Title: 5-brane web diagrams and 5d, 6d SCFTs

Date: Aug 11, 2015 02:00 PM

URL: http://pirsa.org/15080087

Abstract: We study 5d and 6d SCFTs with eight supercharges by using 5-brane web diagrams in Type IIB string theory. There are two important properties of the web diagram. One is that it enables us to compute the exact partition function of the 5d theory on the 5-brane web. Even though the web diagram is not dual to a toric geometry, we develop a technique to compute the partition function by using the topological vertex. The other is that a new class of the 5-brane web diagram is conjectured to imply that the corresponding 5d theory has a 6d superconformal fixed point at high energies. Such a web diagram is called Tao web diagram. By using the Tao web diagram, we conjecture new 5d theories whose UV completion is a 6d SCFT. Moreover, we can show that a brane configuration realizing the 6d SCFT may reduce to the Tao web diagram, giving a direct way of identifying the 6d description of the 5d theory. This construction also indicates various dualities of the 5d theories.

5-brane web diagrams and 5d, 6d SCFTs

Hirotaka Hayashi

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Based on the collaboration with
Sung-Soo Kim, Kimyeong Lee, Masato Taki, Futoshi Yagi
[arXiv:1505.04439] and [To appear]

11th of August, 2015 Perimeter

1. Introduction

- Understanding quantum field theories in higher dimensions, namely d = 5 and 6.
- Those theories are not renormalizable. But certain class of the theories may be sensible if they admit a non-trivial fixed point at UV.
- Six is the maximum dimension for superconformal field theories.

Nahm 78

- 6d SCFTs have mysterious features.

- How can we analyze the 6d SCFTs?

- There is an indirect way to study the 6d SCFTs. It has been suggested that certain 5d supersymmetric field theory has the UV completion as a 6d SCFT.

- Therefore, we may obtain information of a 6d SCFT by computing some (BPS protected) quantity (ex. a partition function) of the corresponding 5d supersymmetric field theory.

- A famous example: E-string theory
  A 6d SCFT on the worldvolume theory on an M5-brane probing the M9-brane wall.

  ⬇ S^1 compactification with Wilson lines

  5d SU(2) gauge theory with 8 flavors.
  (Less flavor cases correspond to 5d SCFTs.)

- An important point is that the information of the 6d E-string theory may be recovered by including all the instanton effects of the 5d theory. Kim, Kim, Lee, Park, Vafa 14

- However, in general, it is difficult to know which 5d theory has the UV completion as a 6d SCFT.

- Ex. Higher rank version of the SU(2) example?

Q1. How can we determine which 5d theory has the UV completion as a 6d SCFT?

Q2. Relating to it, what is the 6d SCFT?

- In fact, 5-brane web diagrams in Type IIB string theory provides a very useful tool to answer the two basic questions!

- Remark:

The use of the 5-brane webs has another advantage. We can compute the exact partition function of the 5d theory by using the topological vertex.

1. Introduction

2. Preliminary: 5d SCFTs from 5-brane webs

3. Analyzing 6d SCFTs from 5-brane webs

4. 5d dualities from 5-brane webs

5. Conclusion

- We construct a 5d supersymmetric field theory with eight supercharges as the worldvolume theory on a 5-brane web.

- The 5-brane configuration in Type IIB string theory.

	0	1	2	3	4	5	6	7	8	9
D5-brane	×	×	×	×	×	×				
NS5-brane	×	×	×	×	×		×			
(p, q) 5-brane	×	×	×	×	×	angle				
7-brane	×	×	×	×	×			×	×	×

Aharony, Hanany 97
Aharony, Hanany, Kol 97
DeWolfe, Iqbal, Hanany, Katz 99

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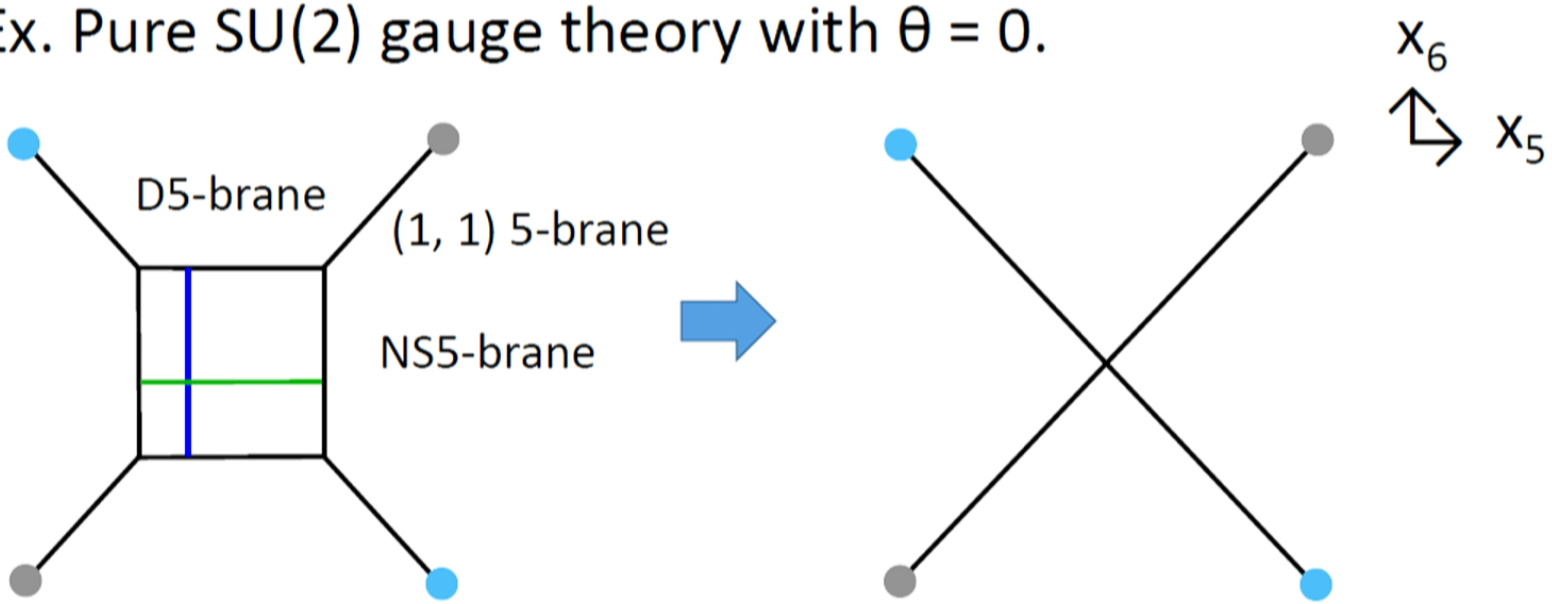
- Ex. Pure SU(2) gauge theory with $\theta = 0$.



- This theory has the fixed point where the gauge coupling is infinitely strong. Seiberg 96
- Enhancement of flavor symmetry: U(1) $\rightarrow$ SU(2) = E_1.

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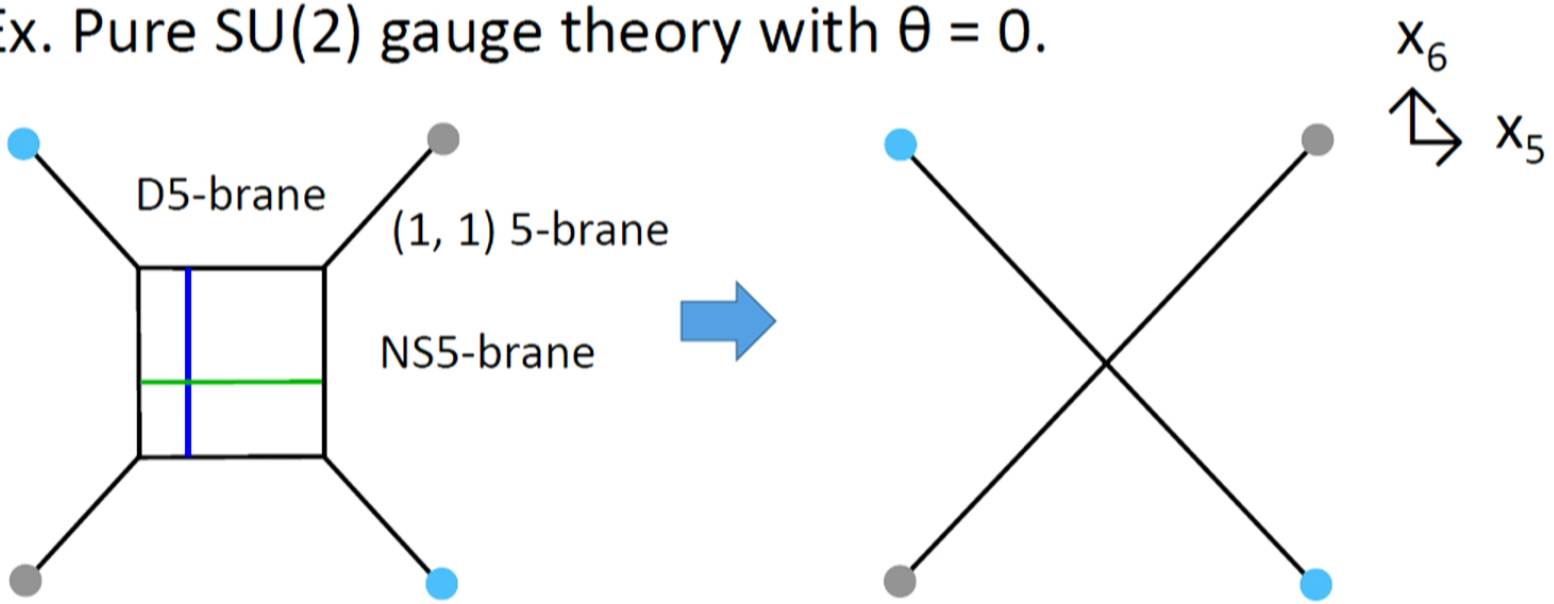

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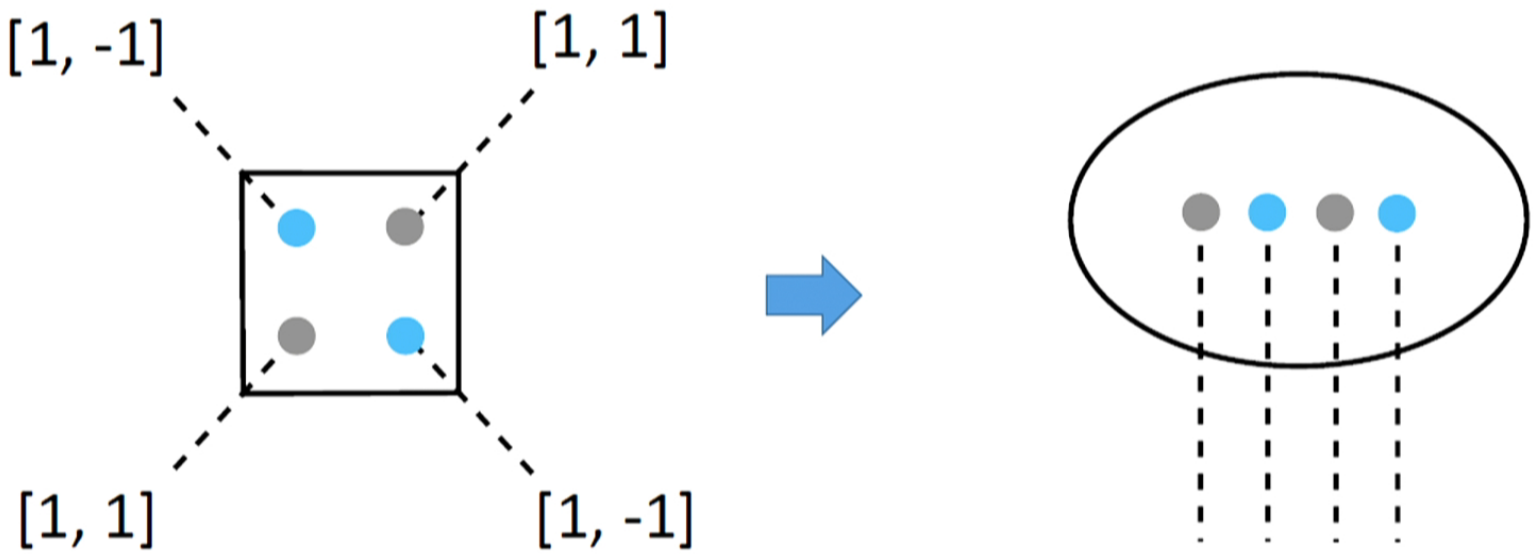
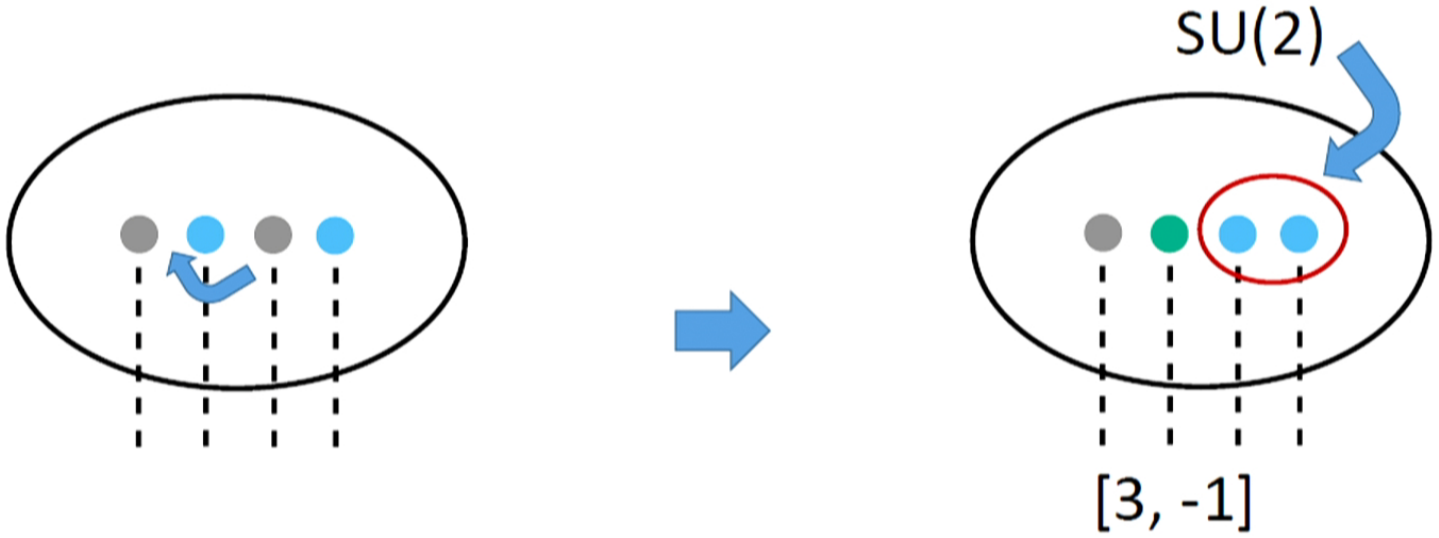
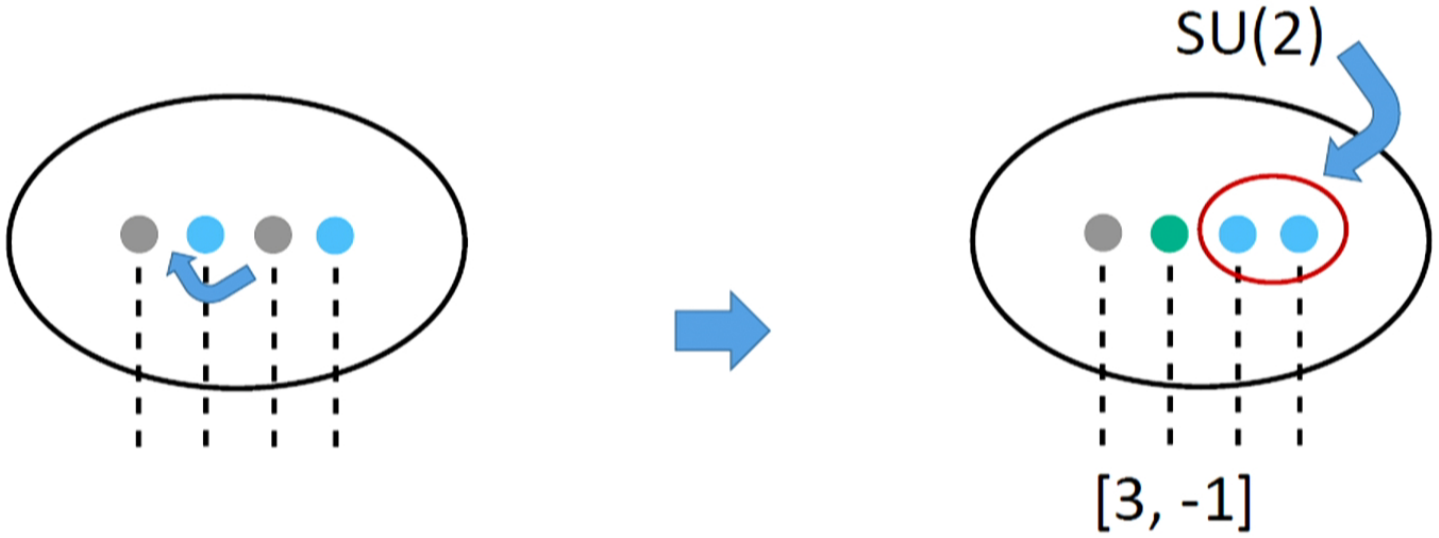
- The global symmetry of the 5d theory can be read off from the 7-brane configuration.





- The monodromy matrix for a [p, q] 7-brane when a 7-brane crosses a branch cut clockwise.

$$M_{p,q} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}$$



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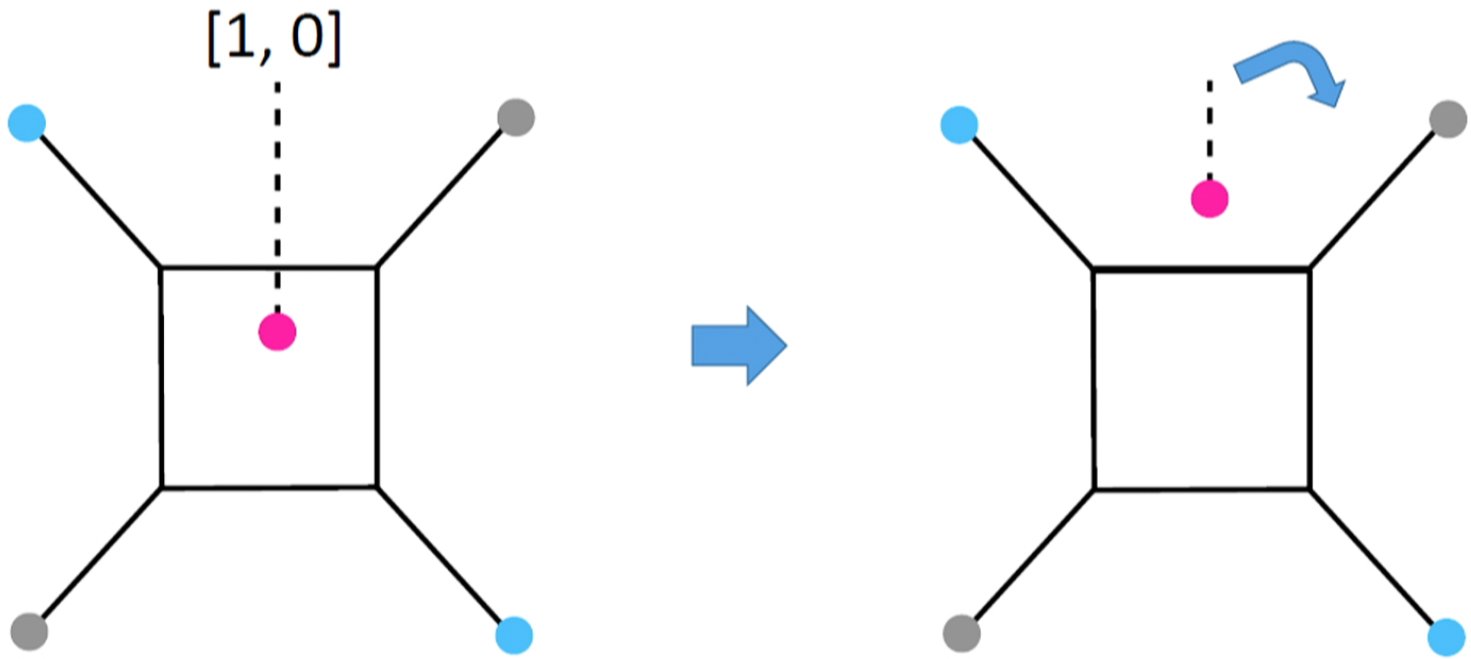
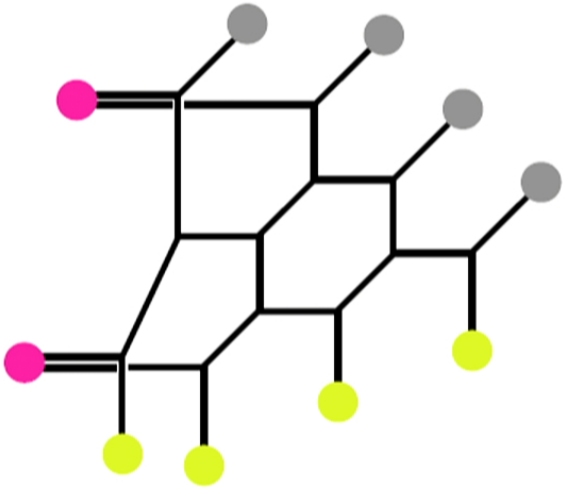
- Ex. 1 flavor : [U(1) x U(1) → SU(2) x U(1) = E_2]

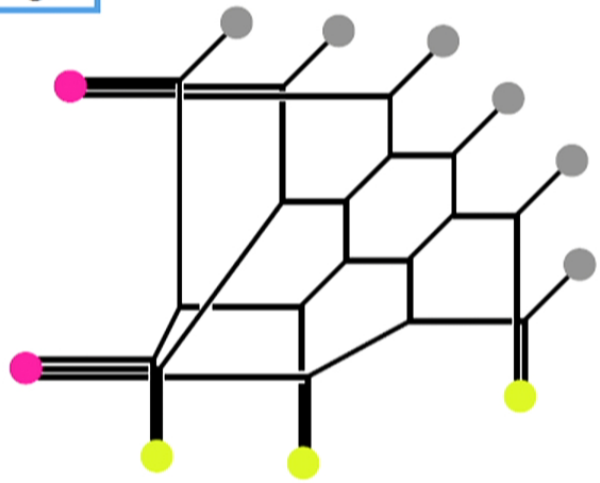


- Further addition of the flavors can be done systematically.

$N_f = 6$ [E_7]



$N_f = 7$ [E_8]



Benini, Benvenuti, Tachikawa 09

- It is possible to construct web diagrams of other gauge theories like a linear quiver of SU(N) gauge groups.

- Recently, the construction was extended by including an O7-plane, which gives USp(2N) , SO(N) gauge theory or SU(N) gauge theory with other matter. Bergman, Zafrir 15

- A conjecture is

The existence of a (finitely large) 5-brane web implies the corresponding 5d theory has the 5d UV fixed point.

3. Analyzing 6d SCFTs from 5-brane webs

Hayashi, Kim, Lee, Taki, Yagi 15

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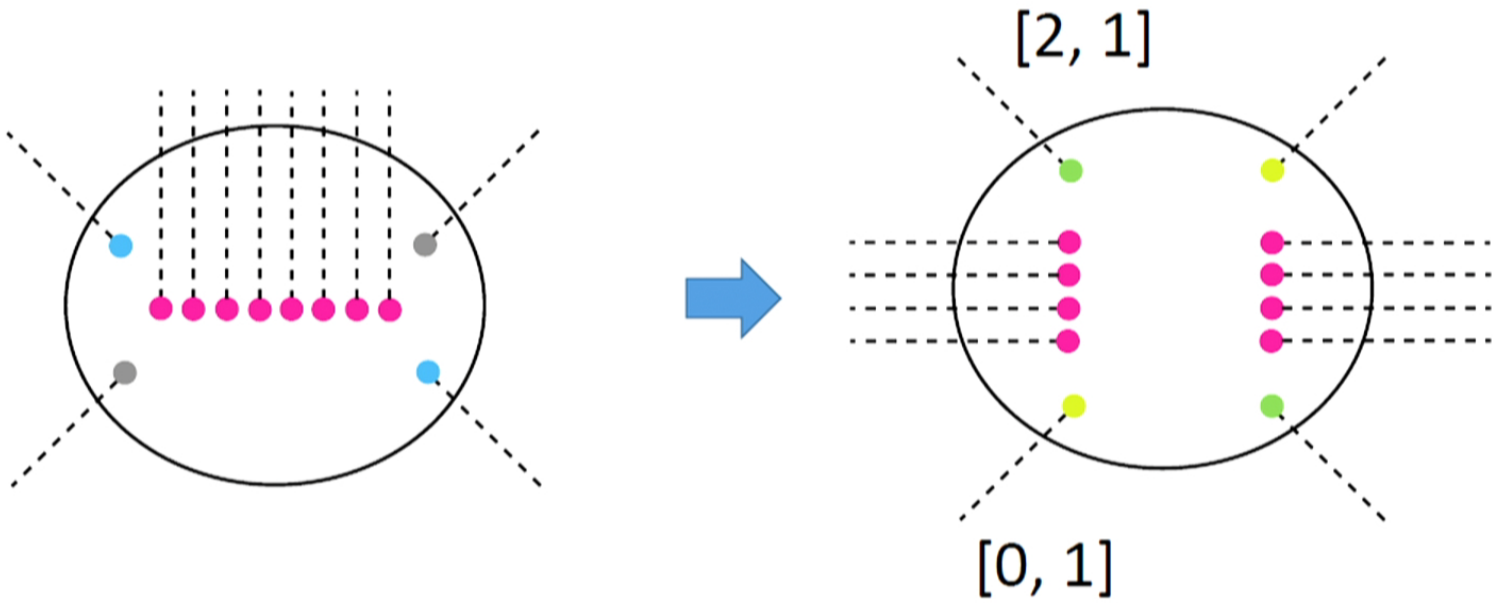
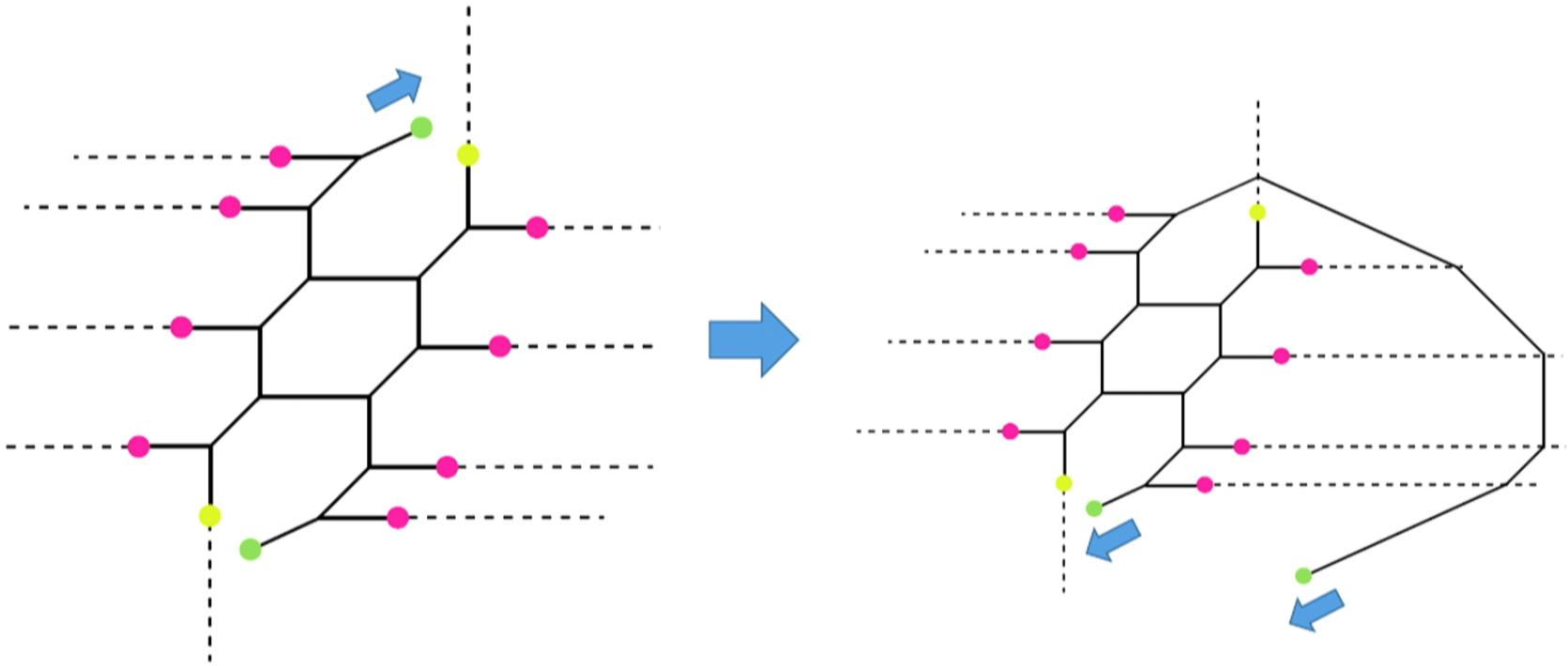
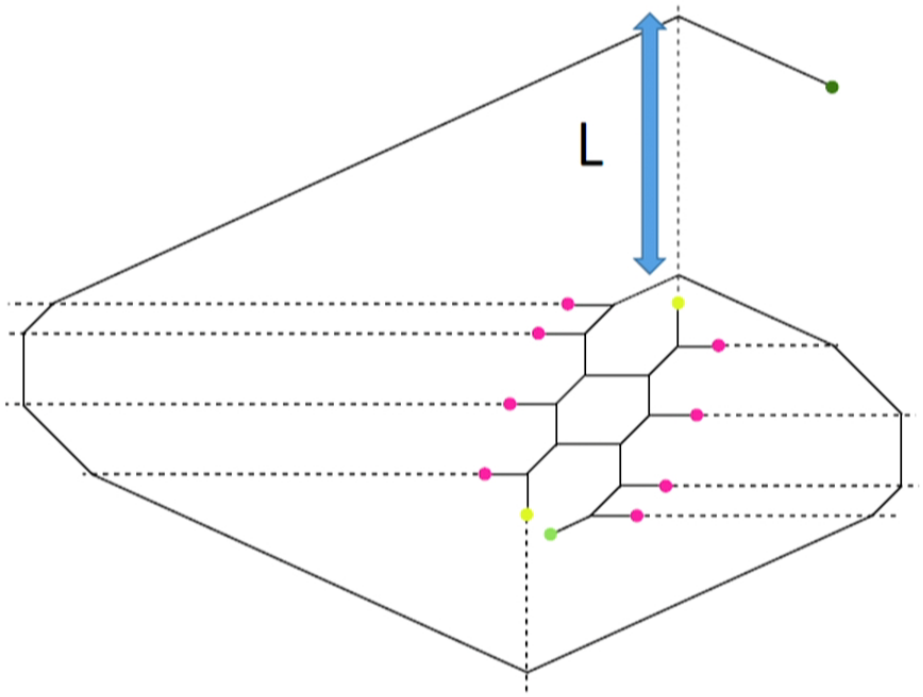
- What happens if we consider the SU(2) gauge theory with 8 flavors? We know that its UV fixed point is the 6d E-string theory.





- The “period” of the Tao web diagram can be identified with the inverse of the radius of the S^1.



$L \approx 1/g^2 \approx R^{-1}$

- Does it corresponds to a 6d SCFT at UV? → Yes!

- Our claim is

Hayashi, Kim, Lee, Taki, Yagi 15
Yonekura 15

6d (D_{n+2}, D_{n+2}) minimal conformal matter theory

↓ S^1 compactification

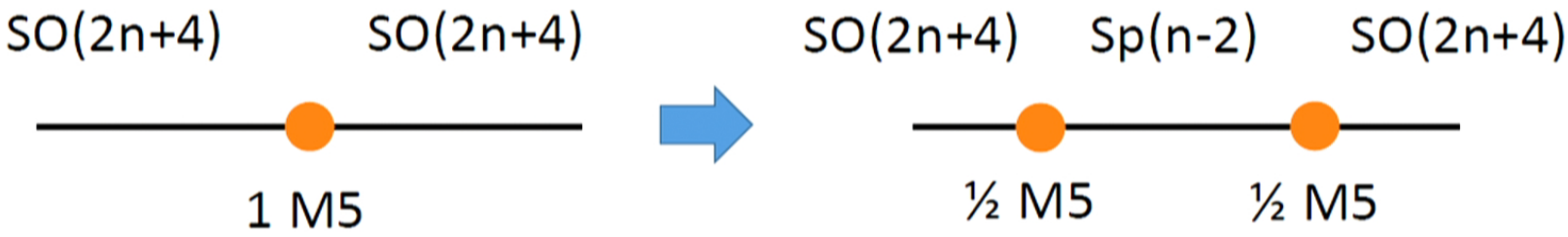
5d SU(n) gauge theory with $N_f = 2n+4$ & CS-level 0

- Evidences:
  1. A derivation by branes (6d → 5d)
  2. Global symmetry matching

- Evidence 1: A derivation by branes

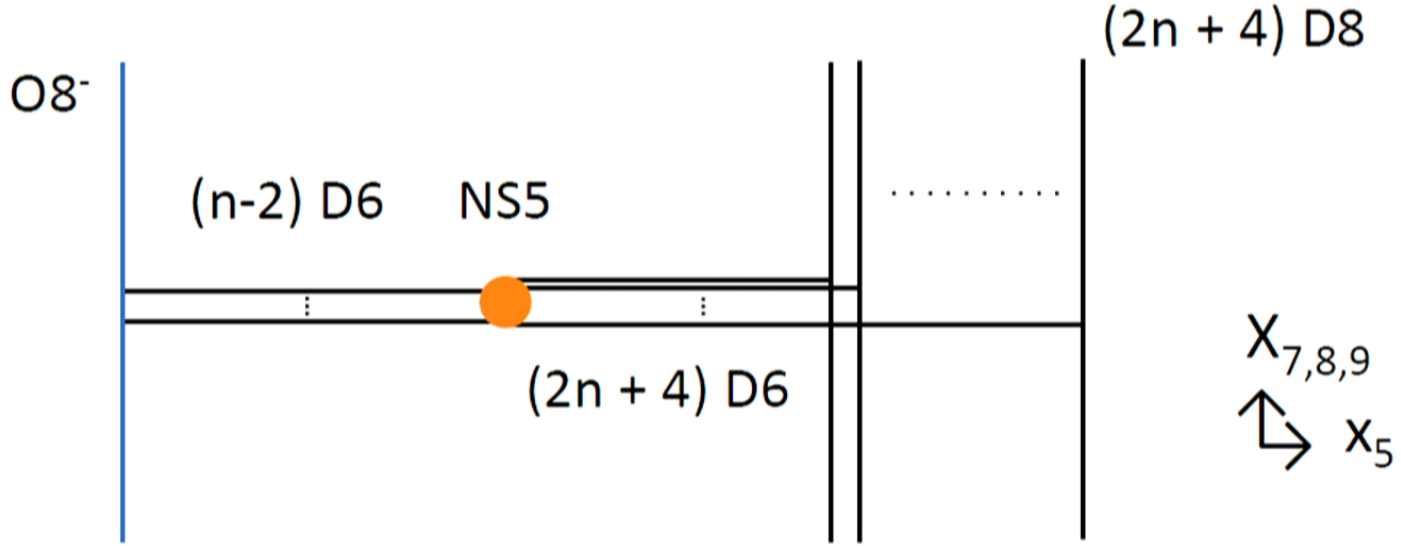
- The 6d (D_{n+2}, D_{n+2}) minimal conformal matter theory is a 6d SCFT realized on a single M5-brane probing the D_{n+2} type singularity.

Del Zotto, Heckman, Tomasiello, Vafa 14



"Tensor branch"

- The brane configuration



- When one fixes the number of the leftmost D6-branes, the number of the next D6-branes is determined. This is equivalent to the anomaly cancellation condition in 6d.

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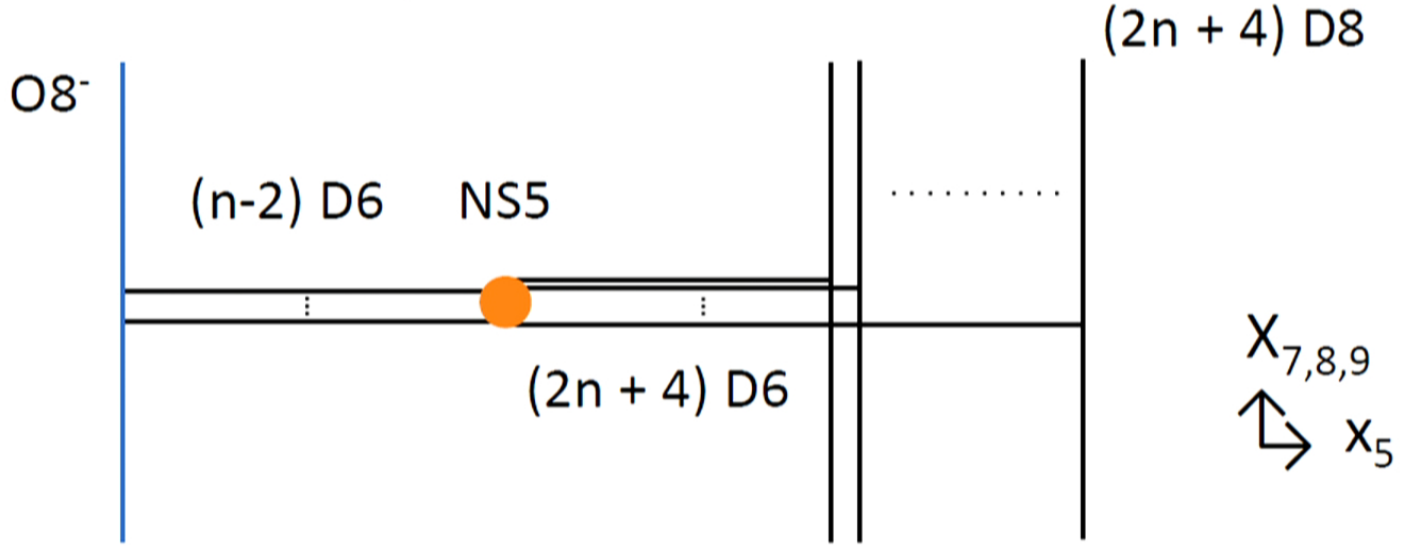
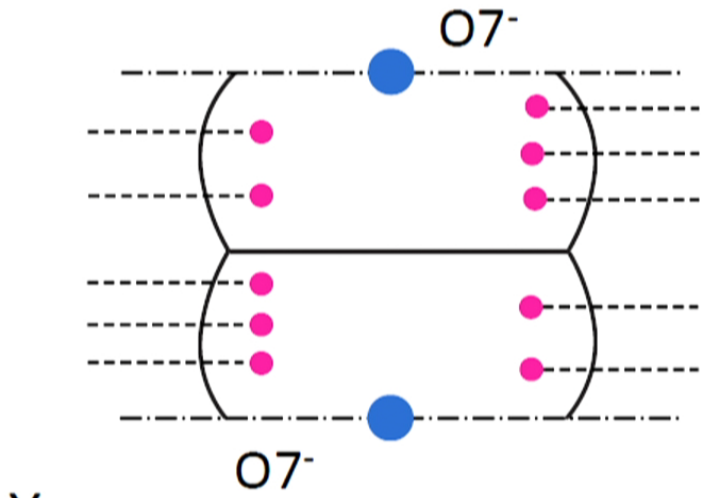
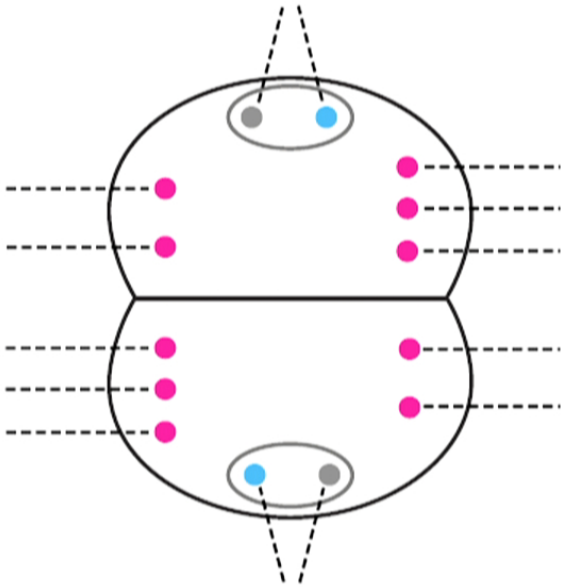


- When one fixes the number of the leftmost D6-branes, the number of the next D6-branes is determined. This is equivalent to the anomaly cancellation condition in 6d.

- We then compactify x_6 on S^1 and perform the T-duality along the direction.

Ex. n = 3





Resolving two O7-planes

- Evidence 2: Global symmetry matching

- The 6d theory in the tensor branch is the Sp(n-2) gauge theory with 2n+4 flavors. The flavor symmetry is SO(4n+8) . Ex. n=3 → SO(20).

- From the viewpoint of 5-brane webs, the global symmetry of the 5d theory can be read off from the 7-branes.
  Ex. Zwiebach, Gaberdiel 97

  $A^n = SU(n)$, $A^nBC = SO(2n)$, $A^{5,7,8}BCC = E_{6,7,8}$

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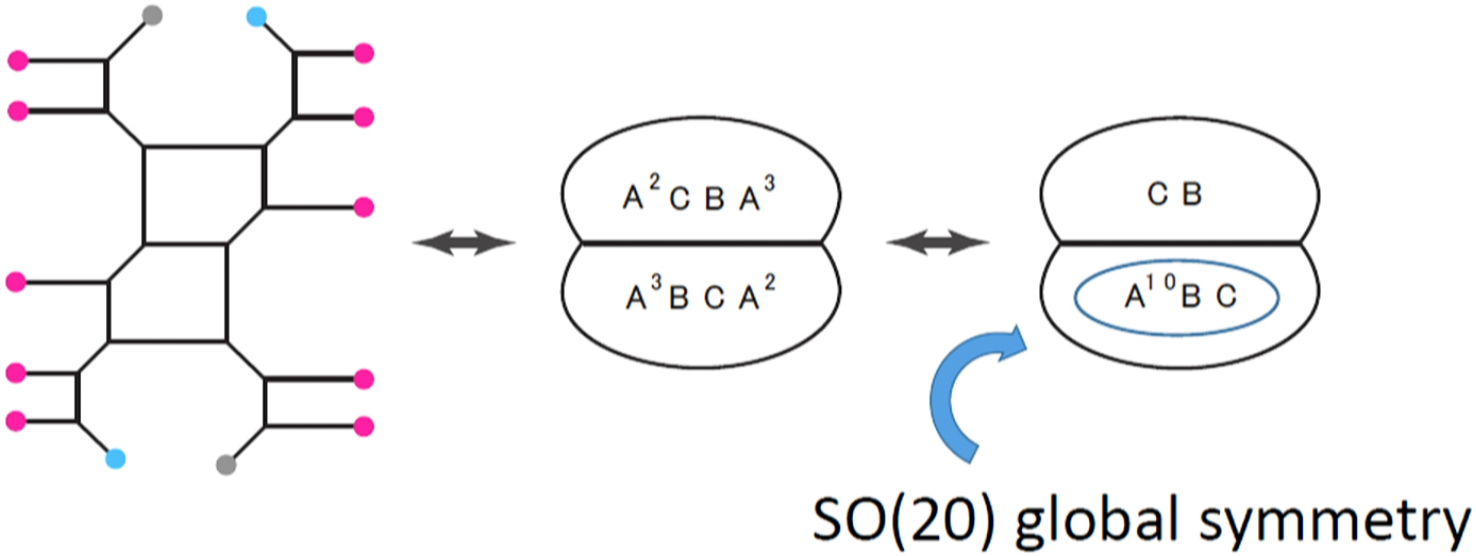
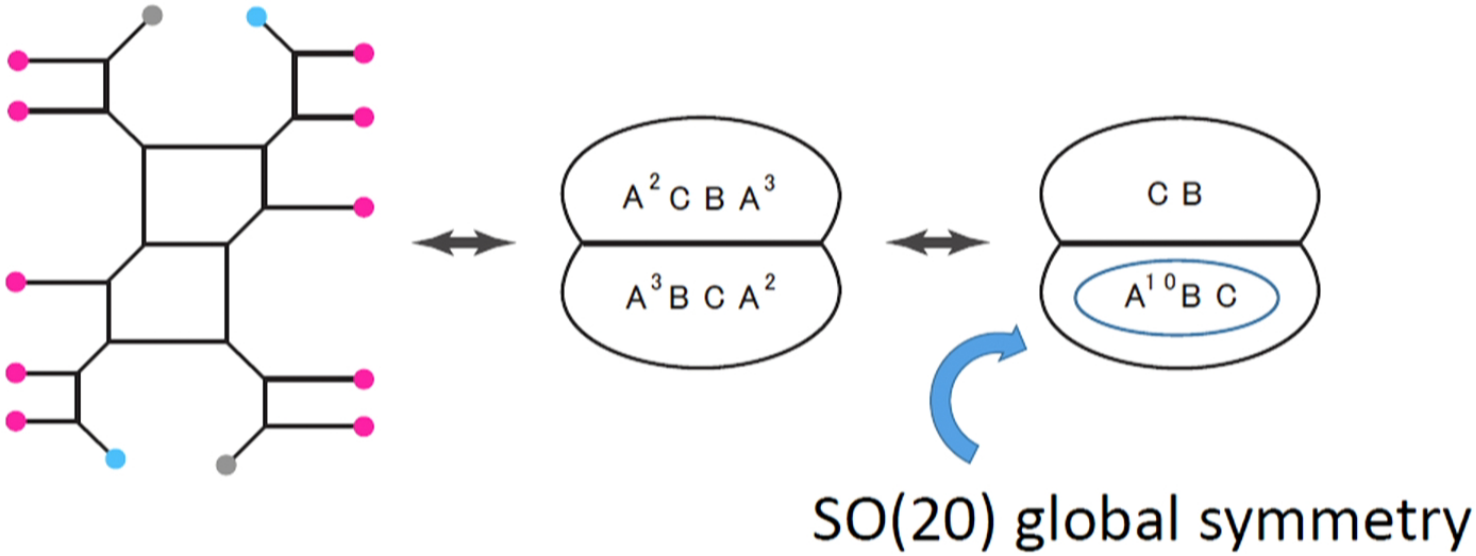
- Ex. SU(3) gauge theory with 10 flavors



- In general, it is possible to show that the 7-brane configuration gives SO(4n+8) x $U(1)_I$ flavor symmetry.

- Ex. SU(3) gauge theory with 10 flavors



- In general, it is possible to show that the 7-brane configuration gives SO(4n+8) x $U(1)_I$ flavor symmetry.

- Remark 2:

By decoupling flavors, we obtain 5d theories whose UV completion is 5d SCFTs.

Flavor symmetry

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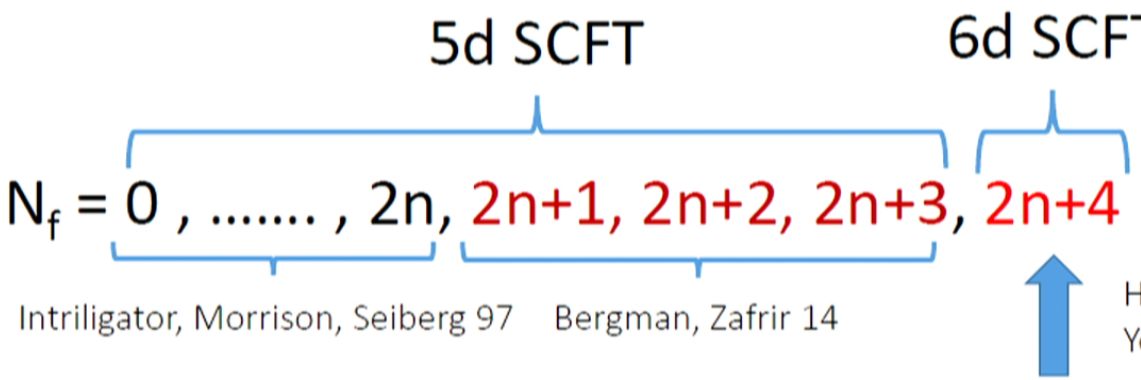
Yonekura 15
Gaiotto, Kim 15

N_f	$G_{\lvert\kappa\rvert}$
$2n+4$	$SO(4n+8)_0$
$2n+3$	$SO(4n+8)_{\frac{1}{2}}$
$2n+2$	$SU(2n+4)_0, \quad \left[SO(4n+4)\times SU(2)\right]_1$
$2n+1$	$\left[SU(2n+2)\times SU(2)\right]_{\frac{1}{2}}, \quad SO(4n+2)_{\frac{3}{2}}$
$2n$	$\left[SU(2n)\times SU(2)\times SU(2)\right]_0, \quad SU(2n+1)_1, \quad SO(4n)_2$

Therefore, the UV completion of the 5d SU(n) gauge theories is



6d (D_{n+2}, D_{n+2}) minimal conformal matter

- It is possible to extend the analysis to more general quiver theories.

Yonekura 15
Hayashi, Kim, Lee, Taki, Yagi [To appear]

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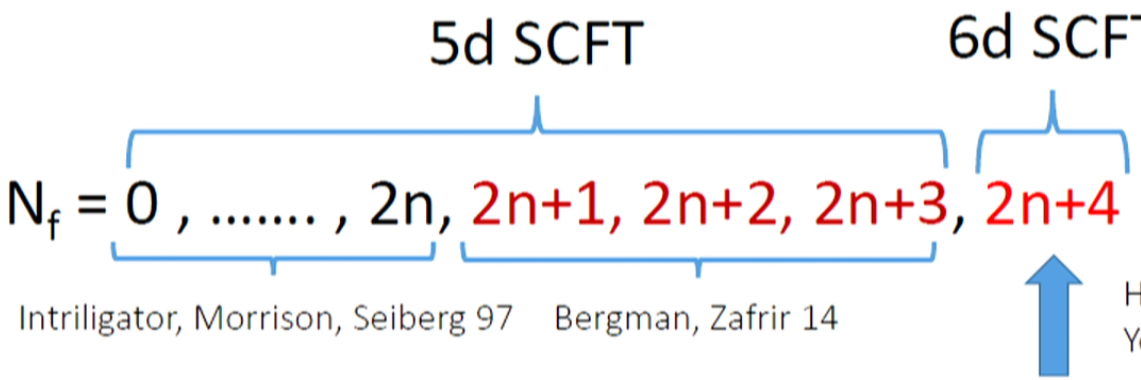

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Yonekura 15
Hayashi, Kim, Lee, Taki, Yagi [To appear]

- The derivation using the brane configuration also implies another interesting 5d dualities.

- 5d SU(n) $\leftrightarrow$ 5d Sp(n-1) duality Gaiotto, Kim 15

5d SU(n) gauge theory with N_f flavors & $\kappa = \pm(N + 2 - N_f/2)$

5d Sp(n-1) gauge theory with N_f flavors

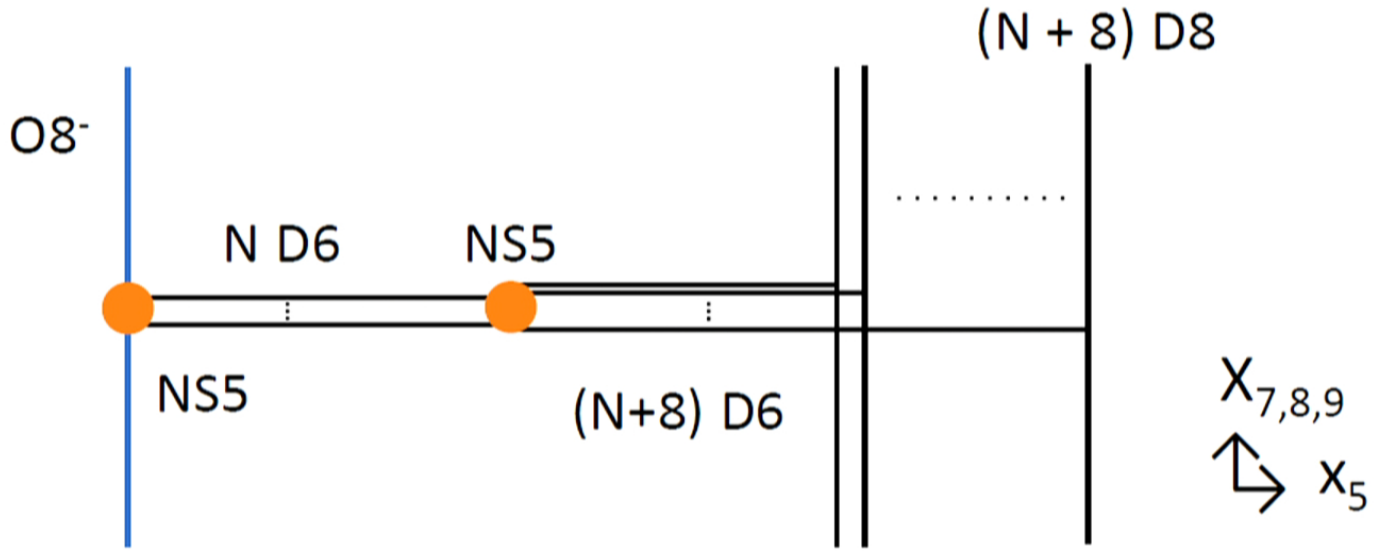
- This implies the 5d SU(n) gauge theory with $N_f = 2n+4$ is dual to the 5d Sp(n-1) gauge theory with $N_f = 2n+4$. Both theories have the same UV completion as the 6d SCFT, namely (D_{n+2}, D_{n+2}) minimal conformal matter.

- By decoupling the same flavors, we can induce the 5d dualities. Note that the Chern – Simons level of the SU(n) gauge theory should take the maximum value since we can only decouple the matter into $+\infty$.

- It is possible to generalize the dualities into another brane configurations.

Hayashi, Kim, Lee, Taki, Yagi [To appear]

- Ex. 6d SU(N) gauge theory with $N_f = N + 8$ and $N_a = 1$.



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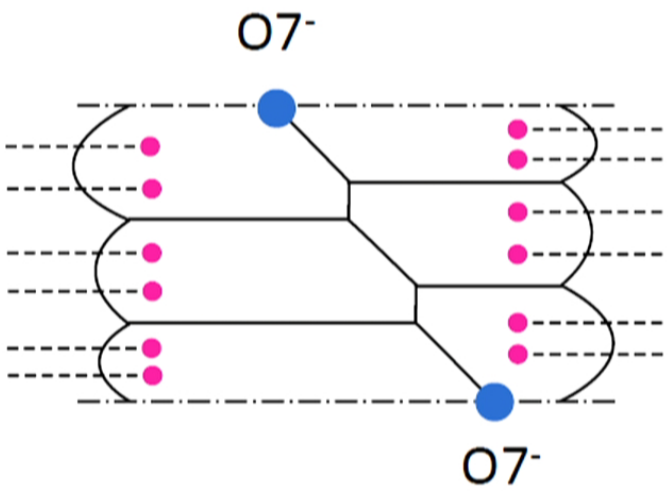
- We then compactify x_6 on S^1 and perform the T-duality along the direction.

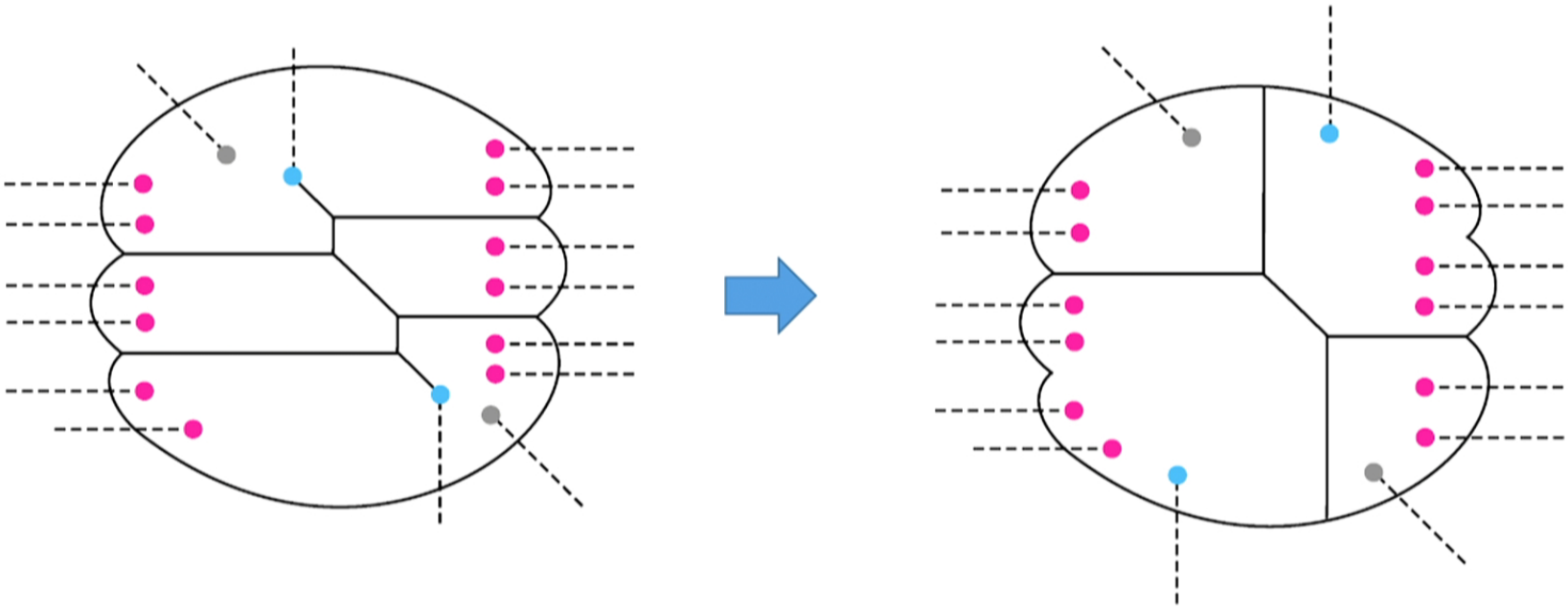
Ex. N = 4

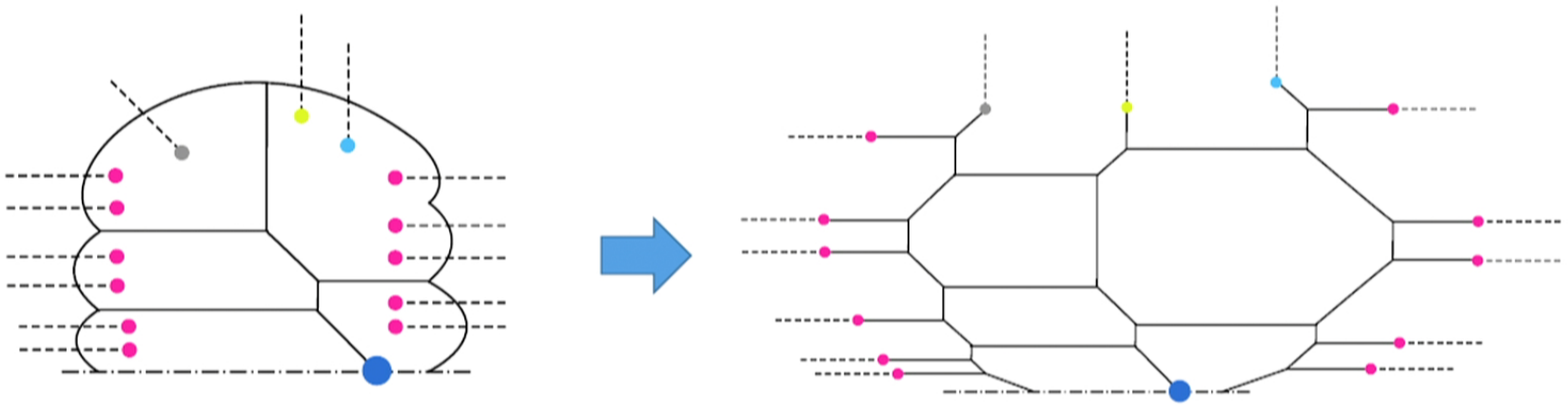


Resolving two O7-planes

Resolving one O7-plane

- Resolving two $O7^-$ - planes.





SU(5) with $N_f = 11$, $N_a = 1$

- This implies that the 5d quiver [5] – SU(3) – SU(3) – [5] is dual to the 5d SU(5) gauge theory with $N_f = 11$ and $N_a = 1$. Both theories have the same UV completion as the 6d SCFT.

- In general, if $N = 2n$, the brane configuration implies [n+3] – SU(n+1) – SU(n+1) – [n+3] is dual to SU(2n+1) with $N_f = 2n+7$ and $N_a = 1$.

- It is possible to generalize the analysis to a general 6d quiver theories, and it implies a lot of 5d dualities.

Hayashi, Kim, Lee, Taki, Yagi [To appear]

- Remarks:

Hayashi, Kim, Lee, Taki, Yagi [To appear]

1. By changing the splitting type of an $O7^-$ - plane, we can obtain another quiver theory, for example,

$$[8] - SU(4) - SU(2) - [2]$$

Therefore, this theory is also dual to $[5] - SU(3) - SU(3) - [5]$ (and similarly to the SU(5) with $N_f = 11$ and $N_a = 1$).

2. S-dualities of the 5d theories also give another dual theories.

- The Tao web diagram implies a 5d theory whose UV completion is a 6d SCFT.
- The Tao web may give a direct connection to the brane configuration of the 6d SCFT.
- Furthermore, the relation between the brane configurations implies various intriguing 5d dualities.