Title: Diffeomorphism-invariant observables and nonlocality

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Abstract: In a theory of gravity, observables must be diffeomorphism-invariant. Such observables are nonlocal, in contrast with the usual formulation of local quantum field theory. Working to leading order in Newtons constant G, I'll describe a construction of diffeomorphism-invariant observables for a scalar field coupled to gravity that closely parallels an analogous construction for charged particles in electrodynamics. These observables acting on the vacuum create scalar particles together with their (linearized) gravitational dressing. The commutator of two such spacelike-separated observables is nonvanishing at order G, and is related to the gravitational potential between two masses.
Based on arXiv:1507.07921 with Steve Giddings.

# Diffeomorphism-invariant observables and nonlocality

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Quantum Information in Quantum Gravity II

ar$\chi$iv:1507.07921 with Steve Giddings

# Local QFT versus GR

Big question: what is a local subsystem in quantum gravity?

In local quantum field theory, we have an algebra of local observables:

$$[\phi(x), \phi(x')] = 0 \qquad \text{when x and x' are spacelike separated}$$

In general relativity (and presumably in quantum gravity) observables must be diffeomorphism-invariant. They can't be local.

Questions:

- If LQFT is a limit of QG, what is $\phi(x)$?
- How big is $[\phi(x), \phi(x')]$?

# Local QFT versus GR

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- How big is $[\phi(x), \phi(x')]$?

Lots of previous work on diffeomorphism-invariant observables!

A “low-brow” approach:

- Scalar matter field $\phi(x)$
- Coupled to linearized gravity
- Flat 3+1 spacetime
- Work perturbatively in $G$

Construct operators $\Phi(x)$ such that

- $\Phi(x) \to \phi(x)$ as $G \to 0$
- $[\Phi(x), \Phi(x')]$ decays as $|x - x'| \to \infty$

First, consider a simpler case: QED.

Gauge-invariant observables for QED first considered by Dirac (1955).

A charged field $\phi$ is not a gauge invariant operator, since

$$\phi \to e^{-iq\Lambda}\phi$$
$$A_\mu \to A_\mu - \partial_\mu \Lambda.$$

But we can dress it with a "Faraday line" to make it gauge invariant

$$\Phi_{W_z}(x) = \phi(x) e^{iq\int_0^\infty ds\, A_z(x+s\hat{z})}.$$

To see how the operator acts on vacuum, consider the commutator

$$[E^z(x), \Phi_{W_z}(x')] = -q\delta^2(\vec{x}_\perp - \vec{x}'_\perp)\theta(z-z')\Phi_{W_z}(x')$$

$\Phi_{W_z}(x)$ creates a charged particle dressed by an electric string.

# Dirac Dressing

Dirac considered a more symmetric gauge-invariant operator:

$$\Phi_D(x) = \phi(x) \exp\left( iq \int d^3\vec{x}' \, \frac{(\vec{x}' - \vec{x})^i}{4\pi|\vec{x}' - \vec{x}|^3} A_i(t, \vec{x}') \right)$$

This creates a Coulomb electric field everywhere outside the lightcone,

$$[A_i(x), \Phi_D(x')] = -\frac{q}{4\pi}(t - t')\frac{(\vec{x}' - \vec{x})_i}{|\vec{x}' - \vec{x}|^3}\Phi_D(x')$$

$$[A_0(x), \Phi_D(x')] = 0$$

**Next:** Construct gravitational analogs of these operators.

# Gravitational Wilson line

Diffeomorphism-invariant prescription:

- Shoot a geodesic in normally from a fixed "platform" at $z = Z$.
- Measure $\phi$ at a fixed proper distance.

Solve the linearized geodesic equation for a curve $x^\mu + s\hat{z}^\mu + v^\mu(s)$:

$$V^\mu_{W_z}(x) = -\int_0^\infty ds\, s\, \Gamma^\mu_{zz}(x + s\hat{z}) + \text{surface term.}$$

$\Phi_{W_z} = \phi + V^\mu_{W_z}(x)\partial_\mu\phi$ creates a particle dressed by a "Wilson line".

Gravitational field is confined to a line at time $t$.

# Gravitational Dirac dressing

To make a symmetric dressing, we can average over all directions:

$$V^\mu_C(x) = \frac{1}{4\pi}\int d^2\Omega\, V^\mu_{W_r}(x) = -\frac{1}{4\pi}\int d^3x' \frac{1}{|x-x'|}\,\Gamma^\mu_{\alpha\beta}\hat{r}^\alpha\hat{r}^\beta$$

For a massive particle at rest $\dot{\phi}(x) = im\phi(x)$ and we have

$$[h_{\mu\nu}(x), \Phi_C(0)] = \frac{\kappa m}{4\pi}\left[\frac{\hat{r}_\mu\hat{r}_\nu}{2r} - \frac{\eta_{\mu\nu}}{4r} - t\frac{\hat{t}_{(\mu}\hat{r}_{\nu)}}{r^2} + \frac{t^2}{2r^3}(q_{\mu\nu} - 3\hat{r}_\mu\hat{r}_\nu)\right]\phi(0)$$

This is just linearized Schwarzschild, in an unusual gauge.

**Next:** Commutators of these observables.

# Commutator - Dirac dressing

To get something more local, consider the Dirac dressing.

Dirac dressing corresponds to a charged particle + Coulomb field, which decays with distance.

Commutator is:

$$[\dot{\Phi}_D(x), \Phi_D(x')] = \frac{iq^2}{4\pi|\vec{x} - \vec{x}'|}\Phi_D(x)\Phi_D(x')$$

Decays with distance - proportional to Coulomb potential between charges.

# Commutator - Gravitational Wilson line

For the gravitational Wilson line, we find many divergent terms:

$$[\dot{\Phi}_{W_z}(x), \Phi_{W_z}(x')] = -i\frac{4G}{\pi^2}\partial_z\phi(x)\partial_z\phi(x')\delta^2(x_\perp - x'_\perp)\int_{\max(z,z')}^{\infty} dz'' + \cdots$$

Similar to the electromagnetic Wilson line, but more indices.

Instead of getting a phase $\propto \phi(x)$ we get a displacement $\propto \partial_\mu\phi$.

Now we are transporting a geodesic through a nontrivial gravitational field.

# Commutators - Gravitational Coulomb

If we consider the static case, $\dot{\phi} = im\phi$, the commutator is

$$[\dot{\Phi}_C(x), \Phi_C(x')] = [\dot{V}_C^0(x), V_C^0(x')]\dot{\phi}(x)\dot{\phi}(x') = i\frac{Gm^2}{|x - x'|}\phi(x)\phi(x')$$

Like electromagnetic case: proportional to the gravitational potential.

Interpretation:

- $\Phi_C(x')$ creates a particle + linearized Schwarzschild field
- Locate point $x'$ in this new geometry using geodesics
- We move off the equal $t = 0$ surface - a $\dot{\phi}$ correction
- For the static particle at rest this gives a factor of $m$

# Commutators - Gravitational Coulomb

We also find a nonvanishing commutator at equal time from $[V_C^0, V_C^i]$

$$[\Phi_C(x), \Phi_C(x')] = -\frac{i\kappa^2}{64\pi}\left[\dot{\phi}(x)\partial_i\phi(x') + \partial_i\phi(x)\dot{\phi}(x')\right]\frac{x^i - x'^i}{|x - x'|} .$$

This is not vanishing at large separation.

Possible interpretation:

- $\Phi_C(x')$ creates a particle + linearized Schwarzschild field.
- Locate point $x'$ in this new geometry using geodesics
- We miss slightly due to focusing - a $\partial_i\phi$ correction to $\phi(x)$

## Summary

Conclusions

- All quantum gravity observables are nonlocal.
- But some are more local than others.
- Natural behaviour of commutators $[\dot{\Phi}(x), \Phi(x')] \sim G m_1 m_2 / r$.

Some questions

- Can we make these observables more local?
- More general backgrounds e.g. AdS, Schwarzschild.
- Connection to asymptotic symmetries?
- What is a subsystem in quantum gravity?

**Thank you!**