

Title: Positivity, negativity, entanglement, and holography

Date: Aug 21, 2015 04:00 PM

URL: <http://pirsa.org/15080083>

Abstract: TBA

PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

POSITIVITY, NEGATIVITY, ENTANGLEMENT, & HOLOGRAPHY
w/ Massimiliano Rota 1406 6989 + 1505 03696
" + E Perlmutter 1506 01679

Entanglement \longleftrightarrow Holography

- does the entanglement have to be EPR type?

POSITIVITY, NEGATIVITY, ENTANGLEMENT, & HOLOGRAPHY
w/ Massimiliano Rota 1406 6989 + 1505 03696
" + E Perlmutter 1506 01679

Entanglement \longleftrightarrow Holography

- does the entanglement have to be EPR type?
- if entanglement were undistillable?
- classical correlations contamination?

Review: Necessity, Entanglement & Holography

- What kind of holographic constraints are to be imposed on QE?
 - Matt's Q: what is special about HEE?

... Necessity, Entanglement & Holography

- What kind of holographic constraints are to be imposed on QE?
- EE {
 - Matt's Q: what is special about HEE?
 - holographic entropy cone?
 - are there alternate measures of QE which provide some intuition of how the holographic map work?

- What kind of holographic constraints are to be imposed on QE?
- EE {
 - Matt's Q: what is special about HEE?
 - holographic entropy cone?
 - are there alternate measures of QE which provide some intuition of how the holographic map work?
- Should every QI notion of entanglement witness have a geometric avatar?

if entanglement were unattainable

Try to distinguish classical correlations from pure QE
using entanglement negativity

Bipartite system $\mathcal{H}_L \otimes \mathcal{H}_R$

density matrix ρ

$$\rho_{\alpha\alpha', \beta\beta'} = \langle r_a l_a | \rho | r_b l_b \rangle$$

EPR type?

Try to distinguish classical correlations from pure QE
using entanglement negativity

Bipartite system: $\mathcal{H}_L \otimes \mathcal{H}_R$

density matrix ρ .

$$\rho_{\alpha\alpha, b\beta} = \langle r_a l_\alpha | \rho | r_b l_\beta \rangle$$

define partial transpose

$$\rho_{\alpha\alpha, b\beta}^T = \rho_{a\beta, b\alpha}$$

onic map work
bitness

define partial transpose

$$\rho_{\alpha\alpha, \beta\beta}^T = \rho_{\beta\beta, \alpha\alpha}$$

Given ρ^T we define 2 measures of entanglement

$$\mathcal{E}(\rho) = \log \|\rho^T\|_1$$

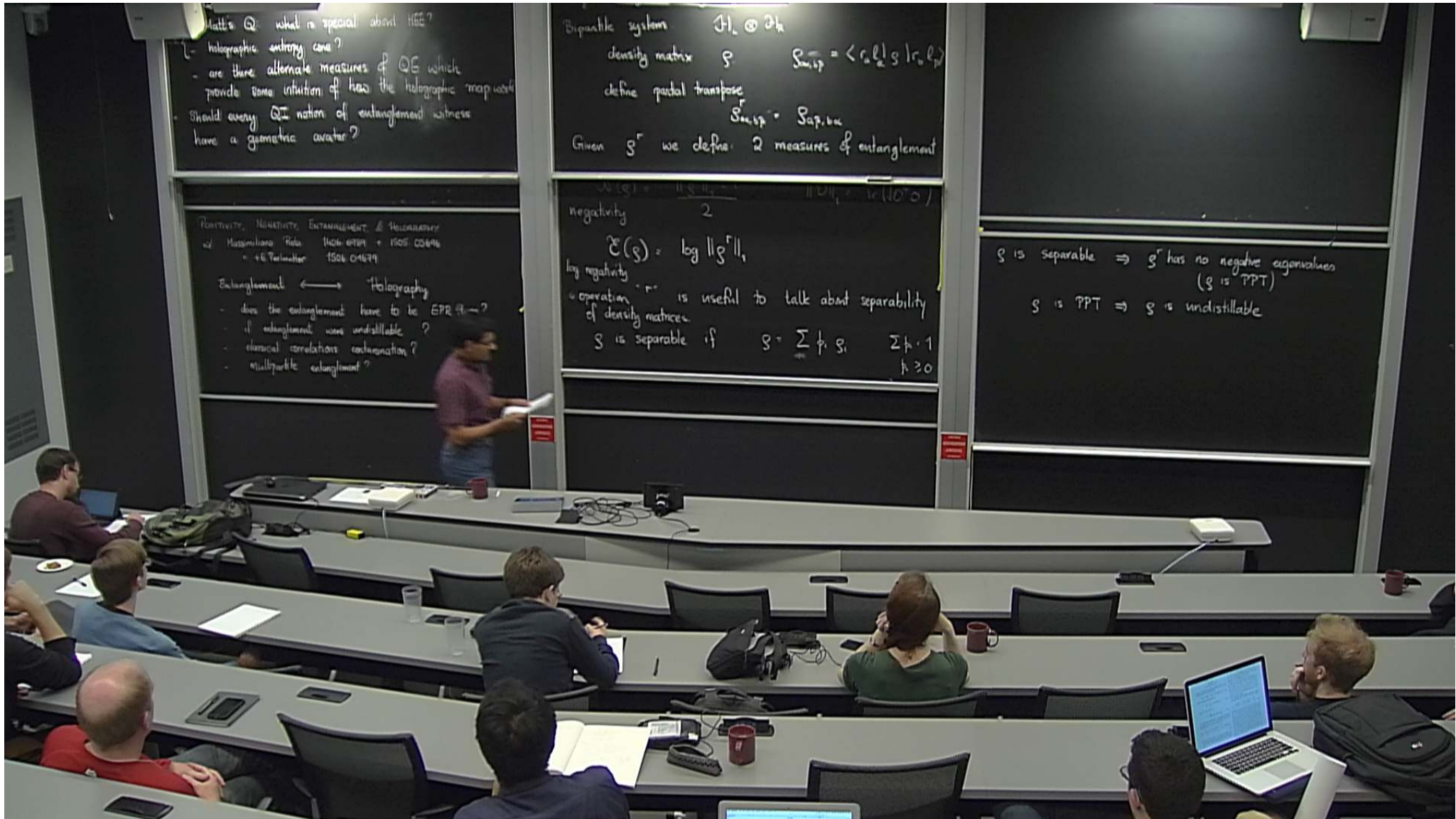
log negativity

operation "T" is useful to talk about separability of density matrices.

$$\rho \text{ is separable if } \rho = \sum p_i \rho_i \quad \begin{matrix} \sum p_i = 1 \\ p_i \geq 0 \end{matrix}$$

R type?





Matt's Q: what is special about HEE?
 - holographic entropy cone?
 - are there alternate measures of QE which provide some intuition of how the holographic map works?
 - Should every QI notion of entanglement witness have a geometric avatar?

Positivity, Negativity, Entanglement & Holography
 w/ Massimiliano Plesa 11/06/09 + 15/05/08/06
 + G. Perelman 15/06/04/07/09

Entanglement \longleftrightarrow Holography
 - does the entanglement have to be EPR-like?
 - if entanglement were undistillable?
 - classical correlations extraction?
 - multipartite entanglement?

Bipartite system $\mathcal{H}_L \otimes \mathcal{H}_R$
 density matrix ρ $S_{\rho, \text{LPT}} = \langle r_L | \rho | r_L \rangle$
 define partial transpose $S_{\rho, \text{LPT}}^T = S_{\rho, \text{LPT}}$
 Given ρ^T we define 2 measures of entanglement

negativity $\frac{1}{2} \|\rho - \rho^T\|_1$
 $\mathcal{N}(\rho) = \log \|\rho^T\|_1$
 log negativity
 operation ρ^T is useful to talk about separability of density matrices.
 ρ is separable if $\rho = \sum p_i \rho_i$ $\sum p_i = 1$
 $p_i \geq 0$

ρ is separable $\Rightarrow \rho^T$ has no negative eigenvalues (ρ is PPT)
 ρ is PPT $\Rightarrow \rho$ is undistillable

ρ is separable $\Rightarrow \rho^T$ has no negative eigenvalues
(ρ is PPT)

ρ is PPT $\Rightarrow \rho$ is undistillable

$\mathcal{N}(\rho)$ gives a measure of the robustness of entanglement

consider ρ & add noise $\rho_s \sim$ separable.

$$\tilde{\rho} \sim \frac{1}{1+s} (\rho + s \rho_s)$$

↑
measure of amount of noise

$$\text{Robustness} \sim \min_{\rho_s} s = 2\mathcal{N}(\rho)$$

entanglement

$$\text{Tr}(\rho^{\otimes 2})$$

$$\text{Robustness} \sim \min_{\rho_S} S = 2J(\rho)$$

separability

$$\sum p_i = 1$$
$$p_i \geq 0$$

Given ρ ($\mathcal{H}_L \otimes \mathcal{H}_R$) we can also consider $\text{Tr}_L \rho = \rho_R$

$$\frac{J(\rho)}{S_R} \quad \frac{\mathcal{E}}{S_R}$$

entanglement

$$\text{Robustness} \sim \min_{S_S} S = 2J(g)$$

$$\text{Tr}(\rho^{\otimes 2})$$

$$\frac{J(g)}{S_R} \quad \frac{\mathcal{E}}{S_R}$$

1 compute \mathcal{E} for a pure state

$$|\psi_{HH}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum e^{-\beta E_a/2} |a\rangle$$

$$g = |\psi_{HH}\rangle \langle \psi_{HH}| \quad \rightarrow \quad \frac{\sum e^{-\beta E_a}}{Z(\beta)} = \beta [F(\beta) - F(\beta/2)]$$

separability

$$\sum p_i = 1$$

$$p_i \geq 0$$

CAUTION

provide some intuition of the EE geometric interpretation

- Should every QI notion of entanglement witness have a geometric avatar?

For pure g $\mathcal{E}(g) = S(g_R)$

In 1+1 CFTs use replica trick, w/ 1-change from usual EE/Renyi computation.

g^T involves 1-swap of twist/anti-twist ops.

provide some intuition of the EE geometric interpretation

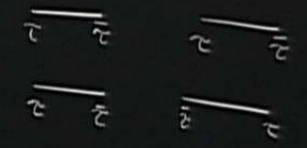
• Should every QI notion of entanglement witness have a geometric avatar?

For pure S $\mathcal{E}(S) = S(S_R)$

In 1+1 CFTs use replica trick, w/ 1-change from usual EE/Renyi computation.

S^{Γ} involves 1-swap of twist/anti-twist ops.

CFT₂ 1-interval $\mathcal{E}(|0\rangle\langle 0|) = \frac{c}{2} \log \frac{L}{\epsilon}$



twist ops.

$\frac{2}{2}$

$\frac{2}{2}$

free theories

$\chi_{(3)}^{\text{scalars}}$ 27

$\chi^{w=4}$ 1.708

holographic theories

0.601 χ^{free}

0.98 $\chi_{\text{free}}^{w=4}$

useful to talk about separability

1
0

CAUTION

twist ops.

$$\begin{array}{c} \hline 2 \\ 2 \end{array}$$

$$\begin{array}{c} \hline 2 \\ 2 \end{array}$$

free theories

$$\chi_{(3)}^{\text{scalars}} = 27$$

$$\chi^{w=4} = 1.708$$

holographic theories

$$0.601 \chi^{\text{free}}$$

$$0.98 \chi_{\text{free}}^{w=4}$$

These were for $|0\rangle_{\text{CFT}} \langle 0|$ partitioned across a spherical ball.

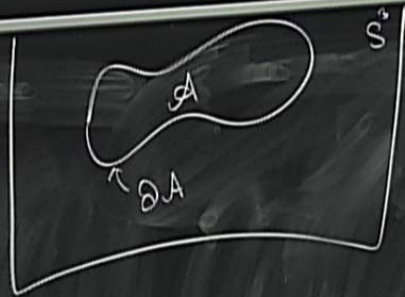


entanglement

Given ρ ($\mathcal{H}_L \otimes \mathcal{H}_R$) we can also consider $\text{Tr}_L \rho = \rho_R$

$$\frac{N(\rho)}{S_R} \quad \frac{Z(\rho)}{S_R}$$

For ∂A_g



$$S^{(n)} = f_a(q) R_{\partial A} + f_b(q) K_{\partial A} + f_c(q) C_{\partial A}$$

f_a, f_b, f_c are h.c. of
Renyi index & the central charges
(a, c)

cross a spherical ball.
separable

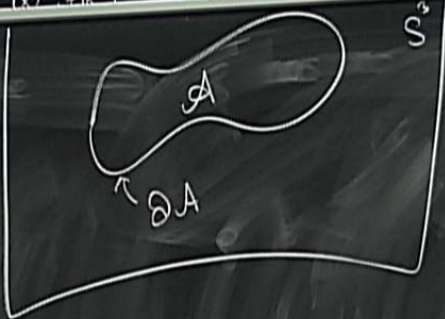
$\Sigma_b = 1$
 $A = 0$

CAUTION

nglement

Given $\mathcal{O} \in (H \otimes H)$ we can also write $\text{Tr} \rho = \mathcal{O}$

For ∂A_g



can be obtained from $\partial A = S^2$

$$S^{(a)} = f_a(q) R_{\partial A} + f_b(q) K_{\partial A} + f_c(q) C_{\partial A}$$

f_a, f_b, f_c are func of Renyi index & the central charges (a, c)

$\chi = \frac{\mathcal{N}_{\text{univ}}}{S_{\text{univ}}}$ can be less than unity if ∂A is of sufficiently high genus in theories w/ $\underline{a > c}$

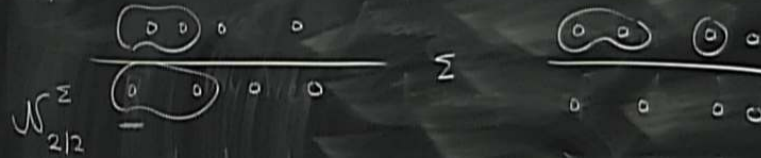
a spherical ball.

provide some intuition of how the holographic map works

- Should every QI notion of entanglement witness have a geometric avatar?

Qubit Experiments

pure states of few qubits



Qubit Experiments

pure states of few qubits



when is monogamy of mutual information respected?

$$I_3 = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{CA} + S_{ABC}$$

$$I_3 \leq 0 \quad ?$$

$$\mathcal{E}(\psi_{HH}) = \frac{\log(Z(1/2))}{Z(P)}$$

- under what conditions is monogamy of \mathcal{S}^2 respected?

$$\mathcal{S}_{A|C}^2 + \mathcal{S}_{A|B}^2 \leq \mathcal{S}_{A|BC}^2$$

holographically one finds situations where Araki-Lieb is saturated

$$|S_A - S_{A^c}| \leq S_{A \cup A^c}$$

happens for states w/ low robustness and large multi-partite entanglement

→ ~~MMI~~ $\mathcal{I} > 0$

when is monogamy of mutual information respected?

$$I_3 = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{CA} + S_{ABC}$$

$$I_3 \leq 0 \quad ?$$

Entanglement \longleftrightarrow Holography

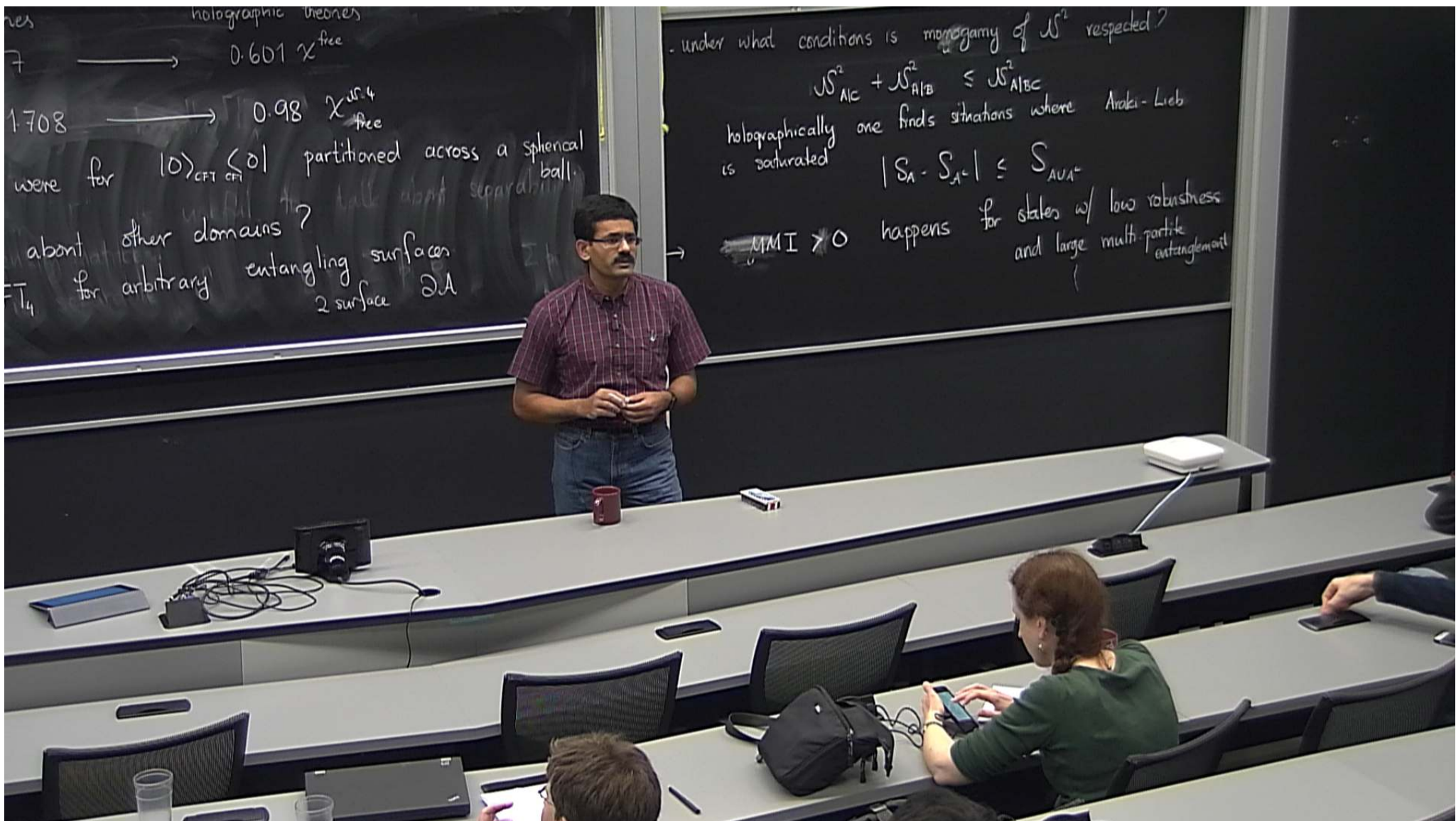
- does the entanglement have to be EPR type?
 - if entanglement were undistillable?
 - classical correlations contamination?
 - multipartite entanglement?
- robust bipartite entanglement seems closer to holographic systems.

free to
 $\chi_{(3)}$ scalars

$\chi^{d=4}$

These

What
CF



holographic theories
7 $\rightarrow 0.601 \chi_{\text{free}}$
1708 $\rightarrow 0.98 \chi_{\text{free}}^{d-4}$
were for $|0\rangle_{\text{CFT}} \langle 0|$ partitioned across a spherical ball.
about other domains?
for arbitrary entangling surfaces
2 surface ∂A

- under what conditions is monogamy of S^2 respected?
 $S_{AC}^2 + S_{AB}^2 \leq S_{ABC}^2$
holographically one finds situations where Araki-Lieb is saturated
 $|S_A - S_B| \leq S_{A \cup B}$
MMI > 0 happens for states w/ low robustness and large multi-partite entanglement