$
Multiply (**) from left by $\langle n^{(0)}|$
$E_n^{(1)} = \langle n^{(0)}|\hat{V}|n^{(0)}\rangle$
Multiply from left by $\langle m^{(0)}|$
$\langle m^{(0)}|n^{(1)}\rangle = \frac{\langle m^{(0)}|V_n|n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}$ $(m \neq n)$
$\Rightarrow |n^{(1)}\rangle = \sum_{m \neq n} \frac{\langle m^{(0)}|V|n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}} |m^{(0)}\rangle$

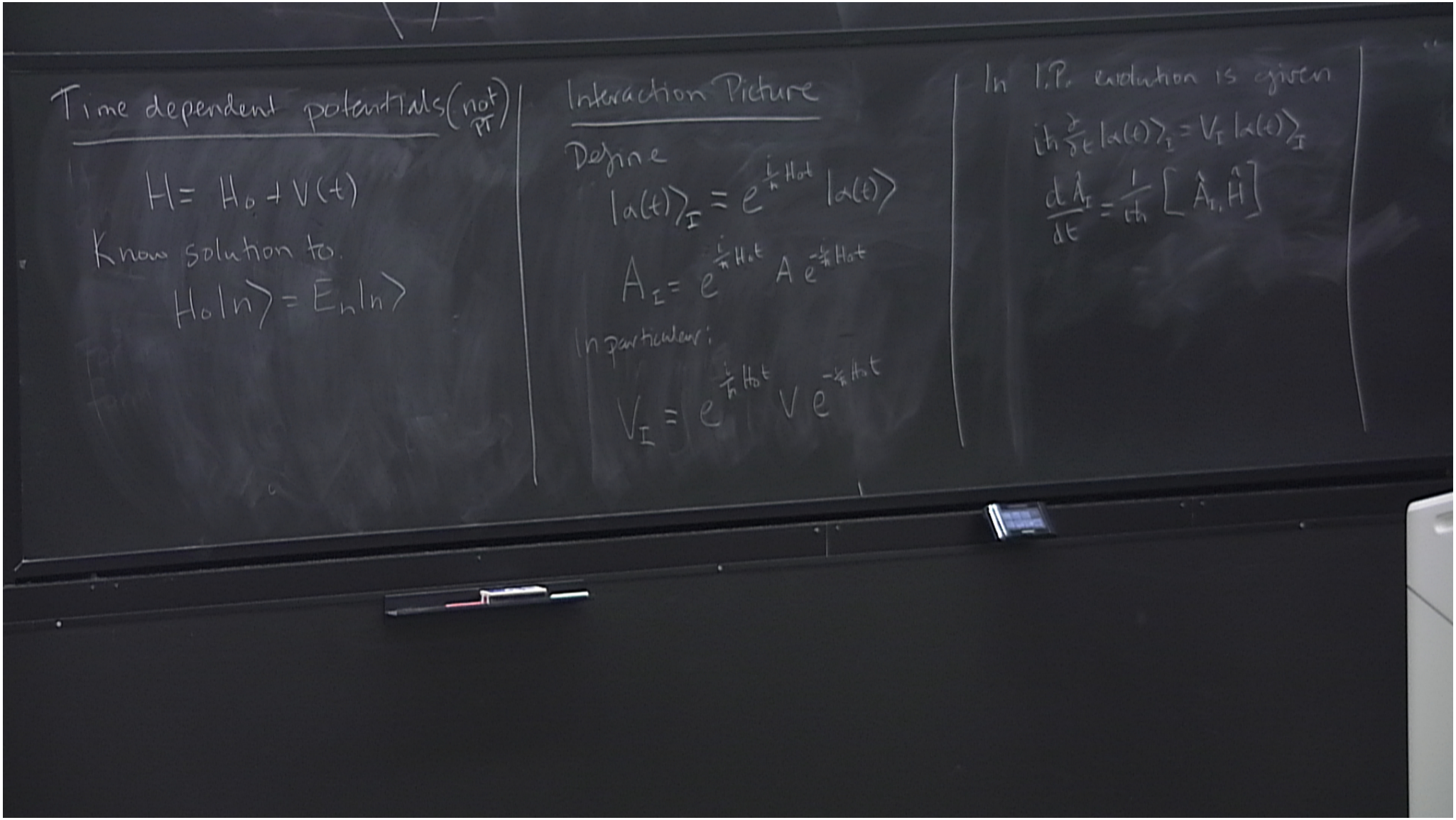

Time dependent potentials

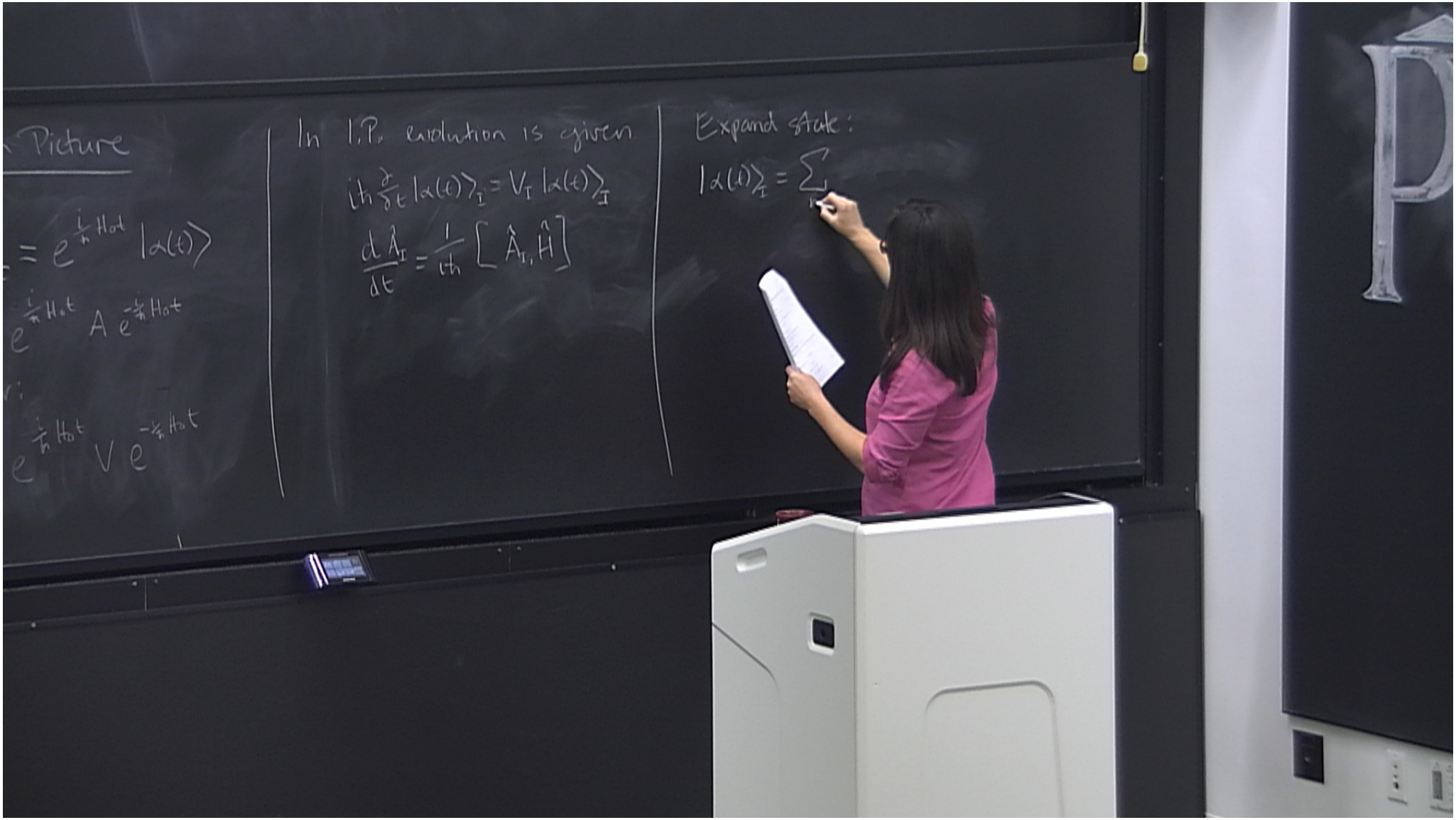

Time dependent potentials
$H = H_0 + V(t)$
Know solution to.
$H_0|n\rangle = E_n|n\rangle$

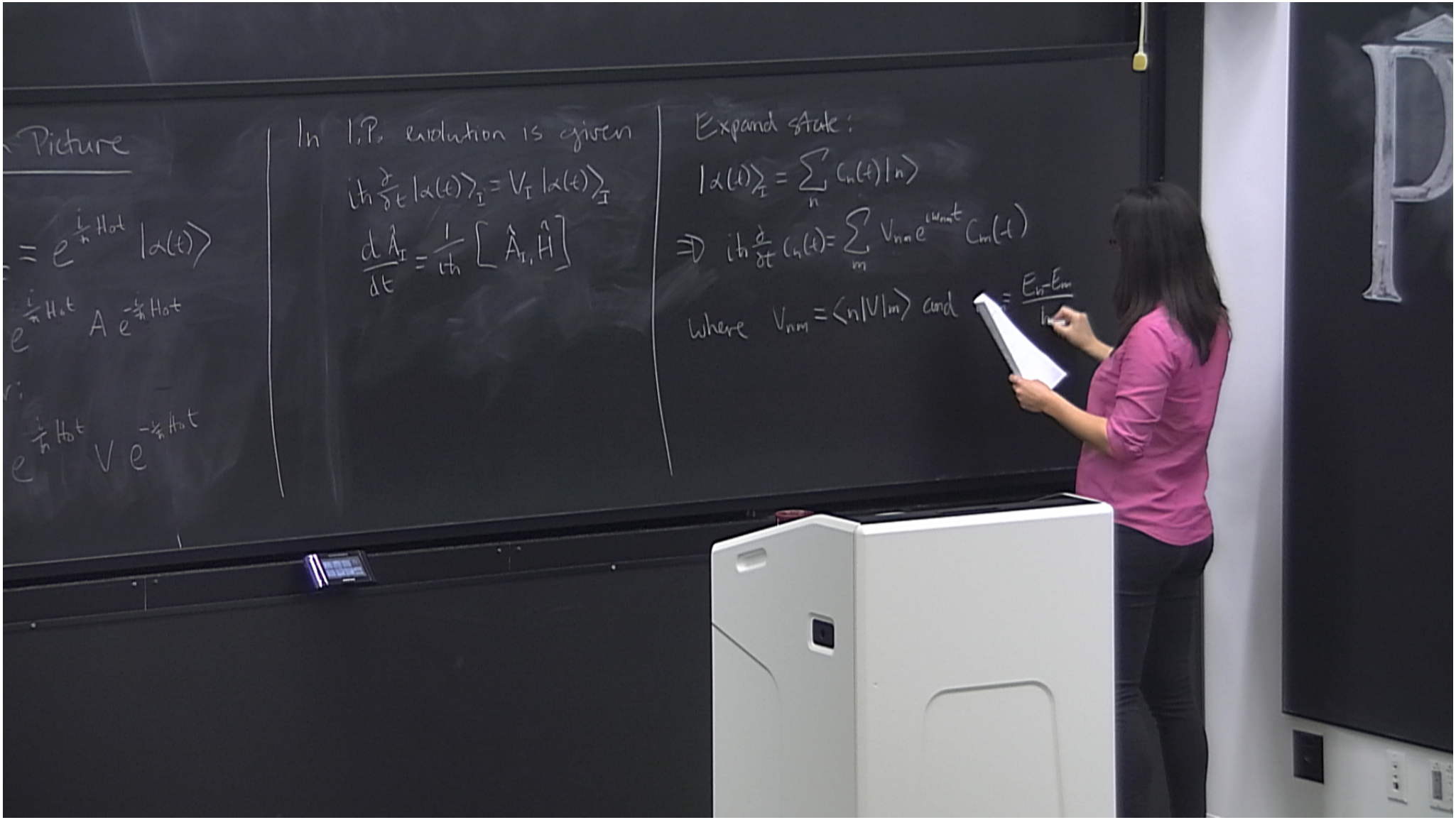

Time dependent potentials (not PT)
$H = H_0 + V(t)$
Know solution to
$H_0|n\rangle = E_n|n\rangle$
Interaction Picture
Define
$|\alpha(t)\rangle_I = e^{i}$

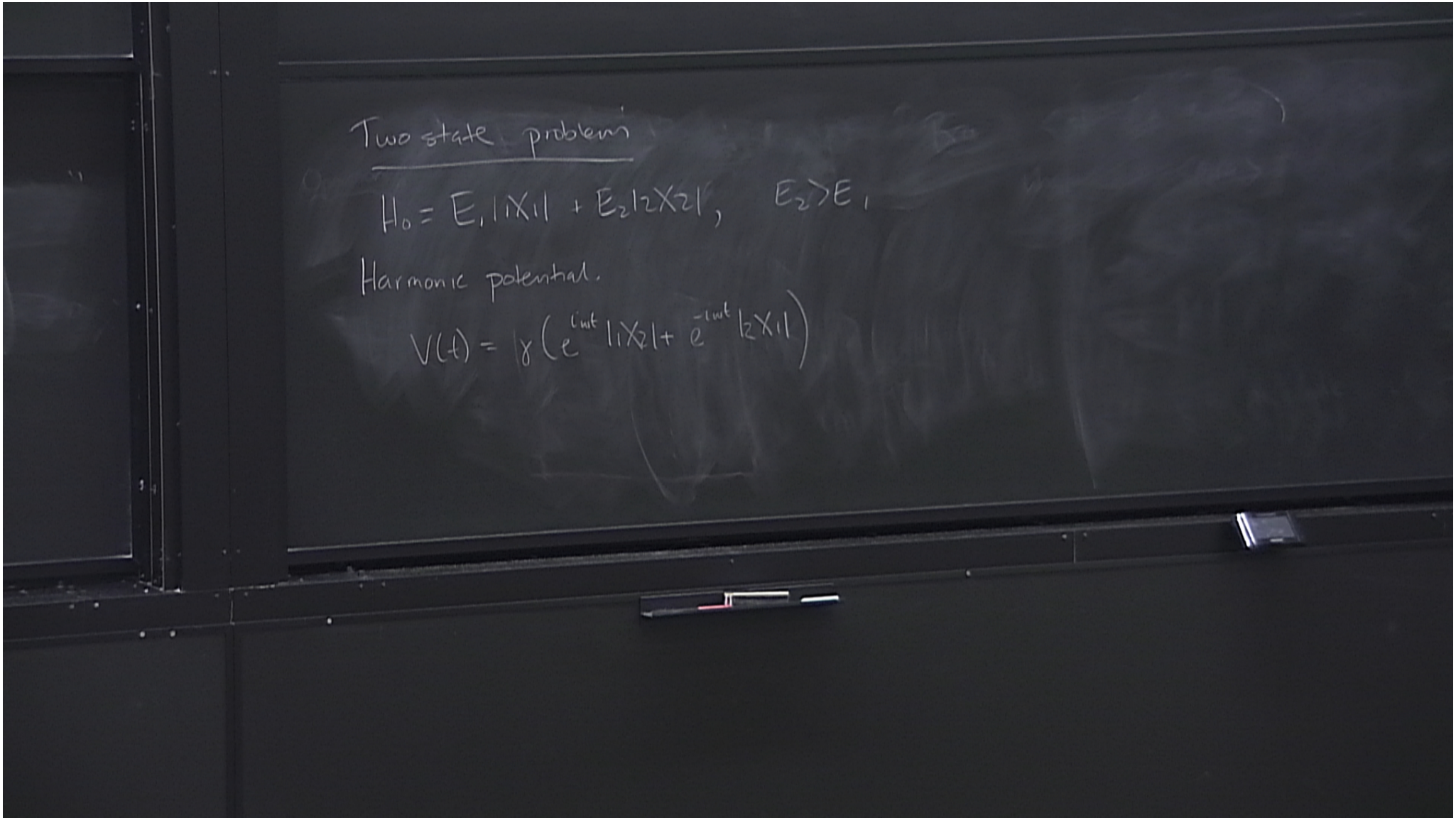

Time dependent potentials (not PT)
$H = H_0 + V(t)$
Know solution to.
$H_0|n\rangle = E_n|n\rangle$
Interaction Picture
Define
$|\alpha(t)\rangle_I = e^{\frac{i}{\hbar}H_0 t}|\alpha(t)\rangle$
$A_I = e^{\frac{i}{\hbar}H_0 t}$
In particular:


Time dependent potentials (not PT)
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Know solution to
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In particular:
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In I.P. evolution is given
$i\hbar\frac{\partial}{\partial t}|\alpha(t)$


Time dependent potentials (not PT)
$H = H_0 + V(t)$
Know solution to.
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In I.P. evolution is given
$i\hbar\frac{\partial}{\partial t}|\alpha(t)\rangle_I = V_I|\alpha(t)\rangle_I$
$\frac{d\hat{A}_I}{dt} = \frac{1}{i\hbar}[\hat{A}_I, \hat{H}]$


Picture
In I.P. evolution is given
Expand state:


Picture
In I.P. evolution is given
Expand state:
Where


Two state problem
$H_0 = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2|, \quad E_2 > E_1$
Harmonic potential.
$V(t) = \gamma\left(e^{i\omega t}|1\rangle\langle 2| + e^{-i\omega t}|2\rangle\langle 1|\right)$