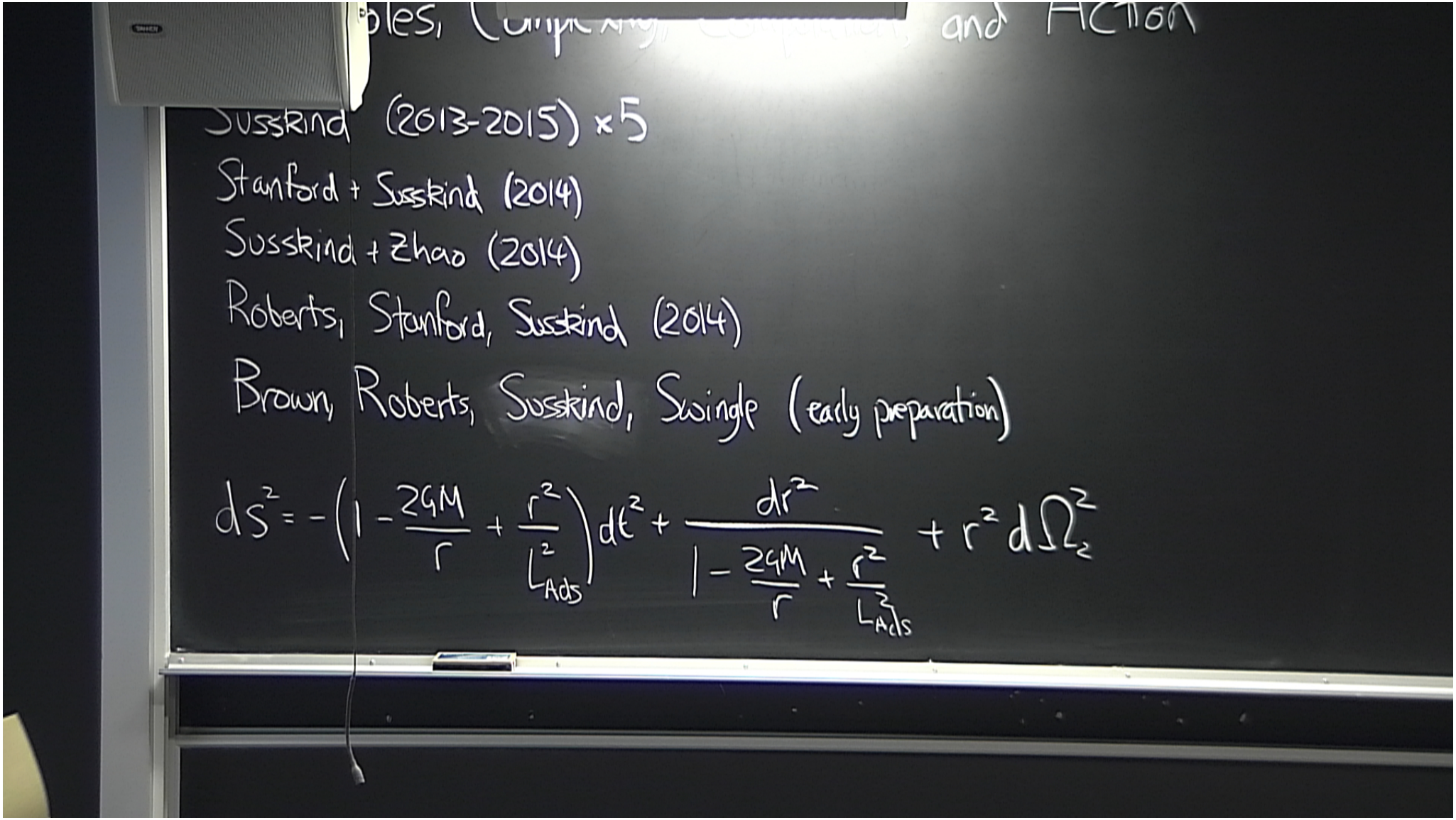
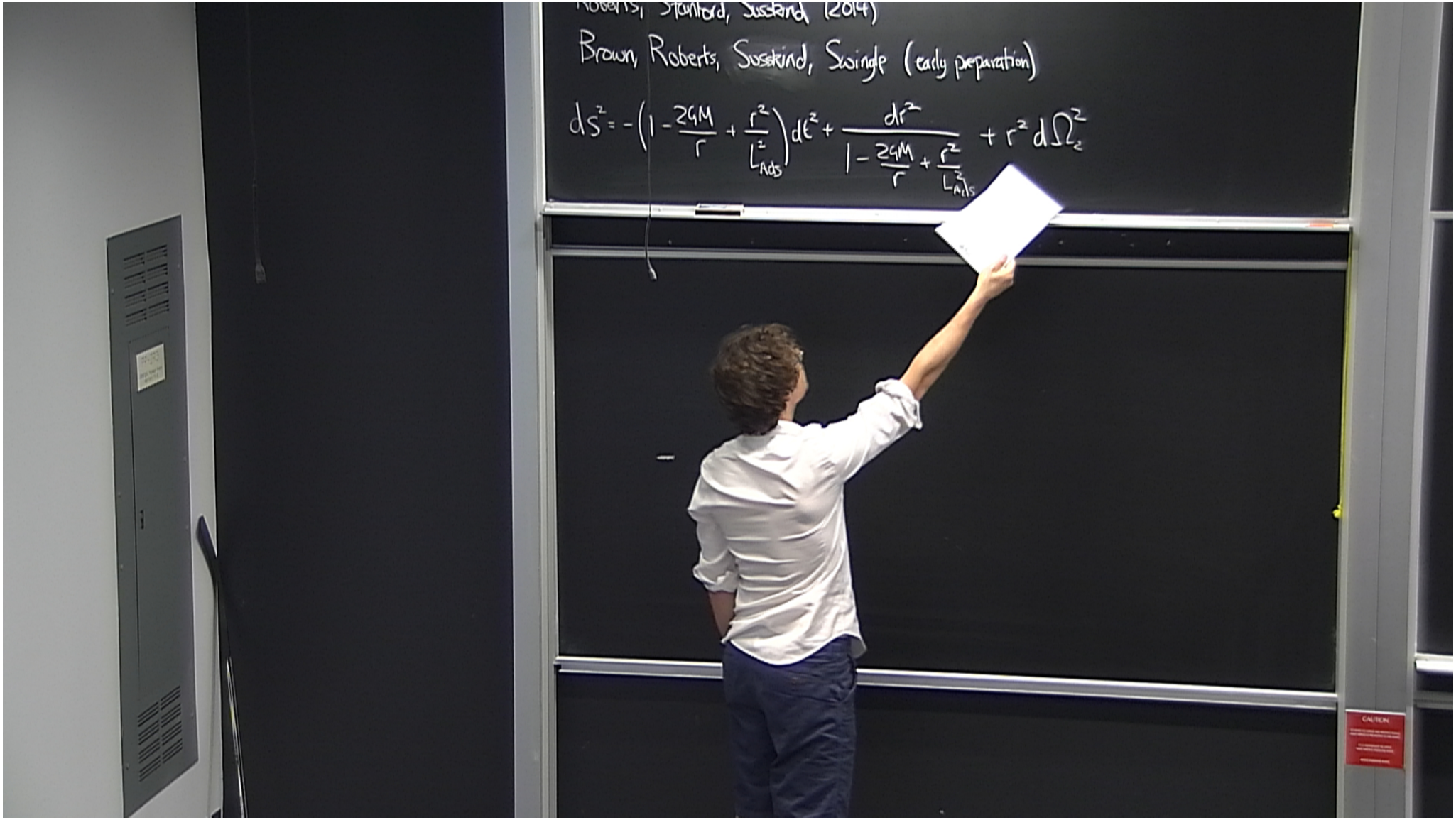
Title: Wormholes and Complexity

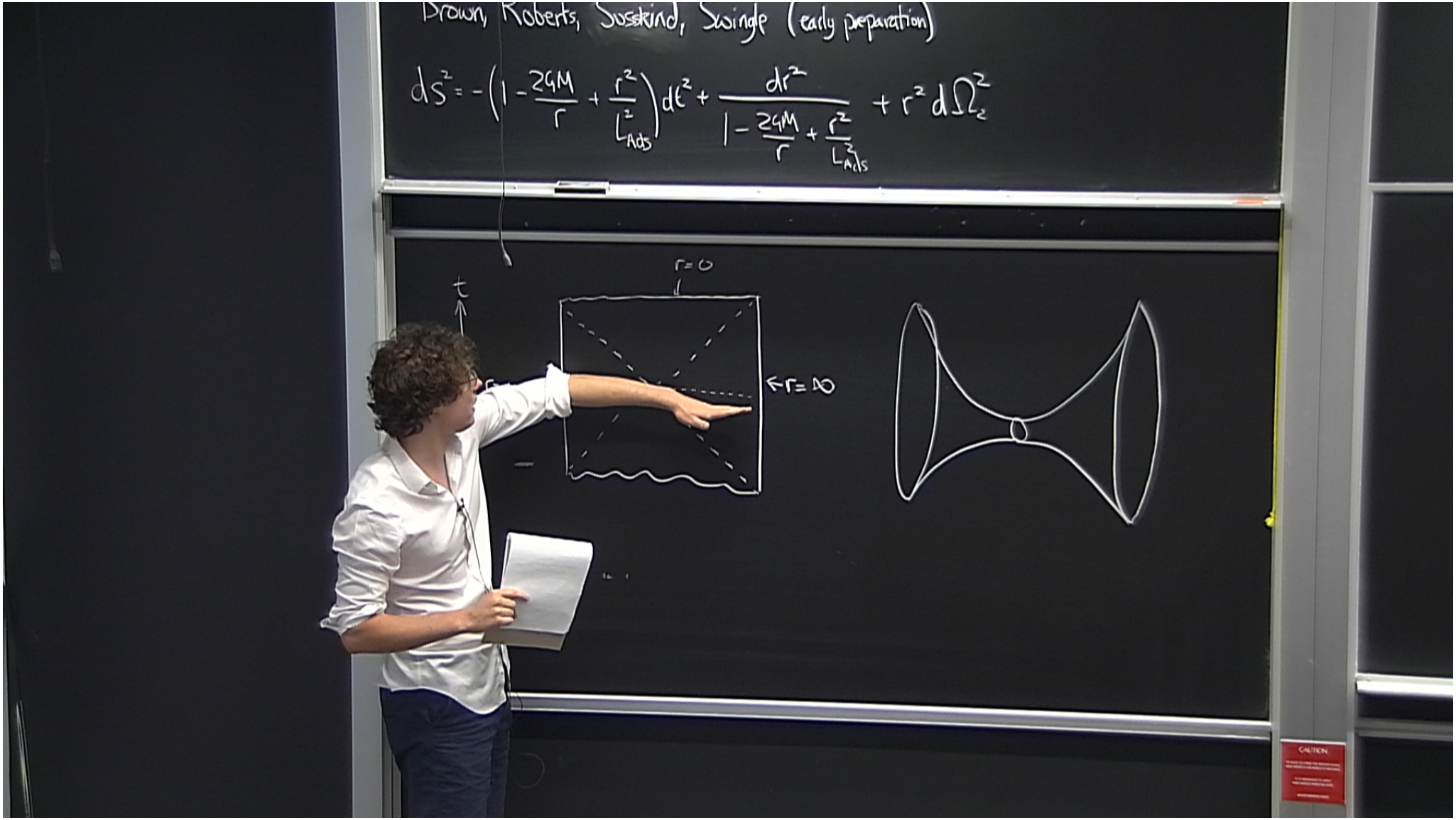
Date: Aug 21, 2015 09:00 AM

URL: http://pirsa.org/15080080

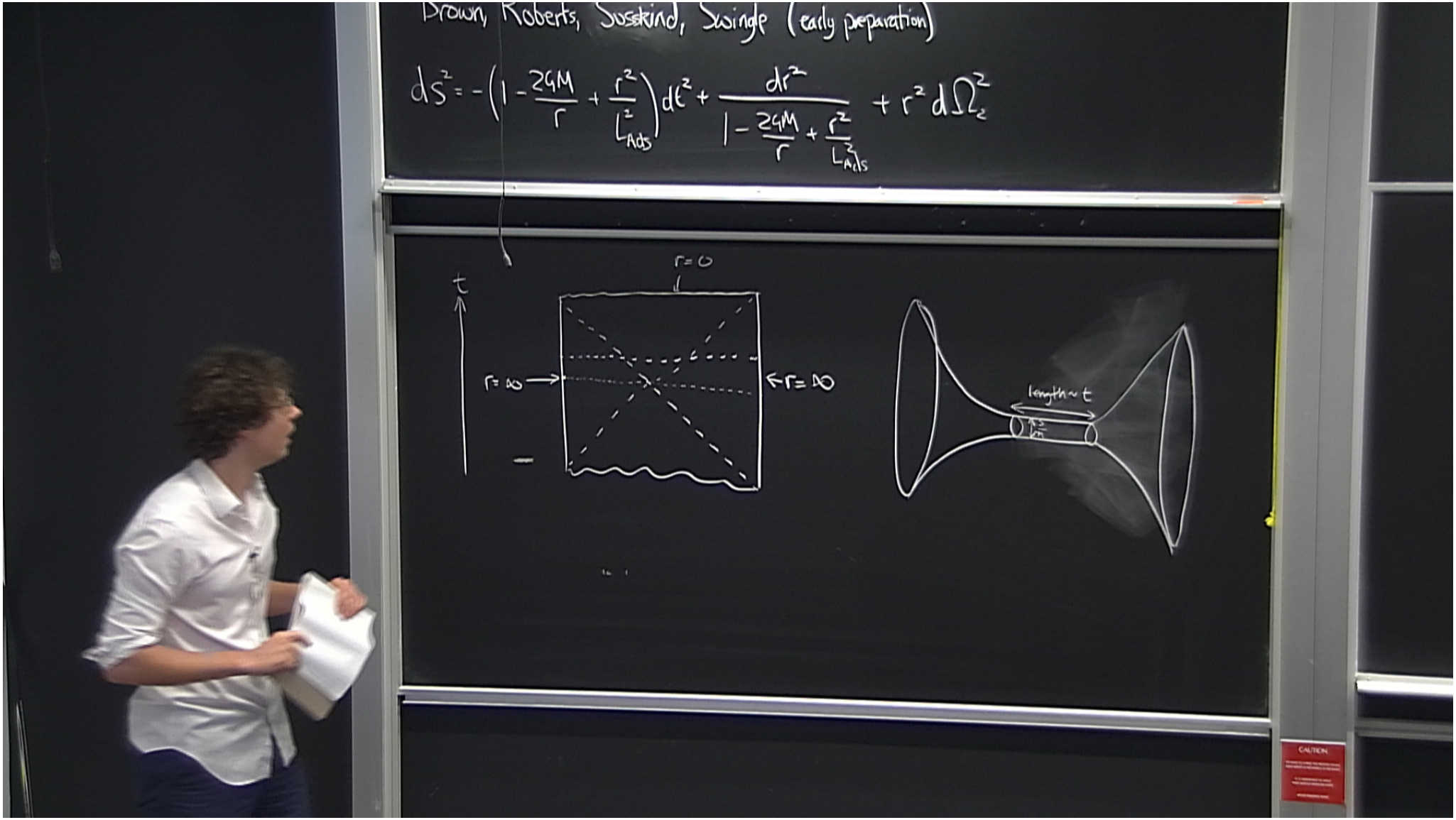
Abstract: I will discuss the connection between wormholes, action, computation and complexity


bles, (Complexity, Computation, and Action
Susskind (2013-2015) ×5
Stanford + Susskind (2014)
Susskind + Zhao (2014)
Roberts, Stanford, Susskind (2014)
Brown, Roberts, Susskind, Swingle (early preparation)
$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}} + r^2 d\Omega_2^2$

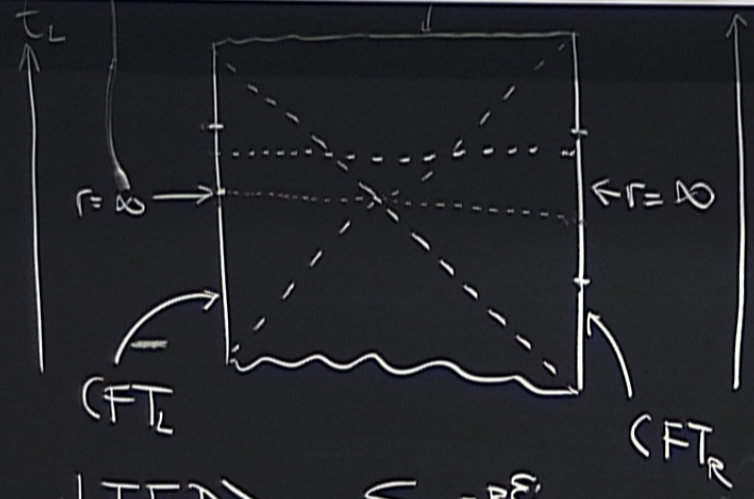

Brown, Roberts, Susskind, Swingle (early preparation)
$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}} + r^2 d\Omega_2^2$

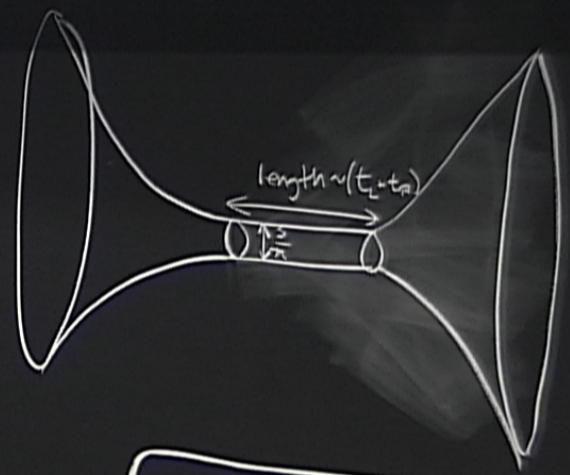

Brown, Roberts, Susskind, Swingle (early preparation)
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r=0
t
r=∞
CAUTION




Brown, Roberts, Susskind, Swingle (early preparation)
$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}} + r^2 d\Omega_2^2$
t
r=0
r=∞
r=∞
length ~ t

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} + \frac{r^2}{L^2_{AdS}}} + r^2 d\Omega_2^2$$

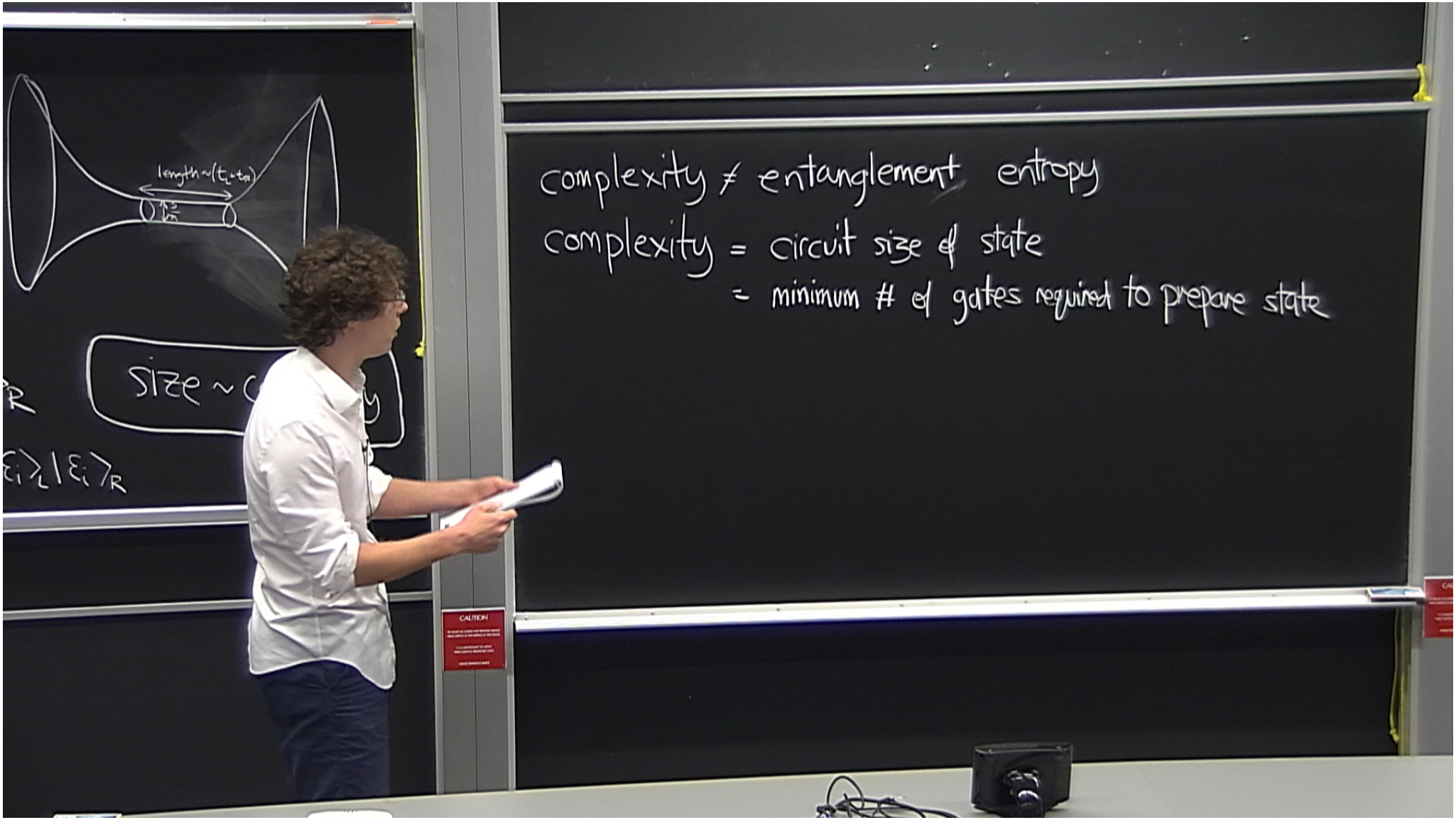


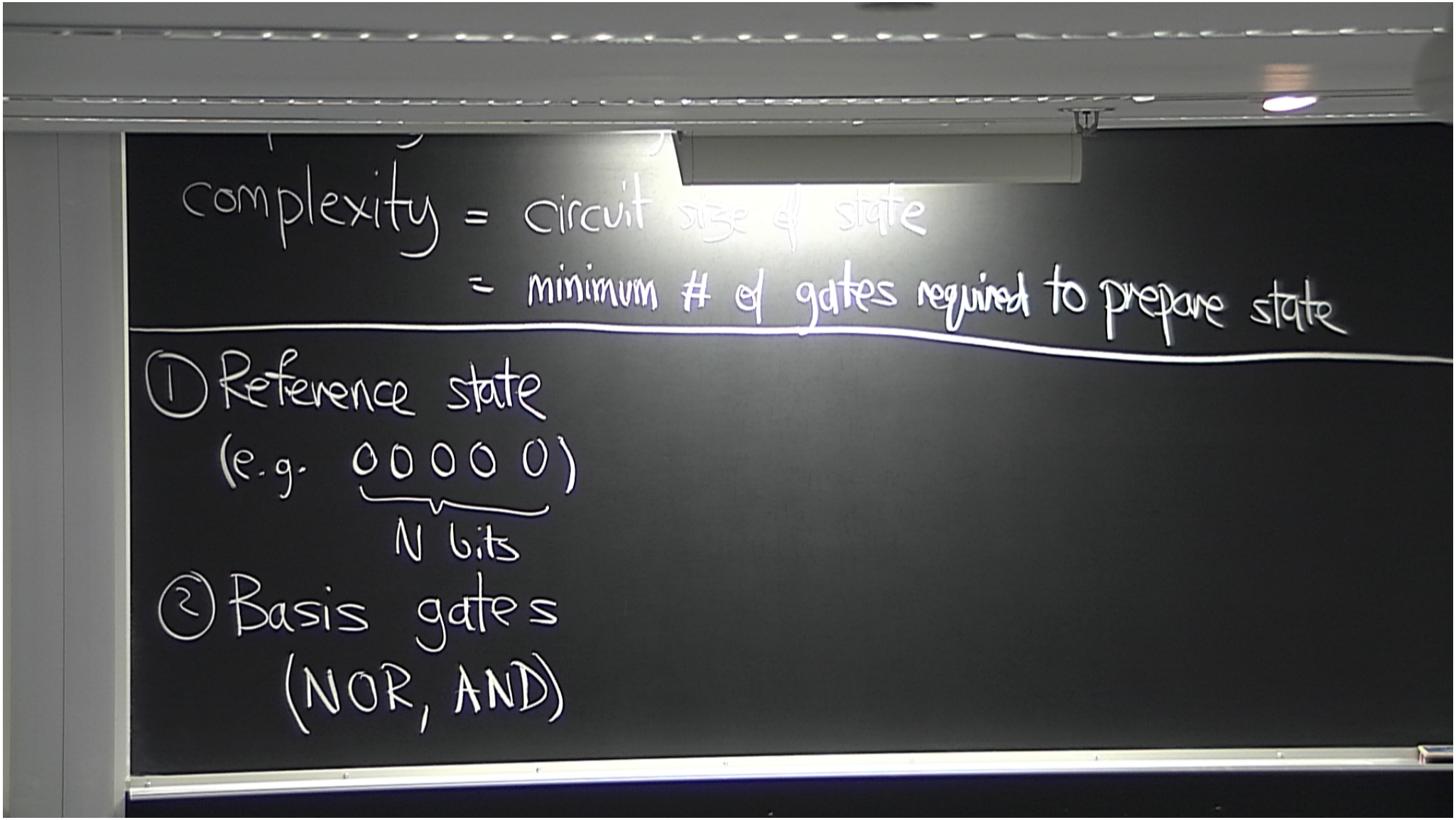


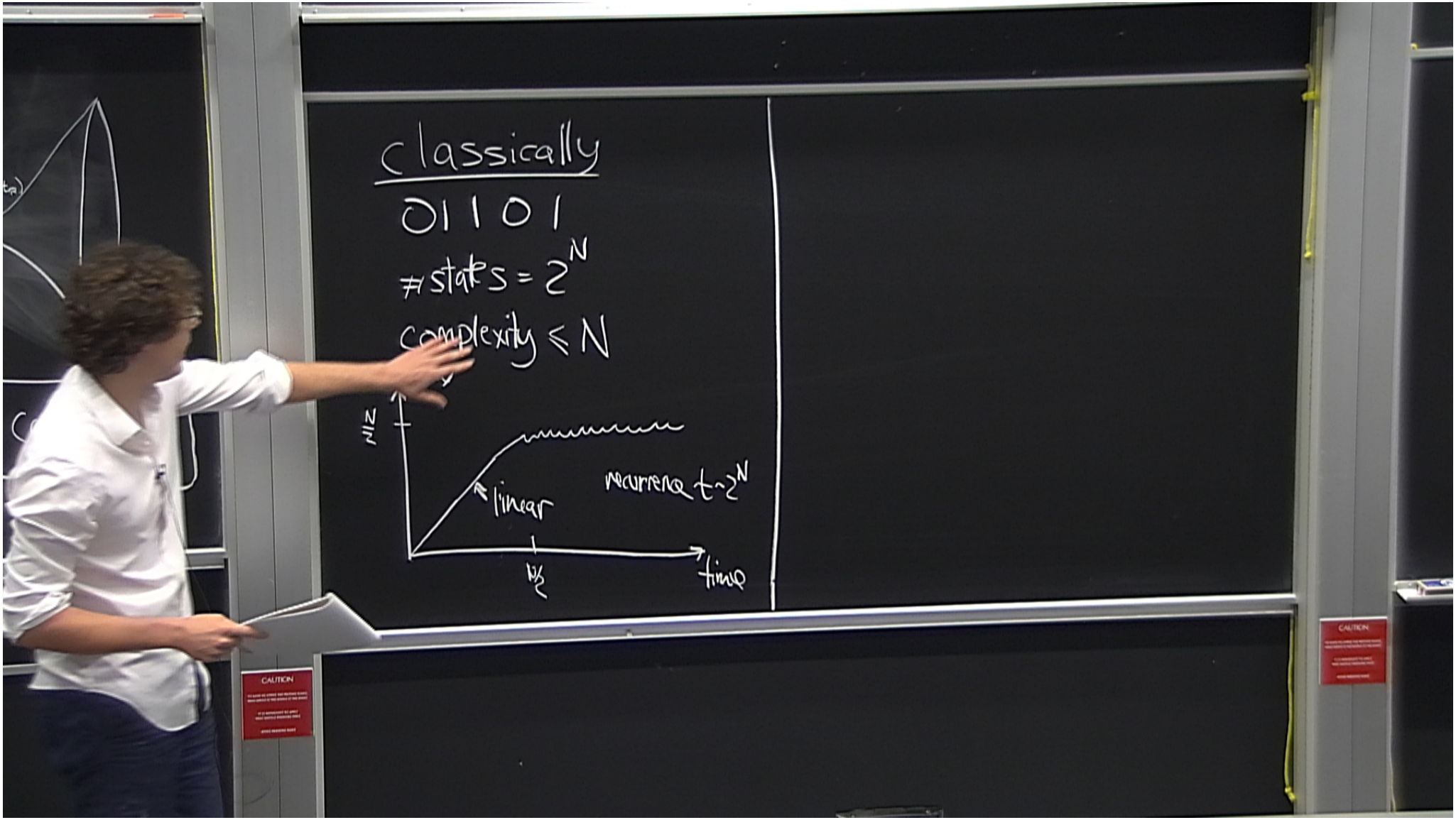
$$|TFD\rangle = \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L |E_i\rangle_R$$

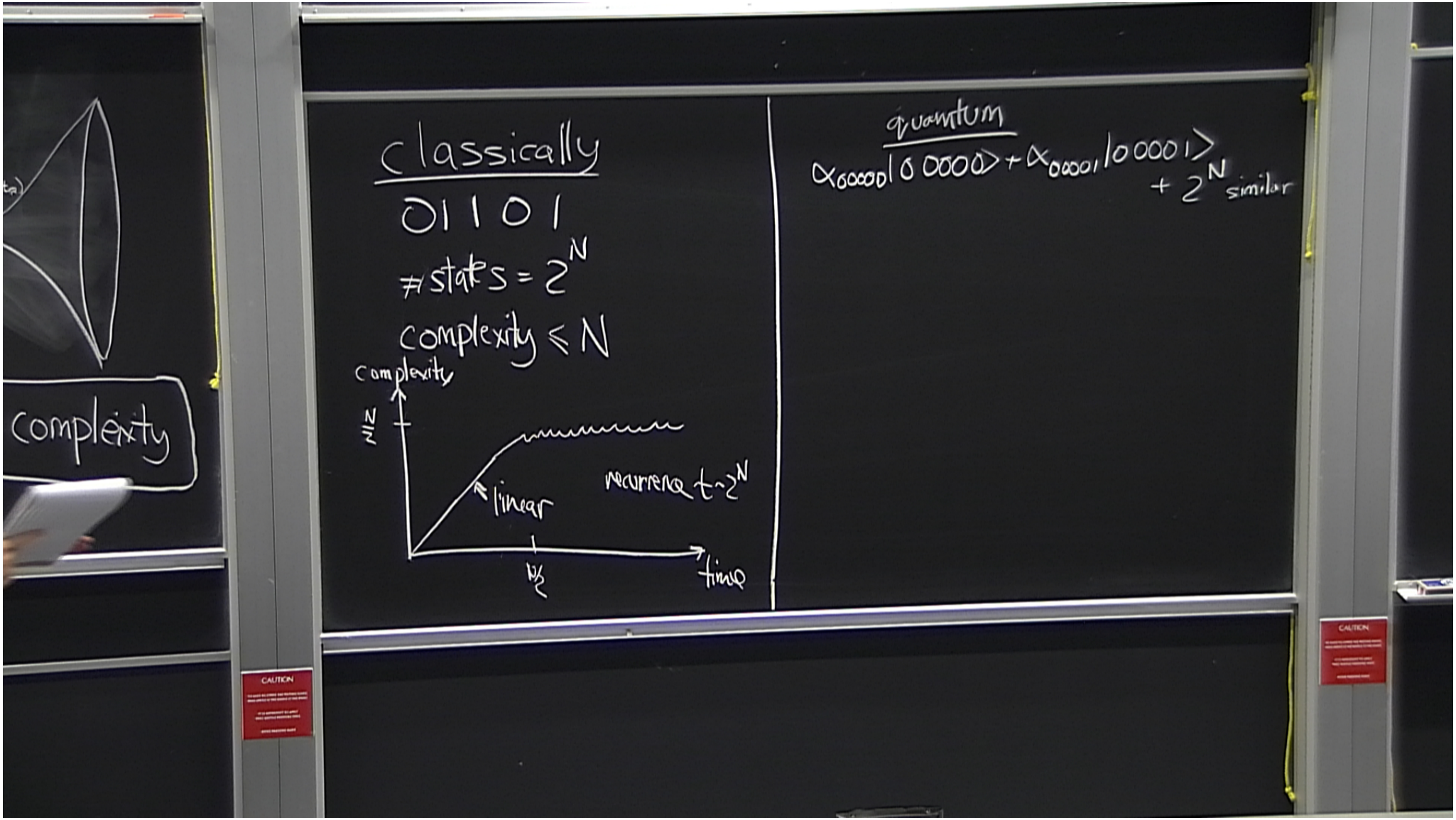
$$|\psi(t_L, t_R)\rangle = \sum_i e^{-\frac{\beta E_i}{2}} e^{iE_i(t_L + t_R)} |E_i\rangle_L |E_i\rangle_R$$

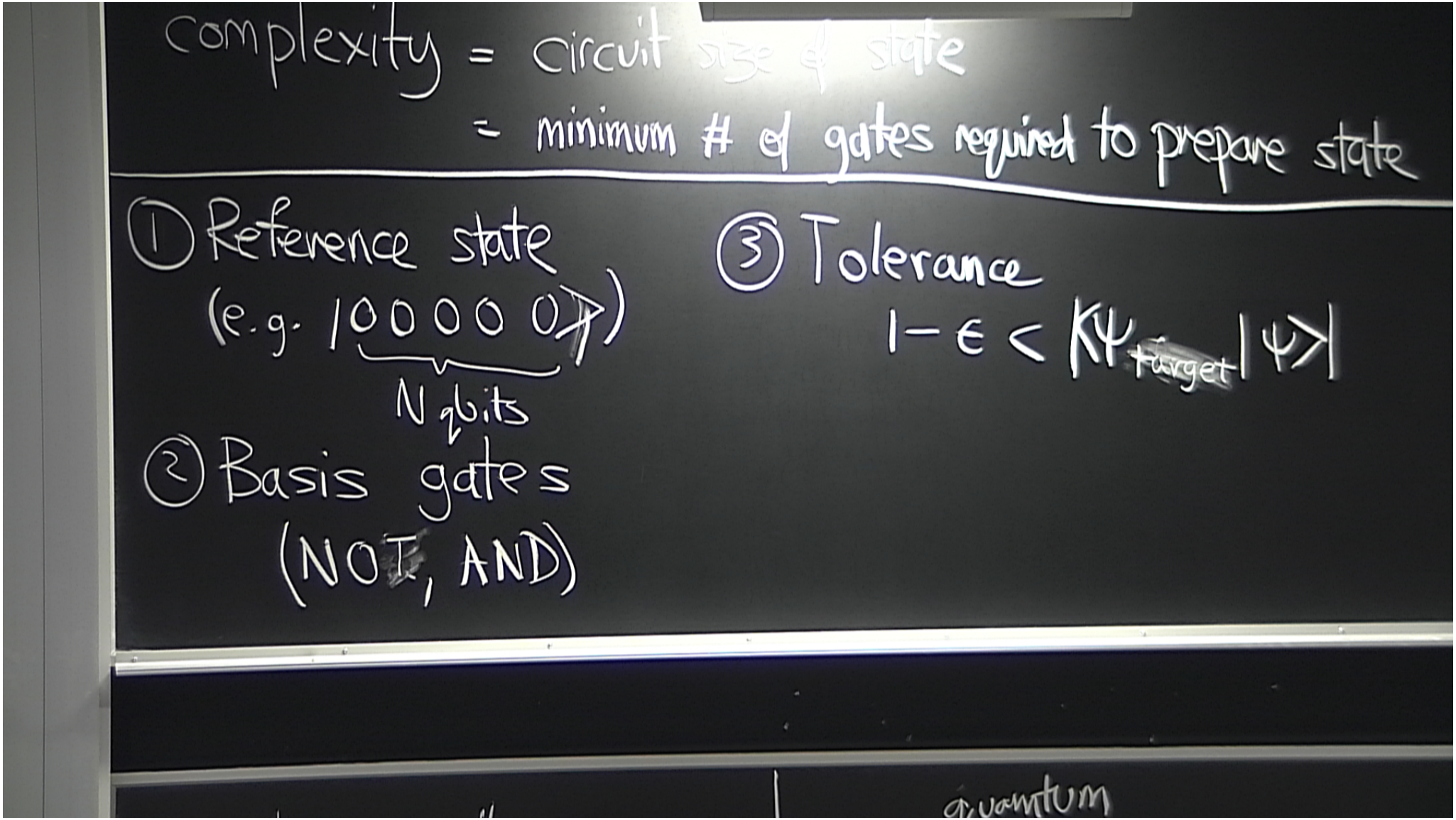
size ~ complexity

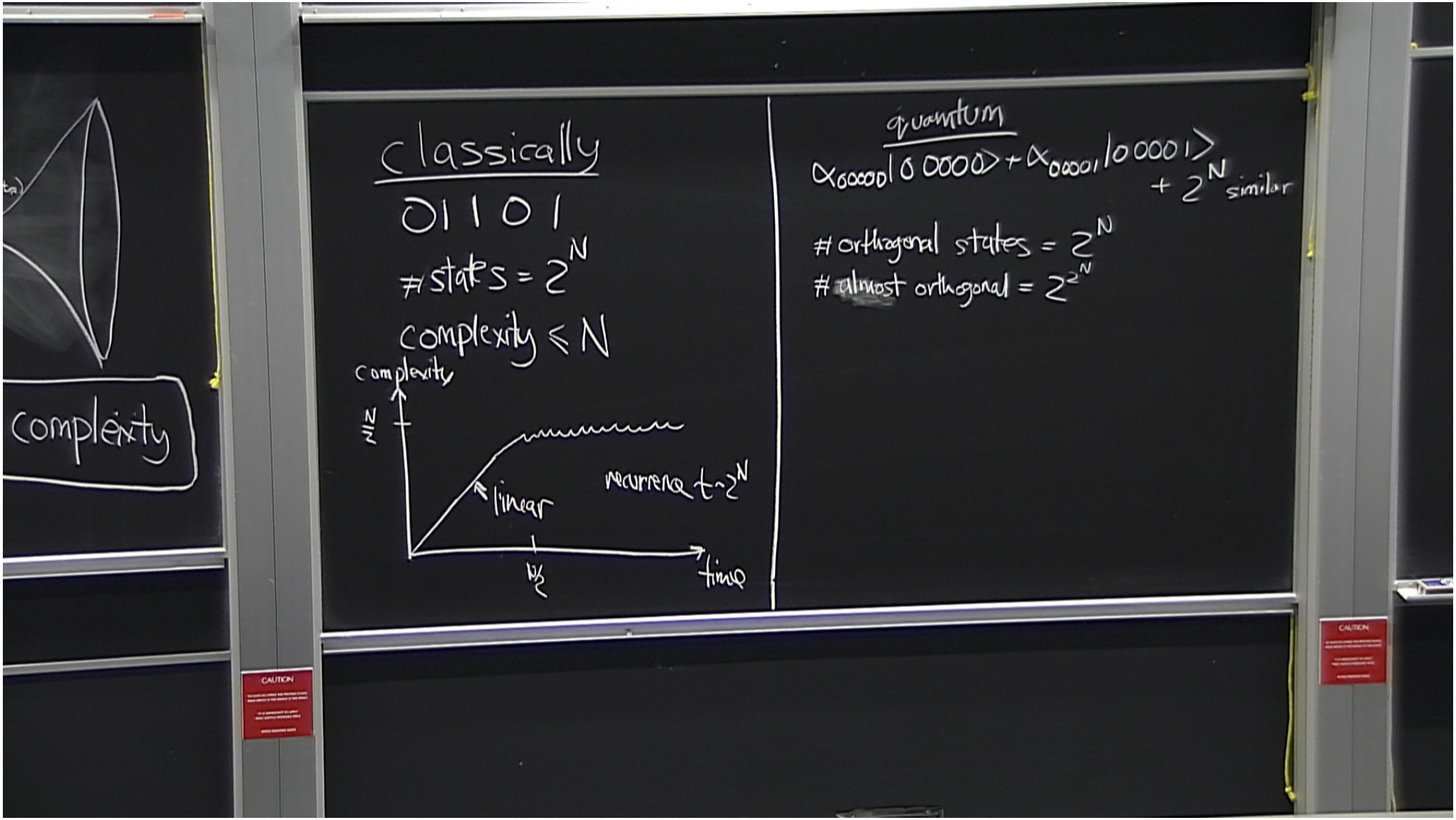

length ~ (t_L + t_R)
size ~
complexity ≠ entanglement entropy
complexity = circuit size of state
= minimum # of gates required to prepare state
CAUTION

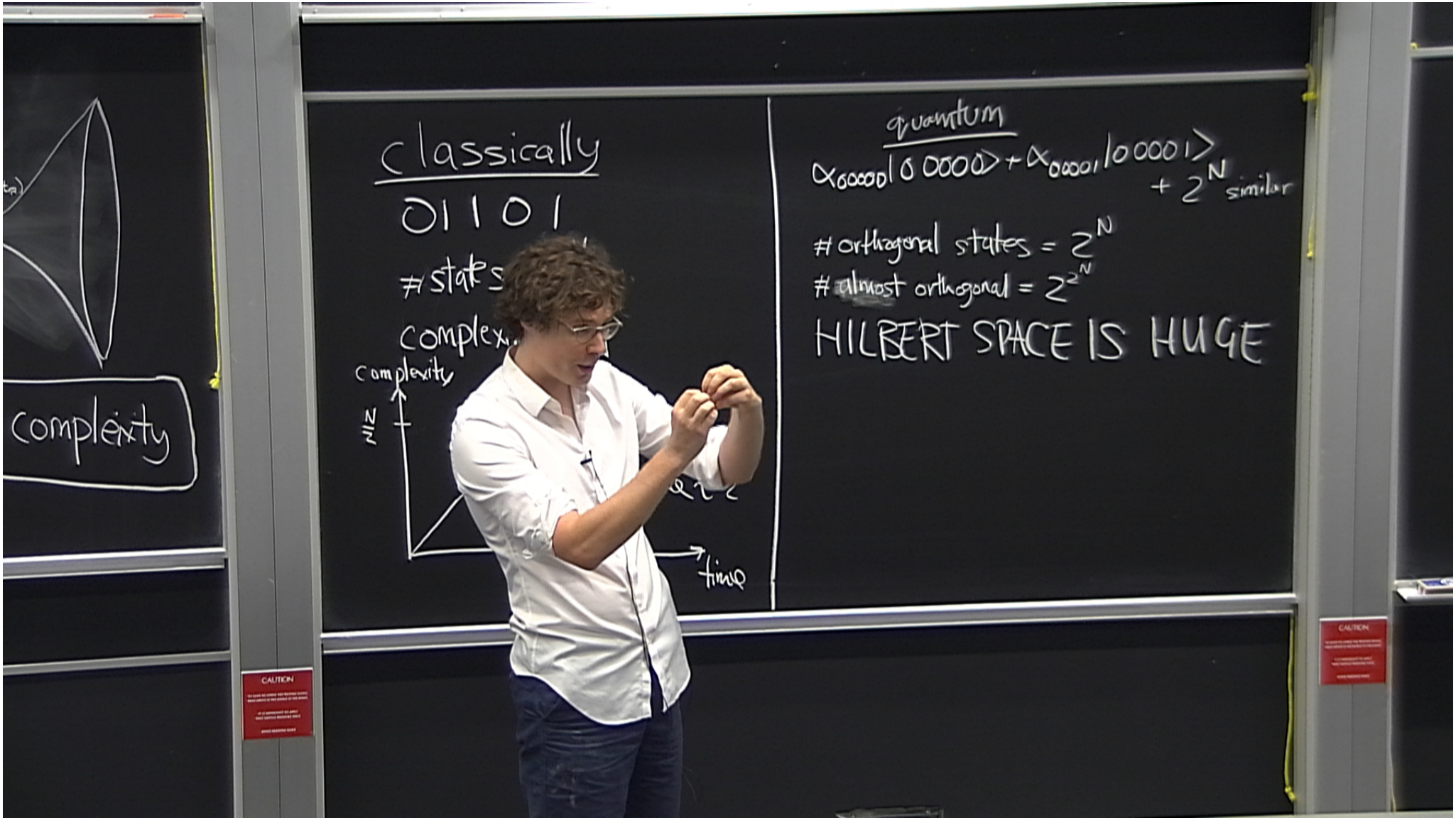

complexity = circuit size of state
= minimum # of gates required to prepare state
① Reference state
(e.g. 00000)
N bits
② Basis gates
(NOR, AND)


classically
01101
#states = 2^N
complexity $\leq N$
N
linear
recurrence $t \sim 2^N$
time


complexity
classically
01101
#states = 2^N
complexity $\leq N$
complexity
N
linear
recurrence $t \sim 2^N$
N/2
time
quantum
$\alpha_{00000}|00000\rangle + \alpha_{00001}|00001\rangle$
$+ 2^N$ similar
CAUTION
CAUTION


complexity = circuit size of state
= minimum # of gates required to prepare state
① Reference state
(e.g. |00000⟩)
N qubits
② Basis gates
(NOT, AND)
③ Tolerance
$1-\epsilon < |\langle\psi_{target}|\psi\rangle|$
quantum

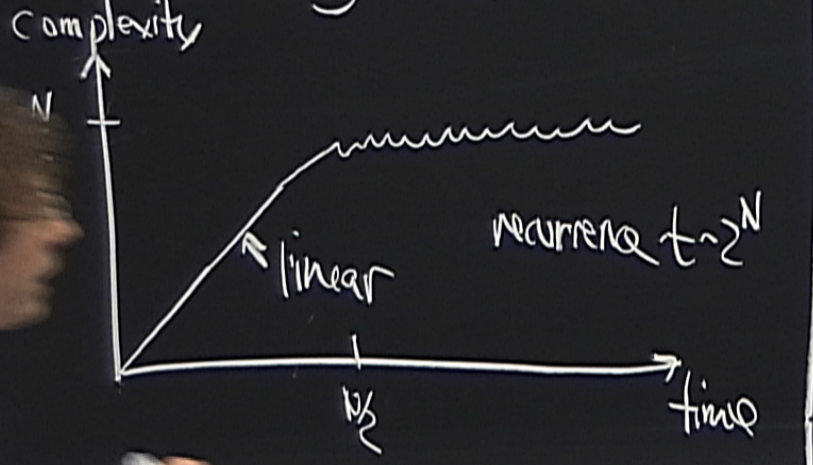

complexity
classically
01101
states = 2^N
complexity $\leq N$
complexity
N
linear
recurrence $t \sim 2^N$
time
quantum
$\alpha_{00000}|00000\rangle + \alpha_{00001}|00001\rangle$
$+ 2^N$ similar
orthogonal states = 2^N
almost orthogonal = 2^{2^N}
CAUTION
CAUTION


complexity
classically
01101
states
complex.
complexity
N
time
quantum
$\alpha_{00000}|00000\rangle + \alpha_{00001}|00001\rangle$
$+ 2^N$ similar
orthogonal states = 2^N
almost orthogonal = 2^{2^N}
HILBERT SPACE IS HUGE
CAUTION
CAUTION

01101

states = 2^N

complexity $\leq N$



$+ 2^N$ similar

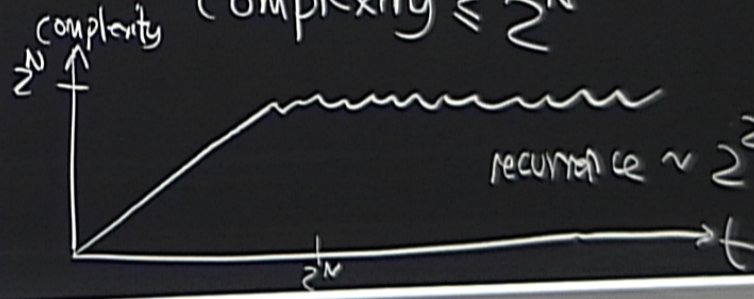
orthogonal states = 2^N

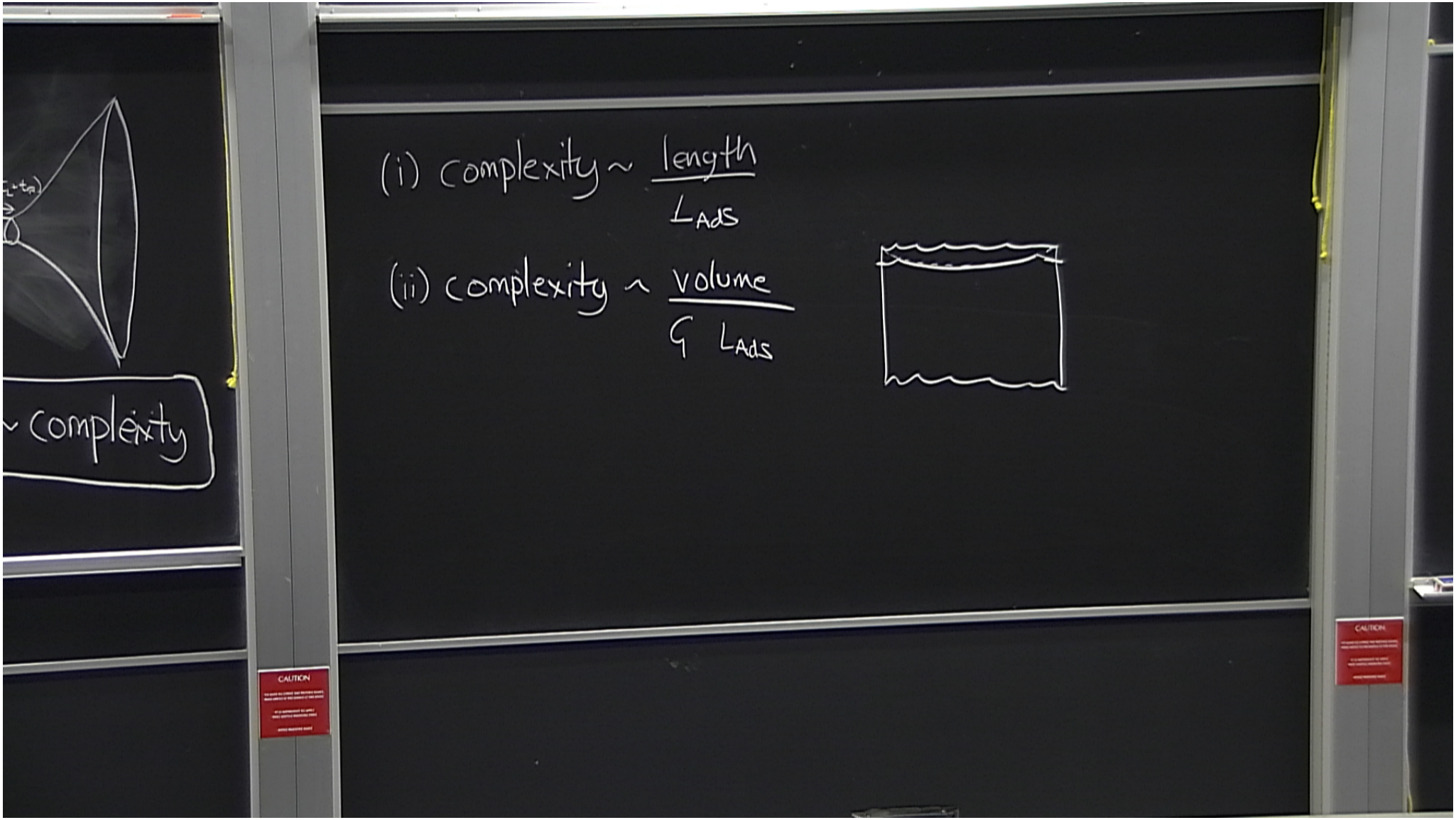
almost orthogonal = 2^{2^N}

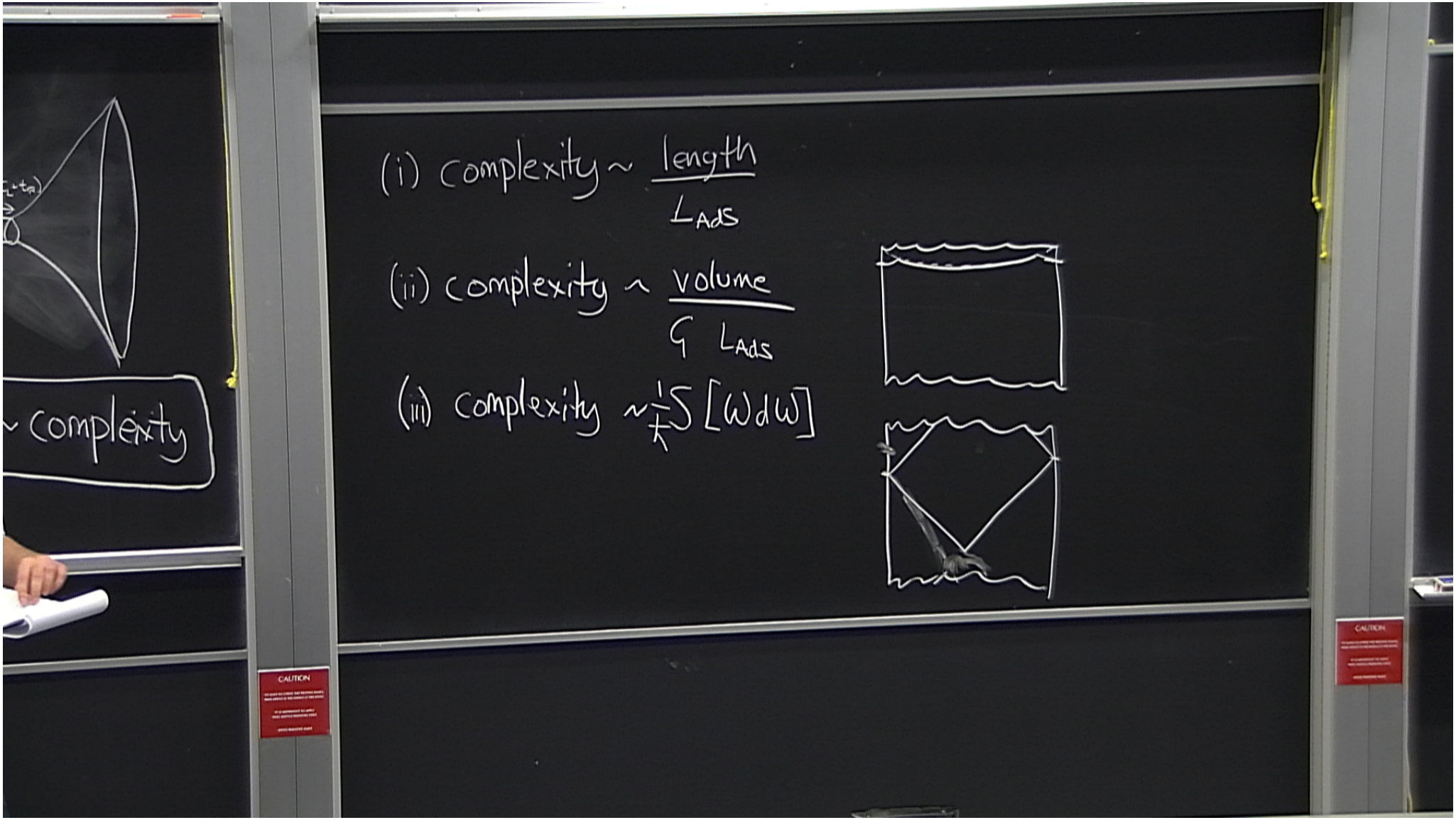
HILBERT SPACE IS HUGE

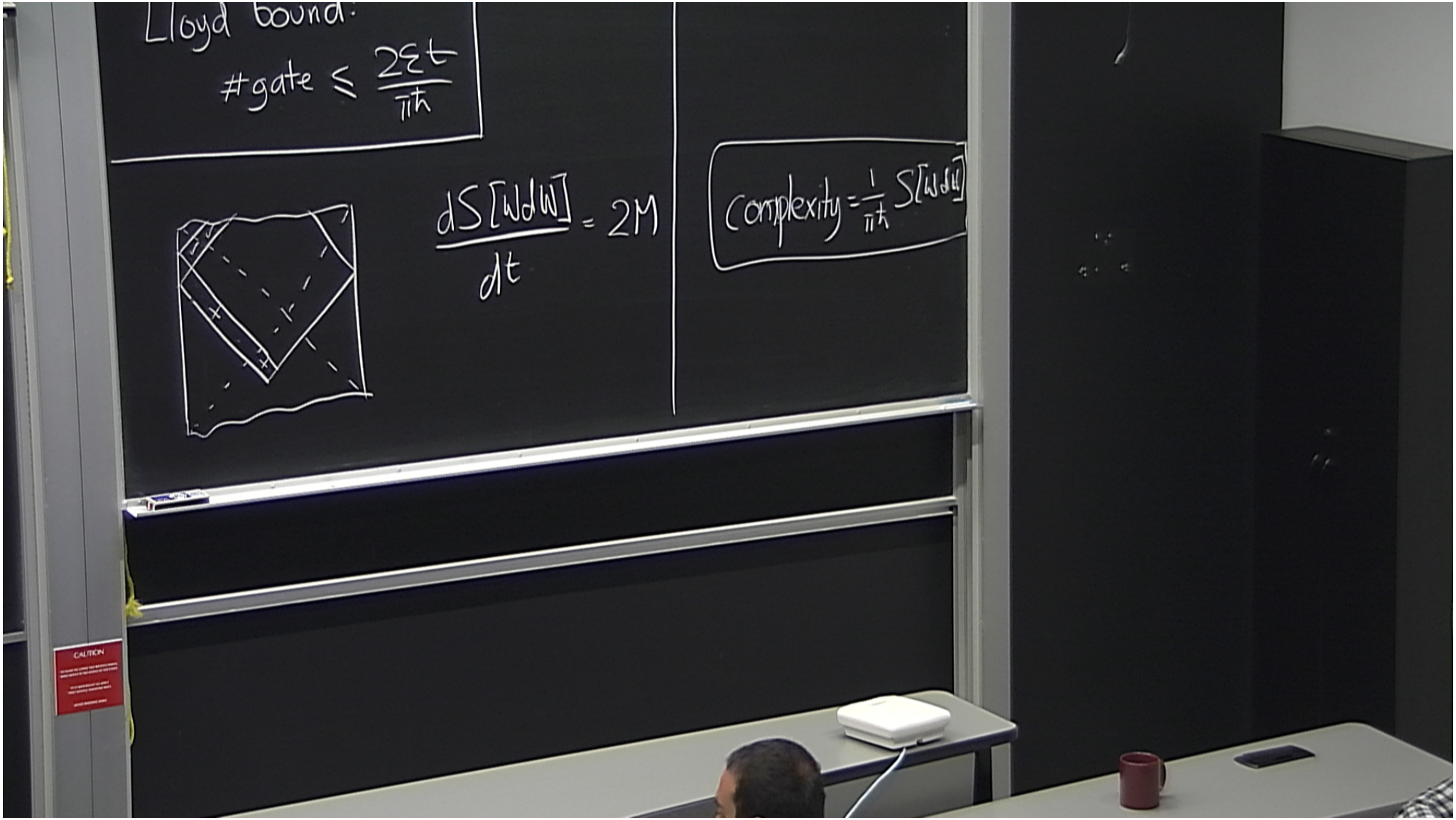
circuits = $\binom{N}{2}^M \sim N^{2M}$

complexity $\leq 2^N$




(i) complexity ~ length / L_{AdS}
(ii) complexity ~ volume / $G L_{AdS}$
~ complexity
CAUTION
CAUTION

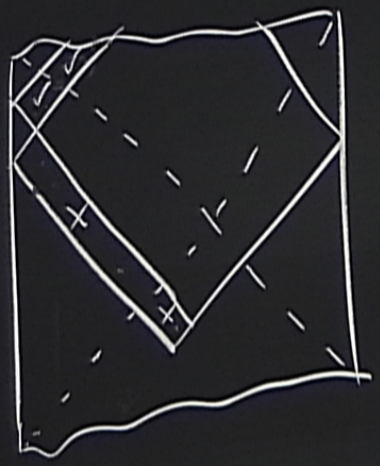

~ complexity
(i) complexity ~ length / L_{AdS}
(ii) complexity ~ volume / $G L_{AdS}$
(iii) complexity ~ $\frac{1}{\hbar} S$ [WdW]
CAUTION
CAUTION


Lloyd bound:
#gate ≤ 2Et/πℏ
dS[WdW]/dt = 2M
Complexity = 1/πℏ S[WdW]
CAUTION


(2013-2015) ×5
Brown, Roberts, Susskind, Swingle (early proposition)
complexity = circuit size of state
= minimum # of gates required to prepare state
① Reference state
(e.g. 10000 ⟩)
N qubits
② Basis gates
③ Tolerance
1 − ϵ < |⟨ψ target | ψ⟩|
CFT
CFT
|TFD⟩ = Σ e^{−βE_n/2} |E_n⟩|E_n⟩
size ~ complexity
(i) complexity ~ length
(ii) complexity ~ volume
Lloyd bound:
#gate ≤ 2Et/πħ
dS[WdW]/dt

Lloyd bound:

$$\#\text{gate} \leq \frac{2Et}{\pi\hbar}$$

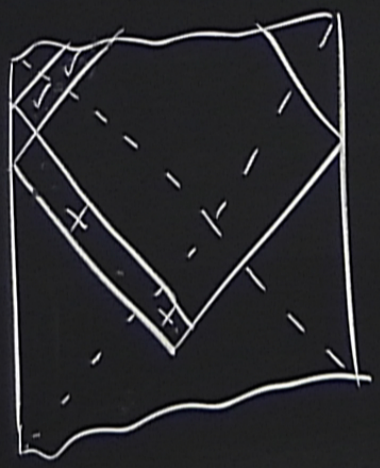


$$\frac{dS[WdW^\dagger]}{dt} \geq M$$

$$\text{Complexity} = \frac{1}{\pi\hbar} S[WdW^\dagger]$$

Lloyd bound:

$$\#\text{gate} \leq \frac{2Et}{\pi\hbar}$$



$$\frac{dS[WdW]}{dt} = 2M$$

$$\text{Complexity} = \frac{1}{\pi\hbar} S[WdW]$$

