Title: Towards a derivation of covariant holographic entanglement

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Abstract: TBA

# Towards a proof of covariant entanglement

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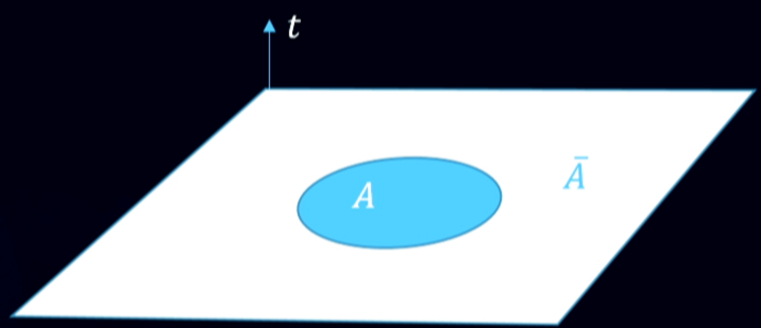
19th August 2015, Perimeter Institute

15XX.XXXXX, IN PROGRESS WITH XI DONG AND MUKUND RANGAMANI

# Entanglement entropy

Divide the system at a fixed time in two sub regions: $A, \bar{A}$

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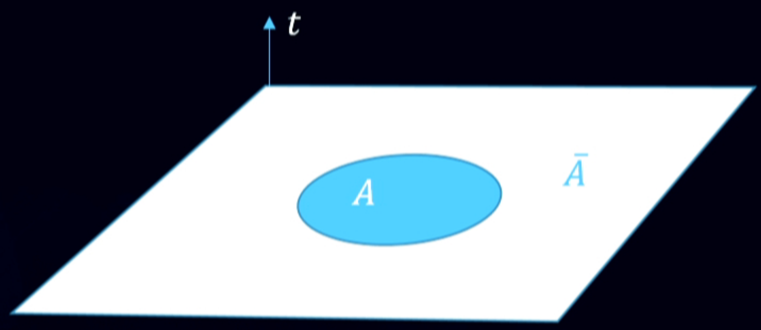


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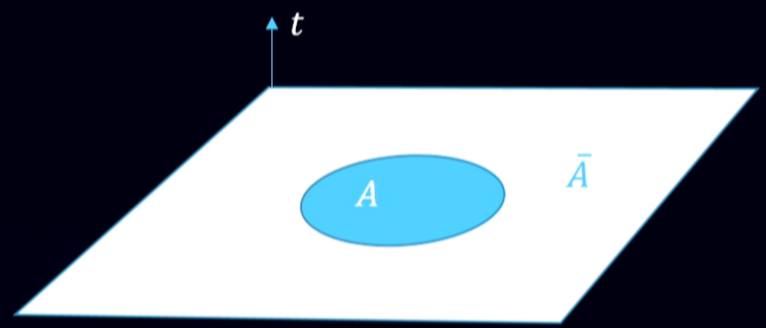
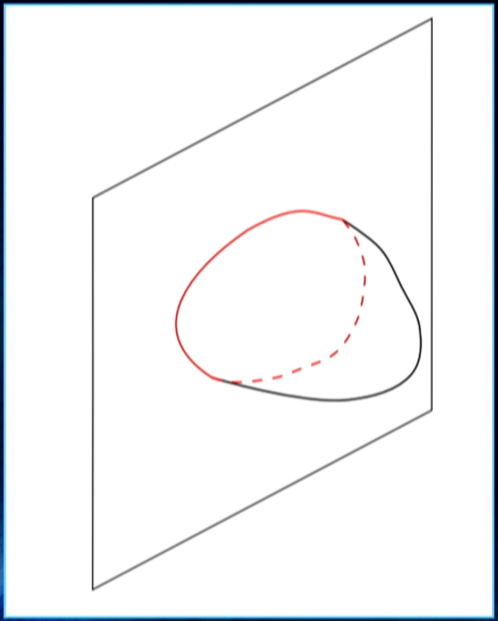



$$S_{EE} = tr\, \rho_A \log \rho_A$$

It is in general very hard to compute.

# Holographic entanglement entropy

[Ryu-Takayanagi] proposed that if the spacetime is static the entanglement entropy of $A$ is given by:



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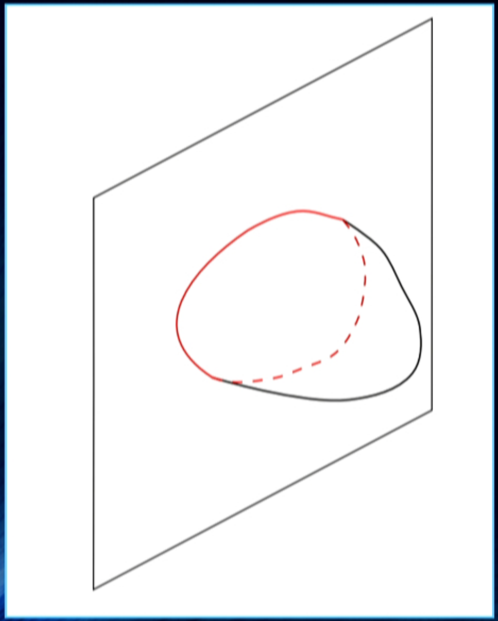


$$S_{EE} = \frac{A_{min}}{4G_N}$$

# Holographic entanglement entropy

For the static case, [AL-Maldacena] used the Euclidean path integral to get the RT formula.

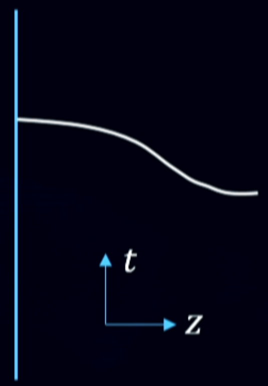
This derivation can't be directly applied to a time dependent case because it assumes time reflection symmetry.

In general, one needs an in-in contour to compute $\rho_A(t)$ .

# *Covariant* entanglement entropy

Time dependent geometries, $g_{\mu\nu}$ is a non trivial function of time.

[Hubeny-Rangamani-Takayanagi]: EE is given by an *extremal* area




$$S_{EE} = \frac{A_{ext}}{4G_N}$$

Time dependence is generic, can't use LM.

# Motivations for proving HRT

- The construction of LM has allowed several generalizations: higher derivatives [Dong,Camps], quantum corrections [Faulkner-AL-Maldacena], ... Generalizations make sense, can we understand the time dependent situation in a similar way?
- LM used Euclidean time, it makes a Lorentzian interpretation complicated. Have a clearer picture.
- Understand directly the meaning of a Lorentzian computation of entanglement entropy can help us understand better what one means with subregion-subregion duality.

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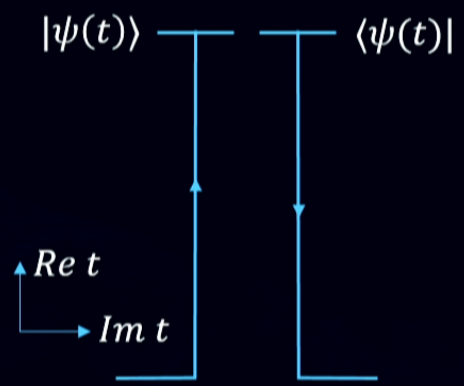
# FIELD THEORY

# Real time path integral

To describe $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ using a path integral, we need to double the fields. It will be constructed with a Schwinger-Keldysh contour.




Normally we think about SK for correlators but it is actually also used to compute effective actions in mixed states [Feynman-Vernon].

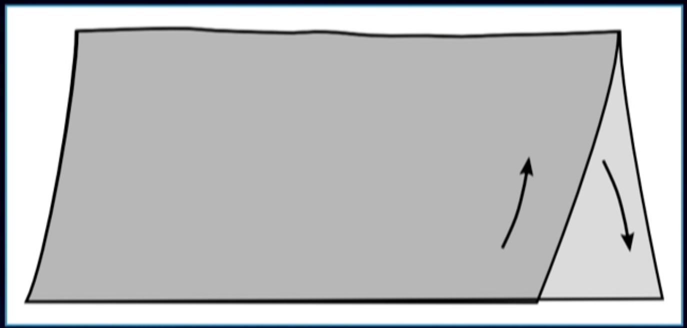
# Real time path integral

Path integral for $S^q = \frac{1}{1-q} \log tr\, \rho_A(t)^q$ , $S_{EE} = S^1$.

In QFT's we think of $\Psi(t, \phi_0^-)$ as the path integral up to $t$, with boundary conditions for the fields there.

"Glue" by summing over the boundary conditions

$$\mathrm{tr}\, \rho(t) = \int [D\phi_0^{\pm}] \Psi(t, \phi_0^-) \overline{\Psi}\, (t, \phi_0^+) \delta(\phi_0^- - \phi_0^+)$$



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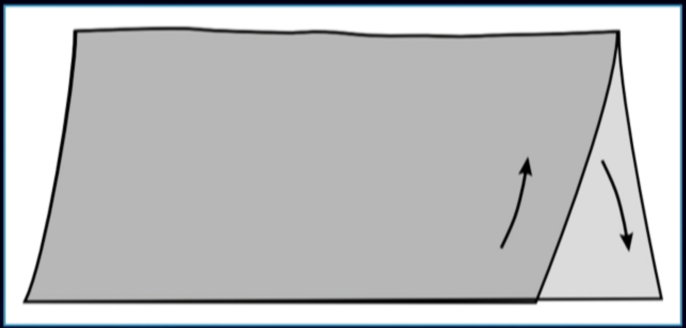
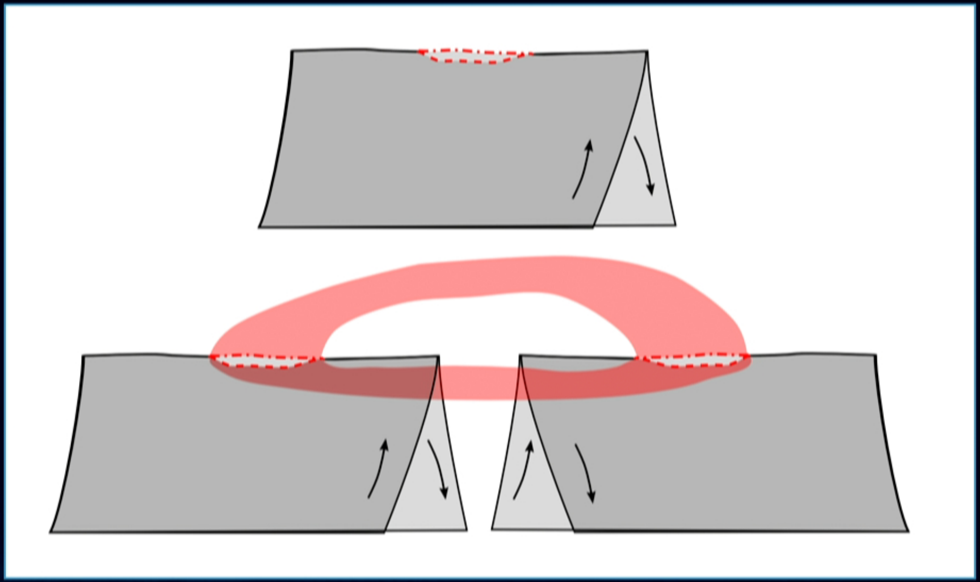


# Reduced density matrix

We can compute $\rho_A$ by gluing in $\bar{A}$ and leaving the $A^-, A^+$ regions untouched.
$tr\ \rho_A^q$ will be given by gluing the remaining $\phi_A^{\pm}$: $\phi_A^{i,-} = \phi_A^{i+1,+}$



# Path integral for the Renyi entropy

The set up is clear, but the geometry is kind of messy: $q$ patches of coordinates $t_j, x_j$.

$\partial A: t_j = 0, x_j = 0$ is a codimension 2 fixed point of the $Z_q$ symmetry that exchanges the replicas (cyclicity of the trace).

From an observer that only has access to $D[A]$, the $Z_q$ symmetric fixed point acts a source of local temperature: $\beta = 2\pi q$. Geometry only smooth (locally flat) if $q = 1$.

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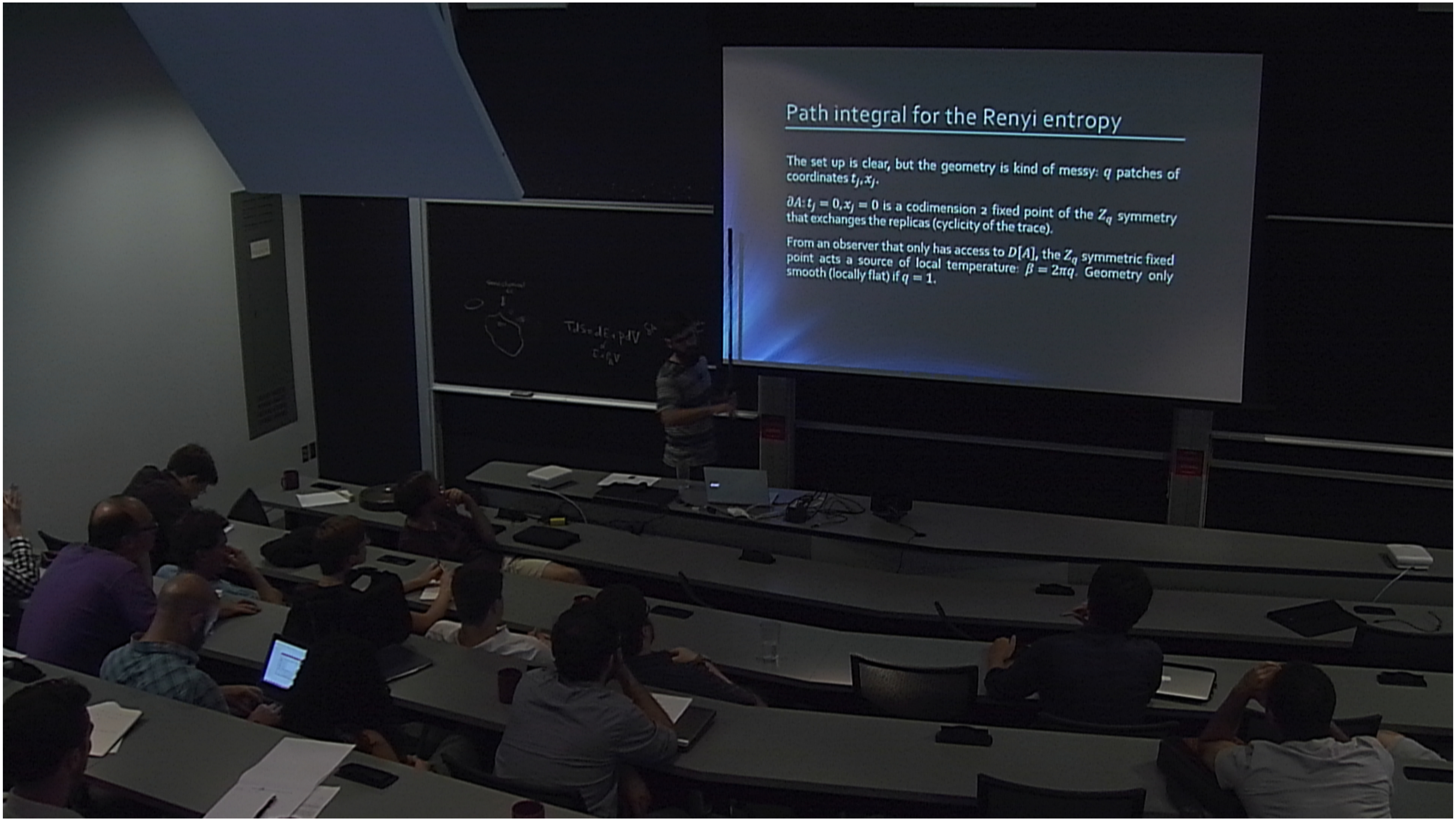
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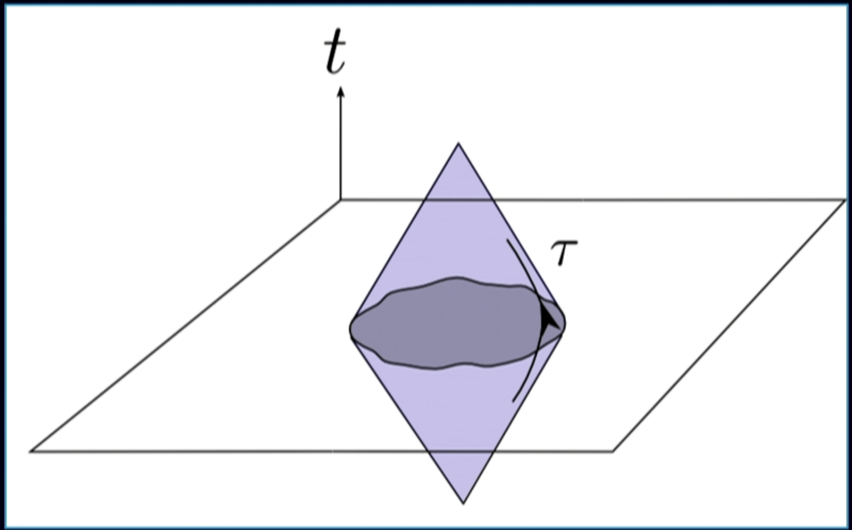
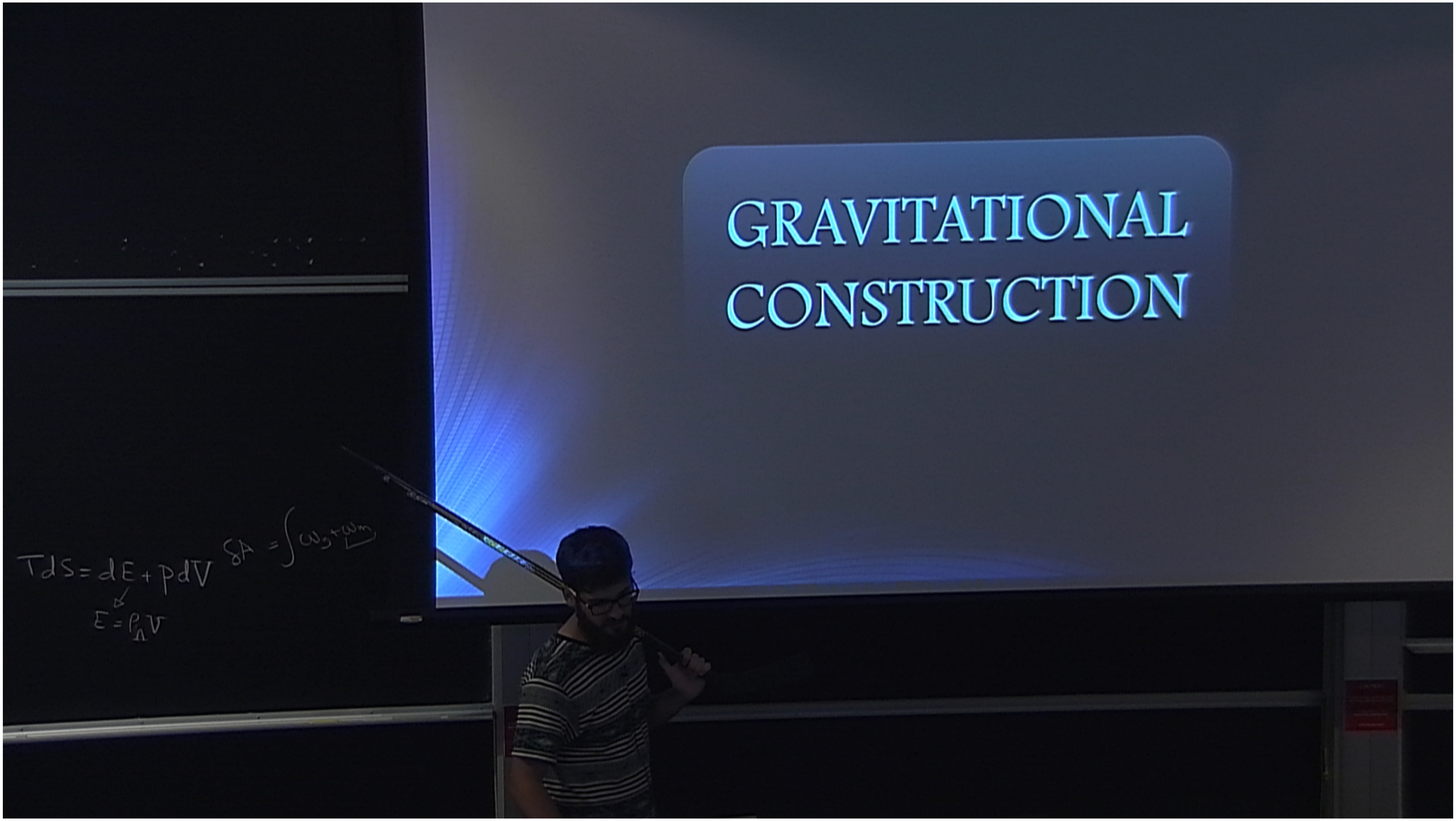



Renyi's: different state where the local Rindler temperature is $2\pi q$.
Equivalently, a complicated replicated geometry.


GRAVITATIONAL
CONSTRUCTION
TdS = dE + pdV
E = ρ_Λ V
δA = ∫ ω_g + ω_m

# Preliminaries

We want to compute $tr\ \rho_A^q$. Roughly two steps:

- Compute $\rho(t)$ in the bulk, dual of Schwinger-Keldish.
- $tr(tr_{\bar{A}}\rho)^q$ in the bulk?

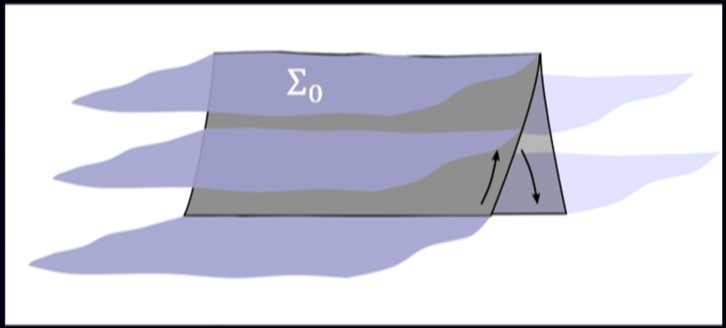
First step is well understood.

Second step is tricky. Use guidance from Euclidean case.

→ Put the two ingredients together.

# Dual of Schwinger-Keldysh

[Skenderis-vanRees] discussed how one should think about the dual of SK contours. They are basically what people call “time folds”, ie filling in the SK contour in the bulk.

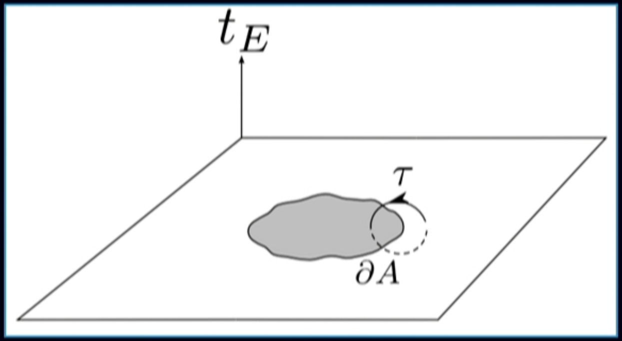



One evolves the bulk up to a Cauchy slice $\Sigma_0$ and then comes back.

The boundary conditions at $\Sigma_0$ are basically continuity of the fields across it.

# Interlude: review of Euclidean case

In [AL,Maldacena], considered the case with $Z_2$ symmetry. In that case, we can go to the Euclidean setup, where have to construct the geometry dual to a boundary with a conical excess $2\pi q$ around $\partial A$.

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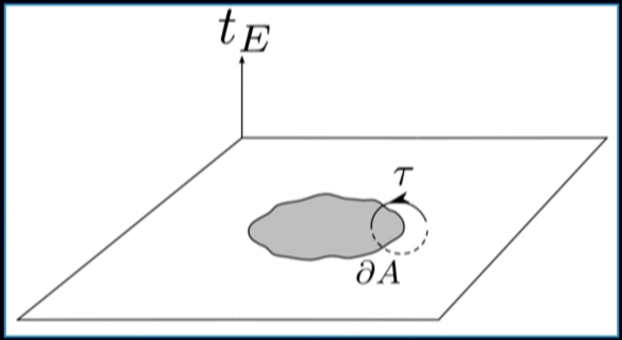


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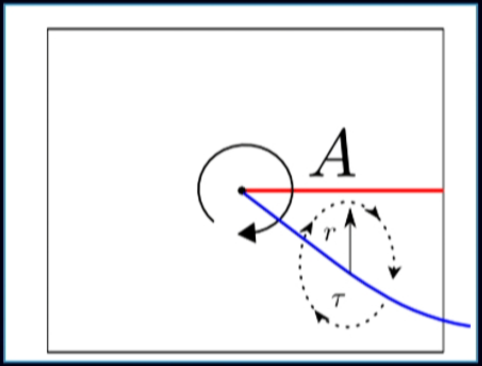
$\tau \to \tau + 2\pi$ moves us from $A$ in one replica to the next. A global $2\pi$ rotation is a symmetry, the $Z_q$ symmetry that exchanges replicas.

The boundary geometry has a $Z_q$ fixed point: $\partial A$.

Assumption: $Z_q$ symmetry extends to the bulk .

Fixed point in the bulk, $\mathcal{E}$.



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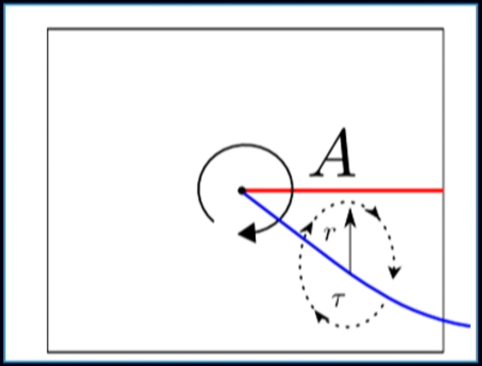

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Dual bulk geometry: smooth, no conical singularity.

$Z_q$ fixed point will be a special codimension two surface in the bulk.

The metric near $\mathcal{E}$ is completely determined by the symmetries:

$$ds^2 = dr^2 + \frac{r^2}{q^2} d\tau^2 + \left(\gamma_{ij} + K^0_{ij} r^q \sin\tau + K^1_{ij} r^q \cos\tau\right) dy_i dy_j + O(r^{2q})$$

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Replica symmetry, same action in each replica $I_q = q\,\hat{I}_q, \tau \in (0,2\pi)$.

$\hat{I}_q$ : original geometry in the presence of codimension 2 fixed point, deficit angle $\frac{2\pi}{q}$.

Smooth parent space, but from this "orbifolded" point of view, it looks singular. Analytic continuation to real $q$ : simply changing the tension of the "cosmic string".

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# Real time $S^q$

Let's pick a spacelike surface in the bulk $\Sigma_0$ where we do SvR.

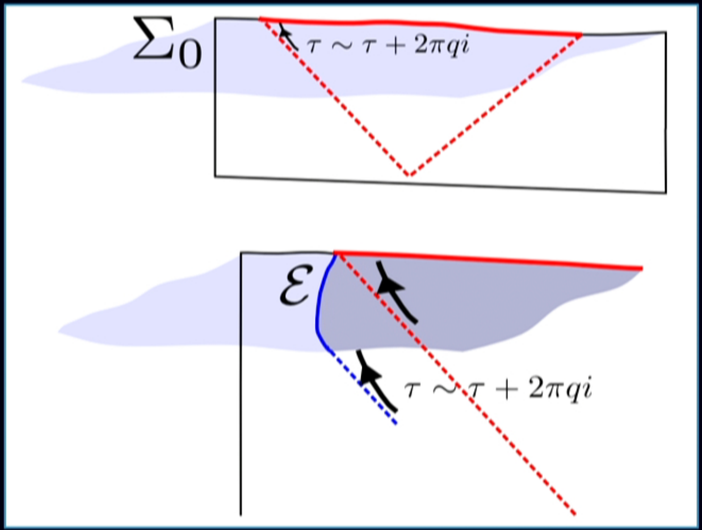
Boundary conditions for fields close to $\partial A$: local temperature $\beta = 2\pi q$.

Global $Z_q$ symmetry, $\partial A$ is a fixed point with respect to this symmetry.

Consider $\mathcal{E} \subset \Sigma_0$, the extension of the fixed point to the bulk.

# Real time $S^q$

Extend $\tau$ direction to the bulk, local temperature $2\pi q$.




These geometries will be real.

# Real time $S^q$

For the Renyis, $Z_q$ symmetry and smoothness constraints the metric near $\mathcal{E}$ :

$$ds^2 = dr^2 - \frac{r^2}{q^2} d\tau^2 + \left(\gamma_{ij} + K^0_{ij} r^q \sinh\tau + K^1_{ij} r^q \cosh\tau\right) dy_i dy_j + O(r^{2q})$$

This is smooth because locally flat.

To compute the action, analytically continue in $q$ by "orbifolding": non-smooth metric, local temperature $2\pi/q$ .

"Kinematics": original SvR geometry with a codimension two defect that changes the local temperature to $\frac{2\pi}{q}$.

# Real time $S^q$

Pick an action. For Einstein, the previous local analysis gives the extremality condition and evaluating the entropy gives $\frac{A}{4G_N}$.

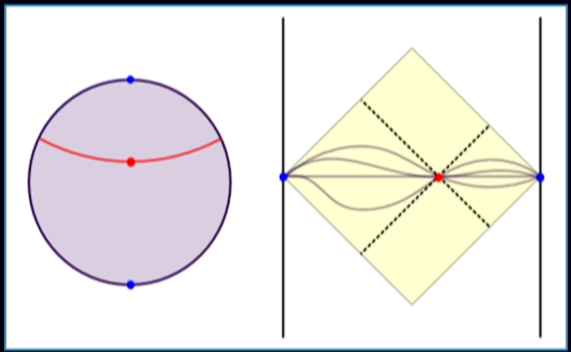
This gives HRT, $\mathcal{E}$ has to be in $\Sigma_0$.

Not all $\Sigma_0$ are good: have to be able to accommodate a singularity.

# Comments

- "Dual" of tracing out in special $\Sigma$'s : those which contain the extremal surface.

Only in these surfaces one can divide $\Sigma = \Sigma_A \cup \Sigma_{\bar{A}}$ in a sensible way
No surprise, same thing happens in the eternal black hole.



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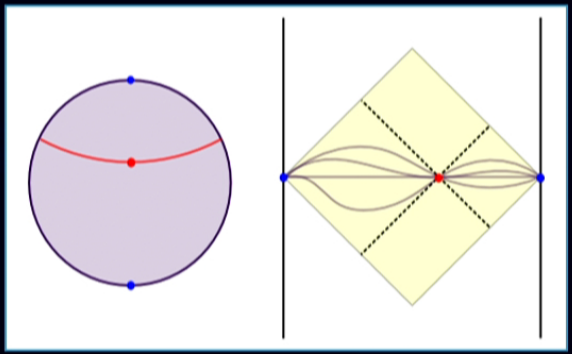

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# Conclusions

- In the $Z_2$ symmetric case one can go to Euclidean space and make sense of the analytic continuation of the replicated geometry in the bulk.
- Lorentzian case: real time path integrals set up the boundary conditions of the problem in terms of changing the local temperature. One can generalize the previous derivation to dynamical situations.
- Intuitive picture of tracing out in the bulk.

# Open questions

- Can one derive (H)RT without using the replica trick? It looks like there should be a way of proving it that just relies in gauge invariance: under what conditions can one split diffeomorphisms in a theory of gravity?
- This looks like "tracing out/cutting" in the bulk. Can one go the other way, ie EPR $\rightarrow$ER ? Under what conditions can one glue stuff in gravity?
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THANK

YOU!!!