Title: Complex Analysis

Date: Aug 11, 2015 03:30 PM

URL: http://pirsa.org/15080088

Abstract:

Tibra ~ Rm 274 tali@...

Complex Analysis

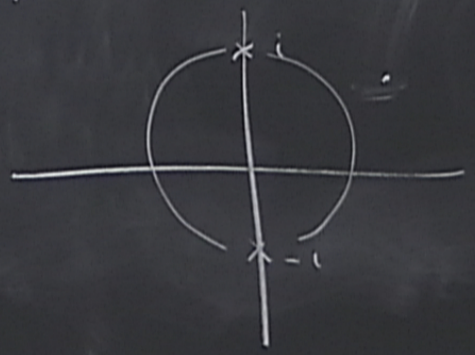
Spiegel, Theory & Problems of Complex Analysis

William T. Shaw. Cmplx Analysis with Mathematica®

Ablowitz & Fokas. Cmplx Analysis. Introduction & Applications

Ronaldo Cardano (1545)

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$$



Quantum Mechanics

$$i\hbar \frac{\partial \psi(x)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(x) + V(x)\psi(x)$$

$\hbar = 1$

$+i\nabla \longrightarrow i\nabla - eA$

$t \longrightarrow -t$

Complex conjugate $\} \psi^*(x)$

Bender and Wu (Phys. Rev. 184, 1231)

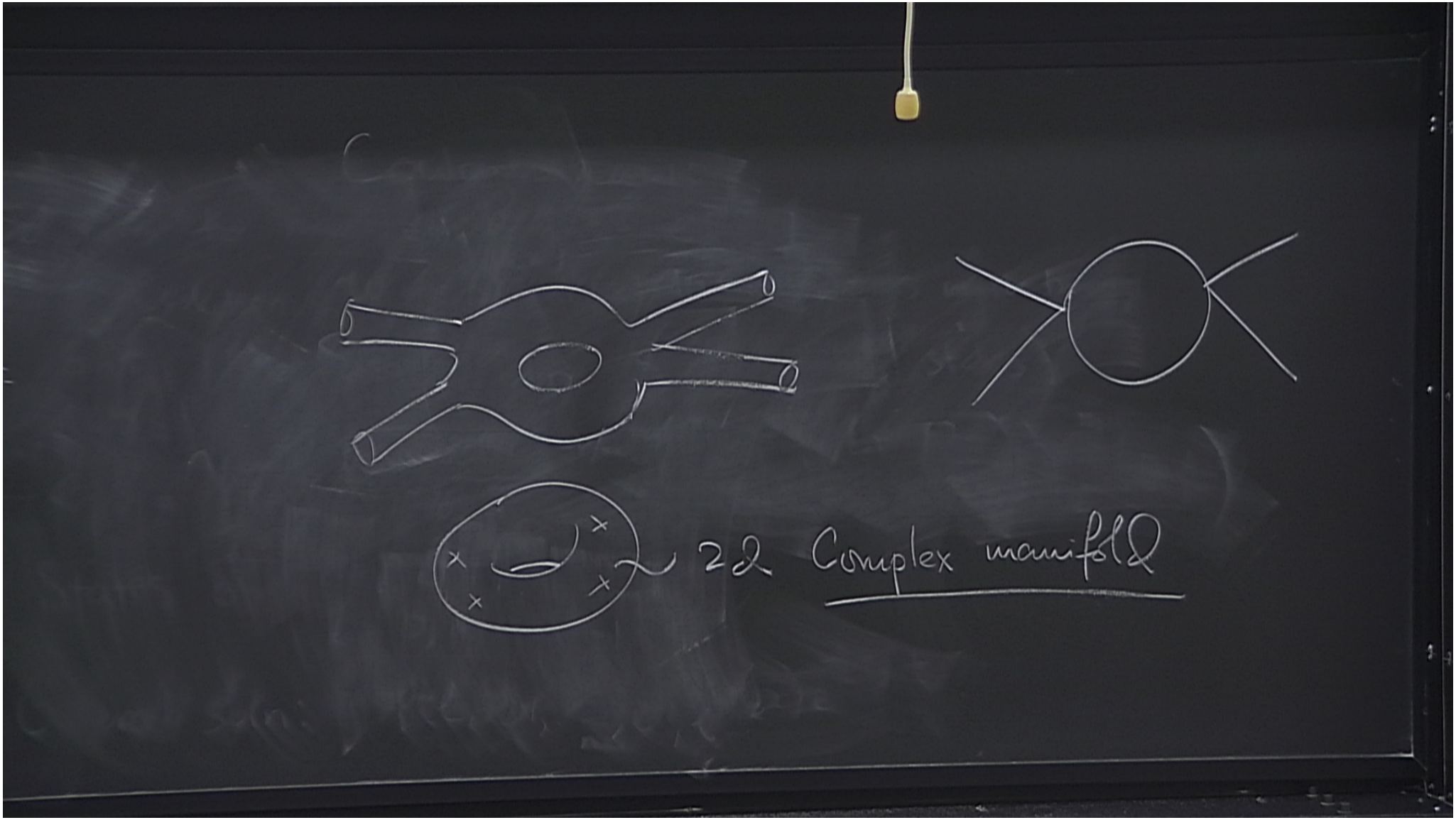
QFT

QR (Brill gr-qc/9507019)

Twistors

String Theory:

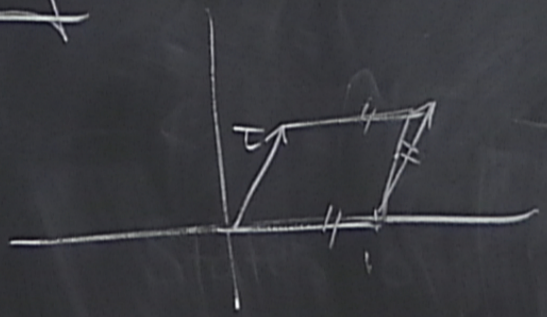


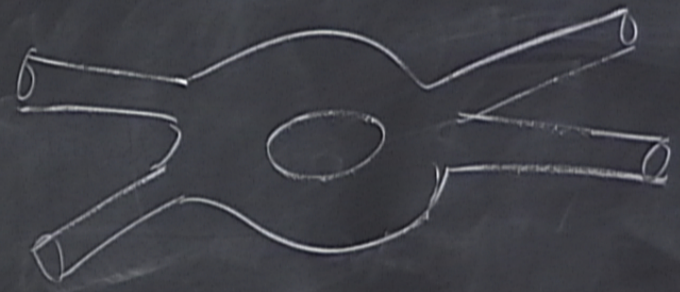

22 Complex manifold

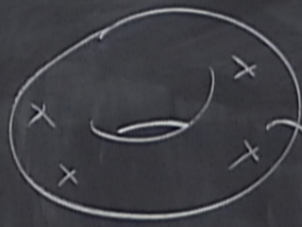
≠ Twistors

String Theory:

$Z = \int_{\tau} d\tau\, e^{-iS}$



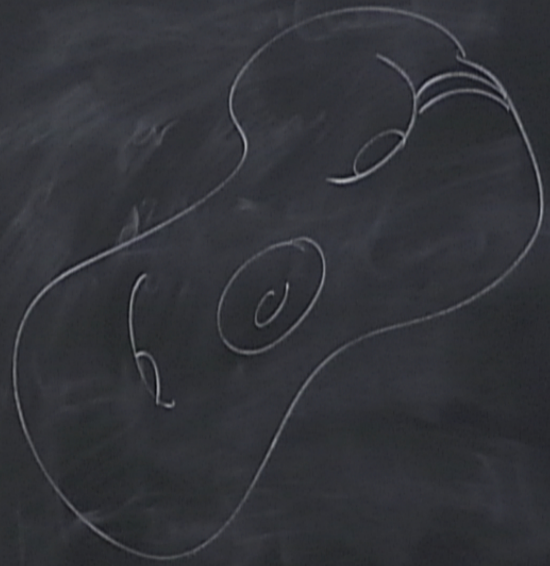


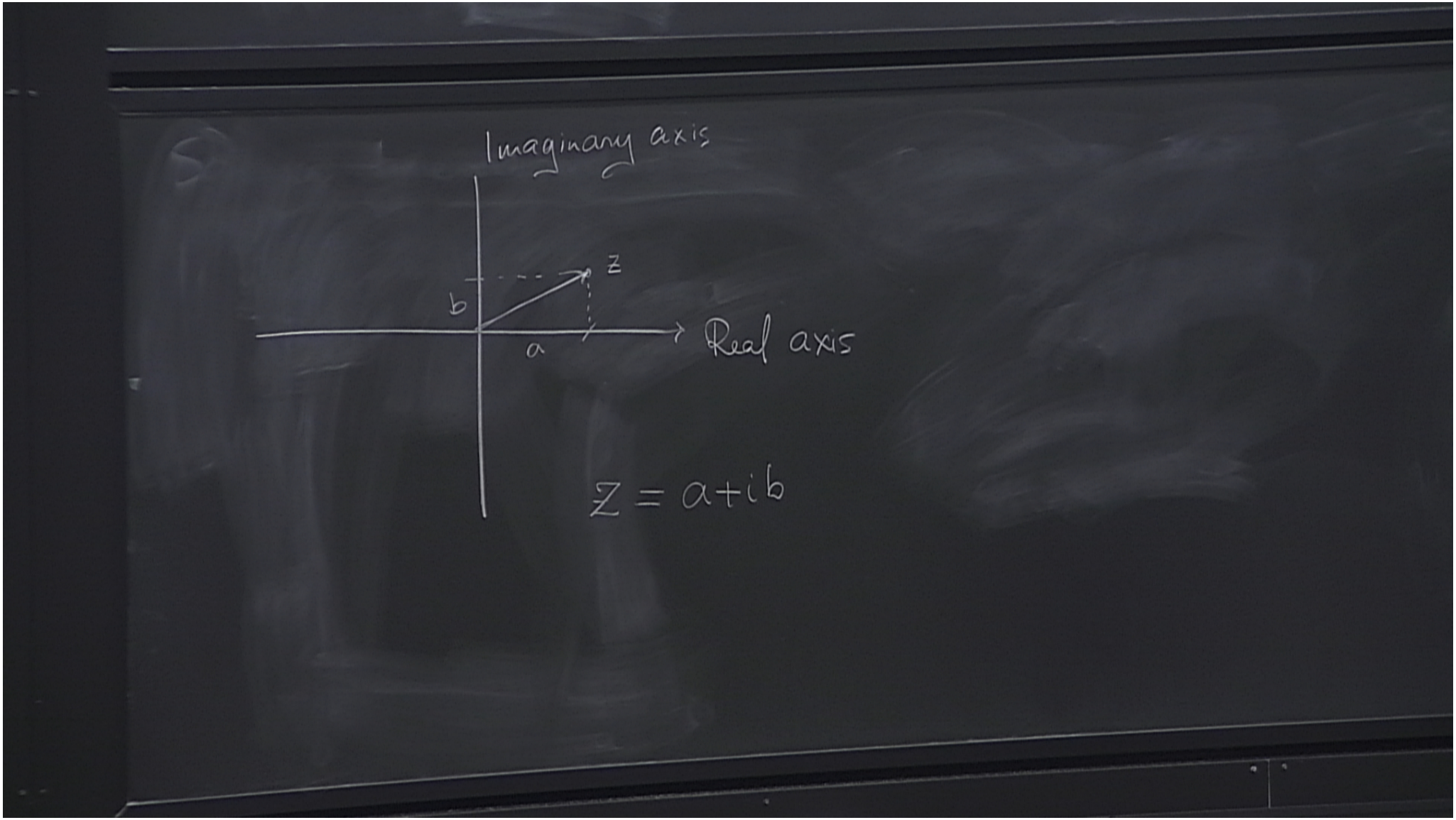


22 Con

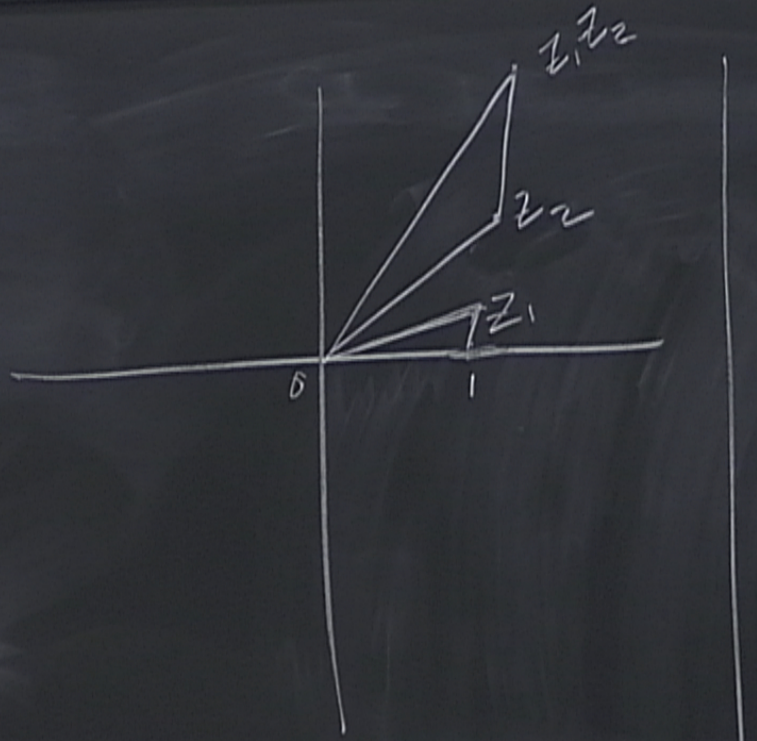
10D = 6D + 4D

↳ Calabi-Yau Manifolds



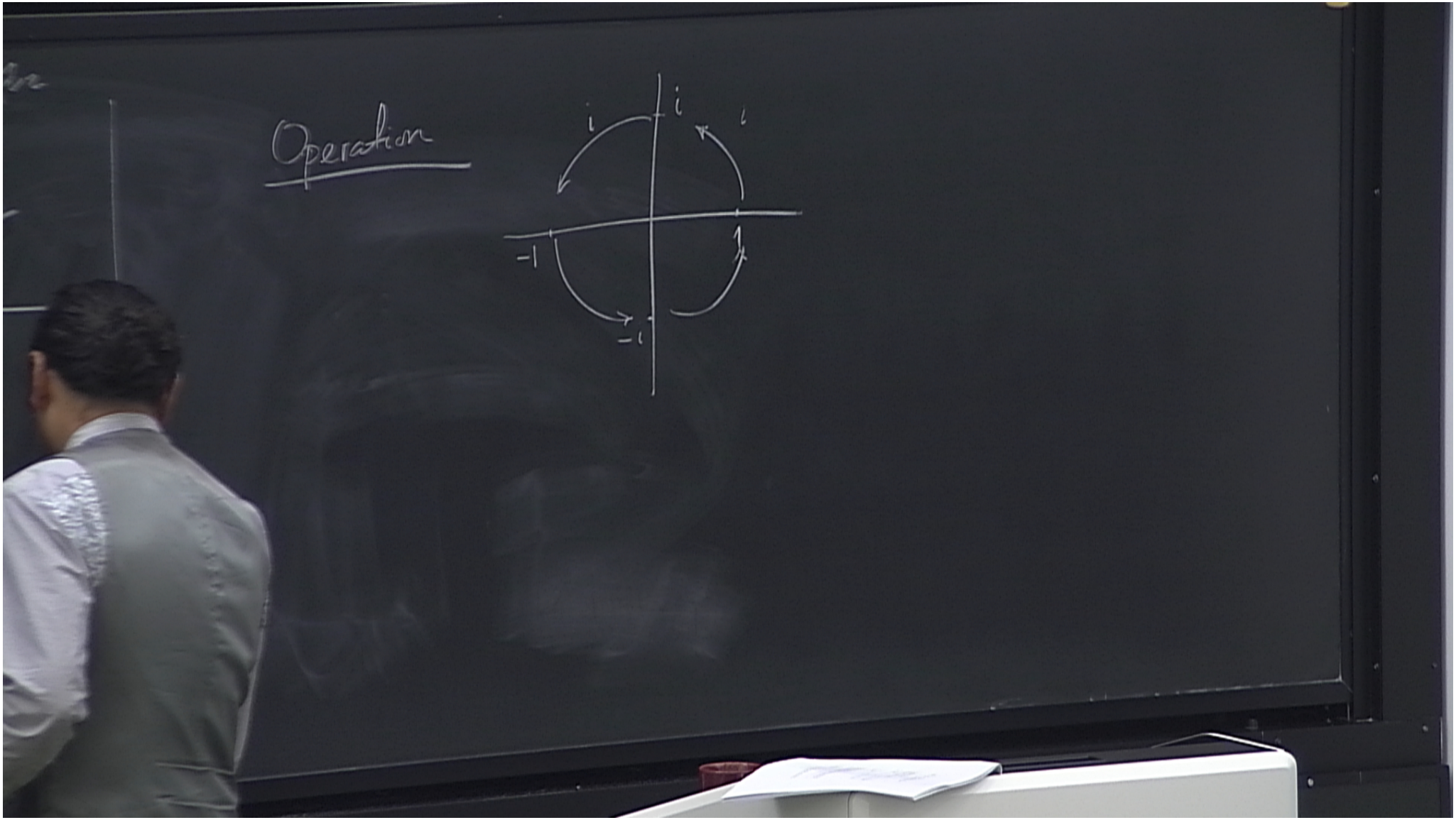

Imaginary axis
z
b
a
Real axis
Z = a+ib

$z_1 z_2$

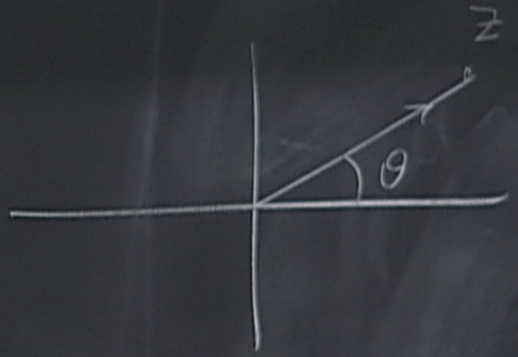


$(0, 1, z_1)$

$(0, z_2, z_1 z_2)$

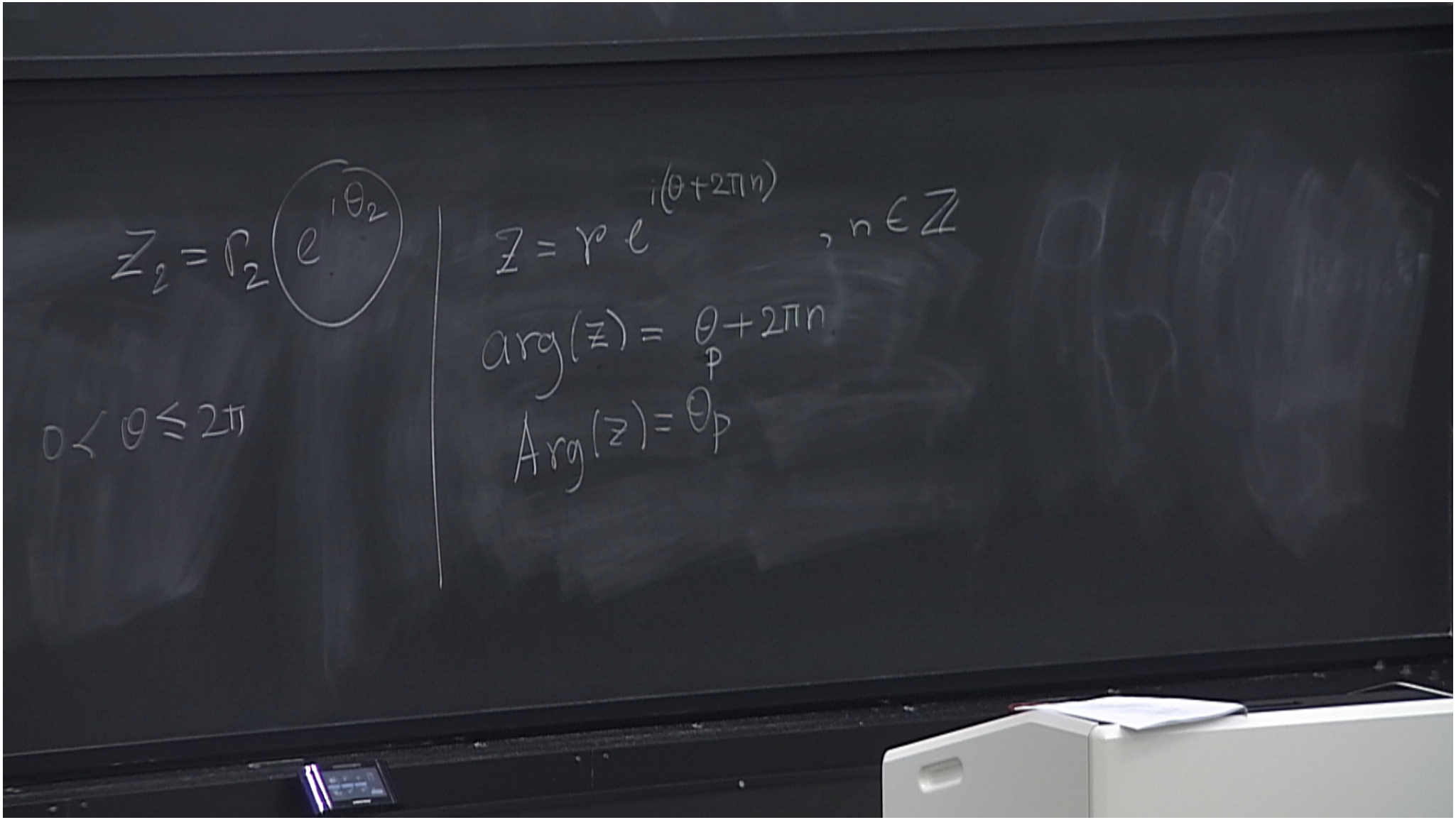

Operation
i
i
i
-1
1
-i

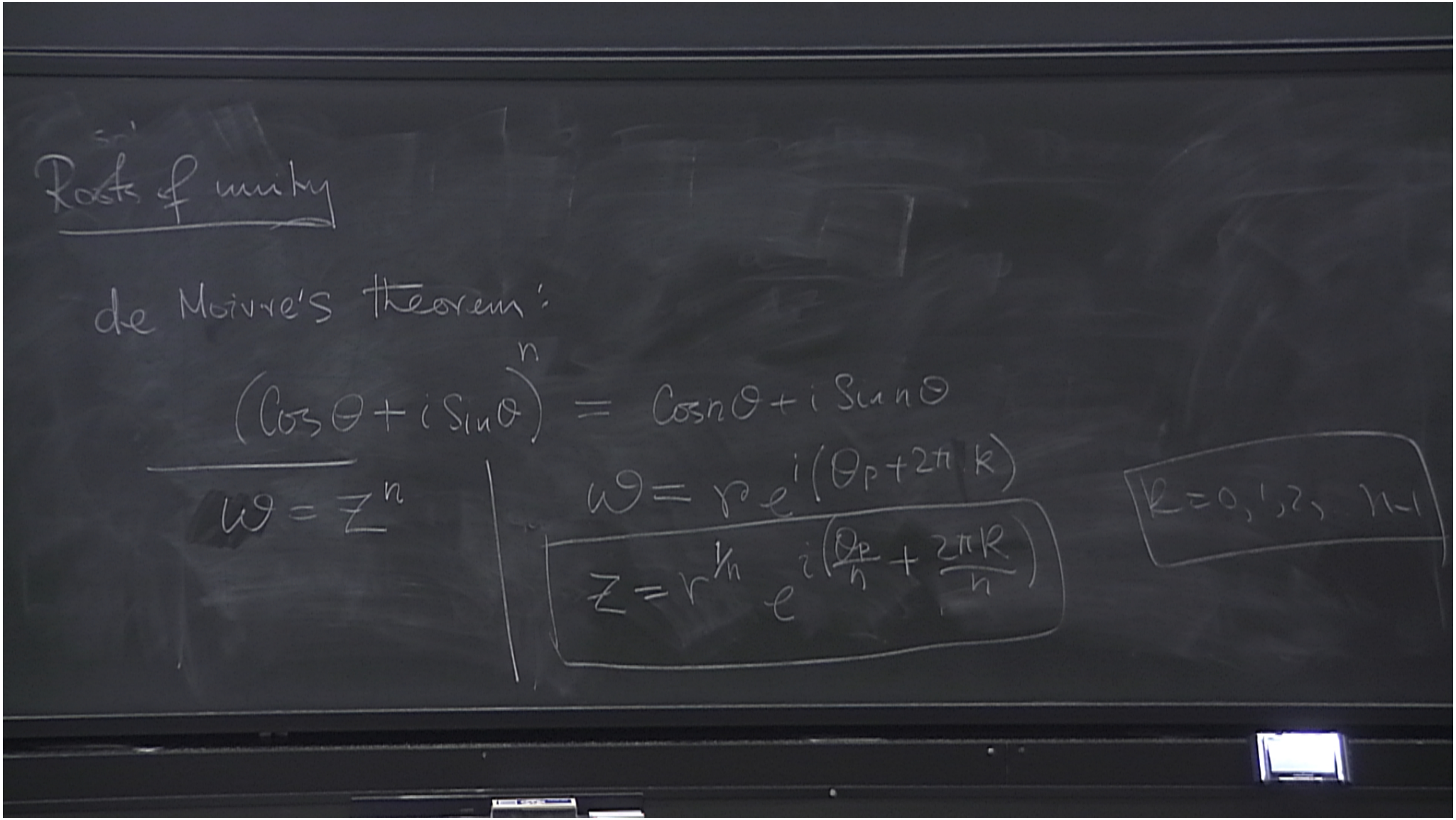
$$z = r(\cos\theta + i\sin\theta)$$



$$= re^{i\theta}$$

$$\Rightarrow \boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$ ~ Euler formula

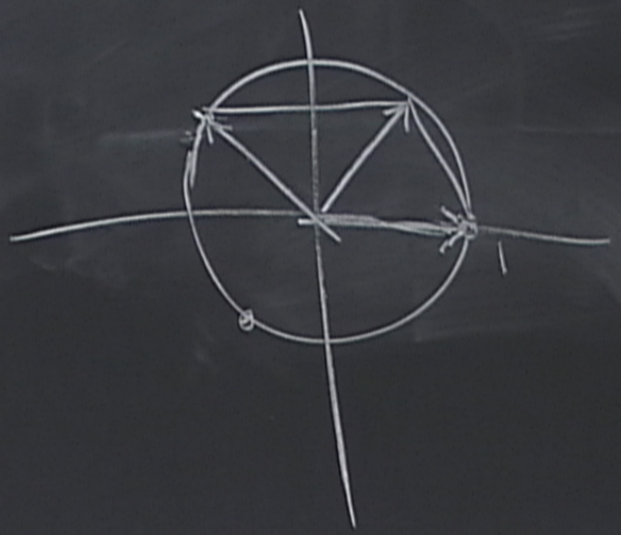

$z_2 = r_2 e^{i\theta_2}$
$0 < \theta \leq 2\pi$
$z = r e^{i(\theta + 2\pi n)}$, $n \in \mathbb{Z}$
$\arg(z) = \theta_p + 2\pi n$
$\mathrm{Arg}(z) = \theta_p$


Roots of unity
de Moivre's theorem:
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
$w = z^n$
$w = r e^{i(\theta_p + 2\pi k)}$
$z = r^{1/n} e^{i\left(\frac{\theta_p}{n} + \frac{2\pi k}{n}\right)}$
$k = 0, 1, 2, \ldots, n-1$

$z^3 = 1$

$z = 1, e^{2i\pi/3}, e^{4i\pi/3}$

$= \omega \qquad = \omega^2$



$1 + \omega = -\omega^2$

$1 + \omega + \omega^2 = 0$

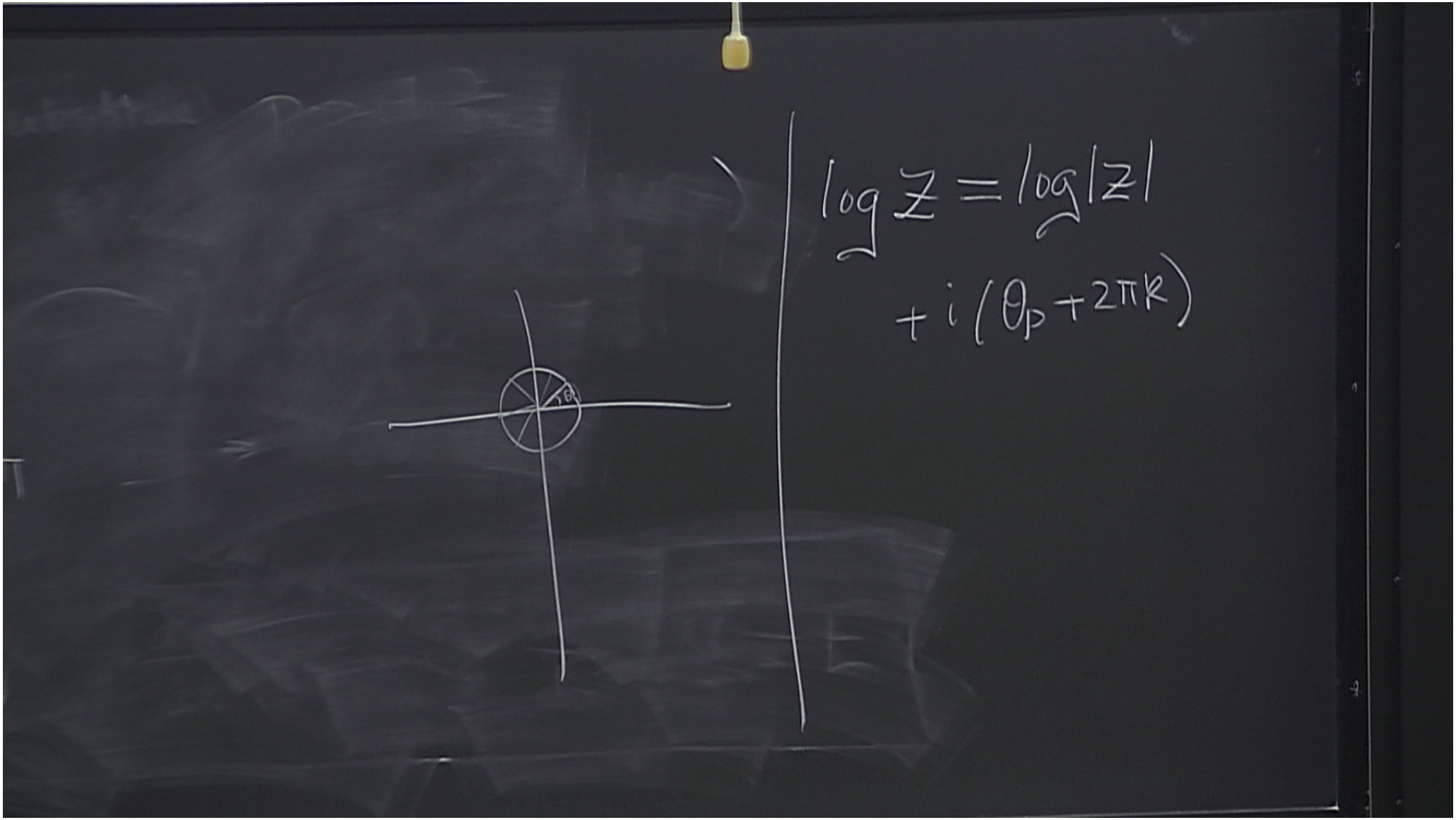
Riemann Surfaces

$$\arg(z) = \theta_p + 2\pi R$$

Riemann Surfaces

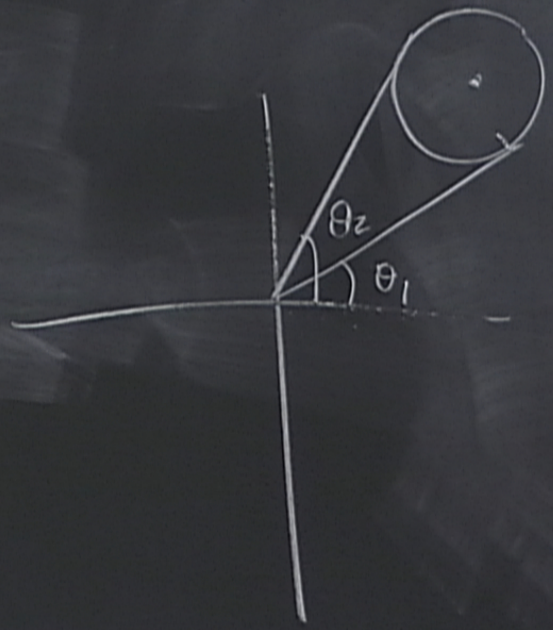
$$\arg(z) = \theta_p + 2\pi R$$

$$\sqrt{z} = \sqrt{r}\, e^{i\theta/2} = \sqrt{r}\, e^{i\frac{\theta_p}{2} + in\pi}$$

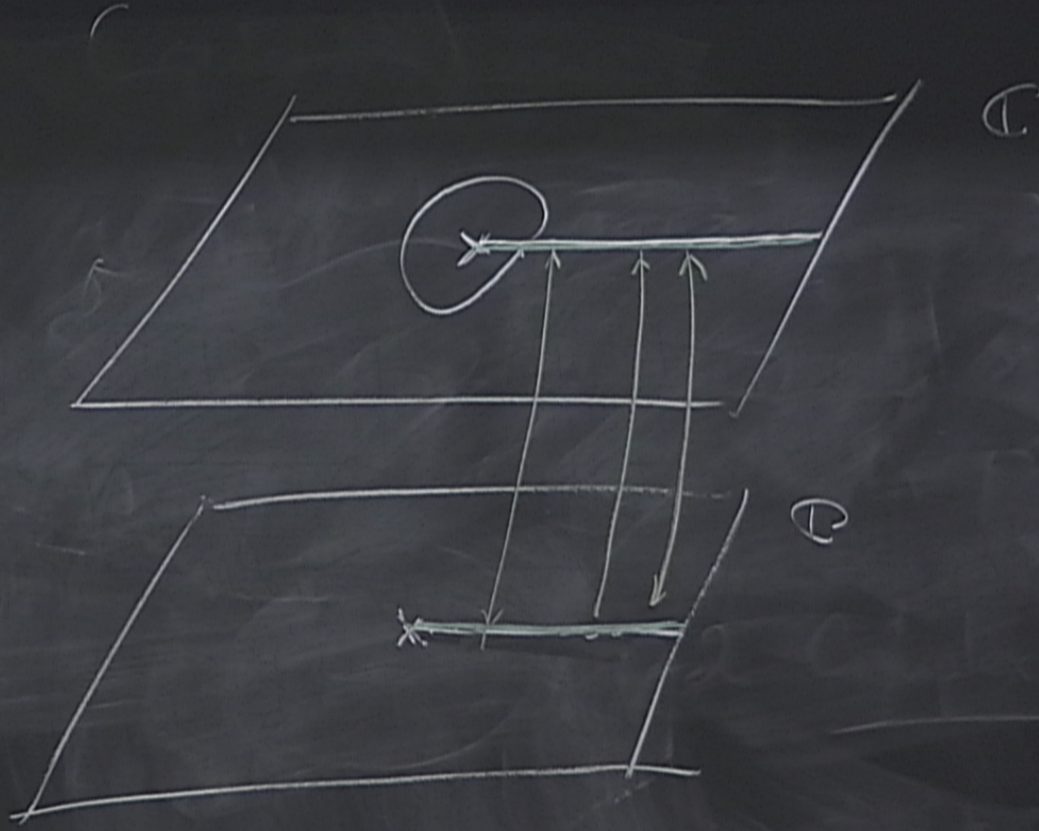

$\log z = \log|z|$
$+ i(\theta_p + 2\pi k)$

$\sqrt{z} = \sqrt{r}\, e^{i\theta/2}$ ~ monodromy around $z = 0$

$(z_0 - z) = z_0 - \epsilon e^{i\theta}$



Branch part

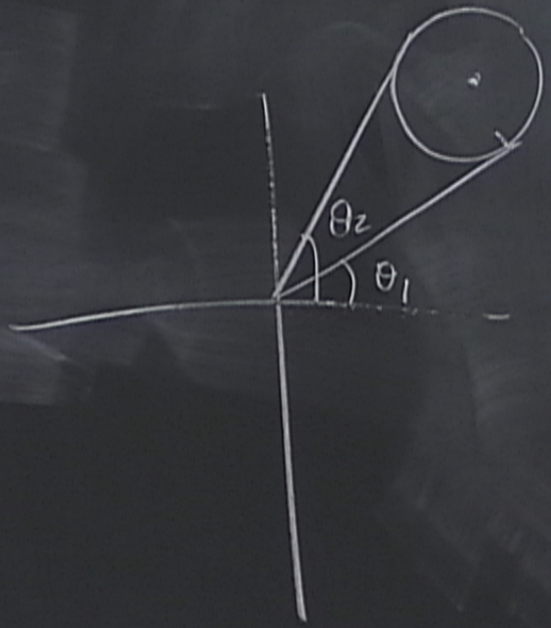


$$\sqrt{z} = \sqrt{r}\, e^{i\theta_p/2}\, e^{i\pi n}$$

$e^{i\theta/2}$

$-\epsilon e^{i\theta}$

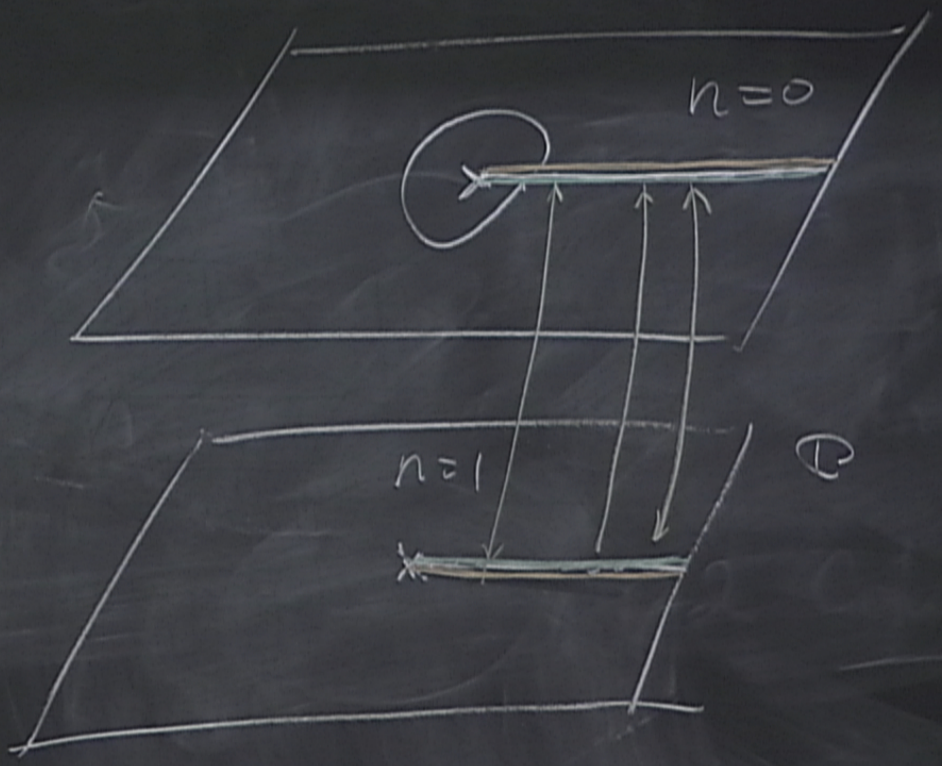
~ monodromy around $z = 0$



Branch pa

Branch point

Branch cut.

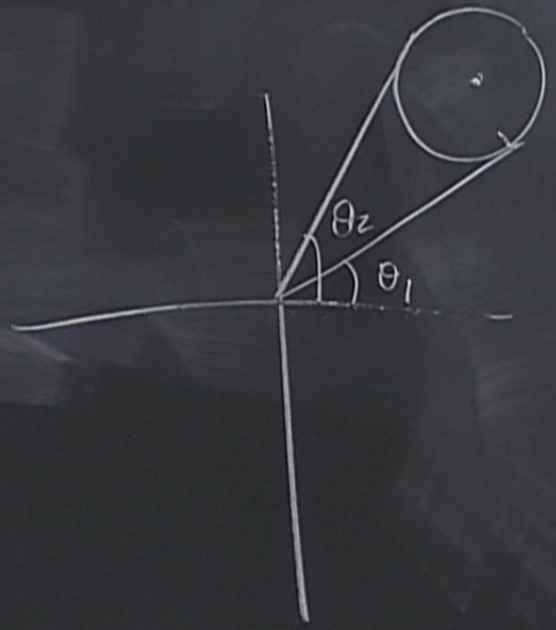


$$\sqrt{z} = \sqrt{r}\, e^{i\theta_p/2}\, e^{i\pi n}$$

$\theta/2$

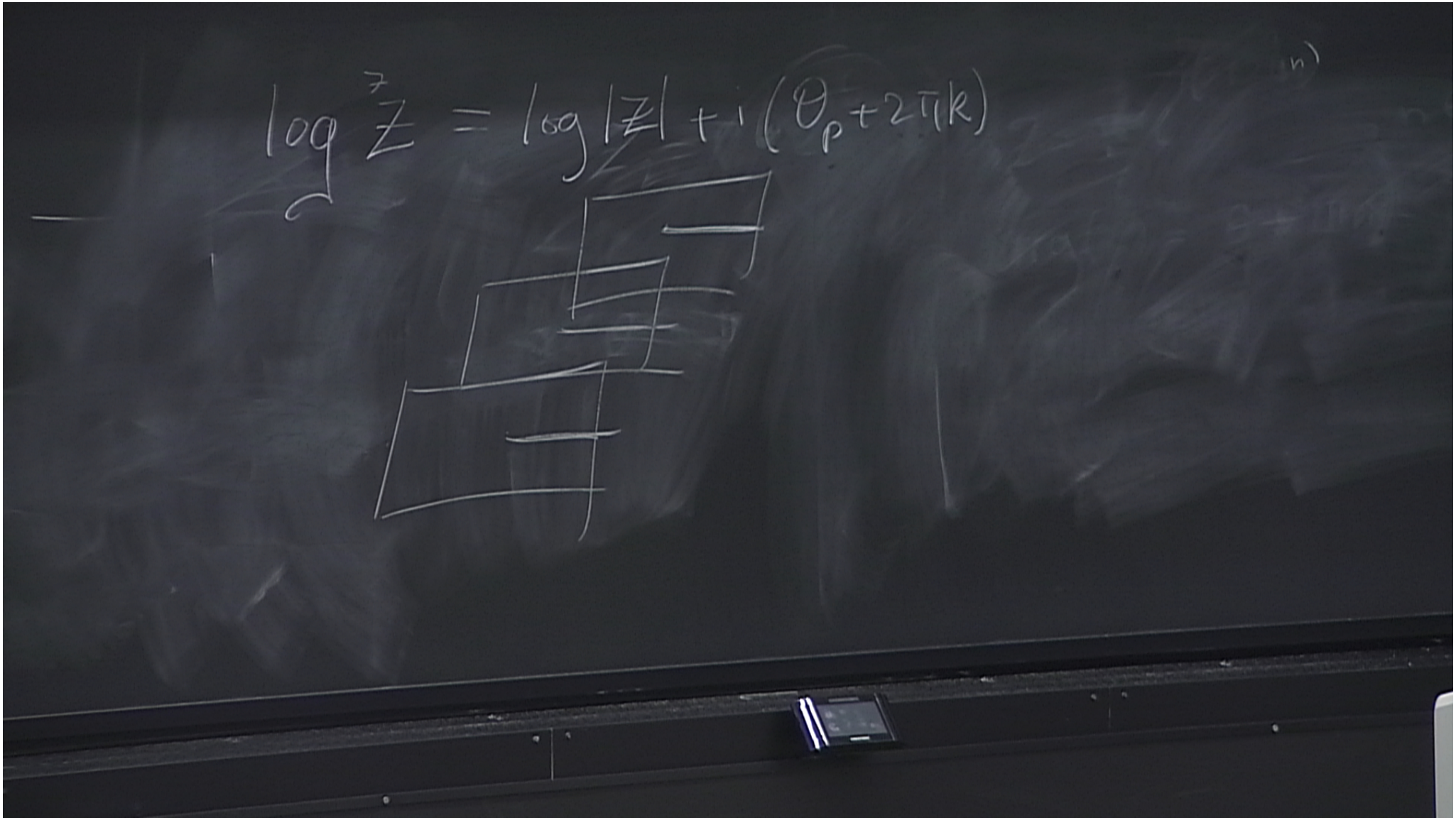
$-\epsilon e^{i\theta}$

~ monodromy around $z = 0$



Branch point

Branch cu


$\log^{\mathbb{Z}} z = \log|z| + i(\theta_p + 2\pi k)$