Title: TBA

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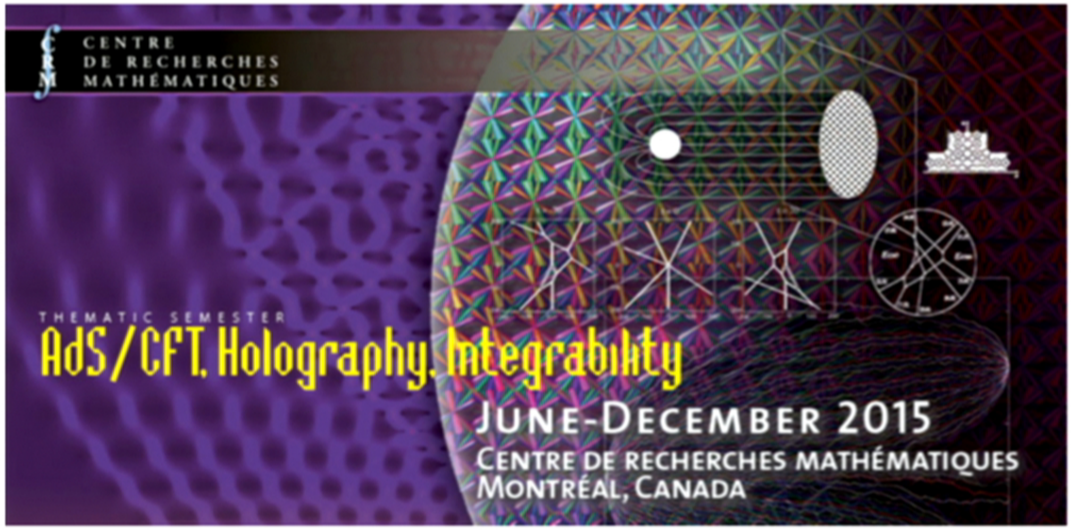
Abstract: TBA

AdS/CFT, Quantum Gravity & Entanglement Workshop



Montreal, Sept 14-16, 2015

Register at `http://www.crm.umontreal.ca/2015/Gravity15/`

Contact: A. Maloney (`maloney@physics.mcgill.ca`)

Entanglement holography

Michal P. Heller
Perimeter Institute for Theoretical Physics, Canada

based on
1508.xxxxx with Jan de Boer, Rob Myers and Yasha Neiman

Holography

AdS/CFT provides the best understood example of the holographic principle.

QG theory in (d+1)-dim = QM theory in d-dim

Within AdS/CFT, emergent direction z is related to energy scale in a dual QFT:

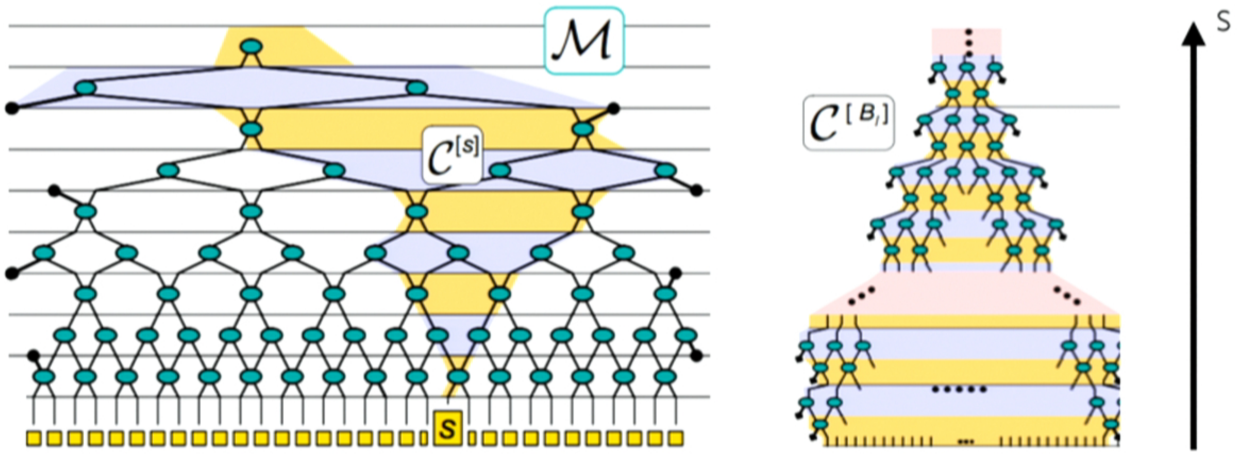
$$ds^2 = \boxed{\frac{1}{z^2} dz^2} + \frac{1}{z^2} \eta_{\mu\nu} dx^\mu dx^\nu \quad \text{with} \quad (z, x^\mu) \to (\lambda\, z, \lambda\, x^\mu)$$

Bulk: Wilsonian RG-flow with z playing the role of (the energy scale)$^{-1}$.

The coarse-graining direction is spacelike.

MERA Vidal quant-ph/0610099

MERA is an example of real-space RG-flow



MERA has "Lorentzian causal structure". The coarse-graining direction is "timelike".

Bény 1110.4872; Evenbly et al. 1307.0831, Czech et al. 1505.05515

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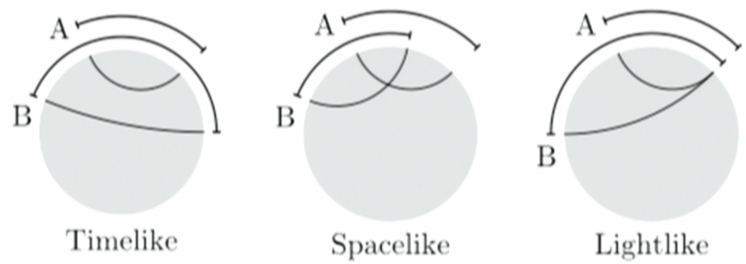
Integral geometry Czech, Lamprou, McCandlish & Sully 1505.05515

One can introduce a partial order on the set of intervals for which we calculate EE



This motivates introducing the light-cone coordinates $u = \theta - \alpha$ and $v = \theta + \alpha$ and considering space with the volume form $\omega = \partial_u \partial_v S \, du \wedge dv$

SSA guarantees $\partial_u \partial_v S \geq 0$. For the vacuum we obtain

$$\omega = \frac{c}{12\sin^2(\alpha)} du \wedge dv$$

Unique conformally-invariant metric compatible with these is

$$ds^2 = \frac{c}{12\sin^2(\alpha)} du\, dv = \frac{c}{12\sin^2(\alpha)} \left(-d\alpha^2 + d\theta^2\right)$$ ← de Sitter$_2$

Question behind this work

Is there a setup in which:

1) scale appears as an emergent time-like direction

and

2) local DOFs in the emergent spacetime can be identified ?

Entanglement first law

Consider small perturbation of some reference density matrix $\rho = \rho_0 + \delta\rho$

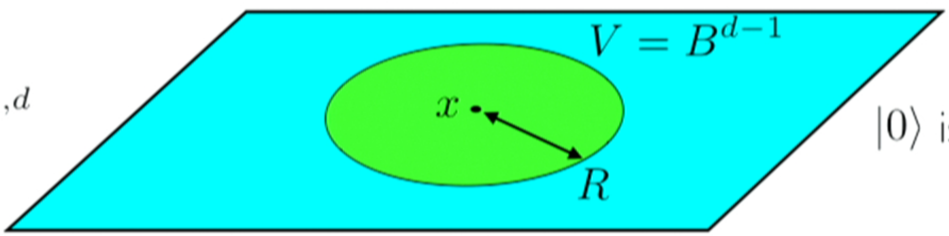
The change in the entropy is equal to the change in <the modular Hamiltonian>

$$\delta S = -\mathrm{tr}\,(\rho \log \rho) - S_0 = \delta \langle H_{mod} \rangle$$

In general, we expect $H_{mod} \equiv \log \rho_0$ to be nonlocal, but for $\rho_0 = \mathrm{tr}_V |0\rangle\langle 0|$:

$$H_{mod} = c' + 2\pi \int_{|\vec{x}-\vec{x}'|^2 \leq R^2} d^{d-1}x' \, \frac{R^2 - |\vec{x}-\vec{x}'|^2}{2R} T_{tt}(x')$$

$t = 0$ of $\mathbb{R}^{1,d}$



$|0\rangle$ is the vacuum$_{\mathrm{CFTd}}$

Casini, Huerta & Myers 1102.0440

Propagation on de Sitter

As a result, the change in the entanglement entropy for small perturbations of $|0\rangle$ is

$$\delta S_B = 2\pi \int_{|\vec{x}-\vec{x}'|^2 \leq R^2} d^{d-1}x' \, \boxed{\frac{R^2 - |\vec{x}-\vec{x}'|^2}{2R}} \langle T_{tt}\rangle(x')$$

see e.g. Xiao 1402.7080

This is the bulk-to-boundary propagator in dS$_d$: $ds^2 = \boxed{-\frac{L^2}{R^2}dR^2} + \frac{L^2}{R^2}d\vec{x}^2$

This implies that δS is a local field in dS$_d$ and obeys the Klein-Gordon equation:

$$\nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0 \quad \text{with} \quad m^2 L^2 = -d$$

Note that the scale R appears here as an emergent time-like coordinate.

How does it work?

$R=0$ corresponds to one of the timelike bdaries in dS_d (say to the future one).

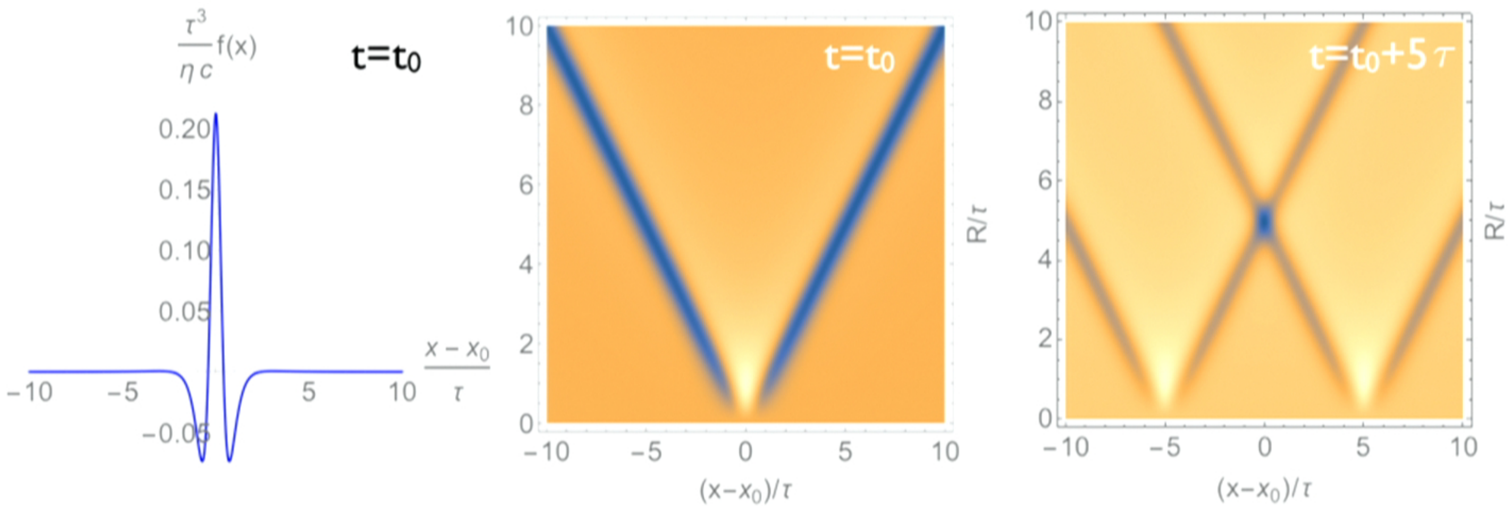
Bdry data:

$$\delta S \overset{R\to 0}{=} F(x)/R + f(x)\,R^d + \dots$$

with

$$F(x)=0 \quad \text{and} \quad f(x) = \frac{\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+3}{2}\right)} \langle T_{tt}(x)\rangle$$

Explicit example in CFT_2: $\delta\rho = \eta\,(|0\rangle\langle\phi| + |\phi\rangle\langle 0|)$ with $|\phi\rangle = T_{tt}(t_0 + i\,\tau, x_0)|0\rangle$



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Comments

$\nabla_a \nabla^a\big|_{dS_d} \delta S_B - m^2 \delta S_B = 0$ in any $\mathrm{CFT_d}$ (large c / strong coupling not needed)

It surfaced before in the studies of HEE & the Einstein equations

Takayanagi et al. 1304.7100 and 1308.3792

It relies only on the applicability of the first law for all values of R

As it is now, it concerns constant time slice configurations in $\mathrm{CFT_d}$

$\nabla_a \nabla^a\big|_{dS_d} \delta S_B - m^2 \delta S_B = 0$ is covariant and applies in any coords in $\mathrm{dS_d}$

Our analysis, as it is now, does not fix the curvature scale of $\mathrm{dS_d}$

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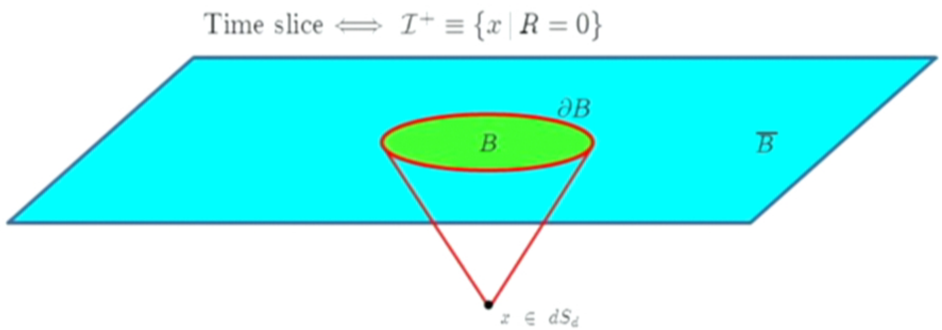
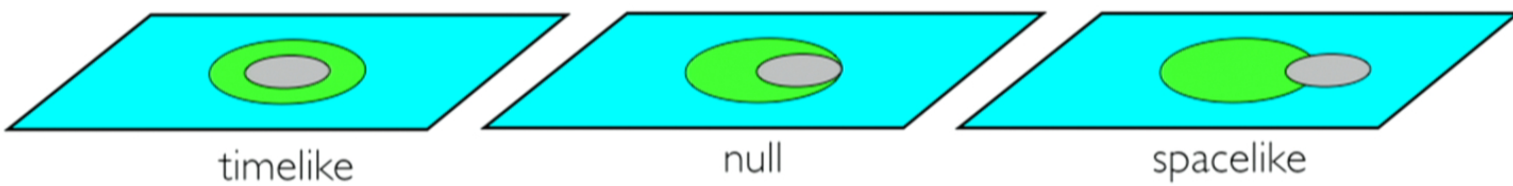
1-to-1 mapping between spheres and dS$_d$

$$\delta S_B = 2\pi \int_{|\vec{x}|^2 \leq R^2} d^{d-1}x' \, \frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} \langle T_{tt} \rangle (x')$$

sphere B in $\mathbb{R}^{d-1}$ maps to a point in dS$_d$:



Causal relations between points in dS$_d$ $\Leftrightarrow$ partial order between B's on t=0:



generalizes Czech et al. 1505.05515

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↓

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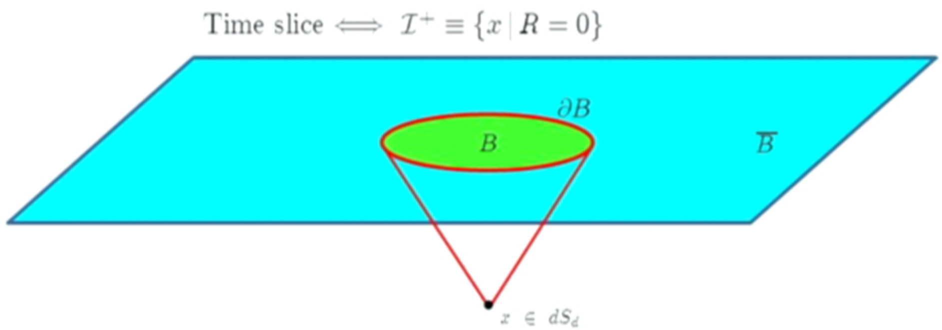
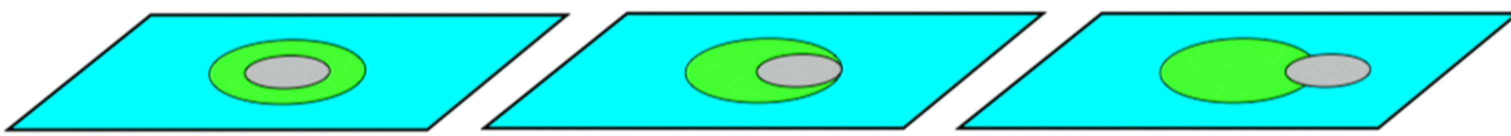
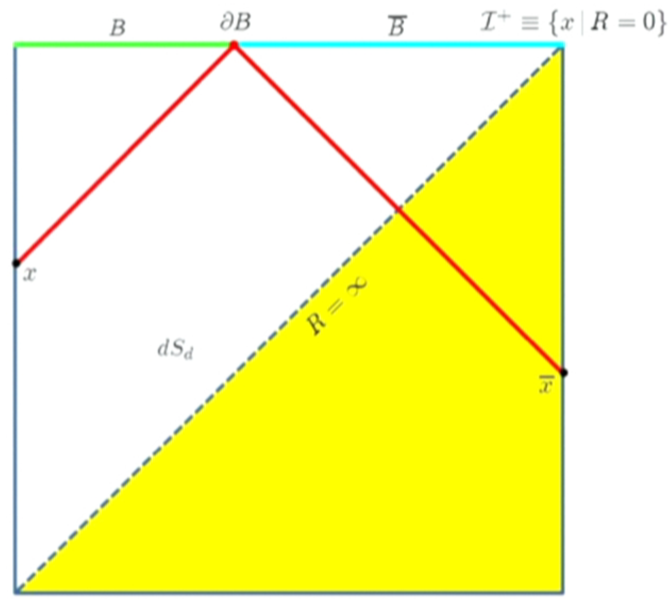


Causal relations between points in dS_d ⇔ partial order between B's on t=0:



generalizes Czech et al. 1505.05515

Elliptic dS$_d$



If $\delta S_B = \delta S_{\bar{B}}$, the field propagates on elliptic dS$_d$: $\delta\rho = \eta\left(|0\rangle\langle\phi| + |\phi\rangle\langle 0|\right)$

If $\delta S_B \neq \delta S_{\bar{B}}$, this is not the case, e.g. $\delta\rho = g e^{-\beta E_1} |E_1\rangle\langle E_1|$ Herzog 1407.1358

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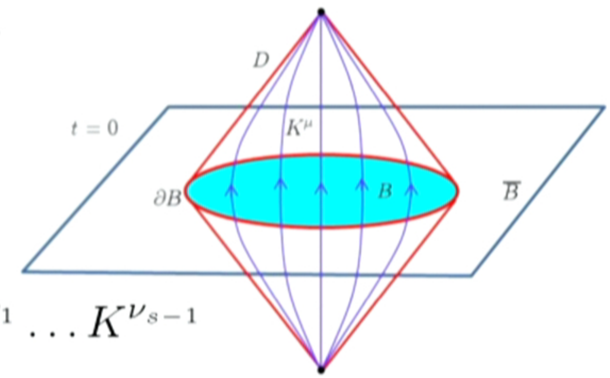
More dynamical scalar fields on dS$_d$

$$H_{mod} = c' + \int_B dB^\mu J_\mu^{(2)} \quad \text{with} \quad J_\mu^{(2)} \equiv T_{\mu\nu} K^\nu$$



$$Q^{(s)} = \int_B dB^\mu J_\mu^{(s)} \quad \text{with} \quad J_\mu^{(s)} = T_{\mu\nu_1 \ldots \nu_{s-1}} K^{\nu_1} \ldots K^{\nu_{s-1}}$$

see Belin et al. 1310.4180 for s=1

see Hijano & Kraus 1406.1804 for d=2 & s>2

$$\delta S_B^{(s)} = (2\pi)^{s-1} \int_B d^{d-1}x' \left(\frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} \right)^{s-1} T_{tt\ldots t}(x')$$

It is clear now that all $\delta S_B^{(s)}$ will be local scalar fields in dS$_d$ and will obey

$$\nabla_a \nabla^a \big|_{dS_d} \delta S_B^{(s)} - m^2 \delta S_B^{(s)} = 0 \quad \text{with} \quad m^2 L^2 = -(s-1)(d+s-2)$$

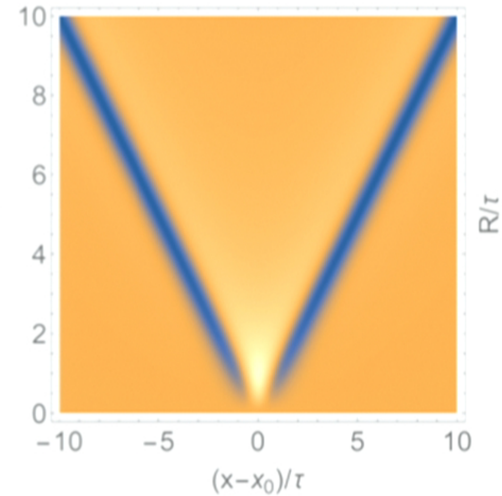
Summary

Entanglement in excited states is organized in a Lorentzian holographic way:

$$\nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0$$

with $m^2 L_{dS_d}^2 = -d$

example:



This statement applies to <u>any CFT</u> in <u>any d</u> provided the first law holds

The statement concerns constant time slices in a CFT

For theories with conserved charges: one dynamical field in dS_d for each charge

Some open problems

Can we describe full CFT in terms of local fields interacting in dS_d (novel dS/CFT)?

Geometry encapsulating time evolution between 2 constant time slices in a CFT?

Does dS_d play the role of the kinematic space / what fixes L_{dS_d}?

Link with MERA / cMERA ?

Does the emergent local Lorentzian propagation persists if ~~conformal symmetry~~.?